Course 2024–2025 in Sustainable Finance
Lecture 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

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[1]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Agenda

Mean-variance optimization problem
Model settings

- An investment universe of n assets
- $w = (w_1, \ldots, w_n)$ is the vector of weights in the portfolio
- The portfolio is fully invested meaning that $\sum_{i=1}^n w_i = \mathbf{1}^\top w = 1$
- $R = (R_1, \ldots, R_n)$ is the vector of asset returns
- We denote by $\mu = \mathbb{E}[R]$ and $\Sigma = \mathbb{E}\left[(R-\mu)(R-\mu)^\top\right]$ the vector of expected returns and the covariance matrix of asset returns

Mean-variance optimization problem
Model setup

We have:

$$R(w) = \sum_{i=1}^{n} w_i R_i = w^\top R$$

The expected return $\mu(w) := \mathbb{E}[R(w)]$ of the portfolio is equal to:

$$\mu(w) = \mathbb{E}\left[w^\top R\right] = w^\top \mathbb{E}[R] = w^\top \mu$$

whereas its variance $\sigma^2(w) := \mathrm{var}(R(w))$ is given by:

$$\begin{aligned}
\sigma^2(w) &= \mathbb{E}\left[(R(w) - \mu(w))(R(w) - \mu(w))^\top\right] \\
&= \mathbb{E}\left[w^\top (R - \mu)(R - \mu)^\top w\right] \\
&= w^\top \Sigma w
\end{aligned}$$

Mean-variance optimization problem

μ- and σ-problems

We can then formulate the investor's financial problem as follows:

1. Maximizing the expected return of the portfolio under a volatility constraint (σ-problem):

$$\max \mu(w) \quad \text{s.t.} \quad \sigma(w) \leq \sigma^\star$$

2. Or minimizing the volatility of the portfolio under a return constraint (μ-problem):

$$\min \sigma(w) \quad \text{s.t.} \quad \mu(w) \geq \mu^\star$$

$\Rightarrow$ The key idea of Markowitz was to transform the original non-linear optimization problems into a quadratic optimization problem

Mean-variance optimization problem

Introducing the quadratic utility function

- The mean-variance (or quadratic) utility function is:

$$\mathcal{U}(w) := \mathbb{E}[R(w)] - \frac{\bar{\gamma}}{2} \operatorname{var}(R(w)) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w$$

where $\bar{\gamma}$ is the absolute risk-aversion parameter

- We obtain the following problem:

$$\begin{aligned} w^\star(\bar{\gamma}) &= \arg\max \left\{ \mathcal{U}(w) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w \right\} \\ & \text{s.t.} \quad \mathbf{1}^\top w = 1 \end{aligned}$$

- $\bar{\gamma} = 0 \Rightarrow$ maximum mean portfolio
- $\bar{\gamma} = \infty \Rightarrow$ minimum variance portfolio:

$$w^\star(\infty) = \arg\min \frac{1}{2} w^\top \Sigma w \quad \text{s.t.} \quad \mathbf{1}^\top w = 1$$

Mean-variance optimization problem

The engineering viewpoint

In practice, professionals formulate the optimization problem as follows:

$$\begin{aligned} w^{\star}(\gamma) &= \arg\min \frac{1}{2} w^{\top} \Sigma w - \gamma w^{\top} \mu \\ \text{s.t.} &\quad \mathbf{1}^{\top} w = 1 \end{aligned}$$

where $\gamma = \bar{\gamma}^{-1}$ is called the risk-tolerance

This is a standard QP problem

Quadratic programming problem

Definition

The formulation of a standard QP problem is:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2} w^{\top} Q w - w^{\top} R \\
&\text{u.c.} \quad \left\{ \begin{array}{l} Aw = B \\ Cw \leq D \\ w^{-} \leq w \leq w^{+} \end{array} \right.
\end{aligned}
$$

$\Rightarrow$ We have $Q = \Sigma$, $R = \gamma\mu$, $A = \mathbf{1}^{\top}$ and $B = 1$

Mean-variance optimization problem

Illustration

Example #1

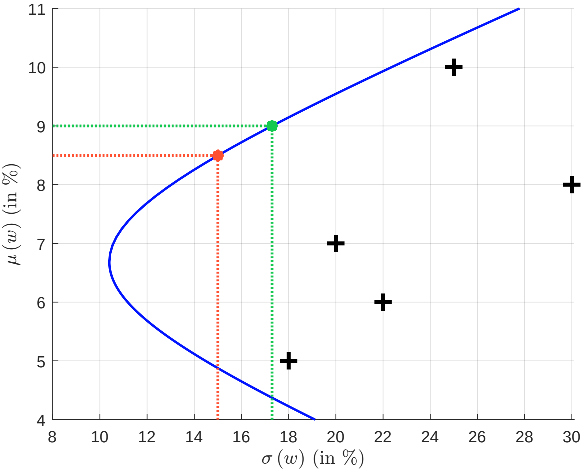
We consider an investment universe of five assets. Their expected returns are equal to 5%, 7%, 6%, 10% and 8% while their volatilities are equal to 18%, 20%, 22%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & & \\ 70\% & 100\% & & & \\ 20\% & 30\% & 100\% & & \\ -30\% & 20\% & 10\% & 100\% & \\ 0\% & 0\% & 0\% & 0\% & 100\% \end{pmatrix}$$

Mean-variance optimization problem

Illustration

Figure 1: Efficient frontier (Example #1)



Mean-variance optimization problem

Illustration

- The GMV portfolio is obtained with $\gamma = 0$
- The solution is:

$$w_{\mathrm{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

- We have:

$$\sigma(w) \geq \sigma(w_{\mathrm{gmv}}) = 10.40\% \qquad \forall w$$

Mean-variance optimization problem
Illustration

Table 1: Solution of the Markowitz optimization problem (in %)

γ	0.00	0.10	0.20	0.50	1.00	5.00
$w_1^\star(\gamma)$	66.35	58.25	50.14	25.84	-14.67	-338.72
$w_2^\star(\gamma)$	-28.52	-22.67	-16.82	0.74	30.00	264.12
$w_3^\star(\gamma)$	15.31	13.30	11.30	5.28	-4.74	-84.93
$w_4^\star(\gamma)$	34.85	37.65	40.44	48.82	62.78	174.50
$w_5^\star(\gamma)$	12.02	13.48	14.94	19.32	26.62	85.03
$\mu(w^\star(\gamma))$	6.69	6.97	7.25	8.09	9.49	20.71
$\sigma(w^\star(\gamma))$	10.40	10.53	10.93	13.35	19.71	84.38

Mean-variance optimization problem

How to solve the μ-problem and the σ-problem?

- We have to find the optimal value of γ such that $\mu\left(w^{\star}\left(\gamma\right)\right)=\mu^{\star}$ or $\sigma\left(w^{\star}\left(\gamma\right)\right)=\sigma^{\star}$
- We use the bisection algorithm
- If we target a portfolio with $\sigma^{\star}=15\%$, we know that $\gamma\in[0.5,1]$. The optimal solution $w^{\star}$ is $(14.06\%, 9.25\%, 2.37\%, 52.88\%, 21.44\%)$ and the bisection algorithm returns $\gamma=0.6455$. In this case, we obtain $\mu\left(w^{\star}\left(\gamma\right)\right)=8.50\%$
- If we consider a μ-problem with $\mu^{\star}=9\%$, we find $\gamma=0.8252$, $w^{\star}=(-0.50\%, 19.77\%, -1.23\%, 57.90\%, 24.07)$ and $\sigma\left(w^{\star}\left(\gamma\right)\right)=17.30\%$

Mean-variance optimization problem
Adding some constraints

- The Lagrange function of the optimization problem is equal to:

$$\mathcal{L}(w;\lambda_0) = \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu + \lambda_0 \left(\mathbf{1}^\top w - 1\right)$$

where λ_0 is the Lagrange coefficients associated with the constraint $\mathbf{1}^\top w = 1$

- The solution $w^\star$ verifies the following first-order conditions:

$$\left\{ \begin{array}{l} \partial_w \mathcal{L}(w;\lambda_0) = \Sigma w - \gamma \mu + \lambda_0 \mathbf{1} = \mathbf{0} \\ \partial_{\lambda_0} \mathcal{L}(w;\lambda_0) = \mathbf{1}^\top w - 1 = 0 \end{array} \right.$$

- We obtain $w = \Sigma^{-1}(\gamma\mu - \lambda_0 \mathbf{1})$. Because $\mathbf{1}^\top w - 1 = 0$, we have $\gamma \mathbf{1}^\top \Sigma^{-1} \mu - \lambda_0 \mathbf{1}^\top \Sigma^{-1} \mathbf{1} = 1$. It follows that:

$$\lambda_0 = \frac{\gamma \mathbf{1}^\top \Sigma^{-1} \mu - 1}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}}$$

Mean-variance optimization problem
Adding some constraints

- The solution is then:

$$\begin{aligned} w^{\star}(\gamma) &= \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}} + \gamma \frac{\left(\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}\right)\Sigma^{-1}\mu - \left(\mathbf{1}^{\top}\Sigma^{-1}\mu\right)\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}} \\ &= w_{\mathrm{gmv}} + \gamma w_{\mathrm{lsp}} \end{aligned}$$

where:

- $w_{\mathrm{gmv}} = \left(\Sigma^{-1}\mathbf{1}\right) / \left(\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}\right)$ is the global minimum variance portfolio
- w_{lsp} is a long/short cash-neutral portfolio such that $\mathbf{1}^{\top} w_{\mathrm{lsp}} = 0$

Mean-variance optimization problem
Adding some constraints

- We could think that a QP solver is not required
- The analytical calculus gives:

$$w_{\mathrm{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

and:

$$w_{\mathrm{lsp}} = (-81.01\%, 58.53\%, -20.05\%, 27.93\%, 14.60\%)$$

- In practice, professionals consider other constraints:

$$\begin{aligned} w^{\star}(\gamma) & = & \arg\min \frac{1}{2} w^{\top} \Sigma w - \gamma w^{\top} \mu \\ & & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top} w = 1 \\ w \in \Omega \end{array} \right. \end{aligned}$$

where $w \in \Omega$ corresponds to the set of restrictions

- No short-selling restriction ($w_i \geq 0$ and $\Omega = [0,1]^n$) and asset bounds ($w_i \leq w^+$) $\Rightarrow$ No analytical solution (because of the KKT conditions) $\Rightarrow$ **QP solver**

The tangency portfolio
Two-fund separation theorem

We consider a combination of the risk-free asset and a portfolio w:

$$R(\tilde{w}) = (1-\alpha) r + \alpha R(w)$$

where:

- r is the return of the risk-free asset
- $\tilde{w} = (\alpha w, 1-\alpha)$ is a vector of dimension $(n+1)$
- $\alpha \geq 0$ is the proportion of the wealth invested in the risky portfolio

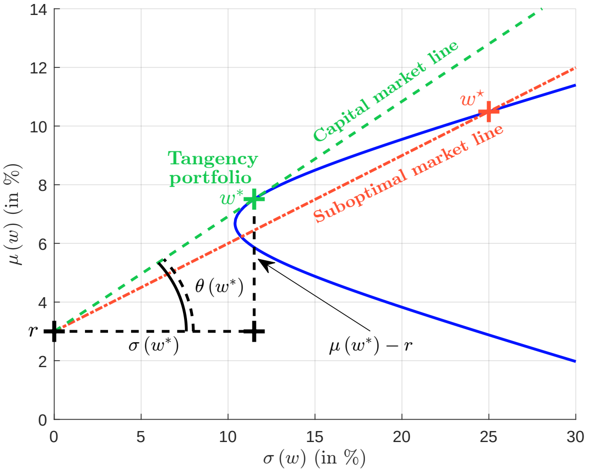
$\Rightarrow$ It follows that $\mu(\tilde{w}) = (1-\alpha) r + \alpha \mu(w) = r + \alpha(\mu(w) - r)$, $\sigma^2(\tilde{w}) = \alpha^2 \sigma^2(w)$ and:

$$\mu(\tilde{w}) = r + \frac{(\mu(w) - r)}{\sigma(w)} \sigma(\tilde{w})$$

The tangency portfolio

Two-fund separation theorem

Figure 2: Capital market line (Example #1)



The tangency portfolio

Two-fund separation theorem

- Let $\mathrm{SR}(w \mid r)$ be the Sharpe ratio of portfolio w:

$$\mathrm{SR}(w \mid r) = \frac{\mu(w) - r}{\sigma(w)}$$

- We have:

$$\frac{\mu(\tilde{w}) - r}{\sigma(\tilde{w})} = \frac{\mu(w) - r}{\sigma(w)} \Leftrightarrow \mathrm{SR}(\tilde{w} \mid r) = \mathrm{SR}(w \mid r)$$

- The tangency portfolio w^* satisfies:

$$w^* = \arg\max \tan\theta(w)$$

The tangency portfolio

Two-fund separation theorem

If we consider our example with $r = 3\%$, the composition of the tangency portfolio is:

$$w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)$$

and we have:

$$\left\{\begin{array}{l} \mu(w^*) = 7.51\% \\ \sigma(w^*) = 11.50\% \\ \mathrm{SR}(w^* \mid r) = 0.39 \\ \theta(w^*) = 21.40 \text{ degrees} \end{array}\right.$$

The tangency portfolio

Augmented optimization problem

- When the risk-free asset belongs to the investment universe, the optimization problem becomes:

$$\begin{aligned} \tilde{w}^\star(\gamma) &= \arg\min \frac{1}{2}\tilde{w}^\top \tilde{\Sigma}\tilde{w} - \gamma \tilde{w}^\top \tilde{\mu} \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^\top \tilde{w} = 1 \\ \tilde{w} \in \Omega \end{array} \right. \end{aligned}$$

  where $\tilde{w} = (w, w_r)$ is the augmented allocation vector of dimension $n+1$

- It follows that:

$$\tilde{\Sigma} = \left(\begin{array}{cc} \Sigma & \mathbf{0} \\ \mathbf{0} & 0 \end{array} \right) \quad \text{and} \quad \tilde{\mu} = \left(\begin{array}{c} \mu \\ r \end{array} \right)$$

The tangency portfolio
Augmented optimization problem

- In the case where $\Omega = \mathbb{R}^{n+1}$, we can show that the optimal solution is equal to:

$$\tilde{w}^{\star}(\gamma) = \underbrace{\alpha \cdot \begin{pmatrix} w^* \\ 0 \end{pmatrix}}_{\text{risky assets}} + \underbrace{(1-\alpha) \cdot \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}}_{\text{risk-free asset}}$$

where w^* is the tangency portfolio:

$$w^* = \frac{\Sigma^{-1}(\mu - r\mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})}$$

- The proportion of risky assets is equal to

$$\alpha = \gamma \mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})$$

- The risk-tolerance coefficient associated to the tangency portfolio is given by:

$$\gamma(w^*) = \frac{1}{\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})}$$

Market equilibrium and CAPM
Risk premium and beta

At the equilibrium, Sharpe (1964) showed that:

$$\pi_i := \mu_i - r = \beta_i \left(\mu\left(w^*\right) - r\right)$$

where π_i is the risk premium of the asset i and:

$$\beta_i = \frac{\operatorname{cov}\left(R_i, R\left(w^*\right)\right)}{\operatorname{var}\left(R\left(w^*\right)\right)}$$

We have:

$$\beta\left(x \mid w\right) = \frac{\sigma\left(x, w\right)}{\sigma^2\left(w\right)} = \frac{x^\top \Sigma w}{w^\top \Sigma w}$$

and:

$$\beta_i = \beta\left(\mathbf{e}_i \mid w\right) = \frac{\mathbf{e}_i^\top \Sigma w}{w^\top \Sigma w} = \frac{\left(\Sigma w\right)_i}{w^\top \Sigma w}$$

Market equilibrium and CAPM

Risk premium and beta

In the case of Example #1, we have:

- $w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)$
- $(\mu(w^*) = 7.51\%, r = 3\%) \Rightarrow \mu(w^*) = 4.51\%$

Table 2: Computation of the beta and risk premia (Example #1)

Portfolio	$\mu(w)$	$\mu(w) - r$	$\beta(w \mid w^*)$	$\pi(w \mid w^*)$
$\mathbf{e}_1$	5.00%	2.00%	0.444	2.00%
$\mathbf{e}_2$	7.00%	4.00%	0.887	4.00%
$\mathbf{e}_3$	6.00%	3.00%	0.665	3.00%
$\mathbf{e}_4$	10.00%	7.00%	1.553	7.00%
$\mathbf{e}_5$	8.00%	5.00%	1.109	5.00%
w_{ew}	7.20%	4.20%	0.932	4.20%
w_{gmv}	6.69%	3.69%	0.817	3.69%

Market equilibrium and CAPM
Risk premium and alpha return

- Jensen (1968) defined the alpha return as:

$$R_{j,t} - r = \alpha_j + \beta_j \left(R_t \left(w_m \right) - r \right) + \varepsilon_{j,t}$$

  where $R_{j,t}$ is the return of the mutual fund j at time t, $R_t \left(w_m \right)$ is the return of the market portfolio and $\varepsilon_{j,t}$ is an idiosyncratic risk

- More generally, the alpha is defined by the difference between the risk premium $\pi \left(w \right)$ of portfolio w and the beta $\beta \left(w \right)$ of the portfolio times the market risk premium π_m:

$$\begin{aligned} \alpha &= \left(\mu \left(w \right) - r \right) - \beta \left(w \mid w_m \right) \left(\mu \left(w_m \right) - r \right) \\ &= \pi \left(w \right) - \beta \left(w \right) \pi_m \end{aligned}$$

Market equilibrium and CAPM

Risk premium and alpha return

In the case of Example #1 & no short-selling constraint, we have:

- $w^{\star} = (33.62\%, 0\%, 8.79\%, 40.65\%, 16.95\%)$
- $(\mu(w^{*}) = 7.63\%, r = 3\%) \Rightarrow \mu(w^{*}) = 4.63\%$

Table 3: Computation of the alpha return (Example #1)

Portfolio	$\mu(w)$	$\mu(w) - r$	$\beta(w \mid w^{*})$	$\pi(w \mid w^{*})$	$\alpha(w \mid w^{*})$
$\mathbf{e}_1$	5.00%	2.00%	0.432	2.00%	0.00%
$\mathbf{e}_2$	7.00%	4.00%	0.970	4.49%	−0.49%
$\mathbf{e}_3$	6.00%	3.00%	0.648	3.00%	0.00%
$\mathbf{e}_4$	10.00%	7.00%	1.512	7.00%	0.00%
$\mathbf{e}_5$	8.00%	5.00%	1.080	5.00%	0.00%
w_{ew}	7.20%	4.20%	0.929	4.30%	−0.10%
w_{gmv}	6.69%	3.69%	0.766	3.55%	0.14%

Portfolio optimization in the presence of a benchmark

Utility function revisited

- b is the benchmark
- The tracking error is:

$$\epsilon = R(w) - R(b) = \sum_{i=1}^{n} w_i R_i - \sum_{i=1}^{n} b_i R_i = w^\top R - b^\top R = (w - b)^\top R$$

- The expected excess return is equal to:

$$\mu(w \mid b) := \mathbb{E}[\epsilon] = (w - b)^\top \mu$$

- The volatility of the tracking error is defined as:

$$\sigma(w \mid b) := \sigma(e) = \sqrt{(w - b)^\top \Sigma (w - b)}$$

Portfolio optimization in the presence of a benchmark

Utility function revisited

- The objective of the investor is then to maximize the expected tracking error with a constraint on the tracking error volatility:

$$w^\star = \arg\max \mu\left(w \mid b\right) \quad \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^\top x = 1 \\ \sigma\left(w \mid b\right) \leq \sigma^\star \end{array} \right.$$

- We have:

$$\begin{array}{rcl} f\left(w \mid b\right) & = & \dfrac{1}{2}\sigma^2\left(w \mid b\right) - \gamma\mu\left(w \mid b\right) \\ & = & \dfrac{1}{2}\left(w-b\right)^\top \Sigma\left(w-b\right) - \gamma\left(w-b\right)^\top \mu \\ & = & \dfrac{1}{2}w^\top \Sigma w - w^\top\left(\gamma\mu + \Sigma b\right) + \underbrace{\dfrac{1}{2}b^\top \Sigma b + \gamma b^\top \mu}_{\text{constant}} \end{array}$$

Portfolio optimization in the presence of a benchmark
QP formulation

We have:

$$
\begin{array}{rcl}
Q & = & \Sigma \\
R & = & \gamma\mu + \Sigma b \\
A & = & \mathbf{1}^\top \\
B & = & 1 \\
C & = & \\
D & = & \\
w^{-} & = & \mathbf{0}_n \text{ (if no short-selling)} \\
w^{+} & = & \mathbf{1}_n \text{ (if no short-selling)}
\end{array}
$$

Portfolio optimization in the presence of a benchmark

Example #2

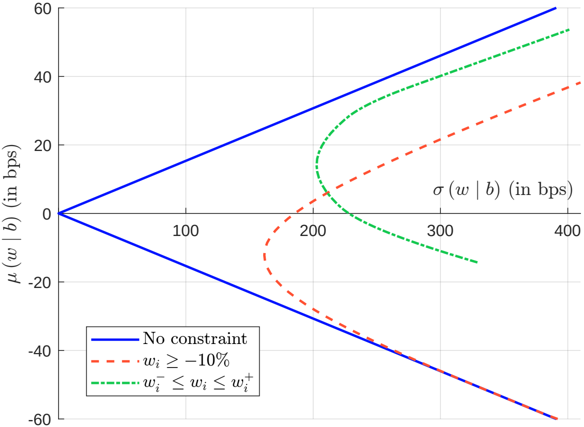
We consider an investment universe of four assets. Their expected returns are equal to 5%, 6.5%, 8% and 6.5% while their volatilities are equal to 15%, 20%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & \\ 10\% & 100\% & & \\ 40\% & 70\% & 100\% & \\ 50\% & 40\% & 80\% & 100\% \end{pmatrix}$$

The benchmark is $b = (60\%, 40\%, 20\%, -20\%)$.

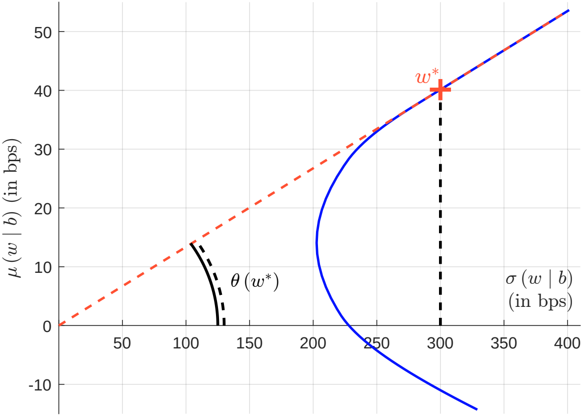
Portfolio optimization in the presence of a benchmark

Figure 3: Efficient frontier with a benchmark (Example #2)



Portfolio optimization in the presence of a benchmark

Figure 4: Tangency portfolio with respect to a benchmark (Example #2)



$\Rightarrow$ the tangency portfolio is equal to $(46.56\%, 33.49\%, 39.95\%, -20.00\%)$

Portfolio optimization in the presence of a benchmark

Information ratio

- We have:

$$\mathrm{IR}\left(w \mid b\right) = \frac{\mu\left(w \mid b\right)}{\sigma\left(w \mid b\right)} = \frac{\left(w - b\right)^\top \mu}{\sqrt{\left(w - b\right)^\top \Sigma \left(w - b\right)}}$$

- If we consider a combination of the benchmark b and the active portfolio w, the composition of the portfolio is:

$$x = \left(1 - \alpha\right) b + \alpha w$$

where $\alpha \geq 0$ is the proportion of wealth invested in the portfolio w

- It follows that:

$$\mu\left(x \mid b\right) = \left(x - b\right)^\top \mu = \alpha \mu\left(w \mid b\right)$$

and:

$$\sigma^2\left(x \mid b\right) = \left(x - b\right)^\top \Sigma \left(x - b\right) = \alpha^2 \sigma^2\left(w \mid b\right)$$

- We deduce that:

$$\mu\left(x \mid b\right) = \mathrm{IR}\left(w \mid b\right) \cdot \sigma\left(x \mid b\right)$$

ESG risk premium

- Expected (or required) returns $\neq$ historical (or realised) returns:

$$\pi_i \neq R_i$$

- Difference between the unconstrained risk premium and the implied risk premium:

$$\pi_i \neq \tilde{\pi}_i$$

The Pastor-Stambaugh-Taylor model
Model settings

- The asset excess returns $\tilde{R} = R - r = \left(\tilde{R}_1, \ldots, \tilde{R}_n\right)$ are normally distributed: $\tilde{R} \sim \mathcal{N}(\pi, \Sigma)$
- Each firm has an ESG characteristic $\mathcal{G}_i$, which is positive for *esg-friendly* (or *green*) firms and negative for *esg-unfriendly* (or *brown*) firms
- $\mathcal{G}_i > 0$ induces positive social impact, while $\mathcal{G}_i < 0$ induces negative externalities on the society
- Economy with a continuum of agents ($j = 1, 2, \ldots, \infty$)
- $w_{i,j}$ is the fraction of the wealth invested by agent j in stock i
- $w_j = (w_{1,j}, \ldots, w_{n,j})$ is the allocation vector of agent j

The Pastor-Stambaugh-Taylor model

Model settings

- The relationship between the initial and terminal wealth W_j and $\tilde{W}_j$ is given by:

$$\tilde{W}_j = \left(1 + r + w_j^\top \tilde{R}\right) W_j$$

- Exponential CARA utility function:

$$\boldsymbol{\mathcal{U}}\left(\tilde{W}_j, w_j\right) = -\exp\left(-\bar{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)$$

where:

  - $\bar{\gamma}_j$ is the absolute risk-aversion
  - $b_j = \varphi_j \boldsymbol{\mathcal{G}}$ is the vector of nonpecuniary benefits ($\varphi_j \geq 0$)

The Pastor-Stambaugh-Taylor model
Optimal portfolio

- The expected utility is equal to:

$$\begin{aligned}
\mathbb{E}\left[\mathcal{U}\left(\tilde{W}_j, w_j\right)\right] &= \mathbb{E}\left[-\exp\left(-\bar{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)\right] \\
&= \mathbb{E}\left[-\exp\left(-\bar{\gamma}_j \left(1 + r + w_j^\top \tilde{R}\right) W_j - w_j^\top b_j W_j\right)\right] \\
&= -e^{-\bar{\gamma}_j (1+r) W_j} \mathbb{E}\left[\exp\left(-\bar{\gamma}_j w_j^\top W_j \left(\tilde{R} + \bar{\gamma}_j^{-1} b_j\right)\right)\right] \\
&= e^{-\bar{\Gamma}_j (1+r)} \mathbb{E}\left[\exp\left(-\bar{\Gamma}_j w_j^\top \left(\tilde{R} + \bar{\gamma}_j^{-1} b_j\right)\right)\right]
\end{aligned}$$

where $\bar{\Gamma}_j = \bar{\gamma}_j W_j$ is the nominal risk aversion

- We notice that $\tilde{R} + \bar{\gamma}_j^{-1} b_j \sim \mathcal{N}\left(\pi + \bar{\gamma}_j^{-1} b_j, \Sigma\right)$ and:

$$-\bar{\Gamma}_j w_j^\top \left(\tilde{R} + \bar{\Gamma}_j^{-1} b_j\right) \sim \mathcal{N}\left(-\bar{\Gamma}_j w_j^\top \left(\pi + \bar{\gamma}_j^{-1} b_j\right), \bar{\Gamma}_j^2 w_j^\top \Sigma w_j\right)$$

The Pastor-Stambaugh-Taylor model
Optimal portfolio

- We deduce that:

$$\mathbb{E}\left[\boldsymbol{\mathcal{U}}\left(\tilde{W}_j, w_j\right)\right] = e^{-\bar{\Gamma}_j(1+r)} \exp\left(-\bar{\Gamma}_j w_j^\top \left(\pi + \bar{\gamma}_j^{-1} b_j\right) + \frac{1}{2}\bar{\Gamma}_j^2 w_j^\top \Sigma w_j\right)$$

- The first-order condition is equal to:

$$-\bar{\Gamma}_j \left(\pi + \bar{\gamma}_j^{-1} b_j\right) + \bar{\Gamma}_j^2 \Sigma w_j = 0$$

Finally, Pastor et al. (2021) concluded that the optimal portfolio is:

$$w_j^\star = \Gamma_j \Sigma^{-1} \left(\pi + \gamma_j b_j\right)$$

where $\Gamma_j = \bar{\Gamma}_j^{-1}$ and $\gamma_j = \bar{\gamma}_j^{-1}$ are the relative nominal and unitary risk-tolerance

The Pastor-Stambaugh-Taylor model

Optimal portfolio

Maximizing the expected utility is equivalent to solve the classical Markowitz QP problem:

$$\begin{aligned} w_j^\star(\gamma_j) &= \arg\min \frac{1}{2} w_j^\top \Sigma w_j - \gamma_j w_j^\top \mu' \\ \text{s.t.} &\quad \mathbf{1}^\top w_j = 1 \end{aligned}$$

where

- $\gamma_j = \bar{\gamma}_j^{-1}$ is the relative risk tolerance
- $\mu' = \mu + \gamma_j b_j$ is the vector of modified expected returns

The Pastor-Stambaugh-Taylor model

Optimal portfolio

Example #3

We consider a universe of n risky assets, where n is an even number. The risk-free rate r is set to 3%. We assume that the Sharpe ratio of these assets is the same and is equal to 20%. The volatility of asset i is equal to $\sigma_i = 0.10 + 0.20 \cdot e^{-n^{-1}\lfloor 0.5i \rfloor}$. The correlation between asset returns is constant: $\mathbb{C} = \mathbb{C}_n(\rho)$. The social impact of the firms is given by the vector $\boldsymbol{\mathcal{G}}$. When $\boldsymbol{\mathcal{G}}$ is not specified, it is equal to the cyclic vector $(+1\%, -1\%, +1\%, \ldots, +1\%, -1\%)$. This implies that half of the firms (green firms) have a positive social impact while the others (brown firms) have a negative impact.

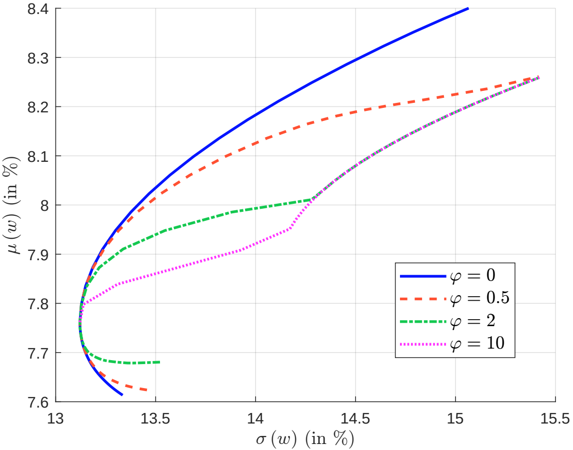
The Pastor-Stambaugh-Taylor model
Optimal portfolio

Table 4: Mean-variance optimized portfolios with ESG preferences (Example #3, $n = 6$, $\rho = 25\%$)

	$\mathcal{G} = (1\%, -1\%, 1\%, -1\%, 1\%, -1\%)$				$\mathcal{G} = (10\%, 5\%, 2\%, 3\%, 25\%, 30\%)$			
φ	0.00%	1.00%	5.00%	50.00%	0.00%	0.50%	1.00%	2.00%
$w_1^\star$	44.97%	48.87%	58.65%	67.48%	44.97%	46.83%	28.69%	0.00%
$w_2^\star$	44.97%	41.06%	19.60%	0.00%	44.97%	37.06%	9.17%	0.00%
$w_3^\star$	5.03%	9.82%	21.75%	32.52%	5.03%	0.00%	0.00%	0.00%
$w_4^\star$	5.03%	0.25%	0.00%	0.00%	5.03%	0.00%	0.00%	0.00%
$w_5^\star$	0.00%	0.00%	0.00%	0.00%	0.00%	0.83%	16.62%	21.09%
$w_6^\star$	0.00%	0.00%	0.00%	0.00%	0.00%	15.28%	45.53%	78.91%
$\mu(w^\star)$	8.33%	8.33%	8.27%	8.22%	8.33%	8.23%	7.79%	7.43%
$\sigma(w^\star)$	20.00%	20.09%	20.07%	21.56%	20.00%	19.33%	16.70%	19.17%
$\mathrm{SR}(w^\star \mid r)$	0.27	0.27	0.26	0.24	0.27	0.27	0.29	0.23

The Pastor-Stambaugh-Taylor model
Optimal portfolio

Figure 5: Efficient frontier with ESG preferences (Example #3, $n = 20$, $\rho = 25\%$)



The Pastor-Stambaugh-Taylor model

Risk premium

- $W = \int W_j \,\mathrm{d}j$
- $\omega_j = W_j / W$ is the market share of the economic agent j
- $W_{i,j} = w_{i,j}^{\star} W_j = w_{i,j}^{\star} \omega_j W$
- We have:

$$W_i = \int_j W_{i,j} \,\mathrm{d}j = \int_j w_{i,j}^{\star} \omega_j W \,\mathrm{d}j$$

- Let $w_m = (w_{1,m}, \ldots, w_{n,m})$ be the market portfolio. We have:

$$w_{i,m} = \frac{W_i}{W} = \int_j w_{i,j}^{\star} \omega_j \,\mathrm{d}j$$

and $\int_j \omega_j \,\mathrm{d}j = 1$

The Pastor-Stambaugh-Taylor model

Risk premium

- The market clearing condition satisfies:

$$\begin{aligned} w_m &= \int_j \omega_j w_j^\star \, \mathrm{d}j \\ &= \int_j \omega_j \Gamma_j \Sigma^{-1} \left(\pi + \gamma_j b_j\right) \mathrm{d}j \\ &= \int_j \omega_j \Gamma_j \Sigma^{-1} \left(\pi + \gamma_j \varphi_j \boldsymbol{\mathcal{G}}\right) \mathrm{d}j \\ &= \left(\int_j \Gamma_j \omega_j \, \mathrm{d}j\right) \Sigma^{-1} \pi + \left(\int_j \omega_j \Gamma_j \psi_j \, \mathrm{d}j\right) \Sigma^{-1} \boldsymbol{\mathcal{G}} \end{aligned}$$

  where $\psi_j = \gamma_j \varphi_j$
- It follows that:

$$w_m = \Gamma_m \Sigma^{-1} \pi + \Gamma_m \psi_m \Sigma^{-1} \boldsymbol{\mathcal{G}}$$

  where $\Gamma_m = \int_j \Gamma_j \omega_j \, \mathrm{d}j$ and $\psi_m = \Gamma_m^{-1} \left(\int_j \omega_j \Gamma_j \psi_j \, \mathrm{d}j\right)$ are the average risk tolerance and the weighted average of ESG preferences

The Pastor-Stambaugh-Taylor model

Risk premium

- The asset risk premia are equal to:

$$\pi = \frac{1}{\Gamma_m} \Sigma w_m - \psi_m \mathcal{G}$$

while the market risk premium is defined as:

$$\begin{aligned} \pi_m &= w_m^\top \pi \\ &= \frac{1}{\Gamma_m} w_m^\top \Sigma w_m - \psi_m w_m^\top \mathcal{G} \\ &= \frac{1}{\Gamma_m} \sigma_m^2 - \psi_m \mathcal{G}_m \end{aligned}$$

where $\sigma_m = \sqrt{w_m^\top \Sigma w_m}$ and $\mathcal{G}_m = w_m^\top \mathcal{G}$ are the volatility and the green intensity (or greenness) of the market portfolio

The Pastor-Stambaugh-Taylor model

Risk premium

- The risk premium including the ESG sentiment is lower than the CAPM risk premium if the market ESG intensity is positive:

$$\mathcal{G}_m > 0 \Longrightarrow \pi_m \leq \pi_m^{\mathrm{capm}}$$

- It is greater than the CAPM risk premium if the market ESG intensity is negative:

$$\mathcal{G}_m < 0 \Longrightarrow \pi_m \geq \pi_m^{\mathrm{capm}}$$

- The gap $\Delta\pi_m^{\mathrm{esg}} := \left|\pi_m - \pi_m^{\mathrm{capm}}\right|$ is an increasing function of the market ESG sentiment ψ_m:

$$\psi_m \nearrow \Longrightarrow \Delta\pi_m^{\mathrm{esg}} \nearrow$$

The Pastor-Stambaugh-Taylor model

Risk premium

If we assume that $\mathcal{G}_m \approx 0$, we have $\Gamma_m = \sigma_m^2 / \pi_m$,

$$\pi = \beta \pi_m - \psi_m \mathcal{G}$$

and:

$$\alpha_i = \pi_i - \beta_i \pi_m = -\psi_m \mathcal{G}_i$$

If $\psi_m > 0$, "green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas" (Pastor et al., 2021).

The Pastor-Stambaugh-Taylor model
Risk premium

Example #4

We consider Example #3. The market is made up of two long-only investors ($j = 1, 2$): a non-ESG investor ($\varphi_1 = 0$) and an ESG investor ($\varphi_2 > 0$). We assume that they have the same risk tolerance γ. We note W_1 and W_2 their financial wealth, which is entirely invested in the risky assets. We assume that $W_1 = W_2 = 1$.

The Pastor-Stambaugh-Taylor model
Risk premium

- The tangency portfolio is equal to:

$$\begin{aligned} w^* &= \frac{\Sigma^{-1}(\mu - r\mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})} \\ &= (15.04\%, 15.04\%, 16.65\%, 16.65\%, 18.31\%, 18.31\%) \end{aligned}$$

- $w_1^\star = w^*$ and $\gamma_1 = 1/\left(\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})\right) = 0.4558$
- $\gamma_2 = \gamma_1$ and:

$$\begin{aligned} w_2^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - \gamma_2 w^\top (\mu + \gamma_2 \varphi_2 \boldsymbol{\mathcal{G}}) \\ & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^\top w = 1 \\ w \geq \mathbf{0} \end{array} \right. \end{aligned}$$

- We obtain

$$w_2^\star = (18.86\%, 11.22\%, 21.33\%, 11.97\%, 23.96\%, 12.65\%)$$

The Pastor-Stambaugh-Taylor model

Risk premium

- The market portfolio is then equal to:

$$\begin{aligned} w_m &= \frac{W_1}{W} w_1^\star + \frac{W_2}{W} w_2^\star \\ &= \left(1 - \omega^{\mathrm{esg}}\right) \cdot w_1^\star + \omega^{\mathrm{esg}} \cdot w_2^\star \end{aligned}$$

- When $W_1 = W_2 = 1$, we obtain

$$\begin{aligned} w_m &= (16.95\%, 13.13\%, 18.99\%, 14.31\%, 21.13\%, 15.48\%) \\ \mu_m &= 7.86\% \\ \sigma_m &= 14.93\% \end{aligned}$$

- We deduce that:

$$\begin{aligned} \beta &= (1.15, 1.05, 1.04, 0.95, 0.95, 0.86) \\ \pi &= (5.58\%, 5.12\%, 5.06\%, 4.61\%, 4.62\%, 4.17\%) \\ \alpha &= (-19.09, 26.19, -19.43, 25.84, -19.72, 25.55) \quad \text{(in bps)} \end{aligned}$$

The Pastor-Stambaugh-Taylor model

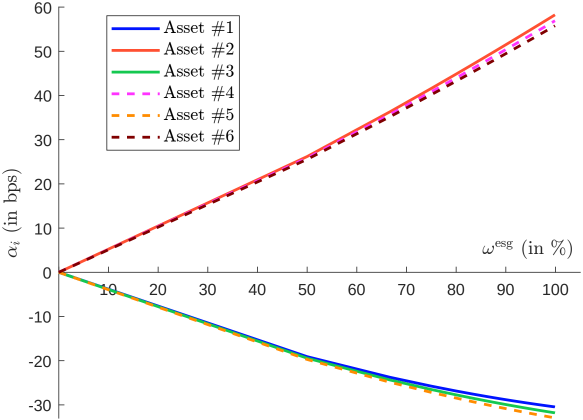
Risk premium

Table 5: Computation of alpha returns (Example #4, $n = 6$, $\rho = 25\%$)

i	Portfolio $w_1^\star$			Portfolio $w_2^\star$				Portfolio w_m			
	w_i (in %)	β_i	π_i (in %)	w_i (in %)	β_i	π_i (in %)	α_i (in bps)	w_i (in %)	β_i	π_i (in %)	α_i (in bps)
1	15.04	1.11	5.39	18.86	1.17	5.69	-30	16.95	1.15	5.58	-19
2	15.04	1.11	5.39	11.22	0.99	4.80	58	13.13	1.05	5.12	26
3	16.65	1.00	4.87	21.33	1.07	5.18	-32	18.99	1.04	5.06	-19
4	16.65	1.00	4.87	11.97	0.88	4.30	57	14.31	0.95	4.61	26
5	18.31	0.91	4.43	23.96	0.98	4.76	-33	21.13	0.95	4.62	-20
6	18.31	0.91	4.43	12.65	0.80	3.87	56	15.48	0.86	4.17	26

The Pastor-Stambaugh-Taylor model
Risk premium

Figure 6: Evolution of the alpha return with respect to the market share of ESG investors (Example #4, $n = 6$, $\rho = 25\%$)



The Pastor-Stambaugh-Taylor model
Risk premium

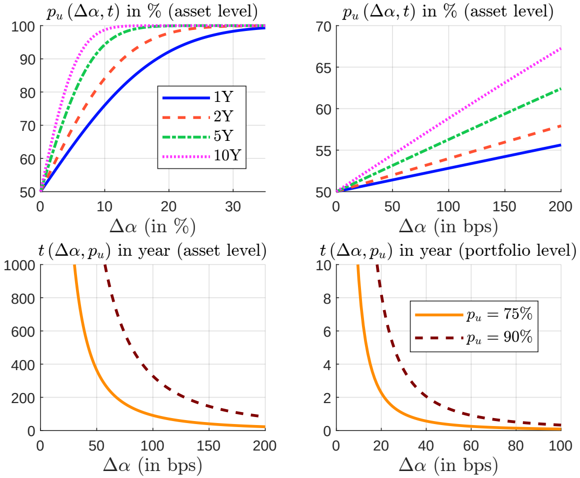
"In equilibrium, green assets have low expected returns because investors enjoy holding them and because green assets hedge climate risk. Green assets nevertheless outperform when positive shocks hit the ESG factor, which captures shifts in customers' tastes for green products and investors' tastes for green holdings." (Pastor et al., 2021).

- **ESG risk premium?**
- **Green risk premium?**

The Pastor-Stambaugh-Taylor model

What does equilibrium mean?

Figure 7: Impact of alpha returns on the underperformance probability



Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

- We have:

$$\begin{pmatrix} \tilde{R} \\ \boldsymbol{\mathcal{S}} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \pi \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma & \Sigma_{\pi,s} \\ \Sigma_{s,\pi} & \Sigma_s \end{pmatrix}\right)$$

- The optimal solution is:

$$w_j^\star = \underbrace{\Gamma_j \Sigma^{-1}\left(\pi + \psi_j \mu_s\right)}_{\text{PST solution}} + \underbrace{\Gamma_j^{-1}\Omega_j\left(\pi + \psi_j \mu_s\right)}_{\text{ESG uncertainty}}$$

Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

- If there is no ESG uncertainty ($\boldsymbol{\mathcal{S}} = \mu_s$ and $\Sigma_s = \mathbf{0}$), the vector of risk premia is given by:

$$\begin{aligned}\pi^{\mathrm{esg}} &= \beta \pi_m - \psi_m \left(\mu_s - \beta \bar{\boldsymbol{\mathcal{S}}}_m\right) \\ &= \pi^{\mathrm{capm}} - \psi_m \left(\mu_s - \beta \bar{\boldsymbol{\mathcal{S}}}_m\right)\end{aligned}$$

- If there is an uncertainty on ESG scores ($\boldsymbol{\mathcal{S}} \neq \mu_s$ and $\Sigma_s \neq \mathbf{0}$), the vector of risk premia becomes:

$$\begin{aligned}\breve{\pi}^{\mathrm{esg}} &= \breve{\beta} \breve{\pi}_m - \psi_m \left(\breve{\mu}_s - \breve{\beta} \breve{\boldsymbol{\mathcal{S}}}_m\right) \\ &= \beta \pi_m + \left(\breve{\beta} - \beta\right) \pi_m - \psi_m \left(\breve{\mu}_s - \breve{\beta} \breve{\boldsymbol{\mathcal{S}}}_m\right)\end{aligned}$$

Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

"In equilibrium, the market premium increases and demand for stocks declines under ESG uncertainty. In addition, the CAPM alpha and effective beta both rise with ESG uncertainty and the negative ESG-alpha relation weakens." (Avramov et al., 2022).

The Pedersen-Fitzgibbons-Pomorski model
Model settings

- $\tilde{R} = R - r \sim \mathcal{N}(\pi, \Sigma)$
- $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_1, \ldots, \boldsymbol{\mathcal{S}}_n)$
- The terminal wealth is $\tilde{W} = \left(1 + r + w^\top \tilde{R}\right) W$
- The model uses the mean-variance utility:

$$\begin{aligned}\boldsymbol{\mathcal{U}}\left(\tilde{W}, w\right) &= \mathbb{E}\left[\tilde{W}\right] - \frac{\bar{\gamma}}{2}\operatorname{var}\left(\tilde{W}\right) + \zeta\left(\boldsymbol{\mathcal{S}}(w)\right) W \\ &= \left(1 + r + w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta\left(w^\top \boldsymbol{\mathcal{S}}\right)\right) W\end{aligned}$$

where ζ is a function that depends on the investor

The Pedersen-Fitzgibbons-Pomorski model

Model settings

- Optimizing the utility function is equivalent to find the mean-variance-esg optimized portfolio:

$$\begin{aligned} w^{\star} &= \arg\max w^{\top}\pi - \frac{\bar{\gamma}}{2} w^{\top}\Sigma w + \zeta\left(w^{\top}\boldsymbol{\mathcal{S}}\right) \\ \text{s.t.} &\quad \mathbf{1}^{\top} w = 1 \end{aligned}$$

- $\sigma(w) = \sqrt{w^{\top}\Sigma w}$
- $\boldsymbol{\mathcal{S}}(w) = w^{\top}\boldsymbol{\mathcal{S}}$

The Pedersen-Fitzgibbons-Pomorski model

Model settings

- The optimization problem can be decomposed as follows:

$$w^{\star} = \arg \left\{ \max_{\bar{\boldsymbol{\mathcal{S}}}} \left\{ \max_{\bar{\sigma}} \left\{ \max_{w} \left\{ f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) \text{ s.t. } w \in \Omega\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) \right\} \right\} \right\} \right\}$$

where:

$$f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) = w^{\top}\pi - \frac{\bar{\gamma}}{2}\sigma^{2}\left(w\right) + \zeta\left(\boldsymbol{\mathcal{S}}\left(w\right)\right)$$

and:

$$\Omega = \left\{ w \in \mathbb{R}^{n} : \mathbf{1}^{\top} w = 1, \sigma\left(w\right) = \bar{\sigma}, \boldsymbol{\mathcal{S}}\left(w\right) = \bar{\boldsymbol{\mathcal{S}}} \right\}$$

The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

- We consider the $\sigma - \boldsymbol{\mathcal{S}}$ problem:

$$
\begin{aligned}
w^{\star}\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) & = & \arg\max w^{\top}\pi \\
& & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top} w = 1 \\ w^{\top}\Sigma w - \bar{\sigma}^{2} = 0 \\ w^{\top}\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = 0 \end{array} \right.
\end{aligned}
$$

- The Lagrange function is:

$$
\mathcal{L}\left(w; \lambda_1, \lambda_2\right) = w^{\top}\pi + \lambda_1\left(w^{\top}\Sigma w - \bar{\sigma}^{2}\right) + \lambda_2\left(w^{\top}\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)
$$

- The first-order condition is:

$$
\frac{\partial\, \mathcal{L}\left(w; \lambda_1, \lambda_2\right)}{\partial\, w} = \pi + 2\lambda_1\Sigma w + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0}
$$

- We deduce that the optimal portfolio is given by:

$$
w = -\frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)
$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- The second constraint $w^\top \left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right) = 0$ implies that:

$$
\begin{aligned}
(*) \quad & \Leftrightarrow \quad \left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right)^\top \frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi + \lambda_2\left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right)\right) = 0 \\
& \Leftrightarrow \quad \lambda_2 = -\frac{\left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right)^\top \Sigma^{-1}\pi}{\left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right)^\top \Sigma^{-1}\left(\mathcal{S} - \bar{\mathcal{S}}\mathbf{1}\right)} \\
& \Leftrightarrow \quad \lambda_2 = \frac{\bar{\mathcal{S}}\left(\mathbf{1}^\top\Sigma^{-1}\pi\right) - \mathcal{S}^\top\Sigma^{-1}\pi}{\mathcal{S}^\top\Sigma^{-1}\mathcal{S} - 2\bar{\mathcal{S}}\left(\mathbf{1}^\top\Sigma^{-1}\mathcal{S}\right) + \bar{\mathcal{S}}^2\left(\mathbf{1}^\top\Sigma^{-1}\mathbf{1}\right)} \\
& \Leftrightarrow \quad \lambda_2 = \frac{C_{1,\pi}\bar{\mathcal{S}} - C_{s,\pi}}{C_{s,s} - 2C_{1,s}\bar{\mathcal{S}} + C_{1,1}\bar{\mathcal{S}}^2}
\end{aligned}
$$

where $C_{x,y}$ is the compact notation for $x^\top\Sigma^{-1}y$ — $C_{1,\pi} = \mathbf{1}^\top\Sigma^{-1}\pi$, $C_{s,\pi} = \mathcal{S}^\top\Sigma^{-1}\pi$, $C_{s,s} = \mathcal{S}^\top\Sigma^{-1}\mathcal{S}$, $C_{1,s} = \mathbf{1}^\top\Sigma^{-1}\mathcal{S}$ and $C_{1,1} = \mathbf{1}^\top\Sigma^{-1}\mathbf{1}$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- Using the first constraint $w^\top \Sigma w - \bar{\sigma}^2 = 0$, we deduce that:

$$\begin{aligned}
\bar{\sigma}^2 &= -\frac{1}{2\lambda_1} w^\top \Sigma \Sigma^{-1} \left(\pi + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1} \right) \right) \\
&= -\frac{1}{2\lambda_1} \left(w^\top \pi + \lambda_2 w^\top \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1} \right) \right) \\
&= -\frac{1}{2\lambda_1} w^\top \pi \\
&= \frac{1}{4\lambda_1^2} \pi^\top \Sigma^{-1} \left(\pi + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1} \right) \right)
\end{aligned}$$

- The first Lagrange coefficient is then equal to ($C_{\pi,\pi} = \pi^\top \Sigma^{-1} \pi$):

$$\begin{aligned}
\lambda_1 &= -\frac{1}{2\bar{\sigma}} \sqrt{\pi^\top \Sigma^{-1} \pi + \lambda_2 \left(\pi^\top \Sigma^{-1} \boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \left(\pi^\top \Sigma^{-1} \mathbf{1} \right) \right)} \\
&= -\frac{1}{2\bar{\sigma}} \sqrt{C_{\pi,\pi} - \frac{\left(C_{1,\pi} \bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi} \right)^2}{C_{s,s} - 2 C_{1,s} \bar{\boldsymbol{\mathcal{S}}} + C_{1,1} \bar{\boldsymbol{\mathcal{S}}}^2}}
\end{aligned}$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- The optimal portfolio is the product of the volatility $\bar{\sigma}$ and the vector $\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)$:

$$\begin{aligned} w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right) &= -\frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi+\lambda_2\left(\boldsymbol{\mathcal{S}}-\bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right) \\ &= \bar{\sigma}\cdot\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right) \end{aligned}$$

where:

$$\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)=\frac{1}{\lambda_1'}\Sigma^{-1}\left(\pi+\lambda_2\left(\boldsymbol{\mathcal{S}}-\bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)$$

and:

$$\lambda_1'=\sqrt{C_{\pi,\pi}-\frac{\left(C_{1,\pi}\bar{\boldsymbol{\mathcal{S}}}-C_{s,\pi}\right)^2}{C_{s,s}-2C_{1,s}\bar{\boldsymbol{\mathcal{S}}}+C_{1,1}\bar{\boldsymbol{\mathcal{S}}}^2}}$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

Example #5

We consider an investment universe of four assets. Their expected returns are equal to 6%, 7%, 8% and 10% while their volatilities are equal to 15%, 20%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & \\ 20\% & 100\% & & \\ 30\% & 50\% & 100\% & \\ 40\% & 60\% & 70\% & 100\% \end{pmatrix}$$

The risk-free rate is set to 2%. The ESG score vector is $\boldsymbol{\mathcal{S}} = (3\%, 2\%, -2\%, -3\%)$.

The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

- We obtain $C_{1,\pi} = 2.4864$, $C_{s,\pi} = 0.0425$, $C_{s,s} = 0.1274$, $C_{1,s} = 1.9801$, $C_{1,1} = 64.1106$ and $C_{\pi,\pi} = 0.1193$
- If we target $\bar{\sigma} = 20\%$ and $\bar{\boldsymbol{\mathcal{S}}} = 1\%$, we deduce that $\lambda_1 = -0.8514$ and $\lambda_2 = -0.1870$
- The optimal portfolio is then:

$$w^{\star}\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) = \begin{pmatrix} 59.31\% \\ 29.52\% \\ 21.76\% \\ 20.72\% \end{pmatrix}$$

- It follows that the portfolio is leveraged since we have $w_r = 1 - \mathbf{1}^\top w = -31.31\%$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- We verify that $\sqrt{w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\Sigma w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)} = 20\%$ and $\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\boldsymbol{\mathcal{S}}\right)\Big/\left(1^{\top}w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)\right) = 1\%$
- We also notice that:

$$\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right) = \begin{pmatrix} 2.9657 \\ 1.4759 \\ 1.0881 \\ 1.0358 \end{pmatrix}$$

  and verify that $w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right) = \bar{\sigma}\cdot\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)$
- The portfolio is then leveraged when $\bar{\sigma} \geq 1/\left(\mathbf{1}^{\top}\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right) = 17.75\%$.

The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

- We rewrite the first-order condition as:

$$
\begin{aligned}
(*) \quad & \Leftrightarrow \quad \pi + 2\lambda_1 \Sigma w + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0} \\
& \Leftrightarrow \quad w^\top \pi + 2\lambda_1 w^\top \Sigma w + \lambda_2 w^\top \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0} \\
& \Leftrightarrow \quad w^\top \pi + 2\lambda_1 \bar{\sigma}^2 = \mathbf{0} \\
& \Leftrightarrow \quad \lambda_1 = -\frac{1}{2}\frac{w^\top \pi}{\bar{\sigma}^2} = -\frac{1}{2}\frac{\mathrm{SR}\left(w \mid r\right)}{\bar{\sigma}}
\end{aligned}
$$

- We deduce that the Sharpe ratio of the optimal portfolio is:

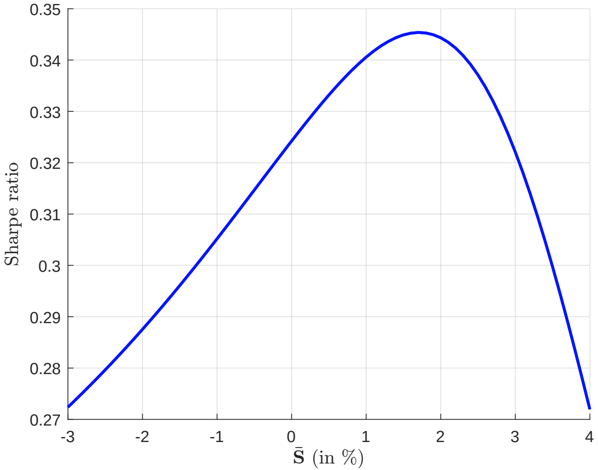
$$
\mathrm{SR}\left(w^\star\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) \mid r\right) = \sqrt{C_{\pi,\pi} - \frac{\left(C_{1,\pi}\bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi}\right)^2}{C_{s,s} - 2C_{1,s}\bar{\boldsymbol{\mathcal{S}}} + C_{1,1}\bar{\boldsymbol{\mathcal{S}}}^2}} = \mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}} \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right)
$$

- It depends on the asset parameters π, Σ, $\boldsymbol{\mathcal{S}}$, the ESG objective $\bar{\boldsymbol{\mathcal{S}}}$ of the investor, but not the volatility target $\bar{\sigma}$

The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

Figure 8: Relationship between $\bar{\mathcal{S}}$ and $\mathrm{SR}\left(\bar{\mathcal{S}} \mid \pi, \Sigma\right)$ (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

Using Example #5

- The Sharpe ratio of the optimal portfolio $w^\star(20\%, 1\%)$ is equal to 0.3406
- We have $\mathrm{SR}(w^\star(\bar{\sigma}, -3\%) \mid r) = 0.2724$, $\mathrm{SR}(w^\star(\bar{\sigma}, -2\%) \mid r) = 0.2875$, $\mathrm{SR}(w^\star(\bar{\sigma}, -1\%) \mid r) = 0.3052$, $\mathrm{SR}(w^\star(\bar{\sigma}, 0\%) \mid r) = 0.3242$, $\mathrm{SR}(w^\star(\bar{\sigma}, 1\%) \mid r) = 0.3406$, $\mathrm{SR}(w^\star(\bar{\sigma}, 2\%) \mid r) = 0.3443$, and $\mathrm{SR}(w^\star(\bar{\sigma}, 3\%) \mid r) = 0.3221$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- The objective function is equal to:

$$\begin{aligned} f\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right);\pi,\Sigma,\boldsymbol{\mathcal{S}}\right) &= \left(\frac{w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\pi}{\bar{\sigma}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \\ &= \mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \end{aligned}$$

- The σ-problem becomes:

$$\begin{aligned} (*) &= \max_{\bar{\sigma}}\left\{\max_{w}\left\{f\left(w;\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\text{ s.t. } w\in\Omega\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)\right\}\right\} \\ &= \max_{\bar{\sigma}}\left\{\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right\} \end{aligned}$$

- The first-order condition is $\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)-\bar{\gamma}\bar{\sigma}=0$ or $\bar{\sigma}=\bar{\gamma}^{-1}\,\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)$

The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

- We have:

$$\begin{aligned} f\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right);\pi,\Sigma,\boldsymbol{\mathcal{S}}\right) &= \bar{\gamma}^{-1}\,\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)- \\ & \quad \frac{1}{2}\bar{\gamma}^{-1}\,\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \\ &= \frac{1}{2}\bar{\gamma}^{-1}\left(\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right) \end{aligned}$$

- We conclude that the $\boldsymbol{\mathcal{S}}$-problem becomes:

$$\boldsymbol{\mathcal{S}}^{\star}=\arg\max_{\bar{\boldsymbol{\mathcal{S}}}}\left\{\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right\}$$

- The optimal portfolio is $w^{\star}=w^{\star}\left(\sigma^{\star},\boldsymbol{\mathcal{S}}^{\star}\right)$ where $\boldsymbol{\mathcal{S}}^{\star}$ is the solution of the $\boldsymbol{\mathcal{S}}$-problem and $\sigma^{\star}=\bar{\gamma}^{-1}\,\mathrm{SR}\left(\boldsymbol{\mathcal{S}}^{\star}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Pedersen *et al.* (2021) distinguished three groups of investors:

- Type-U or ESG-unware investors have no ESG preference and do not use the information of ESG scores
- Type-A or ESG-aware investors have no ESG preference, but they use the ESG scores to update their views on the risk premia
- Type-M or ESG-motivated investors have ESG preferences, implying that they would like to have a high ESG score

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- Type-U investors hold the same portfolio:

$$w_U^\star = \frac{\Sigma^{-1}\pi}{\mathbf{1}^\top \Sigma^{-1}\pi}$$

- Type-A investors choose the optimal portfolio with the highest Sharpe ratio ($\zeta(s) = 0$) $\Rightarrow \boldsymbol{\mathcal{S}}_A^\star$ is the optimal ESG score
- Type-M investors choose an optimal portfolio on the ESG-SR efficient frontier, with:

$$\boldsymbol{\mathcal{S}}_M^\star \geq \boldsymbol{\mathcal{S}}_A^\star$$

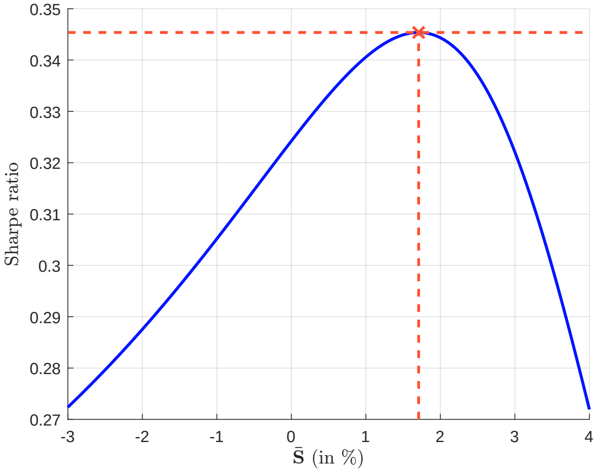
and:

$$\mathrm{SR}\left(\boldsymbol{\mathcal{S}}_M^\star \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) \leq \mathrm{SR}\left(\boldsymbol{\mathcal{S}}_A^\star \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right)$$

The Pedersen-Fitzgibbons-Pomorski model

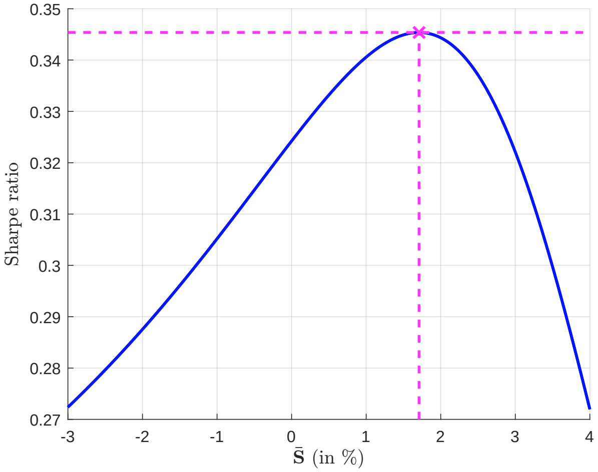
The ESG-SR frontier

Figure 9: Optimal portfolio for type-U investors (Example #5)



The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

Figure 10: Optimal portfolio for type-A investors (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- For type-M investors, we first compute the function $\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right)$:

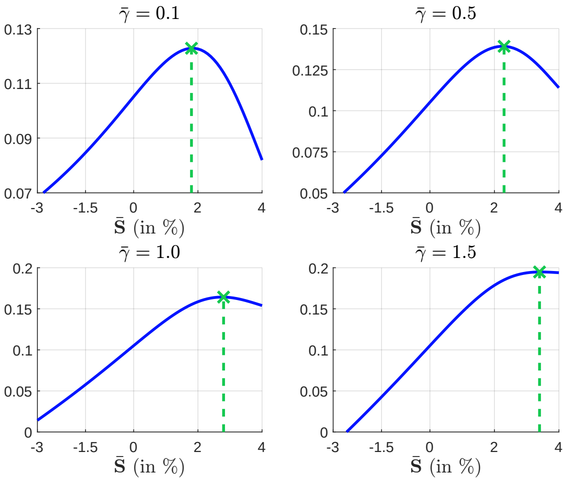
$$\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right) = \mathrm{SR}^2\left(\bar{\boldsymbol{\mathcal{S}}} \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) + 2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)$$

- The optimal portfolio corresponds to the optimal ESG score that maximizes $\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right)$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

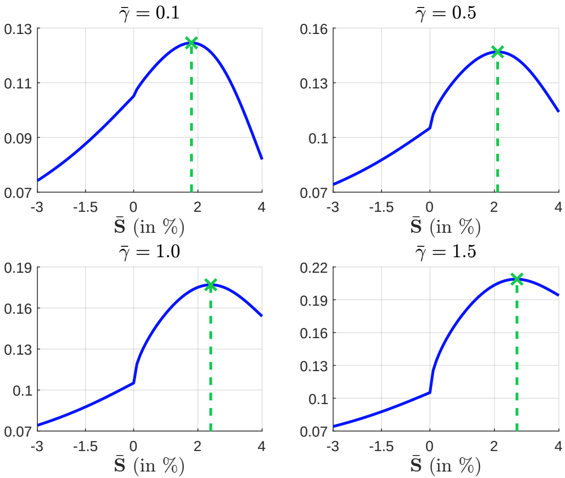
Figure 11: Optimal portfolio for type-M investors when $\zeta(s) = s$ (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Figure 12: Optimal portfolio for type-M investors when $\zeta(s) = 0.2\sqrt{\max(s, 0)}$



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Table 6: Optimal portfolios (Example #5)

Statistics	Type-U	Type-A	Type-M					
			$\zeta(s)=s$			$\zeta(s)=0.2\sqrt{\max(s,0)}$		
$\bar{\gamma}$			0.500	1.000	1.500	0.500	1.000	1.500
$\mathcal{S}(w^\star)$	0.017	0.017	0.023	0.028	0.034	0.021	0.024	0.027
$\sigma(w^\star)$	0.139	0.100	0.682	0.329	0.203	0.687	0.339	0.221
$\mathrm{SR}(w^\star \mid r)$	0.345	0.345	0.341	0.329	0.305	0.343	0.339	0.332
$w_1^\star$	0.524	0.378	3.028	1.623	1.090	2.900	1.542	1.072
$w_2^\star$	0.289	0.208	1.786	1.009	0.718	1.673	0.919	0.660
$w_3^\star$	0.120	0.086	0.383	0.073	−0.056	0.464	0.169	0.065
$w_4^\star$	0.067	0.048	−0.012	−0.144	−0.178	0.106	−0.035	−0.079
$w_r^\star$	0.000	0.280	−4.184	−1.562	−0.574	−4.143	−1.596	−0.718

The Pedersen-Fitzgibbons-Pomorski model

Impact on asset returns

- If $\omega^U = 1$ and $\omega^A = \omega^M = 0$, then unconditional expected returns are given by the CAPM:

$$\mathbb{E}[R_i] - r = \beta_i (\mathbb{E}[R_m] - r)$$

but conditional expected returns depend on the ESG scores:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \beta_i (\mathbb{E}[R_m] - r) + \theta \frac{\boldsymbol{\mathcal{S}}_i - \boldsymbol{\mathcal{S}}_m}{P_i}$$

where P_i is the asset price of asset i

- If $\omega^A = 1$ and $\omega^U = \omega^M = 0$, then the informational value of ESG scores is fully incorporated into asset prices, and we have:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \tilde{\beta}_i (\mathbb{E}[R_m \mid \boldsymbol{\mathcal{S}}] - r)$$

where $\tilde{\beta}_i$ is the ESG-adjusted beta coefficient

- If $\omega^M = 1$ and $\omega^U = \omega^A = 0$, then the conditional expected return is given by:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \tilde{\beta}_i (\mathbb{E}[R_m \mid \boldsymbol{\mathcal{S}}] - r) + \lambda_2 (\boldsymbol{\mathcal{S}}_i - \boldsymbol{\mathcal{S}}_m)$$

The Pedersen-Fitzgibbons-Pomorski model

Impact on asset returns

"If all types of investors exist, then several things can happen. If a security has a higher ESG score, then, everything else equal, its expected return can be ***higher or lower****. A higher ESG score increases the demand for the stock from type-M investors, leading to a higher price and, therefore, a* ***lower required return*** *[...] Companies with poor ESG scores that are down-weighted by type-M investors will have lower prices and higher cost of capital. [...] Furthermore, the force that can increase the expected return is that the higher ESG could be a favorable signal of firm fundamentals, and if many type-U investors ignore this, the fundamental signal perhaps would not be fully reflected in the price [...] A future increase in ESG investing would lead to higher prices for high-ESG stocks [...]. If these flows are unexpected (or not fully captured in the price for other reasons), then high-ESG stocks would experience a* ***return boost*** *during the period of this repricing of ESG. If these flows are expected, then expected returns should not be affected."* (Pedersen et al., 2021)

What is the performance of ESG investing?

According to Coqueret (2022), we can classify the academic studies into four categories:

1. ESG improves performance
2. ESG does not impact performance
3. ESG is financially detrimental
4. The relationship between ESG and performance depends on many factors

What is the performance of ESG investing?

According to Friede *et al.* (2015), the first category dominates the other categories:

> "[...] *The results show that the business case for ESG investing is empirically very well founded. Roughly 90% of studies find a nonnegative ESG – CFP relation. More importantly, the large majority of studies reports positive findings. We highlight that the positive ESG impact on CFP appears stable over time. Promising results are obtained when differentiating for portfolio and non-portfolio studies, regions, and young asset classes for ESG investing such as emerging markets, corporate bonds, and green real estate.*"

⇒ Many dimensions of CFP (cost of capital, G pillar, proxy variables, etc.)

Relationship between ESG and performance in equity markets

We can also find many studies, whose conclusion is more neutral or negative: Barnett and Salomon (2006), Fabozzi *et al.* (2008), Hong and Kacperczyk (2009), Johnson *et al.* (2009), Capelle-Blancard and Monjon (2014), Matos (2020), etc.

$\Rightarrow$ Sin stocks

Mixed results

What is the performance of ESG investing?

- Generally, academic studies that analyze the relationship between ESG and performance are based on long-term historical data, typically the last 20 years or the last 30 years.
- Two issues:
  1. ESG investing was marginal 15+ years ago
  2. ESG data are not robust or relevant before 2010
- The relationship between ESG and performance is dynamic
- Sometimes, ESG may create performance, but sometimes not

Simulated results
Sorted portfolios

Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date t, we rank the stocks according to their Amundi **ESG** z-score $s_{i,t}$
- We form the five quintile portfolios Q_i for $i = 1, \ldots, 5$
- The portfolio Q_i is invested during the period $]t, t+1]$:
    - Q_1 corresponds to the best-in-class portfolio (best scores)
    - Q_5 corresponds to the worst-in-class portfolio (worst scores)
- Quarterly rebalancing
- Universe: MSCI World Index
- Equally-weighted and sector-neutral portfolio (and region-neutral for the world universe)

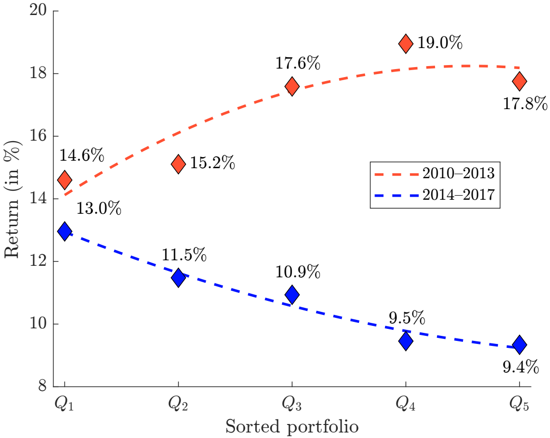
Simulated results
Sorted portfolios

Table 7: An illustrative example

Asset	$\mathcal{S}_i$	Rank	Q_i	Weight
#1	-0.3	6	Q_3	$+50\%$
#2	0.2	5	Q_3	$+50\%$
#3	-1.0	7	Q_4	$+50\%$
#4	1.5	3	Q_2	$+50\%$
#5	-2.9	10	Q_5	$+50\%$
#6	0.8	4	Q_2	$+50\%$
#7	-1.4	8	Q_4	$+50\%$
#8	2.3	2	Q_1	$+50\%$
#9	2.8	1	Q_1	$+50\%$
#10	-2.2	9	Q_5	$+50\%$

Simulated results
Sorted portfolios

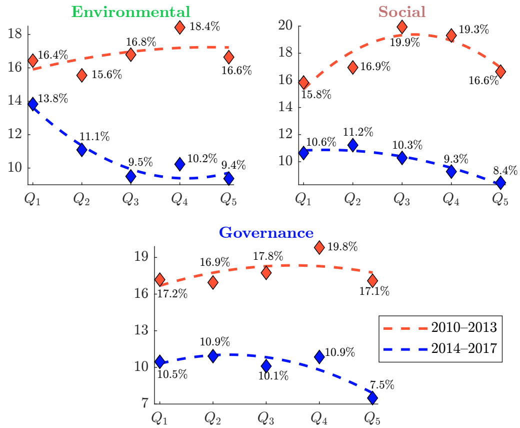
Figure 13: Annualized return of ESG-sorted portfolios (MSCI North America)



Source: Bennani *et al* (2018).

Simulated results
Sorted portfolios

Figure 14: Annualized return of ESG-sorted portfolios (MSCI North America)

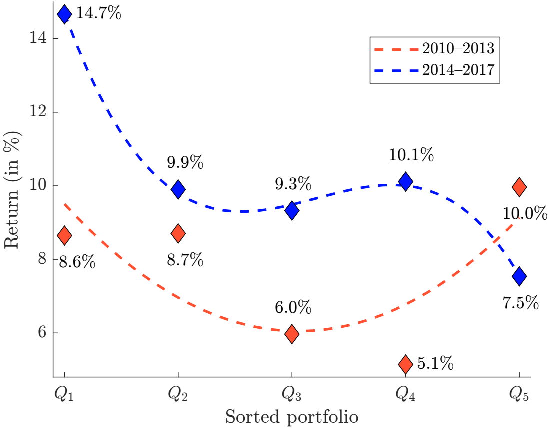


Source: Bennani *et al* (2018).

Simulated results

Sorted portfolios

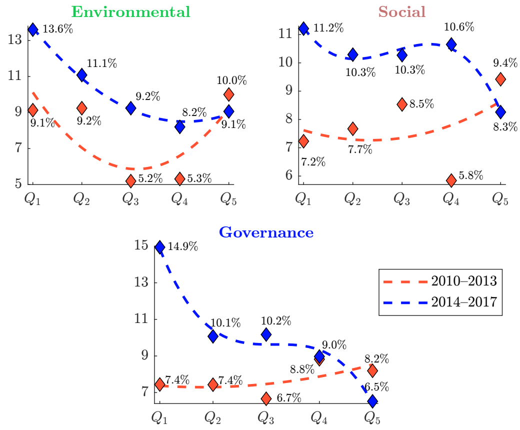
Figure 15: Annualized return of ESG-sorted portfolios (MSCI EMU)



Source: Bennani *et al* (2018).

Simulated results
Sorted portfolios

Figure 16: Annualized return of ESG-sorted portfolios (MSCI EMU)



Source: Bennani *et al* (2018).

Simulated results
Sorted portfolios

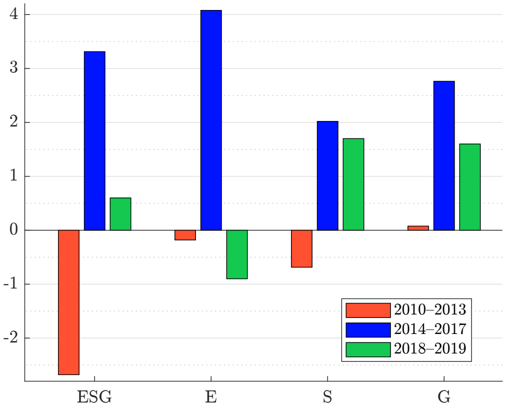
Table 8: Impact of ESG screening on sorted portfolio returns (2010–2017)

Period	Pillar	North America	EMU	Europe-ex-EMU	Japan	World
2010–2013	ESG	−−	−	**0**	+	**0**
	E	−	**0**	+	−	**0**
	S	−	−	**0**	−	−
	G	−	**0**	+	**0**	+
2014–2017	ESG	++	++	**0**	−	+
	E	++	++	−	+	++
	S	+	+	**0**	**0**	+
	G	+	++	**0**	+	++

Source: Bennani *et al* (2018).

Simulated results
Sorted portfolios

Figure 17: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI North America)

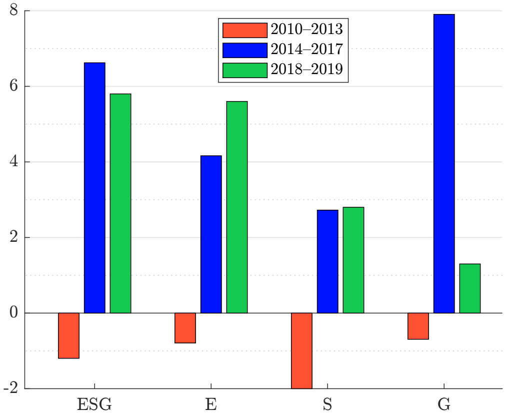


Source: Drei *et al* (2019).

Simulated results

Sorted portfolios

Figure 18: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI EMU)



Source: Drei *et al* (2019).

Simulated results

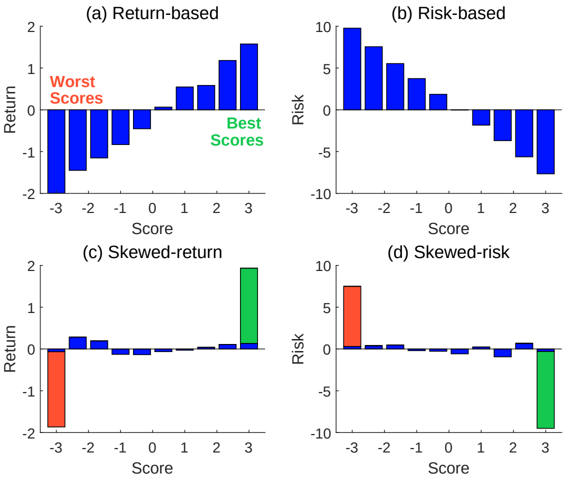
Sorted portfolios

The impact of investment flows

- The 2014 break
  - November 2013: Responsible Investment and the Norwegian Government Pension Fund Global (2013 Strategy Council)
  - Strong mobilization of the largest institutional European investors: NBIM, APG, PGGM, ERAFP, FRR, etc.
  - They are massively invested in European stocks and America stocks:
    $$\text{NBIM} \succ \text{CalPERS} + \text{CalSTRS} + \text{NYSCRF for U.S. stocks}$$
- The 2018–2019 period
  - Implication of U.S. investors continues to be weak
  - Strong mobilization of medium (or tier two) institutional European investors, that have a low exposure on American stocks
  - Mobilization of European investors is not sufficient

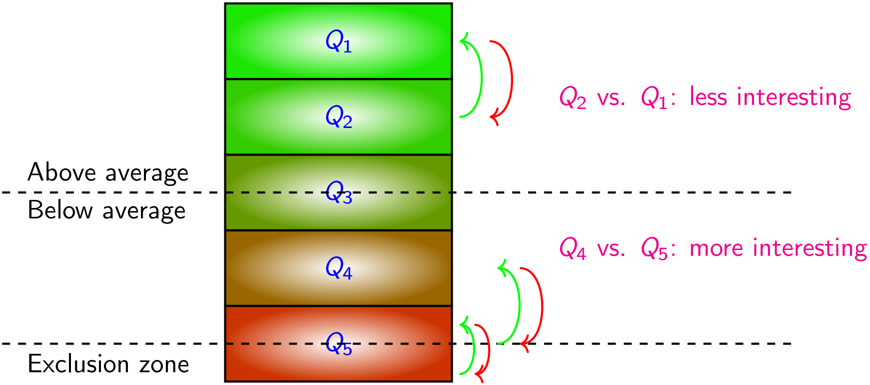
Simulated results
Sorted portfolios

Figure 19: The monotonous assumption of the ESG-performance relationship



Simulated results
Sorted portfolios

Figure 20: How to play ESG momentum?



Simulated results
Optimized portfolios

- We note b the benchmark, $\mathcal{S}$ the vector of ESG scores and Σ the covariance matrix
- We consider the following optimization problem:

$$w^{\star}(\gamma) = \arg\min \frac{1}{2}\sigma^{2}(w \mid b) - \gamma \mathcal{S}(w \mid b)$$

where $\sigma^{2}(w \mid b) = (w - b)^{\top} \Sigma (w - b)$ and $\mathcal{S}(w \mid b)$ are the ex-ante tracking error variance and the ESG excess score of portfolio w with respect to the benchmark b

- Since we have:

$$\mathcal{S}(w \mid b) = (w - b)^{\top} \mathcal{S} = \mathcal{S}(w) - \mathcal{S}(b)$$

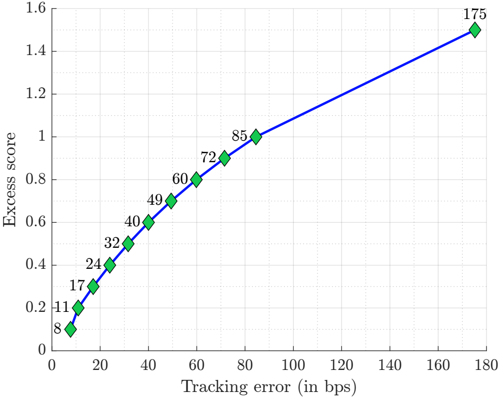
we obtain the following optimization function:

$$w^{\star}(\gamma) = \arg\min \frac{1}{2} w^{\top} \Sigma w - w^{\top} (\gamma \mathcal{S} + \Sigma b)$$

- The QP form is given by $Q = \Sigma$ and $R = \gamma \mathcal{S} + \Sigma b$

Simulated results
Optimized portfolios

Figure 21: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010–2017, global score)

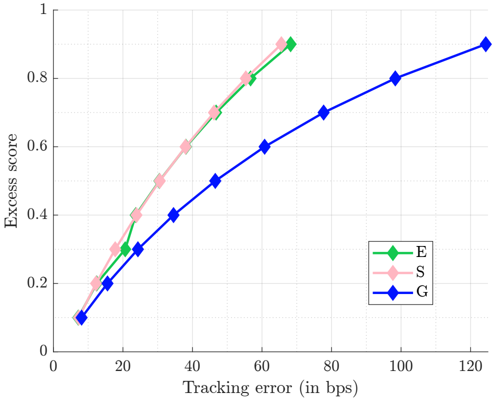


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

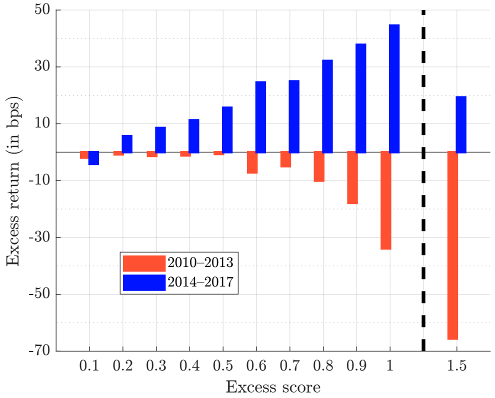
Figure 22: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010–2017, individual pillars)



Source: Bennani *et al* (2018).

Simulated results
Optimized portfolios

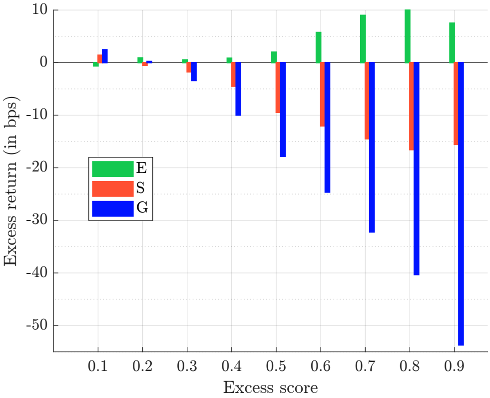
Figure 23: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010–2017, global score)



Source: Bennani *et al* (2018).

Simulated results
Optimized portfolios

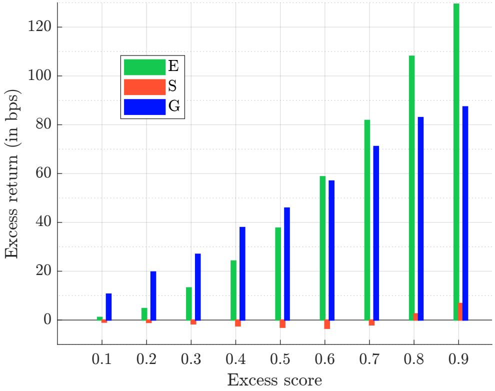
Figure 24: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010–2013, individual pillars)



Source: Bennani *et al* (2018).

Simulated results
Optimized portfolios

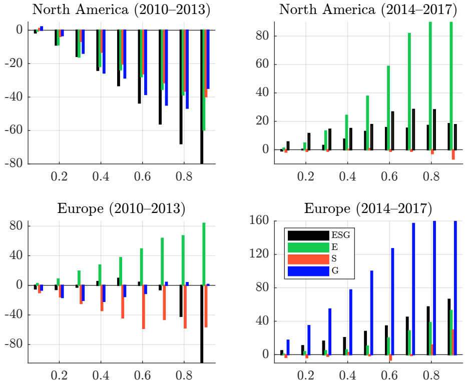
Figure 25: Annualized excess return of ESG-optimized portfolios (MSCI World, 2014–2017, individual pillars)



Source: Bennani *et al* (2018).

Simulated results
Optimized portfolios

Figure 26: Annualized excess return in bps of ESG-optimized portfolios (MSCI North America and EMU, 2010–2017



Source: Bennani *et al* (2018).

Single-factor model

Regression model

The single-factor model is:

$$R_{i,t} = \alpha_{i,j} + \beta_{i,j}\mathcal{F}_{j,t} + \varepsilon_{i,t}$$

where:

- $R_{i,t}$ is the return of stock i at time t
- $\mathcal{F}_{j,t}$ is the value of the j^{th} common risk factor at time t (market, size, value, momentum, low-volatility, quality or ESG)
- $\varepsilon_{i,t}$ is the idiosyncratic risk

The average proportion of the return variance explained by the common factor is given by:

$$\bar{\mathfrak{R}}_j^2 = \frac{1}{n}\sum_{i=1}^{n}\mathfrak{R}_{i,j}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\operatorname{var}\left(\varepsilon_{i,t}\right)}{\operatorname{var}\left(R_{i,t}\right)}\right)$$

Single-factor model

Table 9: Results of cross-section regression with long-only risk factors (single-factor linear regression model, average $\mathfrak{R}^2$)

Factor	North America 2010–2013	North America 2014–2019	Eurozone 2010–2013	Eurozone 2014–2019
Market	40.8%	28.6%	42.8%	36.3%
Size	39.3%	26.1%	37.1%	23.3%
Value	38.9%	26.7%	41.6%	33.6%
Momentum	39.6%	26.3%	40.8%	34.1%
Low-volatility	35.8%	25.1%	38.7%	33.4%
Quality	39.1%	26.6%	42.4%	34.6%
ESG	40.1%	27.4%	42.6%	35.3%

Source: Roncalli (2020).

- Specific risk has increased during the period 2014–2019
- Since 2014, we find that:
  - ESG $\succ$ Value $\succ$ Quality $\succ$ Momentum $\succ$ ... (North America)
  - ESG $\succ$ Quality $\succ$ Momentum $\succ$ Value $\succ$ ... (Eurozone)

Multi-factor model

Regression model

We have:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{m} \beta_{i,j} \mathcal{F}_{j,t} + \varepsilon_{i,t}$$

where m is the number of risk factors

- 1F = market
- 5F = size + value + momentum + low-volatility + quality
- 6F = 5F + ESG

Multi-factor model

Table 10: Results of cross-section regression with long-only risk factors (multi-factor linear regression model, average $\mathfrak{R}^2$)

Model	North America		Eurozone	
	2010–2013	2014–2019	2010–2013	2014–2019
CAPM	40.8%	28.6%	42.8%	36.3%
5F model	46.1%	38.4%	49.5%	45.0%
6F model (5F + ESG)	46.7%	39.7%	50.1%	45.8%

Source: Roncalli (2020).

***p-value statistic for the MSCI Index (time-series, 2014–2019):

- 6F = ~~Size~~, Value, Momentum, Low-volatility, Quality, ~~ESG~~ (North America)
- 6F = Size, Value, Momentum, ~~Low-volatility~~, Quality, ESG (Eurozone)

Factor selection

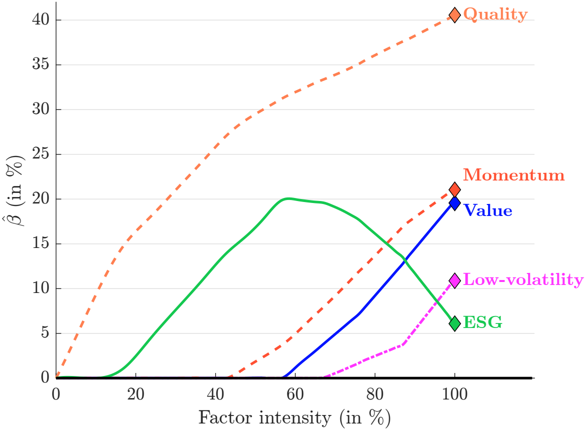
- We use a lasso penalized regression is used in place of the traditional least squares regression:

$$\left\{\hat{\alpha}_i, \hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,m}\right\} = \arg\min \left\{\frac{1}{2}\operatorname{var}\left(\varepsilon_{i,t}\right) + \lambda \left\|\beta_i\right\|_1\right\}$$

- Low-factor intensity ($\lambda \approx \infty$) $\Rightarrow$ we determine which risk factor is the most important
- When the factor intensity reaches 100% ($\lambda = 0$), we obtain the same results calculated previously with the linear regression

Factor selection

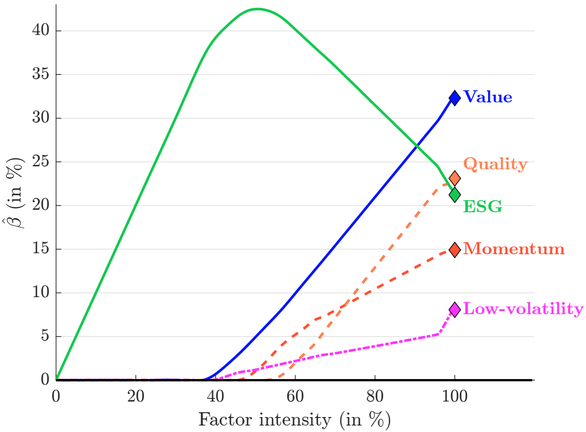
Figure 27: Factor picking (MSCI North America, 2014–2019, global score)



Source: Roncalli (2020).

Factor selection

Figure 28: Factor picking (MSCI EMU, 2014–2019, global score)



Source: Roncalli (2020).

What is the difference between alpha and beta?

α or β?

> *"[...] When an alpha strategy is massively invested, it has an enough impact on the structure of asset prices to become a risk factor.*
> *[...] Indeed, an alpha strategy becomes a common market risk factor once it represents a significant part of investment portfolios and explains the cross-section dispersion of asset returns" (Roncalli, 2020)*

- ESG remains an alpha strategy in North America
- ESG becomes a beta strategy (or a risk factor) in Europe
- Forward looking, ESG will be a beta strategy in North America

Equity indexes

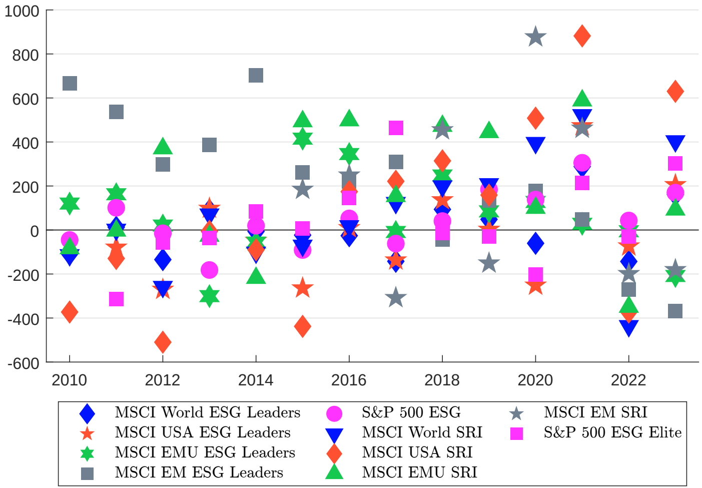
Table 11: Performance of ESG equity indexes (MSCI World, 2010–2023)

Year	Return (in %)			Alpha (in bps)	
	CW	ESG	SRI	ESG	SRI
2010	11.8	10.7	10.6	−109	−114
2011	−5.5	−5.4	−5.5	12	2
2012	15.8	14.5	13.2	−135	−258
2013	26.7	27.6	27.4	89	71
2014	4.9	4.9	3.9	−6	−102
2015	−0.9	−1.1	−1.6	−23	−71
2016	7.5	7.3	7.7	−26	18
2017	22.4	21.0	23.6	−142	124
2018	−8.7	−7.8	−6.7	94	199
2019	27.7	28.2	29.8	48	209
2020	15.9	15.3	19.9	−61	396
2021	21.8	24.7	27.0	288	523
2022	−18.1	−19.6	−22.5	−143	−436
2023	23.8	25.4	27.8	161	404
3Y	7.3	8.0	8.0	67	70
5Y	12.8	13.2	14.4	39	157
7Y	10.7	11.0	12.3	25	161
10Y	8.6	8.7	9.5	11	93

Source: MSCI, Factset & Author's calculation.

Equity indexes

Figure 29: Alpha return of several ESG equity indexes (in bps)



Bond markets ≠ stock markets

Stocks

- ESG scoring is incorporated in portfolio management
- ESG = long-term business risk
  ⇒ strongly impacts the equity
- Portfolio integration
- Managing the business risk

Bonds

- ESG integration is generally limited to exclusions
- ESG lowly impacts the debt
- Portfolio completion
- Fixed income = impact investing
- Development of pure play ESG securities (green and social bonds)

⇒ Stock holders are more ESG sensitive than bond holders because of the capital structure

Bond markets $\neq$ stock markets

ESG investment flows affect asset pricing differently

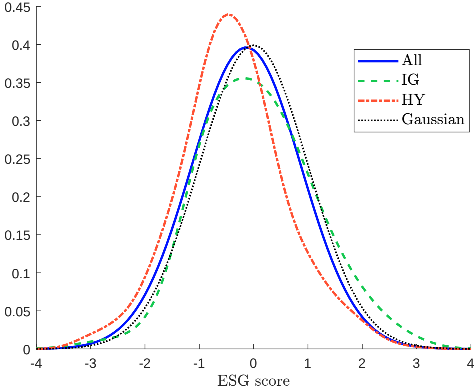
- Impact on carry (coupon effect)?
- Impact on price dynamics (credit spread/mark-to-market effect)?
- Buy-and-hold portfolios $\neq$ managed portfolios

The distinction between IG and HY bonds

- ESG and credit ratings are correlated
- There are more worst-in-class issuers in the HY universe, and best-in-class issuers in the IG universe
- Non-neutrality of the bond universe (bonds $\neq$ stocks)

Bond markets $\neq$ stock markets

Figure 30: Probability density function of ESG scores



Source: Ben Slimane *et al.* (2019).

- The average z-score for IG bonds is positive
- The average z-score for HY bonds is negative

Simulated results
Sorted portfolios

Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date t, we rank the bonds according to their Amundi **ESG** z-score
- We form the five quintile portfolios Q_i for $i = 1, \ldots, 5$
- The portfolio Q_i is invested during the period $]t, t+1]$:
  - Q_1 corresponds to the best-in-class portfolio (best scores)
  - Q_5 corresponds to the worst-in-class portfolio (worst scores)
- Monthly rebalancing
- Universe: ICE (BofAML) Large Cap IG EUR Corporate Bond
- Sector-weighted and sector-neutral portfolio
- Within a sector, bonds are equally-weighted

Simulated results
Sorted portfolios

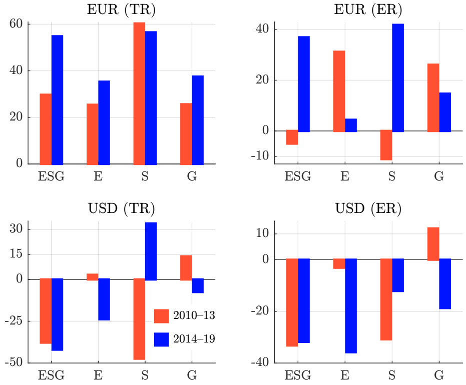


Figure 31: Annualized return in bps of the long short $Q_1 - Q_5$ strategy (IG, 2010–2019)

Source: Ben Slimane *et al.* (2019).

Simulated results

Sorted portfolios

Table 12: Carry statistics (in bps)

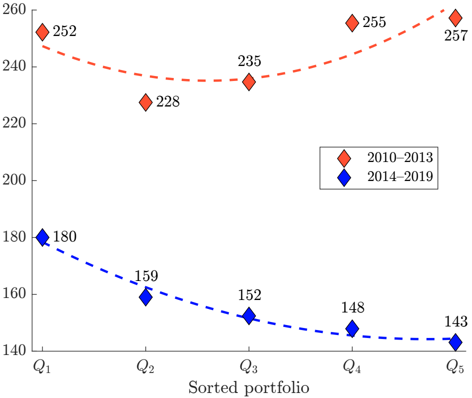
Period	Q_1	Q_5	$Q_1 - Q_5$
2010–2013	175	192	-17
2014–2019	113	128	-15

Source: Ben Slimane *et al.* (2019).

Simulated results

Sorted portfolios

Figure 32: Annualized credit return in bps of ESG sorted portfolios (EUR IG, 2010–2019)



Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

- Portfolio $w = (w_1, \ldots, w_n)$ and benchmark $b = (b_1, \ldots, b_n)$
- ESG score of the portfolio:

$$\mathcal{S}(w) = \sum_{i=1}^{n} w_i \mathcal{S}_i$$

- ESG excess score of portfolio w with respect to benchmark b:

$$\begin{aligned} \mathcal{S}(w \mid b) &= \sum_{i=1}^{n} (w_i - b_i) \mathcal{S}_i \\ &= \mathcal{S}(w) - \mathcal{S}(b) \end{aligned}$$

- z-scores $\Rightarrow \mathcal{S}(w \mid b) > 0$
- Active or tracking risk $\mathcal{R}(w \mid b)$
- The optimization problem becomes:

$$w^{\star}(\gamma) = \arg\min \mathcal{R}(w \mid b) - \gamma \mathcal{S}(w \mid b)$$

Simulated results

Optimized portfolios

- The modified duration risk of portfolio w with respect to benchmark b is:

$$\mathcal{R}_{\mathrm{MD}}(x \mid b) = \sum_{j=1}^{n_S} \left(\left(\sum_{i \in \mathcal{S}ector(j)} w_i \, \mathrm{MD}_i \right) - \left(\sum_{i \in \mathcal{S}ector(j)} b_i \, \mathrm{MD}_i \right) \right)^2$$

  where n_S is the number of sectors and MD_i is the modified duration of bond i

- An alternative is to use the DTS risk measure:

$$\mathcal{R}_{\mathrm{DTS}}(x \mid b) = \sum_{j=1}^{n_S} \left(\left(\sum_{i \in \mathcal{S}ector(j)} w_i \, \mathrm{DTS}_i \right) - \left(\sum_{i \in \mathcal{S}ector(j)} b_i \, \mathrm{DTS}_i \right) \right)^2$$

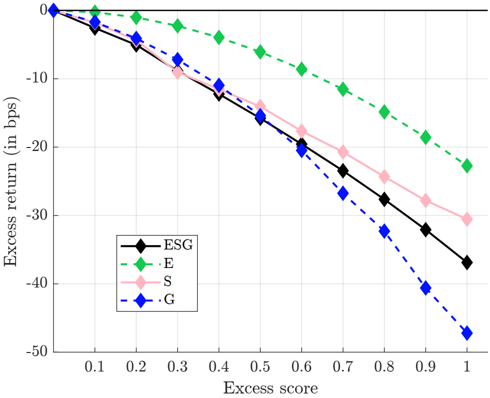
  where DTS_i is the DTS of bond i

- Hybrid approach:

$$\mathcal{R}(w \mid b) = \frac{1}{2}\mathcal{R}_{\mathrm{MD}}(w \mid b) + \frac{1}{2}\mathcal{R}_{\mathrm{DTS}}(w \mid b)$$

Simulated results
Optimized portfolios

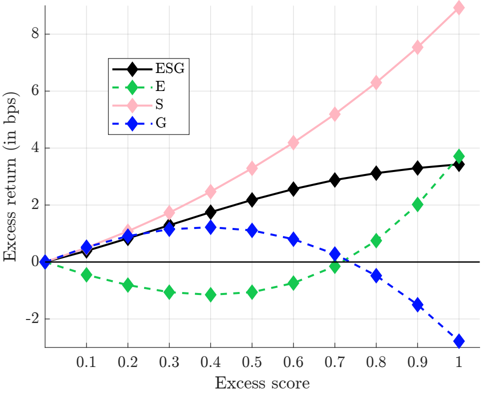
Figure 33: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2010–2013)



Source: Ben Slimane *et al.* (2019).

Simulated results
Optimized portfolios

Figure 34: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2014–2019)

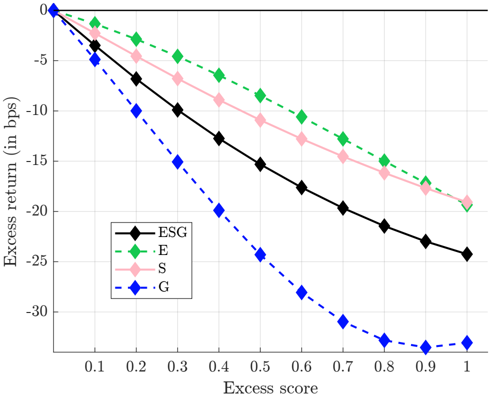


Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

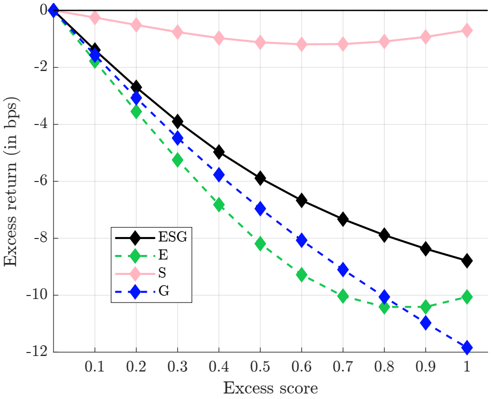
Figure 35: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2010–2013)



Source: Ben Slimane *et al.* (2019).

Simulated results
Optimized portfolios

Figure 36: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2014–2019)



Source: Ben Slimane *et al.* (2019).

Bond indexes

Table 13: Performance of ESG bond indexes (sovereign)

Year	FTSE WGBI Return BM	FTSE WGBI Return ESG	FTSE WGBI Alpha ESG	FTSE EGBI Return BM	FTSE EGBI Return ESG	FTSE EGBI Alpha ESG
2010	4.61	4.31	−30	0.61	4.14	353
2011	6.35	7.05	69	3.41	7.31	391
2012	1.65	3.06	141	10.65	7.39	−326
2013	−4.00	−2.95	105	2.21	−1.40	−362
2014	−0.48	−0.22	26	13.19	11.44	−175
2015	−3.57	−4.85	−128	1.65	0.39	−126
2016	1.60	1.02	−59	3.20	4.00	81
2017	7.49	8.16	67	0.15	−0.47	−62
2018	−0.84	−1.41	−57	0.88	1.65	78
2019	5.90	5.56	−34	6.72	4.45	−227
2020	10.11	10.90	79	5.03	4.11	−92
2021	−6.97	−7.15	−17	−3.54	−3.76	−21
2022	−18.26	−20.00	−173	−18.52	−19.06	−54
2023	5.19	5.69	50	7.90	6.32	−158
3Y	−7.18	−7.75	−57	−5.34	−6.09	−74
5Y	−1.39	−1.67	−29	−1.01	−2.07	−106
7Y	−0.09	−0.29	−20	−0.58	−1.32	−74
10Y	−0.31	−0.62	−31	1.32	0.59	−74

Bond indexes

Table 14: Performance of ESG bond indexes (corporates)

Year	Bloomberg Euro Aggregate Corporate						
	Return				Alpha		
	BM	SRI	S-SRI	ESG-S	SRI	S-SRI	ESG-S
2010	3.07	2.93	2.96		−13	−10	
2011	1.49	1.17	1.43		−32	−5	
2012	13.59	13.99	12.96		40	−63	
2013	2.37	2.49	2.36		12	−1	
2014	8.40	8.31	8.49		−8	10	
2015	−0.56	−0.59	−0.50	−0.59	−3	6	−3
2016	4.73	4.60	4.44	4.60	−13	−29	−13
2017	2.41	2.47	2.48	2.47	6	6	6
2018	−1.25	−1.12	−1.11	−1.12	13	14	13
2019	6.24	6.01	5.92	6.01	−24	−32	−24
2020	2.77	2.69	2.70	2.52	−8	−7	−25
2021	−0.97	−0.96	−0.99	−0.99	1	−2	−2
2022	−13.65	−13.62	−13.48	−13.48	3	16	17
2023	8.19	8.16	8.00	7.99	−3	−18	−20
3Y	−2.56	−2.55	−2.56	−2.56	0	0	0
5Y	0.20	0.15	0.13	0.11	−6	−7	−10
7Y	0.31	0.29	0.28	0.26	−1	−2	−4
10Y	1.44	1.40	1.41		−3	−3	

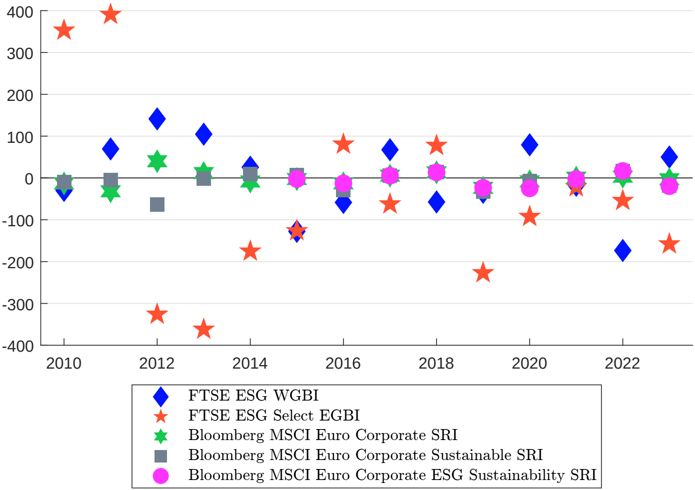
Bond indexes

Table 15: Performance of ESG bond indexes (corporates)

Year	Bloomberg US Corporate							Bloomberg Global High Yield				
	Return				Alpha			Return			Alpha	
	CW	SRI	S-SRI	ESG-S	SRI	S-SRI	ESG-S	CW	SRI	SUS	SRI	SUS
2019								2.30	2.20		−9	
2020								5.73	6.49	6.67	76	94
2021	−1.04	−1.55	9.56	2.34	−51	1 060	338	2.53	0.92	0.48	−161	−205
2022	−15.76	−15.12	−1.10	−13.86	64	1 467	190	−11.05	−13.07	−12.58	−202	−153
2023	8.52	8.44			−8			13.66	14.14	13.59	48	−7

Bond indexes

Figure 37: Alpha returns of European ESG bond indexes (in bps)



Definition

WACC

The weighted average cost of capital (WACC) is equal to:

$$\mathcal{WACC} = \frac{E}{E+D}\mathcal{C}_E + \frac{D}{E+D}\mathcal{C}_D\left(1-\tau_c\right)$$

where:

- $\mathcal{C}_E$ is the cost of equity
- $\mathcal{C}_D$ is the cost of debt
- E is the market value of the firm's equity
- D is the market value of the firm's debt
- τ_c is the corporate tax rate

Definition

ESG preferences and cost of capital

The preference of ESG investors is to finance issuers with higher ESG scores, implying that ESG investors require a lower cost of capital for best-in-class companies than non-ESG investors

We consider a linear model:

$$y_{i,t} = \alpha + \underbrace{\sum_{j=1}^{n_x} \beta_j x_{i,t}^{(j)}}_{\text{ESG variables}} + \underbrace{\sum_{k=1}^{n_z} \gamma_k z_{i,t}^{(k)}}_{\text{Control variables}} + \varepsilon_{i,t}$$

where:

- $y_{i,t}$ is the endogenous variable that depends on the cost of capital of company i at time t
- $x_{i,t}^{(j)}$ is a set of ESG metrics
- $z_{i,t}^{(k)}$ is a set of control variables

Question: $\beta_j < 0$?

Cost of equity

Sharfman and Fernando (2008) used the linear regression model:

$$y_{i,t} = \alpha + \beta \boldsymbol{\mathcal{S}}_{i,t} + \gamma_1 L_{i,t} + \gamma_2 M_{i,t} + \gamma_3 I_{i,t} + \varepsilon_{i,t}$$

where:

- $y_{i,t}$ is one of the cost of capital measures ($\mathcal{C}_E$, $\mathcal{C}_D$ and $\mathcal{WACC}$)
- $(L_{i,t}, M_{i,t}, I_{i,t})$ are the three control variables
- $\boldsymbol{\mathcal{S}}_{i,t}$ is the environmental score

They found that $\hat{\beta}$ has a positive sign for $\mathcal{C}_D$ and a negative sign for $\mathcal{C}_E$ and $\mathcal{WACC}$

Cost of equity

Another seminal study is El Ghoul *et al.* (2011)

- They use different models to estimate the cost of equity
- The control variables include the stock's beta, size, book-to-market ratio, leverage, average long-run growth forecast, and dispersion of analyst forecasts
- For the ESG variables, they use KLD scores: (1) community, (2) diversity, (3) employee relations, (4) environment, (5) human rights and (6) product
- They also consider an aggregate CSR score
- **They found that the average cost of equity for firms with a high CSR score is 56 basis points lower than for firms with a low CSR score**
- They also found that the employee relations, environmental policy, and product strategy dimensions contribute to lower firms' cost of equity, while the other three dimensions (community, diversity, and human rights) do not

Cost of equity

Calculating the cost of equity

In the CAPM, the cost of equity is equal to the expected rate of return:

$$\mathcal{C}_E = r + \beta \left(\mu_m - r \right)$$

where $\pi_m = \mu_m - r$ is the market risk premium, r is the risk-free rate and β is the beta of the stock return relative to the market portfolio return. Once we have assumed a value for π_m, we can easily calculate $\mathcal{C}_E$ using the estimated beta coefficient

Cost of equity

Calculating the cost of equity

The Gordon growth model uses the dividend discount model and assumes that $\mathrm{DPS}\left(t_m\right) = \left(1+g\right)^{\left(t_m - t\right)} \mathrm{DPS}\left(t\right)$ is the dividend per share at time t_m and g is the growth rate. We deduce that:

$$P\left(t\right) = \sum_{t_m = t+1}^{\infty} \frac{\mathrm{DPS}\left(t_m\right)}{\left(1+\varrho\right)^{\left(t_m - t\right)}} = \mathrm{DPS}\left(t\right) \frac{1+g}{\varrho - g}$$

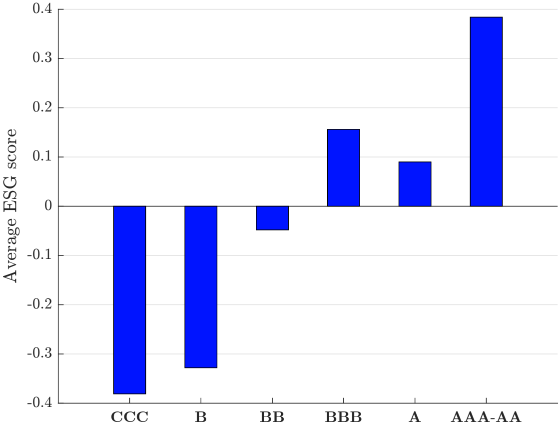
Since $\mathrm{DPS}\left(t+1\right) = \left(1+g\right) \mathrm{DPS}\left(t\right)$, we conclude that:

$$\mathcal{C}_E := \varrho = g + \mathrm{EDY}$$

where $\mathrm{EDY} = \mathrm{DPS}\left(t+1\right) / P\left(t\right)$ is the expected one-year ahead dividend yield.

Correlation between credit ratings and ESG ratings

Figure 38: Average ESG score by credit rating category (2010–2019)



Source: Ben Slimane *et al.* (2019).

An integrated credit-ESG model

We consider the following regression model:

$$\begin{aligned}\ln s_{i,t} &= \alpha_t + \beta \boldsymbol{\mathcal{S}}_{i,t} + \gamma_{md} \,\mathrm{MD}_{i,t} + \sum_{k=1}^{n_{\boldsymbol{\mathcal{S}}ector}} \gamma_{sector}\left(k\right) \boldsymbol{s}_{i,t}\left(k\right) + \\ & \sum_{k=1}^{n_{\mathcal{R}ating}} \gamma_{rating}\left(k\right) \mathcal{R}_{i,t}\left(k\right) + \varepsilon_{i,t}\end{aligned}$$

where:

- $s_{i,t}$ is the yield spread of Bond i at time t
- $\boldsymbol{\mathcal{S}}_{i,t}$ is the ESG score
- $\mathrm{MD}_{i,t}$ is the modified duration
- $\boldsymbol{s}_{i,t}\left(k\right)$ is the dummy variable associated with the k^{th} sector
- $\mathcal{R}_{i,t}\left(k\right)$ is the dummy variable associated with the k^{th} rating
- $\varepsilon_{i,t} \sim \mathcal{N}\left(0, \sigma^2\right)$

An integrated credit-ESG model

Table 16: Results of the panel data regression model (EUR IG corporate bonds, 2010–2019)

	2010–2013				2014–2019			
	ESG	E	S	G	ESG	E	S	G
$\mathfrak{R}^2$ (in %)	60.0	59.4	59.5	60.3	66.3	65.0	65.2	64.6
$\Delta\mathfrak{R}^2$ (in %)	0.6	0.0	0.2	1.0	4.0	2.6	2.9	2.3
$\hat{\beta}$ (in %)	−4.8	−1.1	−2.1	−6.7	−8.9	−7.9	−7.6	−7.7
t-statistic	−31.7	−7.3	−15.6	−38.8	−123.9	−98.4	103.9	−91.8

Source: Ben Slimane *et al.* (2019).

The assumption $\mathcal{H}_0 : \beta_{esg} < 0$ is not rejected

An integrated credit-ESG model

Table 17: Results of the panel data regression model (USD IG corporate bonds, 2010–2019)

	2010–2013				2014–2019			
	ESG	E	S	G	ESG	E	S	G
$\mathfrak{R}^2$ (in %)	52.7	52.8	52.8	53.4	60.6	60.5	60.3	60.9
$\Delta\mathfrak{R}^2$ (in %)	0.0	0.2	0.2	0.7	0.3	0.2	0.0	0.7
$\hat{\beta}$ (in %)	−0.3	2.7	2.6	−7.5	−3.7	−3.1	−0.0	−6.3
t-statistic	−2.1	19.3	20.6	−43.0	−47.7	−39.6	−0.4	−73.2

Source: Ben Slimane *et al.* (2019).

The assumption $\mathcal{H}_0 : \beta_{esg} < 0$ is not rejected in some cases

Cost of debt

The cost of debt is equal to the expected yield spread: $\mathcal{C}_D = \mathbb{E}[s_i]$. We have:

$$\mathcal{C}_D = \mathcal{C}_D(\mathrm{MD}_i, \boldsymbol{s}_i, \mathcal{R}_i)\, \mathcal{C}_D(\boldsymbol{\mathcal{S}}_i)$$

where:

$$\mathcal{C}_D(\mathrm{MD}_i, \boldsymbol{s}_i, \mathcal{R}_i) = \exp\left(\hat{\alpha} + \hat{\gamma}_{md}\,\mathrm{MD}_i + \hat{\gamma}_{sector}(\boldsymbol{s}_i) + \hat{\gamma}_{rating}(\mathcal{R}_i) + \frac{1}{2}\hat{\sigma}^2\right)$$

and:

$$\mathcal{C}_D(\boldsymbol{\mathcal{S}}_i) = \exp\left(\hat{\beta}\,\boldsymbol{\mathcal{S}}_i\right)$$

The cost of debt is the product of two factors:

- The first factor $\mathcal{C}_D(\mathrm{MD}_i, \boldsymbol{s}_i, \mathcal{R}_i)$ is related to the control variables
- The second factor $\mathcal{C}_D(\boldsymbol{\mathcal{S}}_i)$ measures the impact of the ESG score (if the score is zero, $\mathcal{C}_D(\boldsymbol{\mathcal{S}}_i) = 1$ and $\mathcal{C}_D = \mathcal{C}_D(\mathrm{MD}_i, \boldsymbol{s}_i, \mathcal{R}_i)$)

Cost of debt

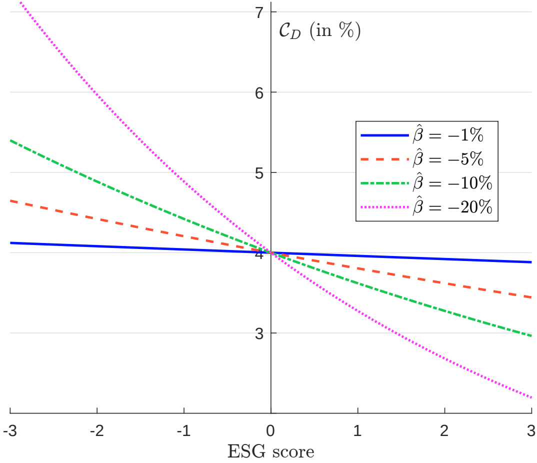
- Suppose two companies issue two bonds with the same characteristics (same maturity, same sector, same rating), but they have two different ESG scores
- The difference in terms of cost of debt is equal to:

$$\Delta \mathcal{C}_D = \bar{\mathcal{C}}_D \left(e^{\hat{\beta} \boldsymbol{\mathcal{S}}_1} - e^{\hat{\beta} \boldsymbol{\mathcal{S}}_2} \right)$$

  where $\bar{\mathcal{C}}_D$ is the average cost of debt (or the cost of debt if the ESG score is zero)
- For example, if one company has an ESG score of -2 and another company has an ESG score of $+1$, the difference in cost of debt is 36 bps if $\hat{\beta} = -3\%$ and $\bar{\mathcal{C}}_D = 4\%$

Cost of debt

Figure 39: Relationship between ESG score and cost of debt ($\bar{\mathcal{C}}_D = 4\%$)



ESG and sovereign risk

Motivation

- Financial analysis **versus/and** extra-financial analysis
- Sovereign risk $\neq$ Corporate risk
- Which ESG metrics are priced and not priced in by the market?
- What is the nexus between ESG analysis and credit analysis?

The economics of sovereign risk

A Tale of Two Countries

- Henry, P.B., and Miller, C. (2009), Institutions versus Policies: A Tale of Two Islands, *American Economic Review*, 99(2), pp. 261-267.
- The example of Barbados and Jamaica
- Why the economic growth of two countries with the same economic development at time t is different 10, 20 or 30 years later?

Sovereign ESG themes

Environmental

- Biodiversity
- Climate change
- Commitment to environmental standards
- Energy mix
- Natural hazard
- Natural hazard outcome
- Non-renewable energy resources
- **Temperature**
- Water management

Social

- Civil unrest
- Demographics
- **Education**
- Gender
- Health
- Human rights
- **Income**
- Labour market standards
- Migration
- Water and electricity access

Governance

- Business environment and R&D
- **Governance effectiveness**
- Infrastructure and mobility
- International relations
- Justice
- **National security**
- **Political stability**

The economics of sovereign risk

Assessment of a country's creditworthiness

- Confidence in the country? Only financial reasons?
- Mellios, C., and Paget-Blanc, E. (2006), Which Factors Determine Sovereign Credit Ratings?, *European Journal of Finance*, 12(4), pp. 361-377 $\Rightarrow$ credit ratings are correlated to the corruption perception index
- Country default risk cannot be summarized by only financial figures!
- Why some rich countries have to pay a credit risk premium?
- How to explain the large differences in Asia?

Single-factor analysis

Endogenous variable

10Y sovereign bond yield

Explanatory variables

- 269 ESG variables grouped into 26 ESG thematics
- 183 indicators come from Verisk Maplecrof database, the 86 remaining metrics were retrieved from the World Bank, ILO, WHO, FAO, UN...
- 6 control variables: GDP Growth, Net Debt, Reserves, Account Balance, Inflation and **Credit Rating**

Panel dimensions

- 67 countries
- 2015–2020

Single-factor analysis

Let $s_{i,t}$ be the bond yield spread of the country i at time t. We consider the following regression model estimated by OLS:

$$s_{i,t} = \alpha + \underbrace{\beta x_{i,t}}_{\text{ESG metric}} + \underbrace{\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)}}_{\substack{\text{Control variables/} \\ \textbf{Fundamental model}}} + \varepsilon_{i,t}$$

and:

$$\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} = \gamma_1 g_{i,t} + \gamma_2 \pi_{i,t} + \gamma_3 d_{i,t} + \gamma_4 ca_{i,t} + \gamma_5 r_{i,t} + \gamma_6 \mathcal{R}_{i,t}$$

where $g_{i,t}$ is the economic growth, $\pi_{i,t}$ is the inflation, $d_{i,t}$ is the debt ratio, $ca_{i,t}$ is the current account balance, $r_{i,t}$ is the reserve adequacy and $\mathcal{R}_{i,t}$ is the credit rating

Single-factor analysis

Table 18: The 7 most relevant variables of the single-factor analysis by pillar

Pillar	Theme	Variable	$\Delta\mathfrak{R}_c^2$
	Climate change	Climate change vulnerability (acute)	5.51%
	Climate change	Climate change exposure (extreme)	4.80%
	Water management	Agricultural water withdrawal	4.02%
E	Climate change	Climate change sensitivity (acute)	3.95%
	Biodiversity	Biodiversity threatening score	3.53%
	Climate change	Climate change exposure (acute)	3.39%
	Climate change	Climate change vulnerability (average)	3.11%
	Human rights	Freedom of assembly	8.74%
	Human rights	Extent of arbitrary unrest	8.04%
	Human rights	Extent of torture and ill treatment	7.63%
S	Labour market standards	Severity of working time violations	7.21%
	Labour market standards	Forced labour violations (extent)	6.10%
	Labour market standards	Child labour (extent)	5.83%
	Migration	Vulnerability of migrant workers	5.83%
	National security	Severity of kidnappings	6.80%
	Business environment and R&D	Ease of access to loans	6.77%
	Infrastructure and mobility	Roads km	6.45%
G	Business environment and R&D	Capacity for innovation	5.65%
	Business environment and R&D	Ethical behaviour of firms	5.37%
	National security	Frequency of kidnappings	5.27%
	Infrastructure and mobility	Physical connectivity	4.94%

Single-factor analysis

Table 19: Summary of the single-factor analysis

Relevance	E	S	G
High	Temperature Climate change Natural hazard outcome	Labour market standards Human rights Migration	Infrastructure and mobility National security Justice
Low	Water management Energy mix	Income Education Water and electricity access	Political stability

Source: Semet *et al.* (2021).

Multi-factor analysis

We consider the following multi-factor regression model:

$$s_{i,t} = \alpha + \underbrace{\sum_{j=1}^{m} \beta_j x_{i,t}^{(j)}}_{\substack{\text{ESG variables/} \\ \textbf{Extra-financial model}}} + \underbrace{\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)}}_{\substack{\text{Control variables/} \\ \textbf{Fundamental model}}} + \varepsilon_{i,t}$$

Multi-factor analysis

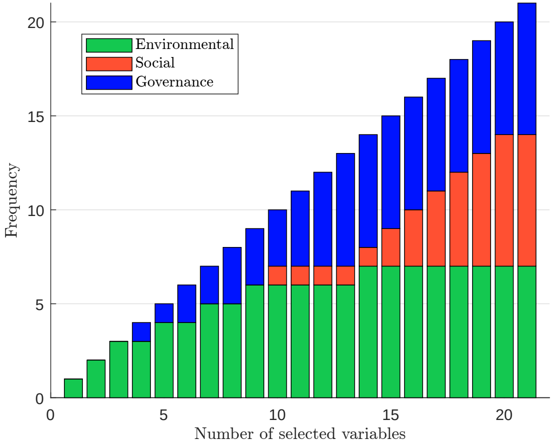
Table 20: Final multi-factor model of sovereign bond yields

Variable	$\hat{\beta}$	t-student	p-value
Intercept α	2.834	15.72***	0.00
GDP growth $g_{i,t}$	0.017	1.37	0.17
Inflation $\pi_{i,t}$	0.048	6.64***	0.00
Debt ratio $d_{i,t}$	−0.001	−1.71*	0.08
Current account balance $ca_{i,t}$	−0.012	−2.45**	0.01
Reserve adequacy $ra_{i,t}$	0.005	0.74	0.45
Rating score $\mathcal{R}_{i,t}$	−0.013	−9.08***	0.00
Exporting across borders (cost)	$4.05e^{-4}$	4.11***	0.00
Severe storm hazard (absolute high extreme)	−0.015	−1.66*	0.09
Capacity for innovation	−0.004	−4.99***	0.00
Ethical behavior of firms	−0.061	−2.79***	0.00
Temperature change	−0.149	−3.50***	0.00
Severity of kidnappings	−0.032	−4.25***	0.00
Drought hazard (absolute high extreme)	$3.33e^{-8}$	2.60***	0.00

Source: Semet *et al.* (2021).

Multi-factor analysis

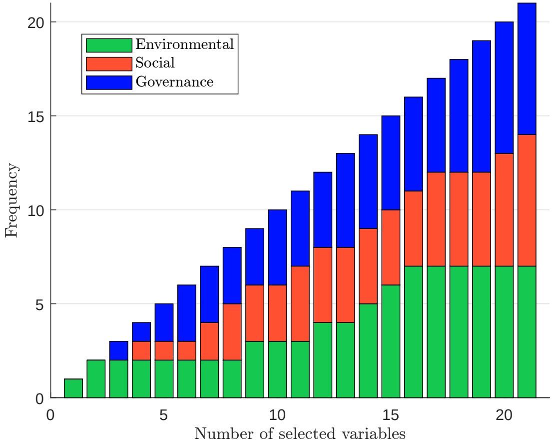
Figure 40: ESG pillar importance for high-income countries



Source: Semet *et al.* (2021).

Multi-factor analysis

Figure 41: ESG pillar importance for middle-income countries



Source: Semet *et al.* (2021).

Multi-factor analysis

Table 21: The most relevant variables of the multi-factor analysis

Category	Rank	Pillar	Variable
High income countries	1	E	Fossil fuel intensity of the economy
	2	E	Temperature change
	3	E	Cooling degree days annual average
	4	G	Capacity for innovation
	5	E	Heat stress (future)
	6	G	Severity of kidnappings
	7	E	Biodiversity threatening score
	8	G	Efficacy of corporate boards
	9	E	Total GHG emissions
	10	S	Significant marginalized group
Middle income countries	1	E	Tsunami hazard
	2	E	Transport infrastructure exposed to natural hazards
	3	G	Severity of kidnappings
	4	S	Discrimination based on LGBT status
	5	G	Air transport departures
	6	G	Exporting across borders (cost)
	7	S	Index of labour standards
	8	S	Vulnerability of migrant workers
	9	E	Paris Agreement
	10	G	Military expenditure (% of GDP)

Source: Semet *et al.* (2021)

Multi-factor analysis

High-income countries

- Transition risk
- S is lagging

Middle-income countries

- Physical risk
- Social issues are priced

Explaining credit ratings with ESG metrics

We consider the logit model:

$$p_{i,t} = \Pr\left\{\mathcal{R}_{i,t} \in \mathcal{UG}\right\} = \mathbf{F}\left(\beta_0 + \sum\nolimits_{j=1}^{n_x} \beta_j x_{i,t}^{(j)}\right)$$

where:

- $\mathcal{UG}$ is the set of upper grading ratings (from AAA to A$-$)
- $\mathcal{R}_{i,t}$ is the rating of country i at time t
- $\mathbf{F}(z) = \dfrac{e^z}{1+e^z}$ is the cumulative function of the logistic distribution
- $x_{i,t}^{(j)}$ is the j^{th} selected ESG variable

Explaining credit ratings with ESG metrics

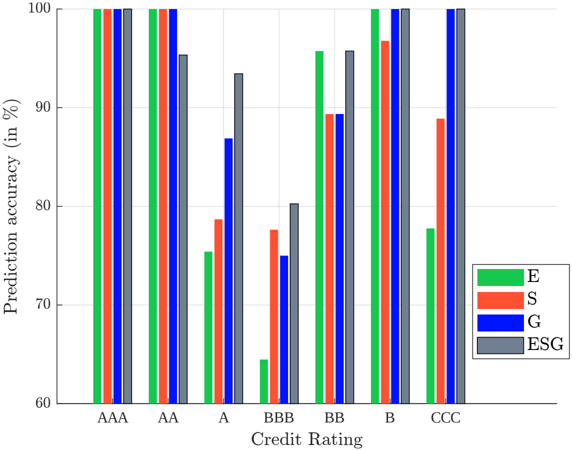
Table 22: Estimated logistic model with the ESG selected variables

Pillar	Variable	$\hat{\theta}_j$	t-student	p-value
E	Domestic regulatory framework	2.881	1.44	0.14
	Climate change vulnerability (average)	0.275	−1.17	0.24
	Water import security (average)	0.717	−0.50	0.61
	Biodiversity threatening score	1.029	0.14	0.88
S	Health expenditure per capita	0.998	−1.10	0.26
	Public dissatisfaction with water quality	1.332	1.41	0.15
	Mean years of schooling of adults	68.298	3.37***	0.00
	Base pay/value added per worker	0.000	−1.07	0.28
	Urban population change (5 years)	3.976	2.95***	0.00
	Basic food stuffs net imports per person	0.990	−2.07**	0.03
	Food import security	0.803	−2.59***	0.00
G	**Government effectiveness index**	1.751	2.37**	0.01
	Venture capital availability	1.099	2.93***	0.00
	Enforcing a contract (time)	0.999	−0.31	0.75
	Paying tax (process)	0.846	−1.47	0.14
	Getting electricity (time)	0.882	−2.95***	0.00

Source: Semet *et al.* (2021).

Explaining credit ratings with ESG metrics

Figure 42: Prediction accuracy of the logistic model



Source: Semet *et al.* (2021).

ESG and sovereign risk

Summary of the results

What is directly priced by the bond market?		What is indirectly priced by credit rating agencies?
E ≻ G ≻ S Significant market-based ESG indicators	$\neq$	G ≻ S ≻ E Relevant CRA-based ESG indicators
• High-income countries Transition risk ≻ Physical risk • Middle-income countries Physical risk ≻ Transition risk		E metrics are second-order variables: • Environmental standards • Water management • Biodiversity • Climate change
S matters for middle-income countries, especially for Gender inequality, Working conditions and Migration		Education, Demographic and Human rights are prominent indicators for the S pillar
National security, Infrastructure and mobility and International relationships are the relevant G metrics		Government effectiveness, Business environment and R&D dominate the G pillar
Fundamental analysis: $\mathfrak{R}_c^2 \approx 70\%$ Extra-financial analysis: $\Delta\mathfrak{R}_c^2 \approx 13.5\%$		Accuracy $> 95\%$ AAA, AA, B, CCC ≻ A ≻ BB ≻ BBB