Improving the Efficiency of the European ETF Market

Implications for Regulators, Providers, Exchanges and Investors

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1The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.
Executive summary

2008-2012: media-oriented (or marketing-oriented) debate

- Focus on problems which are not specified to the ETF market (securities lending, collateralization, synthetic debate, etc.)
- Demonization of the ETF market (systemic risk, contagion, etc.)
- Under the spotlight of the regulation (ESMA, IOSCO, etc.)

2013-2015: investor-oriented debate?

- Going beyond the risk of the ETF market\(^a\).
- Focus on performance, which has been overshadowed by the previous marketing debate.
- Focus on market risk.
- Focus on liquidity.

\(^a\)which is certainly lower than other topics like shadow banking, Basle III liquidity buffers, off-exchange liquidity, regulatory contagion puzzle, etc.
Executive summary

1. The European ETF market is less efficient than the US ETF market.

2. The efficiency measure of an ETF is a function of three main parameters: tracking difference, tracking error and liquidity spread.

3. No standard to measure the 3 main criteria \( \Rightarrow \) Needs a uniform framework to improve the efficiency of the European ETF Market.

4. Needs new tools and rules to judge liquidity depth and to improve trading efficiency

\( \Rightarrow \) Implications for regulators, exchanges, providers and investors.
1. A first look at the European ETF market
   - Mutual funds and ETFs
   - Comparison of European and American markets
   - The European specificity

2. Defining the efficiency of exchange traded funds
   - Specificity of the ETF performance
   - Efficiency measure
   - Numerical results

3. The liquidity issue

4. How to improve the efficiency of the European ETF market?
   - Why?
   - From the regulatory perspective
   - From the exchange (and regulator) perspective
   - From the provider/investor perspective

5. Conclusion
MF, ETF and ETP

Mutual Fund (MF)
A mutual fund is a **collective investment fund** that are regulated and sold to the general public.

Exchange Traded Fund (ETF)
It is a **mutual fund** which trades **intra-day** on a securities exchange.

Exchange Traded Product (ETP)
It is a security that is **derivatively-priced**\(^a\) and that trades intra-day on an exchange. ETPs includes exchange traded funds (ETFs), exchange traded vehicles (ETVs), exchange traded notes (ETNs) and certificates.

\(^a\)The value of an ETP is derived from another investment instruments (commodity, currency, stock price, interest rate, etc.). The ETP is generally benchmarked to an index, a commodity, etc.
The ETF industry

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td># ETFs</td>
<td>1370</td>
<td>1251</td>
</tr>
<tr>
<td># ETFs/ETPs</td>
<td>1992</td>
<td>1531</td>
</tr>
<tr>
<td>ETF assets</td>
<td>388</td>
<td>1578</td>
</tr>
<tr>
<td>ETF/ETP assets</td>
<td>412</td>
<td>1669</td>
</tr>
<tr>
<td>MF assets (2012)</td>
<td>8230</td>
<td>13045</td>
</tr>
<tr>
<td>ETF market share</td>
<td>4.0%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>

⇒ ETF market share:
- Europe = 1.5% in 2007, 4.0% in 2012, 4.0% in Q1 2013.
- US = 4.9% in 2007, 9.3% in 2012, 9.8% in Q1 2013.

Decomposition by asset class

<table>
<thead>
<tr>
<th></th>
<th>Exposure</th>
<th>Europe</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ETF/ETP</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td>67.2%</td>
<td>77.5%</td>
</tr>
<tr>
<td>Fixed Income</td>
<td></td>
<td>18.6%</td>
<td>13.7%</td>
</tr>
<tr>
<td>Commodities</td>
<td></td>
<td>10.8%</td>
<td>5.3%</td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td>3.4%</td>
<td>3.6%</td>
</tr>
<tr>
<td><strong>MF</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td>36.0%</td>
<td>45.0%</td>
</tr>
<tr>
<td>Bond</td>
<td></td>
<td>29.0%</td>
<td>26.0%</td>
</tr>
<tr>
<td>Money market</td>
<td></td>
<td>14.0%</td>
<td>21.0%</td>
</tr>
<tr>
<td>Diversified</td>
<td></td>
<td>16.0%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td>5.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>


⇒ The ETF market is more concentrated in **actively traded** asset classes (Equities, Commodities) than the MF market.
A monopolistic market?

⇒ Top 5 ETF/ETP providers by assets:
  - Europe (49 providers) = IShares (48.1%), DB X (12.0%), Lyxor (10.6%), Source (3.8%), ETF Securities (3.8%).
  - US (57 providers) = IShares (39.2%), SPDR (22.8%), Vanguard (19.7%), PowerShares (5.1%), WisdomTree (2.0%).

Source: ETGFI (November 2013).
A concentrated market?

**Europe**
- The largest ETF is the IShares DAX ETF (22.4 USD Bn).
  - Top 5 = 15.1%
  - Top 10 = 22.0%
  - Top 20 = 30.6%
- 89 ETFs/ETPs > 1 USD Bn
- 186 ETFs/ETPs > 500 USD Mn

**US**
- The largest ETF is the SPDR S&P 500 ETF (163.6 USD Bn).
  - Top 5 = 22.7%
  - Top 10 = 32.2%
  - Top 20 = 43.1%
- 214 ETFs/ETPs > 1 USD Bn
- 332 ETFs/ETPs > 500 USD Mn

Source: ETGFI (November 2013).
The European specificity

- The cross-listing puzzle:
  - 24 exchanges (London Stock Exchange, Deutsche Boerse, Borsa Italiana, NYSE Euronext Paris, SIX, etc.)
  - 5005 listings = 3.7 listings / ETF
- A non-integrated market
  - Some national markets are dominated by national providers, e.g. Lyxor in France & Italy, IShares in Germany, Switzerland and UK, DB X in Germany, etc.
  - Large differences in terms of market share between countries
- Physical vs synthetic (swap) replication
- An institutional investor-oriented market (vs. a balanced market between institutional and retail investors in the US)
- Lack of liquidity
  - ETF/ETP ADV ≃ 3 USD Bn (vs 57 USD Bn in the US)
  - ETF/ETP turnover ≃ 6% of the equity market turnover (vs 25% in the US)

Largest ETFs: between 5 and 14 listings!
Consider an American investor who wants to trade the S&P 500 Index.
- 3 ETFs replicating this index

Consider an European investor who wants to trade the EURO STOXX 50 Index.
- 12 providers
- 22 ETFs replicating this index
- 87 listings

Contrary to the common idea, differences between these ETFs may be large!

⇒ How to compare these different products?
The investment mess

**Figure:** Performance difference between the ETF and the index (EURO STOXX 50)
Defining the efficiency of exchange traded funds

Comparing the performance of mutual funds

- Sharpe ratio ⇒ Mutual funds without benchmark.
- Information ratio ⇒ Mutual funds with benchmark.
- What is the equivalent for ETFs?

1. Current rating systems are not adapted to index funds.
2. The information ratio could not be used to measure the performance of ETFs.
3. The efficiency measure of an exchange traded fund is a function of three parameters: tracking difference, tracking error and liquidity spread.

⇒ Monitoring the efficiency of the European ETF market can be done by monitoring the ETF efficiency measure.
Performance or efficiency?

Fund picking process

- Current rating systems = measure the alpha and its persistence with respect to the right risk factors
  1. How to define the universe of funds?
  2. How to measure the alpha?

- Fund picking is different with passive management.
  1. The categorization of funds is not an issue.
  2. $\alpha$ is not the relevant measure to assess the performance of index funds.

What is a good ETF?

A fund that presents no risk wrt. to the index
(Beta consistency)
The irrelevance of the information ratio for ETFs

The information ratio is equal to:

\[ \text{IR} = \frac{\text{Excess Performance}}{\text{Tracking Error Volatility}} \]

Two drawbacks:

- Excess return **must** be negative for ETFs.
- IR ignores the magnitude of the tracking error volatility.

**Selection criterion based on the information ratio**

<table>
<thead>
<tr>
<th>ETF</th>
<th>Excess Return</th>
<th>Tracking Error Volatility</th>
<th>Information Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>-0.01</td>
<td>0.01</td>
<td>-1.00</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.05</td>
<td>0.07</td>
<td>0.71</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.40</td>
<td>0.50</td>
<td>0.80</td>
</tr>
</tbody>
</table>

\( x_3 \) is the best ETF and \( x_1 \) is the worst ETF!
Defining the efficiency of an ETF

What is a good ETF?

It is a fund that presents the lowest risk in relation to the index that it replicates.

How to characterize the risk?

It is characterized by the future random loss relative to the index that the ETF may incur.

How to measure the efficiency?

It is the risk measure applied to this future relative random loss.

⇒ **Gaussian Value-at-Risk**
The ETF efficiency indicator

**Definition**

The ETF efficiency indicator is equal to:

\[
\text{Efficiency} = \text{Tracking Difference} - \text{Liquidity Spread} - 1.65 \times \text{Tracking Error}
\]

where:

- the tracking difference is the one-year excess return of the ETF with respect to the benchmark;
- the liquidity spread is the average of the daily (two-side) bid/ask spreads using first limit orders;
- the tracking error is the annualized volatility of the daily performance difference between the ETF and the benchmark over a one-year period.
We assume that $TD = 50$ bps, $TE = 40$ bps and $Spread = 20$ bps. The confidence level $\alpha$ is set to 95%.

$\Rightarrow$ The efficiency measure of the ETF is $50 - 20 - 1.65 \times 40 = -35.79$ bps.

$\Rightarrow$ There is a probability of 5% that the investor will face a relative annual loss with respect to the index larger than 35.79 bps.
An illustration

**Table:** The case of EURO STOXX 50 ETFs (December 2012)

<table>
<thead>
<tr>
<th>Rank</th>
<th>TD</th>
<th>Spread</th>
<th>TE</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>65.9</td>
<td>12.3</td>
<td>7.3</td>
<td>41.6</td>
</tr>
<tr>
<td>#2</td>
<td>62.6</td>
<td>9.8</td>
<td>11.9</td>
<td>33.0</td>
</tr>
<tr>
<td>#3</td>
<td>63.5</td>
<td>8.5</td>
<td>14.9</td>
<td>30.5</td>
</tr>
<tr>
<td>#4</td>
<td>58.5</td>
<td>10.4</td>
<td>19.6</td>
<td>15.7</td>
</tr>
<tr>
<td>#5</td>
<td>23.5</td>
<td>15.4</td>
<td>7.3</td>
<td>−3.8</td>
</tr>
</tbody>
</table>

**Table:** The case of MSCI EM ETFs (December 2012)

<table>
<thead>
<tr>
<th>Rank</th>
<th>TD</th>
<th>Spread</th>
<th>TE</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>−80.0</td>
<td>20.7</td>
<td>14.9</td>
<td>−125.3</td>
</tr>
<tr>
<td>#2</td>
<td>−112.3</td>
<td>15.9</td>
<td>12.8</td>
<td>−149.4</td>
</tr>
<tr>
<td>#3</td>
<td>−109.2</td>
<td>50.3</td>
<td>3.9</td>
<td>−166.0</td>
</tr>
<tr>
<td>#4</td>
<td>−107.6</td>
<td>17.9</td>
<td>160.2</td>
<td>−389.8</td>
</tr>
<tr>
<td>#5</td>
<td>−205.8</td>
<td>30.1</td>
<td>150.7</td>
<td>−484.5</td>
</tr>
</tbody>
</table>

(*) All the statistics are expressed in bps.
Table: ETF Efficiency of the 5 best ETF providers (October 2013)

<table>
<thead>
<tr>
<th>Rank</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAC 40</td>
<td>−0.39%</td>
<td>−0.45%</td>
<td>−0.57%</td>
<td>−0.64%</td>
<td>−1.71%</td>
</tr>
<tr>
<td>DAX</td>
<td>−0.23%</td>
<td>−0.28%</td>
<td>−0.38%</td>
<td>−0.45%</td>
<td>−0.51%</td>
</tr>
<tr>
<td>EURO STOXX 50</td>
<td>0.47%</td>
<td>0.40%</td>
<td>0.31%</td>
<td>0.10%</td>
<td>−0.08%</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>−0.37%</td>
<td>−0.50%</td>
<td>−0.54%</td>
<td>−0.59%</td>
<td>−0.81%</td>
</tr>
<tr>
<td>FTSE MIB</td>
<td>−0.04%</td>
<td>−0.19%</td>
<td>−0.39%</td>
<td>−0.43%</td>
<td></td>
</tr>
<tr>
<td>MSCI EM</td>
<td>−0.90%</td>
<td>−1.10%</td>
<td>−1.17%</td>
<td>−1.42%</td>
<td>−3.54%</td>
</tr>
<tr>
<td>MSCI EUROPE</td>
<td>−0.07%</td>
<td>−0.15%</td>
<td>−0.28%</td>
<td>−0.29%</td>
<td>−0.30%</td>
</tr>
<tr>
<td>MSCI USA</td>
<td>0.01%</td>
<td>−0.11%</td>
<td>−0.21%</td>
<td>−0.29%</td>
<td>−0.48%</td>
</tr>
<tr>
<td>MSCI WORLD</td>
<td>−0.24%</td>
<td>−0.33%</td>
<td>−0.43%</td>
<td>−0.46%</td>
<td>−0.57%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.17%</td>
<td>0.10%</td>
<td>0.08%</td>
<td>−0.12%</td>
<td>−0.16%</td>
</tr>
</tbody>
</table>

Remark

The ranking is performed with the Top 20 ETF providers in Europe.
### Results

**Table:** ETF Efficiency of the 5 best ETF providers (December 2013)

<table>
<thead>
<tr>
<th>Rank</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAC 40</td>
<td>−0.38%</td>
<td>−0.42%</td>
<td>−0.56%</td>
<td>−0.69%</td>
<td>−1.47%</td>
</tr>
<tr>
<td>DAX</td>
<td>−0.24%</td>
<td>−0.28%</td>
<td>−0.38%</td>
<td>−0.49%</td>
<td>−0.53%</td>
</tr>
<tr>
<td>EURO STOXX 50</td>
<td>0.66%</td>
<td>0.63%</td>
<td>0.58%</td>
<td>0.56%</td>
<td>0.49%</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>−0.27%</td>
<td>−0.32%</td>
<td>−0.46%</td>
<td>−0.48%</td>
<td>−0.54%</td>
</tr>
<tr>
<td>FTSE MIB</td>
<td>−0.09%</td>
<td>−0.21%</td>
<td>−0.36%</td>
<td>−0.42%</td>
<td>−0.49%</td>
</tr>
<tr>
<td>MSCI EM</td>
<td>−0.91%</td>
<td>−0.92%</td>
<td>−0.95%</td>
<td>−1.00%</td>
<td>−3.02%</td>
</tr>
<tr>
<td>MSCI EUROPE</td>
<td>−0.06%</td>
<td>−0.15%</td>
<td>−0.23%</td>
<td>−0.23%</td>
<td>−0.25%</td>
</tr>
<tr>
<td>MSCI USA</td>
<td>0.01%</td>
<td>−0.06%</td>
<td>−0.15%</td>
<td>−0.18%</td>
<td>−0.20%</td>
</tr>
<tr>
<td>MSCI WORLD</td>
<td>−0.21%</td>
<td>−0.25%</td>
<td>−0.37%</td>
<td>−0.38%</td>
<td>−0.41%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.25%</td>
<td>0.19%</td>
<td>0.17%</td>
<td>0.15%</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

**Remark**

*The ranking is performed with the Top 20 ETF providers in Europe.*
The evolution of the ETF market efficiency

⇒ The efficiency has been improved in the European market?
Defining the ETF market efficiency

Let $\text{eff}_{i,j}$ be the ETF efficiency for the provider $i$ and the indexation $j$. We also note $\text{AUM}_{i,j}$ the corresponding assets under management.

**Definition**

We define the ETF efficiency for the $j^{th}$ indexation by the following weighted average:

$$
\text{eff}_j = \sum_{i=1}^{n} \frac{\text{AUM}_{i,j}}{\sum_{i=1}^{n} \text{AUM}_{i,j}} \cdot \text{eff}_{i,j}
$$

whereas the ETF efficiency for the market is:

$$
\text{eff} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\text{AUM}_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{n} \text{AUM}_{i,j}} \cdot \text{eff}_{i,j}
$$
### Table: ETF Efficiency of some indexations (December 2013)

<table>
<thead>
<tr>
<th>Indexation</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAC 40</td>
<td>−0.44%</td>
</tr>
<tr>
<td>DAX</td>
<td>−0.42%</td>
</tr>
<tr>
<td>EURO STOXX 50</td>
<td>0.25%</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>−0.56%</td>
</tr>
<tr>
<td>FTSE MIB</td>
<td>−0.29%</td>
</tr>
<tr>
<td>MSCI EM</td>
<td>−2.01%</td>
</tr>
<tr>
<td>MSCI EMU</td>
<td>0.13%</td>
</tr>
<tr>
<td>MSCI EUROPE</td>
<td>−0.28%</td>
</tr>
<tr>
<td>MSCI USA</td>
<td>−0.28%</td>
</tr>
<tr>
<td>MSCI WORLD</td>
<td>−0.46%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>−0.09%</td>
</tr>
<tr>
<td><strong>TOTAL(^3)</strong></td>
<td><strong>−0.43%</strong></td>
</tr>
</tbody>
</table>

\(^3\)These indexations represent 37.5% of the European ETF market.
The liquidity issue

Liquidity = the main advantage of ETFs versus index funds:
- The investor can buy or sell the ETF at any time
- The cost is very low
⇒ ETF = a tradable asset (security)

ETF ≃ a liquid stock

BUT

It is not always the case...

Efficiency measure in the multi-period model

If we consider a multi-period model with $m$ trades, the efficiency measure becomes:

$$\text{Efficiency} = \text{TE} - m \times \text{Spread} - 1.65 \times \text{TE}$$

This formula highlights the importance of liquidity for active investors.
The good news

Figure: Average intraday trading volume (EURO STOXX 50)

The intraday liquidity is (almost) uniform.
The bad news

Figure: Statistics of the trading volume (EURO STOXX 50)

Trading volumes are very different among ETFs.
The limit order book

Lyxor EURO STOXX 50 ETF, NYSE Euronext Paris
(December 28, 2012, 14:00:00:056)
What is the information of best limit orders?

**Figure:** Histogram of the daily best limit volume $V^1$ (EURO STOXX 50)

There is no link between best limit volumes and trading volumes.
What is the information of best limit orders?

**Figure:** Scatterplot between the daily volume disequilibrium $D^1$ (EURO STOXX 50)

There is (generally) no correlation between first limit supply/demand.
Computing the Liquidity Spread

Figure: Boxplot of the liquidity spread (EURO STOXX 50)

Liquidity spread may be highly different than the bid-ask spread.
Computing the liquidity spread

**Table:** Median liquidity spread (MSCI World)

<table>
<thead>
<tr>
<th>( N ) (in MEUR)</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
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<tbody>
<tr>
<td>0.0</td>
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<td>9</td>
<td>12</td>
<td>9</td>
<td>13</td>
<td>40</td>
<td>32</td>
<td>22</td>
<td>35</td>
</tr>
<tr>
<td>0.1</td>
<td>13</td>
<td>11</td>
<td>13</td>
<td>9</td>
<td>17</td>
<td>41</td>
<td>33</td>
<td>23</td>
<td>35</td>
</tr>
<tr>
<td>0.3</td>
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<td>19</td>
<td>45</td>
<td>36</td>
<td>23</td>
<td>62</td>
</tr>
<tr>
<td>0.5</td>
<td>16</td>
<td>12</td>
<td>14</td>
<td>10</td>
<td>22</td>
<td>48</td>
<td>42</td>
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<td>91</td>
</tr>
<tr>
<td>0.8</td>
<td>23</td>
<td>13</td>
<td>15</td>
<td>13</td>
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<td>73</td>
<td>136</td>
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<tr>
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<td>43</td>
<td>17</td>
<td>17</td>
<td>19</td>
<td>111</td>
<td>172</td>
<td>110</td>
<td>32</td>
<td>326</td>
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<tr>
<td>2.0</td>
<td>48</td>
<td>18</td>
<td>18</td>
<td>20</td>
<td>130</td>
<td>194</td>
<td>123</td>
<td>33</td>
<td>363</td>
</tr>
</tbody>
</table>
Computing the liquidity notional

**Figure:** Boxplot of the liquidity notional (EURO STOXX 50)
Relationship between spread, volume and liquidity

Figure: Scatterplot between intraday spreads and trading volumes (EURO STOXX 50 ETF #1)
Relationship between spread, volume and liquidity

- Estimation of the upper bound:

\[ \hat{f} = \arg \min \int_0^\infty f(s) \, ds - \lambda \frac{\text{card} \{ V_i : V_i \leq f(S_i) \}}{n} \]

- Estimation of the theoretical curve:

\[ f(s) = \alpha s^\beta \]

- Estimation of the theoretical liquidity measure:

\[ \mathcal{L}^* = \int_0^\infty f(s) g(s) \, ds \]

where \( g(s) \) is the theoretical distribution of the spread.

- Estimation of the empirical liquidity measure:

\[ \hat{\mathcal{L}} = \int_0^\infty \hat{f}(s) \hat{g}(s) \, ds \]

where \( \hat{g}(s) \) is the empirical distribution of the spread.
Computing the liquidity measure

Figure: The preference ordering: $\#2 \succ \#1 \Leftrightarrow \hat{L}(\#2) > \hat{L}(\#1)$
Computing the liquidity measure

\[ \hat{L} = \int_0^\infty \hat{f}(s) \hat{g}(s) \, ds \]
### Computing the liquidity measure

**Table:** Liquidity measure (MSCI WORLD)

<table>
<thead>
<tr>
<th>ETF</th>
<th>$S(N)$</th>
<th>$Q$</th>
<th>$\hat{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 (a)</td>
<td>24.1</td>
<td>20</td>
<td>4.7</td>
</tr>
<tr>
<td>#1 (b)</td>
<td>34.2</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td>#2</td>
<td>12.7</td>
<td>427</td>
<td>24.5</td>
</tr>
<tr>
<td>#3 (a)</td>
<td>13.0</td>
<td>571</td>
<td>10.9</td>
</tr>
<tr>
<td>#3 (b)</td>
<td>14.7</td>
<td>157</td>
<td>9.5</td>
</tr>
<tr>
<td>#4 (a)</td>
<td>12.0</td>
<td>151</td>
<td>10.5</td>
</tr>
<tr>
<td>#4 (b)</td>
<td>21.7</td>
<td>13</td>
<td>2.8</td>
</tr>
<tr>
<td>#5</td>
<td>11.1</td>
<td>16</td>
<td>2.9</td>
</tr>
<tr>
<td>#6</td>
<td>61.0</td>
<td>6</td>
<td>0.0</td>
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<tr>
<td>#7</td>
<td>59.4</td>
<td>13</td>
<td>0.0</td>
</tr>
<tr>
<td>#8</td>
<td>23.7</td>
<td>30</td>
<td>8.9</td>
</tr>
<tr>
<td>#9</td>
<td>163.0</td>
<td>0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
We have:

\[ S_{t}^{\text{Index}} (N) = \sum_{i \in \text{Index}} w_i \cdot S_{t}^{i} (w_i \cdot N) \]

where \( w_i \) is the weight of the stock \( i \) in the index and \( S_{t}^{i} (N_i) \) is the liquidity spread of the stock \( i \) for a given notional \( N_i \).

**Remark**

*The bid-ask spread of the index is then:*

\[ S_{t}^{\text{Index}} (0) = \sum_{i \in \text{Index}} w_i S_{t}^{i} (0) \]

where \( S_{t}^{i} (0) \) is the bid-ask spread of the stock \( i \).
Defining the liquidity spread of the index

Example

We consider the following liquidity spread (expressed in bps):

<table>
<thead>
<tr>
<th>Notional</th>
<th>Stock A</th>
<th>Stock B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 K€</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>300 K€</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>500 K€</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>700 K€</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>1 M€</td>
<td>14</td>
<td>12</td>
</tr>
</tbody>
</table>

If the index is composed by 70% of stock A and 30% of stock B, we have:

$$S_t^{\text{Index}}(0) = 0.70 \cdot 6 + 0.30 \cdot 8 = 6.6 \text{ bps}$$

A notional of 1 M€ for the index ⇒ 700 000 € for the stock A and 300 000 € for the stock B:

$$S_t^{\text{Index}}(1 \text{ M€}) = 0.70 \cdot 12 + 0.30 \cdot 9 = 11.1 \text{ bps}$$
Measuring the liquidity improvement

The liquidity ratio is defined as the ratio between the index spread and the ETF spread:

\[ LR_t (N) = \frac{S_{t,\text{Index}} (N)}{S_{t,\text{ETF}} (N)} \]

- \( LR_t > 1 \) \Rightarrow \text{liquidity improvement}
- \( LR_t < 1 \) \Rightarrow \text{liquidity worsening}
Figure: Boxplot of the intraday liquidity ratio $\mathcal{LR}_t(N)$ (EURO STOXX 50)
ETF / index relationship on liquidity

An example with \( N = 1 \text{ M€} \) and the EURO STOXX 50 index

**Figure: Daily spreads**

**Figure: Intraday spreads**
Comparison with the American ETF market

<table>
<thead>
<tr>
<th>Index</th>
<th>Liquidity Ratio $\mathcal{RL}_t(0)$</th>
<th>ETF</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>6.23</td>
<td>SPDR S&amp;P 500 ETF (SPY)</td>
</tr>
<tr>
<td>NASDAQ 100</td>
<td>3.23</td>
<td>PowerShare Nasdaq ETF (QQQ)</td>
</tr>
<tr>
<td>DJIA</td>
<td>2.39</td>
<td>SPDR DJIA ETF (DIA)</td>
</tr>
</tbody>
</table>
Why improving the efficiency of the European ETF market?

With the end of rebates (UK, Retail Distribution Review or RDR; Netherlands; Italy; etc.) ⇒ Retailization of the investors

How to protect them?

- European market = less efficient than the American market
- Large differences between ETF performances
- Lack of liquidity of some ETFs
The standardization issue
Investors needs and ESMA guidelines on ETFs

To compare the efficiency of ETFs, investors need to easily access the following statistics:

1. The tracking difference
2. The tracking error
3. The liquidity spread

"The annual and half-yearly reports of an index-tracking UCITS should state the size of the tracking error at the end of the period under review. The annual report should provide an explanation of any divergence between the anticipated and realised tracking error for the relevant period. The annual report should also disclose and explain the annual tracking difference between the performance of the UCITS and the performance of the index tracked." (ESMA, 2012).

⇒ Tracking difference and tracking error statistics are identified, but not liquidity spread.
The standardization issue

Annual tracking difference

The difference between the annual return of the *Index-tracking UCITS* and the annual return of the tracked index (ESMA, 2012).

- Some ETFs capitalized dividends;
- Other distribute them;
- Trading dates of the ETF may be not the same as those of pricing dates of the index (end-month and end-year problems)
- etc.

⇒ The issue is then: **How to compute the annual return of the ETF?**

*We need a uniform methodology to compute the tracking difference*, i.e. a formula (or guidelines) to reintroduce the dividends in order to estimate the total return of the ETF.
The standardization issue

Tracking error

The volatility of the difference between the return of the Index-tracking UCITS and the return of the index or indices tracked.

 [...] index-tracking UCITS should indicate the anticipated level of tracking error of the UCITS in normal market conditions. [...] After further analysis and despite the support from most stakeholders, ESMA did not feel necessary to develop precise guidelines for the computation of the tracking-error. (ESMA, 2013).

We need a uniform methodology to compute the tracking error, i.e. a standard formula with a given sampling frequency.
The standardization issue

Tracking error

- Computation of the non-annualized tracking error:

\[ \text{TE}(x \mid b) = \sqrt{\frac{1}{T} \sum_{t=1}^{T} e_t^2} \]

- Including the mean:

\[ e_t = R_t(x) - R_t(b) - \hat{\mu} \text{ with } \hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} (R_t(x) - R_t(b)) \]

- or excluding the mean, i.e. \( e_t = R_t(x) - R_t(b) \)?

- Computation of the annualized tracking error

- Choice of the sampling frequency: daily, weekly, monthly?

- End of the period, overlapping or averaging?
A first look at the European ETF market
Defining the efficiency of exchange traded funds
The liquidity issue
How to improve the efficiency of the European ETF market?
Conclusion

Why?
From the regulatory perspective
From the exchange (and regulator) perspective
From the provider/investor perspective

The standardization issue
Tracking error

\[ TE = 10.0 \text{ bps} / 41.2 \text{ bps} \]

\[ TE = 15.1 \text{ bps} / 18.1 \text{ bps} \]
The standardization issue

Tracking error

Some practices:

- **AMF**$^4$: including the mean – weekly or daily – averaging or not – logarithmic return.
- **AFG**$^5$: including the mean – Friday basis – not averaging.
- **Provider X**: including the mean – monthly – averaging – arithmetic return.
- **Provider Y**: weekly – averaging – arithmetic return – excluding extreme points.
- etc.


The standardization issue

Tracking error

- MSCI World
- Year 2012
- 4 famous ETF providers

Lower TE:
- #1 W, O, -M, L
- #2 W, E, +M, A
- #3 W, E, +M, L
- #4 M, E, +M, L

<table>
<thead>
<tr>
<th>Frequency</th>
<th>LOG</th>
<th>ETF</th>
<th>End of the period</th>
<th>Overlapping</th>
<th>Averaging</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>#1</td>
<td>+ Mean</td>
<td>− Mean</td>
<td>+ Mean</td>
</tr>
<tr>
<td>Daily</td>
<td></td>
<td>#1</td>
<td>1.55</td>
<td>1.96</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#2</td>
<td>1.69</td>
<td>3.29</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>50.80</td>
<td>50.72</td>
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<td></td>
<td></td>
<td></td>
<td>50.50</td>
<td>50.41</td>
<td>50.50</td>
</tr>
</tbody>
</table>

|           |     | #1  | + Mean | − Mean | + Mean | − Mean | + Mean | − Mean |
| Weekly    |     | #1  | 1.39   | 3.02   | 1.34   | 2.97   | 1.35   | 2.98   |
|           |     | #2  | 0.28   | 6.21   | 0.28   | 6.20   | 0.29   | 6.20   |
|           |     | #3  | 6.67   | 6.91   | 7.73   | 8.16   | 7.74   | 8.10   |
|           |     | #4  | 17.89  | 18.36  | 22.23  | 22.53  | 21.71  | 22.02  |
|           |     | ✓   | 1.38   | 3.01   | 1.33   | 2.96   | 1.34   | 2.98   |
|           |     |     | 0.33   | 6.19   | 0.33   | 6.19   | 0.34   | 6.19   |
|           |     |     | 6.65   | 6.89   | 7.69   | 8.11   | 7.70   | 8.05   |
|           |     |     | 17.75  | 18.25  | 21.93  | 22.24  | 21.46  | 21.78  |

|           |     | #1  | + Mean | − Mean | + Mean | − Mean | + Mean | − Mean |
| Monthly   |     | #1  | 2.73   | 6.29   | 2.28   | 5.85   | 2.43   | 5.92   |
|           |     | #2  | 0.45   | 12.67  | 0.46   | 12.62  | 0.47   | 12.61  |
|           |     | #3  | 8.68   | 8.92   | 8.16   | 9.71   | 8.16   | 9.15   |
|           |     | #4  | 33.90  | 31.97  | 17.87  | 19.81  | 17.12  | 18.04  |
|           |     | ✓   | 2.58   | 6.12   | 2.72   | 5.70   | 2.25   | 5.77   |
|           |     |     | 0.51   | 12.48  | 0.47   | 12.47  | 0.47   | 12.43  |
|           |     |     | 8.69   | 8.89   | 8.11   | 9.63   | 8.10   | 9.06   |
|           |     |     | 33.00  | 31.12  | 17.64  | 19.55  | 16.89  | 17.76  |
If the goal of the tracking error is to measure the quality of replication with respect to the index, our works suggest to consider the following rules:

- Arithmetic returns\(^6\).
- Daily frequency\(^7\).
- Without mean\(^8\).

---

\(^6\)Because they measure the true performance, and we are not in a Black-Scholes framework.

\(^7\)Because ETF is an intra-day fund. Using a lower frequency is not consistent with the investment philosophy in ETFs.

\(^8\)Because we don’t want to underestimate the performance difference by taking into account a structural underperformance.
The transparency issue

**On-exchange trading**

The investor trades the ETF via the order book.

\[ \approx 30\% \text{ of all ETF trades} \]

**OTC (or off-exchange) trading**

The investor trades the ETF with a market maker (MM) or an authorized participant (AP).

or

Electronic OTC platform (Tradeweb) with multi-dealer (16) and request-for-quote protocol.

\[ \approx 70\% \text{ of all ETF trades} \]
The transparency issue

The ETF Trading Conundrum:
- European ETF assets: ↑
- European ETF trading volume: ↓

Since 2013, % of on-exchange liquidity decreases (vicious cycle of illiquidity).

- Impact on price formation/discovery.
- No requirement for ETF trade reporting (MiFiD).
- London Stock Exchange / Swiss Stock Exchange = OTC trades are reported.

Challenges to improve the liquidity of European ETFs
- OTC trade reporting
- Consolidated tape or Pan-European platform?
- Transparent rules
The data puzzle

How to have easier access to the (homogeneous) data?

⇒ KID: too complicated.
ETF = a (cheap) access to an index

BUT

ETF \neq \text{the index}

There is a cost to replicate the index

\downarrow

Investors need to be informed about these costs (tracking difference, tracking error, liquidity)

Investors must have easily access to these costs

Improving the efficiency of the ETF market \Rightarrow large benefits for the investors

Specific issues for retail investors
For Further Reading I

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Liquidity Improvement, Exchange Traded Funds Flows and Mispricing.
SSRN, 2013.

CALAMIA A., DEVILLE L., RIVA F.
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ETFGI.
European ETF and ETP industry insights.
November 2013.

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November 2013.
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**European Securities and Markets Authority.**
Guidelines on ETFs and other UCITS issues.  
*ESMA, December 2012.*

**Fuhr D.**
*Journal of Index Investing, 2013.*

**Hassine M., Roncalli T.**
Measuring Performance of Exchange Traded Funds.  
*Journal of Index Investing, 2013.*

**Jaswal A.**
Exchange-Traded Funds: Growth and Challenges.  
*Celent Report, April 2012.*
For Further Reading III

JOHNSON B., BIOY H., KELLETT A., DAVIDSON L.
On the Right Track: Measuring Tracking Efficiency in ETFs.

RONCALLI T., ZHENG B.
Measuring the Liquidity of ETFs: An Application to the European Market.
SSRN, 2014.
A first look at the European ETF market
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How to improve the efficiency of the European ETF market?
Conclusion

The rationale of the information ratio

- $x_2 \succ x_1$ because it has a better excess-return performance
- $x_2 \succ x_3$ because $x_4 \succ x_3$ with:

\[
\begin{align*}
\alpha &= \frac{\sigma(x_3 | b)}{\sigma(x_2 | b)} \\
\end{align*}
\]

Fundamental rule of benchmarked portfolios

\[ x \succ y \iff \text{IR}(x | b) \geq \text{IR}(y | b) \]
The irrelevance of the information ratio for ETFs

- Using the previous rule, we have $x_3 \succ x_2 \Rightarrow x_1 \succ x_2$.

- The problem is that we cannot replicate the benchmark exactly. In real life, we need to use an ETF $x_0$ to proxy the benchmark.

- In the real life, $x_3 \equiv x_4$ and $x_2 \succ x_1$.

- For benchmarked funds with low tracking-error volatility:

$$\text{IR}(x \mid b) > \text{IR}(y \mid b) \not\Rightarrow x \succ y$$

- If we consider the information ratio, investors will never chose the ETF $x_0$!
The ETF efficiency indicator

Notations

We consider a universe of $n$ assets. $\mu$ and $\Sigma$ are the vector of expected returns and the covariance matrix of asset returns. We note $b$ the benchmark (or the index) and $x$ the portfolio. The tracking error is:

$$e = R(x) - R(b) = (x - b)^T R$$

The expected tracking error is then:

$$\mu(x | b) = (x - b)^T \mu$$

whereas tracking error volatility is equal to:

$$\sigma(x | b) = \sqrt{(x - b)^T \Sigma (x - b)}$$
The two-period trading model

- The investor buys the ETF \( x \) at time \( t = 0 \) and sells it at time \( t = 1 \). Note the corresponding tracking error \( e \). The relative PnL of the investor with respect to the benchmark \( b \) is:

\[
\Pi(x \mid b) = e - s(x \mid b)
\]

where \( s(x \mid b) \) is the bid-ask spread of the ETF.

- The loss \( \mathcal{L}(x \mid b) \) of the investor is defined as follows:

\[
\mathcal{L}(x \mid b) = -\Pi(x \mid b)
\]

- The ETF efficiency measure is a risk measure applied to the loss function \( \mathcal{L}(x \mid b) \) of the investor.
The ETF efficiency indicator

What is the more efficient ETF?

![Histograms comparing ETF efficiency](chart)

Thierry Roncalli

Improving the Efficiency of the European ETF Market
The ETF efficiency indicator

Definition

We propose to use the value-at-risk, which is today commonly accepted as a standard risk measure. In this case, the efficiency measure $\zeta_\alpha(x \mid b)$ is defined as follows:

$$\zeta_\alpha(x \mid b) = - \{ \inf \zeta : \Pr \{ \mathcal{L}(x \mid b) \leq \zeta \} \geq \alpha \}$$

Definition

The efficiency measure $\zeta_\alpha(x \mid b)$ of the ETF $x$ with respect to the benchmark $b$ corresponds to:

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - \Phi^{-1}(\alpha) \sigma(x \mid b)$$

where $\mu(x \mid b)$ is the expected value of the tracking error, $s(x \mid b)$ is the bid-ask spread and $\sigma(x \mid b)$ is the volatility of the tracking error\(^{(\ast)}\).

\(^{\ast}\) IOSCO terminology: $\mu(x \mid b) = \text{Tracking Difference (TD)} \& \sigma(x \mid b) = \text{Tracking Error (TE)}$. 
The ETF efficiency indicator

Computation

We assume that $\mu(x \mid b) = 50$ bps, $\sigma(x \mid b) = 40$ bps and $s(x \mid b) = 20$ bps. The confidence level $\alpha$ is set to 95%.

$\Rightarrow$ The efficiency measure of the ETF $\zeta_\alpha(x \mid b)$ is $-35.79$ bps.

$\Rightarrow$ There is a probability of 5% that the investor will face a relative annual loss with respect to the index larger than 35.79 bps.
The ETF efficiency indicator
Impact of parameters on the efficiency measure

Better excess return
$\xi_a(x_2 \mid b) > \xi_a(x_1 \mid b)$

Larger bid–ask
$\xi_a(x_1 \mid b) > \xi_a(x_2 \mid b)$

Larger tracking–error volatility
$\xi_a(x_1 \mid b) > \xi_a(x_2 \mid b)$
The liquidity spread $S_N$

We define $S_{t_j}(N)$ the spread at time $t_j$ weighted by the depth of the market as follows:

$$S_{t_j}(N) = c_{t_j}(N) \frac{\left( \bar{P}^{ASK}_{t_j} - \bar{P}^{BID}_{t_j} \right)}{\bar{P}^{MID}_{t_j}}$$

The bid and ask prices $\bar{P}^{BID}_{t_j}$ and $\bar{P}^{ASK}_{t_j}$ correspond to:

$$\bar{P}^{•}_{t_j} = \frac{\sum_{i=1}^{5} \bar{Q}^{•,i}_{t_j} P^{•,i}_{t_j}}{\sum_{i=1}^{5} \bar{Q}^{•,i}_{t_j}}$$

whereas the quantity $\bar{Q}^{BID,\cdot}_{t_j}$ and $\bar{Q}^{ASK,\cdot}_{t_j}$ are given by the following relationship:

$$\bar{Q}^{•,i}_{t_j} = \max \left( 0, \min \left( Q^{•,i}_{t_j}, Q^{*}_{t_j} - \sum_{k=1}^{i-1} Q^{•,k}_{t_j} \right) \right)$$
The liquidity spread $S_N$

$Q_{t_j}^* = N / P_{t_j}^{MID}$ is the reference quantity to execute in order to target the notional $N$. The factor $c_{t_j}(N)$ is used to treat the case when the trading volume on the order book is lower than the notional $N$. We have:

$$c_{t_j}(N) = \max \left(1, \frac{Q_{t_j}^*}{\min \left(\sum_{i=1}^{5} Q_{t_j}^{ASK,i}, \sum_{i=1}^{5} Q_{t_j}^{BID,i}\right)}\right)$$

The daily spread $S(N)$ corresponds then to intraday spreads weighted by the duration between two ticks:

$$S(N) = \frac{\sum_{t_j=\text{open}}^{\text{close}} S_{t_j}(N)(t_{j+1} - t_j)}{\sum_{t_j=\text{open}}^{\text{close}} (t_{j+1} - t_j)}$$

where $S_{t_j}(N)$ is the spread of the $j^{th}$ tick in order to trade the notional $N$ and $(t_{j+1} - t_j)$ is the elapsed time between two consecutive ticks.
The liquidity spread $S_N$

Table: The limit order book

<table>
<thead>
<tr>
<th>i</th>
<th>Buy orders $Q_{i}^{BID_i}$</th>
<th>$P_{i}^{BID_i}$</th>
<th>Sell orders $Q_{i}^{ASK_i}$</th>
<th>$P_{i}^{ASK_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65201</td>
<td>26.325</td>
<td>70201</td>
<td>26.340</td>
</tr>
<tr>
<td>2</td>
<td>85201</td>
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<td>26.345</td>
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<td>5</td>
<td>20000</td>
<td>26.305</td>
<td>35178</td>
<td>26.360</td>
</tr>
</tbody>
</table>

- The traditional spread is equal to 5.696 bps.
- The mid price is equal to 26.333 €.
- Trading 1 M€ is equivalent to trade 37,976 shares.
The liquidity spread $S_N$

**Table:** Computing the spread for a given notional $N$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$N = 1$ M€</th>
<th>$N = 5$ M€</th>
<th>$N = 10$ M€</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{Q}_{t_j}^\text{BID,i}$</td>
<td>$\bar{Q}_{t_j}^\text{ASK,i}$</td>
<td>$\bar{Q}_{t_j}^\text{BID,i}$</td>
</tr>
<tr>
<td>1</td>
<td>37 976</td>
<td>37 976</td>
<td>65 201</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>85 201</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>39 478</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sum_{i=1}^{5} \bar{Q}_{t_j}^\text{•,i}$</td>
<td>37 976</td>
<td>37 976</td>
<td>189 880</td>
</tr>
<tr>
<td>$c_{t_j}(N)$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$S_{t_j}(N)$</td>
<td>5.696</td>
<td>8.570</td>
<td>12.908</td>
</tr>
</tbody>
</table>
The liquidity spread $S_N$

**Empirical relationship**

**Theoretical relationship**

**Linear assumption if $N > N^+$**
The liquidity notional $N_S$

It is the reverse of the liquidity spread.

The liquidity notional is defined as follows:

$$N_{t_j} (S) = \frac{N_{t_j}^{\text{BID}} \left( \frac{S}{2} \right) + N_{t_j}^{\text{ASK}} \left( \frac{S}{2} \right)}{2}$$

with:

$$N_{t_j}^{\text{BID}} (S) = \sum_{i=1}^{5} P_{t_j}^{\text{BID},i} Q_{t_j}^{\text{BID},i} \mathbb{1} \left\{ P_{t_j}^{\text{BID},i} \geq (1 - S) \cdot P_{t_j}^{\text{MID}} \right\}$$

$$N_{t_j}^{\text{ASK}} (S) = \sum_{i=1}^{5} P_{t_j}^{\text{ASK},i} Q_{t_j}^{\text{ASK},i} \mathbb{1} \left\{ P_{t_j}^{\text{ASK},i} \leq (1 + S) \cdot P_{t_j}^{\text{MID}} \right\}$$