Course 2023-2024 in Financial Risk Management
Lecture 6. Liquidity Risk

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[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

1. **Overview**
   The objective of this course is to understand the theoretical and practical aspects of risk management
2. **Prerequisites**
   M1 Finance or equivalent
3. **ECTS**
   4
4. **Keywords**
   Finance, Risk Management, Applied Mathematics, Statistics
5. **Hours**
   Lectures: 36h, Training sessions: 15h, HomeWork: 30h
6. **Evaluation**
   There will be a final three-hour exam, which is made up of questions and exercises
7. **Course website**
   `http://www.thierry-roncalli.com/RiskManagement.html`

Objective of the course

The objective of the course is twofold:

1. knowing and understanding the financial regulation (banking and others) and the international standards (especially the Basel Accords)
2. being proficient in risk measurement, including the mathematical tools and risk models

Class schedule

Course sessions

- September 15 (6 hours, AM+PM)
- September 22 (6 hours, AM+PM)
- September 19 (6 hours, AM+PM)
- October 6 (6 hours, AM+PM)
- October 13 (6 hours, AM+PM)
- October 27 (6 hours, AM+PM)

Tutorial sessions

- October 20 (3 hours, AM)
- October 20 (3 hours, PM)
- November 10 (3 hours, AM)
- November 10 (3 hours, PM)
- November 17 (3 hours, PM)

Class times: Fridays 9:00am-12:00pm, 1:00pm–4:00pm, University of Evry, Room 209 IDF

Agenda

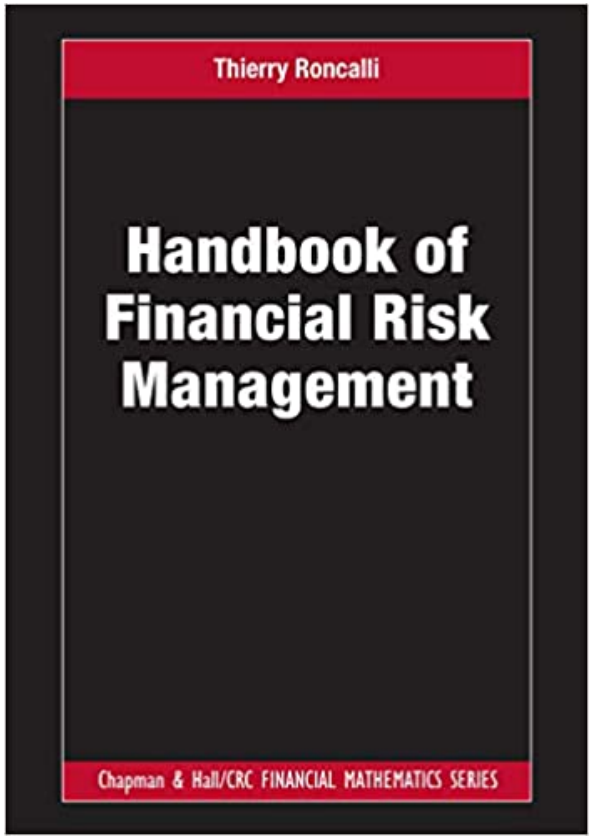
- Lecture 1: Introduction to Financial Risk Management
- Lecture 2: Market Risk
- Lecture 3: Credit Risk
- Lecture 4: Counterparty Credit Risk and Collateral Risk
- Lecture 5: Operational Risk
- Lecture 6: Liquidity Risk
- Lecture 7: Asset Liability Management Risk
- Lecture 8: Model Risk
- Lecture 9: Copulas and Extreme Value Theory
- Lecture 10: Monte Carlo Simulation Methods
- Lecture 11: Stress Testing and Scenario Analysis
- Lecture 12: Credit Scoring Models

Agenda

- Tutorial Session 1: Market Risk
- Tutorial Session 2: Credit Risk
- Tutorial Session 3: Counterparty Credit Risk and Collateral Risk
- Tutorial Session 4: Operational Risk & Asset Liability Management Risk
- Tutorial Session 5: Copulas, EVT & Stress Testing

Textbook

- Roncalli, T. (2020), *Handbook of Financial Risk Management*, Chapman & Hall/CRC Financial Mathematics Series.



Additional materials

- Slides, tutorial exercises and past exams can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagement.html`

- Solutions of exercises can be found in the companion book, which can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagementBook.html`

Agenda

- Lecture 1: Introduction to Financial Risk Management
- Lecture 2: Market Risk
- Lecture 3: Credit Risk
- Lecture 4: Counterparty Credit Risk and Collateral Risk
- Lecture 5: Operational Risk
- **Lecture 6: Liquidity Risk**
- Lecture 7: Asset Liability Management Risk
- Lecture 8: Model Risk
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Bid-ask spread

Definition

The bid-ask quoted spread $\mathbf{S}_t$ is defined by:

$$\mathbf{S}_t = \frac{P_t^{\text{ask}} - P_t^{\text{bid}}}{P_t^{\text{mid}}}$$

where P_t^{ask}, P_t^{bid} and P_t^{mid} are the ask, bid and mid quotes for a given security at time t.

We have:

$$P_t^{\text{mid}} = \frac{P_t^{\text{ask}} + P_t^{\text{bid}}}{2}$$

Bid-ask spread

Table: Snapshot of the limit order book of the Lyxor Euro Stoxx 50 ETF recorded at NYSE Euronext Paris – The corresponding date is 14:00:00 and 56,566 micro seconds on 28 December 2012

i^{th} limit	Buy orders		Sell orders	
	$Q_t^{\text{bid,i}}$	$P_t^{\text{bid,i}}$	$Q_t^{\text{ask,i}}$	$P_t^{\text{ask,i}}$
1	65 201	26.325	70 201	26.340
2	85 201	26.320	116 201	26.345
3	105 201	26.315	107 365	26.350
4	76 500	26.310	35 000	26.355
5	20 000	26.305	35 178	26.360

We have $P_t^{\text{bid}} = 26.325$ and $P_t^{\text{ask}} = 26.340$, implying that the mid price is equal to $P_t^{\text{mid}} = (26.325 + 26.340)/2 = 26.3325$. We deduce that the bid-ask spread is:

$$\mathbf{S}_t = \frac{26.340 - 26.325}{26.3325} = 5.696 \text{ bps}$$

Bid-ask spread

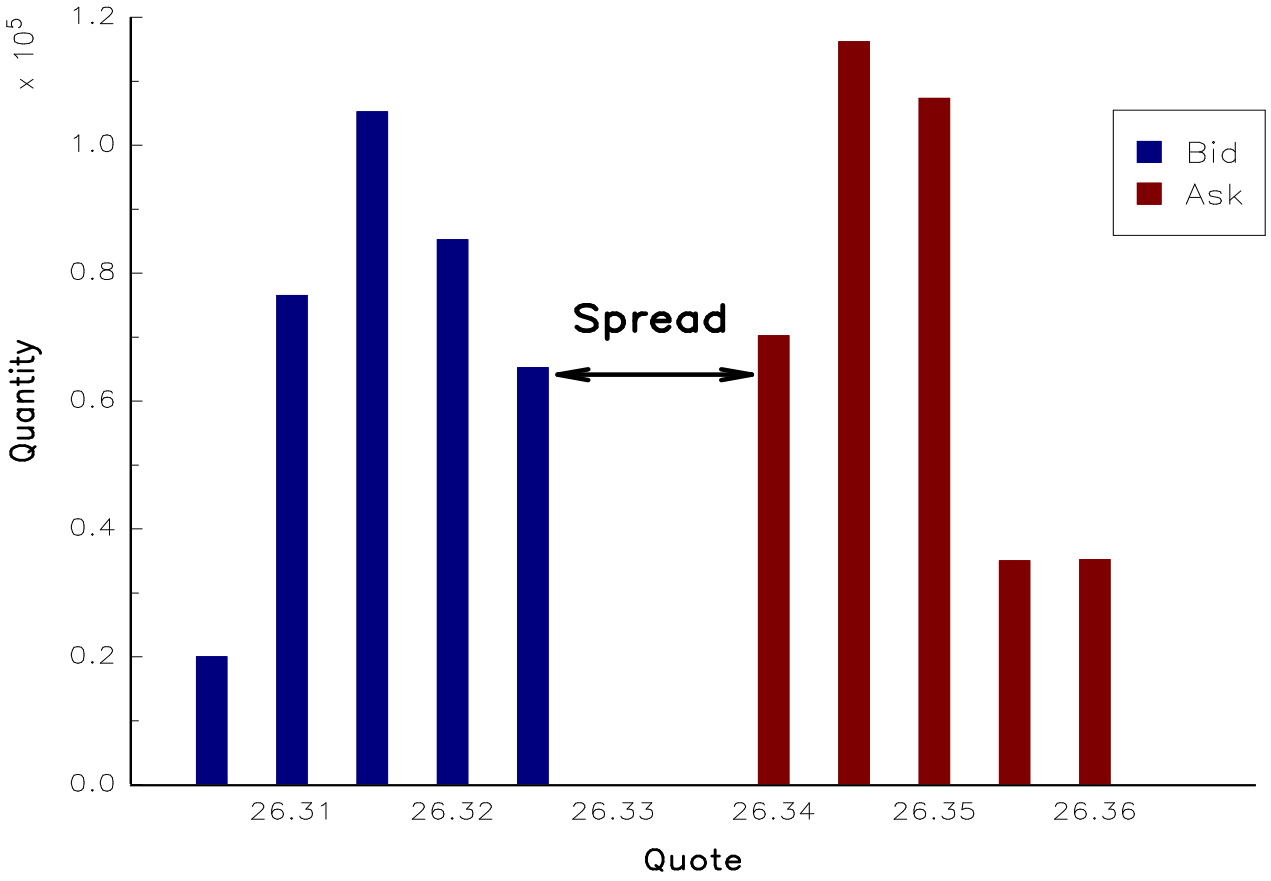

Figure: An example of a limit order book

Bid-ask spread

- The *effective spread* is equal to:

$$\mathbf{S}_\tau^e = 2\left|\frac{P_\tau - P_t^{\text{mid}}}{P_t^{\text{mid}}}\right|$$

where τ is the trade index, P_τ is the price of the τ^{th} trade and P_τ^{mid} is the midpoint of market quote calculated at the time t of the τ^{th} trade

- The *realized spread* is equal to:

$$\mathbf{S}_\tau^r = 2\left|\frac{P_\tau - P_{t+\Delta}^{\text{mid}}}{P_{t+\Delta}^{\text{mid}}}\right|$$

Generally, Δ is set to five minutes

$$\textbf{Price impact} \Rightarrow P_{t+\Delta}^{\text{mid}} \neq P_t^{\text{mid}}$$

Trading volume

The trading volume $\mathbf{V}_t$ indicates the dollar value of the security exchanged during the period t:

$$\mathbf{V}_t = \sum_{\tau \in t} Q_\tau P_\tau$$

where Q_τ and P_τ are the τ^{th} quantity and price traded during the period. Generally, we consider a one-day period and use the following approximation:

$$\mathbf{V}_t \approx Q_t P_t$$

where Q_t is the number of securities traded during the day t and P_t is the closing price of the security.

Turnover

The turnover is the ratio between the trading volume and the free float market capitalization M_t of the asset:

$$\mathbf{T}_t = \frac{\mathbf{V}_t}{M_t} = \frac{\mathbf{V}_t}{N_t P_t}$$

where N_t is the number of outstanding '*floating*' shares

$\Rightarrow$ The asset turnover ratio indicates how many times each share changes hands in a given period

Liquidation ratio

The liquidation ratio $\mathcal{LR}(m)$ measures the proportion of a given position that can be liquidated after m trading days

Liquidation ratio

Computation of the liquidation ratio

We denote $(x_1, \ldots, x_n)$ the number of shares held in the portfolio. For each asset that composes the portfolio, we denote x_i^+ the maximum number of shares for asset i that can be sold during a trading day. The number of shares $x_i(m)$ liquidated after m trading days is defined as follows:

$$x_i(m) = \min\left(\left(x_i - \sum_{k=0}^{m-1} x_i(k)\right)^+, x_i^+\right)$$

with $x_i(0) = 0$. The liquidation ratio $\mathcal{LR}(m)$ is then the proportion of the portfolio liquidated after m trading days:

$$\mathcal{LR}(m) = \frac{\sum_{i=1}^n \sum_{k=0}^m x_i(k) \cdot P_{i,t}}{\sum_{i=1}^n x_i \cdot P_{i,t}}$$

Liquidation ratio

Table: Statistics of the liquidation ratio (size = \$10 bn, liquidation policy = 10% of ADV)

Statistics	SPX	SX5E	DAX	NDX	MSCI EM	MSCI INDIA	MSCI EMU SC
m (in days)	Liquidation ratio $\mathcal{LR}(t)$ in %						
1	88.4	12.3	4.8	40.1	22.1	1.5	3.0
2	99.5	24.7	9.6	72.6	40.6	3.0	6.0
5	100.0	58.8	24.1	99.7	75.9	7.6	14.9
10	100.0	90.1	47.6	99.9	93.9	15.1	29.0
α (in %)	Liquidation time $\mathcal{LR}^{-1}(\alpha)$ in days						
50	1	5	11	2	3	37	21
75	1	7	17	3	5	71	43
90	2	10	23	3	9	110	74
99	2	15	29	5	17	156	455

Source: Roncalli and Weisang (2015).

Liquidation ratio

Table: Statistics of the liquidation ratio (size = \$10 bn, liquidation policy = 30% of ADV)

Statistics	SPX	SX5E	DAX	NDX	MSCI EM	MSCI INDIA	MSCI EMU SC
t (in days)	Liquidation ratio $\mathcal{LR}(t)$ in %						
1	100.0	37.0	14.5	91.0	55.5	4.5	9.0
2	100.0	67.7	28.9	99.8	81.8	9.1	17.8
5	100.0	99.2	68.6	100.0	98.5	22.6	40.4
10	100.0	100.0	99.6	100.0	100.0	43.1	63.2
α (in %)	Liquidation time $\mathcal{LR}^{-1}(\alpha)$ in days						
50	1	2	4	1	1	13	7
75	1	3	6	1	2	24	15
90	1	4	8	1	3	37	25
99	1	5	10	2	6	52	152

Source: Roncalli and Weisang (2015).

Other liquidity measures

- Hui-Heubel liquidity ratio

$$\mathbf{H}_t^2 = \frac{1}{\mathbf{T}_t}\left(\frac{P_t^{\text{high}} - P_t^{\text{low}}}{P_t^{\text{low}}}\right)$$

- Hasbrouck-Schwartz variance ratio

$$\mathbf{VR} = \frac{\text{var}\left(R_{t,t+h}\right)}{\text{var}\left(R_{t,t+1}\right)}$$

- Amihud measure

$$\mathbf{ILLIQ} = \frac{1}{n_t}\sum_t \frac{|R_{t,t+1}|}{\mathbf{V}_t}$$

- Implicit spread of Roll (1984):

$$\tilde{\mathbf{S}} = 2\sqrt{-\text{cov}\left(\Delta P_t, \Delta P_{t-1}\right)}$$

L-CAPM

"[...] there is also broad belief among users of financial liquidity – traders, investors and central bankers – that the principal challenge is not the average level of financial liquidity... but its variability and uncertainty" (Persaud, 2003).

L-CAPM

We note $R_{i,t}$ and $L_{i,t}$ the gross return and the relative (stochastic) illiquidity cost of Asset i. At the equilibrium, Acharya and Pedersen (2005) showed that:

$$\mathbb{E}\left[R_{i,t} - L_{i,t}\right] - r = \tilde{\beta}_i \cdot \left(\mathbb{E}\left[R_{m,t} - L_{m,t}\right] - r\right)$$

where r is the return of the risk-free asset, $R_{m,t}$ and $L_{m,t}$ are the gross return and the illiquidity cost of the market portfolio, and $\tilde{\beta}_i$ is the liquidity-adjusted beta of Asset i:

$$\tilde{\beta}_i = \frac{\operatorname{cov}\left(R_{i,t} - L_{i,t}, R_{m,t} - L_{m,t}\right)}{\operatorname{var}\left(R_{m,t} - L_{m,t}\right)}$$

Asset liability mismatch

"We define funding liquidity as the ability to settle obligations with immediacy. Consequently, a bank is illiquid if it is unable to settle obligations. Legally, a bank is then in default. Given this definition we define funding liquidity risk as the possibility that over a specific horizon the bank will become unable to settle obligations with immediacy" (Drehmann and Nikolaou, 2013).

Relationship between market and funding liquidity risks

"Traders provide market liquidity, and their ability to do so depends on their availability of funding. Conversely, traders' funding, i.e., their capital and margin requirements, depends on the assets' market liquidity. We show that, under certain conditions, margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals" (Brunnermeier and Pedersen, 2009).

Relationship between market and funding liquidity risks

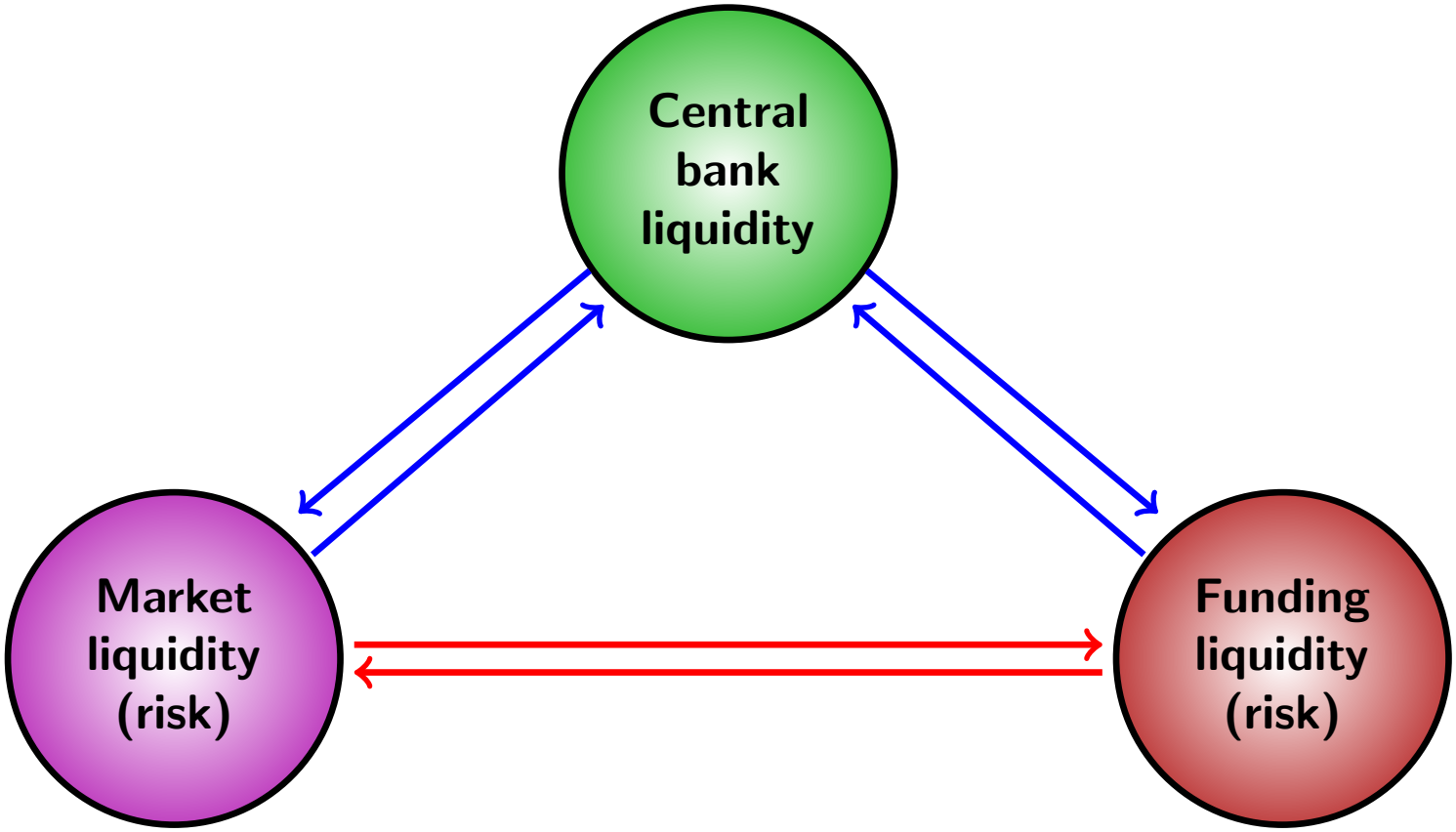

Figure: The liquidity nodes of the financial system

Source: Nikolaou (2009).

Relationship between market and funding liquidity risks

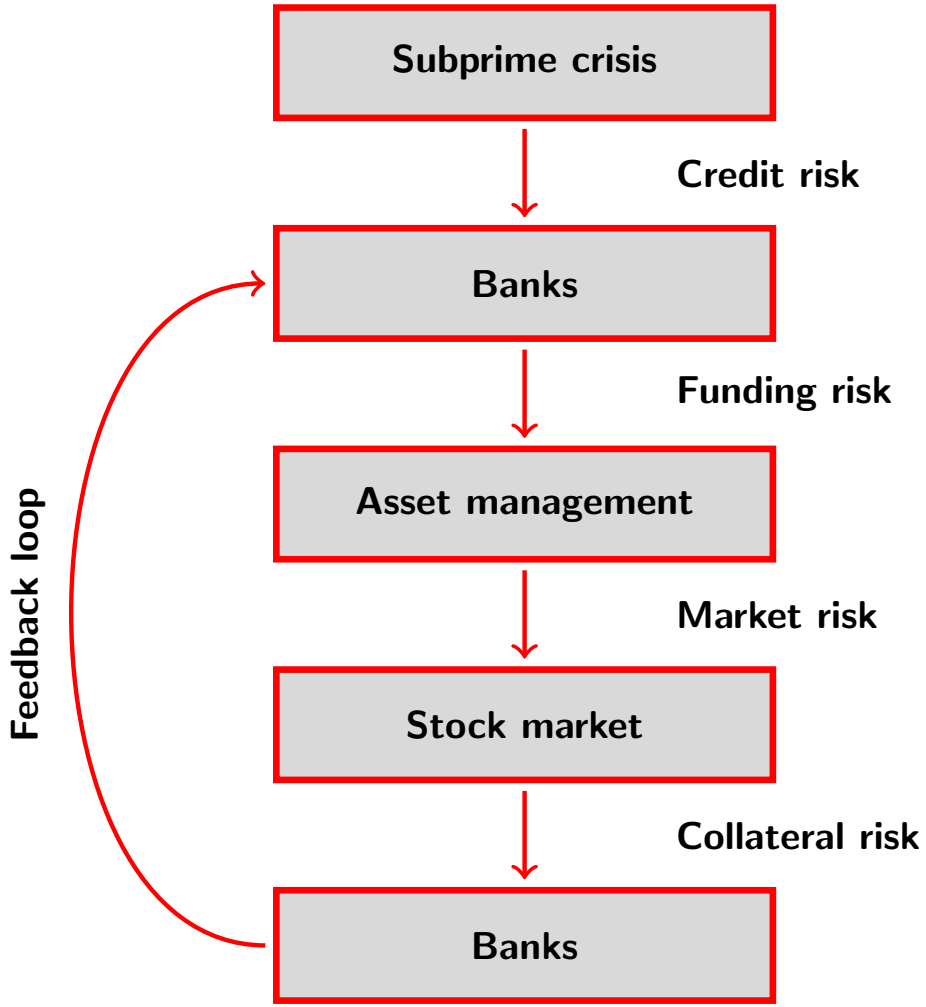

Figure: Spillover effects during the 2008 global financial crisis

Liquidity coverage ratio

The liquidity coverage ratio is defined as:

$$\text{LCR} = \frac{\text{HQLA}}{\text{Total net cash outflows}} \geq 100\%$$

where the numerator is the stock of high quality liquid assets (HQLA) in stressed conditions, and the denominator is the total net cash outflows over the next 30 calendar days

$\Rightarrow$ The underlying idea of the LCR is that the bank has sufficient liquid assets to meet its liquidity needs for the next month

High quality liquid asset

An asset is considered to be a HQLA if it can be easily converted into cash. Therefore, the concept of HQLA is related to asset quality and asset liquidity

Characteristics used by the Basel Committee for defining HQLA:

- fundamental characteristics (low risk, ease and certainty of valuation, low correlation with risky assets, listed on a developed and recognized exchange);
- market-related characteristics (active and sizable market, low volatility, flight to quality).

High quality liquid asset

Table: Stock of HQLA

Level	Description	Haircut
Level 1 assets		
	Coins and bank notes Sovereign, central bank, PSE, and MDB assets qualifying for 0% risk weighting Central bank reserves Domestic sovereign or central bank debt for non-0% risk weighting	0%
Level 2 assets (maximum of 40% of HQLA)		
Level 2A assets		
	Sovereign, central bank, PSE and MDB assets qualifying for 20% risk weighting Corporate debt securities rated AA− or higher Covered bonds rated AA− or higher	15%
Level 2B assets (maximum of 15% of HQLA)		
	RMBS rated AA or higher	25%
	Corporate debt securities rated between A+ and BBB−	50%
	Common equity shares	50%

High quality liquid asset

Level 2 assets are subject to two caps. Let x_{HQLA}, x_1 and x_2 be the value of HQLA, level 1 assets and level 2 assets. We have:

$$\begin{aligned} x_{\text{HQLA}} &= x_1 + x_2 \\ \text{s.t.} &\begin{cases} x_2 = x_{2A} + x_{2B} \\ x_{2A} \leq 0.40 \cdot x_{\text{HQLA}} \\ x_{2B} \leq 0.15 \cdot x_{\text{HQLA}} \end{cases} \end{aligned}$$

We deduce that one trivial solution is:

$$\begin{cases} x^\star_{\text{HQLA}} = \min\left(\frac{5}{3}x_1, x_1 + x_2\right) \\ x^\star_1 = x_1 \\ x^\star_2 = x^\star_{\text{HQLA}} - x^\star_1 \\ x^\star_{2A} = \min\left(x^\star_2, x_{2A}\right) \\ x^\star_{2B} = x^\star_2 - x^\star_{2A} \end{cases}$$

High quality liquid asset

Example

We consider the following assets:

1. Coins and bank notes = \$200 mn
2. Central bank reserves = \$100 mn
3. 20% risk-weighted sovereign debt securities = \$200 mn
4. AA corporate debt securities = \$300 mn
5. Qualifying RMBS = \$200 mn
6. BB+ corporate debt securities = \$500 mn

High quality liquid asset

Table: Solution of the exercise

	Assets	Gross Value	Haircut	Net Value	Capped Value
Level 1 assets	(1) + (2)	300	0%	300	300
Level 2 assets		1 200		825	200
2A	(3) + (4)	500	15%	425	200
2B	(5) + (6)	700		400	0
	(5)	200	25%	150	0
	(6)	500	50%	250	0
Total		1 500		1 125	500

$\Rightarrow$ The stock of HQLA is equal to \$500 mn

Total net cash outflows

The value of total net cash outflows is defined as follows:

$$\text{Total net cash outflows} = \text{Total expected cash outflows} - \min\left(\begin{array}{c}\text{Total expected cash inflows,}\\ 75\% \text{ of total expected cash outflows}\end{array}\right)$$

Total net cash outflows

Table: Cash outflows of the LCR

Liabilities	Description	Rate
Retail deposits		
Demand and term deposits (less than 30 days)		
	Stable deposits covered by deposit insurance	3%
	Stable deposits	5%
	Less stable deposits	10%
Term deposits (with residual maturity greater than 30 days)		0%
Unsecured wholesale funding		
Demand and term deposits (less than 30 days) provided by small business customers		
	Stable deposits	5%
	Less stable deposits	10%
Deposits generated by clearing, custody and cash management		25%
	Portion covered by deposit insurance	5%
Cooperative banks in an institutional network		25%
Corporates, sovereigns, central banks, PSEs and MDBs		40%
	Portion covered by deposit insurance	20%

Total net cash outflows

Table: Cash outflows of the LCR

Liabilities Description	Rate
Secured funding transactions	
With a central bank counterparty	0%
Backed by level 1 assets	0%
Backed by level 2A assets	15%
Backed by non-level 1 or non-level 2A assets with domestic sovereigns, PSEs or MDBs as a counterparty	25%
Backed by level 2B RMBS assets	25%
Backed by other level 2B assets	50%
All other secured funding transactions	100%
Additional requirements	
Margin/collateral calls	$\geq$ 20%
ABCP, SIVs, conduits, SPVs, etc.	100%
Net derivative cash outflows	100%
Other credit/liquidity facilities	$\geq$ 5%

Total net cash outflows

Table: Cash inflows of the LCR

Receivables	Description	Rate
Maturing secured lending transactions		
Backed by level 1 assets		0%
Backed by level 2A assets		15%
Backed by level 2B RMBS assets		25%
Backed by other level 2B assets		50%
Backed by non-HQLAs		100%
Other cash inflows		
Credit/liquidity facilities provided to the bank		0%
Inflows to be received from retail counterparties		50%
Inflows to be received from non-financial wholesale counterparties		50%
Inflows to be received from financial institutions and central banks		100%
Net derivative receivables		100%

Total net cash outflows

Example

The bank has \$500 mn of HQLA. Its main liabilities are:

1. Retail stable deposit = \$17.8 bn (\$15 bn have a government guarantee)
2. Retail term deposit (with a maturity of 6 months) = \$5 bn
3. Stable deposit provided by small business customers = \$1 bn
4. deposit of corporates = \$200 mn

In the next thirty days, the bank also expects to receive \$100 mn of loan repayments, and \$10 mn due to a maturing derivative

Total net cash outflows

- We calculate the expected cash outflows for the next thirty days:

$$\begin{aligned}\text{Cash outflows} &= 3\% \times 15\,000 + 5\% \times 2\,800 + 0\% \times 5\,000 + \\ &\quad 5\% \times 1\,000 + 40\% \times 200 \\ &= \$720 \text{ mn}\end{aligned}$$

- We estimate the cash inflows expected by the bank for the next month:

$$\text{Cash inflows} = 50\% \times 100 + 100\% \times 10 = \$60 \text{ mn}$$

- We deduce that the liquidity coverage ratio of the bank is equal to:

$$\text{LCR} = \frac{500}{720 - 60} = 75.76\%$$

Net stable funding ratio

It is defined as the amount of available stable funding (ASF) relative to the amount of required stable funding (RSF):

$$\text{NSFR} = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%$$

- The available amount of stable funding (ASF) corresponds to the regulatory capital plus some other liabilities
- The required amount of stable funding (RSF) is the sum of weighted assets and off-balance sheet exposures

Leverage ratio

- It is defined as the capital measure divided by the exposure measure
- This ratio must be below 3%
- The capital measure corresponds to the tier 1 capital
- The exposure measure is composed of four main exposures:
  1. On-balance sheet exposures
  2. Derivative exposures
  3. Securities financing transaction (SFT)
  4. Exposures and off-balance sheet items

References


Basel Committee on Banking Supervision (2013)
Basel III: The Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools, January 2013.


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