A Risk Parity Approach to Manage Risk Factors for Strategic Asset Allocation

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Main messages

Questions:

- From asset allocation to risk allocation: what it means?
  - weight budgets ⇒ risk budgets (risk parity & budgeting)
  - asset classes ⇒ risk factors (risk premium & diversification)
- Risk factors: what it means?
  - Statistical factors (e.g. PCA)
  - Market (or portfolio) factors (e.g. Fama-French factors)
  - Economic factors (e.g. growth & inflation)

Results:

- Risk allocation with risk factors is a **complementary** method to risk allocation with asset classes (⇒ another point of view).
- **Risk budgeting with economic risk factors is an appealing and appropriate method for strategic asset allocation.**

⇒ Smart beta is a SAA decision, and <u>not</u> a fund picking decision.

Outline

Some challenges for strategic asset allocation

- Strategic asset allocation and DB plans
- Pension funds, performance objective and economic risk factors
  - Some pension plans have a new objective, e.g. Inflation + 300 bps.
  - How to be sure to define a SAA portfolio which is sensitive to GDP growth?
- SAA, asset classes and risk factors
  - The traditional way to consider SAA with respect to asset classes (Equity, Bonds, Hedge Funds) is out of date.
  - Hedge funds $\Rightarrow \neq$ an asset class, $\neq$ a risk premium.
  - How to invest in equities? CW, smart beta, long-short equity, etc.

The standard linear factor model

- n assets $\{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ and m risk factors $\{\mathcal{F}_1, \ldots, \mathcal{F}_m\}$.
- R_t is the $(n \times 1)$ vector of asset returns at time t and Σ its associated covariance matrix.
- $\mathcal{F}_t$ is the $(m \times 1)$ vector of risk factors at time t and Ω its associated covariance matrix.
- We assume the following linear factor model:

$$R_t = A\mathcal{F}_t + \varepsilon_t$$

with $\mathcal{F}_t$ and ε_t two uncorrelated random vectors. The covariance matrix of ε_t is noted D. We have:

$$\Sigma = A\Omega A^\top + D$$

- The P&L of the portfolio x is:

$$\Pi_t = x^\top R_t = x^\top A\mathcal{F}_t + x^\top \varepsilon_t = y^\top \mathcal{F}_t + \eta_t$$

with $y = A^\top x$ and $\eta_t = x^\top \varepsilon_t$.

The risk allocation principle

Theorem

The risk contributions of common and residual risk factors are[a]:

$$\begin{aligned} \mathrm{RC}(\mathcal{F}_j) &= \left(A^\top x\right)_j \cdot \left(A^+ \frac{\partial \mathcal{R}(x)}{\partial x}\right)_j \\ \mathrm{RC}(\tilde{\mathcal{F}}_j) &= \left(\tilde{B} x\right)_j \cdot \left(\tilde{B} \frac{\partial \mathcal{R}(x)}{\partial x}\right)_j \end{aligned}$$

They satisfy the Euler allocation principle:

$$\sum_{j=1}^{m} \mathrm{RC}(\mathcal{F}_j) + \sum_{j=1}^{n-m} \mathrm{RC}(\tilde{\mathcal{F}}_j) = \mathcal{R}(x)$$

[a] Where A^+ is the Moore-Penrose inverse of A and B^+ is the Kernel of $A^\top$.

An example

Illustration

We consider 4 assets and 3 factors. The loadings matrix is:

$$A = \begin{pmatrix} 0.9 & 0.3 & 0.0 \\ 1.1 & 0.5 & 0.0 \\ 1.2 & 0.0 & 0.7 \\ 0.8 & 0.0 & 0.8 \end{pmatrix}$$

The three factors are uncorrelated and their volatilities are equal to 20%, 5% and 10%. We consider a diagonal matrix D with specific volatilities 10%, 15%, 10% and 15%.

Along assets $\mathcal{A}_1, \ldots, \mathcal{A}_4$

	x_i	$\mathrm{RC}(\mathcal{A}_i)$	$\mathrm{RC}^\star(\mathcal{A}_i)$
$\mathcal{A}_1$	25.00%	4.53%	21.20%
$\mathcal{A}_2$	25.00%	5.84%	27.36%
$\mathcal{A}_3$	25.00%	6.22%	29.15%
$\mathcal{A}_4$	25.00%	4.76%	22.29%
$\sigma(x)$		21.35%	

Along factors $\mathcal{F}_1, \ldots, \mathcal{F}_3$

	y_i	$\mathrm{RC}(\mathcal{F}_i)$	$\mathrm{RC}^\star(\mathcal{F}_i)$
$\mathcal{F}_1$	100.00%	16.60%	77.76%
$\mathcal{F}_2$	20.00%	2.06%	9.65%
$\mathcal{F}_3$	37.50%	2.68%	12.56%
$\tilde{\mathcal{F}}_1$	3.39%	0.01%	0.03%
$\sigma(y)$		21.35%	

Beta contributions versus risk contributions

The linear model is:

$$\begin{pmatrix} R_{1,t} \\ R_{2,t} \\ R_{3,t} \end{pmatrix} = \begin{pmatrix} 0.9 & 0.7 \\ 0.3 & 0.5 \\ 0.8 & -0.2 \end{pmatrix} \begin{pmatrix} \mathcal{F}_{1,t} \\ \mathcal{F}_{2,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}$$

The factor volatilities are equal to 10% and 30%, while the idiosyncratic volatilities are equal to 3%, 5% and 2%.

If we consider the volatility risk measure, we obtain:

Portfolio	$(1/3, 1/3, 1/3)$		$(7/10, 7/10, -4/10)$	
Factor	β	$\mathrm{RC}^\star$	β	$\mathrm{RC}^\star$
$\mathcal{F}_1$	0.67	31%	0.52	3%
$\mathcal{F}_2$	0.33	69%	0.92	97%

The first portfolio has a bigger beta in factor 1 than in factor 2, but about 70% of its risk is explained by the second factor. For the second portfolio, the risk w.r.t the first factor is very small even if its beta is significant.

Issues with risk factor modeling

Betas | Risk Factors | Specific Risks

Asset Returns → $R_t = A \mathcal{F}_t + \epsilon_t$

Estimation

Time-varying Betas

Robustness

Definition

PCA Factors

Market Factors

Economic Factors

Specification

Comprehensive Model

Robustness & stationarity issues !!!

Strategic Asset Allocation

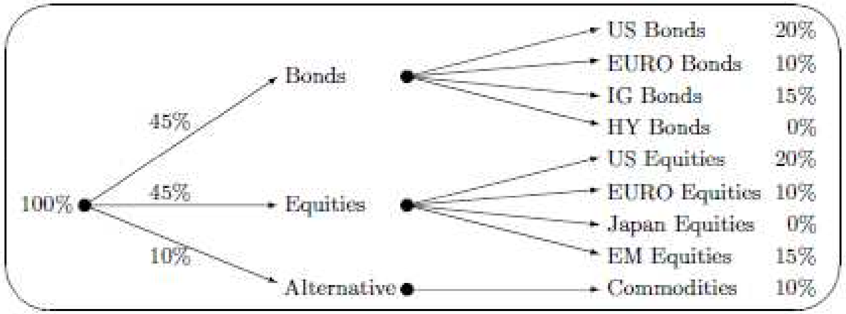
- Long-term investment policy (10-30 years)
- Capturing the risk premia of asset classes (equities, bonds, real estate, natural resources, etc.)
- Top-down macro-economic approach (based on short-run disequilibrium and long-run steady-state)
- Generally computed with portfolio optimization and long-run scenarios

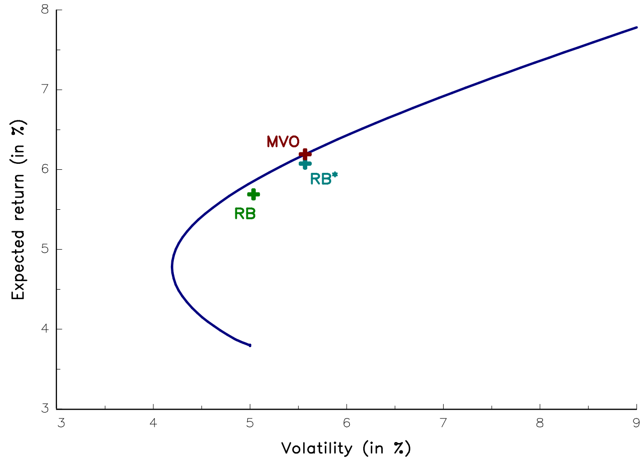
ATP Danish Pension Fund

> *"Like many risk practitioners, ATP follows a portfolio construction methodology that focuses on fundamental economic risks, and on the relative volatility contribution from its five risk classes. [...] The strategic risk allocation is 35% equity risk, 25% inflation risk, 20% interest rate risk, 10% credit risk and 10% commodity risk" (Henrik Gade Jepsen, CIO of ATP, IPE, June 2012).*

These risk budgets are then transformed into asset classes' weights. At the end of Q1 2012, the asset allocation of ATP was also 52% in fixed-income, 15% in credit, 15% in equities, 16% in inflation and 3% in commodities (Source: FTfm, June 10, 2012).

Risk budgeting policy of a pension fund





Asset class	RB		RB*		MVO	
	x_i	RC_i	x_i	RC_i	x_i	RC_i
US Bonds	36.8%	20.0%	45.9%	18.1%	66.7%	25.5%
EURO Bonds	21.8%	10.0%	8.3%	2.4%	0.0%	0.0%
IG Bonds	14.7%	15.0%	13.5%	11.8%	0.0%	0.0%
US Equities	10.2%	20.0%	10.8%	21.4%	7.8%	15.1%
Euro Equities	5.5%	10.0%	6.2%	11.1%	4.4%	7.6%
EM Equities	7.0%	15.0%	11.0%	24.9%	19.7%	49.2%
Commodities	3.9%	10.0%	4.3%	10.3%	1.5%	2.7%

RB* = A BL portfolio with a tracking error of 1% wrt RB / MVO = Markowitz portfolio with the RB* volatility

The framework of risk factor budgeting

- Combining the risk budgeting approach to define the asset allocation and the economic approach to define the factors (Kaya *et al.*, 2011).
- Following Eychenne *et al.* (2011), we consider 7 economic factors grouped into four categories:
  1. activity: gdp & industrial production;
  2. inflation: consumer prices & commodity prices;
  3. interest rate: real interest rate & slope of the yield curve;
  4. currency: real effective exchange rate.
- Quarterly data from Datastream.
- ML estimation using YoY relative variations for the study period Q1 1999 – Q2 2012.
- Risk measure: volatility.

Asset returns, economic factors and risk premia

The Two Economic Pillars

Potential Growth   Inflation

$\Downarrow$

Long-run Returns on Asset Classes

Short Rate $\Rightarrow$ Government Bonds $\Rightarrow$

- Equities
- Corporate Bonds
- Commodities
- Other Asset Classes

Measuring risk factor contributions of SAA portfolios

- 13 AC: equity (US, EU, UK, JP), sovereign bonds (US, EU, UK, JP), corporate bonds (US, EU), High yield (US, EU) and US TIPS.
- Three given portfolios:
  - Portfolio #1 is a balanced stock/bond asset mix.
  - Portfolio #2 is a defensive allocation with 20% invested in equities.
  - Portfolio #3 is an agressive allocation with 80% invested in equities.
- Portfolio #4 is optimized in order to take more inflation risk.

	Equity				Sovereign Bonds				Corp. Bonds		High Yield		TIPS
	US	EU	UK	JP	US	EU	UK	JP	US	EU	US	EU	US
#1	20%	20%	5%	5%	10%	5%	5%	5%	5%	5%	5%	5%	5%
#2	10%	10%			20%	15%	5%	5%	5%	5%	5%	5%	15%
#3	30%	30%	10%	10%	10%	10%							
#4	19.0%	21.7%	6.2%	2.3%		5.9%			24.1%	10.7%	2.6%	7.5%	

Factor	#1	#2	#3	#4
Activity	36.91%	19.18%	51.20%	34.00%
Inflation	12.26%	4.98%	9.31%	20.00%
Interest rate	42.80%	58.66%	32.92%	40.00%
Currency	7.26%	13.04%	5.10%	5.00%
Residual factors	0.77%	4.14%	1.47%	1.00%

Fama-French-Carhart analysis

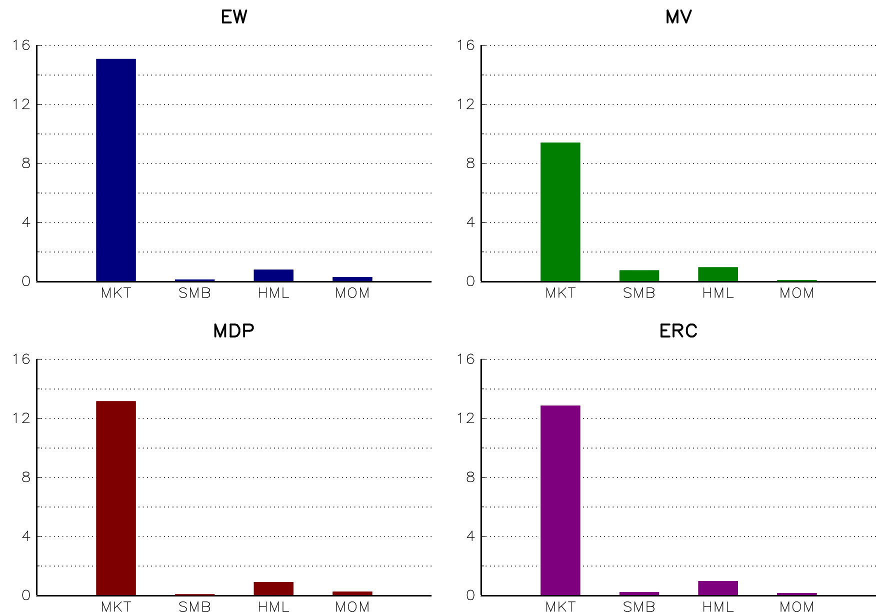
Factor decomposition of (unconstrained) risk-based portfolios

1. Fama-French-Carhart model:
   1. SMB = small cap risk factor
   2. HML = value risk factor
   3. MOM = momentum risk factor
2. S&P 100 portfolios from January 1992 to June 2012

Table : Estimate of the loading matrix A (Jan. 1992 – Jun. 2012)

	MKT	SMB	HML	MOM
CW	0.98	−0.26	−0.06	−0.01
EW	0.99	−0.08	0.22	−0.13
MV	0.58	−0.16	0.18	−0.03
MDP	0.80	−0.04	0.21	−0.11
ERC	0.87	−0.11	0.24	−0.09

Figure : Decomposition of the volatility (in %)



Fama-French-Carhart analysis

Factor decomposition of L/S portfolios

Figure : Absolute risk contribution

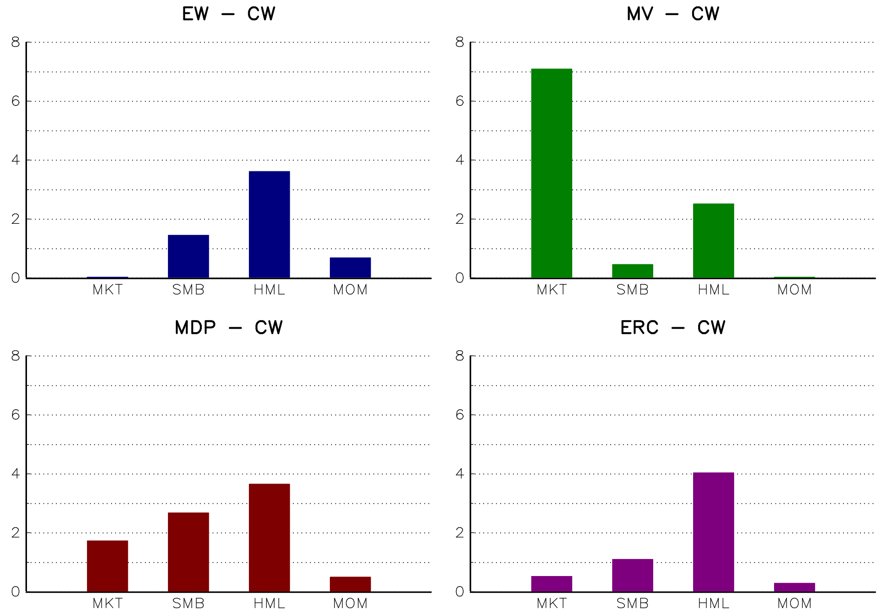
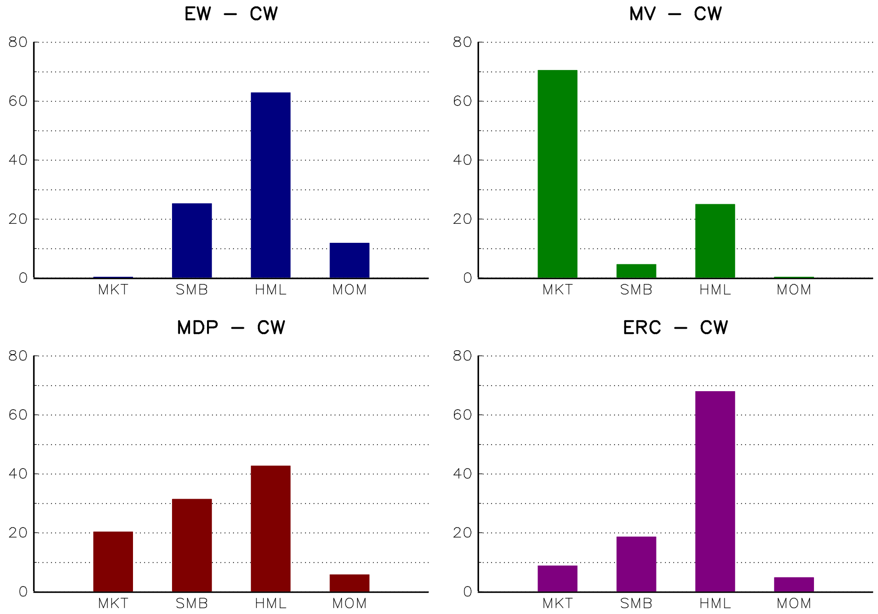


Figure : Relative risk contribution



Analysis with economic risk factors

- Bond-like or equity-like?
- Sensitivity to economic risk factors?
- Behavior with respect to some economic scenarios?

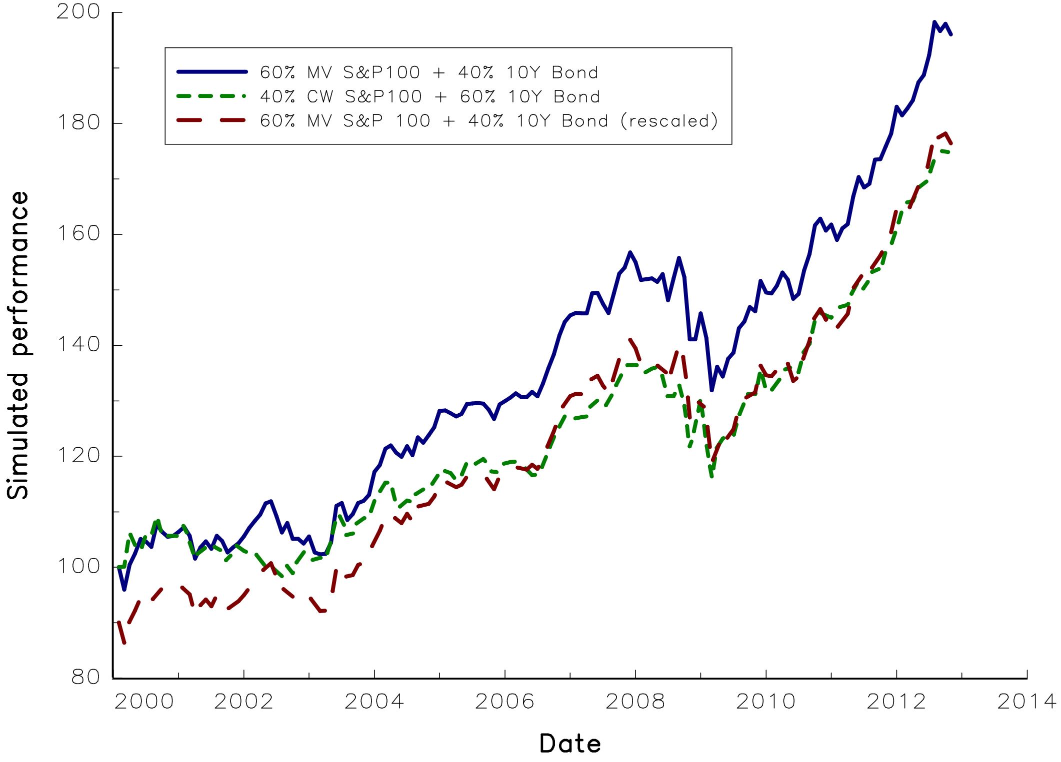
Table : Risk contributions of risk-based S&P 100 indices with respect to economic factors (Q1 1992 – Q2 2012)

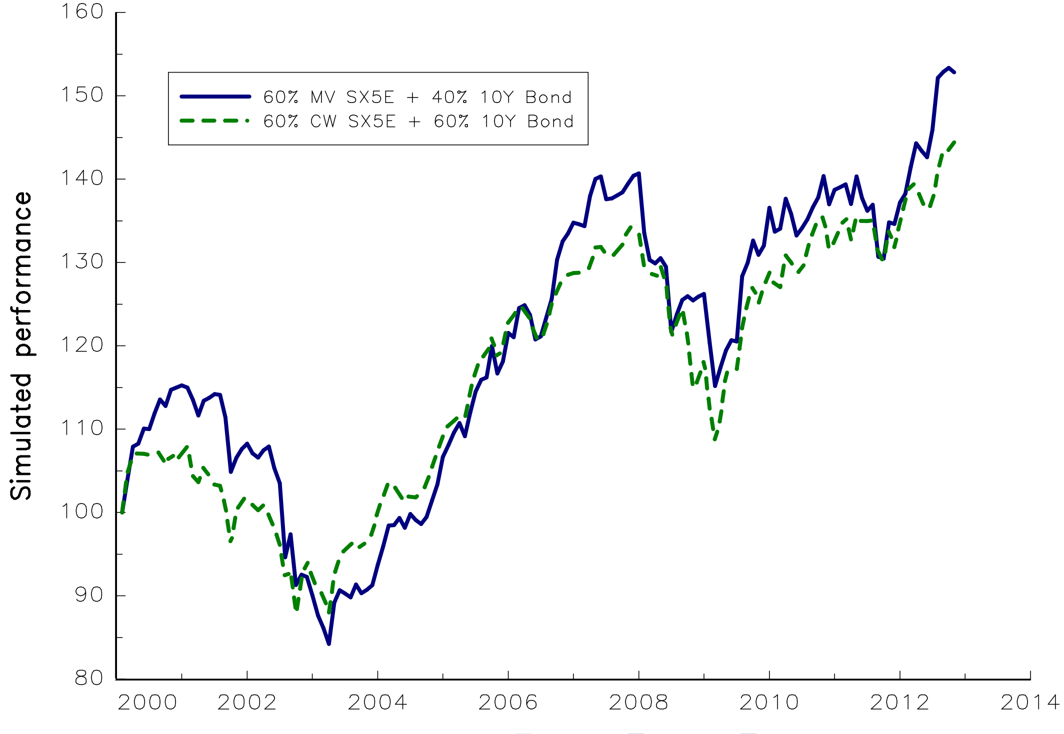
Factor	CW	EW	MV	MDP	ERC
Activity	71.7%	70.0%	29.4%	41.8%	62.1%
Inflation	21.8%	16.7%	4.7%	9.4%	9.5%
Interest rate	6.0%	12.7%	64.7%	46.9%	27.5%
Currency	0.6%	0.6%	1.2%	1.9%	0.9%

Sharpe Style Analysis (Jan. 2000 – Oct. 2012)

S&P 100	CW Equity	Bond	R^2
60% CW + 40% US 10Y	60.00%	40.00%	100%
60% EW + 40% US 10Y	62.58%	37.42%	88%
60% MV + 40% US 10Y	37.79%	62.21%	56%
60% MDP + 40% US 10Y	51.83%	48.17%	72%
60% ERC + 40% US 10Y	54.72%	45.28%	86%

SX5E	CW Equity	Bond	R^2
60% CW + 40% DEM 10Y	60.00%	40.00%	100%
60% EW + 40% DEM 10Y	62.78%	37.22%	96%
60% MV + 40% DEM 10Y	38.69%	61.31%	69%
60% MDP + 40% DEM 10Y	47.31%	52.69%	80%
60% ERC + 40% DEM 10Y	55.40%	44.60%	95%





The strategic asset allocation puzzle

- How to ensure consistency between investing in an asset class (bottom-up approach) and building the strategic asset allocation (top-down approach)?
- How to ensure consistency between smart indices (bottom-up approach) and economic scenarios (top-down approach)?
- What means bond-like?
  - Slope $\neq$ Level
  - The term structure of interest rates is related to the economic cycle
  - What is the lead/lag position of some smart indices with respect to the economic cycle?

Factors	CW	EW	MV	MDP	ERC
Interest rate	6.0%	12.7%	64.7%	46.9%	27.5%
Level	4.5%	10.4%	42.3%	20.5%	20.0%
Slope	1.4%	2.3%	22.4%	26.4%	7.5%

Conclusion

- Risk factor contribution = a powerful tool.
- PCA factors = some drawbacks (not always stable).

- Economic risk factors = make more sense for long-term investment policy $\Rightarrow$ Coherency with the risk premia top-down approach.
- Could be adapted to directional risk measure (e.g. expected shortfall) and to take into account expected returns (Roncalli, 2013).

- Strategic Allocation of Factors = a complementary tool to Strategic Allocation of Assets
- How to use this technology to hedge or be exposed to some economic risks?
- What is the risk premium of economic factors[1]?

[1] Same question with statistical and market risk factors.

For further reading

T. Roncalli.
Introduction to Risk Parity and Budgeting.
Chapman & Hall, 410 pages, July 2013.

T. Roncalli.
Introducing Expected Returns into Risk Parity Portfolios: A New Framework for Tactical and Strategic Asset Allocation.
SSRN, www.ssrn.com/abstract=2321309, July 2013.

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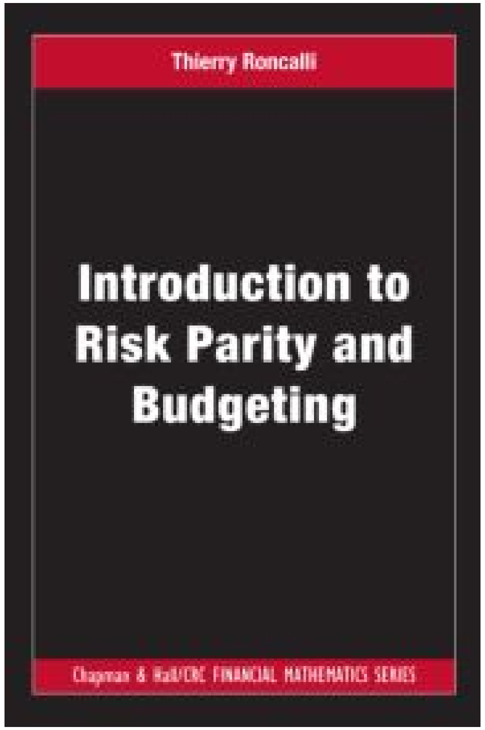
Risk Parity Portfolios with Risk Factors.
SSRN, www.ssrn.com/abstract=2155159, October 2012.



Euler decomposition of risk measure

Let $x = (x_1, \ldots, x_n)$ be the weights of n assets in the portfolio. Let $\mathcal{R}(x_1, \ldots, x_n)$ be a coherent and convex risk measure. We have:

$$\begin{aligned}
\mathcal{R}(x_1, \ldots, x_n) &= \sum_{i=1}^{n} x_i \cdot \frac{\partial \mathcal{R}(x_1, \ldots, x_n)}{\partial x_i} \\
&= \sum_{i=1}^{n} \mathrm{RC}_i(x_1, \ldots, x_n)
\end{aligned}$$

Risk allocation

- How to allocate risk in a fair and effective way (Litterman, 1996)?
- The risk allocation must satisfy some properties (Kalkbrener, 2005):
  - Full allocation
  - RAPM compatible
  - Diversification principle
- The solution is the Euler decomposition (Tasche, 2008).

Risk budgeting (RB) portfolios

Let $b = (b_1, \ldots, b_n)$ be a vector of risk budgets such that $b_i \geq 0$ and $\sum_{i=1}^n b_i = 1$. We consider the following risk allocation problem:

$$\begin{cases} \mathrm{RC}_i(x) = b_i \mathcal{R}(x) \\ x_i \geq 0 \\ \sum_{i=1}^n x_i = 1 \end{cases}$$

Theorem

- The RB portfolio exists and is unique if the risk budgets are strictly positive (and if $\mathcal{R}(x)$ is bounded below)
- The RB portfolio exists and may be not unique if some risk budgets are set to zero
- The RB portfolio may not exist if some risk budgets are negative

These results hold for convex risk measures: volatility, Gaussian VaR & ES, elliptical VaR, non-normal ES, Kernel historical VaR, Cornish-Fisher VaR, etc.

An example of RB portfolio

Illustration

- 3 assets
- Volatilities are respectively 30%, 20% and 15%
- Correlations are set to 80% between the 1st asset and the 2nd asset, 50% between the 1st asset and the 3rd asset and 30% between the 2nd asset and the 3rd asset
- Budgets are set to 50%, 20% and 30%
- For the ERC (Equal Risk Contribution) portfolio, all the assets have the same risk budget

Weight budgeting (or traditional) approach

Asset	Weight	Marginal Risk	Risk Contribution	
			Absolute	Relative
1	**50.00%**	29.40%	14.70%	70.43%
2	**20.00%**	16.63%	3.33%	15.93%
3	**30.00%**	9.49%	2.85%	13.64%
Volatility			20.87%	

Risk budgeting approach

Asset	Weight	Marginal Risk	Risk Contribution	
			Absolute	Relative
1	31.15%	28.08%	8.74%	**50.00%**
2	21.90%	15.97%	3.50%	**20.00%**
3	46.96%	11.17%	5.25%	**30.00%**
Volatility			17.49%	

ERC approach

Asset	Weight	Marginal Risk	Risk Contribution	
			Absolute	Relative
1	19.69%	27.31%	5.38%	**33.33%**
2	32.44%	16.57%	5.38%	**33.33%**
3	47.87%	11.23%	5.38%	**33.33%**
Volatility			16.13%	

The framework

Capital Asset Pricing Model

$$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_{\mathrm{MKT}}] - R_f)$$

where R_{MKT} is the return of the market portfolio.

Fama-French-Carhart model

$$\begin{aligned}\mathbb{E}[R_i] &= \beta_i^{\mathrm{MKT}} \mathbb{E}[R_{\mathrm{MKT}}] \\ &\quad + \beta_i^{\mathrm{SMB}} \mathbb{E}[R_{\mathrm{SMB}}] + \beta_i^{\mathrm{HML}} \mathbb{E}[R_{\mathrm{HML}}] + \beta_i^{\mathrm{MOM}} \mathbb{E}[R_{\mathrm{MOM}}]\end{aligned}$$

where R_{SMB} is the return of small stocks minus the return of large stocks, R_{HML} is the return of stocks with high book-to-market values minus the return of stocks with low book-to-market values and R_{MOM} is the Carhart momentum factor.

Regression analysis

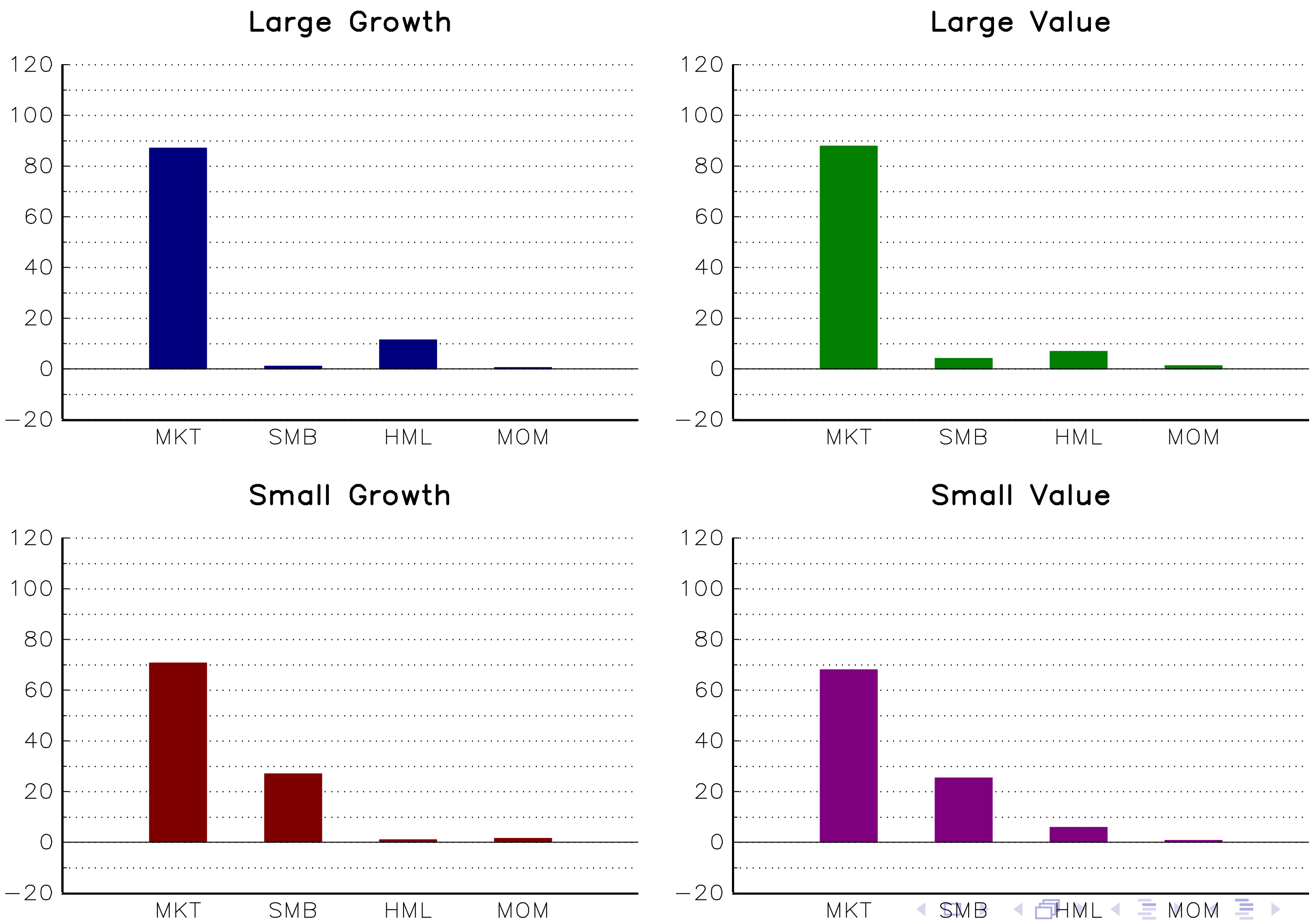
Results$^{(\star)}$ using weekly returns from 1995-2012

Index	β_i^{MKT}	β_i^{SMB}	β_i^{HML}	β_i^{MOM}
MSCI USA Large Growth	1.06	−0.12	−0.38	−0.07
MSCI USA Large Value	0.97	−0.21	0.27	−0.12
MSCI USA Small Growth	1.04	0.64	−0.12	0.15
MSCI USA Small Value	1.01	0.62	0.30	−0.10

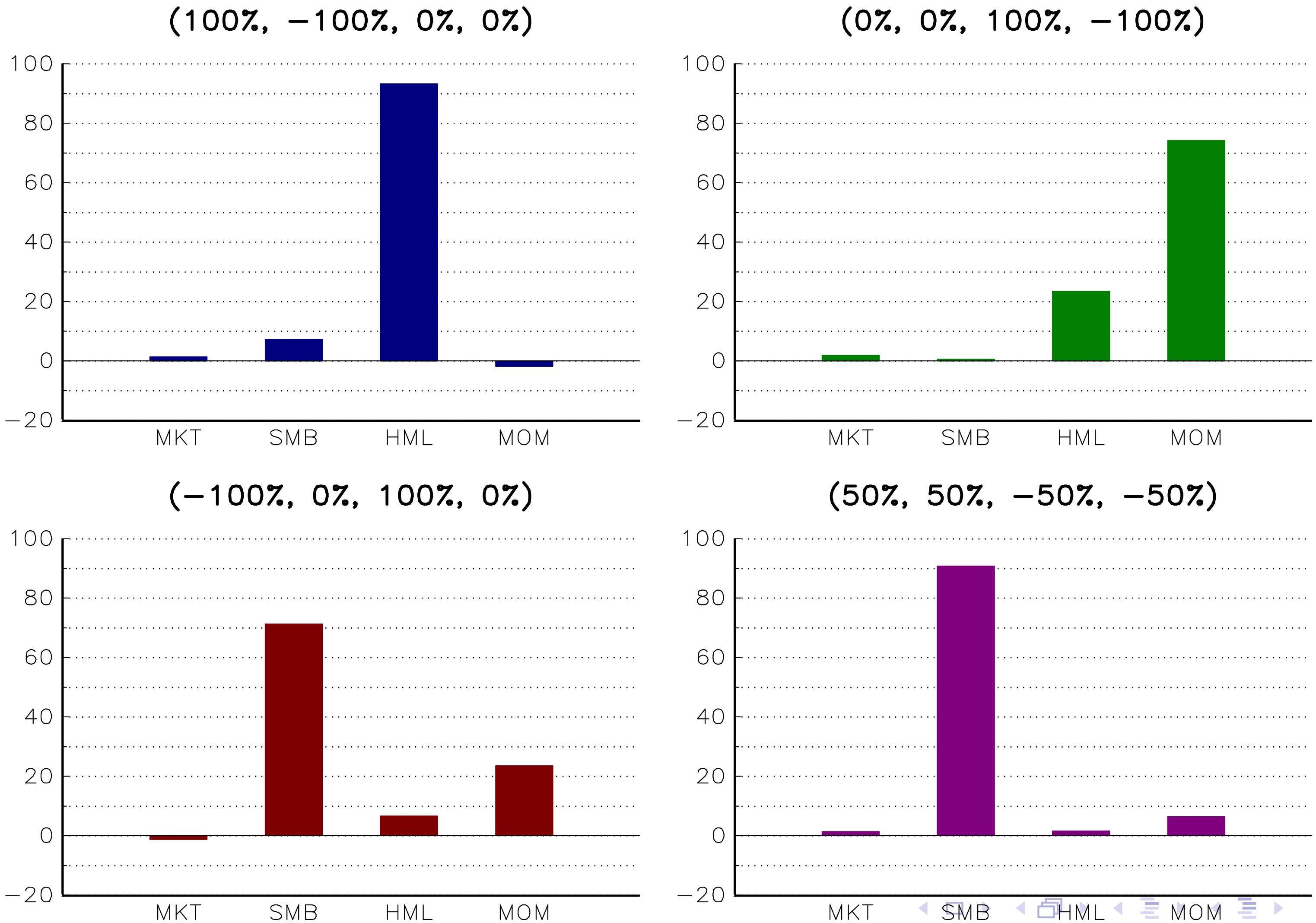
$^{(\star)}$All the estimates are significant at the 95% confidence level.

Question: What is exactly the meaning of these figures?

Risk contributions of long-only portfolios



Risk contributions of long/short portfolios

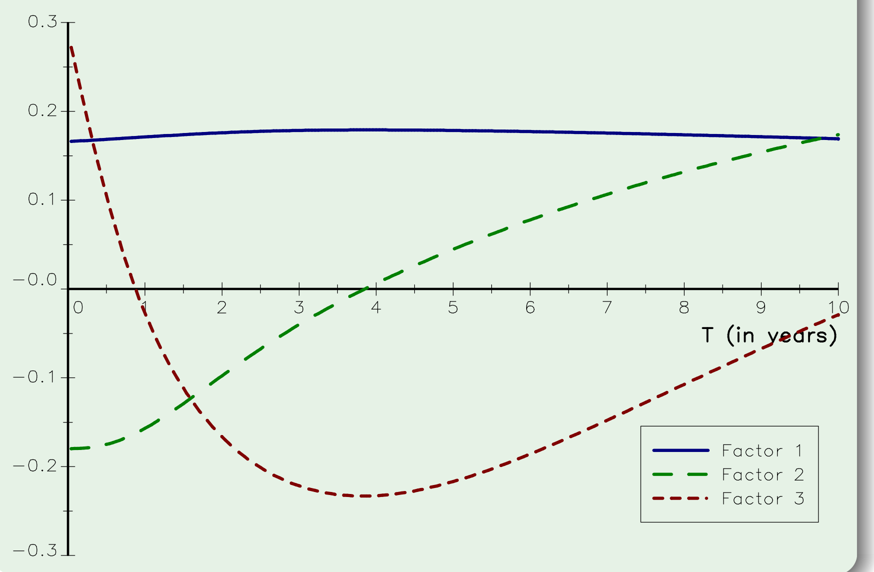


Principal component analysis

PCA factors

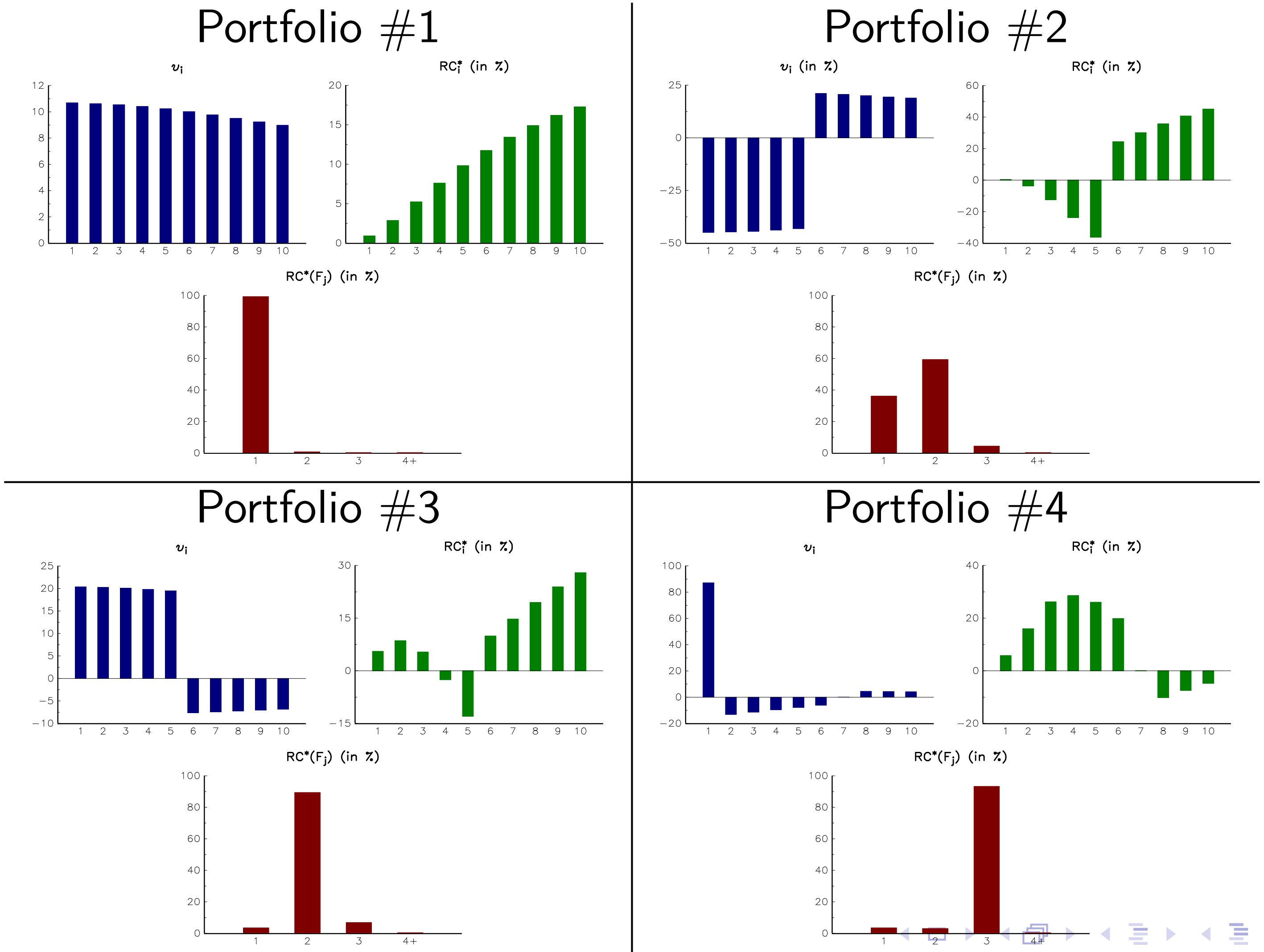
- Level
- Slope
- Convexity

US yield curve (2003-2012)



Portfolio	Maturity (in years) 1	2	3	4	5	6	7	8	9	10
#1	1	1	1	1	1	1	1	1	1	1
#2	−2	−2	−2	−2	−2	1	1	1	1	1
#3	10	10	10	10	10	−4	−4	−4	−4	−4
#4	53	−8	−7	−6	−5	−4	0	3	3	3

Risk decomposition of the portfolios (ZC and PCA)



Barbell portfolios

Maturity	50/50	Cash-neutral	Maturity-W.	Regression-W.
2Y	1.145	0.573	0.859	0.763
5Y	−1.000	−1.000	−1.000	−1.000
10Y	0.316	0.474	0.395	0.422