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Handbook of Financial Risk Management

Chapter 1

Introduction

Box 1

Evolution of financial innovations

1864	Commodity futures
1970	Mortgage-backed securities
1971	Equity index funds
1972	Foreign currency futures
1973	Stock options
1977	Put options
1979	Over-the-counter currency options
1980	Currency swaps
1981	Interest rate swaps
1982	Equity index futures
1983	Equity index options
	Interest rate caps/floors
	Collateralized mortgage obligations
1985	Swaptions
	Asset-backed securities
1987	Path-dependent options (Asian, look-back, etc.)
	Collateralized debt obligations
1992	Catastrophe insurance futures and options
1993	Captions/floortions
	Exchange-traded funds
1994	Credit default swaps
1996	Electricity futures
1997	Weather derivatives
2004	Volatility index futures
2006	Leveraged and inverse ETFs
2008	Green bonds
2009	Crypto currencies

Source: Jorion (2007) and author's research.

TABLE 1.1: Notional outstanding amount of exchange-traded derivatives

	2004	2007	2010	2014	2018
Futures	42.6%	37.9%	34.1%	44.4%	41.2%
Interest rate	99.4%	99.3%	99.2%	99.1%	99.3%
Short-term	94.7%	94.0%	94.9%	93.6%	92.6%
Long-term	5.3%	6.0%	5.1%	6.4%	7.4%
Currency	0.6%	0.7%	0.8%	0.9%	0.7%
Options	57.4%	62.1%	65.9%	55.6%	58.8%
Interest rate	99.8%	99.7%	99.6%	99.6%	99.8%
Short-term	98.2%	98.6%	98.9%	97.7%	98.3%
Long-term	1.9%	1.4%	1.1%	2.3%	1.7%
Currency	0.2%	0.3%	0.4%	0.5%	0.3%
Total (in $ tn)	43.0	71.5	62.3	57.6	94.8

Source: Bank for International Settlement (2019) and author's calculations.

TABLE 1.2: Notional outstanding amount of OTC derivatives

	2004	2007	2010	2014	2018
Forwards	12.9%	11.8%	15.4%	20.2%	24.0%
Swaps	71.1%	73.3%	73.2%	69.4%	65.0%
Options	15.9%	14.9%	11.4%	10.3%	10.8%
Unallocated	0.1%	0.0%	0.0%	0.1%	0.1%
Currency	13.4%	11.4%	11.3%	13.1%	16.1%
Interest rate	79.5%	73.8%	81.9%	82.8%	80.9%
Equity	2.0%	1.6%	1.0%	1.1%	1.2%
Commodity	0.6%	1.6%	0.6%	0.3%	0.4%
Credit	4.5%	11.6%	5.2%	2.7%	1.4%
Unallocated	0.1%	0.0%	0.0%	0.0%	0.0%
Total (in $ tn)	258.6	585.9	601.0	627.8	594.8

Source: Bank for International Settlement (2019) and author's calculations.

Box 2

An history of financial losses

1974	Herstatt Bank: $620 mn (foreign exchange trading)
1994	Metallgesellschaft: $1.3 bn (oil futures)
1994	Orange County: $1.8 bn (reverse repo)
1994	Procter & Gamble: $160 mn (ratchet swap)
1995	Barings Bank: $1.3 bn (stock index futures)
1997	Natwest: $127 mn (swaptions)
1998	LTCM: $4.6 bn (liquidity crisis)
2001	Dexia Bank: $270 mn (corporate bonds)
2006	Amaranth Advisors: $6.5 bn (gaz forward contracts)
2007	Morgan Stanley: $9.0 bn (credit derivatives)
2008	Société Générale: $7.2 bn (rogue trading)
2008	Madoff: $65 bn (fraud)
2011	UBS: $2.0 bn (rogue trading)
2012	JPMorgan Chase: $5.8 bn (credit derivatives)

Source: Jorion (2007) and author's research.

TABLE 1.3: The supervision institutions in finance

	Banks	Insurers	Markets	All sectors
Global	BCBS	IAIS	IOSCO	FSB
EU	EBA/ECB	EIOPA	ESMA	ESFS
US	FDIC/FRB	FIO	SEC	FSOC

TABLE 1.4: The three pillars of the Basel II framework

Pillar 1 Minimum Capital Requirements	Pillar 2 Supervisory Review Process	Pillar 3 Market Discipline
Credit risk Market risk Operational risk	Review & reporting Capital above Pillar 1 Supervisory monitoring	Capital structure Capital adequacy Models & parameters Risk management

TABLE 1.5: Basel III capital requirements

<table>
<tr><th>Capital ratio</th><th>2013</th><th>2014</th><th>2015</th><th>2016</th><th>2017</th><th>2018</th><th>2019</th></tr>
<tr><td>CET1</td><td>3.5%</td><td>4.0%</td><td colspan="4">4.5%</td><td>4.5%</td></tr>
<tr><td>CB</td><td></td><td></td><td></td><td>0.625%</td><td>1.25%</td><td>1.875%</td><td>2.5%</td></tr>
<tr><td>CET1 + CB</td><td>3.5%</td><td>4.0%</td><td>4.5%</td><td>5.125%</td><td>5.75%</td><td>6.375%</td><td>7.0%</td></tr>
<tr><td>Tier 1</td><td>4.5%</td><td>5.5%</td><td colspan="4">6.0%</td><td>6.0%</td></tr>
<tr><td>Total</td><td colspan="6">8.0%</td><td>8.0%</td></tr>
<tr><td>Total + CB</td><td colspan="3">8.0%</td><td>8.625%</td><td>9.25%</td><td>9.875%</td><td>10.5%</td></tr>
<tr><td>CCB</td><td></td><td></td><td></td><td colspan="4">0% – 2.5%</td></tr>
</table>

Source: Basel Committee on Banking Supervision, `www.bis.org/bcbs/basel3.htm`.

Part I

Risk Management in the Financial Sector

Chapter 2

Market Risk

TABLE 2.1: Value of the penalty coefficient ξ for a sample of 250 observations

Zone	Number of exceptions	ξ
Green	0 – 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

TABLE 2.2: Probability distribution (in %) of the number of exceptions ($n = 250$ trading days)

	$\alpha = 99\%$		$\alpha = 98\%$	
m	$\Pr\{N_e = m\}$	$\Pr\{N_e \leq m\}$	$\Pr\{N_e = m\}$	$\Pr\{N_e \leq m\}$
0	8.106	8.106	0.640	0.640
1	20.469	28.575	3.268	3.908
2	25.742	54.317	8.303	12.211
3	21.495	75.812	14.008	26.219
4	13.407	89.219	17.653	43.872
5	6.663	95.882	17.725	61.597
6	2.748	98.630	14.771	76.367
7	0.968	99.597	10.507	86.875
8	0.297	99.894	6.514	93.388
9	0.081	99.975	3.574	96.963
10	0.020	99.995	1.758	98.720

TABLE 2.3: Liquidity horizon (Basel III)

Liquidity class k	Liquidity horizon h_k
1	10
2	20
3	40
4	60
5	120

TABLE 2.4: Scaled expected shortfall

k	Sc_k	Full Current	Reduced Current	Reduced Stress	Full/Stress (not scaled)	Full Stress
1	1	100.00	88.00	112.00	127.27	127.27
2	1	75.00	63.00	83.00	98.81	98.81
3	$\sqrt{2}$	48.08	42.43	66.47	53.27	75.33
4	$\sqrt{2}$	16.97	9.90	12.73	15.43	21.82
5	$\sqrt{6}$	14.70	12.25	17.15	8.40	20.58
Total		135.80	117.31	155.91		180.38

TABLE 2.5: Value of the penalty coefficient ξ in Basel III

Zone	Number of exceptions	ξ
Green	0 – 4	0.00
Amber	5	0.20
	6	0.26
	7	0.33
	8	0.38
	9	0.42
Red	10+	0.50

TABLE 2.6: Computation of the market risk factors $R_{1,s}$ and $R_{2,s}$

s	Date	Apple		Coca-Cola	
		Price	$R_{1,s}$	Price	$R_{2,s}$
1	2015-01-02	109.33	−0.95%	42.14	−0.19%
2	2014-12-31	110.38	−1.90%	42.22	−1.26%
3	2014-12-30	112.52	−1.22%	42.76	−0.23%
4	2014-12-29	113.91	−0.07%	42.86	−0.23%
5	2014-12-26	113.99	1.77%	42.96	0.05%
6	2014-12-24	112.01	−0.47%	42.94	−0.07%
7	2014-12-23	112.54	−0.35%	42.97	1.46%
8	2014-12-22	112.94	1.04%	42.35	0.95%
9	2014-12-19	111.78	−0.77%	41.95	−1.04%
10	2014-12-18	112.65	2.96%	42.39	2.02%

TABLE 2.7: Computation of the simulated P&L $\Pi_s(w)$

s	Date	Apple		Coca-Cola		$\mathrm{MtM}_s(w)$	$\Pi_s(w)$
		$R_{1,s}$	$P_{1,s}$	$R_{2,s}$	$P_{2,s}$		
1	2015-01-02	−0.95%	108.29	−0.19%	42.06	1 924.10	−12.00
2	2014-12-31	−1.90%	107.25	−1.26%	41.61	1 904.66	−31.44
3	2014-12-30	−1.22%	108.00	−0.23%	42.04	1 920.79	−15.31
4	2014-12-29	−0.07%	109.25	−0.23%	42.04	1 933.37	−2.73
5	2014-12-26	1.77%	111.26	0.05%	42.16	1 955.82	19.72
6	2014-12-24	−0.47%	108.82	−0.07%	42.11	1 930.36	−5.74
7	2014-12-23	−0.35%	108.94	1.46%	42.76	1 944.57	8.47
8	2014-12-22	1.04%	110.46	0.95%	42.54	1 955.48	19.38
9	2014-12-19	−0.77%	108.49	−1.04%	41.70	1 918.91	−17.19
10	2014-12-18	2.96%	112.57	2.02%	42.99	1 985.51	49.41
23	2014-12-01	−3.25%	105.78	−0.62%	41.88	1 895.35	−40.75
69	2014-09-25	−3.81%	105.16	−1.16%	41.65	1 884.64	−51.46
85	2014-09-03	−4.22%	104.72	0.34%	42.28	1 892.79	−43.31
108	2014-07-31	−2.60%	106.49	−0.83%	41.79	1 900.68	−35.42
236	2014-01-28	−7.99%	100.59	0.36%	42.29	1 851.76	−84.34
242	2014-01-17	−2.45%	106.65	−1.08%	41.68	1 900.19	−35.91
250	2014-01-07	−0.72%	108.55	0.30%	42.27	1 930.79	−5.31

TABLE 2.8: Scaling factors c_{VaR} and c_{ES}

α (in %)	95.0	96.0	97.0	97.5	98.0	98.5	99.0	99.5
c_{VaR}	1.64	1.75	1.88	1.96	2.05	2.17	**2.33**	2.58
c_{ES}	2.06	2.15	2.27	**2.34**	2.42	2.52	2.67	2.89

TABLE 2.9: Number of exceptions per year for long and short exposures on the S&P 500 index

Year	Long exposure					Short exposure				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
2000	5	5	2	4	4	5	8	4	6	4
2001	4	3	2	3	2	2	4	2	5	2
2002	2	5	2	4	3	5	9	4	6	5
2003	1	0	0	2	0	1	0	1	4	0
2004	2	0	2	6	0	0	0	0	2	1
2005	1	1	2	4	3	1	4	1	6	3
2006	2	4	3	4	4	2	5	3	5	3
2007	6	15	6	10	7	1	9	0	3	7
2008	7	23	5	7	10	4	12	4	3	8
2009	5	0	1	6	0	2	2	2	3	0
2010	7	6	5	8	3	3	5	2	7	3
2011	6	8	6	7	4	2	8	1	6	3
2012	5	1	4	5	0	3	1	2	7	1
2013	4	2	3	9	2	2	2	2	4	1
2014	6	9	7	11	2	2	4	2	2	4

TABLE 2.10: Value of the Cornish-Fisher quantile $\mathfrak{z}\,(99\%; \gamma_1, \gamma_2)$

γ_1	γ_2							
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
−2.00								0.99
−1.00			1.68	1.92	2.15	2.38	2.62	2.85
−0.50		2.10	2.33	2.57	2.80	3.03	3.27	3.50
0.00	2.33	2.56	2.79	3.03	3.26	3.50	3.73	3.96
0.50		2.83	3.07	3.30	3.54	3.77	4.00	4.24
1.00			3.15	3.39	3.62	3.85	4.09	4.32
2.00								3.93

TABLE 2.11: Value of the multiplication factor m_c deduced from the Chebyshev's inequality

α (in %)	90.00	95.00	99.00	99.25	99.50	99.75	99.99
Symmetric	1.74	1.92	3.04	3.36	3.88	5.04	19.01
Asymmetric	2.47	2.72	4.30	4.75	5.49	7.12	26.89

TABLE 2.12: Daily P&L of the long position on the call option when the risk factor is the underlying price

s	R_s (in %)	S_{t+h}	$\mathcal{C}_{t+h}$	Π_s
1	−1.93	98.07	3.09	−104.69
2	−0.69	99.31	3.72	−42.16
3	−0.71	99.29	3.71	−43.22
4	−0.73	99.27	3.70	−44.28
5	1.22	101.22	4.81	67.46
6	1.01	101.01	4.68	54.64
7	1.04	101.04	4.70	56.46
8	1.08	101.08	4.73	58.89
9	−1.61	98.39	3.25	−89.22

TABLE 2.13: Daily P&L of the long position on the call option when the risk factors are the underlying price and the implied volatility

s	R_s (in %)	S_{t+h}	$\Delta\Sigma_s$ (in %)	Σ_{t+h}	$\mathcal{C}_{t+h}$	Π_s
1	−1.93	98.07	−4.42	15.58	2.32	−182.25
2	−0.69	99.31	−1.32	18.68	3.48	−65.61
3	−0.71	99.29	−3.04	16.96	3.17	−97.23
4	−0.73	99.27	2.88	22.88	4.21	6.87
5	1.22	101.22	−0.13	19.87	4.79	65.20
6	1.01	101.01	−0.08	19.92	4.67	53.24
7	1.04	101.04	1.29	21.29	4.93	79.03
8	1.08	101.08	2.93	22.93	5.24	110.21
9	−1.61	98.39	0.85	20.85	3.40	−74.21

TABLE 2.14: Calculation of the P&L based on the Greek sensitivities

s	R_s (in %)	S_{t+h}	Π_s	$\Pi_s^{\boldsymbol{\Delta}}$	$\Pi_s^{\boldsymbol{\Delta}+\boldsymbol{\Gamma}}$	$\Pi_s^{\boldsymbol{\Delta}+\boldsymbol{\Gamma}+\boldsymbol{\Theta}}$
1	−1.93	98.07	−104.69	−108.69	−100.61	−105.09
2	−0.69	99.31	−42.16	−38.86	−37.83	42.30
3	−0.71	99.29	−43.22	−39.98	−38.89	−43.37
4	−0.73	99.27	−44.28	−41.11	−39.96	−44.43
5	1.22	101.22	67.46	68.71	71.93	67.46
6	1.01	101.01	54.64	56.88	59.09	54.61
7	1.04	101.04	56.46	58.57	60.91	56.44
8	1.08	101.08	58.89	60.82	63.35	58.87
9	−1.61	98.39	−89.22	−90.67	−85.05	−89.53
$\mathrm{VaR}_{99\%}(w; \text{one day})$			154.79	171.20	151.16	155.64
$\mathrm{ES}_{97.5\%}(w; \text{one day})$			150.04	165.10	146.37	150.84

TABLE 2.15: Calculation of the P&L using the vega coefficient

s	S_{t+h}	Σ_{t+h}	Π_s	$\Pi_s^{\boldsymbol{\upsilon}}$	$\Pi_s^{\boldsymbol{\Delta}+\boldsymbol{\upsilon}}$	$\Pi_s^{\boldsymbol{\Delta}+\boldsymbol{\Gamma}+\boldsymbol{\upsilon}}$	$\Pi_s^{\boldsymbol{\Delta}+\boldsymbol{\Gamma}+\boldsymbol{\Theta}+\boldsymbol{\upsilon}}$
1	98.07	15.58	−182.25	−79.09	−187.78	−179.71	−184.19
2	99.31	18.68	−65.61	−23.62	−62.48	−61.45	−65.92
3	99.29	16.96	−97.23	−54.40	−94.38	−93.29	−97.77
4	99.27	22.88	6.87	51.54	10.43	11.58	7.10
5	101.22	19.87	65.20	−2.33	66.38	69.61	65.13
6	101.01	19.92	53.24	−1.43	55.45	57.66	53.18
7	101.04	21.29	79.03	23.08	81.65	84.00	79.52
8	101.08	22.93	110.21	52.43	113.25	115.78	111.30
9	98.39	20.85	−74.21	15.21	−75.46	−69.84	−74.32
$\mathrm{VaR}_{99\%}(w; \text{one day})$			181.70	77.57	190.77	179.29	183.76
$\mathrm{ES}_{97.5\%}(w; \text{one day})$			172.09	73.90	184.90	169.34	173.81

TABLE 2.16: The 10 worst scenarios identified by the hybrid method

i	Full pricing		Greeks					
			$\boldsymbol{\Delta}-\boldsymbol{\Gamma}-\boldsymbol{\Theta}-\boldsymbol{\upsilon}$		$\boldsymbol{\Delta}-\boldsymbol{\Theta}$		$\boldsymbol{\Delta}-\boldsymbol{\Theta}-\boldsymbol{\upsilon}$	
	s	Π_s	s	Π_s	s	Π_s	s	Π_s
1	100	−183.86	100	−186.15	182	−187.50	134	−202.08
2	1	−182.25	1	−184.19	169	−176.80	100	−198.22
3	134	−181.15	134	−183.34	27	−174.55	1	−192.26
4	27	−163.01	27	−164.26	134	−170.05	169	−184.32
5	169	−162.82	169	−164.02	69	−157.66	27	−184.04
6	194	−159.46	194	−160.93	108	−150.90	194	−175.36
7	49	−150.25	49	−151.43	194	−149.77	49	−165.41
8	245	−145.43	245	−146.57	49	−147.52	182	−164.96
9	182	−142.21	182	−142.06	186	−145.27	245	−153.37
10	79	−135.55	79	−136.52	100	−137.38	69	−150.68

TABLE 2.17: Risk decomposition of the 99% Gaussian value-at-risk

Asset	w_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
Apple	1093.3	2.83%	30.96	75.14%
Coca-Cola	842.8	1.22%	10.25	24.86%
$\mathcal{R}(w)$			41.21	

TABLE 2.18: Risk decomposition of the 99% Gaussian expected shortfall

Asset	w_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
Apple	1093.3	3.24%	35.47	75.14%
Coca-Cola	842.8	1.39%	11.74	24.86%
$\mathcal{R}(w)$			47.21	

TABLE 2.19: Risk decomposition of the 99% historical value-at-risk

Asset	w_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
Apple	56.47%	77.77	43.92	92.68%
Coca-Cola	43.53%	7.97	3.47	7.32%
$\mathcal{R}(w)$			47.39	

TABLE 2.20: Risk contributions calculated with regularization techniques

Asset	**G**	**R1**	**R2**	**E**	**K**
Apple	30.97	43.92	52.68	35.35	39.21
Coca-Cola	10.25	3.47	2.29	12.03	8.17
$\mathcal{R}(w)$	41.21	47.39	54.96	47.39	47.39

TABLE 2.21: Risk decomposition of the 99% historical expected shortfall

Asset	w_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
Apple	56.47%	114.29	64.54	95.05%
Coca-Cola	43.53%	7.72	3.36	4.95%
$\mathcal{R}(w)$			67.90	

TABLE 2.22: Risk decomposition of the 97.5% historical expected shortfall

Asset	w_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
Apple	56.47%	78.48	44.32	91.31%
Coca-Cola	43.53%	9.69	4.22	8.69%
$\mathcal{R}(w)$			48.53	

Chapter 3

Credit Risk

TABLE 3.1: Debt securities by residence of issuer (in $ bn)

		Dec. 2004	Dec. 2007	Dec. 2010	Dec. 2017
Canada	Gov.	682	841	1 149	1 264
	Fin.	283	450	384	655
	Corp.	212	248	326	477
	Total	1 180	1 544	1 863	2 400
France	Gov.	1 236	1 514	1 838	2 258
	Fin.	968	1 619	1 817	1 618
	Corp.	373	382	483	722
	Total	2 576	3 515	4 138	4 597
Germany	Gov.	1 380	1 717	2 040	1 939
	Fin.	2 296	2 766	2 283	1 550
	Corp.	133	174	168	222
	Total	3 809	4 657	4 491	3 712
Italy	Gov.	1 637	1 928	2 069	2 292
	Fin.	772	1 156	1 403	834
	Corp.	68	95	121	174
	Total	2 477	3 178	3 593	3 299
Japan	Gov.	6 336	6 315	10 173	9 477
	Fin.	2 548	2 775	3 451	2 475
	Corp.	1 012	762	980	742
	Total	9 896	9 852	14 604	12 694
Spain	Gov.	462	498	796	1 186
	Fin.	434	1 385	1 442	785
	Corp.	15	19	19	44
	Total	910	1 901	2 256	2 015
UK	Gov.	798	1 070	1 674	2 785
	Fin.	1 775	3 127	3 061	2 689
	Corp.	452	506	473	533
	Total	3 027	4 706	5 210	6 011
US	Gov.	6 459	7 487	12 072	17 592
	Fin.	12 706	17 604	15 666	15 557
	Corp.	3 004	3 348	3 951	6 137
	Total	22 371	28 695	31 960	39 504

Source: Bank for International Settlement (2019).

TABLE 3.2: Price, yield to maturity and sensitivity of bonds

T	$R_t(T)$	$B_t(T)$	P_t	y	S
1	0.52%	99.48	104.45	0.52%	−104.45
2	0.99%	98.03	107.91	0.98%	−210.86
3	1.42%	95.83	110.50	1.39%	−316.77
4	1.80%	93.04	112.36	1.76%	−420.32
5	2.15%	89.82	113.63	2.08%	−520.16

TABLE 3.3: Impact of a parallel shift of the yield curve on the bond with five-year maturity

ΔR (in bps)	$\breve{P}_t$	ΔP_t	$\hat{P}_t$	ΔP_t	$S \times \Delta y$
−50	116.26	2.63	116.26	2.63	2.60
−30	115.20	1.57	115.20	1.57	1.56
−10	114.15	0.52	114.15	0.52	0.52
0	113.63	0.00	113.63	0.00	0.00
10	113.11	−0.52	113.11	−0.52	−0.52
30	112.08	−1.55	112.08	−1.55	−1.56
50	111.06	−2.57	111.06	−2.57	−2.60

TABLE 3.4: Computation of the credit spread s

$\mathcal{R}$ (in %)	λ (in bps)	PD (in bps)	P_t (in \$)	y (in %)	s (in bps)
0	0	0.0	110.1	3.24	0.0
	10	10.0	109.2	3.34	9.9
	200	198.0	93.5	5.22	198.1
	1000	951.6	50.4	13.13	988.9
40	0	0.0	110.1	3.24	0.0
	10	10.0	109.6	3.30	6.0
	200	198.0	99.9	4.41	117.1
	1000	951.6	73.3	8.23	498.8
80	0	0.0	110.1	3.24	0.0
	10	10.0	109.9	3.26	2.2
	200	198.0	106.4	3.66	41.7
	1000	951.6	96.3	4.85	161.4

TABLE 3.5: US mortgage-backed securities

Year	Agency		Non-agency		Total
	MBS	CMO	CMBS	RMBS	(in $ bn)
			Issuance		
2002	57.5%	23.6%	2.2%	16.7%	2 515
2006	33.6%	11.0%	7.9%	47.5%	2 691
2008	84.2%	10.8%	1.2%	3.8%	1 394
2010	71.0%	24.5%	1.2%	3.3%	2 013
2012	80.1%	16.4%	2.2%	1.3%	2 195
2014	68.7%	19.2%	7.0%	5.1%	1 440
2016	76.3%	15.7%	3.8%	4.2%	2 044
2018	69.2%	16.6%	4.7%	9.5%	1 899
			Outstanding amount		
2002	59.7%	17.4%	5.6%	17.2%	5 289
2006	45.7%	14.9%	8.3%	31.0%	8 390
2008	52.4%	14.0%	8.8%	24.9%	9 467
2010	59.2%	14.6%	8.1%	18.1%	9 258
2012	64.0%	14.8%	7.2%	14.0%	8 838
2014	68.0%	13.7%	7.1%	11.2%	8 842
2016	72.4%	12.3%	5.9%	9.5%	9 023
2018	74.7%	11.3%	5.6%	8.4%	9 732

Source: Securities Industry and Financial Markets Association (2019b,c) and author's calculations.

TABLE 3.6: US asset-backed securities

Year	Auto Loans	CDO & CLO	Credit Cards	Equip-ement	Other	Student Loans	Total (in $ bn)
				Issuance			
2002	34.9%	21.0%	25.2%	2.6%	6.8%	9.5%	269
2006	13.5%	60.1%	9.3%	2.2%	4.6%	10.3%	658
2008	16.5%	37.8%	25.9%	1.3%	5.4%	13.1%	215
2010	46.9%	6.4%	5.2%	7.0%	22.3%	12.3%	126
2012	33.9%	23.1%	12.5%	7.1%	13.7%	9.8%	259
2014	25.2%	35.6%	13.1%	5.2%	17.0%	4.0%	393
2016	28.3%	36.8%	8.3%	4.6%	16.9%	5.1%	325
2018	20.8%	54.3%	6.1%	5.1%	10.1%	3.7%	517
				Outstanding amount			
2002	20.7%	28.6%	32.5%	4.1%	7.5%	6.6%	905
2006	11.8%	49.3%	17.6%	3.1%	6.0%	12.1%	1 657
2008	7.7%	53.5%	17.3%	2.4%	6.2%	13.0%	1 830
2010	7.6%	52.4%	14.4%	2.4%	7.1%	16.1%	1 508
2012	11.0%	48.7%	10.0%	3.3%	8.7%	18.4%	1 280
2014	13.2%	46.8%	10.1%	3.9%	9.8%	16.2%	1 349
2016	13.9%	48.0%	9.3%	3.7%	11.6%	13.5%	1 397
2018	13.3%	48.2%	7.4%	5.0%	16.0%	10.2%	1 677

Source: Securities Industry and Financial Markets Association (2019b,c) and author's calculations.

TABLE 3.7: Price, spread and risky PV01 of CDS contracts

T	$P_t(T)$ $\boldsymbol{c}=10$	$P_t(T)$ $\boldsymbol{c}=100$	$\mathcal{s}$	RPV_{01}
1/2	998	−3 492	30.01	0.499
1	1 992	−6 963	30.02	0.995
2	3 956	−13 811	30.04	1.974
3	5 874	−20 488	30.05	2.929
5	9 527	−33 173	30.08	4.744
7	12 884	−44 804	30.10	6.410
10	17 314	−60 121	30.12	8.604

TABLE 3.8: Price, spread and risky PV01 of CDS contracts (without the accrued premium)

T	$P_t(T)$ $\boldsymbol{c}=10$	$P_t(T)$ $\boldsymbol{c}=100$	$\mathcal{s}$	RPV_{01}
1/2	999	−3 489	30.03	0.499
1	1 993	−6 957	30.04	0.994
2	3 957	−13 799	30.06	1.973
3	5 876	−20 470	30.07	2.927
5	9 530	−33 144	30.10	4.742
7	12 888	−44 764	30.12	6.406
10	17 319	−60 067	30.14	8.598

TABLE 3.9: Calibration of the CDS spread curve using the exponential model

T	$P_t(T)$ $\boldsymbol{c}=10$	$P_t(T)$ $\boldsymbol{c}=100$	$\mathcal{s}$	RPV_{01}	$\hat{\lambda}$
1/2	1 037	−3 454	30.77	0.499	51.28
1	2 146	−6 808	31.57	0.995	52.59
2	4 585	−13 175	33.24	1.973	55.34
3	7 316	−19 026	35.00	2.927	58.25
5	13 631	−28 972	38.80	4.734	64.54
7	21 034	−36 391	42.97	6.380	71.44
10	33 999	−42 691	49.90	8.521	82.92

TABLE 3.10: List of Markit CDX main indices

Index name	Description	n	$\mathcal{R}$
CDX.NA.IG	Investment grade entities	125	40%
CDX.NA.IG.HVOL	High volatility IG entities	30	40%
CDX.NA.XO	Crossover entities	35	40%
CDX.NA.HY	High yield entities	100	30%
CDX.NA.HY.BB	High yield BB entities	37	30%
CDX.NA.HY.B	High yield B entities	46	30%
CDX.EM	EM sovereign issuers	14	25%
LCDX	Secured senior loans	100	70%
MCDX	Municipal bonds	50	80%

Source: Markit (2014).

TABLE 3.11: List of Markit iTraxx main indices

Index name	Description	n	$\mathcal{R}$
iTraxx Europe	European IG entities	125	40%
iTraxx Europe HiVol	European HVOL IG entities	30	40%
iTraxx Europe Crossover	European XO entities	40	40%
iTraxx Asia	Asian (ex-Japan) IG entities	50	40%
iTraxx Asia HY	Asian (ex-Japan) HY entities	20	25%
iTraxx Australia	Australian IG entities	25	40%
iTraxx Japan	Japanese IG entities	50	35%
iTraxx SovX G7	G7 governments	7	40%
iTraxx LevX	European leveraged loans	40	40%

Source: Markit (2014).

TABLE 3.12: Historical spread of CDX/iTraxx indices (in bps)

Date	CDX			iTraxx		
	NA.IG	NA.HY	EM	Europe	Japan	Asia
Dec. 2012	94.1	484.4	208.6	117.0	159.1	108.8
Dec. 2013	62.3	305.6	272.4	70.1	67.5	129.0
Dec. 2014	66.3	357.2	341.0	62.8	67.0	106.0
Sep. 2015	93.6	505.3	381.2	90.6	82.2	160.5

TABLE 3.13: List of Markit credit default tranches

Index name	Tranche				
CDX.NA.IG	0 – 3	3 – 7	7 – 15	15 – 100	
CDX.NA.HY	0 – 10	10 – 15	15 – 25	25 – 35	35 – 100
LCDX	0 – 5	5 – 8	8 – 12	12 – 15	15 – 100
iTraxx Europe	0 – 3	3 – 6	6 – 9	9 – 12	12 – 22	22 – 100
iTraxx Europe XO	0 – 10	10 – 15	15 – 25	25 – 35	35 – 100
iTraxx Asia	0 – 3	3 – 6	6 – 9	9 – 12	12 – 22
iTraxx Australia	0 – 3	3 – 6	6 – 9	9 – 12	12 – 22
iTraxx Japan	0 – 3	3 – 6	6 – 9	9 – 12	12 – 22

Source: Markit (2014).

TABLE 3.14: World's largest banks in 1981 and 1988

	1981		1988	
	Bank	Assets	Bank	Assets
1	Bank of America (US)	115.6	Dai-Ichi Kangyo (JP)	352.5
2	Citicorp (US)	112.7	Sumitomo (JP)	334.7
3	BNP (FR)	106.7	Fuji (JP)	327.8
4	Crédit Agricole (FR)	97.8	Mitsubishi (JP)	317.8
5	Crédit Lyonnais (FR)	93.7	Sanwa (JP)	307.4
6	Barclays (UK)	93.0	Industrial Bank (JP)	261.5
7	Société Générale (FR)	87.0	Norinchukin (JP)	231.7
8	Dai-Ichi Kangyo (JP)	85.5	Crédit Agricole (FR)	214.4
9	Deutsche Bank (DE)	84.5	Tokai (JP)	213.5
10	National Westminster (UK)	82.6	Mitsubishi Trust (JP)	206.0

Source: Tarullo (2008).

TABLE 3.15: Risk weights by category of on-balance sheet assets

RW	Instruments
0%	Cash Claims on central governments and central banks denominated in national currency and funded in that currency Other claims on OECD central governments and central banks Claims† collateralized by cash of OECD government securities
20%	Claims† on multilateral development banks Claims† on banks incorporated in the OECD and claims guaranteed by OECD incorporated banks Claims† on securities firms incorporated in the OECD subject to comparable supervisory and regulatory arrangements Claims† on banks incorporated in countries outside the OECD with a residual maturity of up to one year Claims† on non-domestic OECD public-sector entities Cash items in process of collection
50%	Loans fully secured by mortgage on residential property
100%	Claims on the private sector Claims on banks incorporated outside the OECD with a residual maturity of over one year Claims on central governments outside the OECD and non denominated in national currency All other assets

†or guaranteed by these entities.

Source: BCBS (1988).

TABLE 3.16: Risk weights of the SA approach (Basel II)

Rating		AAA to AA−	A+ to A−	BBB+ to BBB−	BB+ to B−	CCC+ to C	NR
Sovereigns		0%	20%	50%	100%	150%	100%
Banks	1	20%	50%	100%	100%	150%	100%
	2	20%	50%	50%	100%	150%	50%
	2 ST	20%	20%	20%	50%	150%	20%
Corporates				BBB+ to BB−		B+ to C	
		20%	50%	100%		150%	100%
Retail					75%		
Residential mortgages					35%		
Commercial mortgages					100%		

TABLE 3.17: Comparison of risk weights between Basel I and Basel II

Entity	Rating	Maturity	Basel I	Basel II
Sovereign (OECD)	AAA		0%	0%
Sovereign (OECD)	A-		0%	20%
Sovereign	BBB		100%	50%
Bank (OECD)	BBB	2Y	20%	50%
Bank	BBB	2M	100%	20%
Corporate	AA+		100%	20%
Corporate	BBB		100%	100%

TABLE 3.18: Credit rating system of S&P, Moody's and Fitch

	Prime Maximum Safety	High Grade High Quality			Upper Medium Grade		
S&P/Fitch	AAA	AA+	AA	AA−	A+	A	A−
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3

	Lower Medium Grade			Non Investment Grade Speculative		
S&P/Fitch	BBB+	BBB	BBB−	BB+	BB	BB−
Moody's	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3

	Highly Speculative			Substantial Risk	In Poor Standing		Extremely Speculative
S&P/Fitch	B+	B	B−	CCC+	CCC	CCC−	CC
Moody's	B1	B2	B3	Caa1	Caa2	Caa3	Ca

TABLE 3.19: Examples of country's S&P rating

Country	Local currency		Foreign currency	
	Jun. 2009	Oct. 2015	Jun. 2009	Oct. 2015
Argentina	B-	CCC+	B-	SD
Brazil	BBB+	BBB-	BBB-	BB+
China	A+	AA-	A+	AA-
France	AAA	AA	AAA	AA
Italy	A+	BBB-	A+	BBB-
Japan	AA	A+	AA	A+
Russia	BBB+	BBB-	BBB	BB+
Spain	AA+	BBB+	AA+	BBB+
Ukraine	B-	CCC+	CCC+	SD
US	AAA	AA+	AA+	AA+

Source: Standard & Poor's, `www.standardandpoors.com`.

TABLE 3.20: Examples of bank's S&P rating

Bank	Oct. 2001	Jun. 2009	Oct. 2015
Barclays Bank PLC	AA	AA-	A-
Credit Agricole S.A.	AA	AA-	A
Deutsche Bank AG	AA	A+	BBB+
International Industrial Bank	CCC+	BB-	
JPMorgan Chase & Co.	AA-	A+	A
UBS AG	AA+	A+	A

Source: Standard & Poor's, www.standardandpoors.com.

TABLE 3.21: Examples of corporate's S&P rating

Corporate	Jul. 2009	Oct. 2015
Danone	A-	A-
Exxon Mobil Corp.	AAA	AAA
Ford Motor Co.	CCC+	BBB-
General Motors Corp.	D	BBB-
L'Oreal S.A.	NR	NR
Microsoft Corp.	AAA	AAA
Nestle S.A.	AA	AA
The Coca-Cola Co.	A+	AA
Unilever PLC	A+	A+

Source: Standard & Poor's, www.standardandpoors.com.

TABLE 3.22: Standardized supervisory haircuts for collateralized transactions

Rating	Residual Maturity	Sovereigns	Others
AAA to AA−	0−1Y	0.5%	1%
	1−5Y	2%	4%
	5Y+	4%	8%
A+ to BBB−	0−1Y	1%	2%
	1−5Y	3%	6%
	5Y+	6%	12%
BB+ to BB−		15%	
Cash		0%	
Gold		15%	
Main index equities		15%	
Equities listed on a recognized exchange		25%	
FX risk		8%	

TABLE 3.23: An example of internal rating system

<table>
<tr><th colspan="2">Rating</th><th>Degree of risk</th><th>Definition</th><th>Borrower category by self-assessment</th></tr>
<tr><td>1</td><td></td><td>No essential risk</td><td>Extremely high degree of certainty of repayment</td><td rowspan="7">Normal</td></tr>
<tr><td>2</td><td></td><td>Negligible risk</td><td>High degree of certainty of repayment</td></tr>
<tr><td>3</td><td></td><td>Some risk</td><td>Sufficient certainty of repayment</td></tr>
<tr><td>4</td><td>A
B
C</td><td>Better than average</td><td>There is certainty of repayment but substantial changes in the environment in the future may have some impact on this uncertainty</td></tr>
<tr><td>5</td><td>A
B
C</td><td>Average</td><td>There are no problems foreseeable in the future, but a strong likelihood of impact from changes in the environment</td></tr>
<tr><td>6</td><td>A
B
C</td><td>Tolerable</td><td>There are no problems foreseeable in the future, but the future cannot be considered entirely safe</td></tr>
<tr><td>7</td><td></td><td>Lower than average</td><td>There are no problems at the current time but the financial position of the borrower is relatively weak</td></tr>
<tr><td>8</td><td>A
B</td><td>Needs preventive management</td><td>There are problems with lending terms or fulfilment, or the borrower's business conditions are poor or unstable, or there are other factors requiring careful management</td><td>Needs attention</td></tr>
<tr><td>9</td><td></td><td rowspan="3">Needs serious management</td><td>There is a high likelihood of bankruptcy in the future</td><td>In danger of bankruptcy</td></tr>
<tr><td rowspan="2">10</td><td>I</td><td>The borrower is in serious financial straits and "effectively bankrupt"</td><td>Effectively bankruptcy</td></tr>
<tr><td>II</td><td>The borrower is bankrupt</td><td>Bankrupt</td></tr>
</table>

Source: Ieda *et al.* (2000).

TABLE 3.24: Numerical values of $f(\ell)$, $\mathbf{F}(\ell)$ and $\mathbf{F}^{-1}(\alpha)$ when ρ is equal to 10%

ℓ	(in \$ mn)	0.10	1.00	2.00	3.00	4.00	5.00
$\mathbf{F}(\ell)$	(in %)	0.03	16.86	47.98	70.44	83.80	91.26
$f(\ell)$	(in %)	1.04	31.19	27.74	17.39	9.90	5.43
α	(in %)	10.00	25.00	50.00	75.00	90.00	95.00
$\mathbf{F}^{-1}(\alpha)$	(in \$ mn)	0.77	1.25	2.07	3.28	4.78	5.90

TABLE 3.25: Percentage change in capital requirements under CP2 proposals

		SA	FIRB	AIRB
G10	Group 1	6%	14%	−5%
	Group 2	1%		
EU	Group 1	6%	10%	−1%
	Group 2	−1%		
Others		5%		

Source: BCBS (2001b).

TABLE 3.26: IRB risk weights (in %) for corporate exposures

Maturity		M = 1		M = 2.5		M = 2.5 (SME)	
LGD		45%	75%	45%	75%	45%	75%
PD (in %)	0.10	18.7	31.1	29.7	49.4	23.3	38.8
	0.50	52.2	86.9	69.6	116.0	54.9	91.5
	1.00	73.3	122.1	92.3	153.9	72.4	120.7
	2.00	95.8	159.6	114.9	191.4	88.5	147.6
	5.00	131.9	219.8	149.9	249.8	112.3	187.1
	10.00	175.8	292.9	193.1	321.8	146.5	244.2
	20.00	223.0	371.6	238.2	397.1	188.4	314.0

TABLE 3.27: IRB risk weights (in %) for retail exposures

		Mortgage		Revolving		Other retail	
LGD		45%	25%	45%	85%	45%	85%
PD (in %)	0.10	10.7	5.9	2.7	5.1	11.2	21.1
	0.50	35.1	19.5	10.0	19.0	32.4	61.1
	1.00	56.4	31.3	17.2	32.5	45.8	86.5
	2.00	87.9	48.9	28.9	54.6	58.0	109.5
	5.00	148.2	82.3	54.7	103.4	66.4	125.5
	10.00	204.4	113.6	83.9	158.5	75.5	142.7
	20.00	253.1	140.6	118.0	222.9	100.3	189.4

TABLE 3.28: Risk weights of the SA approach (ECRA, Basel III)

Rating		AAA to AA−	A+ to A−	BBB+ to BBB−	BB+ to B−	CCC+ to C	NR
Sovereigns		0%	20%	50%	100%	150%	100%
PSE	1	20%	50%	100%	100%	150%	100%
	2	20%	50%	50%	100%	150%	50%
MDB		20%	30%	50%	100%	150%	50%
Banks	2	20%	30%	50%	100%	150%	SCRA
	2 ST	20%	20%	20%	50%	150%	SCRA
	Covered	10%	20%	20%	50%	100%	(*)
Corporates		20%	50%	75%	100%	150%	100%
Retail				75%			

(*) For unrated covered bonds, the risk weight is generally half of the risk weight of the issuing bank.

TABLE 3.29: Risk weights of the SA approach (ECRA, Basel III)

Residential real estate			Commercial real estate		
Cash flows	ND	D	Cash flows	ND	D
$\text{LTV} \leq 50$	20%	30%	$\text{LTV} \leq 60$	$\min(60\%, \text{RW}_C)$	70%
$50 < \text{LTV} \leq 60$	25%	35%			
$60 < \text{LTV} \leq 80$	30%	45%	$60 < \text{LTV} \leq 80$	RW_C	90%
$80 < \text{LTV} \leq 90$	40%	60%	$\text{LTV} > 80$	RW_C	110%
$90 < \text{LTV} \leq 100$	50%	75%			
$\text{LTV} > 100$	70%	105%			

TABLE 3.30: Standardized supervisory haircuts for collateralized transactions (Basel III)

Rating	Residual Maturity	Sovereigns	Others	Securitization exposures
AAA to AA−	0−1Y	0.5%	1%	2%
	1−3Y	2%	3%	8%
	3−5Y	2%	4%	8%
	5Y−10Y	4%	6%	16%
	10Y+	4%	12%	16%
A+ to BBB−	0−1Y	1%	2%	4%
	1−3Y	3%	4%	12%
	3−5Y	3%	6%	12%
	5Y−10Y	6%	12%	24%
	10Y+	6%	20%	24%
BB+ to BB−		15%		
Cash			0%	
Gold			20%	
Main index equities			20%	
Equities listed on a recognized exchange			30%	
FX risk			8%	

TABLE 3.31: Value of the parameters α, β, γ, δ and ϵ (SEC-IRBA)

Category	Senior	Granularity	α	β	γ	δ	ϵ
Wholesale	✓	$N \geq 25$	0.00	3.56	−1.85	0.55	0.07
	✓	$N < 25$	0.11	2.61	−2.91	0.68	0.07
		$N \geq 25$	0.16	2.87	−1.03	0.21	0.07
		$N < 25$	0.22	2.35	−2.46	0.48	0.07
Retail	✓		0.00	0.00	−7.48	0.71	0.24
			0.00	0.00	−5.78	0.55	0.27

TABLE 3.32: Common survival functions

Model	$\mathbf{S}(t)$	$\lambda(t)$
Exponential	$\exp(-\lambda t)$	λ
Weibull	$\exp(-\lambda t^{\gamma})$	$\lambda\gamma t^{\gamma-1}$
Log-normal	$1 - \Phi(\gamma \ln(\lambda t))$	$\gamma t^{-1}\phi(\gamma \ln(\lambda t)) / (1 - \Phi(\gamma \ln(\lambda t)))$
Log-logistic	$1/\left(1 + \lambda t^{\frac{1}{\gamma}}\right)$	$\lambda\gamma^{-1} t^{\frac{1}{\gamma}} / \left(t + \lambda t^{1+\frac{1}{\gamma}}\right)$
Gompertz	$\exp(\lambda(1 - e^{\gamma t}))$	$\lambda\gamma \exp(\gamma t)$

TABLE 3.33: Calibrated piecewise exponential model from CDS prices

Maturity (in years)	#1	#2	#3
1	83.3	83.3	582.9
3	110.1	110.1	637.5
5	140.3	235.0	702.0
7	182.1	289.6	589.4
10	194.1	241.9	498.5

TABLE 3.34: Example of credit migration matrix (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	92.82	6.50	0.56	0.06	0.06	0.00	0.00	0.00
AA	0.63	91.87	6.64	0.65	0.06	0.11	0.04	0.00
A	0.08	2.26	91.66	5.11	0.61	0.23	0.01	0.04
BBB	0.05	0.27	5.84	87.74	4.74	0.98	0.16	0.22
BB	0.04	0.11	0.64	7.85	81.14	8.27	0.89	1.06
B	0.00	0.11	0.30	0.42	6.75	83.07	3.86	5.49
CCC	0.19	0.00	0.38	0.75	2.44	12.03	60.71	23.50
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Source: Kavvathas (2001).

TABLE 3.35: Two-year transition probability matrix P^2 (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	86.20	12.02	1.47	0.18	0.11	0.01	0.00	0.00
AA	1.17	84.59	12.23	1.51	0.18	0.22	0.07	0.02
A	0.16	4.17	84.47	9.23	1.31	0.51	0.04	0.11
BBB	0.10	0.63	10.53	77.66	8.11	2.10	0.32	0.56
BB	0.08	0.24	1.60	13.33	66.79	13.77	1.59	2.60
B	0.01	0.21	0.61	1.29	11.20	70.03	5.61	11.03
CCC	0.29	0.04	0.68	1.37	4.31	17.51	37.34	38.45
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

TABLE 3.36: Five-year transition probability matrix P^5 (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	69.23	23.85	5.49	0.96	0.31	0.12	0.02	0.03
AA	2.35	66.96	24.14	4.76	0.86	0.62	0.13	0.19
A	0.43	8.26	68.17	17.34	3.53	1.55	0.18	0.55
BBB	0.24	1.96	19.69	56.62	13.19	5.32	0.75	2.22
BB	0.17	0.73	5.17	21.23	40.72	20.53	2.71	8.74
B	0.07	0.47	1.73	4.67	16.53	44.95	5.91	25.68
CCC	0.38	0.24	1.37	2.92	7.13	18.51	9.92	59.53
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

TABLE 3.37: Markov generator $\hat{\Lambda}$ (in bps)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	−747.49	703.67	35.21	3.04	6.56	−0.79	−0.22	0.02
AA	67.94	−859.31	722.46	51.60	2.57	10.95	4.92	−1.13
A	7.69	245.59	−898.16	567.70	53.96	20.65	−0.22	2.80
BBB	5.07	21.53	650.21	−1352.28	557.64	85.56	16.08	16.19
BB	4.22	10.22	41.74	930.55	−2159.67	999.62	97.35	75.96
B	−0.84	11.83	30.11	8.71	818.31	−1936.82	539.18	529.52
CCC	25.11	−2.89	44.11	84.87	272.05	1678.69	−5043.00	2941.06
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3.38: Markov generator $\breve{\Lambda}$ (in bps)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	−745.85	699.11	38.57	2.80	6.27	−0.70	−0.16	−0.05
AA	67.54	−855.70	716.56	54.37	2.81	10.81	4.62	−1.01
A	7.77	243.62	−891.46	560.45	56.33	20.70	0.07	2.53
BBB	5.06	22.68	641.55	−1335.03	542.46	91.05	16.09	16.15
BB	4.18	10.12	48.00	903.40	−2111.65	965.71	98.28	81.96
B	−0.56	11.61	29.31	19.39	789.99	−1887.69	491.46	546.49
CCC	23.33	−1.94	42.22	81.25	272.44	1530.66	−4725.22	2777.25
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3.39: Markov generator $\bar{\Lambda}$ (in bps)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	−748.50	703.67	35.21	3.04	6.56	0.00	0.00	0.02
AA	67.94	−860.44	722.46	51.60	2.57	10.95	4.92	0.00
A	7.69	245.59	−898.38	567.70	53.96	20.65	0.00	2.80
BBB	5.07	21.53	650.21	−1352.28	557.64	85.56	16.08	16.19
BB	4.22	10.22	41.74	930.55	−2159.67	999.62	97.35	75.96
B	0.00	11.83	30.11	8.71	818.31	−1937.66	539.18	529.52
CCC	25.11	0.00	44.11	84.87	272.05	1678.69	−5045.89	2941.06
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3.40: Markov generator $\tilde{\Lambda}$ (in bps)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	−747.99	703.19	35.19	3.04	6.55	0.00	0.00	0.02
AA	67.90	−859.88	721.98	51.57	2.57	10.94	4.92	0.00
A	7.69	245.56	−898.27	567.63	53.95	20.65	0.00	2.80
BBB	5.07	21.53	650.21	−1352.28	557.64	85.56	16.08	16.19
BB	4.22	10.22	41.74	930.55	−2159.67	999.62	97.35	75.96
B	0.00	11.83	30.10	8.71	818.14	−1937.24	539.06	529.40
CCC	25.10	0.00	44.10	84.84	271.97	1678.21	−5044.45	2940.22
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3.41: 207-day transition probability matrix (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	95.85	3.81	0.27	0.03	0.04	0.00	0.00	0.00
AA	0.37	95.28	3.90	0.34	0.03	0.06	0.02	0.00
A	0.04	1.33	95.12	3.03	0.33	0.12	0.00	0.02
BBB	0.03	0.14	3.47	92.75	2.88	0.53	0.09	0.11
BB	0.02	0.06	0.31	4.79	88.67	5.09	0.53	0.53
B	0.00	0.06	0.17	0.16	4.16	89.84	2.52	3.08
CCC	0.12	0.01	0.23	0.45	1.45	7.86	75.24	14.64
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

TABLE 3.42: Discrete default correlation in % ($\lambda_1 = 100$ bps, $\lambda_2 = 50$ bps, Normal copula with $\rho = 20\%$)

t_1 / t_2	1	2	3	4	5	10	25	50
1	2.0	2.4	2.7	2.9	3.1	3.6	4.2	4.5
2	2.3	2.9	3.3	3.6	3.8	4.5	5.3	5.7
3	2.6	3.2	3.6	4.0	4.2	5.0	6.0	6.5
4	2.7	3.4	3.9	4.2	4.5	5.4	6.5	7.1
5	2.9	3.6	4.1	4.5	4.8	5.7	6.9	7.5
10	3.2	4.1	4.7	5.1	5.5	6.6	8.2	9.1
25	3.4	4.5	5.1	5.7	6.1	7.5	9.6	10.9
50	3.3	4.4	5.1	5.6	6.1	7.6	9.9	11.5

TABLE 3.43: Discrete default correlation in % ($\lambda_1 = 100$ bps, $\lambda_2 = 50$ bps, Student's t copula with $\rho = 20\%$ and $\nu = 4$)

t_1 / t_2	1	2	3	4	5	10	25	50
1	13.9	14.5	14.5	14.3	14.0	12.6	9.8	7.2
2	12.8	14.3	14.8	14.9	14.9	14.3	11.9	9.2
3	11.9	13.7	14.5	14.9	15.1	15.0	13.1	10.4
4	11.2	13.1	14.1	14.6	14.9	15.3	13.8	11.3
5	10.6	12.6	13.7	14.3	14.7	15.4	14.3	11.9
10	8.5	10.5	11.8	12.6	13.3	14.8	15.2	13.6
25	5.5	7.2	8.3	9.2	9.9	11.9	14.0	14.3
50	3.3	4.5	5.3	5.9	6.5	8.3	11.0	12.6

TABLE 3.44: Discrete default correlation in % ($\lambda_1 = 20\%$, $\lambda_2 = 10\%$, Normal copula with $\rho = 20\%$)

t_1 / t_2	1	2	3	4	5	10	25	50
1	8.8	10.2	10.7	11.0	11.1	10.4	6.6	2.4
2	9.4	11.0	11.8	12.1	12.3	11.9	7.9	3.1
3	9.3	11.0	11.9	12.4	12.7	12.5	8.6	3.4
4	9.0	10.8	11.7	12.2	12.6	12.6	8.9	3.7
5	8.6	10.4	11.3	11.9	12.3	12.4	9.0	3.8
10	6.3	7.8	8.7	9.3	9.7	10.3	8.1	3.7
25	1.9	2.4	2.8	3.1	3.3	3.8	3.5	1.9
50	0.2	0.3	0.3	0.3	0.4	0.5	0.5	0.3

TABLE 3.45: Statistics of the default rate (in %))

ρ	$\mu(f_t)$	$\sigma(f_t)$	$Q_\alpha(f_t)$						
			1%	10%	25%	50%	75%	90%	99%
0%	20.0	1.3	17.1	18.4	19.1	20.0	20.8	21.6	23.0
20%	20.0	13.0	1.7	5.6	10.0	17.4	27.3	38.3	59.0
50%	20.0	21.7	0.0	0.6	3.1	11.7	30.3	53.8	87.3
90%	20.0	33.2	0.0	0.0	0.0	0.4	26.3	88.2	100.0

TABLE 3.46: Estimation of canonical default correlations

Sector	CISC model	Basel model
Aerospace/Automobile	11.2%	11.6%
Consumer/Service sector	8.7%	7.5%
Energy/Natural resources	21.3%	11.5%
Financial institutions	15.7%	12.2%
Forest/Building products	6.8%	14.5%
Health	8.3%	9.2%
High technology	6.8%	4.7%
Insurance	12.2%	7.6%
Leisure time/Media	7.0%	7.0%
Real estate	35.9%	27.7%
Telecom	27.1%	34.3%
Transportation	6.8%	8.3%
Utilities	18.3%	21.2%
Inter-sector	6.8%	✓

Source: Demey *et al.* (2004).

TABLE 3.47: Granularity adjustment $\text{GA}^\star$ (in %)

n		50	100	500	50	100	500
Parameters	α	$\text{LGD} \sim \mathcal{U}_{[0,1]}$			$\text{LGD} = 50\%$		
$\text{PD} = 10\%$, $\rho = 10\%$	90%	13.8	7.4	1.6	12.5	6.8	1.2
	99%	19.3	10.0	2.1	13.3	6.2	1.2
	99.9%	21.5	10.9	2.3	12.2	6.9	1.6
$\text{PD} = 10\%$, $\rho = 20\%$	90%	8.1	4.2	0.9	2.7	2.7	0.9
	99%	10.3	5.3	1.1	6.7	4.1	0.6
	99.9%	11.3	5.6	1.2	6.5	2.8	0.6
$\text{PD} = 1\%$, $\rho = 20\%$	90%	43.7	23.5	5.0	60.1	20.1	4.0
	99%	36.7	18.8	3.9	32.9	19.6	3.7
	99.9%	30.2	15.3	3.1	23.7	9.9	1.7

Chapter 4

Counterparty Credit Risk and Collateral Risk

TABLE 4.1: Counterparty exposure of Bank A

t	1	2	3	4	5	6	7	8
No netting	7	17	8	0	2	3	10	20
Global netting	1	9	0	0	0	0	0	4
Partial netting	2	15	8	0	0	0	5	12

TABLE 4.2: Counterparty exposure of Bank B

t	1	2	3	4	5	6	7	8
No netting	6	8	12	17	19	17	14	16
Global netting	0	0	4	17	17	14	4	0
Partial netting	1	6	12	17	17	14	9	8

TABLE 4.3: Mark-to-market and counterparty exposure of the call option

t	Scenario #1				Scenario #2			
	$S(t)$	$\mathcal{C}(t)$	$\mathrm{MtM}(t)$	$e(t)$	$S(t)$	$\mathcal{C}(t)$	$\mathrm{MtM}(t)$	$e(t)$
1M	97.58	8.44	−2 013	0	91.63	5.36	−5 092	0
2M	98.19	8.25	−2 199	0	89.17	3.89	−6 564	0
3M	95.59	6.26	−4 188	0	97.60	7.35	−3 099	0
4M	106.97	12.97	2 519	2 519	97.59	6.77	−3 683	0
5M	104.95	10.83	382	382	96.29	5.48	−4 970	0
6M	110.73	14.68	4 232	4 232	97.14	5.29	−5 157	0
7M	113.20	16.15	5 700	5 700	107.71	11.55	1 098	1 098
8M	102.04	6.69	−3 761	0	105.71	9.27	−1 182	0
9M	115.76	17.25	6 802	6 802	107.87	10.18	−272	0
10M	103.58	5.96	−4 487	0	108.40	9.82	−630	0
11M	104.28	5.41	−5 043	0	104.68	5.73	−4 720	0
1Y	104.80	4.80	−5 646	0	115.46	15.46	5 013	5 013

TABLE 4.4: Capital charge of counterparty credit risk under the FIRB approach

	PD	1%	2%	3%	4%	5%
Basel II	ρ (PD) (in %)	19.28	16.41	14.68	13.62	12.99
	$\mathcal{K}$ (in \$ mn)	4.12	5.38	6.18	6.82	7.42
Basel III	ρ (PD) (in %)	24.10	20.52	18.35	17.03	16.23
	$\mathcal{K}$ (in \$ mn)	5.26	6.69	7.55	8.25	8.89
	$\Delta\mathcal{K}$ (in %)	27.77	24.29	22.26	20.89	19.88

TABLE 4.5: Regulatory add-on factors for the current exposure method

Residual Maturity	Fixed Income	FX and Gold	Equity	Precious Metals	Other Commodities
0−1Y	0.0%	1.0%	8.0%	7.0%	10.0%
1Y−5Y	0.5%	5.0%	8.0%	7.0%	12.0%
5Y+	1.5%	7.5%	10.0%	8.0%	15.0%

TABLE 4.6: Supervisory parameters for the SA-CCR approach

Asset class		$\mathcal{SF}_j$	ρ_k			Σ_i
Interest rate	0−1Y	0.50%	100%			50%
	1Y−5Y	0.50%	70%	100%		50%
	5Y+	0.50%	30%	70%	100%	50%
Foreign exchange		4.00%				15%
Credit	AAA	0.38%		50%		100%
	AA	0.38%		50%		100%
	A	0.42%		50%		100%
	BBB	0.54%		50%		100%
	BB	1.06%		50%		100%
	B	1.60%		50%		100%
	CCC	6.00%		50%		100%
	IG index	0.38%		80%		80%
	SG index	1.06%		80%		80%
Equity	Single name	32.00%		50%		120%
	Index	20.00%		80%		75%
Commodity	Electricity	40.00%		40%		150%
	Oil & gas	18.00%		40%		70%
	Metals	18.00%		40%		70%
	Agricultural	18.00%		40%		70%
	Other	18.00%		40%		70%

Source: BCBS (2014b).

TABLE 4.7: Supervisory risk weights (BA-CVA)

Sector	Credit quality	
	IG	HY/NR
Sovereign	0.5%	3.0%
Local government	1.0%	4.0%
Financial	5.0%	12.0%
Basic material, energy, industrial, agriculture, manufacturing, mining and quarrying	3.0%	7.0%
Consumer goods and services, transportation and storage, administrative and support service activities	3.0%	8.5%
Technology, telecommunication	2.0%	5.5%
Health care, utilities, professional and technical activities	1.5%	5.0%
Other sector	5.0%	12.0%

Source: BCBS (2017c).

Chapter 5

Operational Risk

TABLE 5.1: Internal losses larger than €20 000 per year

Year	pre 2002	2002	2003	2004	2005	2006	2007
n_L	14 017	10 216	13 691	22 152	33 216	36 386	36 622
L (in € bn)	3.8	12.1	4.6	7.2	9.7	7.4	7.9
n_B	24	35	55	68	108	115	117

Source: BCBS (2009d).

TABLE 5.2: Mapping of business lines for operational risk

Level 1	Level 2	β_j
Corporate Finance[†]	Corporate Finance Municipal/Government Finance Merchant Banking Advisory Services	18%
Trading & Sales[‡]	Sales Market Making Proprietary Positions Treasury	18%
Retail Banking	Retail Banking Private Banking Card Services	12%
Commercial Banking[♮]	Commercial Banking	12%
Payment & Settlement	External Clients	18%
Agency Services	Custody Corporate Agency Corporate Trust	15%
Asset Management	Discretionary Fund Management Non-Discretionary Fund Management	12%
Retail Brokerage	Retail Brokerage	12%

[†] Mergers and acquisitions, underwriting, securitization, syndications, IPO, debt placements.
[‡] Buying and selling of securities and derivatives, own position securities, lending and repos, brokerage.
[♮] Project finance, real estate, export finance, trade finance, factoring, leasing, lending, guarantees, bills of exchange.

TABLE 5.3: Distribution of annualized operational losses (in %)

Business line	Event type 1	2	3	4	5	6	7	All
Corporate Finance	0.2	0.1	0.6	93.7	0.0	0.0	5.4	28.0
Trading & Sales	11.0	0.3	2.3	29.0	0.2	1.8	55.3	13.6
Retail Banking	6.3	19.4	9.8	40.4	1.1	1.5	21.4	32.0
Commercial Banking	11.4	15.2	3.1	35.5	0.4	1.7	32.6	7.6
Payment & Settlement	2.8	7.1	0.9	7.3	3.2	2.3	76.4	2.6
Agency Services	1.0	3.2	0.7	36.0	18.2	6.0	35.0	2.6
Asset Management	11.1	1.0	2.5	30.8	0.3	1.5	52.8	2.5
Retail Brokerage	18.1	1.4	6.3	59.5	0.1	0.2	14.4	5.1
Unallocated	6.5	2.8	28.4	28.3	6.5	1.3	26.2	6.0
All	6.1	8.0	6.0	52.4	1.4	1.2	24.9	100.0

Source: BCBS (2009d).

TABLE 5.4: Density function, mean and variance of parametric probability distribution

Distribution	$f(x;\theta)$	$\mathbb{E}[X]$	$\operatorname{var}(X)$
$\mathcal{G}(\alpha,\beta)$	$\dfrac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}$	$\dfrac{\alpha}{\beta}$	$\dfrac{\alpha}{\beta^2}$
$\mathcal{LG}(\alpha,\beta)$	$\dfrac{\beta^\alpha (\ln x)^{\alpha-1}}{x^{\beta+1}\Gamma(\alpha)}$	$\left(\dfrac{\beta}{\beta-1}\right)^\alpha$ if $\beta>1$	$\left(\dfrac{\beta}{\beta-2}\right)^\alpha - \left(\dfrac{\beta}{\beta-1}\right)^{2\alpha}$ if $\beta>2$
$\mathcal{LL}(\alpha,\beta)$	$\dfrac{\beta(x/\alpha)^{\beta-1}}{\alpha\left(1+(x/\alpha)^\beta\right)^2}$	$\dfrac{\alpha\pi}{\beta\sin(\pi/\beta)}$ if $\beta>1$	$\alpha^2\left(\dfrac{2\pi}{\beta\sin(2\pi/\beta)}-\dfrac{\pi^2}{\beta^2\sin^2(\pi/\beta)}\right)$ if $\beta>2$
$\mathcal{LN}\left(\mu,\sigma^2\right)$	$\dfrac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right)$	$\exp\left(\mu+\dfrac{1}{2}\sigma^2\right)$	$\exp\left(2\mu+\sigma^2\right)\left(\exp\left(\sigma^2\right)-1\right)$
$\mathcal{GEV}(\mu,\sigma,\xi)$	$\dfrac{1}{\sigma}\left[1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right]^{-(1+1/\xi)} \exp\left\{-\left[1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$	$\mu+\dfrac{\sigma}{\xi}\left(\Gamma(1-\xi)-1\right)$ if $\xi<1$	$\dfrac{\sigma^2}{\xi^2}\left(\Gamma(1-2\xi)-\Gamma^2(1-\xi)\right)$ if $\xi<\frac{1}{2}$
$\mathcal{P}(\alpha,x_-)$	$\dfrac{\alpha x_-^\alpha}{x^{\alpha+1}}$	$\dfrac{\alpha x_-}{\alpha-1}$ if $\alpha>1$	$\dfrac{\alpha x_-^2}{(\alpha-1)^2(\alpha-2)}$ if $\alpha>2$

TABLE 5.5: Comparison of the capital-at-risk calculated with Panjer recursion and Monte Carlo simulations

α	Panjer recursion (a)	(b)	(c)	(d)	Monte Carlo simulations (a)	(b)	(c)	(d)
90%	2400	4500	11000	91000	2350	4908	11648	93677
95%	2900	6500	19000	120000	2896	6913	19063	123569
99%	4300	13500	52000	231000	4274	13711	51908	233567
99.5%	4900	18000	77000	308000	4958	17844	77754	310172
99.9%	6800	32500	182000	604000	6773	32574	185950	604756

Chapter 6

Liquidity Risk

TABLE 6.1: An example of a limit order book

i^{th} limit	Buy orders		Sell orders	
	$Q_t^{\text{bid,i}}$	$P_t^{\text{bid,i}}$	$Q_t^{\text{ask,i}}$	$P_t^{\text{ask,i}}$
1	65 201	26.325	70 201	26.340
2	85 201	26.320	116 201	26.345
3	105 201	26.315	107 365	26.350
4	76 500	26.310	35 000	26.355
5	20 000	26.305	35 178	26.360

TABLE 6.2: Statistics of the liquidation ratio (size = \$10 bn, liquidation policy = 10% of ADV)

Statistics	SPX	SX5E	DAX	NDX	MSCI EM	MSCI INDIA	MSCI EMU SC
m (in days)	Liquidation ratio $\mathcal{LR}(t)$ in %						
1	88.4	12.3	4.8	40.1	22.1	1.5	3.0
2	99.5	24.7	9.6	72.6	40.6	3.0	6.0
5	100.0	58.8	24.1	99.7	75.9	7.6	14.9
10	100.0	90.1	47.6	99.9	93.9	15.1	29.0
α (in %)	Liquidation time $\mathcal{LR}^{-1}(\alpha)$ in days						
50	1	5	11	2	3	37	21
75	1	7	17	3	5	71	43
90	2	10	23	3	9	110	74
99	2	15	29	5	17	156	455

Source: Roncalli and Weisang (2015).

TABLE 6.3: Statistics of the liquidation ratio (size = $10 bn, liquidation policy = 30% of ADV)

Statistics	SPX	SX5E	DAX	NDX	MSCI EM	MSCI INDIA	MSCI EMU SC
t (in days)	Liquidation ratio $\mathcal{LR}(t)$ in %						
1	100.0	37.0	14.5	91.0	55.5	4.5	9.0
2	100.0	67.7	28.9	99.8	81.8	9.1	17.8
5	100.0	99.2	68.6	100.0	98.5	22.6	40.4
10	100.0	100.0	99.6	100.0	100.0	43.1	63.2
α (in %)	Liquidation time $\mathcal{LR}^{-1}(\alpha)$ in days						
50	1	2	4	1	1	13	7
75	1	3	6	1	2	24	15
90	1	4	8	1	3	37	25
99	1	5	10	2	6	52	152

Source: Roncalli and Weisang (2015).

TABLE 6.4: Stock of HQLA

Level	Description	Haircut
Level 1 assets		
	Coins and bank notes Sovereign, central bank, PSE, and MDB assets qualifying for 0% risk weighting Central bank reserves Domestic sovereign or central bank debt for non-0% risk weighting	0%
Level 2 assets (maximum of 40% of HQLA)		
Level 2A assets		
	Sovereign, central bank, PSE and MDB assets qualifying for 20% risk weighting Corporate debt securities rated AA− or higher Covered bonds rated AA− or higher	15%
Level 2B assets (maximum of 15% of HQLA)		
	RMBS rated AA or higher	25%
	Corporate debt securities rated between A+ and BBB−	50%
	Common equity shares	50%

Source: BCBS (2013a).

TABLE 6.5: Cash outflows of the LCR

Liabilities	Description	Rate
Retail deposits		
Demand and term deposits (less than 30 days)		
	Stable deposits covered by deposit insurance	3%
	Stable deposits	5%
	Less stable deposits	10%
Term deposits (with residual maturity greater than 30 days)		0%
Unsecured wholesale funding		
Demand and term deposits (less than 30 days) provided by small business customers		
	Stable deposits	5%
	Less stable deposits	10%
Deposits generated by clearing, custody and cash management		25%
	Portion covered by deposit insurance	5%
Cooperative banks in an institutional network		25%
Corporates, sovereigns, central banks, PSEs and MDBs		40%
	Portion covered by deposit insurance	20%
Secured funding transactions		
With a central bank counterparty		0%
Backed by level 1 assets		0%
Backed by level 2A assets		15%
Backed by non-level 1 or non-level 2A assets with domestic sovereigns, PSEs or MDBs as a counterparty		25%
Backed by level 2B RMBS assets		25%
Backed by other level 2B assets		50%
All other secured funding transactions		100%
Additional requirements		
Margin/collateral calls		$\geq$ 20%
ABCP, SIVs, conduits, SPVs, etc.		100%
Net derivative cash outflows		100%
Other credit/liquidity facilities		$\geq$ 5%

Source: BCBS (2013a).

TABLE 6.6: Cash inflows of the LCR

Receivables Description	Rate
Maturing secured lending transactions	
Backed by level 1 assets	0%
Backed by level 2A assets	15%
Backed by level 2B RMBS assets	25%
Backed by other level 2B assets	50%
Backed by non-HQLAs	100%
Other cash inflows	
Credit/liquidity facilities provided to the bank	0%
Inflows to be received from retail counterparties	50%
Inflows to be received from non-financial wholesale counterparties	50%
Inflows to be received from financial institutions and central banks	100%
Net derivative receivables	100%

Source: BCBS (2013a).

Chapter 7

Asset Liability Management Risk

TABLE 7.1: Assets and liabilities of FDIC-insured commercial banks and savings institutions (Amounts in $ bn)

Total Assets	**17 943**
Loans secured by real estate	**4 888**
1-4 Family residential mortgages	2 119
Nonfarm nonresidential	1 446
Construction and development	350
Home equity lines	376
Multifamily residential real estate	430
Farmland	105
Real estate loans in foreign offices	62
Commercial & industrial loans	**2 165**
Loans to individuals	**1 743**
Credit cards	903
Other loans to individuals	839
Auto loans	455
Farm loans	82
Loans to depository institutions	84
Loans to foreign gov. & official inst.	11
Obligations of states in the U.S.	188
Other loans	862
Lease financing receivables	133
Gross total loans and leases	10 155
Less: Unearned income	2
Total loans and leases	10 152
Less: Reserve for losses	125
Net loans and leases	**10 028**
Securities	**3 723**
Available for sale (fair value)	2 590
Held to maturity (amortized cost)	1 129
U.S. Treasury securities	549
Mortgage-backed securities	2 187
State and municipal securities	330
Equity securities	3
Cash & due from depos. instit.	**1 694**
Fed. funds sold and reverse repos	622
Bank premises and fixed assets	130
Other real estate owned	7
Trading account assets	572
Intangible assets	399
Goodwill	334
Other Assets	769

Total liabilities and capital	**17 943**
Deposits	**13 866**
Foreign office deposits	1 253
Domestic office deposits	12 613
Interest-bearing deposits	9 477
Noninterest-bearing deposits	3 136
Estimated insured deposits	7 483
Time deposits	1 971
Brokered deposits	1 071
Federal funds purchased & repos	240
FHLB advances	571
Other borrowed money	557
Subordinated debt	69
Trading account liabilities	236
Other liabilities	381
Total liabilities	**15 921**
Total equity capital	**2 023**
Total bank equity capital	2 019
Perpetual preferred stock	9
Common stock	43
Surplus	1 277
Undivided profits	759
Other comprehensive income	-68
Net unrealized P&L on AFS	0

Source: Federal Deposit Insurance Corporation (2019), `www.fdic.gov/bank/analytical/qbp`.

TABLE 7.2: A simplified balance sheet

Assets	Liabilities
Cash	Due to central banks
Loans and leases	Deposits
Mortgages	Deposit accounts
Consumer credit	Savings
Credit cards	Term deposits
Interbank loans	Interbank funding
Investment securities	Short-term debt
Sovereign bonds	Subordinated debt
Corporate bonds	Reserves
Other assets	Equity capital

TABLE 7.3: Annual income and expense of FDIC-insured commercial banks and savings institutions (Amounts in $ mn)

Total interest income	**660 988**
Domestic office loans	492 201
Foreign office loans	21 965
Lease financing receivables	5 192
Balances due from depository institutions	24 954
Securities	92 908
Trading accounts	11 025
Federal funds sold	8 347
Other interest income	4 397
Total interest expense	**119 799**
Domestic office deposits	74 781
Foreign office deposits	8 877
Federal funds purchased	4 108
Trading liabilities and other borrowed money	28 629
Subordinated notes and debentures	2 780
Net interest income	**541 189**
Provision for loan and lease losses	**49 998**
Total noninterest income	**266 165**
Fiduciary activities	37 525
Service charges on deposit accounts	35 745
Trading account gains and fees	26 755
Interest rate exposures	7 148
Foreign exchange exposures	12 666
Equity security and index exposures	4 750
Commodity and other exposures	1 299
Credit exposures	367
Investment banking, advisory, brokerage and underwriting fees and commissions	12 522
Venture capital revenue	60
Net servicing fees	10 680
Net securitization income	230
Insurance commission fees and income	4 574
Net gains (losses) on sales of loans	12 593
Net gains (losses) on sales of other real estate owned	-99
Net gains (losses) on sales of other assets (except securities)	1 644
Other noninterest income	123 938
Total noninterest expense	**459 322**
Salaries and employee benefits	217 654
Premises and equipment expense	45 667
Other noninterest expense	190 944
Amortization expense and goodwill impairment losses	5 058
Securities gains (losses)	328
Income (loss) before income taxes and extraordinary items	**298 362**
Applicable income taxes	61 058
Extraordinary gains (losses), net	-267
Net charge-offs	47 479
Cash dividends	164 704
Retained earnings	72 045
Net operating income	**237 059**

Source: Federal Deposit Insurance Corporation (2019), `www.fdic.gov/bank/analytical/qbp`.

TABLE 7.4: A simplified income statement

	Interest income
$-$	Interest expenses
$=$	Net interest income
$+$	Non-interest income
$=$	Gross income
$-$	Operating expenses
$=$	Net income
$-$	Provisions
$=$	Earnings before tax
$-$	Income tax
$=$	Profit after tax

TABLE 7.5: Computation of the liquidity gap

Period	0	1	2	3	4	5	6	7	8	9	10	11	12
Loans	120	110	100	90	80	70	60	50	40	30	20	10	0
Assets	120	110	100	90	80	70	60	50	40	30	20	10	0
Debt #1	65	65	65										
Debt #2	10	10	10	10	10	10	10	10					
Debt #3	5	5	5	5	5	5	5	5	5	5	5	5	5
Debt (total)	80	80	80	15	15	15	15	15	5	5	5	5	5
Equity	40	40	40	40	40	40	40	40	40	40	40	40	40
Liabilities	120	120	120	55	55	55	55	55	45	45	45	45	45
$\mathcal{LG}(t)$	0	10	20	-35	-25	-15	-5	5	5	15	25	35	45

TABLE 7.6: Repayment schedule of the constant amortization mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	15.00	5.00	10.00	10.00	90.00
2	90.00	14.50	4.50	10.00	20.00	80.00
3	80.00	14.00	4.00	10.00	30.00	70.00
4	70.00	13.50	3.50	10.00	40.00	60.00
5	60.00	13.00	3.00	10.00	50.00	50.00
6	50.00	12.50	2.50	10.00	60.00	40.00
7	40.00	12.00	2.00	10.00	70.00	30.00
8	30.00	11.50	1.50	10.00	80.00	20.00
9	20.00	11.00	1.00	10.00	90.00	10.00
10	10.00	10.50	0.50	10.00	100.00	0.00

TABLE 7.7: Repayment schedule of the constant payment mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	12.95	5.00	7.95	7.95	92.05
2	92.05	12.95	4.60	8.35	16.30	83.70
3	83.70	12.95	4.19	8.77	25.06	74.94
4	74.94	12.95	3.75	9.20	34.27	65.73
5	65.73	12.95	3.29	9.66	43.93	56.07
6	56.07	12.95	2.80	10.15	54.08	45.92
7	45.92	12.95	2.30	10.65	64.73	35.27
8	35.27	12.95	1.76	11.19	75.92	24.08
9	24.08	12.95	1.20	11.75	87.67	12.33
10	12.33	12.95	0.62	12.33	100.00	0.00

TABLE 7.8: Repayment schedule of the bullet repayment mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	5.00	5.00	0.00	0.00	100.00
2	100.00	5.00	5.00	0.00	0.00	100.00
3	100.00	5.00	5.00	0.00	0.00	100.00
4	100.00	5.00	5.00	0.00	0.00	100.00
5	100.00	5.00	5.00	0.00	0.00	100.00
6	100.00	5.00	5.00	0.00	0.00	100.00
7	100.00	5.00	5.00	0.00	0.00	100.00
8	100.00	5.00	5.00	0.00	0.00	100.00
9	100.00	5.00	5.00	0.00	0.00	100.00
10	100.00	105.00	5.00	100.00	100.00	0.00

TABLE 7.9: Computation of the liquidity gap (mixed schedule)

t	Assets					Liabilities					$\mathcal{LG}_t$
	#1	#2	#3	#4	A_t	#1	#2	#3	#4	L_t	
1	99.4	49.9	39.6	110	298.8	119.2	78.7	70	30	297.9	−0.92
2	98.7	49.7	39.2	110	297.6	118.5	77.3	70	30	295.8	−1.83
3	98.1	49.6	38.8	110	296.4	117.7	76.0	70	30	293.7	−2.75
4	97.4	49.5	38.3	110	295.2	116.9	74.7	70	30	291.6	−3.66
5	96.8	49.3	37.9	110	294.0	116.1	73.3	70	30	289.4	−4.58
6	96.1	49.2	37.5	110	292.8	115.3	72.0	70	30	287.3	−5.49
7	95.4	49.1	37.1	110	291.6	114.5	70.7	70	30	285.2	−6.41
8	94.8	48.9	36.7	110	290.4	113.7	69.3	70	30	283.1	−7.32
9	94.1	48.8	36.3	110	289.2	112.9	68.0	70	30	280.9	−8.24
10	93.4	48.7	35.8	110	287.9	112.1	66.7	70	30	278.8	−9.15
11	92.8	48.5	35.4	110	286.7	111.3	65.3	70	30	276.7	−10.06
12	92.1	48.4	35.0	110	285.5	110.5	64.0	70	30	274.5	−10.97
0	100.0	50.0	40.0	110	300.0	120.0	80.0	70	30	300.0	0.00
1	92.1	48.4	35.0	110	285.5	110.5	64.0	70	30	274.5	−10.97
2	83.8	46.7	30.0	110	270.4	100.5	48.0	70	30	248.5	−21.90
3	75.0	44.8	25.0	110	254.8	90.1	32.0	70	30	222.1	−32.76
4	65.9	42.7	20.0	110	238.6	79.0	16.0	70	30	195.0	−43.55
5	56.2	40.5	15.0	110	221.7	67.4		70	30	167.4	−54.27
6	46.1	38.1	10.0	110	204.2	55.3		70	30	155.3	−48.91
7	35.4	35.5	5.0		75.9	42.5		70	30	142.5	66.56
8	24.2	32.7			56.9	29.0		70	30	129.0	72.12
9	12.4	29.7			42.1	14.9		70	30	114.9	72.81
10		26.4			26.4				30	30.0	3.62
11		22.8			22.8				30	30.0	7.19
12		18.9			18.9				30	30.0	11.06
13		14.8			14.8				30	30.0	15.24
14		10.2			10.2				30	30.0	19.77
15		5.3			5.3				30	30.0	24.68
16					0.0				30	30.0	30.00

TABLE 7.10: Relationship between the new production and the outstanding amount

s	NP (s)	$\mathbf{S}(s,7)$	NP $(s,7)$	$\mathbf{S}(s,10)$	NP $(s,10)$	$\mathbf{S}(s,12)$	NP $(s,12)$
1	110	0.301	33.13	0.165	18.18	0.111	12.19
2	125	0.368	45.98	0.202	25.24	0.135	16.92
3	95	0.449	42.69	0.247	23.43	0.165	15.70
4	75	0.549	41.16	0.301	22.59	0.202	15.14
5	137	0.670	91.83	0.368	50.40	0.247	33.78
6	125	0.819	102.34	0.449	56.17	0.301	37.65
7	115	1.000	115.00	0.549	63.11	0.368	42.31
8	152			0.670	101.89	0.449	68.30
9	147			0.819	120.35	0.549	80.68
10	159			1.000	159.00	0.670	106.58
11	152					0.819	124.45
12	167					1.000	167.00
$N(t)$			472.14		640.36		720.69

TABLE 7.11: Amortization function and liquidity duration of the three amortization schemes

Amortization	$\mathbf{S}(t,u)$	$\mathcal{D}(t)$
Bullet	$\mathbb{1}\{t \le u < t+m\}$	m
Constant	$\mathbb{1}\{t \le u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)$	$\frac{m}{2}$
Exponential	$e^{-\lambda(u-t)}$	$\frac{1}{\lambda}$
Amortization	$\mathbf{S}^{\star}(t,u)$	$\mathcal{D}^{\star}(t)$
Bullet	$\mathbb{1}\{t \le u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)$	$\frac{m}{2}$
Constant	$\mathbb{1}\{t \le u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)^2$	$\frac{m}{3}$
Exponential	$e^{-\lambda(u-t)}$	$\frac{1}{\lambda}$
Amortization	$\mathrm{d}N(t)$	
Bullet	$\mathrm{d}N(t) = (\mathrm{NP}(t) - \mathrm{NP}(t-m))\ \mathrm{d}t$	
Constant	$\mathrm{d}N(t) = \left(\mathrm{NP}(t) - \frac{1}{m}\int_{t-m}^{t} \mathrm{NP}(s)\ \mathrm{d}s\right)\ \mathrm{d}t$	
Exponential	$\mathrm{d}N(t) = (\mathrm{NP}(t) - \lambda N(t))\ \mathrm{d}t$	

TABLE 7.12: Cap on core deposits and maximum average maturity

Category	Cap ω^+	Cap t^+
Retail transactional	90%	5.0
Retail non-transactional	70%	4.5
Wholesale	50%	4.0

TABLE 7.13: Economic value of the assets

Bucket	t_k	$\mathrm{CF}_0(t_k)$	$R_0(t_k)$	$\mathrm{EV}_0(t_k)$
6	0.875	200	1.55%	197.31
11	4.50	700	3.37%	601.53
17	12.50	100	5.71%	48.98
EV_0				847.82

TABLE 7.14: Economic value of the pure liabilities

Bucket	t_k	$\mathrm{CF}_0(t_k)$	$R_0(t_k)$	$\mathrm{EV}_0(t_k)$
1	0.0028	100	1.00%	100.00
5	0.625	50	1.39%	49.57
9	2.50	450	2.44%	423.35
10	3.50	100	2.93%	90.26
14	7.50	100	4.46%	71.56
EV_0				734.73

TABLE 7.15: Stressed economic value of equity

Bucket	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$
			Assets			
$R_s(t_6)$	3.55%	−0.45%	0.24%	3.30%	3.96%	−0.87%
$R_s(t_{11})$	5.37%	1.37%	3.65%	3.54%	4.34%	2.40%
$R_s(t_{17})$	7.71%	3.71%	6.92%	4.96%	5.84%	5.58%
$\mathrm{EV}_s(t_6)$	193.89	200.80	199.57	194.31	193.20	201.52
$\mathrm{EV}_s(t_{11})$	549.76	658.18	594.03	596.91	575.74	628.48
$\mathrm{EV}_s(t_{17})$	38.15	62.89	42.13	53.83	48.18	49.79
EV_s	781.79	921.87	835.74	845.05	817.11	879.79
			Pure liabilities			
$R_s(t_1)$	3.00%	−1.00%	−0.95%	3.40%	4.00%	−2.00%
$R_s(t_5)$	3.39%	−0.61%	−0.08%	3.32%	3.96%	−1.17%
$R_s(t_9)$	4.44%	0.44%	2.03%	3.31%	4.05%	0.84%
$R_s(t_{10})$	4.93%	0.93%	2.90%	3.40%	4.18%	1.68%
$R_s(t_{14})$	6.46%	2.46%	5.31%	4.07%	4.92%	4.00%
$\mathrm{EV}_s(t_1)$	99.99	100.00	100.00	99.99	99.99	100.01
$\mathrm{EV}_s(t_5)$	48.95	50.19	50.02	48.97	48.78	50.37
$\mathrm{EV}_s(t_9)$	402.70	445.05	427.77	414.27	406.69	440.69
$\mathrm{EV}_s(t_{10})$	84.16	96.80	90.34	88.77	86.39	94.30
$\mathrm{EV}_s(t_{14})$	61.59	83.14	67.17	73.70	69.13	74.07
EV_s	697.39	775.18	735.31	725.71	710.98	759.43
			Equity			
EVE_s	84.41	146.68	100.43	119.34	106.13	120.37
$\Delta\,\mathrm{EVE}_s$	28.69	−33.58	12.67	−6.24	6.97	−7.27

TABLE 7.16: Impact of a parallel shift of the yield curve

	Discrete-time		Continuous-time	
ΔR	+1%	−1%	+1%	−1%
$V(R+\Delta R)$	816.69	887.52	812.78	886.09
ΔV	−34.38	36.75	−35.57	37.74
$V(y+\Delta R)$	815.64	888.42	811.94	887.02
ΔV	−35.13	37.64	−36.41	38.67
Approximation	−36.35	36.35	−37.51	37.51

TABLE 7.17: Bank balance sheet after immunization of the duration gap

Assets	V_j	$\mathcal{D}_j$	Liabilities	V_j	$\mathcal{D}_j$
Cash	5	0.0	Deposits	40	3.2
Loans	40	1.5	CDs	20	0.8
Mortgages	40	6.0	Debt	10.48	1.7
Securities	15	3.8	Zero-coupon bond	19.52	10.0
			Equity capital	10	0.0
Total	100		Total	100	

TABLE 7.18: Interest income schedule and liquidity gap

$u-t$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Loan A	7.50	7.50	7.50	7.50	7.50	7.50		
Loan B	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
IR revenues	13.25	13.25	13.25	13.25	13.25	13.25	6.25	6.25
Debt C	6.00	6.00	6.00	6.00				
Equity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
IR expenses	6.00	6.00	6.00	6.00	0.00	0.00	0.00	0.00
NII (t,u)	7.25	7.25	7.25	7.25	13.25	13.25	6.25	6.25
$\mathcal{LG}(t,u)$	0	0	0	0	-800	-800	-300	-300

TABLE 7.19: Balance sheet under the constraint of a zero liquidity gap

	$u-t$	1.25	1.50	1.75	2.00
Approach #1	Debt D	500	500		
	Debt E	300	300	300	300
Approach #2	Loan F			500	500
	Debt G	800	800	800	800

TABLE 7.20: Optimal refinancing rule $\delta^\star$

M (in KUSD)	$\delta^\star$	r	$\delta^\star$	σ	$\delta^\star$	λ	$\delta^\star$
10	612	1%	101	1%	79	2%	89
100	198	2%	103	2%	110	5%	98
250	150	5%	110	3%	133	10%	110
500	131	8%	116	5%	171	15%	120
1 000	121	10%	120	10%	239	20%	128

TABLE 7.21: Impact of a new mortgage rate (100 KUSD, 5%, 10-year)

i	A	$\mathfrak{D}_A$ (in \$)		$\mathfrak{D}_R$	$\mathfrak{C}$	$\mathfrak{N}$
(in %)	(in \$)	Monthly	Annually	(in %)	(in %)	(in years)
5.0	1 061					
4.5	1 036	24	291	2.3	2.9	9.67
4.0	1 012	48	578	4.5	5.8	9.42
3.5	989	72	862	6.8	8.6	9.17
3.0	966	95	1 141	9.0	11.4	8.92
2.5	943	118	1 415	11.1	14.2	8.75
2.0	920	141	1 686	13.2	16.9	8.50
1.5	898	163	1 953	15.3	19.5	8.33
1.0	876	185	2 215	17.4	22.2	8.17
0.5	855	206	2 474	19.4	24.7	8.00

TABLE 7.22: Impact of a new mortgage rate (100 KUSD, 5%, 20-year)

i (in %)	A (in \$)	$\mathfrak{D}_A$ (in \$) Monthly	Annually	$\mathfrak{D}_R$ (in %)	$\mathfrak{C}$ (in %)	$\mathfrak{N}$ (in years)
5.0	660					
4.5	633	27	328	4.1	6.6	18.67
4.0	606	54	648	8.2	13.0	17.58
3.5	580	80	960	12.1	19.2	16.67
3.0	555	105	1 264	16.0	25.3	15.83
2.5	530	130	1 561	19.7	31.2	15.17
2.0	506	154	1 849	23.3	37.0	14.50
1.5	483	177	2 129	26.9	42.6	14.00
1.0	460	200	2 401	30.3	48.0	13.50
0.5	438	222	2 664	33.6	53.3	13.00

TABLE 7.23: Impact of a new mortgage rate (100 KUSD, 10%, 10-year)

i (in %)	A (in \$)	$\mathfrak{D}_A$ (in \$) Monthly	Annually	$\mathfrak{D}_R$ (in %)	$\mathfrak{C}$ (in %)	$\mathfrak{N}$ (in years)
10.0	1 322					
9.0	1 267	55	657	4.1	6.6	9.33
8.0	1 213	108	1 299	8.2	13.0	8.75
7.0	1 161	160	1 925	12.1	19.3	8.33
6.0	1 110	211	2 536	16.0	25.4	7.92
5.0	1 061	261	3 130	19.7	31.3	7.58
4.0	1 012	309	3 709	23.3	37.1	7.25
3.0	966	356	4 271	26.9	42.7	6.92
2.0	920	401	4 816	30.4	48.2	6.67
1.0	876	445	5 346	33.7	53.5	6.50

TABLE 7.24: Conditional prepayment rates in June 2018 by coupon rate and issuance date

Year	2012	2013	2014	2015	2016	2017	2018
Coupon = 3%	9.6%	10.2%	10.9%	10.0%	8.7%	5.3%	3.1%
Coupon = 4.5%	16.1%	15.8%	16.6%	17.9%	17.4%	12.8%	5.3%
Difference	6.5%	5.6%	5.7%	8.0%	8.7%	7.6%	2.2%

Source: RiskSpan dataset, FHFA (2018) & author's calculations.

Chapter 8

Systemic Risk and Shadow Banking System

TABLE 8.1: List of global systemically important banks (November 2018)

Agricultural Bank of China	Bank of America	Bank of China
Bank of New York Mellon	Barclays	BNP Paribas
China Construction Bank	Citigroup	Credit Suisse
Deutsche Bank	Goldman Sachs	Crédit Agricole
BPCE	HSBC	ICBC
ING Bank	JP Morgan Chase	Mitsubishi UFJ FG
Mizuho FG	Morgan Stanley	Royal Bank of Canada
Santander	Société Générale	Standard Chartered
State Street	Sumitomo Mitsui FG	UBS
UniCredit	Wells Fargo	

Source: FSB (2018b), 2018 List of Global Systemically Important Banks.

TABLE 8.2: List of global systemically important insurers (November 2016)

Aegon	Allianz	AIG
Aviva	AXA	MetLife
Ping An Group	Prudential Financial	Prudential plc

Source: FSB (2016), 2016 List of Global Systemically Important Insurers.

TABLE 8.3: Scoring system of G-SIBs

	Category		Indicator	Weight
1	Size	1	Total exposures	$1/5$
2	Interconnectedness	2	Intra-financial system assets	$1/15$
		3	Intra-financial system liabilities	$1/15$
		4	Securities outstanding	$1/15$
3	Substitutability/financial institution infrastructure	5	Payment activity	$1/15$
		6	Assets under custody	$1/15$
		7	Underwritten transactions in debt and equity markets	$1/15$
4	Complexity	8	Notional amount of OTC derivatives	$1/15$
		9	Trading and AFS securities	$1/15$
		10	Level 3 assets	$1/15$
5	Cross-jurisdictional activity	11	Cross-jurisdictional claims	$1/10$
		12	Cross-jurisdictional liabilities	$1/10$

TABLE 8.4: An example of calculating the G-SIB score

Category	Indicator	Indicator value[(1)]	Sample total[(1)]	Indicator score[(2)]	Category score[(2)]
Size	Total exposures	2,032	66,313	306	306
Interconnectedness	Intra-financial system assets	205	7,718	266	370
	Intra-financial system liabilities	435	7,831	556	
	Securities outstanding	314	10,836	290	
Substitutability/financial insitution infrastructure	Payment activity	49,557	1,850,755	268	369
	Assets under custody	4,181	100,012	418	
	Underwritten transactions in debt and equity markets	189	4,487	422	
Complexity	Notional amount of OTC derivatives	39,104	639,988	611	505
	Trading and AFS securities	185	3,311	559	
	Level 3 assets	21	595	346	
Cross-jurisdictional activity	Cross-jurisdictional claims	877	15,801	555	485
	Cross-jurisdictional liabilities	584	14,094	414	
Final score					407

[(1)] The figures are expressed in billion of EUR.

[(2)] The figures are expressed in bps.

Source: BCBS (2014), G-SIB Framework: Denominators; BNP Paribas (2014), Disclosure for G-SIBs indicators as of 31 December 2013.

TABLE 8.5: Risk decomposition of the 95% systemic expected shortfall

Bank	w_i (in \$ bn)	MES_i (in %)	SES_i (in \$ bn)	$\beta_i(w)$ (in bps)	$\beta_i(w^\star)$
1	139	19.28	26.80	28.55	0.84
2	75	22.49	16.87	33.29	0.98
3	81	29.48	23.88	43.64	1.29
$\mathrm{ES}_\alpha(w)$			67.55		

TABLE 8.6: Calculation of the 95% CoVaR measure

Bank	w_i (in \$ bn)	$\mathrm{VaR}_\alpha(L_i)$ (in %)	(in \$ bn)	$\mathrm{CoVaR}_i(\mathcal{E})$ $\mathcal{D}_i = 1$	$\mathcal{D}_i = 0$	$\Delta\,\mathrm{CoVaR}_i$ (in \$ bn)
1	139	16.45	22.86	69.48	19.13	50.35
2	75	19.74	14.80	71.44	22.50	48.94
3	81	24.67	19.98	67.69	16.37	51.32

TABLE 8.7: Calculation of the SRISK measure ($\mathbb{S} = -40\%$)

Bank	$\mathrm{MES}_{i,t}$ (in %)	$\vartheta_{i,t}$	$\mathrm{SRISK}_{i,t}$ (in \$ bn)	(in %)
1	41.18	1.22	69.47	22.11
2	42.35	1.63	105.93	33.70
3	35.20	0.36	33.11	10.54
4	30.59	0.88	105.77	33.65

TABLE 8.8: Impact of the stress $\mathbb{S}$ on SRISK

Bank	$\mathbb{S} = -20\%$ (in \$ bn)	(in %)	$\mathbb{S} = -40\%$ (in \$ bn)	(in %)	$\mathbb{S} = -60\%$ (in \$ bn)	(in %)
1	58.7	22.6	69.5	22.1	80.3	21.7
2	93.3	36.0	105.9	33.7	118.6	32.1
3	18.4	7.1	33.1	10.5	47.8	13.0
4	88.9	34.3	105.8	33.7	122.7	33.2

TABLE 8.9: Systemic risk contributions in America (2015-11-27)

Rank	institution	$\mathrm{SRISK}_{i,t}$ (in \$ bn)	(in %)	$\mathrm{MES}_{i,t}$ (in %)	$\mathcal{LR}_{i,t}$
1	Bank of America	49.7	10.75	2.75	11.42
2	Citigroup	44.0	9.52	3.23	10.83
3	JP Morgan Chase	42.6	9.22	3.09	9.74
4	Prudential Financial	37.6	8.13	3.07	19.64
5	MetLife	33.9	7.33	2.85	15.40
6	Morgan Stanley	28.6	6.20	3.50	12.60
7	Banco do Brasil	24.1	5.22	4.00	29.45
8	Goldman Sachs	20.3	4.38	3.21	10.51
9	Manulife Financial	20.1	4.36	3.43	15.04
10	Power Corp of Canada	16.2	3.50	2.82	26.81

Source: Volatility Institute (2015), `vlab.stern.nyu.edu`.

TABLE 8.10: Systemic risk contributions in Europe (2015-11-27)

Rank	institution	$\text{SRISK}_{i,t}$ (in \$ bn)	(in %)	$\text{MES}_{i,t}$ (in %)	$\mathcal{LR}_{i,t}$
1	BNP Paribas	94.1	8.63	3.42	33.41
2	Crédit Agricole	88.1	8.09	4.22	59.34
3	Barclays	86.3	7.92	4.31	36.60
4	Deutsche Bank	86.1	7.90	4.32	53.61
5	Société Générale	61.3	5.63	3.85	38.74
6	Royal Bank of Scotland	39.5	3.63	3.15	24.23
7	Banco Santander	38.3	3.51	3.79	18.57
8	HSBC	34.5	3.16	2.49	15.96
9	UniCredit	33.1	3.04	3.58	27.21
10	London Stock Exchange	31.3	2.87	2.90	52.67

Source: Volatility Institute (2015), `vlab.stern.nyu.edu`.

TABLE 8.11: Systemic risk contributions in Asia (2015-11-27)

Rank	institution	$\text{SRISK}_{i,t}$ (in \$ bn)	(in %)	$\text{MES}_{i,t}$ (in %)	$\mathcal{LR}_{i,t}$
1	Mitsubishi UFJ FG	121.5	9.45	2.41	24.80
2	China Construction Bank	117.3	9.12	2.61	17.01
3	Bank of China	94.5	7.35	2.53	15.21
4	Mizuho FG	93.7	7.29	2.10	31.84
5	Agricultural Bank of China	92.0	7.16	0.66	19.20
6	Sumitomo Mitsui FG	85.7	6.67	2.71	26.99
7	ICBC	58.4	4.54	0.84	13.80
8	Bank of Communications	45.0	3.50	2.47	16.89
9	Industrial Bank	29.4	2.29	1.38	17.94
10	National Australia Bank	27.4	2.13	3.27	13.48

Source: Volatility Institute (2015), `vlab.stern.nyu.edu`.

TABLE 8.12: Assets of financial institutions (in $ tn)

Year	CB	Banks	PFI	IC	PF	OFI	FA	Total	MUNFI	
2002	4.7	52.6	11.2	14.9	11.9	32.4	0.2	127.8	59.2	46.3%
2003	5.5	62.2	12.0	19.3	13.8	39.9	0.3	152.9	73.0	47.7%
2004	6.4	73.1	12.1	22.6	15.3	46.3	0.3	176.0	84.2	47.8%
2005	6.8	76.9	11.9	21.4	16.5	49.9	0.2	183.7	87.8	47.8%
2006	7.7	89.5	11.9	25.3	18.3	60.6	0.3	213.6	104.2	48.8%
2007	10.1	110.7	13.0	29.8	19.8	73.4	0.3	257.1	123.0	47.8%
2008	14.5	123.3	14.2	21.2	19.4	65.8	0.4	258.9	106.5	41.1%
2009	15.1	124.1	14.6	23.7	21.9	70.6	0.6	270.6	116.2	43.0%
2010	16.7	129.8	14.8	25.4	24.4	74.8	0.6	286.5	124.6	43.5%
2011	20.3	139.2	15.0	26.2	25.4	75.7	0.7	302.5	127.3	42.1%
2012	22.4	143.5	15.0	27.9	27.4	83.2	0.8	320.1	138.4	43.3%
2013	23.0	142.0	14.7	28.6	28.9	90.9	0.8	328.9	148.4	45.1%
2014	23.2	138.9	14.7	28.8	29.6	94.9	0.8	330.9	153.3	46.3%
2015	23.6	133.5	15.1	28.2	29.6	94.3	0.7	325.0	152.0	46.8%
2016	26.2	137.8	16.0	29.1	31.0	99.2	0.7	339.9	159.3	46.9%

Source: FSB (2018a) and author's calculations.

TABLE 8.13: Assets of OFIs (in $ tn)

Year	MMF	HF	IF	REIT	TC	FC	BD	SFV	CCP	CFI	Other
2002	3.2	0.0	5.6	0.2	0.0	2.3	3.2	2.3	0.0	2.0	13.6
2003	3.3	0.0	7.5	0.2	0.0	2.7	3.8	2.7	0.0	2.3	17.3
2004	3.4	0.0	8.9	0.4	0.0	2.9	4.6	3.3	0.0	2.7	20.3
2005	3.4	0.0	10.0	0.5	0.0	2.8	5.0	4.2	0.0	3.5	20.6
2006	4.0	0.0	12.6	0.6	0.1	2.9	5.7	5.1	0.0	3.8	26.0
2007	5.1	0.5	16.2	0.6	0.1	3.0	6.5	6.5	0.0	4.4	30.4
2008	6.0	0.6	17.0	0.6	0.3	3.2	9.3	6.4	0.1	4.2	18.1
2009	5.5	0.6	21.9	0.7	0.5	3.6	7.9	9.0	0.5	4.3	16.0
2010	4.8	0.8	25.0	0.8	0.7	3.8	8.7	7.6	0.5	4.6	17.5
2011	4.5	1.5	24.3	1.0	1.0	3.9	9.1	6.7	0.5	4.4	18.9
2012	4.4	2.5	28.8	1.3	1.5	3.5	9.3	6.2	0.5	4.6	20.5
2013	4.5	2.9	33.5	1.4	2.2	3.3	9.1	5.7	0.4	4.7	23.1
2014	4.7	3.5	35.3	1.5	2.7	3.4	9.6	5.1	0.4	4.5	24.1
2015	5.1	3.5	35.1	1.5	2.9	3.4	8.7	4.7	0.4	4.5	24.5
2016	5.0	3.7	37.8	1.6	3.4	3.4	8.7	4.5	0.4	5.1	25.7

Source: FSB (2018a) and author's calculations.

TABLE 8.14: Wholesale funding

		Banks		OFIs	
		2011	2016	2011	2016
Funding (% of balance sheet)	Repo	5.82%	5.52%	6.99%	4.14%
	ST wholesale	4.74%	5.01%	2.91%	4.04%
	LT wholesale	6.94%	7.03%	9.10%	6.45%
Repo (in $ tn)	Assets	3.33	4.16	3.05	4.01
	Liabilities	4.58	4.72	2.92	3.19
	Net position	−0.60	−0.58	0.14	0.83

TABLE 8.15: Classification of the shadow banking system by economic functions

Economic Function	Definition	Typical entity types
EF1	Management of collective investment vehicles with features that are susceptible to runs	Fixed-income funds, mixed funds, credit hedge funds, real estate funds
EF2	Loan provision that is dependent on short-term funding	Finance companies, leasing, factoring and consumer credit companies
EF3	Intermediation of market activities that is dependent on short-term funding or on secured funding of client assets	Broker-dealers, securities finance companies
EF4	Facilitation of credit creation	Credit insurance companies, financial guarantors, monolines
EF5	Securitization-based credit intermediation and funding of financial entities	Securitization vehicles, structured finance vehicles, asset-backed vehicles

Source: FSB (2018a).

TABLE 8.16: Size of the narrow shadow banking (in $ tn)

Year	2010	2011	2012	2013	2014	2015	2016
Banks	129.8	139.2	143.5	142.0	138.9	133.5	137.8
OFIs	74.8	75.7	83.2	90.9	94.9	94.3	99.2
Shadow banking	28.4	30.2	32.9	35.6	39.0	42.0	45.2

Source: FSB (2018a) and author's calculation.

Part II

Mathematical and Statistical Tools

Chapter 9

Model Risk of Exotic Derivatives

TABLE 9.1: Impact of the dividend on the option premium

	Put option				Call option			
S_0 / δ	0.00	0.02	0.05	0.07	0.00	0.02	0.05	0.07
90	1.28	1.44	1.73	1.94	13.50	12.67	11.48	10.72
100	4.42	4.83	5.50	5.97	6.89	6.31	5.50	5.00
110	10.19	10.87	11.91	12.63	2.91	2.59	2.16	1.90

TABLE 9.2: An example of delta hedging strategy (negative P&L)

i	t_i	$S(t_i)$	$\boldsymbol{\Delta}(t_{i-1})$	$V_S(t_i)$	$V_B(t_i)$	$V(t_i)$	$\mathcal{C}(t_i, S(t_i))$	$X(t_i)$	$\Pi(t_i)$
0	0.00	100.00	0.00	0.00	590.90	590.90	590.90	0.00	0.00
1	0.02	95.63	58.59	5603.15	−5273.36	329.79	350.22	20.43	−20.43
2	0.04	95.67	43.72	4182.80	−3854.96	327.84	336.15	8.31	−8.31
3	0.06	94.18	43.24	4072.36	−3812.62	259.75	260.57	0.82	−0.82
4	0.08	92.73	37.29	3457.72	−3255.16	202.55	196.22	−6.33	6.33
5	0.10	96.59	31.34	3027.23	−2706.31	320.93	326.47	5.54	−5.54
6	0.12	101.68	44.63	4537.99	−3993.73	544.26	582.71	38.45	−38.45
7	0.13	101.41	63.39	6428.19	−5906.72	521.47	545.64	24.17	−24.17
8	0.15	100.22	62.36	6249.97	−5808.29	441.68	453.62	11.94	−11.94
9	0.17	99.32	57.57	5718.25	−5333.51	384.74	382.58	−2.16	2.16
10	0.19	101.64	53.46	5433.52	−4929.49	504.03	495.99	−8.04	8.04
11	0.21	101.81	63.27	6441.30	−5932.22	509.08	483.87	−25.21	25.21
12	0.23	102.62	64.10	6578.19	−6022.97	555.22	513.53	−41.69	41.69
13	0.25	107.56	67.97	7311.26	−6426.42	884.84	876.68	−8.16	8.16
14	0.27	102.05	86.90	8867.94	−8470.05	397.89	424.07	26.18	−26.18
15	0.29	100.88	66.19	6677.01	−6362.67	314.34	321.76	7.41	−7.41
16	0.31	106.90	59.86	6399.37	−5730.15	669.21	756.02	86.80	−86.80
17	0.33	107.66	90.32	9723.75	−8994.54	729.22	806.47	77.25	−77.25
18	0.35	101.79	94.74	9643.97	−9480.00	163.96	276.24	112.27	−112.27
19	0.37	101.76	69.88	7111.04	−6955.85	155.19	228.08	72.89	−72.89
20	0.38	101.83	75.10	7647.28	−7494.04	153.24	183.00	29.76	−29.76

TABLE 9.3: An example of delta hedging strategy (positive P&L)

i	t_i	$S(t_i)$	$\boldsymbol{\Delta}(t_{i-1})$	$V_S(t_i)$	$V_B(t_i)$	$V(t_i)$	$\mathcal{C}(t_i, S(t_i))$	$X(t_i)$	$\Pi(t_i)$
0	0.00	100.00	0.00	0.00	590.90	590.90	590.90	0.00	0.00
1	0.02	98.50	58.59	5771.31	−5273.36	497.95	489.70	−8.25	8.25
2	0.04	97.00	53.45	5184.51	−4771.31	413.19	396.75	−16.44	16.44
3	0.06	95.47	47.89	4571.99	−4236.14	335.85	311.62	−24.24	24.24
4	0.08	98.17	41.87	4110.19	−3664.81	445.38	419.94	−25.44	25.44
5	0.10	100.48	51.10	5134.88	−4575.85	559.03	528.68	−30.35	30.35
6	0.12	102.92	59.19	6092.33	−5394.04	698.28	664.00	−34.29	34.29
7	0.13	105.50	67.69	7140.94	−6274.05	866.89	829.99	−36.90	36.90
8	0.15	101.81	76.13	7750.53	−7171.44	579.09	550.21	−28.88	28.88
9	0.17	100.65	63.86	6427.97	−5928.66	499.31	457.48	−41.83	41.83
10	0.19	98.86	59.15	5847.59	−5459.40	388.19	337.04	−51.15	51.15
11	0.21	99.26	50.91	5053.11	−4649.03	404.09	335.31	−68.78	68.78
12	0.23	101.78	52.25	5317.65	−4786.50	531.15	458.03	−73.12	73.12
13	0.25	99.28	64.14	6367.78	−6002.74	365.03	288.19	−76.84	76.84
14	0.27	99.19	51.19	5077.96	−4722.07	355.89	257.52	−98.36	98.36
15	0.29	95.53	49.97	4773.36	−4604.77	168.59	92.40	−76.18	76.18
16	0.31	98.02	26.47	2594.85	−2362.61	232.23	148.05	−84.19	84.19
17	0.33	97.03	39.61	3843.35	−3653.84	189.51	83.97	−105.54	105.54
18	0.35	96.64	29.34	2835.17	−2659.65	175.51	44.51	−131.01	131.01
19	0.37	95.01	21.11	2005.37	−1866.05	139.32	3.75	−135.56	135.56
20	0.38	93.67	3.62	338.73	−204.45	134.27	0.00	−134.27	134.27

TABLE 9.4: Implied volatility $\Sigma(T, K)$

K	90	95	98	100	101	102	105	110
$\Sigma_1(T, K)$	20.00	20.01	19.99	20.0	20.01	19.99	20.00	20.00
$\Sigma_2(T, K)$	26.18	23.41	21.24	20.0	19.14	18.90	18.69	19.14
$\Sigma_3(T, K)$	24.53	21.95	20.68	20.0	19.93	20.20	20.95	23.43

TABLE 9.5: Calibration of $\Sigma^{\text{flat}}(K, T_n)$, $\Sigma^{\text{spot}}(K, T_i)$ and γ_i

T_n	$\Sigma^{\text{flat}}(K, T_n)$	T_i	$\Sigma^{\text{spot}}(K, T_i)$	T_i	γ_i
6M	5.000%	3M	5.000%	3M	5.000%
9M	5.083%	6M	5.199%	6M	5.391%
12M	5.130%	9M	5.449%	9M	5.918%
15M	5.158%	12M	5.497%	12M	5.637%
18M	5.192%	15M	5.557%	15M	5.794%
21M	5.214%	18M	5.616%	18M	5.899%

TABLE 9.6: Error of the SLN implied volatility formula (in bps)

K	($\alpha = 22, \sigma = 25\%$)			($\alpha = -70, \sigma = 12\%$)		
	1M	1Y	5Y	1M	1Y	5Y
80	1.0	11.1	57.0	−0.9	−12.9	−66.0
90	0.7	10.6	54.1	−1.0	−11.9	−61.4
100	0.9	10.2	51.6	−1.1	−11.3	−57.3
110	1.0	9.7	49.6	−0.8	−10.8	−53.8
120	0.7	9.3	47.7	−0.6	−10.3	−51.3

TABLE 9.7: Calibrated parameters of the mixed SLN model

Model	#1	#2	#3
σ_1	16.5%	8.2%	10.2%
σ_2	7.3%	17.2%	21.7%
α_1	−53.3	−289.7	−145.2
α_1	−53.3	19.6	47.4
p	50.0%	25.0%	50.0%

TABLE 9.8: Price of the binary call option ($\alpha = -50, \sigma = 15\%$)

K	$\Sigma(T,K)$	$\omega(T,K)$	BS	SLN	SM
80	23.64	−5.47	0.8087	0.8184	0.8184
90	23.14	−4.57	0.6761	0.6895	0.6895
100	22.72	−3.87	0.5160	0.5306	0.5306
110	22.36	−3.34	0.3582	0.3715	0.3715
120	22.05	−2.92	0.2271	0.2374	0.2374

TABLE 9.9: Price of the binary call option ($\alpha = 50, \sigma = 40\%$)

K	$\Sigma(T,K)$	$\omega(T,K)$	BS	SLN	SM
80	16.71	17.25	0.8937	0.8780	0.8780
90	18.21	13.13	0.7390	0.7055	0.7055
100	19.39	10.51	0.5364	0.4971	0.4971
110	20.34	8.69	0.3546	0.3202	0.3202
120	21.14	7.35	0.2209	0.1953	0.1953

TABLE 9.10: Calibration set

				$T_m = 1/12$					
K_i	87.0	92.0	96.0	98.0	100.0	103.0	106.0	110.0	116.0
$\Sigma(T_m, K_i)$	13.7	13.7	13.3	13.2	13.0	13.1	13.2	13.5	13.5
				$T_m = 3/12$					
K_i	77.0	85.0	93.0	97.0	101.0	106.0	111.0	121.0	134.0
$\Sigma(T_m, K_i)$	14.9	14.9	14.1	14.0	13.5	13.8	14.2	15.1	15.1
				$T_m = 6/12$					
K_i	66.0	78.0	89.0	96.0	102.0	111.0	119.0	136.0	161.0
$\Sigma(T_m, K_i)$	16.8	16.8	15.5	15.0	14.5	15.0	15.5	16.8	16.8
				$T_m = 1$					
K_i	53.0	69.0	86.0	96.0	104.0	119.0	133.0	166.0	217.0
$\Sigma(T_m, K_i)$	19.0	19.0	17.0	16.0	15.5	16.5	17.5	18.5	18.5
				$T_m = 2$					
K_i	37.0	56.0	80.0	96.0	103.0	137.0	163.0	229.0	347.0
$\Sigma(T_m, K_i)$	21.9	21.9	20.0	18.5	18.5	19.0	19.5	20.8	20.8

TABLE 9.11: Barrier option pricing with the local volatility model

Option	Payoff	LV	BS-PDE		BS-RR	
			Σ_1	Σ_2	Σ_1	Σ_2
Call	$(S(T) - K)^+$	8.85	8.96	8.78	8.96	8.78
Put	$(K - S(T))^+$	3.97	4.08	3.90	4.08	3.90
DOC	$\mathbb{1}\{S(t) > L\} \cdot (S(T) - K)^+$	7.98	8.14	8.05	8.11	8.02
DOP	$\mathbb{1}\{S(t) > L\} \cdot (K - S(T))^+$	0.26	0.27	0.28	0.25	0.27
UOC	$\mathbb{1}\{S(t) < H\} \cdot (S(T) - K)^+$	0.99	0.88	0.94	0.83	0.89
UOP	$\mathbb{1}\{S(t) < H\} \cdot (K - S(T))^+$	3.81	3.90	3.75	3.89	3.74
KOC	$\mathbb{1}\{S(t) \in [L, H]\} \cdot (S(T) - K)^+$	0.65	0.56	0.64	0.52	0.59
KOP	$\mathbb{1}\{S(t) \in [L, H]\} \cdot (K - S(T))^+$	0.20	0.20	0.22	0.19	0.21
BCC	$\mathbb{1}\{S(T) \geq K\}$	0.58	0.56	0.57	0.56	0.57
BCP	$\mathbb{1}\{S(T) \leq K\}$	0.37	0.39	0.38	0.39	0.38

TABLE 9.12: Calibration of the parameter β in the SABR model

Rate	Level		Difference		Empirical quantile of $\hat{\beta}_{t,t+h}$				
	$\hat{\beta}$	R_c^2	$\hat{\beta}$	R_c^2	10%	25%	50%	75%	90%
1y1y	−0.06	0.91	0.59	0.15	−2.01	−0.14	0.71	1.00	2.17
1y5y	−0.29	0.87	0.32	0.27	−1.80	−0.28	0.73	1.11	2.76
1y10y	−0.37	0.80	0.34	0.22	−2.04	−0.23	0.71	1.11	2.69
5y1y	0.42	0.29	0.35	0.22	−1.58	−0.31	0.71	1.00	2.38
5y5y	−0.01	0.73	0.23	0.28	−2.12	−0.36	0.61	1.00	2.52
5y10y	−0.10	0.69	0.27	0.23	−1.99	−0.30	0.70	1.05	2.58
10y1y	0.96	0.00	0.28	0.20	−1.88	−0.20	0.80	1.07	2.43
10y5y	−0.10	0.65	0.28	0.20	−2.02	−0.29	0.73	1.02	2.76
10y10y	−0.47	0.73	0.27	0.20	−1.71	−0.24	0.85	1.07	2.93

TABLE 9.13: Impact of the dividend on the option price

	Call			Put		
$D(t_1)$	(#1)	(#2)	(#3)	(#1)	(#2)	(#3)
0	14.23	14.23	14.23	9.35	9.35	9.35
3	12.46	12.81	12.69	10.51	10.86	10.64
5	11.34	11.92	11.69	11.34	11.92	11.59
10	8.78	9.93	9.42	13.66	14.80	14.20

TABLE 9.14: Impact of the correlation on the basket option price

	α_1	1.0	1.0	0.5	0.5	0.1
	α_2	−1.0	−1.0	0.5	0.5	0.1
	K	1	5	100	110	−5
	−0.90	20.41	18.23	5.39	0.66	24.78
	−0.75	19.81	17.62	6.06	1.35	24.78
	−0.50	18.76	16.55	6.97	2.31	24.78
	−0.25	17.61	15.37	7.73	3.12	24.78
ρ	0.00	16.35	14.08	8.39	3.83	24.78
	0.25	14.94	12.61	8.99	4.46	24.78
	0.50	13.30	10.88	9.54	5.05	24.78
	0.75	11.29	8.66	10.05	5.59	24.78
	0.90	9.78	6.81	10.34	5.90	24.78

TABLE 9.15: Relationship between the basket option price and the correlation parameter ρ

Option type	Payoff	Increasing	Decreasing
Spread	$(S_2 - S_1 - K)^+$		✓
Basket	$(\alpha_1 S_1 + \alpha_2 S_2 - K)^+$	$\alpha_1\alpha_2 > 0$	$\alpha_1\alpha_2 < 0$
Max	$(\max(S_1, S_2) - K)^+$		✓
Min	$(\min(S_1, S_2) - K)^+$	✓	
Best-of call/call	$\max\left((S_1 - K_1)^+, (S_2 - K_2)^+\right)$		✓
Best-of put/put	$\max\left((K_1 - S_1)^+, (K_2 - S_2)^+\right)$		✓
Worst-of call/call	$\min\left((S_1 - K_1)^+, (S_2 - K_2)^+\right)$	✓	
Worst-of put/put	$\min\left((K_1 - S_1)^+, (K_2 - S_2)^+\right)$	✓	

Chapter 10

Statistical Inference and Model Estimation

TABLE 10.1: Data of the linear regression problem

i	y	x_1	x_2	x_3	x_4
1	1.5	1.0	2.4	3.6	0.3
2	20.4	1.0	1.1	3.8	5.9
3	17.1	1.0	5.1	6.3	6.1
4	30.9	1.0	2.7	2.4	9.5
5	22.2	1.0	3.3	3.0	7.4
6	9.1	1.0	1.0	5.4	4.9
7	39.2	1.0	9.6	2.8	8.1
8	3.1	1.0	2.9	4.4	1.0
9	7.2	1.0	4.2	5.6	1.7
10	27.6	1.0	8.1	1.7	5.4

TABLE 10.2: Results of the linear regression

Parameter	Estimate	Standard error	t-statistic	p-value
β_1	3.4461	3.9183	0.8795	0.4130
β_2	1.5442	0.3289	4.6943	0.0033
β_3	−1.6454	0.6543	−2.5146	0.0457
β_4	2.8951	0.3071	9.4264	0.0001

TABLE 10.3: Results of the conditional Gaussian regression

Y	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	σ	R_c^2
X_1	2.974		0.335	0.774	0.148	19.01%	96.39%
X_2	−7.205	2.667		−1.949	−0.308	53.59%	92.82%
X_3	−4.017	1.000	−0.317		−0.133	21.60%	81.33%
X_4	−4.273	2.091	−0.545	−1.455		71.35%	49.09%

TABLE 10.4: Results of the maximum likelihood estimation

Parameter	Estimate	Standard error	t-statistic	p-value
α	1.8356	0.6990	2.6258	0.0236
β	1.2478	0.4483	2.7834	0.0178

TABLE 10.5: Non-linear models

Model	Function $g(x,\beta)$
Exponential (gowth)	$y_i = \beta_1 e^{\beta_2 x_i} + u_i$
Exponential (decay)	$y_i = y_- + (y_+ - y_-)\, e^{-\beta x_i} + u_i$
Hyperbola	$y_i = (\beta_1 x_i) / (\beta_2 + x_i)$
Sine	$y_i = \beta_1 + \beta_2 \sin(\beta_3 x_i + \beta_4) + u_i$
Boltzmann	$y_i = y_- + (y_+ - y_-) / \left(1 + e^{\frac{\beta_1 - x_i}{\beta_2}}\right)$
Damped sine	$y_i = \beta_1 e^{-\beta_2 x_i} \sin(\beta_3 x_i + \beta_4) + u_i$

TABLE 10.6: Insulation life in hours at various test temperatures

Motorette	1	2	3	4	5
150°	8 064*	8 064*	8 064*	8 064*	8 064*
170°	1 764	2 772	3 444	3 542	3 780
190°	408	408	1 344	1 344	1 440
220°	408	408	504	504	504
Motorette	6	7	8	9	10
150°	8 064*	8 064*	8 064*	8 064*	8 064*
170°	4 860	5 196	5 448*	5 448*	5 448*
190°	1 680*	1 680*	1 680*	1 680*	1 680*
220°	528*	528*	528*	528*	528*

Source: Schmee and Hahn (1979).
An asterisk * indicates that the test has been stopped without the failure of the motorette, implying that the observation is censored.

TABLE 10.7: Expected failure time $\mathbb{E}[Z_i \mid Z_i \geq c_i]$ obtained with the EM algorithm

Motorette	1	2	3	4	5
150°	17 447*	17 447*	17 447*	17 447*	17 447*
170°	1 764	2 772	3 444	3 542	3 780
190°	408	408	1 344	1 344	1 440
220°	408	408	504	504	504
Motorette	6	7	8	9	10
150°	17 447*	17 447*	17 447*	17 447*	17 447*
170°	4 860	5 196	8 574*	8 574*	8 574*
190°	2 862*	2 862*	2 862*	2 862*	2 862*
220°	850*	850*	850*	850*	850*

The censored data represented by an asterisk * are replaced by the value of $\mathbb{E}[Z_i \mid Z_i \geq c_i]$ calculated by the EM algorithm at the last iteration.

TABLE 10.8: Comparison of GMM and SMM estimates

Method	n_S	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\sigma}(\hat{\mu})$	$\hat{\sigma}(\hat{\sigma})$	$\hat{\sigma}_{\mathrm{MC}}(\hat{\mu})$	$\hat{\sigma}_{\mathrm{MC}}(\hat{\sigma})$
GMM		−0.109	0.836	0.306	0.165		
SMM	25	−0.115	0.880	0.373	0.218	0.214	0.340
SMM	200	−0.102	0.847	0.315	0.172	0.070	0.062

TABLE 10.9: Critical values of the ADF test

Significance level	10%	5%	1%
τ	−1.62	−1.94	−2.56
τ_μ	−2.57	−2.86	−3.43
τ_τ	−3.13	−3.41	−3.96

TABLE 10.10: Critical values of the KPSS test

Significance level	10%	5%	1%
η_μ	0.347	0.463	0.739
η_τ	0.119	0.146	0.216

TABLE 10.11: Spectral window functions

Name	$W(u)$
Bartlett	$1-\lvert u\rvert$
Parzen	$\left(1-6\lvert u\rvert^2+6\lvert u\rvert^3\right)\cdot\mathbb{1}\left\{\lvert u\rvert<\frac{1}{2}\right\}+$ $2\left(1-\lvert u\rvert\right)^3\cdot\mathbb{1}\left\{\lvert u\rvert\geq\frac{1}{2}\right\}$
Tukey	$1-2a+2a\cos(\pi u)$
Rectangular	1
Daniell	$(\pi u)^{-1}\sin(\pi u)$
Priestley	$3\left((\pi u)^{-3}\sin(\pi u)-(\pi u)^{-1}\cos(\pi u)\right)$

TABLE 10.12: Autocorrelation function $\rho_y(k)$ (in %) of FWN and AR(1) processes

k	FWN process			AR(1) process		
d / ϕ	0.2	0.45	0.499	0.50	0.90	0.999
0	100.00	100.00	100.00	100.00	100.00	100.00
1	25.00	81.82	99.60	50.00	90.00	99.90
5	9.65	69.90	99.29	3.13	59.05	99.50
10	6.37	65.22	99.15	0.10	34.87	99.00
100	1.60	51.81	98.69	0.00	0.00	90.48
500	0.61	44.11	98.38	0.00	0.00	60.64
1000	0.40	41.15	98.24	0.00	0.00	36.77
5000	0.15	35.04	97.93	0.00	0.00	0.67
10000	0.10	32.69	97.79	0.00	0.00	0.00

TABLE 10.13: Value of the energy ratio $\mathcal{ER}_m$ (in %)

m	1	2	3	4	5	10	25	50	75	100	2 500
$\mathcal{ER}_m$	34.8	69.5	73.8	78.1	79.9	87.4	94.4	96.5	97.3	97.5	100.0

TABLE 10.14: Numerical example

$$\mathbf{X}^\top \mathbf{X} = \begin{pmatrix} 28.88 & 23.15 & 20.75 & 21.60 & 22.97 \\ 23.15 & 35.01 & 24.73 & 25.14 & 24.73 \\ 20.75 & 24.73 & 28.23 & 22.42 & 21.63 \\ 21.60 & 25.14 & 22.42 & 32.22 & 24.17 \\ 22.97 & 24.73 & 21.63 & 24.17 & 33.10 \end{pmatrix} \quad \mathbf{X}^\top \mathbf{Y} = \begin{pmatrix} 100.53 \\ 136.62 \\ 128.20 \\ 146.07 \\ 117.01 \end{pmatrix}$$

TABLE 10.15: Data of the Tobit example

i	1	2	3	4	5	6	7	8	9	10
y_i	4.0	0.0	0.5	0.0	0.0	17.4	18.0	0.0	0.0	9.7
$x_{1,i}$	−4.3	−9.2	−2.8	−2.7	−8.4	2.0	5.3	−8.1	0.9	−7.8
$x_{2,i}$	−1.2	−5.5	1.8	−3.4	2.9	9.3	9.1	6.8	−6.3	4.3
i	11	12	13	14	15	16	17	18	19	20
y_i	9.7	1.8	6.5	26.1	0.0	5.0	21.6	6.2	9.9	1.4
$x_{1,i}$	0.5	6.7	−0.9	4.0	−8.6	2.3	7.1	7.3	9.3	−0.2
$x_{2,i}$	5.3	−8.6	2.1	9.2	−8.7	−7.9	9.2	−7.5	−4.4	1.7
i	21	22	23	24	25	26	27	28	29	30
y_i	5.0	0.0	0.0	18.1	0.0	7.7	0.0	0.0	0.0	4.0
$x_{1,i}$	−3.5	−1.5	−2.6	8.4	8.6	5.8	−7.9	0.9	−7.3	2.3
$x_{2,i}$	0.1	−4.5	−8.9	3.8	−8.5	8.3	1.8	−6.2	8.4	6.7

Chapter 11

Copulas and Dependence Modeling

TABLE 11.1: Archimedean copula functions

Copula	$\varphi(u)$	$\mathbf{C}(u_1, u_2)$
$\mathbf{C}^{\perp}$	$-\ln u$	$u_1 u_2$
Clayton	$u^{-\theta} - 1$	$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$
Frank	$-\ln \dfrac{e^{-\theta u} - 1}{e^{-\theta} - 1}$	$-\dfrac{1}{\theta} \ln \left(1 + \dfrac{\left(e^{-\theta u_1} - 1\right)\left(e^{-\theta u_2} - 1\right)}{e^{-\theta} - 1}\right)$
Gumbel	$(-\ln u)^{\theta}$	$\exp\left(-\left(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta}\right)^{1/\theta}\right)$
Joe	$-\ln\left(1 - (1-u)^{\theta}\right)$	$1 - \left(\bar{u}_1^{\theta} + \bar{u}_2^{\theta} - \bar{u}_1^{\theta}\bar{u}_2^{\theta}\right)^{1/\theta}$

TABLE 11.2: Values in % of the upper tail dependence λ^+ for the Student's t copula

ν	Parameter ρ (in %)					
	-70.00	-50.00	0.00	50.00	70.00	90.00
1	7.80	13.40	29.29	50.00	61.27	77.64
2	2.59	5.77	18.17	39.10	51.95	71.77
3	0.89	2.57	11.61	31.25	44.81	67.02
4	0.31	1.17	7.56	25.32	39.07	62.98
6	0.04	0.25	3.31	17.05	30.31	56.30
10	0.00	0.01	0.69	8.19	19.11	46.27
∞	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 11.3: Matrix of linear correlations $\hat{\rho}_{i,j}$

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	57.8	100.0			
Sovereign	-34.0	-32.6	100.0		
Credit	-15.1	-28.6	69.3	100.0	
Commodity	51.8	34.3	-22.3	-14.4	100.0

TABLE 11.4: Matrix of parameters $\hat{\rho}_{i,j}$ estimated using Kendall's tau

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	57.7	100.0			
Sovereign	−31.8	−32.1	100.0		
Credit	−17.6	−33.8	73.9	100.0	
Commodity	43.4	30.3	−19.6	−15.2	100.0

TABLE 11.5: Matrix of parameters $\hat{\rho}_{i,j}$ estimated using Spearman's rho

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	55.4	100.0			
Sovereign	−31.0	−31.3	100.0		
Credit	−17.1	−32.7	73.0	100.0	
Commodity	42.4	29.4	−19.2	−14.9	100.0

TABLE 11.6: Omnibus estimate $\hat{\rho}$ (Normal copula)

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	56.4	100.0			
Sovereign	−32.5	−32.1	100.0		
Credit	−16.3	−30.3	70.2	100.0	
Commodity	46.5	30.7	−21.1	−14.7	100.0

TABLE 11.7: Omnibus estimate $\hat{\rho}$ (Student's t copula with $\nu = 1$)

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	47.1	100.0			
Sovereign	−20.3	−18.9	100.0		
Credit	−9.3	−22.1	57.6	100.0	
Commodity	28.0	17.1	−7.4	−6.2	100.0

TABLE 11.8: Omnibus estimate $\hat{\rho}$ (Student's t copula with $\nu = 4$)

	EU Equity	US Equity	Sovereign	Credit	Commodity
EU Equity	100.0				
US Equity	59.6	100.0			
Sovereign	−31.5	−31.9	100.0		
Credit	−18.3	−32.9	71.3	100.0	
Commodity	43.0	30.5	−17.2	−13.4	100.0

Chapter 12

Extreme Value Theory

TABLE 12.1: ML estimate of σ (in bps) for the probability distribution $\mathbf{t}_1$

Size n	Order i									
	1	2	3	4	5	6	7	8	9	10
1	50									
2	48	49								
3	44	54	44							
4	41	53	53	41						
5	38	52	55	51	37					
6	35	51	56	56	48	33				
7	32	49	55	56	55	45	29			
8	31	48	53	55	54	50	43	26		
9	29	46	55	56	57	55	49	40	25	
10	28	43	53	58	57	56	53	48	37	20

TABLE 12.2: ML estimate of σ (in bps) for the probability distribution $\mathbf{t}_6$

Size n	Order i									
	1	2	3	4	5	6	7	8	9	10
1	88									
2	89	87								
3	91	91	85							
4	95	92	89	87						
5	98	99	87	90	88					
6	101	104	95	88	92	89				
7	101	112	100	88	94	95	89			
8	102	116	103	89	85	89	98	89		
9	105	121	117	97	85	86	94	101	88	
10	105	123	120	108	91	87	92	99	104	88

TABLE 12.3: ML estimate of σ (in bps) for the probability distribution $\mathbf{t}_\infty$

Size n	Order i									
	1	2	3	4	5	6	7	8	9	10
1	125									
2	125	124								
3	136	116	129							
4	147	116	112	140						
5	155	133	103	114	150					
6	163	142	118	107	122	157				
7	171	152	125	105	117	134	162			
8	175	165	130	106	99	111	139	170		
9	180	174	155	122	95	99	128	152	171	
10	183	182	162	136	110	100	111	127	155	181

TABLE 12.4: Maximum domain of attraction and normalizing constants of some distribution functions

Distribution	$\mathbf{G}(x)$	a_n	b_n
$\mathcal{E}(\lambda)$	$\mathbf{\Lambda}$	λ^{-1}	$\lambda^{-1}\ln n$
$\mathcal{G}(\alpha,\beta)$	$\mathbf{\Lambda}$	β^{-1}	$\beta^{-1}\left(\ln n + (\alpha-1)\ln(\ln n) - \ln\Gamma(\alpha)\right)$
$\mathcal{N}(0,1)$	$\mathbf{\Lambda}$	$(2\ln n)^{-1/2}$	$\dfrac{4\ln n - \ln 4\pi - \ln(\ln n)}{2\sqrt{2\ln n}}$
$\mathcal{LN}(\mu,\sigma^2)$	$\mathbf{\Lambda}$	$\sigma(2\ln n)^{-1/2}\, b_n$	$\exp\left(\mu + \sigma\left(\dfrac{4\ln n - \ln 4\pi + \ln(\ln n)}{2\sqrt{2\ln n}}\right)\right)$
$\mathcal{P}(\alpha, x_-)$	$\mathbf{\Phi}_\alpha$	$x_- n^{1/\alpha}$	0
$\mathcal{LG}(\alpha,\beta)$	$\mathbf{\Phi}_\beta$	$\dfrac{\left(n(\ln n)^{\alpha-1}\right)^{1/\beta}}{\Gamma(\alpha)}$	0
t_ν	$\mathbf{\Phi}_\nu$	$\mathbf{T}_\nu^{-1}\left(1-n^{-1}\right)$	0
$\mathcal{U}_{[0,1]}$	$\mathbf{\Psi}_1$	n^{-1}	1
$\mathcal{B}(\alpha,\beta)$	$\mathbf{\Psi}_\alpha$	$\left(\dfrac{n\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta+1)}\right)^{-1/\beta}$	1

Source: Embrechts *et al.* (1997).

TABLE 12.5: Comparing Gaussian, historical and GEV value-at-risk measures

VaR	α	Long US	Long EM	Long US Short EM	Long EM Short US
Gaussian	99.0%	2.88%	2.83%	3.06%	3.03%
	99.5%	3.19%	3.14%	3.39%	3.36%
	99.9%	3.83%	3.77%	4.06%	4.03%
Historical	99.0%	3.46%	3.61%	3.37%	3.81%
	99.5%	4.66%	4.73%	3.99%	4.74%
	99.9%	7.74%	7.87%	6.45%	7.27%
GEV	99.0%	2.64%	2.61%	2.72%	2.93%
	99.5%	3.48%	3.46%	3.41%	3.82%
	99.9%	5.91%	6.05%	5.35%	6.60%

TABLE 12.6: Estimation of the generalized Pareto distribution

Parameter	Long US	Long EM	Long US Short EM	Long EM Short US
$\hat{a}$	0.834	1.029	0.394	0.904
$\hat{b}$	0.160	0.132	0.239	0.142
$\hat{\sigma}$	0.719	0.909	0.318	0.792
$\hat{\xi}$	0.138	0.117	0.193	0.124

TABLE 12.7: Estimating value-at-risk and expected shortfall risk measures using the generalized Pareto distribution

Risk measure	α	Long US	Long EM	Long US Short EM	Long EM Short US
VaR	99.0%	3.20%	3.42%	2.56%	3.43%
	99.5%	3.84%	4.20%	2.88%	4.13%
	99.9%	5.60%	6.26%	3.80%	6.02%
ES	99.0%	4.22%	4.64%	3.09%	4.54%
	99.5%	4.97%	5.52%	3.48%	5.34%
	99.9%	7.01%	7.86%	4.62%	7.49%

TABLE 12.8: List of extreme value copulas

Copula	θ	$\mathbf{C}(u_1, u_2)$	$A(w)$
$\mathbf{C}^{\perp}$		$u_1 u_2$	1
Gumbel	$[1, \infty)$	$\exp\left(-\left(\tilde{u}_1^{\theta} + \tilde{u}_2^{\theta}\right)^{1/\theta}\right)$	$\left(w^{\theta} + (1-w)^{\theta}\right)^{1/\theta}$
Gumbel II	$[0, 1]$	$u_1 u_2 \exp\left(\theta \dfrac{\tilde{u}_1 \tilde{u}_2}{\tilde{u}_1 + \tilde{u}_2}\right)$	$\theta w^2 - \theta w + 1$
Galambos	$[0, \infty)$	$u_1 u_2 \exp\left(\left(\tilde{u}_1^{-\theta} + \tilde{u}_2^{-\theta}\right)^{-1/\theta}\right)$	$1 - \left(w^{-\theta} + (1-w)^{-\theta}\right)^{-1/\theta}$
Hüsler-Reiss	$[0, \infty)$	$\exp\left(-\tilde{u}_1 \vartheta(u_1, u_2; \theta) - \tilde{u}_2 \vartheta(u_2, u_1; \theta)\right)$	$w\kappa(w; \theta) + (1-w)\kappa(1-w; \theta)$
Marshall-Olkin	$[0, 1]^2$	$u_1^{1-\theta_1} u_2^{1-\theta_2} \min\left(u_1^{\theta_1}, u_2^{\theta_2}\right)$	$\max(1 - \theta_1 w, 1 - \theta_2(1-w))$
$\mathbf{C}^{+}$		$\min(u_1, u_2)$	$\max(w, 1-w)$

$\vartheta(u_1, u_2; \theta) = \Phi\left(\frac{1}{\theta} + \frac{\theta}{2} \ln(\ln u_1 / \ln u_2)\right)$
$\kappa(w; \theta) = \vartheta(w, 1-w; \theta)$

Source: Ghoudi *et al.* (1998).

Chapter 13

Monte Carlo Simulation Methods

TABLE 13.1: Simulation of 10 uniform pseudorandom numbers

n	x_n	u_n	x_n	u_n
0	1	0.000000	123 456	0.000057
1	16 807	0.000008	2 074 924 992	0.966212
2	282 475 249	0.131538	277 396 911	0.129173
3	1 622 650 073	0.755605	22 885 540	0.010657
4	984 943 658	0.458650	237 697 967	0.110687
5	1 144 108 930	0.532767	670 147 949	0.312062
6	470 211 272	0.218959	1 772 333 975	0.825307
7	101 027 544	0.047045	2 018 933 935	0.940139
8	1 457 850 878	0.678865	1 981 022 945	0.922486
9	1 458 777 923	0.679296	466 173 527	0.217079
10	2 007 237 709	0.934693	958 124 033	0.446161

TABLE 13.2: Simulation of the piecewise exponential model

u_i	$t^\star_{m-1}$	$S\left(t^\star_{m-1}\right)$	λ_m	t_i
0.9950	0	1.0000	0.05	0.1003
0.3035	5	0.6907	0.12	11.8531
0.5429	5	0.6907	0.12	7.0069
0.9140	1	0.9512	0.08	1.4991
0.7127	1	0.9512	0.08	4.6087

TABLE 13.3: Simulation of the standard Gaussian distribution using the acceptance-rejection algorithm

u_1	u_2	x	v	test	z
0.9662	0.1291	9.3820	0.0000	reject	
0.0106	0.1106	−30.0181	0.0000	reject	
0.3120	0.8253	−0.6705	0.9544	accept	−0.6705
0.9401	0.9224	5.2511	0.0000	reject	
0.2170	0.4461	−1.2323	0.9717	accept	−1.2323
0.6324	0.0676	0.4417	0.8936	accept	0.4417
0.6577	0.1344	0.5404	0.9204	accept	0.5404
0.1596	0.6670	−1.8244	0.6756	accept	−1.8244
0.4183	0.3872	−0.2625	0.8513	accept	−0.2625
0.9625	0.0752	8.4490	0.0000	reject	

TABLE 13.4: Simulation of the Clayton copula

Random uniform variates		Clayton copula			
		$\theta = 0.01$		$\theta = 1.5$	
v_1	v_2	u_1	u_2	u_1	u_2
0.2837	0.4351	0.2837	0.4342	0.2837	0.3296
0.0386	0.2208	0.0386	0.2134	0.0386	0.0297
0.3594	0.5902	0.3594	0.5901	0.3594	0.5123
0.3612	0.3268	0.3612	0.3267	0.3612	0.3247
0.0797	0.6479	0.0797	0.6436	0.0797	0.1704

TABLE 13.5: Linear regression between the Asian call option and the control variates

$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2	$1 - R^2$
−51.482	0.036	0.538			90.7%	9.3%
−24.025	−0.346	0.595	0.548		96.5%	3.5%
−4.141	0.069		0.410		81.1%	18.9%
−38.727		0.428	0.174		92.9%	7.1%
−1.559	−0.040	0.054	0.111	0.905	99.8%	0.2%

TABLE 13.6: Variance ratio (in %) when $a = 1$

b	Correlation ρ (in %)								
	−90.0	−75.0	−50.0	−25.0	0.0	25.0	50.0	75.0	90.0
0.0	3.2	8.0	16.1	24.5	33.3	43.0	54.0	67.8	79.8
1.0	2.9	7.3	14.8	22.5	30.6	39.3	48.9	60.0	67.6
2.0	2.2	5.6	11.3	17.2	23.3	29.6	35.9	41.9	48.8
3.0	1.4	3.4	7.0	10.7	14.4	18.1	21.4	24.6	

TABLE 13.7: Comparison between MC and STR estimators

$\varphi(x)$		x	x^2	$(1+\cos(\pi x))/2$
I		0.50000	0.33333	0.50000
$n_S \operatorname{var}\left(\hat{I}_{\mathrm{MC}}\right)$		0.08333	0.08890	0.12501
$n_S \operatorname{var}\left(\hat{I}_{\mathrm{STR}}\right)$	$p(j)$	0.00083	0.00113	0.00105
$n_S \operatorname{var}\left(\hat{I}_{\mathrm{STR}}\right)$	$q^{\star}(j)$	0.00083	0.00083	0.00084

TABLE 13.8: Pricing of the spread option using quasi-Monte Carlo methods

n_S	10^2	10^3	10^4	10^5	10^6	5×10^6
LCG (1)	4.3988	5.9173	5.8050	5.8326	5.8215	5.8139
LCG (2)	6.1504	6.1640	5.8370	5.8219	5.8265	5.8198
LCG (3)	6.1469	5.7811	5.8125	5.8015	5.8142	5.8197
Hammersley (1)	32.7510	26.5326	21.5500	16.1155	9.0914	5.8199
Hammersley (2)	32.9082	26.4629	21.5465	16.1149	9.0914	5.8199
Halton (1)	8.6256	6.1205	5.8493	5.8228	5.8209	5.8208
Halton (2)	10.6415	6.0526	5.8544	5.8246	5.8208	5.8207
Halton (3)	8.5292	6.0575	5.8474	5.8235	5.8212	5.8208
Sobol	5.7181	5.7598	5.8163	5.8190	5.8198	5.8198
Faure	5.7256	5.7718	5.8157	5.8192	5.8197	5.8198

Chapter 14

Stress Testing and Scenario Analysis

TABLE 14.1: Worst historical scenarios of the S&P 500 index

Sc.	1D		1W		1M	
1	1987-10-19	−20.47	1987-10-19	−27.33	2008-10-27	−30.02
2	2008-10-15	−9.03	2008-10-09	−18.34	1987-10-26	−28.89
3	2008-12-01	−8.93	2008-11-20	−17.43	2009-03-09	−22.11
4	2008-09-29	−8.79	2008-10-27	−13.85	2002-07-23	−19.65
5	1987-10-26	−8.28	2011-08-08	−13.01	2001-09-21	−16.89
Sc.	**2M**		**3M**		**6M**	
1	2008-11-20	−37.66	2008-11-20	−41.11	2009-03-09	−46.64
2	1987-10-26	−31.95	1987-11-30	−30.17	1974-09-13	−34.33
3	2002-07-23	−27.29	1974-09-13	−28.59	2002-10-09	−31.29
4	2009-03-06	−26.89	2002-07-23	−27.55	1962-06-27	−26.59
5	1962-06-22	−23.05	2009-03-09	−25.63	1970-05-26	−25.45

TABLE 14.2: Probability (in %) associated to the return period $\mathcal{T}$ in years

Return period	1	5	10	20	30	50
Daily	0.3846	0.0769	0.0385	0.0192	0.0128	0.0077
Weekly	1.9231	0.3846	0.1923	0.0962	0.0641	0.0385
Monthly	8.3333	1.6667	0.8333	0.4167	0.2778	0.1667
$1-\alpha_{\mathrm{GEV}}$	7.6923	1.5385	0.7692	0.3846	0.2564	0.1538

TABLE 14.3: GEV parameter estimates (in %) of MSCI USA and MSCI EMU indices

Parameter	Long position		Short position	
	MSCI USA	MSCI EMU	MSCI USA	MSCI EMU
μ	1.242	1.572	1.317	1.599
σ	0.720	0.844	0.577	0.730
ξ	19.363	21.603	26.341	26.494

TABLE 14.4: Stress scenarios (in %) of MSCI USA and MSCI EMU indices

Year	Long position		Short position	
	MSCI USA	MSCI EMU	MSCI USA	MSCI EMU
5	−5.86	−7.27	5.69	7.16
10	−7.06	−8.83	7.01	8.84
25	−8.92	−11.29	9.17	11.60
50	−10.56	−13.49	11.18	14.17
75	−11.62	−14.94	12.54	15.91
100	−12.43	−16.05	13.59	17.26
Extreme statistic	−9.51	−10.94	11.04	10.87
$\mathcal{T}^{\star}$	32.49	22.24	47.87	20.03

TABLE 14.5: Upper and lower bounds of the return time $\mathcal{T}$ (in years)

n (in days)	$\mathcal{T}_1$	$\mathcal{T}_2$	Lower bound	Upper bound
1	5	5	5	6500
5	5	5	5	1300
20	5	5	5	325
260	5	5	5	25
260	10	5	10	50
260	1	1	1	1

TABLE 14.6: Stress scenario of the probability of default

t	g_t	π_t	u_t	$\mathbb{E}[\mathrm{PD}_t \mid \mathbb{S}(X)]$	$q_{90\%}(\mathbb{S}(X))$
0	2.00	2.00	5.00	7.90	12.78
1	−6.00	2.00	6.00	11.45	18.26
2	−7.00	1.00	7.00	12.47	19.79
3	−9.00	1.00	9.00	14.03	22.14
4	−7.00	1.00	10.00	13.12	20.78
5	−7.00	2.00	11.00	13.01	20.59
6	−6.00	2.00	10.00	12.26	19.49
7	−4.00	4.00	9.00	10.49	16.80
8	−2.00	3.00	8.00	9.70	15.58
9	−1.00	3.00	7.00	9.11	14.68
10	2.00	3.00	6.00	7.82	12.68
11	4.00	3.00	6.00	7.14	11.60
12	4.00	3.00	6.00	7.14	11.60

Chapter 15

Credit Scoring Models

TABLE 15.1: An example of risk factors for consumer credit

Character	Age of applicant
	Marital status
	Number of children
	Educational background
	Time with bank
	Time at present address
Capacity	Annual income
	Current living expenses
	Current debts
	Time with employer
Capital	Purpose of the loan
	Home status
	Saving account
Condition	Maturity of the loan
	Paid interests

TABLE 15.2: Dynamic programming matrices $\boldsymbol{J}$ and $\boldsymbol{C}$

state	$\mathcal{J}(1, s(1))$	$\mathcal{J}(1, s(2))$	$\mathcal{J}(1, s(3))$	$c(1)$	$c(2)$
1	7.6059	4.0714	0.0256	4	7
2	7.7167	3.8618	0.1053	6	7
3	9.0239	3.7048	0.2432	6	7
4	10.4945	3.6059	0.4444	6	7
5	10.5450	3.5714	0.8065	6	7
6	5.9231	5.4945	0.0000	7	7
7	4.7576	4.0000	3.5714	8	8
8	$-\infty$	3.0000	3.0000	1	9
9	$-\infty$	$-\infty$	2.0000	1	1

TABLE 15.3: Data of the lasso regression problem

i	y	x_1	x_2	x_3	x_4	x_5
1	3.1	2.8	4.3	0.3	2.2	3.5
2	24.9	5.9	3.6	3.2	0.7	6.4
3	27.3	6.0	9.6	7.6	9.5	0.9
4	25.4	8.4	5.4	1.8	1.0	7.1
5	46.1	5.2	7.6	8.3	0.6	4.5
6	45.7	6.0	7.0	9.6	0.6	0.6
7	47.4	6.1	1.0	8.5	9.6	8.6
8	−1.8	1.2	9.6	2.7	4.8	5.8
9	20.8	3.2	5.0	4.2	2.7	3.6
10	6.8	0.5	9.2	6.9	9.3	0.7
11	12.9	7.9	9.1	1.0	5.9	5.4
12	37.0	1.8	1.3	9.2	6.1	8.3
13	14.7	7.4	5.6	0.9	5.6	3.9
14	−3.2	2.3	6.6	0.0	3.6	6.4
15	44.3	7.7	2.2	6.5	1.3	0.7

TABLE 15.4: Results of the lasso regression

λ	0.0	0.9	2.5	5.5	7.5
$\hat{\beta}_1(\lambda)$	0.4586	0.4022	0.3163	0.1130	
$\hat{\beta}_2(\lambda)$	−0.1849	−0.2005	−0.1411		
$\hat{\beta}_3(\lambda)$	0.8336	0.7265	0.5953	0.3951	0.2462
$\hat{\beta}_4(\lambda)$	−0.1893	−0.1102			
$\hat{\beta}_5(\lambda)$	0.0931				
$\left\Vert \hat{\beta}(\lambda) \right\Vert_1$	1.7595	1.4395	1.0527	0.5081	0.2462
$\mathrm{RSS}\left(\hat{\beta}(\lambda)\right)$	0.0118	0.0304	0.1180	0.4076	0.6306
R_c^2	0.9874	0.9674	0.8735	0.5633	0.3244
$\mathrm{df}^{(\mathrm{model})}$	5	4	3	2	1

TABLE 15.5: Data of the ridge regression problem

i	y	x_1	x_2	x_3	x_4	x_5
1	−23.0	−8.0	6.0	−12.7	9.5	−7.5
2	−21.0	−6.5	11.1	5.4	6.6	6.7
3	−5.0	−14.4	−13.3	−3.2	0.8	1.0
4	−39.6	−6.7	26.0	11.5	15.5	6.5
5	5.8	2.3	−7.1	−4.6	7.0	−0.6
6	13.6	2.0	−13.0	−13.3	−0.9	−8.6
7	14.0	10.7	−4.9	−23.1	2.5	19.0
8	−5.2	−8.5	1.0	4.2	−11.5	12.9
9	6.9	3.4	4.9	9.5	−12.8	11.0
10	−5.2	0.0	5.1	−14.3	−3.8	−10.0
11	0.0	1.0	4.0	14.1	−3.5	−23.6
12	3.0	2.4	1.6	−1.2	−4.8	−9.2
13	9.2	−0.1	−10.6	16.0	7.5	5.8
14	26.1	15.2	2.5	5.3	−18.0	10.4
15	−6.3	−19.2	−20.7	−5.1	3.9	−13.8
16	11.5	10.1	1.7	−12.1	−2.7	13.9
17	4.8	3.8	0.8	2.7	1.0	14.4
18	35.2	23.1	1.2	−5.0	−16.1	3.3
19	14.0	13.1	6.6	1.6	−7.4	−3.5
20	−21.4	−19.0	0.7	0.8	−2.7	11.3

TABLE 15.6: Data of the clustering problem

i	X_1	X_2	X_3	X_4	X_5
1	17.6	19.6	19.8	20.4	28.8
2	13.2	17.5	17.5	17.4	24.2
3	35.9	25.4	32.4	25.0	40.7
4	28.1	24.0	25.1	28.7	26.7
5	23.5	23.6	23.7	14.3	18.1
6	36.5	30.3	29.5	32.0	29.5
7	14.0	23.9	18.3	19.2	17.2
8	36.7	29.0	30.3	21.1	28.7
9	31.2	19.4	29.9	33.3	23.8
10	17.0	20.5	23.8	16.0	19.7

TABLE 15.7: Optimal centroids $\mu_j^\star$ for 2 and 3 clusters

$\mu_j^\star$	X_1	X_2	X_3	X_4	X_5
			$n_C = 2$		
$\mu_1^\star$	17.06	21.02	20.62	17.46	21.60
$\mu_2^\star$	33.68	25.62	29.44	28.02	29.88
			$n_C = 3$		
$\mu_1^\star$	17.06	21.02	20.62	17.46	21.60
$\mu_2^\star$	36.37	28.23	30.73	26.03	32.97
$\mu_3^\star$	29.65	21.70	27.50	31.00	25.25

TABLE 15.8: Agglomerative hierarchical clustering (single linkage)

$L^{(s)}$	$D^{(s)}$	$(i^{\star}, i'^{\star})$	$\mathcal{C}^{(s)}$
1	7.571	$(5,10)$	$\{5,10\}$
2	7.695	$(1,2)$	$\{1,2\}$
3	8.204	$(5,7)$	$\{5,7,10\}$
4	9.131	$(4,9)$	$\{4,9\}$
5	9.238	$(1,5)$	$\{1,2,5,7,10\}$
6	11.037	$(6,8)$	$\{6,8\}$
7	12.179	$(4,6)$	$\{4,6,8,9\}$
8	13.312	$(3,4)$	$\{3,4,6,8,9\}$
9	15.199	$(1,3)$	$\{1,2,3,4,5,6,7,8,9,10\}$

TABLE 15.9: Eigendecomposition of the covariance matrix

	β_1	β_2	β_3	β_4
X_1	0.18	−0.20	−0.57	0.77
X_2	0.33	0.58	−0.63	−0.40
X_3	0.53	−0.73	−0.13	−0.41
X_4	0.76	0.31	0.50	0.27
λ_j	5.92	2.31	1.31	0.46

TABLE 15.10: Eigendecomposition of the correlation matrix

	β_1	β_2	β_3	β_4
X_1	0.48	−0.44	−0.65	−0.40
X_2	0.44	0.67	−0.40	0.45
X_3	0.53	−0.51	0.38	0.57
X_4	0.55	0.33	0.53	−0.56
λ_j	2.06	0.97	0.73	0.23

TABLE 15.11: Principal component analysis of Example 166

Factor	Z_1	Z_2	Z_3	Z_4	Z_5
λ_j	3.4173	0.7271	0.5548	0.2783	0.0226
$\mathbb{Q}_j$	68.35%	14.54%	11.10%	5.57%	0.45%
$\mathbb{Q}_j^\star$	68.35%	82.89%	93.98%	99.55%	100.00%
Matrix B of eigenvectors					
X_1	0.5295	−0.1015	−0.0567	−0.2554	0.8006
X_2	0.3894	−0.7546	−0.0500	0.4855	−0.2019
X_3	0.5044	−0.0188	−0.0247	−0.6650	−0.5499
X_4	0.3952	0.5318	−0.6238	0.3995	−0.1107
X_5	0.3967	0.3702	0.7775	0.3120	−0.0609
Correlation between X_k and Z_j					
X_1	97.88%	−8.66%	−4.22%	−13.47%	12.03%
X_2	71.98%	−64.35%	−3.72%	25.61%	−3.03%
X_3	93.25%	−1.60%	−1.84%	−35.08%	−8.27%
X_4	73.06%	45.35%	−46.46%	21.07%	−1.66%
X_5	73.34%	31.57%	57.91%	16.46%	−0.92%
Quality of representation of each variable $\mathbb{Q}_{k,j}$					
X_1	95.81%	0.75%	0.18%	1.82%	1.45%
X_2	51.81%	41.40%	0.14%	6.56%	0.09%
X_3	86.95%	0.03%	0.03%	12.31%	0.68%
X_4	53.38%	20.57%	21.59%	4.44%	0.03%
X_5	53.78%	9.96%	33.54%	2.71%	0.01%
Contribution of each variable $\mathbb{C}_{k,j}$					
X_1	28.04%	1.03%	0.32%	6.52%	64.09%
X_2	15.16%	56.94%	0.25%	23.57%	4.08%
X_3	25.44%	0.04%	0.06%	44.22%	30.24%
X_4	15.62%	28.29%	38.91%	15.96%	1.23%
X_5	15.74%	13.70%	60.46%	9.73%	0.37%

TABLE 15.12: Data of the classification problem

i	$\mathcal{C}_j$	X_1	X_2	i	$\mathcal{C}_j$	X_1	X_2	i	$\mathcal{C}_j$	X_1	X_2
1	1	1.03	2.85	12	2	3.70	5.08	23	3	3.55	0.58
2	1	0.20	3.30	13	2	2.81	1.99	24	3	3.86	1.83
3	1	1.69	3.73	14	2	3.66	2.61	25	3	5.39	0.47
4	1	0.98	3.52	15	2	5.63	4.19	26	3	3.15	−0.18
5	1	0.98	5.15	16	2	3.35	3.64	27	3	4.93	1.91
6	1	3.47	6.56	17	2	2.97	3.55	28	3	3.87	2.61
7	1	3.94	4.68	18	2	3.16	2.92	29	3	4.09	1.43
8	1	1.55	5.99	19	3	3.00	0.98	30	3	3.80	2.11
9	1	1.15	3.60	20	3	3.09	1.99	31	3	2.79	2.10
10	2	1.20	2.27	21	3	5.45	0.60	32	3	4.49	2.71
11	2	3.66	5.49	22	3	3.59	−0.46	33	3	3.51	1.82

TABLE 15.13: Parameter estimation of the discriminant analysis

Class	$\mathcal{C}_1$		$\mathcal{C}_2$		$\mathcal{C}_3$	
$\hat{\pi}_j$	0.273		0.273		0.455	
$\hat{\mu}_j$	1.666	4.376	3.349	3.527	3.904	1.367
$\hat{\Sigma}_j$	1.525	0.929	1.326	0.752	0.694	−0.031
	0.929	1.663	0.752	1.484	−0.031	0.960

TABLE 15.14: Computation of the discriminant scores $S_j(x)$

i	QDA			LDA			LDA^2		
	$S_1(x)$	$S_2(x)$	$S_3(x)$	$S_1(x)$	$S_2(x)$	$S_3(x)$	$S_1(x)$	$S_2(x)$	$S_3(x)$
1	−2.28	−3.69	−7.49	0.21	−0.96	−0.79	6.93	5.60	5.76
2	−2.28	−6.36	−12.10	−0.26	−2.17	−2.34	1.38	−2.13	−1.89
3	−1.76	−3.13	−6.79	2.84	2.16	1.71	12.13	12.01	11.38
4	−1.80	−4.43	−8.88	1.35	0.09	−0.22	7.73	6.20	5.93
5	−2.36	−7.75	−13.70	4.32	2.93	1.45	8.12	5.54	4.76
6	−3.16	−5.63	−14.68	10.75	11.36	8.95	14.82	13.99	12.96
7	−3.79	−1.92	−6.32	8.06	9.22	8.15	17.36	19.03	17.89
8	−2.85	−8.43	−15.23	6.73	5.76	3.70	10.47	8.09	7.15
9	−1.74	−4.12	−8.37	1.76	0.64	0.27	8.94	7.77	7.39
10	−3.14	−3.21	−6.17	−0.58	−1.56	−0.98	6.59	5.55	6.15
11	−2.87	−3.01	−9.45	9.10	9.96	8.31	16.89	17.65	16.42
12	−3.04	−2.38	−7.77	8.42	9.34	7.98	17.28	18.50	17.28
13	−6.32	−2.29	−1.62	1.41	1.82	2.64	12.48	13.94	14.46
14	−6.91	−2.07	−1.42	3.86	4.94	5.34	15.15	17.41	17.34
15	−9.79	−3.62	−7.12	9.79	12.43	11.75	12.58	14.01	13.50
16	−3.90	−1.47	−3.44	5.25	5.99	5.65	16.84	18.82	18.03
17	−3.31	−1.55	−3.61	4.50	4.92	4.63	16.25	17.95	17.21
18	−4.84	−1.60	−2.19	3.65	4.28	4.45	15.51	17.48	17.14
19	−10.21	−4.12	−1.27	−0.13	0.52	2.06	8.98	9.99	11.70
20	−7.05	−2.41	−1.24	1.85	2.50	3.32	12.99	14.72	15.22
21	−23.11	−11.16	−2.56	2.98	5.75	7.61	3.79	4.57	7.26
22	−19.22	−9.53	−2.42	−1.84	−0.57	2.01	1.81	1.53	5.51
23	−13.86	−5.92	−1.01	−0.01	1.15	2.98	7.65	8.67	10.95
24	−10.01	−3.43	−0.70	2.75	4.07	5.02	12.84	14.95	15.65
25	−23.48	−11.44	−2.54	2.65	5.38	7.33	3.40	4.09	6.95
26	−15.87	−7.59	−2.30	−2.01	−1.14	1.23	3.19	3.02	6.50
27	−14.09	−5.40	−1.52	4.56	6.78	7.70	11.17	13.24	14.08
28	−7.55	−2.27	−1.39	4.18	5.45	5.85	15.10	17.44	17.40
29	−12.40	−4.67	−0.61	2.38	3.92	5.17	11.21	13.14	14.33
30	−8.85	−2.87	−0.88	3.17	4.41	5.17	13.77	15.97	16.37
31	−5.97	−2.17	−1.72	1.58	1.97	2.70	12.78	14.26	14.67
32	−9.40	−2.97	−1.81	5.33	7.11	7.46	14.55	16.95	16.93
33	−8.84	−3.01	−0.80	2.19	3.21	4.16	12.82	14.77	15.45

TABLE 15.15: Data of program effectiveness

OBS	GPA	TUCE	PSI	GRD	OBS	GPA	TUCE	PSI	GRD
1	2.66	20	0	0	17	2.75	25	0	0
2	2.89	22	0	0	18	2.83	19	0	0
3	3.28	24	0	0	19	3.12	23	1	0
4	2.92	12	0	0	20	3.16	25	1	1
5	4.00	21	0	1	21	2.06	22	1	0
6	2.86	17	0	0	22	3.62	28	1	1
7	2.76	17	0	0	23	2.89	14	1	0
8	2.87	21	0	0	24	3.51	26	1	0
9	3.03	25	0	0	25	3.54	24	1	1
10	3.92	29	0	1	26	2.83	27	1	1
11	2.63	20	0	0	27	3.39	17	1	1
12	3.32	23	0	0	28	2.67	24	1	0
13	3.57	23	0	0	29	3.65	21	1	1
14	3.26	25	0	1	30	4.00	23	1	1
15	3.53	26	0	0	31	3.10	21	1	0
16	2.74	19	0	0	32	2.39	19	1	1

Source: Greene (2017), Table F14.1 and Spector and Mazzeo (1980).

TABLE 15.16: Results of the logistic regression

Parameter	Estimate	Standard error	t-statistic	p-value
β_0	−13.0214	4.9313	−2.6405	0.0134
β_1	2.8261	1.2629	2.2377	0.0334
β_2	0.0952	0.1415	0.6722	0.5069
β_3	2.3787	1.0646	2.2344	0.0336

TABLE 15.17: Estimated probability $\hat{p}_i = \Pr\{\mathrm{GRD}_i = 1\}$

OBS	Logit	NN	OBS	Logit	NN
1	2.658	2.658	17	5.363	5.363
2	5.950	5.950	18	3.859	3.859
3	18.726	18.726	19	58.987	58.987
4	2.590	2.590	20	66.079	66.079
5	56.989	56.989	21	6.138	6.138
6	3.486	3.486	22	90.485	90.485
7	2.650	2.650	23	24.177	24.177
8	5.156	5.156	24	85.209	85.209
9	11.113	11.113	25	83.829	83.829
10	69.351	69.351	26	48.113	48.113
11	2.447	2.447	27	63.542	63.542
12	19.000	19.000	28	30.722	30.722
13	32.224	32.224	29	84.170	84.170
14	19.321	19.321	30	94.534	94.534
15	36.099	36.099	31	52.912	52.912
16	3.018	3.018	32	11.103	11.103

TABLE 15.18: An example of linearly separable observations

i	1	2	3	4	5	6	7
$x_{i,1}$	0.5	2.7	2.7	1.7	1.5	2.3	4.0
$x_{i,2}$	2.5	4.2	2.0	4.2	0.7	5.3	6.9
y_i	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$

i	8	9	10	11	12	13	14	15
$x_{i,1}$	6.4	7.7	8.8	7.4	6.5	8.3	6.0	5.0
$x_{i,2}$	4.5	2.2	6.0	6.5	1.7	1.3	1.3	0.5
y_i	-1	-1	-1	-1	-1	-1	-1	-1

TABLE 15.19: Soft margin classification with $C = 0.05$

i	y_i	$x_{i,1}$	$x_{i,2}$	$\hat{\alpha}_i$	$\hat{\xi}_i$
1	$+1$	0.5	2.5	0.000	0.000
2	$+1$	2.7	4.2	0.039	0.000
3	$+1$	2.7	2.0	0.050	0.369
4	$+1$	1.7	4.2	0.000	0.000
5	$+1$	1.5	0.7	0.050	0.038
6	$+1$	2.3	5.3	0.000	0.000
7	$+1$	4.0	6.9	0.050	0.143
8	-1	6.4	4.5	0.050	0.354
9	-1	7.7	2.2	0.000	0.000
10	-1	8.8	6.0	0.000	0.000
11	-1	7.4	6.5	0.050	0.231
12	-1	6.5	1.7	0.000	0.000
13	-1	8.3	1.3	0.000	0.000
14	-1	6.0	1.3	0.039	0.000
15	-1	5.0	0.5	0.050	0.324
16	$+1$	6.0	5.0	0.050	1.379
17	-1	2.0	2.0	0.050	1.952

TABLE 15.20: Comparison of OLS, LAD and SVM estimates

$\hat{\beta}_k$	OLS	LAD	LS-SVM ($C = 1, \varepsilon = 1$)	ε-SVM ($C = 1, \varepsilon = 1$)	LS-SVM ($C = \infty, \varepsilon = 0$)	ε-SVM ($C = \infty, \varepsilon = 0$)
$\hat{\beta}_0$	3.446	2.331	3.389	3.262	3.446	2.331
$\hat{\beta}_1$	1.544	1.893	1.542	1.631	1.544	1.893
$\hat{\beta}_2$	-1.645	-1.735	-1.616	-1.526	-1.645	-1.735
$\hat{\beta}_3$	2.895	2.908	2.885	2.726	2.895	2.908

TABLE 15.21: Illustration of the boosting algorithm ($n_S = 2$)

i	y_i	x_i	$w_{i,1}$ (in %)	$\hat{y}_{i,1}$	$\vartheta_{i,1}$	$w_{i,2}$ (in %)	$\hat{y}_{i,2}$	$\vartheta_{i,2}$	$\hat{y}_i$	ϑ_i
1	1	0.597	5.00	1		4.55	1		1	
2	1	1.496	5.00	1		4.55	1		1	
3	−1	−0.914	5.00	1	✓	5.56	−1		−1	
4	−1	−0.497	5.00	1	✓	5.56	1	✓	1	✓
5	−1	0.493	5.00	1	✓	5.56	1	✓	1	✓
6	1	0.841	5.00	1		4.55	1		1	
7	−1	−0.885	5.00	1	✓	5.56	1	✓	1	✓
8	1	1.418	5.00	1		4.55	1		1	
9	−1	−0.183	5.00	1	✓	5.56	1	✓	1	✓
10	−1	−1.298	5.00	1	✓	5.56	−1		−1	
11	1	−0.324	5.00	1		4.55	1		1	
12	1	−1.454	5.00	−1	✓	5.56	−1	✓	−1	✓
13	1	−0.270	5.00	1		4.55	1		1	
14	1	−0.770	5.00	1		4.55	1		1	
15	1	0.232	5.00	1		4.55	1		1	
16	−1	0.970	5.00	1	✓	5.56	1	✓	1	✓
17	−1	1.196	5.00	1	✓	5.56	1	✓	1	✓
18	1	0.578	5.00	1		4.55	1		1	
19	1	−0.686	5.00	1		4.55	1		1	
20	1	−0.590	5.00	1		4.55	1		1	

TABLE 15.22: Estimated model at each boosting iteration ($n_S = 5$)

s	1	2	3	4	5
$\hat{\beta}_{0,s}$	0.4133	0.2334	−0.0771	0.0009	0.0103
$\hat{\beta}_{1,s}$	0.2976	0.2558	0.0278	0.0277	−0.0751
$\mathcal{L}_{(s)}$	0.4500	0.3889	0.4805	0.4741	0.5000
ω_s	0.2007	0.4520	0.0780	0.1038	0.0000

TABLE 15.23: Confusion matrix of three scoring systems and three cut-off values s

Score	$s = 100$		$s = 200$		$s = 500$	
S_1	386	616	698	1 304	1 330	3 672
	1 614	7 384	1 302	6 696	670	4 328
S_2	372	632	700	1 304	1 386	3 616
	1 628	7 368	1 300	6 696	614	4 384
S_3	382	616	656	1 344	1 378	3 624
	1 618	7 384	1 344	6 656	622	4 376
Perfect	1 000	0	2 000	0	2 000	3 000
	1 000	8 000	0	8 000	0	5 000

TABLE 15.24: Binary classification ratios (in %) of the three scoring systems

Score	s	TPR	FNR	TNR	FPR	PPV	ACC	F_1
	100	92.3	7.7	19.3	80.7	82.1	77.7	86.9
S_1	200	83.7	16.3	34.9	65.1	83.7	73.9	83.7
	500	54.1	45.9	66.5	33.5	86.6	56.6	66.6
	100	92.1	7.9	18.6	81.4	81.9	77.4	86.7
S_2	200	83.7	16.3	35.0	65.0	83.7	74.0	83.7
	500	54.8	45.2	69.3	30.7	87.7	57.7	67.5
	100	92.3	7.7	19.1	80.9	82.0	77.7	86.9
S_3	200	83.2	16.8	32.8	67.2	83.2	73.1	83.2
	500	54.7	45.3	68.9	31.1	87.6	57.5	67.3
	100	100.0	0.0	50.0	50.0	88.9	90.0	94.1
Perfect	200	100.0	0.0	100.0	0.0	100.0	100.0	100.0
	500	62.5	37.5	100.0	0.0	100.0	70.0	76.9

TABLE 15.25: Best scoring system

Cut-off	TPR	FNR	TNR	FPR	PPV	ACC	F_1
100	S_1/S_3	S_1/S_3	S_1	S_1	S_1	S_1	S_1
200	S_1/S_2	S_1/S_2	S_2	S_2	S_2	S_2	S_2
500	S_2	S_2	S_2	S_2	S_2	S_2	S_2

Conclusion

Appendix A

Technical Appendix

TABLE A.1: Value x_n of the last knot in Gauss-Laguerre and Gauss-Hermite quadratures

n	Laguerre	Hermite
4	9.3951	1.6507
8	22.8631	2.9306
16	51.7012	4.6887
32	111.7514	7.1258
100	374.9841	13.4065
200	767.8000	19.3300