Risk Management Lessons from Madoff Fraud[*]

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Abstract

In December 2008, as the financial and economic crisis continued on its devastating course, a new scandal erupted. After the 1998's failure of Long-Term Capital Management, Madoff's fraud once again discredits the hedge funds industry. This scandal is however of a different kind. Indeed, Madoff's firm is not a standard hedge fund but a developed Ponzi scheme. By explaining Madoff's system and exploring the reasons for its collapse, this paper draws risk management lessons from this fraud, especially for operational risk management. Present day risk management processes have partially failed to prevent the Madoff scandal. This paper presents the issues for risk capital requirements raised by the collapse of the Madoff scheme. Implications for due diligence processes, including the use of quantitative replication to assess the credibility of the performance of a hedge fund, are also considered. Finally, consideration is given to the regulatory and standardizing approaches of the hedge fund industry as a response to frauds similar to that of Madoff.

Keywords: Madoff fraud, Ponzi scheme, operational risk, due diligence, supervision, hedge funds, bull spread strategy, split strike conversion.

JEL classification: G1, G3.

In December 2008, as the financial and economic crisis continued on its devastating course, a new scandal bursts. On the night of the 11th, from Thursday to Friday, the 12th of December, a financial fraud amounting to up to US\$ 50 billion is uncovered through the confession of its perpetrator: the financier Bernard L. Madoff. Needless to say, the unprecedented scope of the fraud is beyond the reach of other recent scandals of the financial system — a system in such a bad shape that it prompted President-elect Barack Obama to say on December 18th that "we fell asleep at the wheel". On this occasion, we find ourselves facing a swindle whose origins are at least as old as the idea of a stock exchange. The system developed by Madoff's firm, commonly known as a Ponzi scheme, was "pyramidal" in the sense that the returns on investment of yesterday's

investors were paid out of the deposits of today's investors rather than the true product of their investments. The system went on for years, however with the advent of the financial crisis, several of Madoff's customers asked to withdraw their capital provoking the collapse of the entire scheme. In a way, this series of events once again substantiates Warren Buffet's famous aphorism[1]: "it's only when the tide goes out that you see who has been swimming naked."

How can the extent of this con be explained then at a time of high technologies, coercive rules in risk management and the immediate diffusion of information? In the words of Alan Greenspan, "risk management involves judgment as well as science, and the science is based on the past behavior of markets, which is not an infallible guide to the future". Certainly, he is right, but today, the past provides us with a formidable occasion to improve our "guide to the future". For this reason, if justifiable charges exist against the different actors (including both the system and the science), we believe one must seize the occasion and scientifically improve the system. The inquiry into the system's failure and the circumstances that lead to it must go beyond accusations of dysfunction. It needs a scientific demonstration of how the system failed, and what necessary consequences must be drawn to avoid future scandals.

In 1998 already, the hedge fund Long Term Capital Management affair had helped to point out the failings of market risk management. Philippe Jorion [28] had thus proposed a very interesting reading of LTCM's failure (see also Edwards [14]). Madoff's fraud further highlights the failure of operational risk management. More precisely, it illustrates the failure of the quantitative measures used in the management of this risk by the asset management industry — a risk which is perhaps underestimated since asset management firms are considered by the Basel Committee as low risk activities. This paper can thus be understood as building on these previous contributions to the improvement of risk management by the asset management industry. Next, we provide the plan of the paper.

In section 1, we clarify the primary reasons which allowed Madoff's system to operate for several years. In section 2, we explore Madoff's firm strategy by showing firstly how difficult and nearly impossible it is to replicate its advertised strategy, and secondly, the fundamental reasons behind the scheme's collapse in December 2008 through a detailed analysis of the mechanism of a Ponzi scheme. In section 3, we build on these investigations to draw new lessons for operational risk capital requirements. Finally, in section 4, we unravel the implications of Madoff's fraud for the regulators and the investment industry.

1   The story

According to the title of an article [13] in "The Economist" from December 20th, 2008, the scheme revealed on December 12th is the "con of the century". How and why was Bernard Leon Madoff Investment Securities (BMIS), Madoff's firm, presented as one of the most respectable and irreproachable firms, able to originate and sustain such a scheme remains even now somewhat puzzling. In this section, we review the mechanism of the fraud, before exposing some of the reasons thought to underpin the "success" of Madoff's scheme.

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[1] Allegedly, Warren Buffet [33] added in August 2008: "Wall Street has been kind of a nudist beach."

1.1 Mechanism of the fraud

This financial scandal is not the work of quantitative speculators building complex financial products, of which buyers have little understanding, as was the case with the *subprime* crisis[2] and many of the products resulted from the process called *securitization.* It is not either the result of the clever stratagem of some dishonest trader trying to avoid controls to invest more than is reasonable. No, this "con of the century", on the level of Luxembourg's GDP, stemmed from one of the simplest and one of the oldest embezzling schemes: a Ponzi scheme. Bhattacharya [7] dates its origin back to the scandal of Scotsman John Law's Compagnie des Indes in France in 1719. However, the financier who gave his name, Charles Ponzi, went to jail in 1920 for having swindled approximately 40,000 of his customers. His idea was simple. Advertising an investment strategy relying on the arbitrage of international mail coupons between Italy and the USA, he promised a 50% return on investment in 90 days to his investors. In truth, he used the capital invested by his newest investors to pay high and quasi-insured returns to his older ones. The depreciations of his investments were then filled by the contributions of the new customers. The scheme lasted for approximated six months before it was uncovered.

To be fair, there exist many kinds of Ponzi schemes not all of which are fraudulent (see e.g. Bhattacharya [7]). Almost every country uses some form of a Ponzi scheme. When rolling-over their debt, states are indeed asking new lenders for the sum they need to pay-off their previous lenders. This insures against short-term shocks. At the state level, especially in the case of rich countries, this technique is viable because the default risk is negligible. Yet, when it is a firm such as BMIS and when credit facilities become scarce, then it is a serious collapse.

Besides its simplicity, one of the puzzling issues of Madoff's fraud is the profile of its victims. Those are not simple individuals disconnected from the financial markets and its remote mechanisms. On the contrary, they are, for the most part, investors who presumably have an expert knowledge of the trade of financial investment: rich private individuals and large financial institutions[3]. The conception of such a system, which operated for certainly more than 15 years, while Ponzi was able to swindle for only 6 months at a time when new information technologies were non-existent and the training of investors far less developed, seems today incomprehensible. It happened nonetheless. In the following, we contend that the three core tenets to the existence of this fraud are: Madoff's reputation; a high and attractive performance; and the concurrent holding of several positions which facilitated the implementation of Madoff's system.

1.2 Madoff's reputation

The first reason explaining the fraud is the reputation of its mastermind which inspired great confidence to its investors. To understand its origins, it is necessary to go back over the biography of this 70-year-old man. Swimming instructor on Long Island, Madoff created his investment firm at the age of 22 with only $ 5,000 of his personal savings. His following success consequently embodied the American dream and provided Madoff with the aura of a great financier. Moreover, he was president of the NASDAQ, in 1990-1991, an institution which he reformed in depth to become one of the most successful stock exchanges. Madoff was also a philanthropist who founded

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[2]The *subprime* crisis was caused by the sudden illiquidity of Asset-Backed Securities (ABS) built on the debt of some American households which had obtained mortgage loans unreasonable for their incomes.

[3]A list of Madoff's victims compiled by the Wall Street Journal on January 28, 2008 is available at `http://s.wsj.net/public/resources/documents/st_madoff_victims_20081215.html`

the *Madoff Family Foundation*. His ethics was praised by many, and many of his investors were charity organizations. In January 2008, his firm even claimed on its website that Madoff himself had "a personal interest in maintaining the unblemished record of value, fair-dealing, and high ethical standards that has always been the firm's hallmark". The personal reputation of its founder is not, however, the only factor of the incredible success of this scheme. Being registered to the Securities and Exchange Commission (SEC) and passing without incident the various inquiries made by the regulator of American finance could only inspire confidence in BMIS and contributed to reinforce its credibility. Thus, one of the first reasons to explain Madoff fraud ensued from the irrational behavior of the agents who based their investment decisions not on some scientific study but on the reputation of the fund's manager[4]. Perhaps, some of his customers were informed of Madoff's scheme, but it seems as certain that, given their large number, all were not informed. It illustrates that there will always be "sheep to be shorn" as Charles Kindleberger wrote [30]. In financial markets, confidence and reputation remain essential factors to understand the mechanisms of a fraudulent Ponzi scheme as well as those leading to the development of a speculative bubble.

1.3 A highly attractive performance

The second reason we find behind the success of Madoff's fraud is the attraction that Madoff's performance must have had on investors. Given its performance, we could even be astonished by the fact that this fraud is not more impressive. Two opposed reasons can explain this: first of all, due diligence processes (we will describe these more precisely in section 4.1) have put Madoff's funds into black lists; the second reason is due to Madoff himself. Indeed, Madoff had introduced a hard and cruel selection for new customers to invest in his funds: they must have the authorization of Madoff. It permits to assert his talent and to lure new customers who thought that they are specially selected thanks to their own characteristics.

Since Madoff's performance is not public information, we used the following 6 funds, all known feeders of Madoff's fund to proxy and understand his performance: Fairfield Sentry Ltd (FFS), Kingate Global Fund Ltd (KING), Optimal Strategic US Equity Ltd (OPTI), Santa Clara I Fund (SANTA), LuxAlpha Sicav - American Selection (LUX) and Herald Fund SPC - USA Segregated Portfolio One (HRLD). Their characteristics are reported in Table 1. The study period begins in January 31th, 1990 and ends in October 31st, 2008. Notice their absolute yearly rates of returns close to the 10% mark for relatively low corresponding volatility leading to uncannily high Sharpe ratios, at least 5 times greater that the S&P500's. Of the 156 months of data available to us for the FFS feeder fund, only 5 months exhibit a negative performance. Besides, Madoff's management fees were non-existent: Madoff claimed that the fees perceived on the trading deals via his broker's activities were amply sufficient. He told his customers he was letting them in to share in the benefits of his expertise in securities valuation. Like Ponzi in 1920, the so-called arbitrage was at the heart of the performance. These phenomenal performances, clearly visible on a classic portfolio optimization display (cf. Figure 2), were nevertheless often called into question because of their weak volatility. They are indeed very smooth (cf. Figure 1).

BMIS performances should be very similar to those of these six feeder funds. It is no wonder then that Madoff could attract the unaware or the "no-so-attentive" investors. But, what could

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[4]We refer to [27] for more details on how Madoff operates.

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Figure 1: Comparison of funds invested in Madoff with traditional asset classes
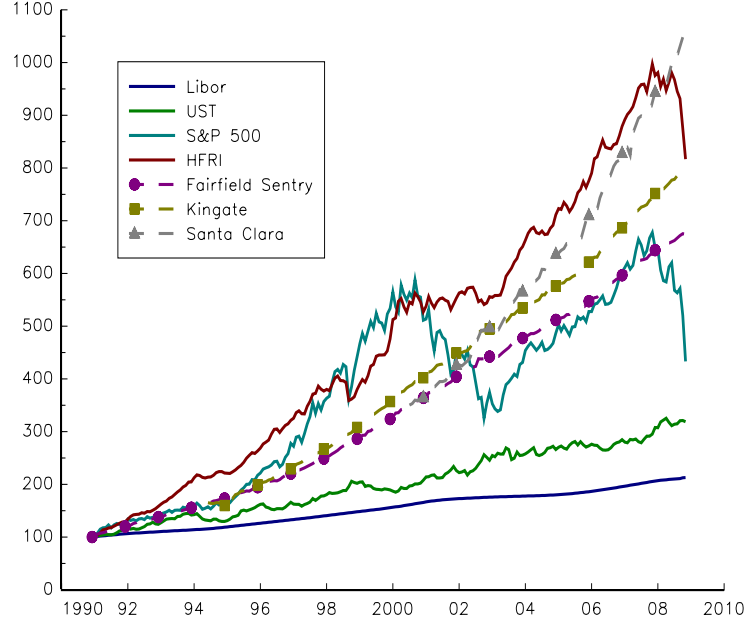

Table 1: Statistics of funds invested in Madoff

$\hat{\mu}_{1Y}$ is the annualized performance, $\hat{\sigma}_{1Y}$ the yearly volatility, s the Sharpe ratio, γ_1 the skewness and γ_2 the excess kurtosis. D_{1M}, D_{3M} and D_{6M} are respectively the drawdown for one, three and six months and $D_{\max}$ is the maximum drawdown over the entire period. H is the hit ratio of monthly positive returns. All statistics are expressed in percents, except for the statistics s, γ_1 and γ_2.

	UST	S&P 500	HFRI	FFS	KING	OPTI	SANTA	LUX	HRLD
$\hat{\mu}_{1Y}$	6.68	8.52	12.42	11.24	11.56	10.96	14.19	8.29	7.27
$\hat{\sigma}_{1Y}$	6.80	14.28	7.08	3.81	4.71	2.69	4.63	1.53	1.69
s	0.33	0.28	1.10	1.75	1.47	2.48	2.26	2.73	1.87
γ_1	-0.32	-0.76	-0.81	4.70	6.14	0.87	1.06	0.48	0.53
γ_2	0.74	1.79	2.99	39.96	59.70	0.27	2.07	0.07	0.24
D_{1M}	-7.09	-16.80	-8.70	-0.55	-2.30	-0.39	-1.87	-0.19	-0.37
D_{3M}	-8.52	-23.11	-13.60	-0.17	-4.53	-0.64	-1.89	0.85	0.56
D_{6M}	-8.83	-29.28	-15.14	0.75	-4.72	0.26	-0.79	2.50	1.85
$D_{\max}$	-10.62	-44.73	-18.13	-0.55	-5.54	-0.64	-2.01	-0.19	-0.37
H	64.65	64.65	73.49	93.37	92.22	91.43	74.75	98.18	88.89

Figure 2: Portfolio optimization
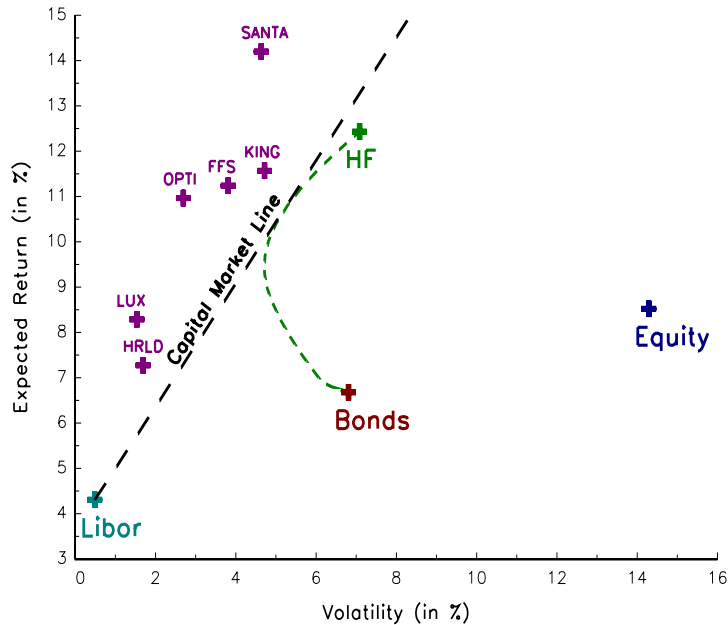
be said of the professional hedge fund managers who fed funds to the scheme? Once again, the point here is not to blindly attack managers about some alleged investment decisions, but rather to point out that by surfing on the alternative investments' trend of these last years, Madoff has further blackened the image of an industry often characterized by its lack of transparency (see e.g. [36]). Indeed, in the recent financial climate, this fraud cannot but stress the need for urgent regulatory change. We shall examine this issue in section 4.

1.4 Holding several posts concurrently: broker, fund manager and custodian

The final explanation we will give to explain the unhindered development of the fraud is the fact that BMIS was concurrently a broker, a fund manager and also the custodian of its funds. Conversely to what is practiced in Europe (cf. the European "Undertakings for Collective Investment in Transferable Securities" or UCITS III), American regulations do not prohibit an asset management firm to be the custodian of its own managed funds. In the European "Product Directive" it is stipulated, for the safekeeping of assets, that:

> "The depositary must act independently of the other parties to the UCITS and solely in the interest of the unit-holders."

To understand the potential conflicts of interest, one needs only to look at the structure of Madoff's fraud. Among Madoff's customers, one must first separate prominent billionaires, universities and charity organizations from financial institutions. The latter can be large banks, which lent money to hedge funds which then placed it either directly at BMIS or, more often than not, invested it in feeder funds. Feeder funds are structures which collect capital from banks, hedge funds and rich private individuals to entrust them to a fund manager — in this case Madoff. They are thus intermediaries and distributors. They can be classified into two categories:

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1. Offshore firms of average size and of Luxembourgian or also Irish right, among them: Fairfield Greenwich Group with Fairfield Sentry fund ($ 7.5 billion); Tremont Capital Management ($ 3.1 billion); Kingate Management with Kingate Global Fund ($ 2.8 billion); Ascot Partners ($ 1.8 billion); and, Access International Advisors ($ 1.5 billions in LuxAlpha). These firms are generally opaque and base their clientele on an extensive network.

2. Large banks managing funds of hedge funds (FoHF) including for example: Santander Optimal (2.33 billion euros), Union Bancaire Privée (796 millions euros), etc. What is marketed by these managers is the solidity of the balance sheet and the system of controls via due diligence processes.

Madoff operated mostly as a fund manager by delegation of funds obtained through these distributors. Although, these latter had large international banks as custodians (UBS for Lux-Alpha, HSBC for Thema), these institutions also delegated to Madoff the custody of the assets of the managed funds. Thus, in the end, Madoff was the manager and the custodian of his own funds by means of this double delegation. The fraudulent scheme thus largely benefited from lesser scrutiny since the control over the management or the custody of the assets all remained in BMIS's house.

Notwithstanding, while under the French law (ARIA 3), the independence of the manager from the custodian of the assets (who is obligated to restore assets in case of failure, while it is only an obligation of supervision in the Anglo-Saxon world) is mandatory for a hedge fund to be eligible to inclusion in a fund of hedge funds's portfolio, according to the French Autorité des Marchés Financiers — the French regulator, counterpart to the American SEC — some of French managed funds were affected by the collapse of Madoff's scheme. Disregarding the independence rule normally raises a red flag in the due diligence process. We are therefore facing a failure of the system. The question we shall ask in section 4 is whether new laws must be promulgated or whether a stricter enforcement of those really existing already would be sufficient to prevent future scandals.

2 How Madoff lost the capital

One frequently asked question is: "how could Madoff lose US$ 50 billion?" At first sight, this seems impossible and incredible. One could think that it suffices to invest the capital at the risk-free rate in order to easily cover redemptions except in the case where they are very large. In fact, we will show in this section that this is not true, and we estimate that a figure of 20% of redemptions in November is sufficient to explain Madoff's default. But before explaining the collapse of Madoff, we first present the alleged investment strategy sold by Madoff.

2.1 Understanding the Madoff investment strategy

Madoff used a strategy called "Split Strike Conversions" or "Bull-Spread". This strategy is constituted by three positions: a long position on the risky asset S_t, a short position on the call option $C(K_C)$ and and a long position on the put option $P(K_P)$ with $K_P < K_C$. The reader can find in Bernard and Boyle [6] a comprehensive study of the strategy. We also present in Appendix A some results on this strategy. We report here a backtest of this strategy since March 1994 on S&P 500 using a monthly rebalancing frequency. We consider symmetric strikes, that is $K_C = (1 + \kappa) S_0$ and $K_P = (1 - \kappa) S_0$ using the at-the-money (ATM) volatility to price the

options[5]. We reported the results in Figure 3. First, notice that the backtests are very far from the performance of the FFS feeder fund. Second, remark that the Bull-Spread strategy with $\kappa = 1\%$ or $\kappa = 2\%$ produces higher volatility than FFS. To give you an idea, to match the volatility of FSS and our backtest, one has to use $\kappa = 0.65\%$. In this case, however, the performance of the Bull-Spread strategy would be close to the Libor's. In Figure 4, we reported the scatterplot of returns of FFS, S&P 500 and our backtests. Note that the monthly returns of FFS are not correlated with the monthly returns of S&P 500 nor with these of our backtests. In order to reconcile the FFS performance with our backtest, one has to assume that Madoff has a good stock picking process. For example, if we consider that the stock picking process generates an out-performance of 8.5% by year, we obtain a backtest with similar returns on average[6] (see Figure 5). But if the impressive returns obtained by Madoff are explained by his stock selection process (or by market timing), one may wonder why Madoff did not launch a pure L/S equity fund instead of a complex derivatives strategy. Notice also that the previous analysis and back-test have been certainly done by some quants and explain why Madoff was on the blacklist of some banks, hedge funds or financial institutions. In a recent paper, Bernard and Boyle [6] draw the same conclusion: "*Madoff's returns lie well outside their theoretical bounds and should have raised suspicious about Madoff's performance*".

Figure 3: Backtest of the Bull-Spread strategy on S&P 500 index
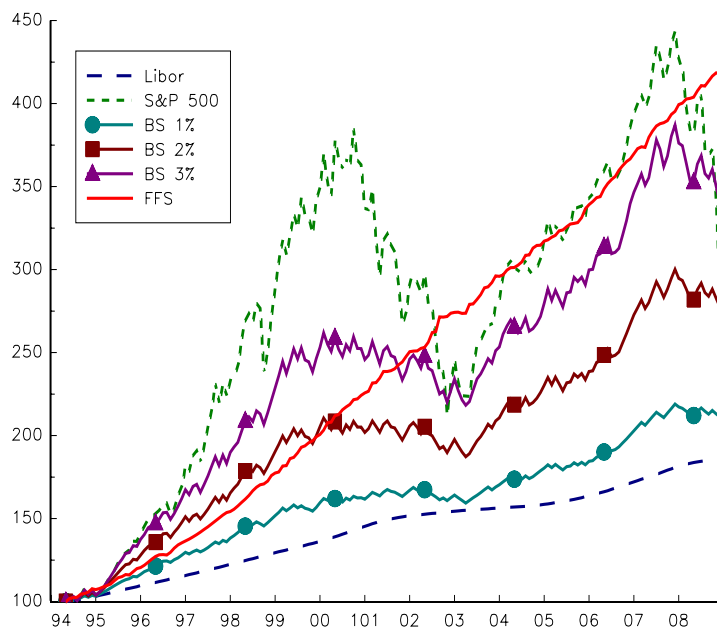
2.2 Explaining the collapse of Madoff

We show here that the collapse of Madoff is mainly due to the liquidity crisis on hedge funds. Using the data of the six Madoff feeder funds (which represents an amount of 15.5 US$ billion), we estimate that a figure of 20% of redemptions in November suffices to explain the collapse of Madoff.

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[5]We do not take into account smile effects meaning that our results are more favorable than in real life.

[6]Note that it has nevertheless 30% more volatility.

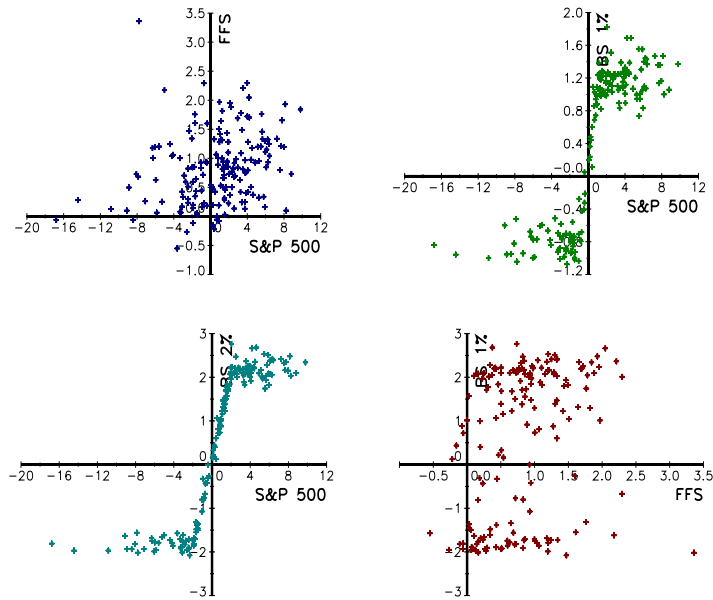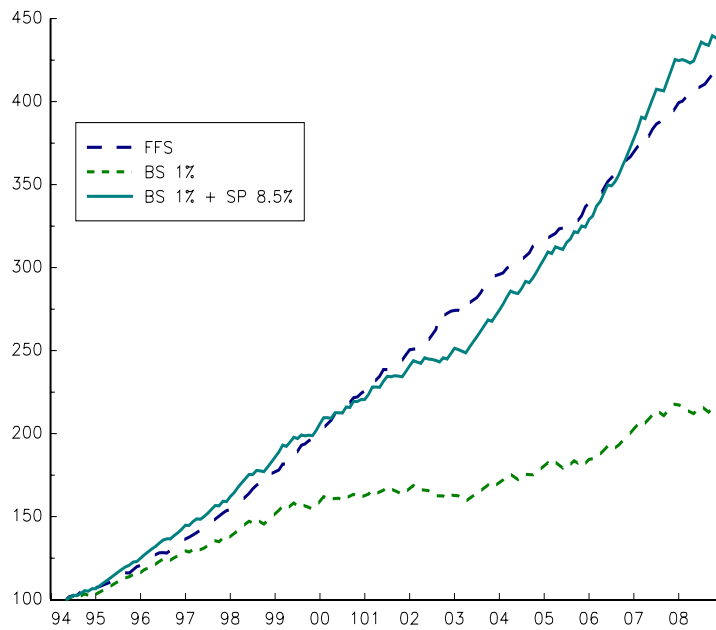Figure 4: Comparison of monthly returns

Figure 5: Introducing stock picking in the Bull-Spread strategy

Consider the Ponzi scheme framework developed in Appendix B. In order to estimate the net flows rate $\delta_t = \lambda_t^+ - \lambda_t^-$, we describe the net asset value V_t and the assets under management F_t using the following system of equations

$$\begin{cases} \mathrm{d}V_t = \mu_t V_t \, \mathrm{d}t \\ \mathrm{d}F_t = \mu_t F_t \, \mathrm{d}t + \left(\lambda_t^+ - \lambda_t^- \right) F_t \, \mathrm{d}t \end{cases}$$

Combining the two equations, one obtains

$$\delta_t \, \mathrm{d}t = \frac{\mathrm{d}F_t}{F_t} - \frac{\mathrm{d}V_t}{V_t}$$

After estimating δ_t^i for each fund i, one may compute the amount Δ_t of net flows by

$$\Delta_t = \sum_{i=1}^{6} \int_0^t \delta_t^i F_t^i \, \mathrm{d}t$$

We have reported δ_t^i and $\mathrm{d}\Delta_t$ in Figure 6. Notice the large outflows since the end of August. In September, our estimate shows an amount of outflows of US\$ 240 millions, while the estimate for October is about US\$790 millions. Thus, for these two months, the amount of redemption is at least bigger than one billion dollars. This figure may nevertheless not explain Madoff's collapse by itself.

Figure 6: Estimating the net flows rate δ_t and the monthly net flows amount $\mathrm{d}\Delta_t$
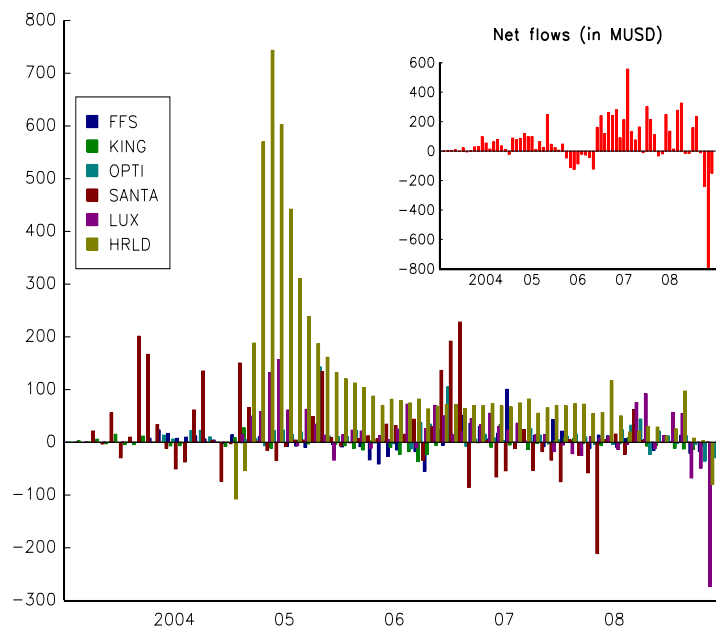
Madoff defaulted because there were no more capital in the end. Of course, the main reason is redemptions. But we must also take into account the fees. Let us note G_t the gross asset value. To obtain the net asset value V_t, we have to consider the management fees m_t and the performance fees p_t. Because management fees are senior on performance fees, we proceed in two steps.

1. First, we compute $G_t^\star$ by subtracting the management fees

$$\mathrm{d}G_t^\star = \mathrm{d}G_t - m_t\,\mathrm{d}t$$

2. Second, we deduce the net asset value by subtracting the performance fees in the following way

$$V_t = V_{t_\bullet} \times \left(\frac{G_t^\star}{G_{t_\bullet}^\star} - p_t \times \left(\frac{G_t^\star}{G_{t_\bullet}^\star} - \frac{B_t}{B_{t_k}} \right)^+ \right)$$

where $t_\bullet = t_p(t)$ is the last settlement date of performance fees before t. For example, if we assume that the performance fees are paid at the end of each year, $t_\bullet$ is equal to $31/12/2007$ if t is October 31st, 2008. B_t is the value of the benchmark, and we take performance fees only if the strategy has a return bigger than the benchmark's one.

Given a sequence of $\{G_t\}_0^T$, we may define the historical series of net asset value $\{V_t\}_0^T$. But the reverse is also true. Given the $\{V_t\}_0^T$, we may estimate $\{G_t\}_0^T$ by iterations. We may also estimate the amount of fees generated by the fund

$$W_t = \int_0^t \left(\frac{\mathrm{d}G_t}{G_t} - \frac{\mathrm{d}V_t}{V_t} \right) F_t$$

In order to clearly understand how we proceed, consider the feeder fund FFS in Figure 7. The top-left graph represents its net asset value. Using $m_t = 1\%$, $p_t = 20\%$ and $B_t = 1$ (performance fees are paid if the fund has a return above zero), we deduce the gross asset value in the top-right graph. Using the assets under managements (bottom-left), we finally compute the total fees W_t. Thus, we estimate for example that the total fees generated by FFS between January 2003 and October 2008 is about 900 millions dollars.

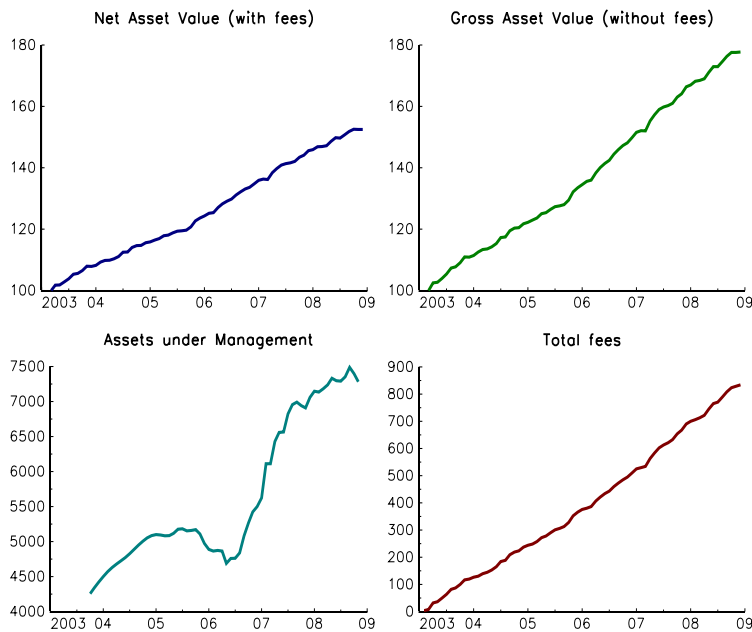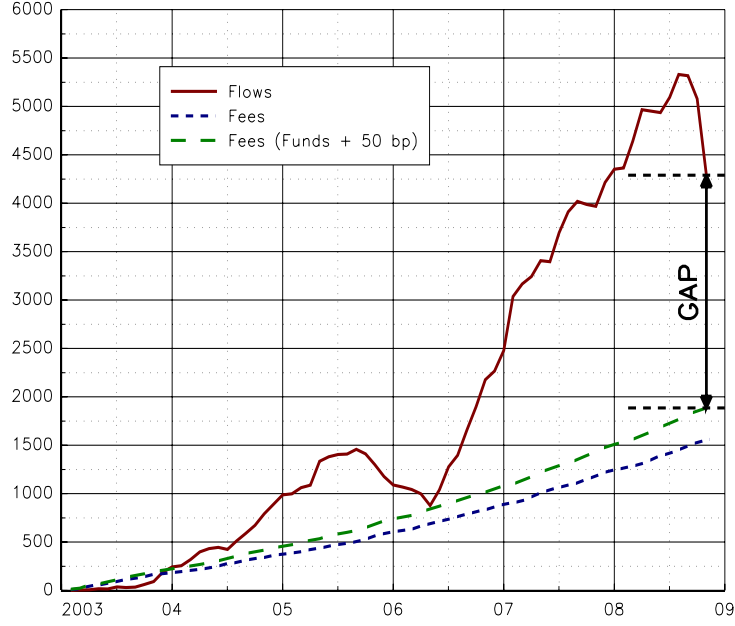Figure 7: An example of fees computing

Figure 8: Estimating the gap in November 2008

Let's now consider our Ponzi scheme model. Assuming that $r_t = 0$ and $K_0 = 0$, the value of the capital for the 6 funds is then

$$K_t = \Delta_t - W_t = \sum_{i=1}^{6} \int_0^t \delta_t^i F_t^i \, dt - \sum_{i=1}^{6} \int_0^t \left(\frac{dG_t^i}{G_t^i} - \frac{dV_t^i}{V_t^i} \right) F_t^i$$

In Figure 8, we reported Δ_t and W_t. Starting in January 2003, we estimate Δ_t at about US\$ 4.3 billions, while W_t is around US\$ 1.5 billions[7]. These fees do not, however, include Madoff's remuneration and other expensive fees. If we add .50% of hidden fees, the estimated fees to pay out of the assets under management between January 2003 and October 2008 for the six funds is US\$ 1.9 billions. The remaining capital in October 2008 is also estimated at US\$ 2.4 billions dollars, which represent 15% of assets under management. Of course, this number is certainly underestimated (because $r_t > 0$ and $K_0 > 0$). It gives nonetheless an idea of the cause of Madoff's failure. In November 2008, because of the Hf liquidity crisis, Madoff certainly had to face a big amount of redemptions (perhaps higher than 20%), but he had not enough capital to pay them. Without the HF crisis of October 2008, Madoff's Ponzi scheme could certainly have continued a long time.

Another interesting thing of note is that the main contributor to Madoff's default is the LuxAlpha Sicav fund. By leaving the hedge funds' world and entering the European UCITS III format, and therefore providing liquidity to its investors, it dragged Bernard Madoff in a position where he could not face up to the liquidity crisis. Had he stayed in a hedge fund format and had he applied a gate, Madoff could have perhaps not avoided the default, but would have certainly survived longer.

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[7]The numbers presented here are purely illustrative. Note that they only concern the assets of the six feeder funds and not the entire wealth managed by Madoff.

3 New lessons for operational risk capital requirements

The Madoff fraud concerns the operational risk. More precisely, depending on the relationship between Madoff and the victim institution, it may be classified into different Basel II event type categories (cf. Table 2):

- Some financial institutions have launched or distributed Madoff feeder funds.

  - In case of internal complicity within the financial institution, it then clearly classifies as a type 1 risk. That is an internal fraud.
  - In case of no internal complicity, it is an external fraud. It also concerns financial institutions that have a direct exposure to Madoff funds.

- Finally, some financial institutions will certainly choose to compensate their clients for Madoff-related losses because of potential mis-selling or advisory failures. In this case, the loss is a type 4 risk. That is, the loss may be classified into "clients, products & business practices."

Table 2: The loss event type classification in Basel II

	Event-Type Category (Level 1)
1	Internal fraud
2	External fraud
3	Employment practices and workplace safety
4	Clients, products & business practices
5	Damage to physical assets
6	Business disruption and system failures
7	Execution, delivery & process management

Making the difference between internal and external fraud is easy. However, making the difference between external fraud and clients, products, & business practice is more difficult when the financial institution has launched products related to Madoff funds. Let us consider the example of *Banco Santander*. The Spanish bank has distributed the Optimal Strategic hedge fund to their private banking and institutional clients. At the end of January, Banco Santander announced that it was offering its private banking clients 1.38 billion euros in compensation for Madoff-related losses. From the point of view of Banco Santander, it represents an external fraud. But, it is also a loss which pertains to a type 4 event defined as follows:

> *"Losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product."*

3.1 A new beta for the asset management industry?

In Basel II, capital must be used to cover for operational risk. In the Standardized Approach, banks' activities are divided into eight business lines presented in Table 3. The capital charge for each business line is calculated by multiplying the gross income by a factor assigned to that business line and denoted beta. For the Asset Management line, the beta coefficient is fixed to

12%. For example, for a gross income of US\$ 1 Bn, the yearly capital charge for operational risk is US\$ 120 Mns.

Table 3: The SA approach in Basel II

Business Line	β factor
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

Notice that *Asset Management* is classified as a low operational risk category along with *Retail Banking* and *Retail Brokerage*. To understand why, it is best to reflect on how Basel II was calibrated some years ago. In January 2001, the Basel Committee on Banking Supervision (henceforth BCBS) published the second consultative paper [2] and proposed to add capital for operational risk. During 2001, BCBS conducted two major data collection exercises called Quantitative Impact Study: QIS 2 and QIS 2.5. The results for operational risk were published in January 2002 [4] and in March 2003 [5]. In September 2001, BCBS released a new working paper on operational risk [3]. In this new paper, one can already find the overall framework for operational risk of Basel II. In particular, it contains some results on collected loss data. Using different ranking schemes, it appears that *"Retail Banking tends to be ranked low, while Trading and Sales, Payment and Settlement, and Agency Services and Custody tend to be ranked high"*. On average, Asset Management comes in fourth position[8]. Using these data, Moscadelli (2004) [35] calibrated the betas and found that Asset Management comes in third position (see Table 14 in [35]). In the third Consultative Paper published in April 2003, BCBS confirmed a low beta of 12% for Asset Management.

The reaction of the Asset Management industry was generally positive as exemplified by the letter of the European Asset Management Association dated from January 29, 2003.

> *"We accept the need for investment management firms to hold a minimum amount of regulatory capital, but remain of the view, which is supported by the 2001 EAMA Study [19], that the business of investment management does not pose the same systemic risk issues as banking, and that regulatory capital should not be the principal safeguard against operational risk."*

They therefore proposed the reinforcement of risk management principles in Asset Management (as in the UCITS III directive), but suggested a maximum capital charge of 10 million Euros. Similar conclusions may be found in the report [10].

In June 2008, BCBS initiated a new Loss Data Collection Exercise (LDCE). The results are not yet known, but we could ask ourselves whether there is need for a new beta for the Asset

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[8]from less risky to more risky business lines.

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Management line, especially after Madoff and similar frauds (one could think also of the alleged twin scandal to Madoff's perpetrated by Robert Allen Stanford's firms and revealed on February 17, 2009) which would not be integrated in the LDCE results. One must notice that this new LDCE will certainly show that operational risks have increased since the last LDCE, but most probably because of the financial crisis for most business lines. What about the asset management industry in particular? Conspicuously, the Asset Management category has not suffered from more operational risk losses than Trading & Sales. Nonetheless, one may think that the Madoff scandal should negatively impact the new beta for Asset Management relatively to the Retail Banking category, because the financial crisis reveals a riskier behavior of the asset management industry essentially due to the use of complex financial instruments and vehicles, like Madoff's funds. But the reality is more complex because Private Banking is generally classified in the Retail Banking category and not in the Asset Management category. In this case, BCBS has several options to redefine the SA approach:

- first, it may do nothing considering that Madoff fraud was already taken into account by the existing betas;

- second, it may conduct a new LDCE in order to measure the impact of Madoff fraud and revise the existing betas;

- finally, it may modify the business lines classification by creating a Private Banking category.

Anyway, the problem is set to BCBS, which will certainly conduct a new LDCE to measure the impact of the Madoff fraud in terms of operational risk.

3.2 Impact on Advanced Measurement Approach (AMA) capital

According to Basel II, supervisors may authorize a bank to compute its capital charge using an internal model. In general, this model is based on the Loss Distribution Approach [17, 18]. In this approach, the operational risk loss for one business line and one risk type is defined by

$$L = \sum_{n=0}^{N} \ell_n$$

where N is the number of losses for the next year and $\{\ell_1, \ldots, \ell_N\}$ are the amounts of the individual losses. In LDA, it is assumed that N is drawn from a yearly frequency distribution $\mathbf{F}$, and ℓ_n are i.i.d. random variates from the severity distribution $\mathbf{G}$. The distribution $\mathbf{H}$ of L is then obtained as the convolution of $\mathbf{G}$ by $\mathbf{F}$. The capital charge is therefore computed as the quantile of $\mathbf{H}$ at the 99.9% confidence level:

$$\mathrm{CaR}\left(\alpha\right) = \mathbf{H}^{-1}\left(\alpha\right) = \inf\left\{x \mid \mathbf{G}\left(x\right) \geq \alpha\right\}$$

with $\alpha = 99.9\%$.

When a bank faces a big loss, it has no impact on the frequency distribution $\mathbf{F}$ but may have a big impact on the estimation of the severity distribution $\mathbf{G}$. In this case, the distribution $\mathbf{H}$ will change accordingly, and the computed capital charge will be very different with respect to past years. Thus, banks exposed to the Madoff fraud will certainly have to compute additional capital under the Advanced Measurement Approach.

Let us consider an example. Assume that ℓ_n and N are respectively distributed according to a log-normal $\mathcal{LN}(7, 2.5)$ and a Poisson distribution $\mathcal{P}_{20}$. The capital charge CaR for this business line and this event type is about US$ 19 Mns. Let us suppose that a big loss occurs for this business line and this event type, e.g. US$ 100 Mns. We can compute the impact of this big loss on the new capital charge CaR*. If we assume the use of a database of 5 years length to estimate the severity distribution, we can compute the ratio CaR* / CaR. The density of this ratio is reported in Figure 9. Notice that the mean of this ratio is close to 3.5, but it may take larger value. This simple example illustrates how important may be the impact on the capital charge computed with an internal model when facing a relatively big loss.

Figure 9: Density of the ratio CaR* / CaR
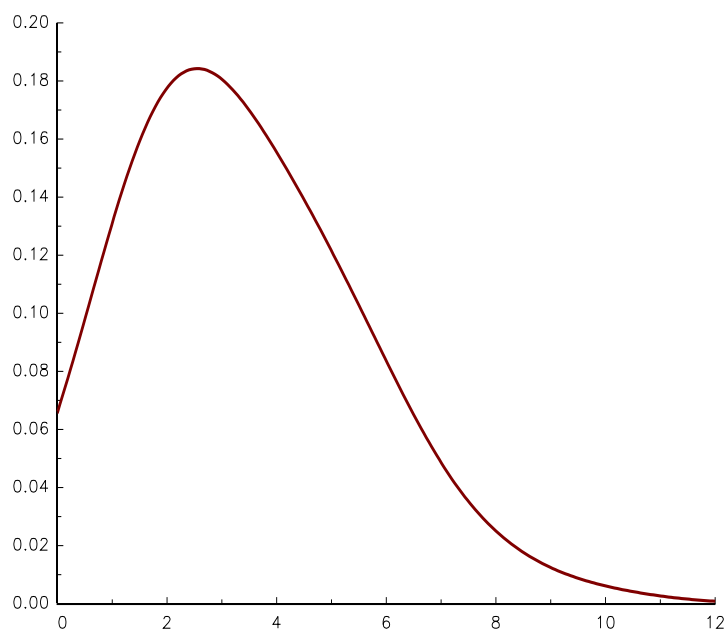
4   Implications for regulators and the investment industry

At first glance, it seems the extent of the Madoff affair calls inevitably for new regulations guaranteeing hedge funds' investors better information on their investments. Already the calls for higher disclosure requirements on the part of hedge fund managers about their operational and, possibly also, their investment processes have gained in strength and in validity and, perhaps grudgingly, in support. While we certainly agree that it is a desirable end — at least for investors — one must question what impact such measures would truly have, especially in the case of interest to us: funds of hedge funds (FoHF) managers and similar professional investors. Indeed, one can be surprised at the mention of the enormous amounts lost by professional experts in Madoff's scheme[9].

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[9]Among the losers, the eight biggest, all professional investors, lost more than $1 billion each to Madoff's scheme (cf. `http://s.wsj.net/public/resources/documents/st_madoff_victims_20081215.html`), and the list of professional investors having lost hundreds of millions is unfortunately and surprisingly long.

While the stunning features of this affair have already left a mark on the image of hedge funds, and will possibly have significant regulatory consequences for the industry, in terms of management of the risk borne by HF investors, the financial crisis and the hedge fund crisis that ensued also bring forth relevant issues. Among them, some of the more salient pertain to the due diligence process as it is the single most important part of the investment process [26]. In truth, it is profoundly disturbing that retail investors could have been impacted by Madoff (albeit often through means of complex investment schemes) whereas they are normally barred from investing in such funds, and while there were ways to surmise Madoff's true nature and that for a decade [34]. We hasten to add that it is not our goal to point out the failings of some individuals while we comfortably take advantage of the benefit of hindsight. To the contrary, what we pursue here is the identification of the features of the due diligence process that could be improved, and make propositions. Thus, after having examined in the sections above the mechanism of Madoff scheme, the reasons that lead to its demise and the lessons which must be drawn from it in terms of operational risk capital requirements, we now want to look at the implications the Madoff affair will have on regulators and on the investment industry. We proceed in this section by looking first at the due diligence process and the components which may need rethinking. Second, we discuss the future of hedge funds, before turning to notions of deregulation, supervision and coordination.

4.1   Rethinking due diligence processes

Hedge fund *due diligence* is the process of monitoring and reviewing the operation and management of hedge fund managers with the objective of identifying managers with whom to invest, and, later, monitoring those managers in order to insure alignment with the investors' interests [8, 43]. One can further distinguish between operational and financial due diligence, each process designed to monitor the corresponding risk item of investing with a particular manager. Although due diligence is recognized by professional associations as a "best practice", on a global basis, there is no legal requirement even for professional investors to perform due diligence before any investment in a particular hedge fund [40, p. 23]. It is only recently that the suggestion has been made to include the process in a regulatory framework [39].

By nature, due diligence is part of a fund of hedge funds (FoHF) manager's value proposition [26, p. 57]. Thus, although no global legal framework requires due diligence[10], the process is nevertheless often carried out by professional managers and advisors. Despite the fact that "best practices" have lately been charted through consultation of the industry and resulted in a tentative to set standards (cf. [25]) or promote legislation (cf. [39]), no definite set of guidelines or rules yet exists to perform a thorough due diligence[11]. In light of the roughly 40% of the HF industry constituted of funds of hedge funds, it is obvious to the authors that due diligence deserves careful consideration in the aftermath of the Madoff scandal and the failure of part of the industry to detect the enormous operational risk it represented. Moreover, operational due diligence has been found to be a significant source of out-performance for FoHFs [8]. Thus, we believe that a careful discussion would be beneficial for the industry as well.

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[10]We have to qualify this point. If no global legal requirement exists, there are nonetheless some local regulations enforcing the practice with different degrees of procedures, among them Hong-Kong, France, Germany and Ireland [40, p. 24].

[11]A very helpful tool put together by several organizations representing the industry provides a good overview of the general framework around hedge fund investors and professionals: http://www.hedgefundmatrix.com/

One of the difficulties behind reaching an agreement on the constitutive elements of a due diligence process certainly stems from the broad and varied landscape of the hedge fund industry — as broad and varied as hedge fund investors can be. Nevertheless, a general understanding is that due diligence is labor intensive and "more art than science" [26, p. 42]. The due diligence process should be comprised of two legs: a qualitative and a quantitative investigation, and should assess both operational and financial risks for the investors. Although one will probably not find it mentioned anywhere, there exists an implicit double dichotomy, qualitative/quantitative vs. operational/financial risks, possibly because of the inherent difficulty of assessing quantitatively the operational risk and the widespread use of quantitative measures such as Sharpe ratios, maximum drawdown, etc. to assess financial risks. The qualitative part of the due diligence process often comprises of a thorough investigation of the background of the employees of the fund, including their qualifications, experience, criminal records, and anything that could indicate past misconduct. The structure of the fund is also investigated, where accent is put on potential conflicts of interest, and procedures to handle orders, trades, or risk management. Thorough due diligence often includes at least an on-site visit per year to the manager to assess first-hand the hedge fund operations.

In truth, the role of quantitative assessment is often played down in regard of the common disclaimer "past performance is not indicative of future results" (cf. [26, p. 57]) . Certainly, this is common sense. Yet, if there is one thing that the Madoff affair and quantitative analysis exemplify (cf. Appendix A), it is that past performance could be indicative of past misdoings. And, while the Madoff scandal illustrates well how both financial and operational risks are connected, it is primarily for investors, as discussed above, a case of operational risk realization. One dimension in the due diligence process which therefore needs re-thinking is the role of quantitative assessment. For sure, the role of the qualitative part is as relevant now as it was before, and the standards of disclosure put forth (in particular [1] and [25]) provide a strong basis on which to judge of the soundness of hedge funds. Yet, the case of LuxAlpha Sicav, one of Madoff's feeders settled in Luxembourg, and thus subject to European regulation, perhaps demonstrates that compliance with regulations and disclosure standards are no guarantee against operational risk for investors. Our point is less that current practice has weaknesses, for there is no perfect risk detection process. Rather, it is that quantitative analysis could have detected early on the fraud[12] and that we must make sure that the next Madoff-like scheme is detected and caught in its early stages.

Quantitative measures, in particular of the kind proposed by [31, 12, 21], may help cast light on suspicious funds. Nonetheless, we cannot claim that quantitative analysis is the panacea to these problems. First, as explained by Lo and colleagues, there are valid reasons to obtain unusual values for each of those statistics. Second, certainly, quantitative methods are not armored against bubbles of exuberant overconfidence in the wake of a miraculous Madoff — bubbles which were almost certainly fed by the man's reputation, and so far record. The Madoff case is clear evidence of a bias which overtook many investors in whose minds performance overshadows risks. Against this, no measure can possibly withstand alone. Yet again, the explanation is not as simple as investors looking the other way. A non negligible number of Madoff's investors were gullible only because they had limited understanding and experience of complex trading strategies. This is as certain as the fact that a significant number of sophisticated investors did not buy into Madoff's marketing material. To be fair, the case calls for quantitative analysis

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[12]Indeed, it was [34] as is now widely known, and some in the industry refused to deal with Madoff.

in the sense of tentative replications and "feasibility" studies. This type of exercise is similar in nature to the practice of replicating hedge fund strategies that is found in some hedge fund "clones". As a matter of fact, as demonstrated in Section 2 and Appendix A, only a carefully conducted replication study of Madoff's strategy would have clearly pointed out the reasons why it is virtually impossible to produce the type of track records that Madoff's feeder funds exhibited. Without replication of the strategy, there is no definite answer. Furthermore, as Foster and colleagues demonstrated [16], it is impossible to distinguish with a 100% confidence between outright con artists and reliable managers from their track records.

What is the point, then, one may ask, of such a discussion? We think it necessary to call for a re-thinking of the due diligence process, and more specifically to stress on a call to use the tools provided by the quantitative methods — not as a partisan campaign serving the limited interests of the "quants", but because these considerations have been strangely absent from any of the regulatory or standardizing framework we have read lately. Indeed, we hope that the Madoff case will convince any professional investor still doubting of the benefits of conducting thorough quantitative due diligence, as a risk management tool for one, but also as a marketing product. Indeed, one may wonder: "What did the managers of Madoff's feeder funds actually showed to the due diligence investigators visiting them about their investment process?" Certainly, it was not a replication study of Madoff's performance.

With this last point, we touch on the central elements of the value proposition of FoHF managers, and the constituents of most justifications of their additional services. Indeed, a sound process of funds selection is two-pronged: thorough due diligence and constant monitoring of risks. If we insist above on the importance of quantitative assessment for due diligence, one must not forget that monitoring risks, as well as to some extent portfolios' construction, is hardly realized without the tools of a quantitative approach. With regard to the Madoff's case, and for the reasons just cited, one may wonder how any fund of hedge funds invested in the scheme could monitor and manage their risks but by means of quantitative tools. We hasten to mention that Madoff's operation had been blacklisted by many asset managers, and many respectable funds of hedge funds managers, in all likelihood because it could not sustain the scrutiny of quantitative analysis. Again, our purpose is not to point at the failings of a few but, on the contrary, to hint at the strong advantages of the quantitative methods, despite their esoteric quality. Our only conclusion is therefore an advice to investors to stress on more quantitative measures and processes.

Unfortunately, there is little hope that at first the introduction of more quantitative analysis will provide information understandable and useful to every investors, particularly retail investors (a key point raised during consultations with the industry [40, p. 26]). But, no positive change will come without providing better and more accurate information, while protecting the industry's business — a point already made by Lo when he called almost a decade ago for the development of better quantitative assessment tools [31]. Furthermore, as emerged during the consultation process with professionals [40, p. 27], concerning the selection of funds for FoHFs' portfolios, there is still a debate between a principles-based approach or strict eligibility criteria. Perhaps, the tools of quantitative assessment can provide an escape route by enabling the compromise between principles-based approach and strict eligibility criteria. For example, one could think of strict quantitative eligibility criteria of which violation would require managers to explain in details the reasons regularly.

4.2 The future of hedge funds

Under the economic duress of 2008, the hedge fund industry as a whole faced its worst year ever with a yearly loss of 18.30%, while funds of hedge funds alone did a little worst with 19.97%, according to Hedge Fund Research's HFRI indices. The liquidity crisis also forced investors to withdraw their investments forcing the industry to redeem enormous amounts of money with total estimates as high as 50% of the Assets Under Management (AUM) of the entire industry [15]. Other more conservative estimates put the total amount of redemptions around US$382 billion for the entire industry for the year 2008 [45]. All in all, it is an industry significantly smaller than a year earlier (in terms of AUM) — 39% smaller according to [45] — that is now facing the year 2009. Conjointly, there is significant talk and pressure on both sides of the Atlantic Ocean to enforce more regulation on hedge funds[13]. It is difficult to judge with precision where the future of the hedge fund industry lays exactly. The Madoff affair is only one of many factors threatening to ignite in a conflagration which may or may not destroy the industry but will certainly change significantly its landscape. In this section, we review what we think are the most significant of them, including calls for better transparency and regulation of the industry.

Impact of broad financial and economic changes

One cannot imagine the future of hedge funds without considering them in the broader economic context. The one objective which regulators will likely pursue in the wake of the financial crisis is to provide better regulations to preserve future economic and financial stability. There are two possible dimensions in the general regulatory framework whose changes might affect hedge funds. The first pertains to the relationship that hedge funds keep with financial institutions as borrowers, makers of securities transactions and counterparts in derivative trades. New regulation on the risk capital requirements for these institutions will likely have impacts on hedge funds that are not easy to foresee. Second, and connected with the previous dimension, new regulation on the trading of securities, beyond the current limitations on insider trading, compliance with anti-takeover laws, and disclosure requirements for large stakes in thinly traded securities, could seriously change the landscape in which hedge funds operate. Following the debacle of the subprime crisis and the current financial fiasco, there will likely be a push toward stricter regulation and control mechanisms for specific investment products like credit default swaps, collateralized-debt obligations and other derivatives. Similarly, trading restrictions, concerning for example short selling, will probably undergo once again close scrutiny. Part of the incertitude associated with the future of hedge funds is therefore "tightly coupled" to their situation in the global financial system as advanced sophisticated investment vehicles [32]. As the system changes, so will the hedge funds adapt. Yet, many outcomes remain unclear.

Any debate on the subject is complicated still by the absence of any in-depth study of the social costs and benefits to hedge fund trading. There are nonetheless dimensions specific to the hedge fund industry which we could examine. Ever since the collapse of LTCM in 1998, there have been discussions as to whether hedge fund activity increases the systemic risk (cf. [14], [28] for more detailed discussions of the LTCM collapse; and [43, pp. 24-26] and [32] for a review of the systemic risk dimensions of the hedge fund industry). While there has been building evidence of a possible increase of systemic risk due to hedge funds' activities [11, 12, 29], although often

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[13]Cf. e.g., http://online.wsj.com/article/SB122627663064812111.html?mod=todays_us_money_and_investing in the U.S., and http://www.manifesto2009.pes.org/en/debate/post/801 for an example of voices calling for more regulation in Europe.

indirect or incidental, the complexity of the issues at hand would undermine any attempt to discuss them thoroughly in the forum provided by this paper. Still, by appraising the hedge fund industry by the function it performs rather than the form it takes, there are several broad predictions which one can make to the possible directions the industry will take.

Transparency and monitorship

As a function, enabled by the increasing use of complex financial instruments over the past two decades, the hedge fund industry has provided an ever larger pool of investors with access to a set of dynamic nonlinear trading strategies. One can see in the ever increasing amount of AUM in the past two decades a clear sign that the hedge fund industry offers to its "customers" a service not provided by other more regulated investment vehicles, like mutual funds or CTAs (e.g., [31], [37]). Insofar as this raison d'être remains, the hedge fund industry will not likely disappear. It is with this in mind that the idea of increasing the transparency of the industry and heightening the regulation requirements must be examined.

With respect to the idea of mandatory registration of hedge fund managers, Brown and colleagues, taking advantage of the recent failed attempt[14] by the American SEC to enforce registration of hedge funds [9], investigated this issue in the broader context of general investors and more specifically in relation to the operational risk. Their findings can be summed up in three points. First, since operational risk indicators are conditionally correlated to conflict of interests, it suggests that greater disclosure of those conflicts of interests is of value to investors. Second, as operational risk indicators are correlated to lower leverage and concentrated ownership, it seems that a disclosure requirement would be redundant to the lenders and equity investors in hedge funds. Finally, and conversely to the first point, it seems that the existence of operational risk factors have no ex-post impact on the flow-performance relationship[15]. It means that investors either lack the information or do not regard it as material. While the benefits of HF registration are aligned with protecting the investors' interests, it falls short of providing a safe guard against dangerous behaviors and their incidental promotion by specious performance profiles — Madoff being a case in point. In other words, registration is only the first step to prevent scandals akin to Madoff's as it provides a firm handle on who needs monitoring, yet this approach must be complemented by other mechanisms in order to protect investors.

If the idea of monitorship is certainly attractive, it would require strong and resourceful regulators in every country where hedge funds are active. Monitoring hedge funds individually would require that the regulators devote an amount of resources comparable to those of the industry itself to be efficient. This is hardly sustainable, at least under the form of a systematic monitoring undertaken by a single agency. Instead, one should try to use market forces both as a dissemination medium and an enforcement engine behind better transparency standards. Some authors have argued for the creation of an entity comparable to the U.S. National Transportation Safety Board for the financial system (cf. [21], [12], [11] and [32]) as a first step toward better information dissemination among investors. This institution would be in charge of investigating,

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[14]In 2003 already, the SEC attempted to enforce the registration of a majority of hedge fund managers by reinterpreting the definition of 'client' to an investment adviser. The new rule and rule amendments under the Investment Advisers Act of 1940 would have required HF managers to register as investment adviser by February 1, 2006 [9]. This attempt was later dismissed by the U.S Court of Appeals for the District of Columbia on June 23, 2006.

[15]The flow-performance relationship is the name given to the nonlinear and positive relationship that exists between performance of a fund and the amounts invested.

reporting and archiving the financial crashes. Lo [32] also called for adaptive regulations taking as an example the case of OTC contracts with the establishment of organized markets and clearing corporations whenever those markets exceeds some predefined thresholds. While these measures would bring in more transparency to the hedge fund industry by providing a clearer picture of both the mechanisms that lead to financial disasters — note that the institution of a "Capital Market Safety Board" seems well indicated to handle the Madoff scandal — or the murky market of OTC contracts, we believe that there already exist trends in or tangent to the industry that should be encouraged as they provide a clearer risk picture.

Opposition to increased transparency requirements for hedge funds mainly stems from the "trade secret" argument. That is, more transparency is a threat to the competitive edge of the hedge fund manager. While it may be hard to dismiss the argument, its value is certainly diminished in view of the cries for simpler and clearer finance brought by the current general economic and financial disaster from the general public. For this very reason, more regulation is likely to be inevitable, and perhaps not unbecoming to the situation. In line with what we argued in the sections above on thorough due diligence, increased transparency could probably be achieved by imposing a standard of reporting to hedge funds, comprising of a mandatory registration with disclosure of any conflict of interests, and also containing for example, to be meaningful in any way, enough information for the quantitative replication of the hedge fund's strategy. The format of such a change would most probably lead to a partial exposition of the industry's positions and tactical bets.

There are several general remarks one could make about its implications. First as noted in [26, p. 60], regulation brings normalization, and thus enforce the predictable features of any organization. One possibility such scenario could lead to is an increased systemic instability in period of financial distress where part of the market bets against the known position of one of its actors[16]. Second, as some in the industry noted [40, p. 30], the nature of the hedge fund investment is to delegate to the skills of the manager rather than constantly monitor, and it is a valid question to ask whether, in general, investors would be pleased by such an outcome.

"Retailization" of the industry and standardization pressures

At this point, notwithstanding the variety of hedge fund investors, it may be necessary to classify these into two groups whose characteristics can help derive further implications for the future of hedge funds. On the one hand, fiduciary investors, acting in the stead of their clients, might display greater need for transparency as they could be held responsible for the poor or fraudulent performances of their investments. On the other, non-fiduciary investors acting in their own name and to some extent willing to give out more freedom to hedge fund managers in terms of operational and financial reporting, as long as performances are consistent and reward the risks.

It is important to recognize among hedge fund investors the growing importance during the last decade of institutional investors seeking alternative to the traditional investments offered to them. While some investors might be looking for absolute performance, a growing trend is to invest in hedge fund "clones." Considering the poor performance of the replication process, these

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[16]Those situations are known to happen. For example one could cite the one we have seen for banks in the first half of 2008 which were forced to de-lever when the market knew about it, a situation which already happened to some HFs in Q3 1998 in the wake of LTCM.

investors are often looking for a risk-profile similar to hedge funds in a perspective of portfolio optimization.

The "retailization" of hedge funds [11] ensuing from the bigger role of institutional players thus increases the pressure for transparency. As such, the progress of hedge fund replication [41] may lead to *de facto* "standardization" of the main hedge fund strategies or categories. This trend toward index-like products [20] is likely to be supported by its more transparent allocation methodology and its simpler and cheaper fee structure. Similarly, the advent and the likely increased popularity of managed account platforms of hedge funds for institutional investors may force a significant part of fund of hedge funds toward more standardized offers. The main advantage of those platforms is their transparency and the increased liquidity they offer to their clients. If these platforms have noticeably suffered since October 2008, it is likely because of the very advantages they offer. Investors looking for cash have taken advantage of the platforms' format to get it quickly rather than through the more tedious process of withdrawing from traditional hedge funds structure. Once the crisis has passed, the advantages they offer in terms of risk management will certainly make them very attractive to institutional investors and private bankers alike.

In a way, these new "standards" would fill the regulatory gap which exists between hedge funds and their more regulated competitors, as Oesterle argued for in the past [37]. One main advantage of standardized products is that, hopefully, with standardization progressively comes the knowledge and understanding of their advantages and pitfalls. The overall impact of this trend would be to somewhat deflate the bubble of investors who came to hedge fund investments in search of "alternatives" to the "tedious" trend-following behavior of classic investments. The remnants of investors of the non standard highly risky HF strategies would be constituted of hard-core non fiduciary investors, ideally knowledgeable about the risks of the non standard strategies, but with full ownership of their investment decisions anyhow.

Noticeably, this possible "standardization" of a significant part of the hedge fund industry is not incompatible with the adaptive regulation perspective put forth by Lo [32]. Indeed, one could imagine that the more standardized products are in time cleared and sold in organized markets. Similarly, as advanced dynamic strategies at the edge of hedge funds become increasingly popular, the pressure for standardization rises. The question remaining of course is will we set the thresholds for regulatory regulations low enough to control the risks and will that be enough to tame the systemic risk created by the existence of players "too big to fail".

Finally, besides the standardizing pressures on some strategies that have become "classic" in the alternative investment world the future of part of the industry is coupled more tightly to the debate on regulation and begs for further reflection. One can further separate this remaining part of the hedge fund industry into two groups: a group of strategies (e.g., 130/30, Long/Short Equity, etc.) for which some regulated formats exist already, at least in Europe, and other black-box strategies more akin to private equity than alternative investment. For the latter, very little can actually be said on regulation except restricting their access to well-informed and wealthy investors, and possibly also the number of their investors. Conversely, the debate on the former group is complicated by preexisting national regulations, their disparity around the globe and the existence, especially in Europe, of procedures of passporting between national regulations. We review those issues in more details in the next section.

4.3   Deregulation, supervision and coordination

Even though some aspects of the future of the hedge fund industry the current crisis and the Madoff scandal will engender remain undecided, one thing is nonetheless certain. The current circumstances have spurred the debate on the need for regulations, and the Madoff affair is a case in point. In this section, we examine the circumstances surrounding the debate, particularly the dimensions of deregulation, supervision and coordination as we believe the answer cannot simply be more regulation. If one could argue that the lack of regulations on hedge funds' disclosure requirement have made it easier for the scheme to develop and prosper for so many years, the problem is in fact almost certainly more complex. First, as we have alluded to above, all the consequences of more regulation for hedge funds are not necessarily known or even desirable. Second, and more important, we have to recognize that regulations already exist, and perhaps even more noticeable, that the field is not leveled on this particular aspect [38].

Deregulation

With the Madoff case though, proponents of deregulation could make the argument that even some of the strictest regulations were not capable of impeding its development. Indeed, one of the striking aspects of the organization of Madoff's scheme is the case of the LuxAlpha fund — one of Madoff's feeder funds — which was created under Luxembourgian regulation regime. As such, and contrary to the other feeders of Madoff's fund, LuxAlpha is subject to strict regulations in terms of disclosure, control and risk management. In Luxembourg, contrary to the Cayman or the British Virgin Islands, the agreement procedures to set up a fund and market it to investors are far more stringent. In other words, since stringent regulations have failed, in the same place where other less requiring regulators have failed too, one could ask what is in fact the benefit of regulation.

We believe however that deregulation is not a direction one should consider. It would in all likelihood induce in the end too high of an economic and financial cost. There is little doubt that without the existence of regulation many more frauds similar to the Madoff scheme would exist and perhaps proliferate. Even though situations are not comparable, one can nonetheless take lessons from the situation of Albania at the beginning of the 1990s where lax regulatory conditions and weak enforcement agencies allowed the development of a state-wide swindling scheme that brought the country's economy to its knees. Of course, no regulation is perfect; and regulations can only help inasmuch as they are enforced. Nevertheless, the occasional failure of the enforcing agencies should not be seen as proof that regulations are worthless. That is not to say that current regulations could not benefit from improvement. Certainly, Madoff, as well as new uncovered scandals, are evidence enough that things can be improved.

Madoff's scheme illustrates in particular the problem of national regulations "arbitrage", fund passporting and the lack of coordination and supervision which exist between markets[17]. In the following, we focus specifically on these issues, and, assuming that the current circumstances will lead to a major redesign of the regulatory framework, propose tentative solutions.

Supervision

Before we pursue, we need to acknowledge that some national regulations already allow some regulated investment vehicles to engage in strategies long considered to be uniquely reserved for

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[17]Note that the recent Stanford scandal is another example of this arbitrage between regulations.

hedge funds. This is the case in Europe for example. We certainly support the adaptation of regulations to the changing landscape of the industry. This form of "convergence" between asset management and hedge fund industries is desirable in the sense that many alternative investment strategies could certainly benefit to the clients of the more classic asset management industry without necessarily inducing more risk. Nonetheless, if, as we have argued above, there exist pressures toward a standardization of part of the offer of the hedge fund industry, we have also pointed that these pressures are not universal. And, unfortunately, by allowing funds practicing very different strategies, mutual funds and absolute returns hedge funds, to coexist under the same regulatory format and share risk requirements, it is possible that it increases the risk of confusion for investors, and thus diminishes the usefulness of the regulatory framework.

Our point here is not to incriminate existing regulatory dispositions, but rather to point that any tentative to regulate the hedge fund industry should not blur the distinction between classic asset management and nonlinear dynamic (hedge fund) strategies. Thus, we contend here that any tentative at supervising the hedge fund industry should accommodate for these standardization pressures as well as the need for better investor protection by offering at least two different registered and regulated formats to the classic and the alternative asset management industries. Contrary to Oesterle [37], we believe that the current regulations of mutual funds should remain intact (including any recent adjustment that allowed more flexibility to the managers in their strategy). As an example, in Europe, we understand the UCITS III format as a well suited overall for this purpose. On the other hand, regulators should offer to hedge funds a regulated format sharing the requirements of mutual funds in terms of risk management, deontology, registration, reporting, valuation, or disclosure of strategies, but allowing for more maneuverability in terms of leverage, liquidity or invested assets classes. The rationale is two-fold. First, if standardization of part of the industry is probably an unstoppable trend at this point, it could nevertheless benefit from, and perhaps be accelerated by, a frame provided by regulators. Second, however, the latter should keep in mind that the purpose of a regulated HF format should not be to capture the standardization trend — which by essence needs few harmonizing constraints — but to attract some non standard strategies in a regulated format amenable to the protection of all investors, particularly retail investors.

Coordination

The idea of regulations "arbitrage" is to create a fund in a place where the regulations are relatively lax, and then use existing agreements on the marketing of foreign funds in the more strictly regulated national markets to attract and commercialize the funds to investors. Note that the problem is heightened in Europe by the existence of UCITS III. We want to stress on the importance of harmonizing the formats between regulations, and thus the importance of coordination between regulators. We hasten to point out that efforts in this direction exist already, in particular for funds of hedge funds [39] (cf. also http://www.hedgefundmatrix.com/). Furthermore, beside the need to coordinate the regulatory formats, coordination should also be required in order to set a clear delineation between regulated and unregulated zones. One major problem to a fully effective implementation of any regulatory framework is that even if the investment vehicles are completely regulated because of the existence of fiscal paradises the funds' portfolios may themselves be outside the scope of the regulators. This issue is particularly relevant for funds of hedge funds. Coordination is therefore key to the promotion of the industry's best practices, comprising, but not limited to, rigorous due diligence and risk management processes as we have advocated above.

The promotion of best practices is in our view perhaps the only way to preserve the purpose of any new regulation, and even perhaps better in some subject areas than regulation itself. Indeed, if one sees the purpose of regulation as protecting investors and the general public from any deviant behaviors, one must also acknowledge that regulations need to be adaptable [32], particularly in the face of the rapid changes of international finance. That is to say it is perhaps possible to regulate every single aspect of the financial investment landscape as we know it today. Yet, the very structural phenomenon which lead to the rise of the hedge fund industry would only suffer a set back rather than be understood and directed as a force toward a better use of financial innovations.

Finally, let's summarize the points we made in this section on deregulation, supervision and coordination. First of all, one needs to maintain a clear delineation of the differences between classic asset management and hedge funds, even though a significant part of the hedge fund industry will likely progress toward more standardized and index-like product offers. Second, pursuing the creation of two different registration formats should on the one hand provide a regulatory framework to offer to institutional and other fiduciary investors the formats they need to preserve their duties while benefiting from some complex strategies; on the other hand, diminish the attraction of black-boxed strategies by providing transparently to all investors the access to nonlinear payoffs. Wealthy and knowledgeable individual investors should still have the opportunity however to invest in hedge funds with highly risky strategies — outside of the regulated formats proposed above — as long as the total size of these strategies do not induce an increased systemic risk. Coordination is also needed among regulators to ensure that this delineation between regulated formats remains consistent, as well as to promote the industry's best practices in terms of due diligence and risk management.

5   Conclusion

The Madoff fraud is exceptional and will presumably have great impacts on the hedge funds industry's regulation and its risk management processes. A fraud of this extent is so rare that to prevent other scandals, it is necessary to understand it scientifically. This was the objective of this paper. Furthermore, the hedge fund industry has profoundly changed since its first major crisis in 1969, or even the 1998 failure of LTCM. Nowadays, democratization of these investment vehicles has spread. This phenomenon cannot be ignored. Moreover, beecause of the hedge fund industry's tight relationship with the economy, and because frauds akin to Madoff's could in a worst case scenario take down key players, new responses commensurate with the issues at stake must be sought.

One remarkable trait of the Madoff story is its alleged longevity and more surprisingly the fact that, without the subprime crisis and its consecutive liquidity default, the fraud would probably still be ongoing. We tried to understand how Madoff could have supported and sustained his reputation and how he could have survived in a well regulated environment. We therefore appealed for improvement of the rules of operational risk management to take into account in their implementation the peculiarities of asset management and private banking. We also showed that with quantitative tools, it was possible to detect Madoff's fraud. These tools — essentially replication tools — could improve due diligence processes.

New responses should also help in the fight against the lack of transparency, which, as we illustrated in this paper, was one major factor in allowing the development of the Madoff scheme.

We advocated for the dissemination of best practices in order to increase the chances of survival of this industry — which despite its shortcomings is essential to the asset management universe. Another solution we considered would be to increase regulation; but to do so without taking into account the industry's idiosyncrasies would be inappropriate and likely too expensive. Good standards exist already — for example, in Europe, UCITS III is a well suited registration format which probably needs to be completed by other similar formats specific to hedge funds. Therefore, to avoid other Madoff-type frauds, we proposed to develop current legislation along two avenues. First, we promoted the standardization of the hedge fund industry. Had Madoff's strategy been compared with well known standards, the scheme could possibly have been uncovered earlier. Second, we proposed to create a new registration format to clarify the industry without compromising its specificity. Our approach is motivated by the constatation that unlike previous scandals — e.g. LTCM which was essentially based on excessive leverages — the Madoff fraud (and possibly others of the same kind) took advantage of the relative opaqueness which reigns in the hedge fund industry to attract investors. Hence, regulation should primarily be aimed at avoiding investors' confusion and providing protection against fraudulent behavior. The new format we put forth, which would be accessible to a majority of hedge funds — but not to black-boxes — would bear the same rules of risk management, reporting, disclosure, etc. This, we hope, should avoid frauds and contagion, and restore the hedge fund industry's image.

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A The bull-spread or 'split strike conversion' strategy

A.1 Description of the strategy

Let S_t be the price of a risky asset. We denote by $C(K_C)$ the price of a call option written on this risky asset with strike K_C. $P(K_P)$ is the price of the put option with strike K_P. The bull-spread strategy is composed of three positions

1. a long position on the risky asset S_t;

2. a short position on the call option $C(K_C)$;

3. and a long position on the put option $P(K_P)$ with $K_P < K_C$.

We consider a period with two dates 0 and T. The payoff function of this strategy is then

$$
\begin{aligned}
\text{PnL} \quad = \quad & (S_T - S_0) + \\
& C(K_C) - \max(0, S_T - K_C) + \\
& \max(0, K_P - S_T) - P(K_P)
\end{aligned}
$$

We obtain 3 cases

$$
\text{PnL} = \begin{cases}
(K_P - S_0) + C(K_C) - P(K_P) & \text{if} \quad S_T \leq K_P \\
(S_T - S_0) + C(K_C) - P(K_P) & \text{if} \quad K_P < S_T < K_C \\
(K_C - S_0) + C(K_C) - P(K_P) & \text{if} \quad S_T \geq K_C
\end{cases}
$$

We represented the payoff function[18] in Figure 10. Because the payoff functions of the bull spread strategy and the long only strategy are different, their distribution functions may not be compared (see Figure 11).

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[18]We assumed that $S_0 = 100$. The strikes of the options are respectively $K_C = 103$ and $K_P = 93$. The maturity T of the options is equal to one year. We considered that the implied volatility Σ is 30%. And we assumed no dividends and an interest rate equal to 0%.

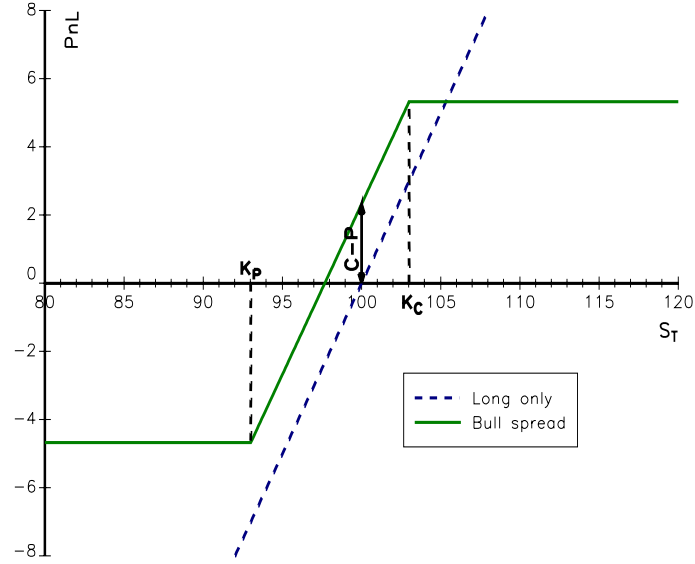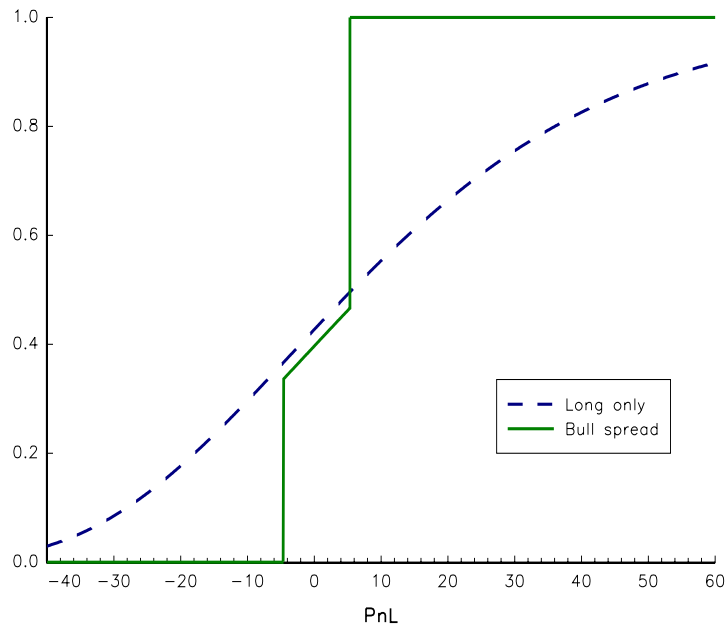Figure 10: Payoff functions of the long only strategy and the bull spread strategy

Figure 11: Cumulative distribution function of the PnL for the long only and bull spread strategies

A special case of the bull spread is when one completely finances the buying of the put option by selling the call. It means that $C(K_C) = P(K_P)$. Given a strike K_C for the call, you may also find the strike K_P which verifies $C(K_C) = P(K_p)$, and vice versa. This is done by solving a non-linear equation. Let $C_{\mathrm{BS}}(K, \Sigma, T)$ and $P_{\mathrm{BS}}(K, \Sigma, T)$ be respectively the call and put prices. We define $K_P = \kappa(K_C)$ as follows

$$K_P = \{K : P_{\mathrm{BS}}(K, \Sigma, T) = C_{\mathrm{BS}}(K_C, \Sigma, T)\}$$

We represented the function $K_P = \kappa(K_C)$ for the previous set of parameters (but with a one month maturity) in Figure 12. Notice that when taking into account the smile, the profiles of the strikes are less favorable to the bull-spread strategy. Indeed, the implied strike for the put with smile is below the one computed with no smile.

Figure 12: Relationship $K_P = \kappa(K_C)$
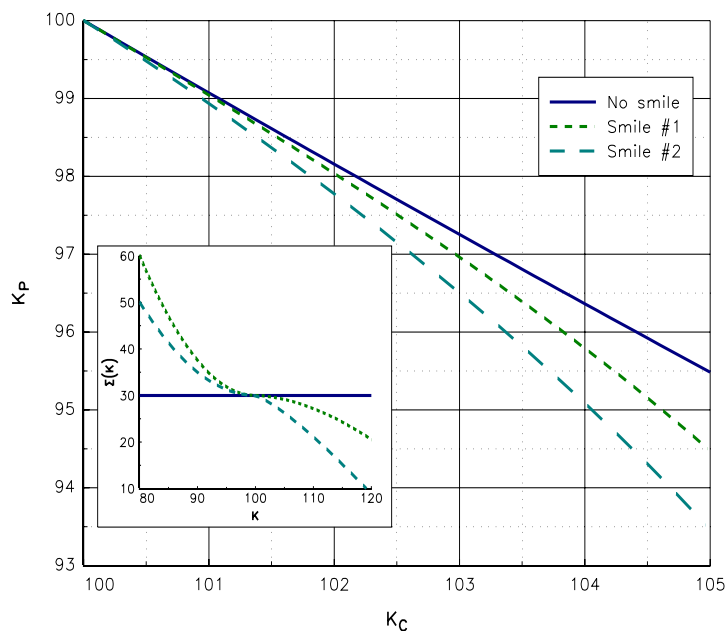
A.2 Rationale of the strategy

There are two main arguments (put forth by Madoff) for investing in a bull-spread strategy.

1. The first one is that a bull spread strategy will have a larger Sharpe ratio than the long only strategy, because, even if it produces smaller performance, the volatility will be considerably lower than the volatility of the long only strategy;

2. The second argument is that we may benefit from the alpha produced by the stock picking process by writing the options on the index but investing in a basket of stocks instead of the entire index.

Let us consider the first argument. Assuming that the price of the index follows a geometric brownian motion,

$$\mathrm{d}S_t = \mu S_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t \qquad\qquad (\text{A-1})$$

with the previous parameters, $\mu = 10\%$ and $\sigma = \Sigma = 30\%$, we obtain the sharpe ratios[19] for several sets of parameters K_C and K_P in Table 4. The sharpe ratio of the long only strategy is obtained when $K_C \to \infty$ and $K_P \to 0$. It is equal to $\left(\mu - \frac{1}{2}\sigma^2 - r_0\right)\big/\sigma$. Notice that in this example the bull-spread strategy produces higher Sharpe ratios. However, in real life, the underlying price is not log-normal. In particular, one can observe a skew in the probability distribution function of the returns. In the case of the equity market, it is known that positive returns are certainly more frequent than negative returns. However, the magnitude of negative returns is generally higher than the one of positive returns. Suppose that the one-year return $r_S = \ln S_1 - \ln S_0$ is given by the following model

$$r_S = \begin{cases} \mu_+ + \sigma_+\varepsilon & \text{with probability } p \\ \mu_- + \sigma_-\varepsilon & \text{with probability } q = 1-p \end{cases} \qquad\qquad (\text{A-2})$$

with $\varepsilon \sim \mathcal{N}(0,1)$. Computations give us

$$\begin{aligned} \mathbb{E}[r] &= p\mu_+ + (1-p)\mu_- \\ \sigma^2[r] &= \left(p\left(\mu_+^2 + \sigma_+^2\right) + (1-p)\left(\mu_-^2 + \sigma_-^2\right)\right) - \left(p\mu_+ + (1-p)\mu_-\right)^2 \end{aligned}$$

Stylized facts impose that $p \gg q$ and $|\mu_+| \ll |\mu_-|$. Knowing the parameters μ and σ of the log-normal model (A-1), it is easy to calibrate the five parameters p, μ_+, μ_-, σ_+ and σ_- of the skew model (A-2) by matching the first two moments. For example, with $\mu = 10\%$ and $\sigma = 30\%$, one solution is: $p = \frac{2}{3}$, $\mu_+ = 21.2\%$, $\mu_- = -25.9\%$, and $\sigma_+ = \sigma_- = 20.2\%$. With these parameters, we may compute the Sharpe ratio of the bull-spread strategy. Results are reported in Table 4. We verify that we obtain the same Sharpe ratio than in the case of the log-normal model since we have the same first two moments. As in the case of log-normal model, the Sharpe ratio increases when the call and put strikes tends to the initial spot price. But the most important thing is that Sharpe ratios are bigger in the skew model than in the log-normal model.

Table 4: Sharpe ratio of the bull spread strategy

K_C	K_P	Model (A-1)	Model (A-2)	Model (A-3) case (1)	case (2)	case (3)
101	99	0.265	0.328	2.319	0.594	0.290
102	98	0.263	0.325	1.795	0.660	0.310
103	97	0.260	0.322	1.481	0.729	0.330
104	96	0.258	0.319	1.276	0.801	0.351
105	95	0.255	0.316	1.132	0.872	0.371
107	93	0.251	0.310	0.947	1.005	0.411
110	90	0.244	0.300	0.792	1.136	0.470
∞	0	0.183	0.183	0.350	0.650	0.650

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[19]Computed using logarithmic returns.

Remark 1 *We must be careful with the previous argument because the skew property of the returns is also taken into account by the smile effect and the cost of put options will therefore be higher.*

The second argument is to write the option position on the index S_t but to build the linear position with a basket of stocks B_t. We may for example suppose that one has a stock picking process which creates alpha, and one wants to use it to improve his or her performance. In this case, the PnL of the strategy becomes

$$
\begin{aligned}
\text{PnL} \quad = \quad & (B_T - B_0) + \\
& C(K_C) - \max(0, S_T - K_C) + \\
& \max(0, K_P - S_T) - P(K_P)
\end{aligned}
$$

Let us assume that

$$
\begin{cases}
\mathrm{d}S_t = \mu_S S_t \, \mathrm{d}t + \sigma_S S_t \, \mathrm{d}W_t^S \\
\mathrm{d}B_t = \mu_B B_t \, \mathrm{d}t + \sigma_B B_t \, \mathrm{d}W_t^B
\end{cases}
\tag{A-3}
$$

with $\langle W_t^S, W_t^B \rangle = \rho \, \mathrm{d}t$. We may distinguish between different cases

1. if $\rho = 1$, $\sigma_B = \sigma_S$ and $\mu_B > \mu_S$, the stock picking process creates alpha every time because we have $\forall t_2 > t_1$:
$$
\ln B_{t_2} - \ln B_{t_1} > \ln S_{t_2} - \ln S_{t_1}
$$

2. if $\rho = 1$, $\sigma_B < \sigma_S$ and $\mu_B > \mu_S$, the stock picking process creates alpha on average with same observed volatility

$$
\begin{aligned}
\mathbb{E}\left[\ln B_{t_2} - \ln B_{t_1}\right] \quad &> \quad \mathbb{E}\left[\ln S_{t_2} - \ln S_{t_1}\right] \\
\mathrm{var}\left[\ln B_{t_2} - \ln B_{t_1}\right] \quad &= \quad \mathrm{var}\left[\ln B_{t_2} - \ln B_{t_1}\right]
\end{aligned}
$$

3. if $\rho < 1$, $\sigma_B < \sigma_S$ and $\mu_B > \mu_S$, the stock picking process creates alpha on average;

4. if $\sigma_B > \sigma_S$ and $\mu_B < \mu_S$, the stock picking process does not create alpha on average.

Consider our previous example and let's illustrate the first three cases using the following values for the parameters: $\mu_B = 15\%$, $\sigma_B = 30\%$ and $\rho = 1$ for the case (1), $\mu_B = 15\%$, $\sigma_B = 20\%$ and $\rho = 1$ for the case (2), and $\mu_B = 15\%$, $\sigma_B = 20\%$ and $\rho = 0.85$ for the case (3). We reported the results in Table 4. For case (1), we obtain very high Sharpe ratios especially when the two strikes are very close. In the second case, we obtain strange results. At first, the Sharpe ratio increases as we move away from the ATM strikes and then decreases. In the third case, the Sharpe ratio is smaller than the long only strategy. These last two cases are very interesting because the bull-spread strategy faces correlation risk and volatility risk. Notice also that the PnL of the bull-spread strategy is now not bounded and the volatility of the PnL is higher. In case (2), we have a larger dispersion of the PnL which explains that the Sharpe ratio is smaller than case (1). In case (3), the basket may go up whereas the index may go down (and vice versa). This correlation risk has a negative impact on the Sharpe ratio.

To conclude, the two arguments used to explain the appealing performance for the bull-spread strategy provide in fact mitigated results. First, the fact that asset returns are more frequently positive than negative is good for the strategy, but it also implies a higher cost of the put option.

Second, even if you are a good stock picker, you may face correlation risk which can be dramatic for the performance of the strategy. However, if you are able to eliminate this correlation risk, you may effectively create impressive Sharpe ratios. But, this will mean that you are always right (in your stock selection) and a simple strategy with a long position on your stock selection and a short position on the index is enough to create similar results.

B   Modeling Ponzi schemes in investment management

Ponzi schemes have been studied in economics principally in the case of sovereign debt [7, 24] or rational expectations [44, 22, 42]. However, to our knowledge, they have not been formalized in the case of investment management. In this section, we propose a simplified model to understand the mechanism of Ponzi schemes in investment management and to model the default time of such schemes.

Let F_t and K_t be respectively the value of the investment fund (assets under management) and the value of the capital at time t. We assume that the return of the capital is r_t whereas the gross return of the fund is denoted by μ_t. We suppose that we have at time t subscription and redemption amounts given by K_t^+ and K_t^-. The dynamics of F_t and K_t are then

$$\begin{cases} \mathrm{d}K_t = r_t K_t \, \mathrm{d}t + K_t^+ - K_t^- \\ \mathrm{d}F_t = \mu_t F_t \, \mathrm{d}t + K_t^+ - K_t^- \end{cases}$$

The first equation means there are two components of the capital growth,

1. the first component is related to the return on the capital;

2. whereas the second component is composed of jumps corresponding to subscription/redemption on the fund.

The dynamic of the value of the fund is similar to the dynamic of the capital except that the return of the fund is μ_t. Of course, μ_t is bigger than r_t in order to attract investors. In a first approach, we will assume that subscriptions and redemptions are proportional to the value of the fund F_t : $K_t^+ = \lambda_t^+ F_t \, \mathrm{d}t$ and $K_t^- = \lambda_t^- F_t \, \mathrm{d}t$. λ_t^+ and λ_t^- may be viewed as instantaneous intensity rates. With these specifications, the model becomes

$$\begin{cases} \mathrm{d}K_t = r_t K_t \, \mathrm{d}t + \left(\lambda_t^+ - \lambda_t^- \right) F_t \, \mathrm{d}t \\ \mathrm{d}F_t = \mu_t F_t \, \mathrm{d}t + \left(\lambda_t^+ - \lambda_t^- \right) F_t \, \mathrm{d}t \end{cases} \tag{B-4}$$

with $K_0 = F_0$. At the launch date, the assets under management are equal to the capital. The default τ of the fund is also defined by the first hitting time: $\tau = \inf \{ t > 0 : K_t < 0 \}$. Let $X = (K_t, F_t)$. We have

$$\begin{cases} \mathrm{d}X_t = A_t X_t \, \mathrm{d}t \\ X_0 = K_0 \mathbf{1} \end{cases}$$

with

$$A_t = \begin{pmatrix} r_t & \lambda_t^+ - \lambda_t^- \\ 0 & \mu_t + \lambda_t^+ - \lambda_t^- \end{pmatrix}$$

A solution is

$$X_t = K_0 \left(e^{\int_0^t A_s \, \mathrm{d}s} \right) \mathbf{1} \tag{B-5}$$

In the case where the parameters r_t, μ_t, λ_t^+ and λ_t^- are constant, the solution reduces to $X_t = K_0 \left(e^{At} \right) \mathbf{1}$. The behavior of τ is related to the eigenvalues of the A matrix. In particular, it is obvious to show that the fund does not default if the subscription rate is greater than the redemption rate: $\lambda^+ > \lambda^-$. Otherwise, the fund may collapse because the cash outflows are bigger than the capital appreciation (or cash inflows).

Let us consider an example with $r_t = 5\%$, $\mu_t = 10\%$, $\lambda_t^+ = 5\%$ and $\lambda_t^- = 0.02 \times \mathbf{1}_{\{t<5\}} + \lambda \times \mathbf{1}_{\{t \geq 5\}}$ The rates of return on capital and of the fund are respectively 5% and 10% per year. The inflows rate is 5% per year whereas we assume that the redemptions are time-varying. During the first 5 years, the outflows rate is equal to 2% by year. After 5 years, we assume that the outflows rate is constant but bigger than or equal to the inflows rate. We consider four cases : (a) $\lambda = 5\%$, (b) $\lambda = 11\%$, (c) $\lambda = 15\%$ and (d) $\lambda = 50\%$. Results are reported in Figure 13. In the first case (a), the condition $\lambda_t^+ - \lambda_t^- > 0$ is verified and the fund does not default. In the second case, the redemption rate is 11% whereas the subscription rate is 5%. The value of the fund continue to growth. Nevertheless, the fund defaults after 19 years. In the case (c), both the assets under management and the capital decrease. The default time is about 15 years. The case (d) is similar to case (c) but the redemption rate is bigger due for example to a financial crisis or a panic of investors. In this case, the default is quick with only 8 years (that is three years after the redemption rate went up to λ.

Figure 13: Simulation of the default time τ
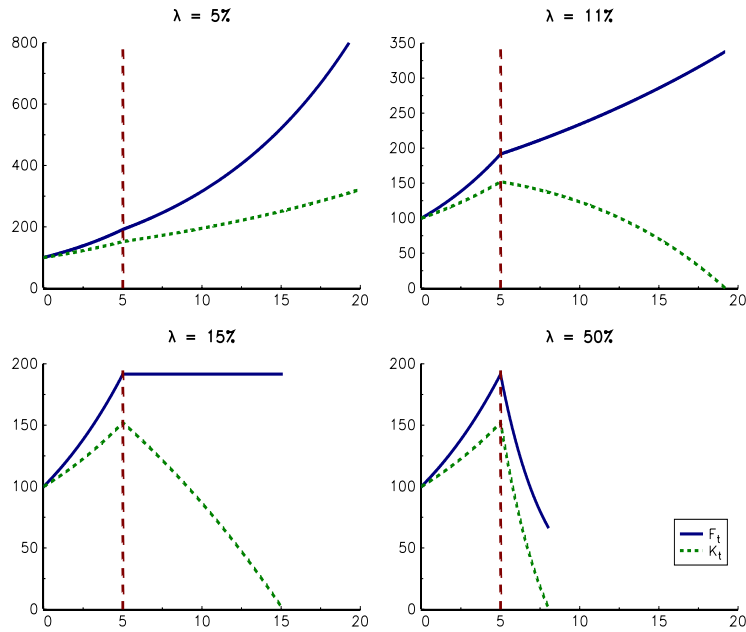
The previous model is incomplete however because it does not take into account the fees. The investors pay to the manager of the fund some fees in order to manage the fund (management fees) and to perform well (performance fees). If we assume no performance fees, the model becomes now

$$\begin{cases} \mathrm{d}K_t = r_t K_t \, \mathrm{d}t + \left(\lambda_t^+ - \lambda_t^- \right) F_t \, \mathrm{d}t - m_t F_t \, \mathrm{d}t \\ \mathrm{d}F_t = \left(\mu_t - m_t \right) F_t \, \mathrm{d}t + \left(\lambda_t^+ - \lambda_t^- \right) F_t \, \mathrm{d}t \end{cases}$$

The solution is similar to (B-5) except that the A_t matrix is now specified as follows

$$A_t = \begin{pmatrix} r_t & \lambda_t^+ - \lambda_t^- - m_t \\ 0 & \mu_t - m_t + \lambda_t^+ - \lambda_t^- \end{pmatrix}$$

We see now that even if the net flows are positive — $\lambda_t^+ - \lambda_t^- > 0$ — the fund may default because the capital is used to pay the manager. Moreover, even if the capital appreciation is greater than the management fees ($r_t > m_t$), the fund may collapse because fees are paid on the basis of the assets under management. Let us consider the previous example with $m_t = 2\%$ (see Figure 14). The fund defaults now even in the case (a). And the default time are all smaller

Figure 14: The impact of management fees
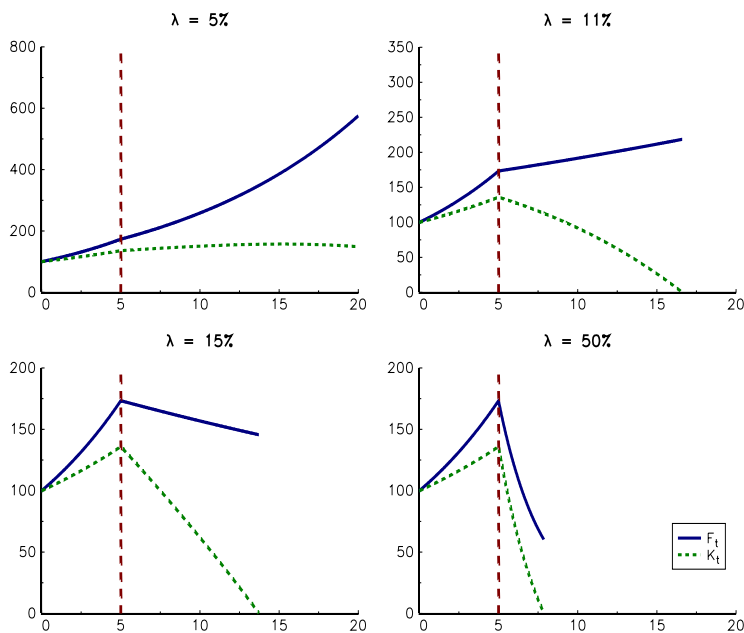
than previously. In reality, the parameters r_t, μ_t, λ_t^+ and λ_t^- are stochastic. In particular, the parameter μ_t must change in order to create noise for the investors. Let us consider the case (b) with $m_t = 2\%$ and $\mu_t \sim \mathcal{N}(\mu, \sigma)$ with $\mu = \sigma = 10\%$. We have represented the density of the default time τ in Figure 15 and reported one simulation run. We verify that assets under management are not smooth. Indeed, the Ponzi scheme may not be viable if net flows are positive because of management fees. We may avoid default only if $m_t < \lambda_t^+ - \lambda_t^-$. Otherwise, default is certain. It is obvious that the default time is a negative function of m_t and μ_t. With higher fees, you quicken the default because you use more capital to remunerate the manager. In the same way, if μ_t is large, AUM grow more quickly and more fees are generated. This fact is illustrated in Figure 16 with $r_t = 5\%$, $\lambda_t^+ = 5\%$ and $\lambda_t^- = 3\%$.

C   Data

We report in Tables 5, 6 and 7 a description of the six feeder funds which may be found in Bloomberg.
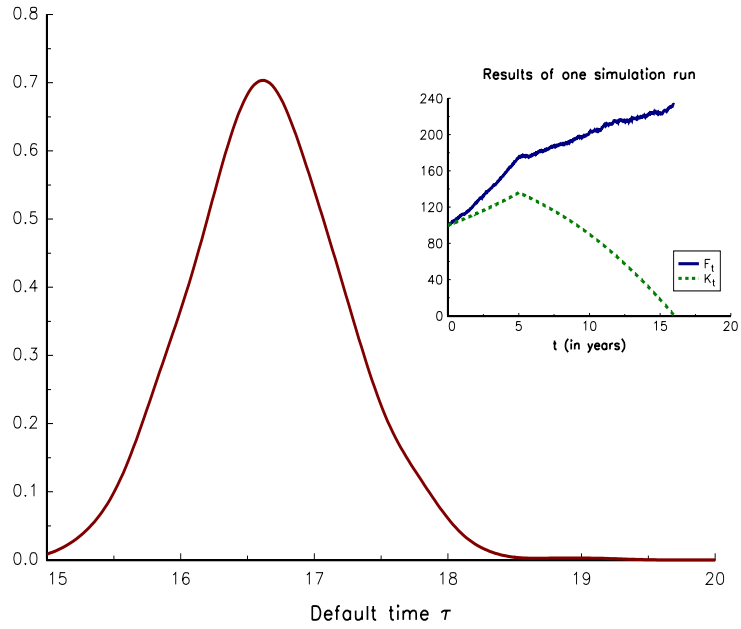
Figure 15: Density of the default time τ

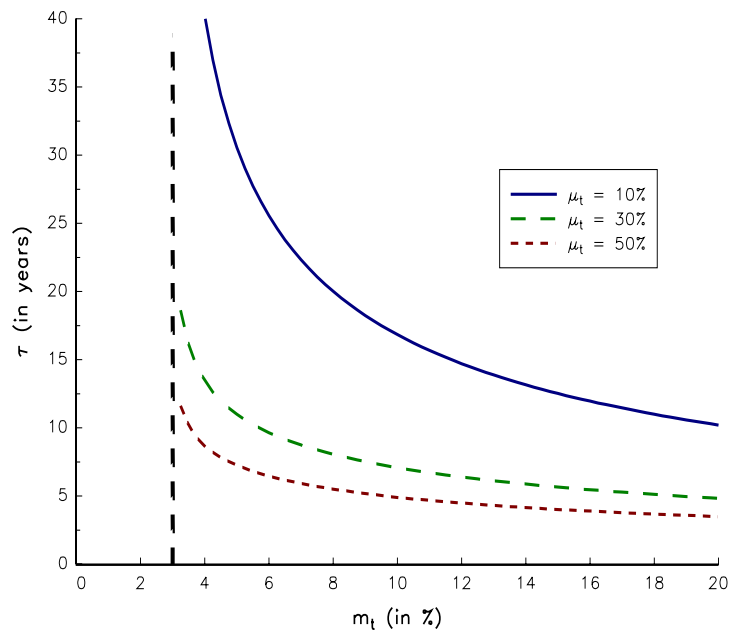Figure 16: Default time with respect to management fees

Table 5: Description of the fund's strategy (I)

Ticker	Fund name	Fund Manager	Currency	Inception date	Min. investment
FAISENI VI	Fairfield Sentry Ltd	Fairfield Greenwich Bermuda Ltd	USD	01/12/1990	100 000
KINGLBB VI	Kingate Global Fund Ltd	Kingate Management Ltd	USD	01/03/1994	250 000
OPTSTRU BM	Optimal Multiadvisors Ltd - Optimal Strategic US Equity Ltd	Optimal Investment Services SA	USD	31/01/1997	50 000
SANCLHA KY	Santa Clara I Fund	Fix Asset Management	USD	30/06/2000	1 000 000
LUXAMSA LX	LUXALPHA SICAV - American Selection	Access Management Luxembourg	USD	31/03/2004	100 000
HERUS1U KY	Herald Fund SPC - USA Segregated Portfolio One	Herald Asset Management Ltd	USD	01/04/2004	50 000

Table 6: Description of the fund's strategy (II)

Ticker	Asset class	Strategy	Style	Fund assets date	Fund assets	Mgt. fees	Perf. fees
FAISENI VI	Alternative	Equity Market Neutral	Equity Fundamental Market Neutral	31/10/2008	7 283	1	20
KINGLBB VI	Alternative	Equity Market Neutral	Equity Statistical Arbitrage	28/11/2008	2 728	1.5	0
OPTSTRU BM	Alternative	Equity Directional	Short Biased Equity	28/11/2008	3 091	2.15	0
SANCLHA KY	Alternative	Equity Market Neutral	Equity Statistical Arbitrage	31/03/2008	193	2	20
LUXAMSA LX	Asset Allocation	Geographically Focused		17/11/2008	1 409	0.8	16
HERUS1U KY	Alternative	Multi-Strategy	Multi-Style	31/10/2008	2 196	2	10

Table 7: Description of the fund's strategy (III)

Ticker	Description of the strategy
FAISENI VI	Fairfield Sentry Ltd is a hedge fund incorporated in the British Virgin Islands. The Fund's objective is to achieve capital appreciation. The Fund employs an options trading strategy described as a 'split strike conversion' strategy. The strategy has defined risk and profit parameters which may be ascertained when the positions are established.
KINGLBB VI	Kingate Global Fund Limited is an open-end fund incorporated in the British Virgin Islands. The primary objective of the Fund is long-term capital growth by allocating USD Share capital to a selected investment advisor. The Fund invests in a non-traditional investment strategy that is a variation of the traditional "option conversion" strategies of the purchasing of equity shares.
OPTSTRU BM	Optimal Multiadvisors Ltd - Optimal Strategic US Equity Ltd is an open-end fund incorporated in the Bahamas. The Fund's objective is capital appreciation. The Fund invests in a single investment managers that invests primarily in equity securities of S&P 500 companies.
SANCLHA KY	Santa Clara I Fund is an open-end investment company incorporated in the Cayman Islands. The Fund's objective is to achieve 14% annualized capital appreciation and to be profitable both in rising and in declining markets. The Fund invests in one or more funds or managed accounts that successfully use option strategies.
LUXAMSA LX	LUXALPHA SICAV - American Selection is an open-end investment fund incorporated in Luxembourg. The Fund's objective is to provide investors with an opportunity to invest mainly in transferable securities and to provide a consistent performance. The Fund invests according to the principle of risk diversification in equities listed on the New York Stock Exchange and US government securities.
HERUS1U KY	Herald Fund SPC - USA Segregated Portfolio One is an open-end fund incorporated in the Cayman Islands. The Fund's objective is to seek long-term capital growth while minimizing risk through an active trading style. The Fund invests in a wide range of financial instruments, primarily in equities and equity-related instruments, fixed income and other debt-related instruments.

Source: Bloomberg.