Introduction to Risk Parity and Budgeting

Appendix A – Technical Appendix

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Instructors may find the description of the book at the following addresses:

http://www.crcpress.com/product/isbn/9781482207156

http://www.thierry-roncalli.com/RiskParityBook.html

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Figure: Example of building a bivariate probability distribution with a copula function

$F_1 = IG(2,1.5)$

$F_2 = B(2,2)$

Density of the copula C
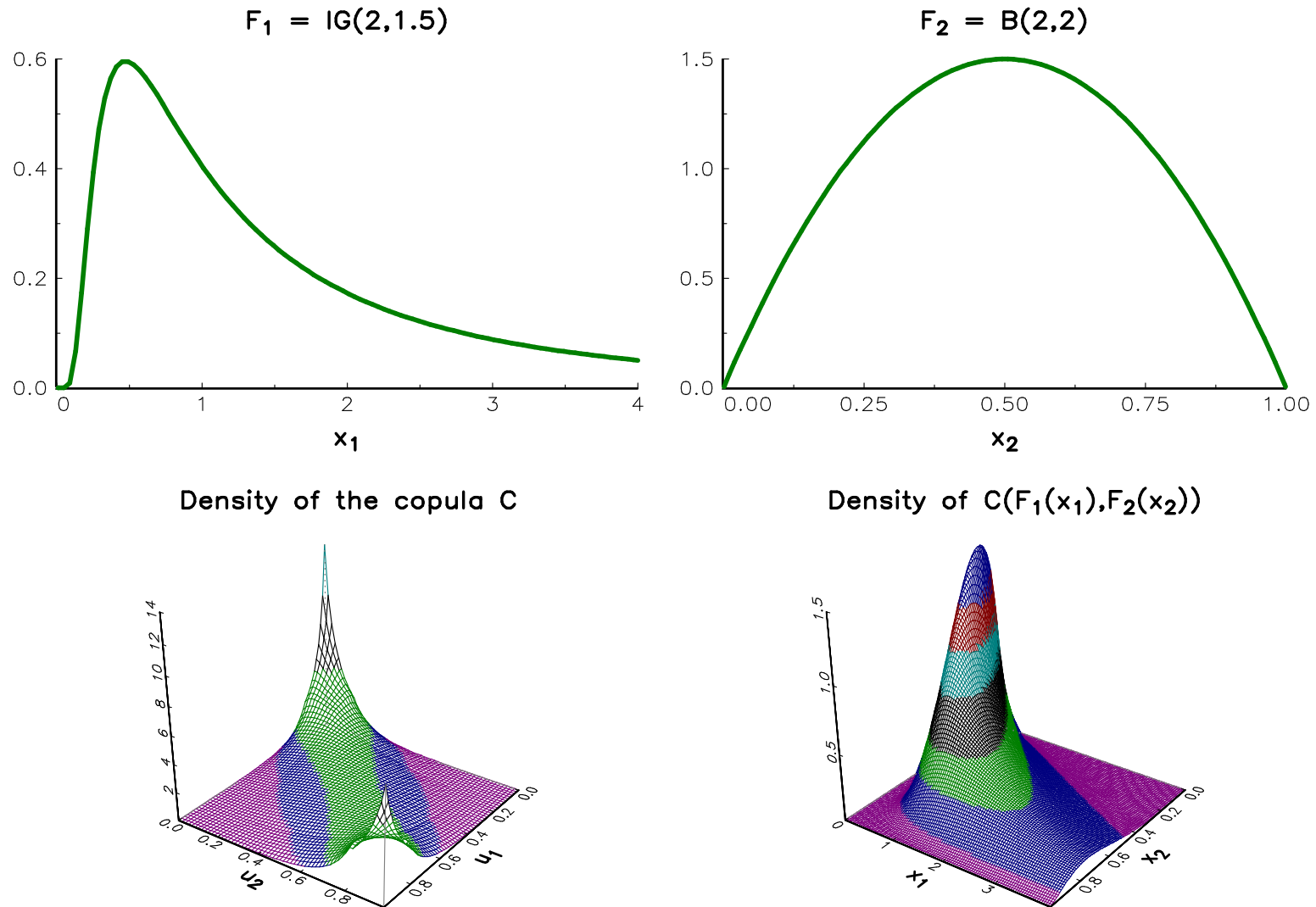
Density of $C(F_1(x_1),F_2(x_2))$

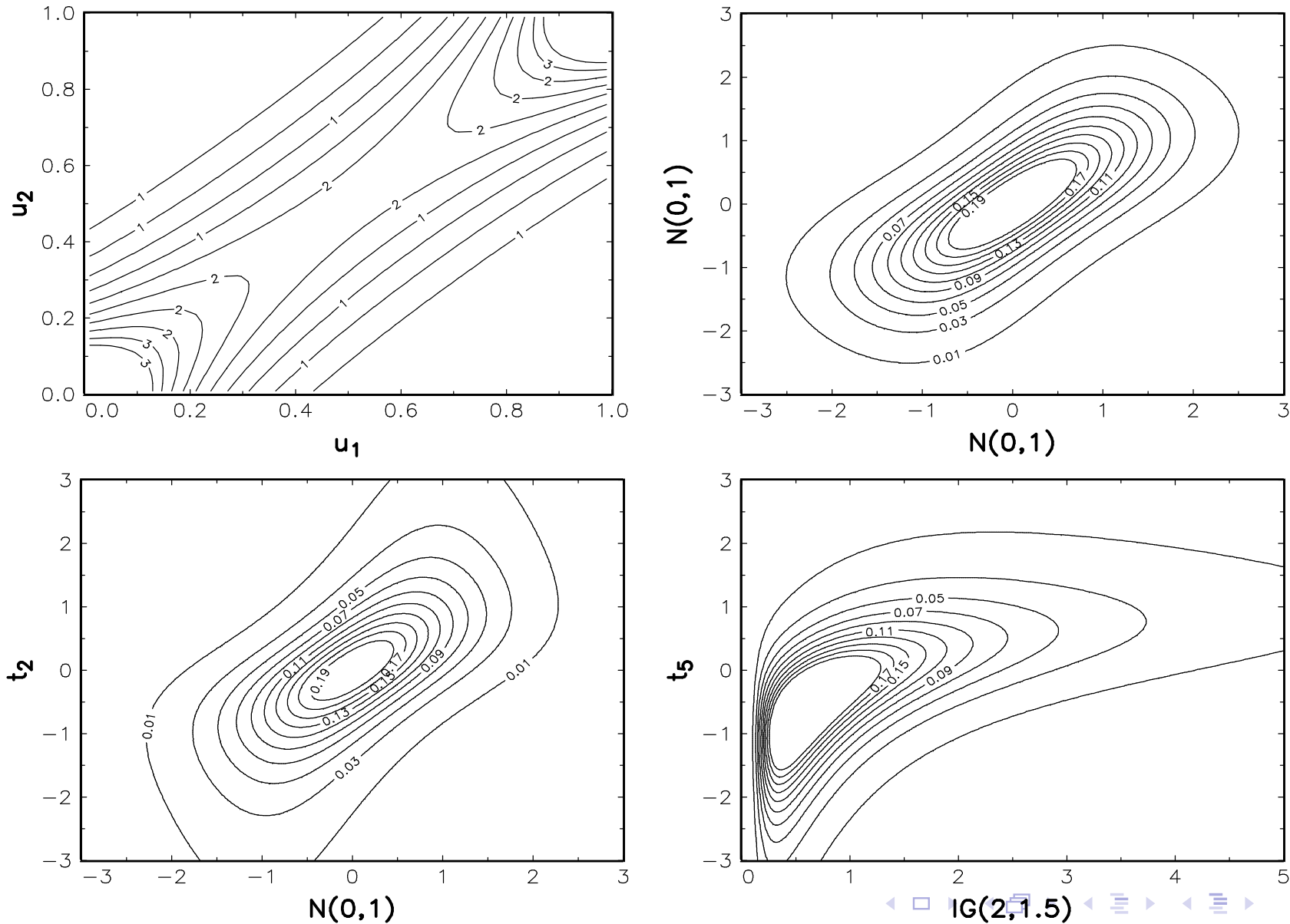Figure: Level curves of bivariate distributions (Frank copula)

Figure: Level curves of bivariate distributions (Gumbel copula)

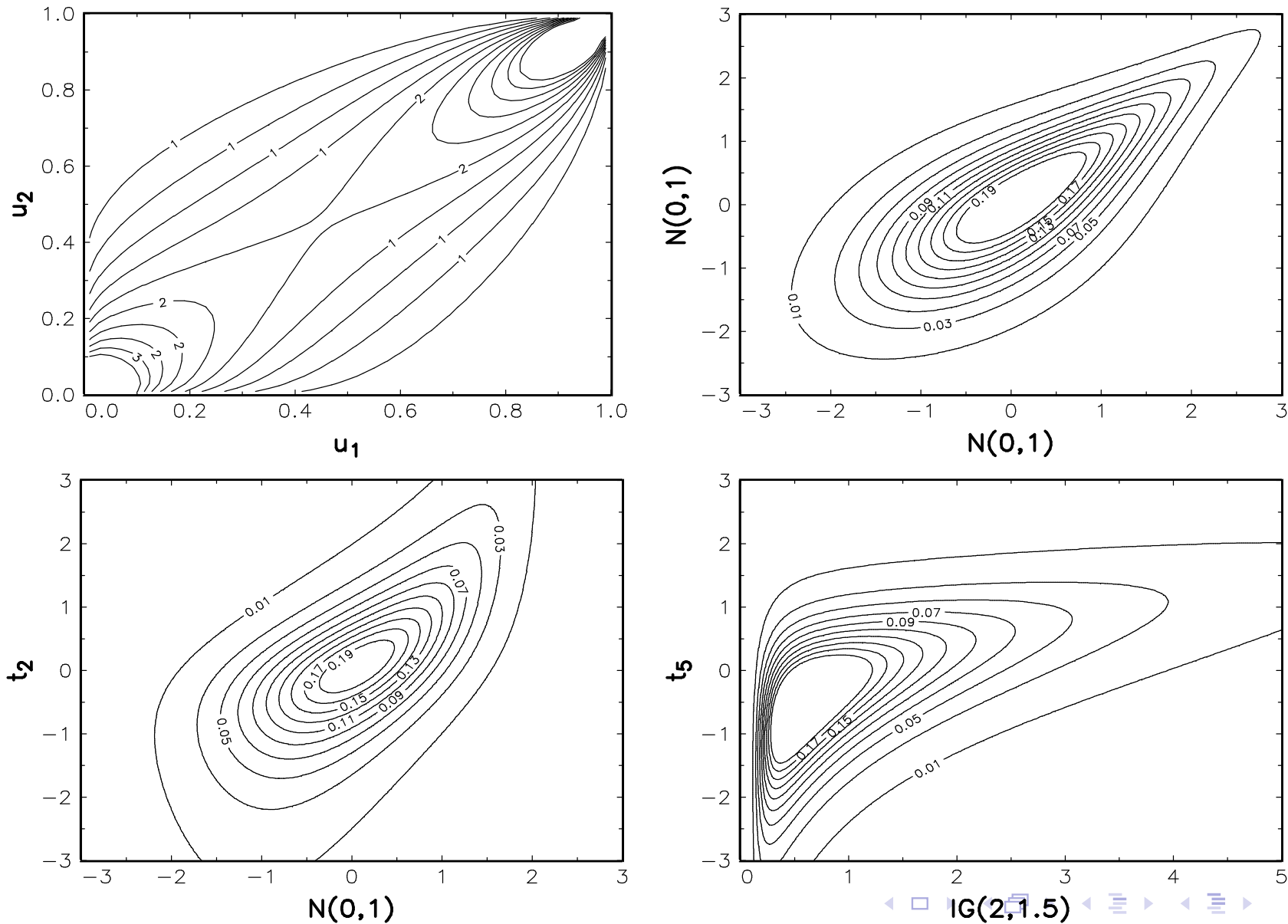Table: Examples of Archimedean copula functions

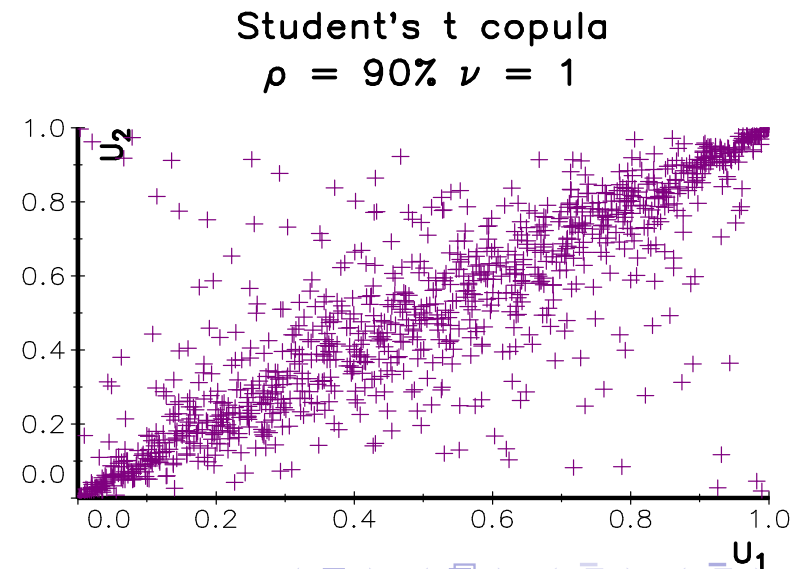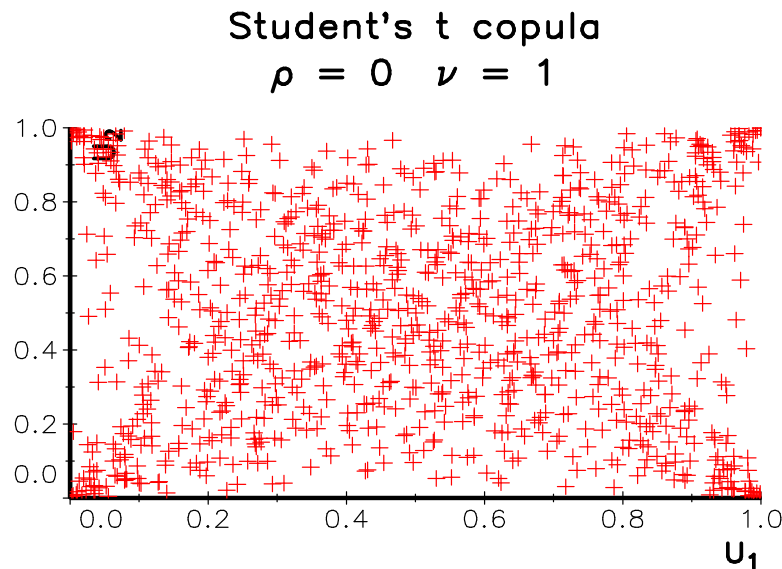| Copula	$\varphi(u)$	$\mathbf{C}(u_1, u_2)$
$C^{\perp}$	$-\ln u$	$u_1 u_2$
GumbelGumbel copula	$(-\ln u)^{\theta}$	$\exp\left(-\left(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta}\right)^{1/\theta}\right)$
FrankFrank copula	$-\ln \frac{e^{-\theta u} - 1}{e^{-\theta} - 1}$	$-\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{e^{-\theta} - 1}\right)$
Joe	$-\ln\left(1 - (1-u)^{\theta}\right)$	$1 - \left(\bar{u}_1^{\theta} + \bar{u}_2^{\theta} - \bar{u}_1^{\theta}\bar{u}_2^{\theta}\right)^{1/\theta}$
ClaytonClayton copula	$u^{-\theta} - 1$	$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$

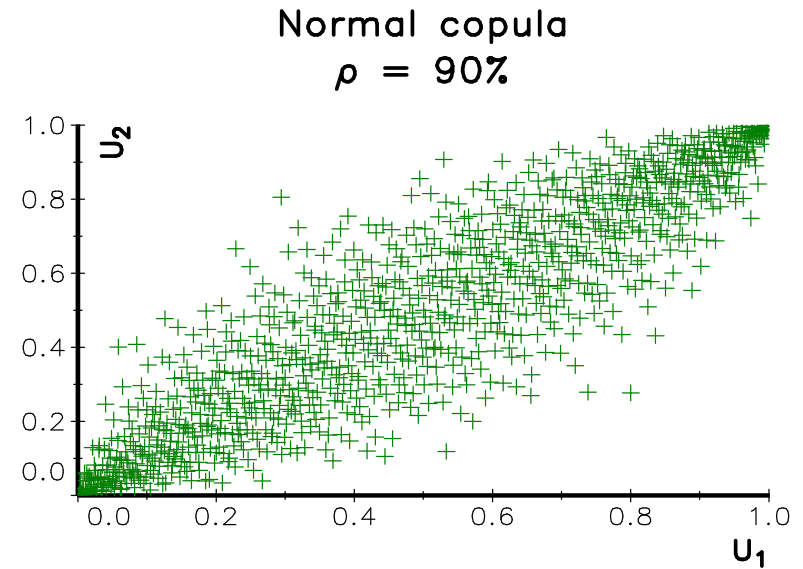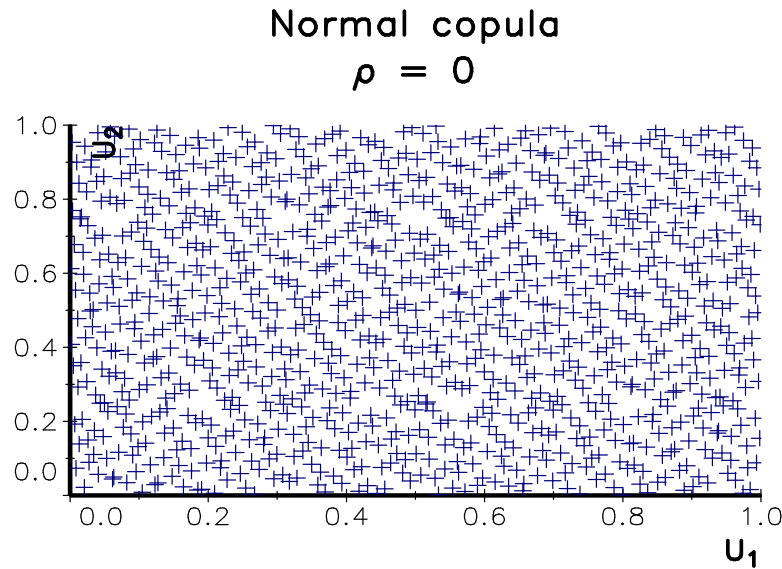Figure: Comparison of normal and t copulas
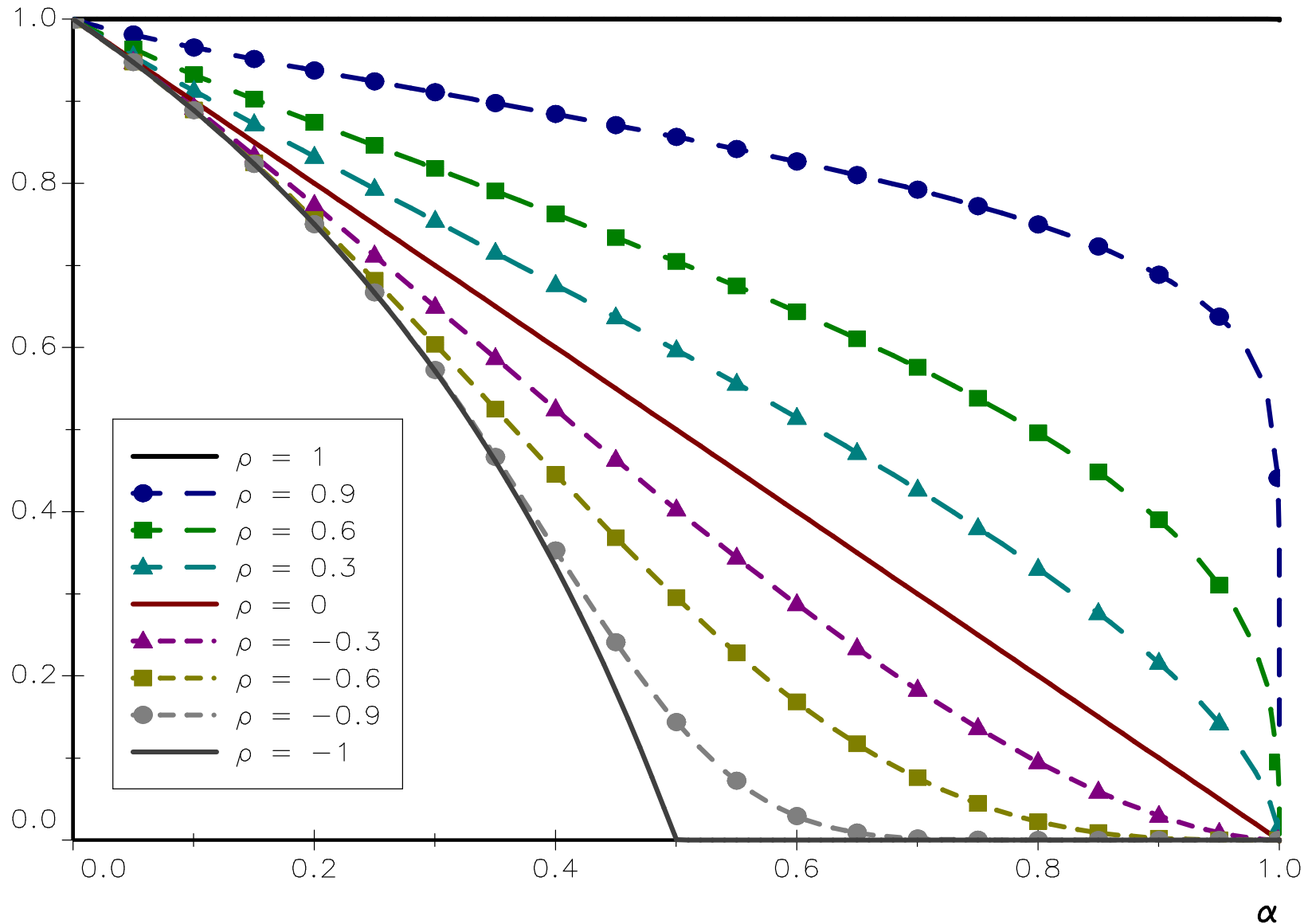
Figure: Quantile-quantile dependence measure for the normal copula

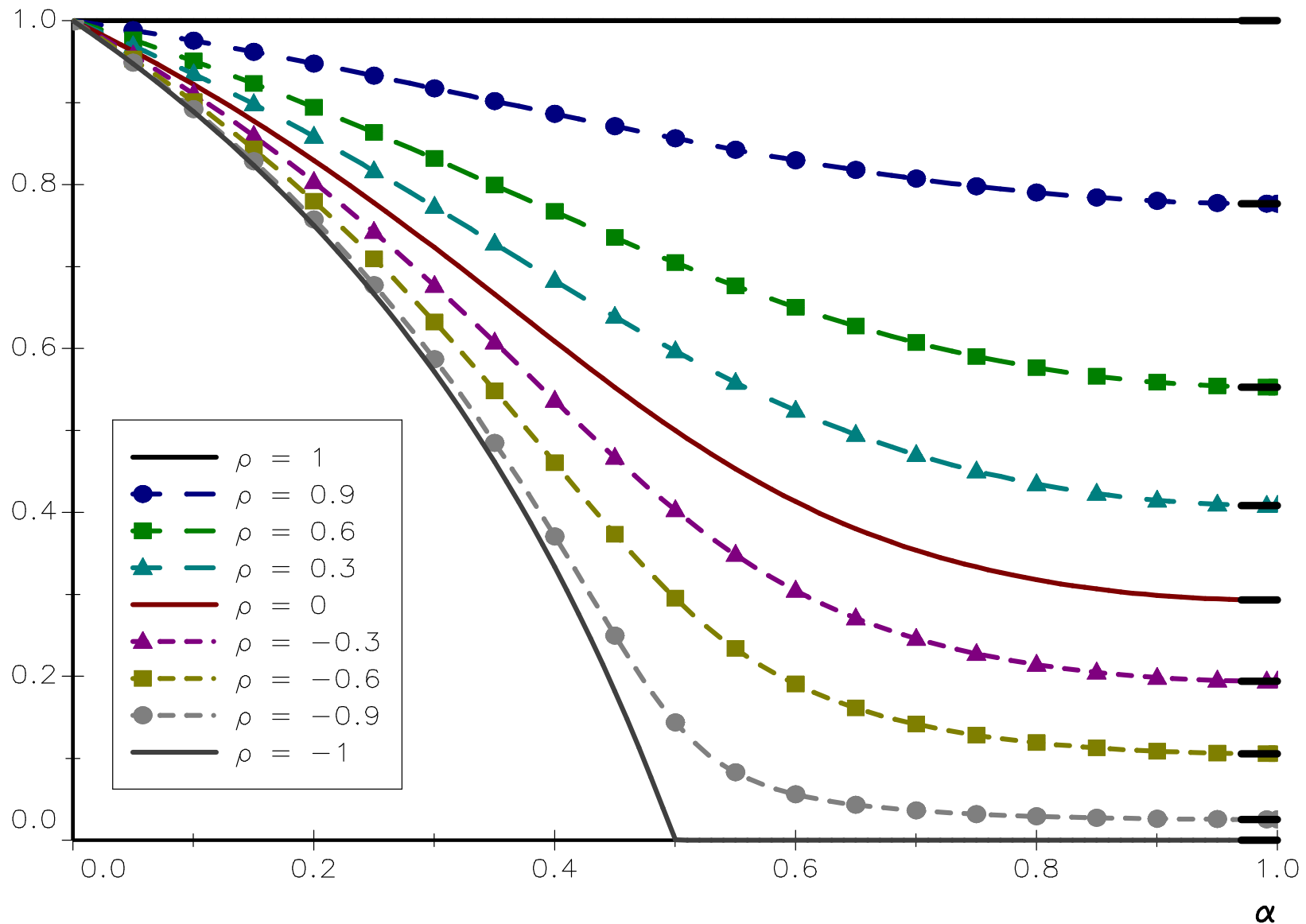Figure: Quantile-quantile dependence measure for the t_1 copula

Table: Calibration of the lifestyle fund profiles ($T = 10$ years, $\rho_{S,B} = 20\%$)

Profile	$\alpha_C^\star$	$\alpha_B^\star$	$\alpha_S^\star$	$\hat{\gamma}$
Safety	-0.03	99.88	0.15	867
Defensive	-3.02	85.52	17.50	7.20
Balanced	-8.55	59.07	49.48	2.55
Dynamic	-14.07	32.61	81.46	1.55

Table: Calibration of the lifestyle fund profiles ($T = 10$ years, $\rho_{S,B} = -20\%$)

Profile	$\alpha_C^\star$	$\alpha_B^\star$	$\alpha_S^\star$	$\hat{\gamma}$
Safety	-0.08	99.93	0.15	867
Defensive	-10.59	90.62	19.98	6.72
Balanced	-26.16	76.82	49.34	2.72
Dynamic	-41.72	63.03	78.70	1.71

Figure: Sensitivity of the equity allocation $\alpha_S^\star$ (in %) in lifestyle funds
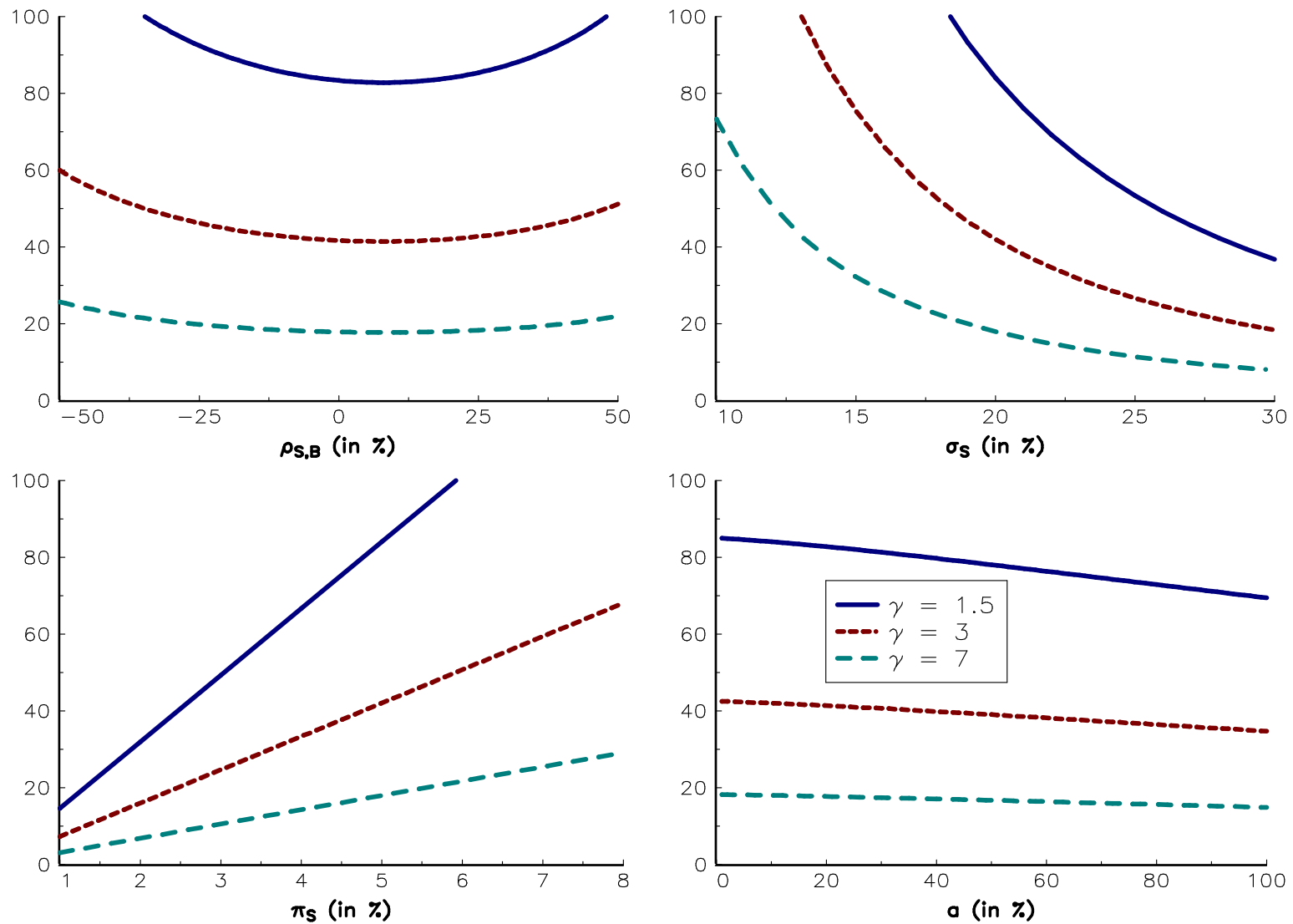
Figure: Influence of the parameters on the glide path of target-date funds

Risk premium of the risky asset ↓

Volatility of the risky asset ↑

Current wealth ↑

Risk aversion ↑

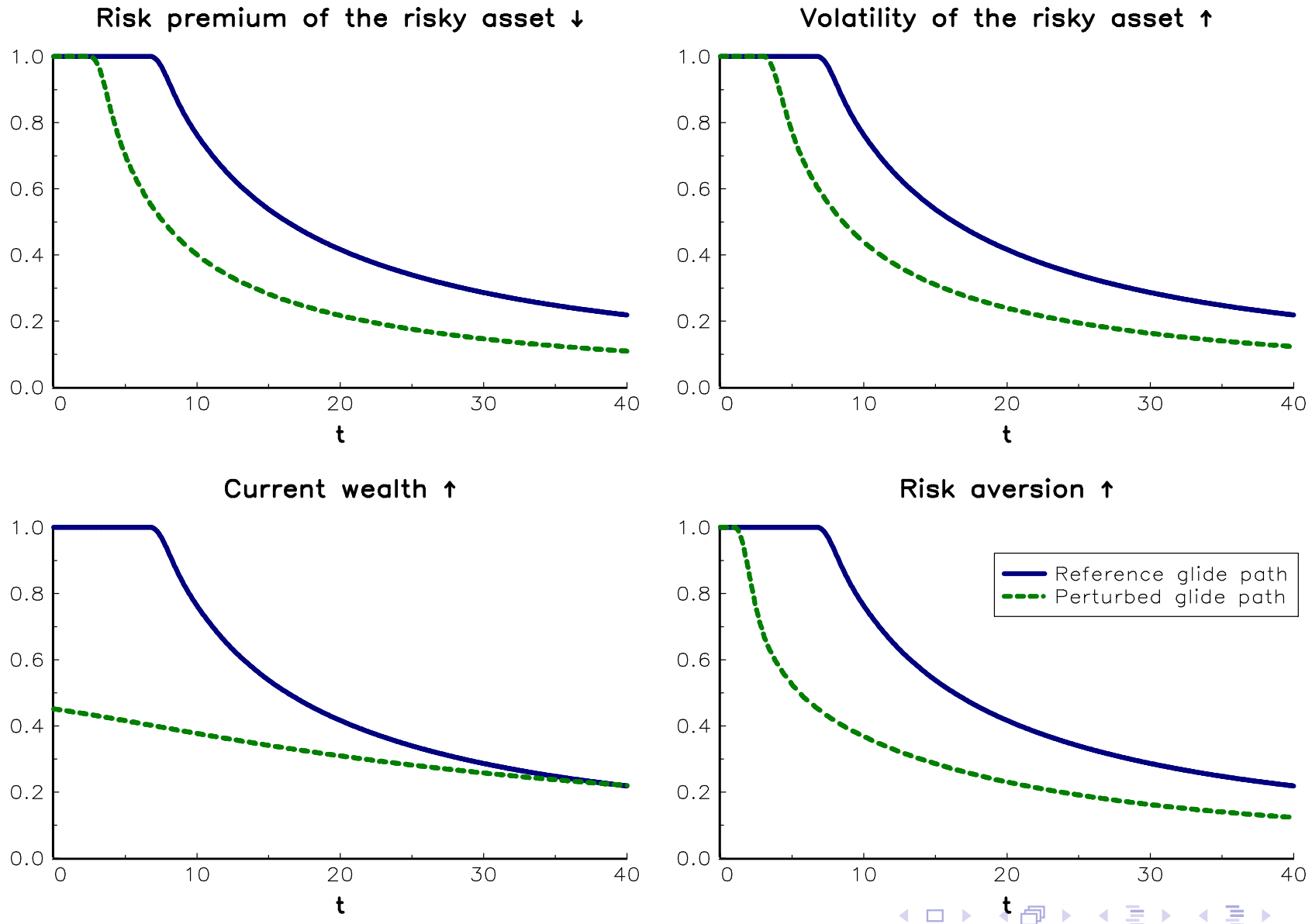Reference glide path
Perturbed glide path
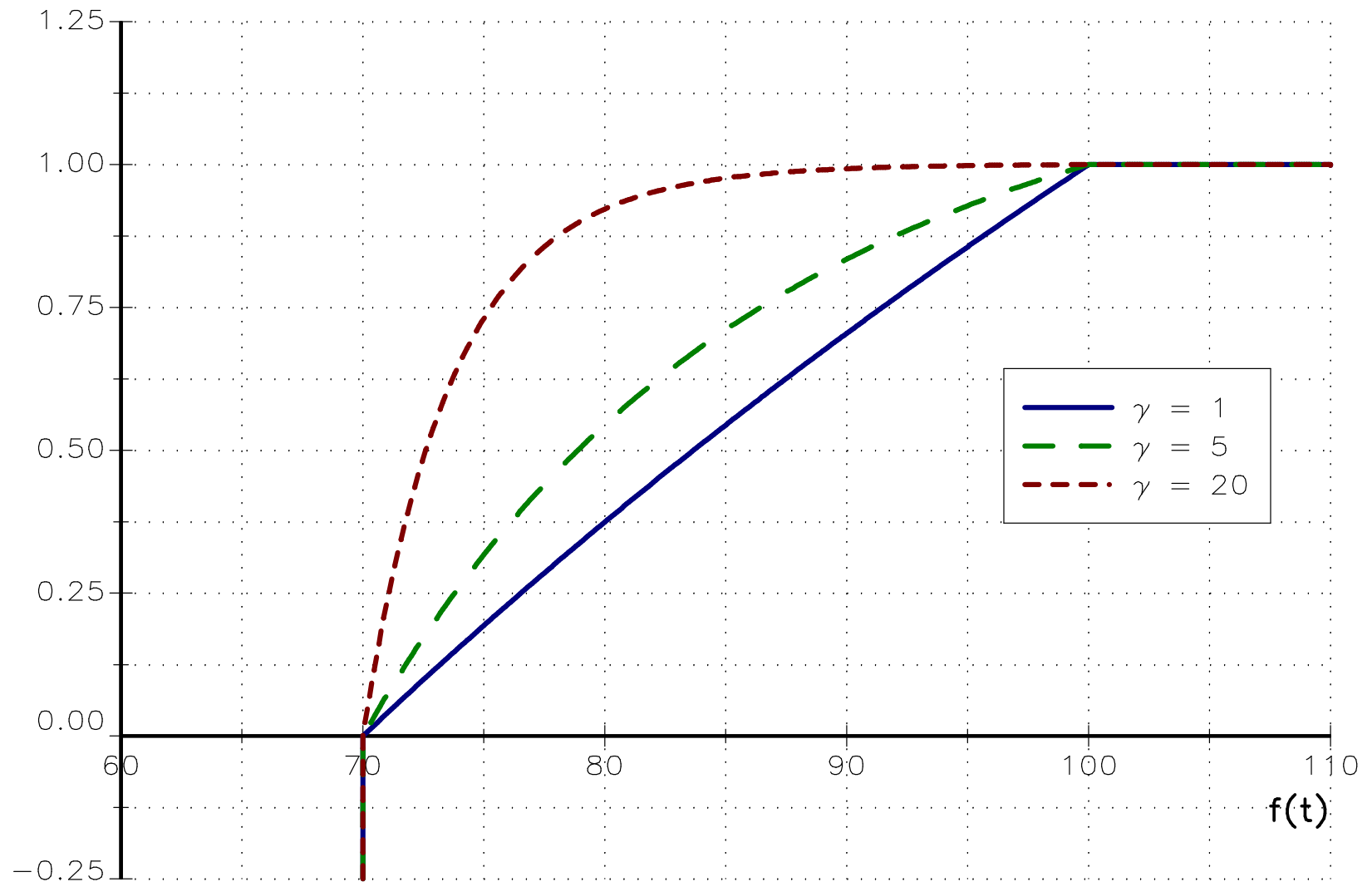
Figure: Example of the LDI utility function

Figure: Optimal exposure $\alpha^{\star}(t)$ (in %) in the LDI portfolio