A Framework For Structuring a Blended Finance Fund*

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Abstract

This paper aims to provide a comprehensive framework for structuring blended finance funds, which are becoming increasingly important mechanisms for channeling private capital toward impactful projects in developing countries. The paper explores several key dimensions. First, we clarify the definition of blended finance, emphasizing the strategic use of concessional capital alongside private investment to promote sustainable development goals. We also examine the motivations, roles, and utility functions of various stakeholders, including public investors, asset managers, and private investors. Second, the paper analyzes the common use of two-tranche structures, in which senior tranches are allocated to private investors and junior tranches are held by sponsoring entities. Specifically, we demonstrate the economic rationale behind this tiered structure and its relevance in blended finance contexts. Third, the paper presents an analytical framework for calibrating two- and three-tier fund structures, offering detailed insights into credit risk modeling, cash flow generation, and the concessionality premium.

Through the use of theoretical modeling, illustrative examples, and Monte Carlo simulations, this paper contributes to a more comprehensive understanding of how structured blended finance funds can effectively balance public interests with private return expectations. We particularly focus on the critical structuring process that creates differentiated investment profiles aligned with the diverse objectives of stakeholders. The importance of a well-calibrated asset-liability structure is emphasized, taking into account factors such as asset characteristics, fund maturity, and levels of concessionality. Finally, we introduce key metrics for evaluating tranche-level risk-return profiles and underscore the value of benchmarking in assessing the performance of structured blended finance funds.

Keywords: Blended finance, sustainable development goals, impact investing, development finance institution, junior-senior structure, leverage, concessionality, mezzanine, risk premium, calibration, senior protection mechanism, copula, Monte Carlo simulation.

JEL Classification: F21, G11, G23, Q01.

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1 Introduction

Blended finance is an investment approach in which the public sector leverages private sector capital for projects that have a strong social or environmental impact but may not initially attract private investors due to perceived risks. Capital mobilization is typically achieved through the use of concessional funding. By "*public sector*", we refer to development finance institutions (DFIs) and multilateral development banks (MDBs), which are primarily owned by governments and have a mandate to serve public interests, such as reducing poverty, promoting sustainable development, and addressing climate change[1]. The current growing interest in blended finance is driven by the recent development of ESG and climate investing. While blended finance was niche twenty years ago, it has become more widespread, and its mechanisms are now well-known among institutional investors. Furthermore, blended finance benefits from the growing interest in impact investing and investors' focus on SDGs, climate, and biodiversity risks.

Among the various forms of blended finance products, two-tranche structures have become very popular in recent years. In this case, concessional capital is used to create senior tranches that are safer for private investors, while junior tranches (with higher risks) are allocated to public sector investors. The design of these structured funds is complex because the pay-through structure creates asymmetries in the risk/return profiles of different investors. In particular, many parameters come into play, such as the nature of the assets, the management of the underlying portfolio, the utility functions of public and private investors, the maturity of the fund, and so on. This explains why the market for structured blended finance is concentrated among a few public investors, asset managers, and private investors.

The junior-senior structure may evoke comparisons to collateralized debt/loan obligations (CDOs/CLOs) in the banking sector[2]. While the credit risk modeling involved is similar, there are several key reasons why the similarities end there. CDOs are typically sponsored by commercial banks that want to repackage and transfer existing credit risk. In contrast, structured blended finance (SBF) funds are initiated by development finance institutions to invest in new assets. Banks offload existing loans to a special purpose vehicle in CDOs, whereas SBF portfolio managers actively acquire assets for the fund. CDO sponsors generally prefer to invest in senior tranches, whereas SBF sponsors take the equity tranche to improve the risk-return profile for senior tranche investors. Finally, CDOs are often driven by regulatory capital arbitrage. SBF funds, on the other hand, are explicitly designed to achieve development impact and support the SDGs. These differences between CDOs and SBFs become even more pronounced when comparing the utility functions of their respective sponsors. Typically, the primary objective of a CDO sponsor is to transfer credit risk to the market. In contrast, the sponsor of an SBF fund has a dual objective. First, they aim to finance sustainable projects by offering a concessionality premium. Second, they seek to mobilize private capital to amplify the impact of these projects. As a result, concepts such as the concessionality rate and leverage ratio are central to blended finance instruments. This dual mandate makes the utility function of an SBF sponsor relatively complex, extending beyond a traditional risk-return optimization framework.

[1] Blended finance is sometimes equated with public-private partnerships (PPPs). However, they are two different concepts. A public-private partnership typically involves collaboration between public and private entities, often for the purpose of building infrastructure, where the expertise and technical skills are primarily provided by the private sector, without necessarily involving concessional funding. In contrast, in blended finance, the expertise and skills are provided primarily by public investors.

[2] In general, CDOs are structured with a junior-mezzanine-senior tranche hierarchy, which results in a more complex design. For now, however, we will set this issue aside because including a mezzanine tranche in a structured blended finance fund is a significant consideration.

Traditional structuring approaches must therefore be adapted to reflect the sponsor's utility function. In contrast, the utility function of investors in the senior tranche usually aligns with that of investment-grade institutional investors — more specifically, those targeting AAA or AA-rated bond characteristics[3]. Maximizing the impact of catalytic capital is equivalent to maximizing the leverage ratio. However, the risk borne by the senior tranche increases with higher leverage. While the sponsor and the senior investor may share the same impact-oriented (extra-financial) objective, their financial objectives diverge. For this reason, the objective function must incorporate a constraint reflecting the maximum acceptable risk for senior investors. Furthermore, the structuring must account for the concessionality dimension, which affects the asset side of the investment. Thus, the optimal structuring challenge is not merely a liability-side problem, but rather, a broader asset-liability structuring problem. As we will demonstrate in this article, this has significant implications for both the junior-senior capital structure and the composition of the asset portfolio.

The concept of structured blended finance raises several important considerations. One key issue is determining when a pay-through structure is preferable to a pass-through structure. Although a pay-through structure is generally considered superior to a pass-through structure for mobilizing catalytic capital and attracting private investment, it is unclear whether it results in a more favorable concessionality premium. Another area of analysis concerns the impact of the asset portfolio on the properties of the junior-senior structure. Factors such as maturity (long vs. short), portfolio diversification (across countries and industries), asset concentration (few vs. more assets), and the nature of the underlying credit risk (crossover vs. high-yield assets) can all influence the effectiveness and resilience of the structured finance vehicle. Cash flow design is also a central aspect. Modifying the modeling of cash flows may improve the structure's performance and alignment with investor expectations. This consideration naturally leads to the question of whether a mezzanine tranche should be included. Beyond the technical design, this raises deeper questions about the purpose of the mezzanine layer and the utility function of mezzanine investors. Ultimately, these questions highlight a key insight: the optimal structuring of the liability side cannot be decoupled from the composition of the asset side. The two are inherently interdependent, so thoughtful design must consider this dynamic to maximize the efficiency and impact of the blended finance structure. Although these technical questions are important, they should not distract from the primary goal of blended finance, which is to support the achievement of the sustainable development goals and financing impact-oriented investments. This includes addressing biodiversity loss and promoting new financial instruments, such as outcome bonds. However, the long-term development of blended finance depends on its ability to deliver value through effective structuring. Therefore, all stakeholders — sponsors, asset managers, and investors — must understand how pay-through structures function within blended finance vehicles to align financial and impact objectives.

This paper is organized as follows. In Section Two, we define structured blended finance funds, presenting their various structures, providing an overview of the blended finance market, and explaining the associated portfolio management approaches. Section Three introduces the mathematical tools used to design the asset-liability structure of a blended finance fund, with a focus on credit risk modeling and cash flow modeling. The sensitivity of the asset-liability structure to various factors is assessed through Monte Carlo simulations, which help in the assessment and calibration of key parameters. Section Four explores the relationship between investor preferences and optimal structuring. It examines the connections between risk aversion and risk premium, as well as between risk-neutral and historical

[3] Although many senior investors are responsible investors and view their participation in blended finance deals as part of their ESG stewardship strategy, they generally expect returns comparable to those of traditional investments. For this reason, a standard utility function can be assumed for senior investors.

probability measures. Within this context, we analyze junior-senior structures, the introduction of mezzanine tranches, various senior protection mechanisms, and the concessionality premium. Finally, Section Five offers concluding remarks.

2 What is a structured blended finance fund?

To define a structured blended finance (SBF) fund, we must first understand what blended finance is and what it is not. In particular, it is important to understand the motivations and roles of the different parties (public investors, portfolio managers, structurers, private investors). We then identify the assets that fits the public sector's utility function and explore the different ways to manage the underlying portfolio. Finally, we focus on the structuring process and explain the mechanisms created by the capital structure.

2.1 Definition of blended finance

Blended finance lacks a generally accepted definition at the official level. For example, OECD (2018, page 22) defines blended finance as "*the strategic use of development finance for the mobilisation of additional finance towards sustainable development in developing countries.*" This definition highlights three key dimensions:

1. Strategic use: Blended finance is integrated into public policies, with an emphasis on intentionality and planning.

2. Mobilization of additional finance: It aims to attract new sources of funding rather than substitute traditional investments, acting as a complementary mechanism.

3. Focus on sustainable development in developing countries: The ultimate goal is to support sustainable development initiatives in these regions.

However, this definition is too broad to accurately capture the concept of blended finance. In particular, it overlooks a critical fourth dimension:

4. The role of concessional finance: Concessional finance involves funding provided on terms that are more favorable than market conditions, such as below-market interest rates or extended repayment periods.

To address this gap, DFI Working Group (2017, page 3) offers a more nuanced definition, describing blended finance as "*combining concessional finance from donors or third parties alongside DFIs' normal own account finance and/or commercial finance from other investors, to develop private sector markets, address the Sustainable Development Goals (SDGs), and mobilise private resources.*" This definition explicitly incorporates concessional finance, which is central to understanding the mechanisms and objectives of blended finance.

2.1.1 What makes it a blended finance deal?

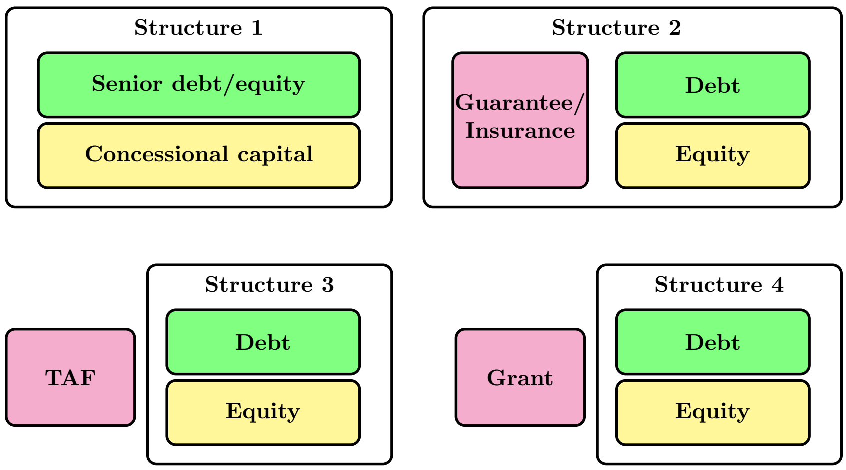
While the definition of blended finance varies from organization to organization, the primary objective — mobilizing additional private investment to achieve sustainable development — is widely accepted. As a result, blended finance is typically analyzed along two key aspects: the level of concessionality and the mix of actors involved. According to European Bank for Reconstruction and Development (2013, page 3), concessional finance refers to "*financial products, including loans, guarantees, and equity investments, provided on terms that are clearly more favourable than those explicitly available from the market.*" These concessional terms often include lower interest rates compared to commercial loans, extended repayment periods, or grace periods during which principal or interest payments are deferred. Such terms are tailored to the specific financial circumstances of the borrower, making debt service more manageable. In addition, concessional (or first-loss) capital plays a critical role in de-risking development projects by absorbing part of the financial risk. This risk-sharing mechanism is essential to incentivize private sector investment in sectors or projects that might otherwise be considered too risky. The second key aspect of blended finance is the pool of investors and stakeholders involved in the project. Typically, a blended finance initiative is led by a development finance institution (DFI) or group of DFIs. DFIs bring the necessary knowledge, expertise and technical skills to design, implement and manage the project effectively. Private investors are then brought in to provide additional financing, although they may not have the same level of technical expertise as the DFIs. A critical component of blended finance is then leverage. This refers to the ability of DFIs to leverage private sector funding relative to public sector contributions. The ratio of private to public sector funding is the key measure of the DFI's success in mobilizing resources. Because DFIs operate with limited annual budgets, achieving high leverage allows them to support a greater number of projects than they could otherwise fund on their own. In particular, the roles of the public and private sectors in blended finance differ from those in traditional public-private partnerships (PPPs). In PPPs, the private sector typically plays a lead role, often bringing specialized expertise and taking primary responsibility for project delivery. In blended finance, however, the public sector — represented by DFIs — plays a more central role in steering the project and mitigating risk to attract private investment. However, this perspective overlooks the potential role of another private sector participant. Public institutions must bear a significant portion of the risk to incentivize private capital participation. This differentiated approach to risk sharing is often achieved through structured financial products, which play a central role in blended finance mechanisms. To facilitate this structuring, DFIs may engage a structurer and/or an asset manager to design and manage the investment product, particularly in the context of structured blended finance funds. The critical importance of structuring is highlighted in the definition of blended finance provided by Convergence, the global network for blended finance:

> "*Blended finance is a* <u>*structuring approach*</u> *that allows organizations with different objectives to invest alongside each other while achieving their own objectives (whether financial return, social impact, or a blend of both). The main investment barriers for private investors addressed by blended finance are (i) high perceived and real risk and (ii) poor returns for the risk relative to comparable investments. Blended finance creates investable opportunities in developing countries which leads to more development impact. Blended finance is not an investment approach, instrument, or end solution. It is also different from impact investing. Impact investing is an investment approach, and impact investors often participate in blended finance structures.*" (Convergence, 2024, `https://www.convergence.finance/blended-finance`).

For Convergence, blended finance is fundamentally a structuring approach because it brings together investors with different utility functions. In this case, structuring is central to creating asymmetric and differentiated investment profiles that meet the different needs and objectives of public and private investors. This highlights one of the key challenges of blended finance: reconciling the different objectives and levels of expertise of public and private investors, while ensuring that they can effectively collaborate within a single investment framework.

Remark 1. *Blended finance differs from impact investing for two main reasons. First,* *impact investing is a broader concept that encompasses a wider range of investment strategies and cannot be reduced to blended finance alone. Second, private investors do not always fit the criteria for impact investing, as they may not be willing to take the same level of risk or prioritize impact as public investors. However, it's important to note that many blended finance deals do indeed qualify as impact investments because of their explicit focus on achieving positive social or environmental outcomes. Therefore, when assessing the nature of a blended finance transaction, it's important to distinguish between the specific deal or project and the motivations of the individual investors involved (Roncalli, 2025a, page 355).*

Figure 1: Common blended finance structures



Source: www.convergence.finance/blended-finance.

2.1.2 Blended finance structures

As noted above, concessionality is at the heart of the blended finance concept. Blended finance therefore falls under the category of structured finance to achieve this concessionality. According to Convergence (2024), there are four widely accepted blended finance structures:

1. Concessional capital
   Public and philanthropic institutions often offer funding at terms that are more favorable than market conditions, such as lower interest rates or longer repayment periods. This funding, known as concessional capital, is strategically positioned within the financial structure, typically as subordinated debt or equity. By reducing the overall cost of financing, concessional capital enables projects with social, environmental or development objectives to become financially viable. It also provides a risk buffer for private investors, encouraging their participation by limiting their exposure to potential losses.

2. Credit enhancement
   Public and philanthropic organizations provide credit enhancement tools, such as guarantees or risk insurance, on favorable terms. A guarantee is a commitment to cover losses or repay investments under certain circumstances, such as when a borrower defaults. This reduces the perceived risk to private investors, making investments more attractive and lowering the overall cost of financing. Similarly, risk insurance protects against certain uncertainties, such as political instability, expropriation, or breach of

contract, that might deter private sector participation. Together, these mechanisms create a more secure investment environment.

3. Technical assistance facility (TAF)
   A TAF is a grant-based mechanism that provides targeted assistance to improve the design and implementation of development projects. It provides specialized expertise in areas such as project design, institutional strengthening, capacity building, and policy development. By addressing key challenges, building necessary skills, and creating supportive infrastructure, TAF helps ensure that projects are both effective and sustainable. This form of assistance is particularly valuable in complex or underdeveloped regions where additional support is critical to success.

4. Design funding
   This type of funding is provided in the early stages of a project to support activities such as feasibility studies, strategy development, and detailed project design. By ensuring that projects are thoroughly planned and aligned with their intended goals, design funding helps mitigate risks and lays a strong foundation for implementation. Identifying and addressing potential challenges early increases the likelihood of successful outcomes and long-term impact.

These four structures are illustrated in Figure 1. By definition, concessionality is directly evident in the first and second structures, while it takes other forms in the third and fourth structures. In the latter case, it is more appropriate to refer to development assistance as a means of creating favorable conditions for private investment. In Table 1, we report the proportion of blended finance deals corresponding to these structures. The market is dominated by the first structure. It is also important to note that these structures are not mutually exclusive. For example, a blended finance deal may involve both concessional capital and technical assistance funds.

Table 1: Proportion of blended finance deals by structure (2021–2023)

Structure	1	2	3	4
Agriculture	82%	11%	19%	4%
Energy	73%	27%	26%	11%
Financial services	56%	44%	21%	2%
Health & education	90%	14%	7%	10%
Infrastructure (non-energy)	74%	23%	14%	2%

Source: Convergence (2024).

Compared to traditional finance, blended finance is distinguished by two key features: the concessionality rate and the private sector leverage ratio. Broadly speaking, we define the concessionality rate $\mathcal{CR}$ as the difference between the market reference price (*i.e.*, the cost of financing on fully commercial terms and all the favorable conditions) and the concessional price (*i.e.*, the actual cost of the project under the blended financing arrangement), expressed as a percentage of the total project cost[4]:

$$\mathcal{CR} = \frac{\text{Market Price} - \text{Concessional Price}}{\text{Total Project Cost}}$$

[4] There are various definitions and measures of concessionality (Buiter and Schankerman, 2002). Another widely used approach is the grant equivalent (or grant element) method. This method quantifies the financial value of concessional terms as an absolute monetary amount, calculated as the project amount extended minus the total discounted repayments (Scott, 2017).

Table 2 shows the concessionality rate $\mathcal{CR}$ for IFC blended finance projects. While one might assume that publishing concessionality rates is a common practice, this is not the case. Such statistics are rarely disclosed, which makes the IFC's approach highly original and noteworthy. The leverage ratio is the second key metric used to assess the blended characteristics of a blended finance deal. According to Convergence (2024), the leverage ratio is defined as the proportion of concessional capital (provided below market terms) to all commercially priced capital (market-rate capital) within a financial transaction. Commercial capital includes contributions from private, public, and philanthropic sources. Convergence (2024, page 39) reports that, on average, every dollar of concessional capital mobilizes $4.1 in commercially priced capital. In addition, transactions over $1 billion achieve a leverage ratio of 7.6, while transactions categorized under Structure 1 (concessional debt and equity) have the highest average leverage ratio of 4.3, outperforming other structures, such as guarantees and grants.

Table 2: IFC's average concessionality rate (2010–2023)

By product	Level	By facility theme	3.7%
Senior debt	3.3%	Agriculture	3.1%
Subordinated debt	3.7%	Climate	2.9%
Guarantee	5.7%	SME finance	0.8%
Equity	2.4%	Health	5.2%
Performance incentive	1.7%	Fragile economies/ Vulnerable states	6.9%
Local currency	11.7%		

Source: www.ifc.org/en/what-we-do/sector-expertise/blended-finance/how-blended-finance-works.

2.2 The market of blended finance

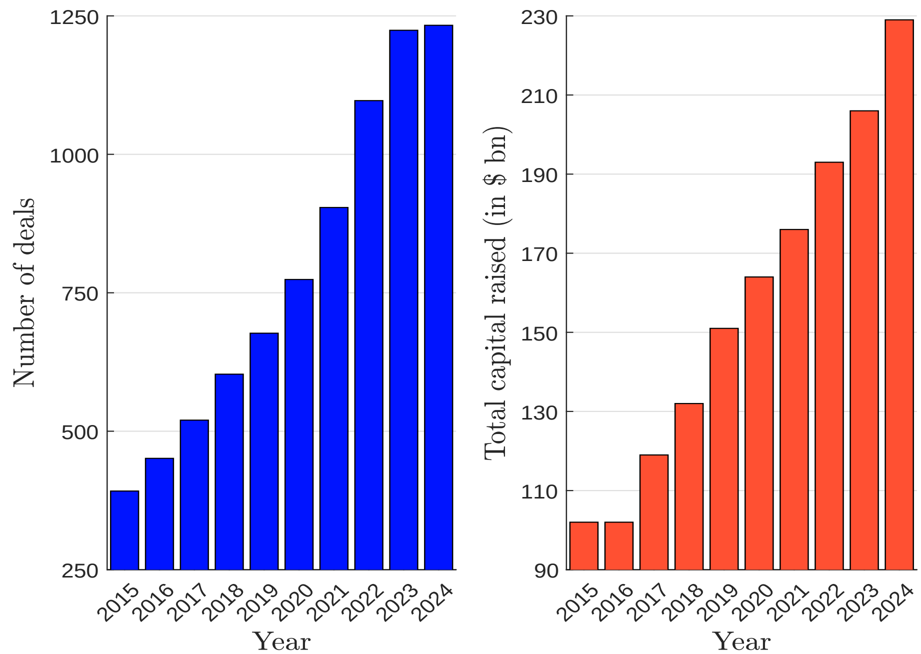
2.2.1 Global overview

Convergence (2024) reports that blended finance has mobilized $230 billion for sustainable development in developing countries by June 2024 (Figure 2). This figure represents 1 200 transactions with a median deal size of $64 million. While most transactions (70%) are concessional loans, enhancing credit and technical assistance account for 25% each. Geographically, sub-Saharan Africa dominates the market with 50% of transactions, followed by Asia (27%) and Latin America (17%). However, the market is skewed towards smaller deals, with 35% of transactions under $25 million and only 15% exceeding $250 million. In terms of sectors, energy, financial services, and agriculture each account for more than 20% of blended finance transactions. In terms of investors, Convergence (2024) notes that 65% of blended finance investors are from the private sector, but the majority have only participated in one blended finance transaction. The public and philanthropic sectors represent 19% and 16% of investors, respectively. The most active public investors are the International Finance Corporation (IFC), the Netherlands Development Finance Company (FMO), the US International Development Finance Corporation (DFC), the US Agency for International Development (USAID), and the European Investment Bank (EIB), while the top philanthropic investors include the Shell Foundation, the Bill & Melinda Gates Foundation, the Omidyar Network, Oikocredit, and the Rockefeller Foundation.

2.2.2 The case of equity/debt SBF funds

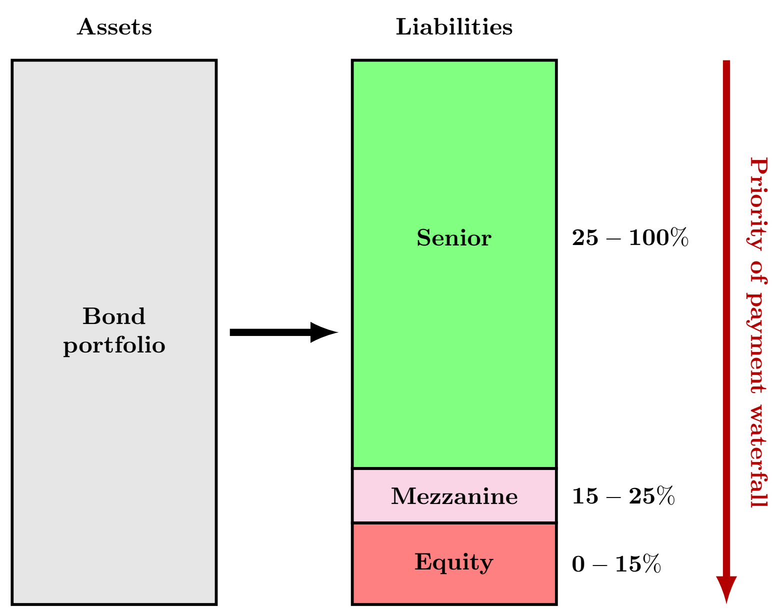
A deeper analysis of the market, the deals and the investors reveals that the large blended finance deals form a hyper-specialized market. Less than forty deals are larger than $1

Figure 2: Growth of blended finance activities



Source: Convergence (2024) & Authors' calculations.

Figure 3: An example of an equity/debt SBF fund



billion, and they were done by a small group of investors. This article focuses on these large deals for two key reasons. First, their substantial size makes them the most effective type of blended finance product for mobilizing significant amounts of private capital. Second, the majority of these deals share a similar capital structure, aligning with Structure 1 or Structure 2 in the classification proposed by Convergence (2024). More specifically, this research examines equity/debt structured funds, an example of which is shown in Figure 3. A typical blended finance vehicle is a junior-mezzanine-senior pay-through structure. In this approach, the fund invests in debt instruments typically rated BBB, BB, B or CCC, with cash flows varying from tranche to tranche. The junior or equity tranche is most exposed to default risk, while the senior tranche is relatively protected and can achieve investment grade ratings, allowing institutional investors restricted by investment grade mandates to invest in the senior notes.

In Table 3, we have listed some blended finance funds and the corresponding tranche width. We can see that the tranches vary significantly from fund to fund. First, some funds use a junior-mezzanine-senior structure, while others use a junior-senior structure. Second, the range for the senior tranche varies from 50% to 92%. How do we explain these large differences? One reason could be that the portfolio assets do not have the same risk profile. In fact, we cannot use the same structure whether the average rating of the portfolio assets is BBB or B. A second reason could be that investors' preferences are not the same. It is obvious that the preferences of the senior holder are different from the preferences of the junior holder, but it can also be assumed that senior investors investing in two blended finance funds do not have the same preferences. Another important element is the preferences and the motivation of the sponsor of the blended finance fund. There is a trade-off between the rating of the senior tranche and the size of the junior tranche. A large junior tranche is one way to obtain a better rating for the senior tranche and to attract Tier 1 investors with capital requirements to invest in the senior tranche. In this case, the leverage ratio can be lower than for a product with a small junior tranche, which implies a lower rating for the senior tranche.

2.3 Blended finance assets

According to Convergence (2024), climate blended finance transactions in 2023 are categorized into five primary financial vehicle types: bonds/notes (14%), direct private equity and debt financing to companies (21%), facilities[5] (3%), limited partnership private equity and debt funds (27%), and greenfield and brownfield infrastructure projects (36%). Depending on the strategy, a structured blended finance fund typically utilizes a combination of these financial instruments to achieve its financial return and risk mitigation objectives. Through public or private equity investments, a fund takes a stake in a company or project. With private equity, the ownership stake can be substantial, allowing investors to take an active role in the management of the business. Similarly, various debt instruments can be employed. These may be publicly traded (*e.g.*, corporate bonds) or privately issued (*e.g.*, loans or private placements). Table 4 presents the main types of bonds.

2.4 Portfolio management

A blended finance fund typically goes through two main stages in its life cycle: the investment (or ramp-up) period and the run-off (or liquidation) period. In some cases, a third phase, known as the reinvestment period, occurs between these two phases (Figure 4). During the investment period, also known as the fundraising or ramp-up period, the fund gradually

[5] A facility is an earmarked allocation of public development resources combined with private capital at the vehicle level for deployment to a specific recipient or intervention.

Table 3: Junior/mezzanine/senior structure of some blended finance funds

Fund	Junior	Mezzanine	Senior	Fund term	Asset manager & investors
Amundi Plant EGO Fund	6.25%	3.75%	90%	12	Amundi/IFC
Amundi Planet II	8%		92%	10	Amundi
Asia Climate Strategy	15%		85%	10	responsAbility Investments AG
Beyond the Grid (BTG) Solar Fund	10%	22%	68%	5	Mirova (SunFunder)
Emerging Africa Infrastructure Fund	32%		68%	Open-end	Ninety One/PIDG
Emerging Market Climate Action Fund	30%		70%	17	Allianz GI/EIB
GAIA	21%		79%	30	MUFG/FinDev Canada/CFM/Pollination
IFC & Sida MCPP Infrastructure Fund	10%		90%	25	IFC/Sida/Alliance/Axa/Prudential
Land Degradation Neutrality (LDN) Fund				15	Mirova/EIB
Mirova Gigaton Fund	15%	35%	50%	15	Mirova
SDG Loan Fund	10%		90%	25	Allianz GI/FMO
Vertelo/GCF	13%	14%	73%	10	Macquarie

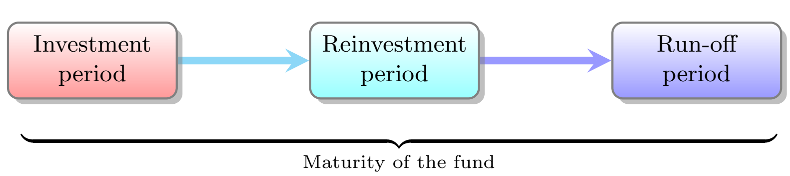
Source: Sustainable Markets Initiative (2024), EIB Knowledge Lab (2024) & Authors' research.

Table 4: List of bonds used in structured blended finance funds

Bond Type	Purpose
Development impact bonds (DIBs)	Finance development projects with outcomes-based repayment linked to social or environmental impact
Social impact bonds (SIBs)	Finance social programs with repayment based on achieving predefined social outcomes
Green bonds	Finance projects with environmental benefits, such as renewable energy or pollution control
Sustainability bonds	Finance projects addressing both environmental and social challenges
Climate bonds	Raise funds for projects that mitigate or adapt to climate change
Senior/mezzanine debt bonds	Provide capital for blended finance projects with varying risk/return profiles between senior and mezzanine debt
Project/infrastructure bonds	Finance specific infrastructure or development projects with a social or environmental focus
Impact-linked finance bonds	Link bond repayment or return to achieving specific impact targets
Concessional bonds	Provide affordable financing for development projects with a social impact, often at favorable terms

deploys capital into investments. The fund manager selects investments based on two key objectives: achieving the targeted risk-return profile and meeting social and environmental impact objectives. If suitable investment opportunities are not immediately available, the fund may temporarily allocate capital to highly liquid, low-risk assets such as government bonds. These assets can be easily sold when more permanent investments are identified. Some funds have a reinvestment period after the initial investment period. During this time, the fund may reinvest income from its initial investments — such as maturing bonds, dividends, or capital gains — in new opportunities or existing projects. Fund managers may also rebalance the portfolio to maintain alignment with the fund's objectives, risk parameters, and diversification requirements, with a particular focus on investments that generate greater social or environmental impact. The run-off period marks the final stage of the fund as it exits investments and winds down operations. The focus shifts from deploying capital to liquidating investments and collecting returns. The fund stops making new investments and distributes all realized net income and divestment proceeds, including capital gains and redemptions, to investors. This phase continues until all investments have been liquidated and the fund's obligations have been met, culminating in the closure of the fund.

Figure 4: Life cycle of a structured blended finance fund



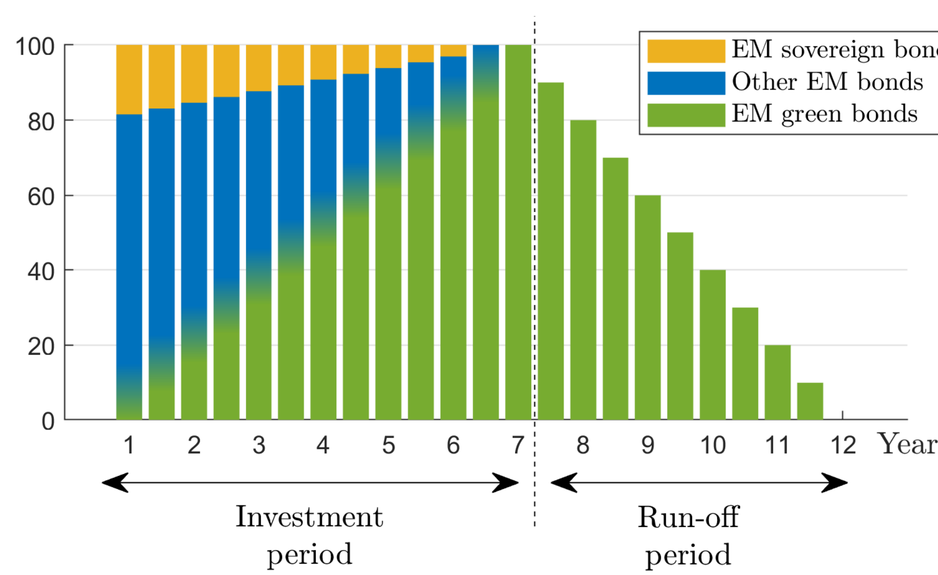
The fact that the fund needs a ramp-up period to find assets and may invest in non-market securities explains why many SBF funds are closed-end funds. This is typically the case when the maturity of the fund is long, for example 30 years, or when the asset portfolio consists of infrastructure projects. In fact, there are few open-end funds, generally invested in private loans or microcredit. In such cases, the fund can always find new projects to finance. The Emerging Africa Infrastructure Fund (EAIF) is the only open-end fund listed in Table 3. Established over two decades ago, EAIF is a private debt fund that provides long-term commercial debt across nine key infrastructure sectors in Africa.

The fund term, or the maturity of the fund, corresponds to the investment period plus the run-off period. Table 3 shows that most fund terms exceed 10 years. In fact, both the investment and run-off periods have a minimum duration of five years. Regarding the investment process, we distinguish between two approaches. The first approach is to call funds on the basis of the projects undertaken. The second approach is to replace an existing portfolio with assets that are closely aligned with the target assets. An example of this is the AP EGO fund:

> "*Overall, the AP EGO fund has projected that the origination of green bonds and the replacement rate of other assets by green bonds will take place at an annual rate of 15%, with the investment period spread out over seven years (see Figure 5). In the first few years of the fund, the bulk of the assets are expected to be plain bonds issued by emerging market banks, together with a small fraction of sovereign bonds. By year seven of the fund, the asset pool is expected to be 100% green bonds, which will be amortized over the remaining five year period of the fund.*" (Bolton *et al.*, 2020, page 33).

This approach has the advantage that full capital is available from year zero and sends a strong signal to the target market for issuance programs.

Figure 5: Life cycle of a blended finance fund with a ramp-up period



Source: Bolton *et al.* (2020, Figure 2, page 34).

3 Designing and structuring a blended finance fund

To understand the operational mechanics of a SBF fund, this section outlines its underlying engineering. We consider a diversified portfolio of sustainable bonds (such as green, blue, and forest bonds), sustainable projects (such as green infrastructure initiatives), and private assets (including loans and private debt). Each asset is assumed to generate a predictable income stream, such as coupon payments for bonds. While other risks such as currency and interest rate fluctuations are recognized, our primary focus is on credit risk modeling. We first present the methodology for assessing credit risk and then the framework for modeling cash flows within the fund. Finally, we provide simulations to illustrate the fund's pay-through mechanisms.

3.1 Credit risk modeling

3.1.1 Default time

According to Roncalli (2020, Chapter 2), there are three main approaches to modeling default times: intensity-based survival function, transition probability matrix, and structural credit risk models. This last category is not considered here because it is not appropriate for the assets of a blended finance fund.

Survival function Let $\boldsymbol{\tau}$ be the default time of an issuer. The survival function is defined as follows:

$$\mathbf{S}(t) = \Pr\{\boldsymbol{\tau} > t\} = 1 - \mathbf{F}(t)$$

where $\mathbf{F}$ is the cumulative distribution function of $\boldsymbol{\tau}$. We deduce that the probability density function is $f(t) = -\partial_t \mathbf{S}(t)$. The hazard function (or the default intensity) $\lambda(t)$ represents the instantaneous default rate, given that the default has not occurred before time t. It is defined as $\lambda(t) = \dfrac{f(t)}{\mathbf{S}(t)} = -\dfrac{\partial_t \mathbf{S}(t)}{\mathbf{S}(t)} = -\dfrac{\partial \ln \mathbf{S}(t)}{\partial t}$, which implies:

$$\mathbf{S}(t) = \exp\left(-\int_0^t \lambda(u)\,\mathrm{d}u\right) \tag{1}$$

From Equation (1), we can derive the probability of default.

Table 5: Cumulative probability of default (in %)

t	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.01	0.01	0.03	0.05	0.08	0.10	0.12	0.12	0.13
AA	0.02	0.06	0.12	0.20	0.30	0.40	0.50	0.55	0.60	0.70
A	0.05	0.15	0.30	0.50	0.70	0.95	1.20	1.45	1.70	1.90
BBB	0.15	0.40	0.75	1.10	1.45	1.80	2.20	2.60	3.00	3.40
BB	0.90	2.50	4.40	6.30	8.15	9.80	11.35	12.80	14.25	15.70
B	3.20	7.70	12.35	16.65	20.55	24.05	27.15	29.95	32.45	34.70
CCC	9.10	16.20	22.35	27.70	32.40	36.40	39.80	42.90	45.80	48.25

Rating agencies provide the average cumulative defaults per rating on an annual basis. Table 5 gives an example of the cumulative probability of default for global corporate bonds over a 40-year history. For example, the average cumulative default over 5 years for a BB-rated company is 8.15%. Assuming a constant intensity $\lambda(t) = \lambda$, we get $\mathbf{S}(t) = \exp(-\lambda t)$

and deduce the implied constant default intensity[6] for each maturity t. Results are given in Table 6. It is clear that the default intensity is not constant over time. In fact, it is an increasing function for high credit ratings (above BB) and a decreasing function for low credit ratings (below BB).

Table 6: Implied constant default intensity (in %)

t	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.01
AA	0.02	0.03	0.04	0.05	0.06	0.07	0.07	0.07	0.07	0.07
A	0.05	0.08	0.10	0.13	0.14	0.16	0.17	0.18	0.19	0.19
BBB	0.15	0.20	0.25	0.28	0.29	0.30	0.32	0.33	0.34	0.35
BB	0.90	1.27	1.50	1.63	1.70	1.72	1.72	1.71	1.71	1.71
B	3.25	4.01	4.39	4.55	4.60	4.58	4.53	4.45	4.36	4.26
CCC	9.54	8.84	8.43	8.11	7.83	7.54	7.25	7.00	6.81	6.59

Another approach is to assume that the default intensity is piecewise linear, which is a common practice in the market (Roncalli, 2020, page 202). In this case, we assume that $\lambda(u) = \lambda_t$ for $t \in [t-1, t)$. We have:

$$\mathbf{S}(t) = \exp\left(-\int_0^{t-1} \lambda(u)\, \mathrm{d}u - \int_{t-1}^{t} \lambda_t\, \mathrm{d}u\right) = \mathbf{S}(t-1)\, e^{-\lambda_t}$$

It follows that an estimate of λ_t is given by:

$$\hat{\lambda}_t = \ln \frac{\mathbf{S}(t-1)}{\mathbf{S}(t)}$$

Applying this approach to Table 5, we obtain the results shown in Table 7.

Table 7: Piecewise linear default intensity (in %)

t	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.01	0.00	0.02	0.02	0.03	0.02	0.02	0.00	0.01
AA	0.02	0.04	0.06	0.08	0.10	0.10	0.10	0.05	0.05	0.10
A	0.05	0.10	0.15	0.20	0.20	0.25	0.25	0.25	0.25	0.20
BBB	0.15	0.25	0.35	0.35	0.35	0.36	0.41	0.41	0.41	0.41
BB	0.90	1.63	1.97	2.01	1.99	1.81	1.73	1.65	1.68	1.71
B	3.25	4.76	5.17	5.03	4.79	4.51	4.17	3.92	3.63	3.39
CCC	9.54	8.13	7.62	7.14	6.72	6.10	5.49	5.29	5.21	4.63

Transition probability matrix When dealing with risk classes and modeling credit rating migration, it is useful to model a transition probability matrix. We consider a time-homogeneous Markov chain $\boldsymbol{R}$, with a transition matrix $P = \left(p_{i,j}\right)$. We define $\mathcal{S} = \{1, 2, \ldots, K\}$ as the state space of the ratings and $p_{i,j}$ as the probability that the issuer migrates from rating i to rating j. We generally assume that K is the absorbing state (or the default state), meaning that any issuer reaching this state remains in this state. Let

[6] We have $\hat{\lambda} = -\ln\left(1 - \mathbf{F}(t)\right)/t$ where $\mathbf{F}(t)$ is the cumulative probability of default.

$\boldsymbol{R}(t)$ denote the state at time t. We define $p(s,i;t,j)$ as the probability that the issuer reaches the state j at time t, given that it was in the state i at time s. We have:

$$p(s,i;t,j) = \Pr\left\{\boldsymbol{R}(t) = j \mid \boldsymbol{R}(s) = i\right\} = p_{i,j}^{(t-s)}$$

Because of the Markov property, this probability only depends on the duration between s and t . It is given by the Chapman-Kolmogorov forward equation $P^{(n+m)} = P^{(n)}P^{(m)}$ with the convention $P^{(0)} = I_K$. We deduce that:

$$p(t,i;t+n,j) = p_{i,j}^{(n)} = \mathbf{e}_i^\top P^n \mathbf{e}_j$$

The survival function $\mathbf{S}_i(t)$ of an issuer whose initial rating is state i is given by:

$$\mathbf{S}_i(t) = 1 - \Pr\left\{\boldsymbol{R}(t) = K \mid \boldsymbol{R}(0) = i\right\} = 1 - \mathbf{e}_i^\top P^t \mathbf{e}_K$$

In the piecewise exponential model, the survival function has the following expression $\mathbf{S}(t) = \mathbf{S}(t-1)e^{-\lambda_t}$, which implies that:

$$\hat{\lambda}_{i,t} = \ln\left(\frac{1 - \mathbf{e}_i^\top P^{t-1}\mathbf{e}_K}{1 - \mathbf{e}_i^\top P^t \mathbf{e}_K}\right) \tag{2}$$

The previous analysis can be extended to continuous time $t \in \mathbb{R}_+$. In this case, the transition probability matrix satisfies the following relationship:

$$P(t) = \exp(t\Lambda) \tag{3}$$

where Λ is the Markov generator matrix, and $\exp(A)$ is the matrix exponential of A. It follows that:

$$\mathbf{S}_i(t) = 1 - \mathbf{e}_i^\top \exp(t\Lambda)\,\mathbf{e}_K \tag{4}$$

and[7]:

$$\lambda_i(t) = \frac{\mathbf{e}_i^\top \Lambda \exp(t\Lambda)\,\mathbf{e}_K}{1 - \mathbf{e}_i^\top \exp(t\Lambda)\,\mathbf{e}_K} \tag{5}$$

In practice, computing $\mathbf{S}_i(t)$ requires estimating Λ. The natural estimator is $\hat{\Lambda} = \ln\big(P(1)\big)$ where $P(1)$ is the one-year transition probability matrix calibrated by rating agencies[8].

Table 8: One-year credit migration matrix $P(1)$ (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	91.17	8.25	0.45	0.07	0.03	0.01	0.01	0.01
AA	0.73	89.79	8.91	0.44	0.06	0.03	0.02	0.02
A	0.04	2.50	91.50	5.31	0.45	0.11	0.05	0.04
BBB	0.02	0.12	3.86	91.53	3.48	0.67	0.17	0.15
BB	0.01	0.02	0.41	6.51	83.47	7.73	0.95	0.90
B	0.01	0.02	0.13	0.45	5.33	82.56	8.13	3.37
CCC	0.01	0.02	0.03	0.07	0.29	6.45	83.31	9.82
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

[7] We use the notation $\lambda_i(t)$ instead of $\lambda_{i,t}$ because $\lambda_i(t)$ is now a function rather than a scalar parameter.

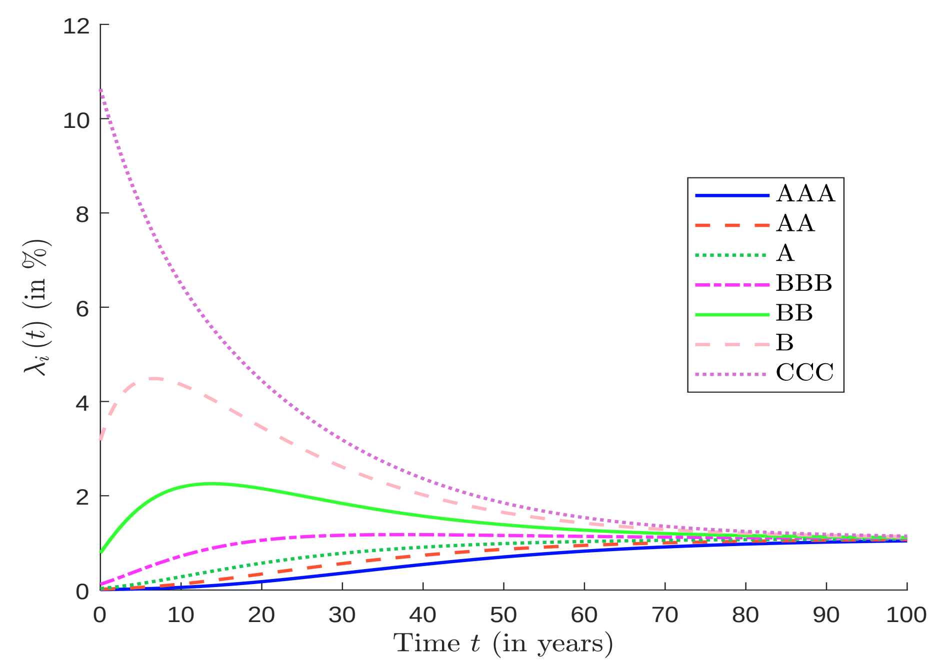
[8] We also consider the estimators defined by Israel *et al.* (2001), which are more robust and ensure that $\hat{\Lambda}$ is a valid Markov generator.

Table 9: Markov generator Λ (in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	−9.281	9.122	0.046	0.057	0.030	0.008	0.010	0.009
AA	0.806	−10.943	9.837	0.198	0.041	0.026	0.019	0.017
A	0.032	2.758	−9.141	5.793	0.395	0.086	0.046	0.032
BBB	0.021	0.073	4.213	−9.117	3.960	0.582	0.147	0.121
BB	0.010	0.013	0.304	7.436	−18.520	9.289	0.681	0.786
B	0.011	0.019	0.129	0.276	6.428	−19.849	9.811	3.176
CCC	0.011	0.021	0.026	0.059	0.098	7.796	−18.642	10.630
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Consider the one-year credit migration matrix $P(t)$ given in Table 8. We estimate the Markov generator $\Lambda(t)$ and obtain the results shown in Table 9. Finally, we compute the default intensity function $\lambda_i(t)$ for the different ratings using Equation (5). In Figure 6, we observe that the default intensity increases over time t for high ratings (AAA, AA, A and BBB). For medium ratings (BB and B), the default intensity first increases and then decreases, while for the lowest rating it decreases (CCC). This behavior is expected, as conditional credit risk tends to increase for good ratings and decrease for poor ratings. In fact, credit risk increases over time for AAA-rated issuers, while it decreases for CCC-rated issuers because they either default quickly or survive long enough to improve their credit situation. This implies that the long-term default intensity $\lambda_i(\infty)$ does not depend on the initial rating i and is equal to the one-year default frequency. In this example, we obtain $\lambda_i(\infty) = 1.09\%$.

Figure 6: Default intensity function $\lambda_i(t)$



3.1.2 Default correlations

The joint distribution of default times has a significant impact on the risk profile of different tranches within a SBF fund. In particular, this impact varies between junior and senior tranches. As a result, accurate modeling of the dependence of default times is critical when designing a blended finance product. As Roncalli (2020, page 227) points out, the concept of default correlations is not straightforward, as it can refer to multiple definitions that capture different aspects of dependence. For example, Roncalli (2020) distinguishes between *canonical correlations*, *default time correlations*, *discrete default correlations*, *asset correlations*, and *equity correlations*. Another key challenge is to estimate the dependence between defaults. These issues are discussed in detail in the following sections.

Discrete default correlations Let D_i be the random variable representing the default event of issuer i. Since we have $D_i = \mathbb{1}\{\boldsymbol{\tau} \leq t\}$ where t is the time horizon, it follows that D_i is a Bernoulli random variable with parameter p_i. This implies that the default indicator function D_i is related to the survival function $\mathbf{S}_i(t)$ as follows:

$$p_i = \Pr\{D_i = 1\} = \Pr\left\{\mathbb{1}\{\boldsymbol{\tau} \leq t\}\right\} = \mathbb{E}\left[\mathbb{1}\{\boldsymbol{\tau} \leq t\}\right] = 1 - \mathbf{S}_i(t)$$

The discrete default correlation between two issuers i and j is defined as:

$$\rho_{i,j} = \frac{\operatorname{cov}\left(D_i, D_j\right)}{\sigma(D_i)\,\sigma\left(D_j\right)} = \frac{p_{i,j} - p_i p_j}{\sqrt{p_i(1-p_i)\,p_j\left(1-p_j\right)}} \tag{6}$$

where $p_{i,j}$ is the joint default probability. From Equation (6), we deduce that:

$$p_{i,j} = p_i p_j + \rho_{i,j}\sqrt{p_i(1-p_i)\,p_j\left(1-p_j\right)}$$

Note that the joint default probability is a linear function of $\rho_{i,j}$. Therefore, it reaches its maximum when $\rho_{i,j}$ is equal to 100%. More surprisingly, the joint default probability is not a monotonic function of the individual probabilities, as shown in Figure 7. $p_{i,j}$ reaches its maximum when $p_i < 1$, except in the independent case ($\rho_{i,j} = 0$).

Copula models A convenient way to capture the dependence between defaults and to model the joint default probability is through copula models. Specifically, we can show that the multivariate survival function has a canonical representation:

$$\mathbf{S}(t_1, \ldots, t_n) = \mathbf{C}\left(\mathbf{S}_1(t_1), \ldots, \mathbf{S}_n(t_n)\right) \tag{7}$$

where $\mathbf{S}_i(t_i) = \Pr\left\{\mathbb{1}\{\boldsymbol{\tau}_i \leq t_i\}\right\}$ is the marginal survival function of issuer i, and $\mathbf{C}(u_1, \ldots, u_n)$ is the survival copula function. In practice, professionals commonly use two copula families to model joint defaults: the Gaussian copula and the Student t copula. Both copula functions require the specification of a parameter matrix corresponding to the correlation matrix $\mathbb{C}$ of standardized Gaussian random variables — in the bivariate case, we denote the copula correlation parameter by $\rho_{\mathbf{C}}$. However, this copula correlation is not the same as the discrete default correlation. They are related by the following formula[9]:

$$\rho_{i,j} = \frac{\mathbf{C}\left(1-p_i, 1-p_j; \rho_{\mathbf{C}}\right) + p_i + p_j - \left(1 + p_i p_j\right)}{\sqrt{p_i(1-p_i)\,p_j\left(1-p_j\right)}}$$

[9]We have $p_{i,j} = \Pr\left\{D_i = 1, D_j = 1\right\} = \Pr\left\{D_j = 1\right\} - \Pr\left\{D_i = 0, D_j = 1\right\} = \Pr\left\{D_j = 1\right\} + \Pr\{D_i = 1\} - 1 + \Pr\left\{D_i = 0, D_j = 0\right\} = p_i + p_j + \mathbf{C}\left(1-p_i, 1-p_j; \rho_{\mathbf{C}}\right) - 1$.

Figure 7: Joint default probability for different values of $\rho_{i,j}$ ($p_i = 10\%$)

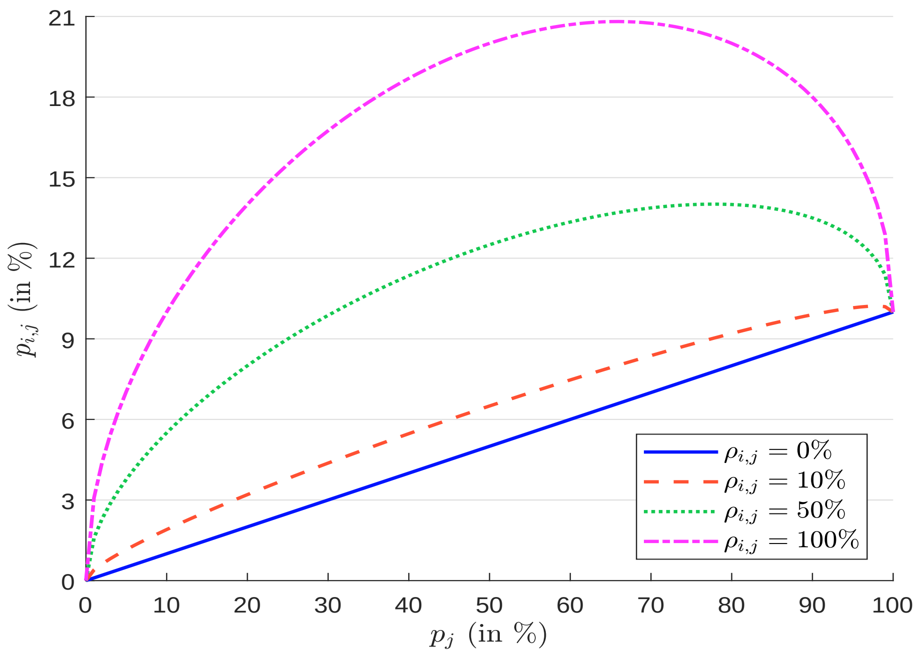


Figure 8: Joint default probability for different values of $\rho_{\mathbf{C}}$ ($p_i = 10\%$, $p_j = 20\%$)

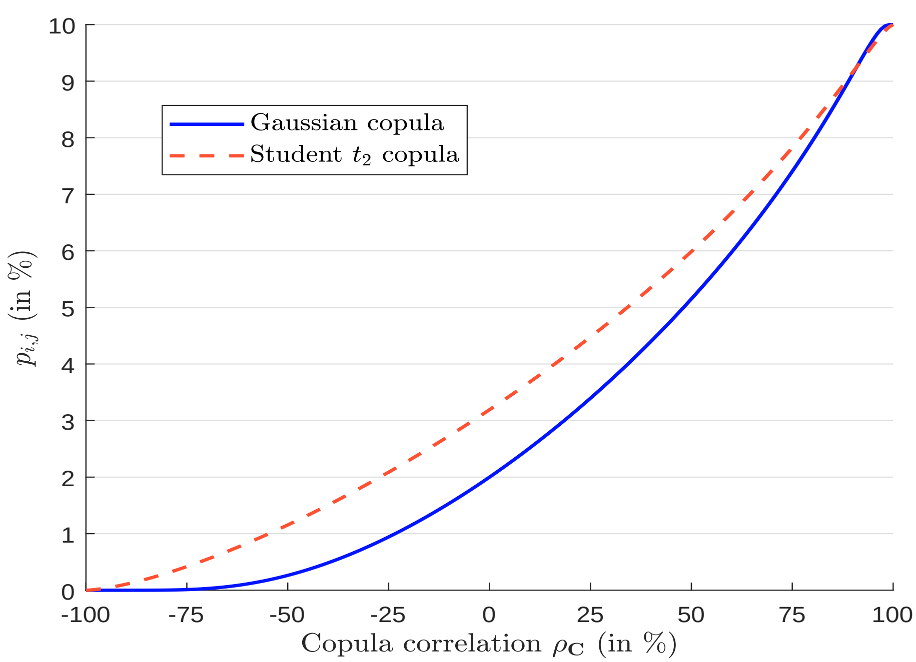


Figure 8 illustrates the relationship[10] between the copula parameter $\rho_{\mathbf{C}}$ and the joint default probability $p_{i,j}$. In particular, the copula correlation has a significant impact on tranche pricing. Holders of the senior tranche prefer a low joint default probability $p_{i,j}$, as this reduces the likelihood that they will have to cover losses through the protection leg. The underlying mechanism is as follows. When the copula correlation is high, defaults tend to occur in clusters rather than being spread out over time. As a result, if a few assets in the portfolio default, many others are likely to default as well. This significantly increases the risk that losses will be severe enough to to breach the protection provided by the junior and mezzanine tranches, ultimately affecting the senior tranche. Therefore, the choice of the copula correlation matrix $\rho_{\mathbf{C}}$ is a critical decision when structuring a blended finance fund, as it directly influences the risk profile of the fund and the stability of the senior tranche.

Factor models Another approach to modeling default correlations is through factor models. Consider the one-factor model of Merton (1974) and Vasicek (1991). Let Z_i denote the normalized asset value of entity i. In the Merton model, default occurs when Z_i falls below a given barrier B_i:

$$D_i = 1 \Leftrightarrow Z_i < B_i$$

Assuming that Z_i is Gaussian, we deduce that:

$$p_i = \Pr\{D_i = 1\} = \Pr\{Z_i < B_i\} = \Phi(B_i)$$

The value of the barrier B_i is then equal to $\Phi^{-1}(p_i)$. We assume that the asset value Z_i depends on a common risk factor X and an idiosyncratic risk factor ε_i, such that:

$$Z_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$$

where X and ε_i are two independent standard normal random variables. This implies that the correlation between asset values is given by $\rho\left(Z_i, Z_j\right) = \rho$, meaning that ρ represents the constant asset correlation. It follows that the joint default probability is:

$$p_{1,2} = \Pr\{D_1 = 1, D_2 = 1\} = \Pr\{Z_1 \le B_1, Z_2 \le B_2\} = \Phi_2(B_1, B_2; \rho)$$

because $Z = (Z_1, \ldots, Z_n) \sim \mathcal{N}\left(0_n, \mathbb{C}_n(\rho)\right)$. Given that $B_i = \Phi^{-1}(p_i)$, we deduce that the copula associated with the default indicator functions is a Gaussian copula, whose parameters are the constant correlation matrix $\mathbb{C}_n(\rho)$:

$$p_{1,2} = \Phi\left(\Phi^{-1}(p_1), \Phi^{-1}(p_2); \rho\right) = \mathbf{C}(p_1, p_2; \rho)$$

Let us now consider the dependence between survival times:

$$\begin{array}{rcl}\mathbf{S}(t_1, t_2) & = & \Pr\{\boldsymbol{\tau}_1 > t_1, \boldsymbol{\tau}_2 > t_2\} \\ & = & \Pr\left\{Z_1 > \Phi^{-1}(p_1), Z_2 > \Phi^{-1}(p_2)\right\} \\ & = & \mathbf{C}(1-p_1, 1-p_2; \rho) \\ & = & \mathbf{C}\left(\mathbf{S}_1(t_1), \mathbf{S}_2(t_2); \rho\right)\end{array}$$

Thus, the dependence between the default times is again governed by the Gaussian copula with the parameter matrix $\mathbb{C}_n(\rho)$.

[10] We observe that the relationship varies depending on the copula family. In general, the joint default probability obtained with a Student t copula is higher than that obtained with a Gaussian copula because the Student t copula captures stronger dependence, especially in the tails.

The extension to the multi-factor model is straightforward. The default of issuer i occurs when Z_i falls below the threshold B_i: $D_i = 1 \Leftrightarrow Z_i < B_i = \Phi^{-1}(p_i)$. The asset value Z_i depends on a set of m common risk factors X_j and an idiosyncratic risk factor ε_i:

$$Z_i = \sum_{j=1}^{m} A_{i,j} X_j + \sqrt{1 - \sum_{j=1}^{m} A_{i,j}^2 \varepsilon_i}$$

Demey *et al.* (2004) consider a restricted version of this model, assuming that the default of an issuer is impacted by a common economic risk factor $X \sim \mathcal{N}(0,1)$ and a class-specific risk factor $X_j \sim \mathcal{N}(0,1)$. If issuer i belongs to the j^{th} class ($i \in \mathcal{C}_j$), the asset value follows:

$$Z_i = \sqrt{\rho} X + \sqrt{\rho_j - \rho} X_j + \sqrt{1 - \rho_j} \varepsilon_i$$

where ρ is the inter-class correlation, and ρ_j is the intra-class correlation for class $\mathcal{C}_j$. If issuers are ranked by class, we can show that the copula between default times is Gaussian with the following parameter matrix:

$$\mathbb{C} = \begin{pmatrix} \mathbb{C}_{n_1}(\rho_1) & \rho \mathbf{1}_{n_1,n_2} & \cdots & \rho \mathbf{1}_{n_1,n_m} \\ \rho \mathbf{1}_{n_2,n_1} & \mathbb{C}_{n_2}(\rho_2) & & \vdots \\ \vdots & & \ddots & \\ \rho \mathbf{1}_{n_m,n_1} & \cdots & & \mathbb{C}_{n_m}(\rho_m) \end{pmatrix}$$

where n_j is the number of issuers in class $\mathcal{C}_j$. Demey *et al.* (2004) apply this model by assuming that the risk classes are sectors. In the case of blended finance, another approach is to assume that the risk classes are countries or regions.

Estimation While the estimation of default probabilities is relatively straightforward, the estimation of default correlations is a challenging task due to the limited availability of data and the large number of parameters that need be calibrated. To address this issue, we follow Demey *et al.* (2004) and use the Merton-Vasicek multi-factor model, where issuers are classified into homogeneous risk classes:

$$Z_i = \sqrt{\rho} X + \sqrt{\rho_j - \rho} X_j + \sqrt{1 - \rho_j} \varepsilon_i$$

Demey *et al.* (2004) show that the asymptotic likelihood for observation t is given by:

$$L_t(\theta) = \int_0^1 \mathrm{d}x \prod_{j=1}^{m} \phi\left(f_j(x)\right) \frac{\sqrt{1-\rho_j}}{\sqrt{\rho_j - \rho}} \frac{1}{\phi\left(\Phi^{-1}\left(\mu_{j,t}\right)\right)} \tag{8}$$

where:

$$f_j(x) = \frac{B_j - \sqrt{1-\rho_j}\,\Phi^{-1}\left(\mu_{j,t}\right) - \sqrt{\rho}\,\Phi^{-1}(x)}{\sqrt{\rho_j - \rho}}$$

and $\mu_{j,t}$ is the observed default rate in class $\mathcal{C}_j$ at year t. In Equation (8), the default barriers can either be assumed to be known — $B_j = \Phi^{-1}(p_j)$ where $p_{j,t}$ is the default probability of class $\mathcal{C}_j$ — or they must be estimated. In the first case, the parameter vector to be estimated is $\theta = (\rho, \rho_1, \ldots, \rho_m)$, which contains the $m+1$ inter- and intra-class correlations. In the second case, $\theta = (\rho, \rho_1, \ldots, \rho_m, B_1, \ldots, B_m)$ additionally includes the default barriers. Finally, θ is estimated by maximizing the log-likelihood function:

$$\hat{\theta} = \arg\max \sum_t \ln L_t(\theta)$$

Several rating agencies, including Moody's, publish annual default data with regional breakdowns. While Moody's discloses the total number of issuers, it doesn't provide regional distributions. To calculate region-specific issuer counts and default rates, we assumed that the issuer distribution mirrors that of the Bloomberg Global Aggregate and Global High Yield Bond indices. As shown in Table 10, the intra-region correlation estimates across the five regions range from 8.12% to 16.10%, while the inter-region correlation is 4.24%.

Table 10: Default correlation estimate (1986 – 2023)

Region	Weight (in %)	Correlation (in %)
Africa and Middle East	3.34	8.12
Asia Pacific	13.94	13.27
Europe	31.17	16.10
Latin America	3.73	11.95
North America	47.83	9.49
Inter-region		4.24

Impact of the default correlation on the portfolio default rate As previously explained, investors are very sensitive to assumptions about default correlations. To illustrate this, consider a homogeneous portfolio of n issuers, each with the same probability of default p. The default rate of the portfolio is defined as:

$$\mu_n(p;\rho_D) = \frac{1}{n}\sum_{i=1}^{n} D_i$$

where $\mu_n(p;\rho_D)$ is the mean of correlated Bernoulli random variables. It follows that:

$$\mathbb{E}\left[\mu_n(p;\rho_D)\right] = p$$

and its variance is given by[11]:

$$\operatorname{var}\left(\mu_n(p;\rho_D)\right) = p(1-p)\left(\frac{1}{n} + \frac{(n-1)}{n}\rho_D\right)$$

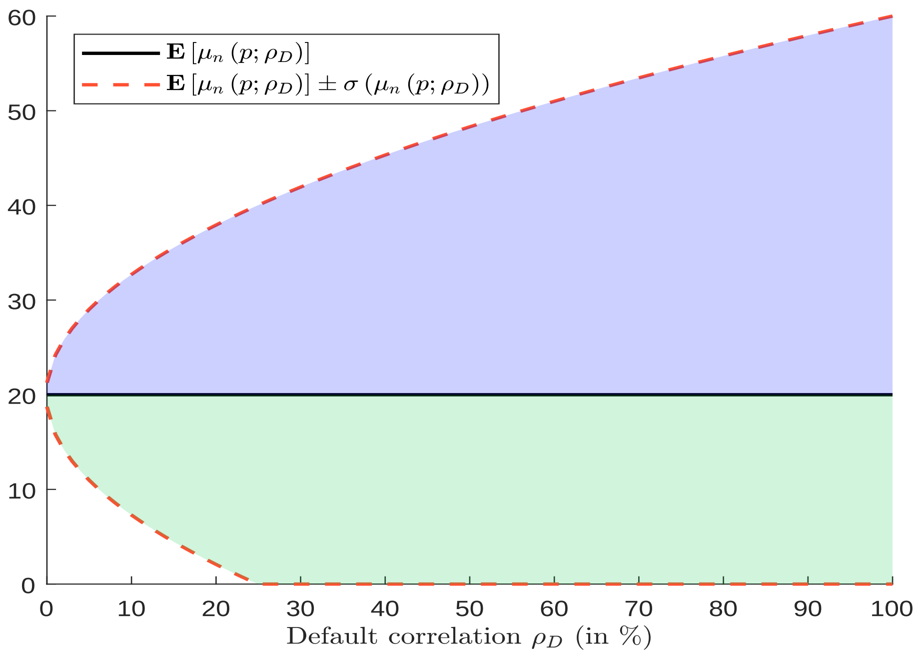
where ρ_D is the correlation between D_i and D_j. While the expected default rate of the portfolio remains independent of the default correlation, the variance is significantly affected by it. In particular, higher default correlation leads to higher variance, which increases the

[11] We have:

$$\begin{aligned}
\operatorname{var}\left(\mu_n(p;\rho)\right) &= \mathbb{E}\left[\left(\frac{1}{n}\sum_{i=1}^{n}(D_i - p)\right)^2\right] \\
&= \frac{1}{n^2}\left(\sum_{i=1}^{n}\mathbb{E}\left[(D_i-p)^2\right] + \sum_{i\neq j}\mathbb{E}\left[(D_i-p)(D_j-p)\right]\right) \\
&= \frac{1}{n^2}\left(np(1-p) + n(n-1)\rho_D p(1-p)\right) \\
&= \frac{p(1-p)}{n} + \frac{(n-1)}{n}\rho_D p(1-p)
\end{aligned}$$

risk of the senior tranche. However, these results can be misleading because the volatility of the portfolio default rate depends on the square root function of the default correlation (Figure 9).

Figure 9: Mean and standard deviation of the portfolio default rate (in %) — $n = 1\,000$ & $p = 20\%$



This suggests a monotonous convex relationship between default correlation and the credit risk of the senior tranche[12]. In reality, this is not the case. The nature of the credit risk in the senior tranche is fundamentally different and does not vary continuously with the default correlation. In fact, when the default correlation is equal to 0%, $\mu_n(p, 0)$ follows a scaled binomial distribution $\mathcal{B}(n, p)$ that converges to the Gaussian distribution as $n \to \infty$:

$$\lim_{n\to\infty} \mu_n(p, 0) \sim \mathcal{N}\left(p, \frac{p(1-p)}{n}\right)$$

Conversely, when the default correlation is 100%, $\mu_n(p, 1)$ follows a Bernoulli distribution $\mathcal{B}(p)$:

$$\lim_{n\to\infty} \mu_n(p, 1) \sim \mathcal{B}(p)$$

This shows that the distribution of $\mu_n(p; \rho_D)$ changes significantly with the default correlation, highlighting its crucial role in structuring a blended finance fund. Figure 10 shows the evolution of the probability distribution of the default rate as the default correlation changes. When defaults are independent ($\rho_D = 0$), the portfolio default rate follows a bell-shaped distribution similar to a Gaussian curve. As default correlation increases, the distribution of $\mu_n(p; \rho_D)$ becomes skewed to the right and takes on a log-normal shape. At a correlation level of 50%, the distribution has high kurtosis, giving it a hyperbolic shape. As we approach the limit case $\rho_D = 1$, the distribution converges to the Bernoulli shape, reflecting an all-or-nothing outcome — either no defaults or a complete default of the entire portfolio.

[12] That is, the risk of capital loss associated with the senior tranche.

Figure 10: Probability distribution of the portfolio default rate (in %) — $n = 1\,000$ & $p = 20\%$

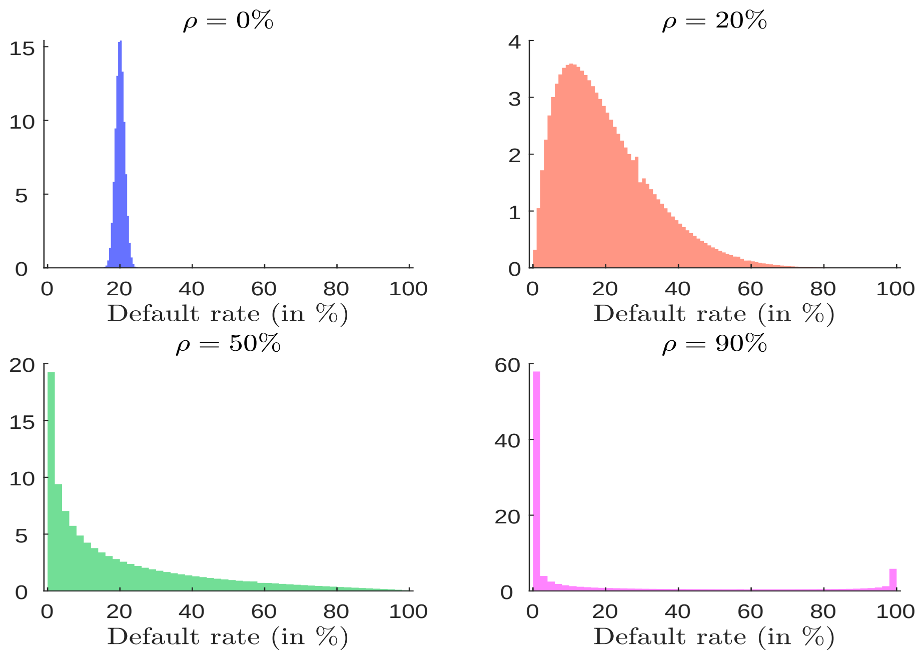
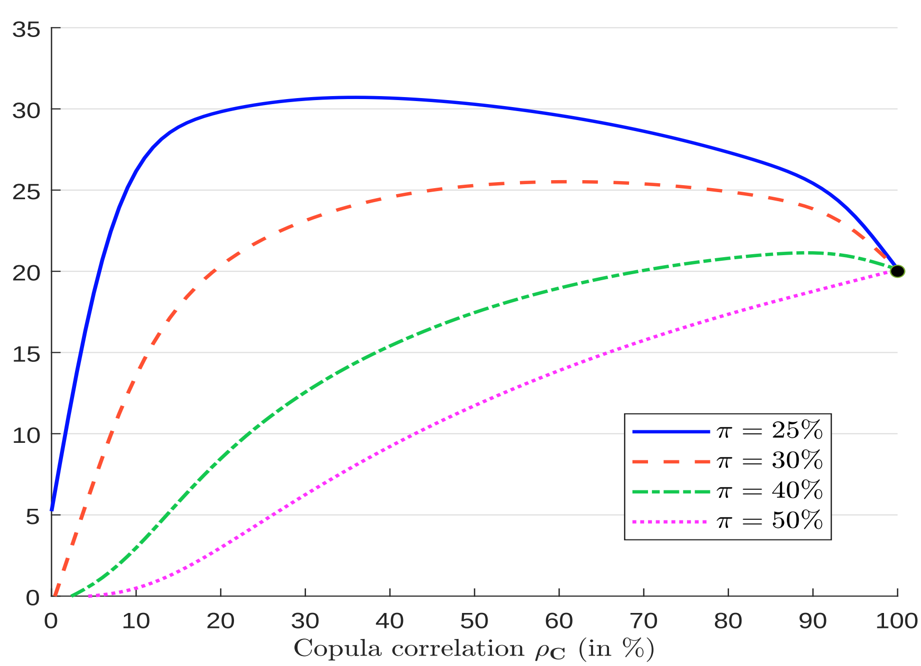


Figure 11: Threshold probability $\Pr\left\{\mu_n\left(p, \rho_{\mathbf{C}}\right) \geq \pi\right\}$



Remark 2. *The non-monotonicity property of the credit risk of the senior tranche is illustrated in Figure 11, where we calculate the threshold probability* $\Pr\left\{\mu_n\left(p,\rho_{\mathbf{C}}\right)\geq\pi\right\}$. *Depending on the value of* π*, the probability can either increase and then decrease or remain increasing.*

3.1.3 Recovery rate

When an issuer defaults, creditors typically do not lose their entire investment, but may recover a portion of the principal and/or accrued interest. Recovery can take various forms, including cash, new financial instruments (such as debt or equity), assets, proceeds from the sale of assets, or a combination of these.

The recovery rate represents the percentage of principal that investors can expect to recover in the event of default. It varies significantly depending on factors such as the seniority of the debt, the existence of collateral and the financial condition of the issuer at the time of default. In general, secured or senior bonds have higher recovery rates than subordinated or unsecured bonds. For example, senior bondholders receive priority in bankruptcy proceedings, while subordinated bondholders or equity holders may receive little or nothing. If a bond is secured by assets (such as real estate or machinery), the recovery rate tends to be higher because creditors can seize the collateral. In addition, companies with more assets relative to liabilities tend to have higher recovery rates.

Table 11: Recovery rate in % by instrument type in the US bond market (1987–2023)

Bonds	Mean	Median	Std-dev.	$\frac{\text{std-dev.}}{\text{mean}}$
Senior secured	58.1	58.7	33.2	57.2
Senior unsecured	44.8	41.9	32.8	73.2
Senior subordinated	29.9	18.1	32.1	107.5
Other subordinated	22.8	9.2	29.6	129.9
All bonds	40.4	31.8	34.1	84.6

Source: S&P Global (2023, Table 2).

Rating agencies regularly publish reports on recovery rates as part of default studies. Based on S&P's 2023 U.S. recovery study (Table 11), several general trends can be observed in the U.S. market:

- The long-term average recovery rate is approximately 40%.
- Recovery rates increase with the seniority of the debt.
- Lower seniority instruments tend to have a wider range of recoveries than higher seniority instruments.
- The standard deviation scaled by the mean increases for instruments lower in the capital structure.

Market practice is to assume constant recovery rates[13]. This assumption is particularly valid when the portfolio is infinitely fine-grained — that is, when credit risk is well diversified and there is no concentration in any single issuer. Otherwise, the recovery rate must be

[13] The recovery rate of 40% is commonly used for CDS, bonds, and CDOs, provided the assets are not in a special situation.

considered stochastic. In such cases, it is generally assumed to follow a beta distribution: $\boldsymbol{\mathcal{R}} \sim \mathcal{B}(\alpha, \beta)$. Its density function is given by:

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathfrak{B}(\alpha, \beta)}$$

where $\mathfrak{B}(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,\mathrm{d}t = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ is the beta function. The mean and the variance are:

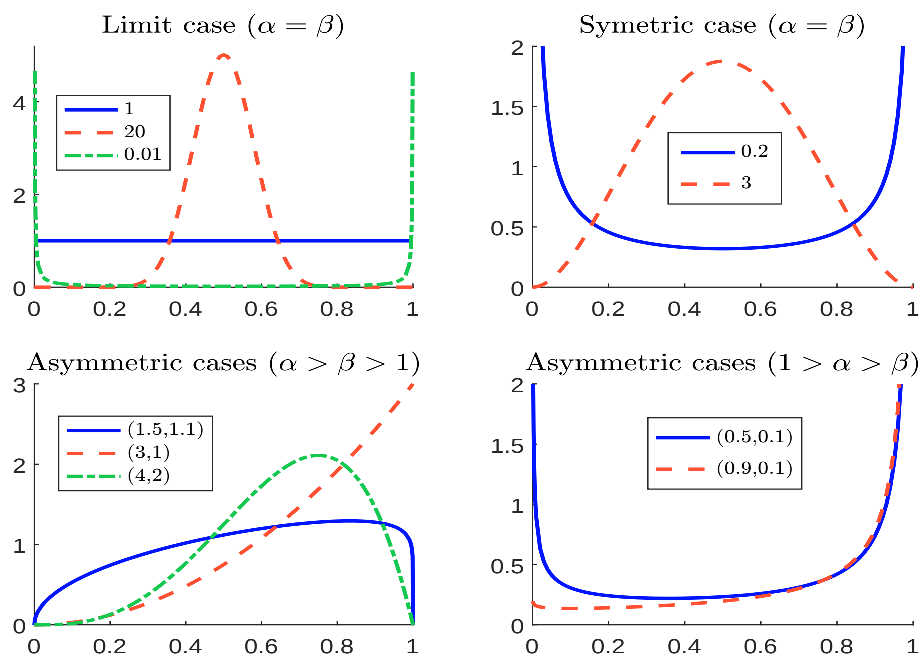
$$\mathbb{E}[\boldsymbol{\mathcal{R}}] = \frac{\alpha}{\alpha+\beta}$$

and:

$$\operatorname{var}(\boldsymbol{\mathcal{R}}) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

When α and β are greater than 1, the distribution has a mode at $x_{\mathrm{mode}} = (\alpha-1)/(\alpha+\beta-2)$.

Figure 12: Probability density function of the beta distribution $\mathcal{B}(\alpha, \beta)$



This probability distribution is very flexible and allows to obtain different shapes, which are shown in Figure 12:

- if $\alpha = 1$ and $\beta = 1$, we get the uniform distribution. If $\alpha \to \infty$ and $\beta \to \infty$, we get the Dirac distribution at the point $x = 0.5$. If one parameter goes to zero, we get a Bernoulli distribution.
- if $\alpha = \beta$, the distribution is symmetric around $x = 0.5$. We have a bell curve when the two parameters α and β are greater than 1, and a U-shape curve when the two parameters α and β are less than 1.
- if $\alpha > \beta$, the skewness is negative and the distribution is left-skewed. If $\alpha < \beta$, the skewness is positive and the distribution is right-skewed.

Remark 3. *Incorporating a stochastic recovery rate is challenging in terms of both implementation and calibration. Therefore, it should only be used in specific situations, such as when a portfolio is not well-diversified.*

3.2 Cash flow modeling

We now turn to the issue of cash flow modeling, which is an essential component of a structured blended finance fund. Several cash flow streams need to be distinguished:

- Investment phase
  During this phase, fund managers purchase assets (*e.g.*, green bonds) to deploy the capital raised during the fundraising process. The pace of capital deployment depends on the asset issuance pipeline.

- Uninvested capital
  The fund manager must define an investment strategy for uninvested proceeds.

- Income generation
  Throughout the life of the fund, investments generate income in the form of coupons, dividends or revenues (*e.g.*, annual or quarterly coupons from green bonds).

- Asset maturity and reinvestment
  As assets mature (*e.g.*, green bonds), they generate principal repayments that must be reinvested in new assets to maintain the fund's portfolio.

- Fees
  The fee policy must be defined. It concerns administrative and management fees that apply during the life of the portfolio[14]. For example, management fees may be applied to the beginning balance or to the invested nominal amounts at each period.

- Run-off period
  A redemption policy must be established for maturing assets. For example, principal repayments can be prorated according to the beginning balance. Alternatively, a waterfall structure can be implemented where the senior tranche is repaid first, followed by the junior tranche. In the waterfall scenario, the senior tranche may be repaid before the fund reaches maturity, particularly if the junior tranche is large.

The above cash flows form the basis of a structured blended finance fund.

Remark 4. *In practice, the cash flow modeling may be more complex and take into account several other mechanisms. For example, fees may be charged for a technical assistance facility (TAF) during the ramp-up period. A loss carryforward mechanism may also be included to provide additional protection to the senior tranche. In this case, there is no payment to the junior tranche until credit losses have been absorbed. The fund may also have a mezzanine tranche that receives a higher coupon than the senior tranche.*

3.2.1 Internal rate of return and credit loss calculation

The internal rate of return (IRR) of tranche k is defined as the interest rate at which the discounted value of net cash flows equals zero:

$$\sum_{t=0}^{T} \frac{\boldsymbol{\mathcal{CF}}_k(t)}{(1+\mathrm{IRR}_k)^t} = 0 \tag{9}$$

[14]Carried interest — performance-based compensation typically earned by private equity managers — introduces complexity, as it depends on performance and may reference certain KPIs that can create circular dependencies.

where the net cash flows for tranche k consist of dividends paid to investors plus principal repayments, minus the initial investments:

$$\boldsymbol{\mathcal{CF}}_k(t) = \mathrm{Div}_k(t) + \mathrm{RePay}_k(t) - \mathrm{Inv}_k(t)$$

From Equation (9), we deduce that:

$$\sum_{t=0}^{T} \frac{\mathrm{Div}_k(t) + \mathrm{RePay}_k(t)}{(1+\mathrm{IRR}_k)^t} = \sum_{t=0}^{T} \frac{\mathrm{Inv}_k(t)}{(1+\mathrm{IRR}_k)^t}$$

To simulate the cash flow streams, we also need to calculate the credit loss of the portfolio:

$$\mathcal{L}oss(t) = \sum_{i=1}^{n} (1 - \boldsymbol{\mathcal{R}}_i) \cdot N_i \cdot \mathbb{1}\{\boldsymbol{\tau}_i \leq t\}$$

where n is the number of assets, N_i and $\boldsymbol{\mathcal{R}}_i$ are the notional and recovery rate associated with asset i, and $\boldsymbol{\tau}_i$ is the default time of issuer i. The credit loss of tranche k is then equal to:

$$\begin{aligned}
\mathcal{L}oss_k(t) &= \left(\mathcal{L}oss(t) - A_k\right) \cdot \mathbb{1}\left\{A_k \leq \mathcal{L}oss(t) \leq D_k\right\} + (D_k - A_k) \cdot \mathbb{1}\left\{\mathcal{L}oss(t) \geq D_k\right\} \\
&= \min\Big(D_k - A_k, \max\left(\mathcal{L}oss(t) - A_k, 0\right)\Big)
\end{aligned}$$

where A_k and D_k are the attachment and detachment points of tranche k, respectively. We deduce that the principal repayment of tranche k at the maturity T is the difference between the initial investments and the credit loss:

$$\mathrm{RePay}_k(T) = \sum_{t=0}^{T} \mathrm{Inv}_k(t) - \mathcal{L}oss_k(T)$$

There are, of course, many variations on the above cash flow streams.

3.2.2 Examples of asset and liability cash flows

Let's look at some examples to better understand the cash flow streams of assets and liabilities in a blended finance fund. We assume a maturity of 8 years and an initial investment of 1 000, which is fully allocated at time $t = 0$ into bonds paying a coupon of 8%. The detachment point of the junior tranche is set at 10%, which means that the junior tranche has a size of 100, while the senior tranche has a size of 900. The coupon promised to the senior tranche is 5%.

In the case of no default, we obtain the results in Table 12. The portfolio generates an attributable net income of $1\,000 \times 8\% = 80$. Since the coupon of the senior tranche is 5%, the holders of this tranche receive an annual dividend of $900 \times 5\% = 45$, while the holders of the junior tranche receive an annual dividend of $80 - 45 = 35$. We deduce that the IRR of the senior tranche is 5% and the IRR of the junior tranche is 35% due to the leverage effect.

We now consider a variation of the previous example, where 100% of the capital cannot be deployed at the inception date $t = 0$, and two defaults occur at $t = 2$ and $t = 6$. 70% of the capital is deployed at date $t = 0$ and 30% at date $t = 1$. Assuming the interest rate on the cash is 3%, the net income attributable to the first year is equal to 65 ($700 \times 8\%$ plus $300 \times 3\%$). Since there is a credit loss of 100 at date $t = 2$, the beginning balance is 900 at date $t = 3$. This explains the net income of 72 for this period. Using the cash flow stream given in Table 13, we calculate that the internal rate of return is 3.65% for the senior tranche and 20.60% for the junior tranche. Since the credit loss is equal to 200, the capital repayment for the senior and junior tranches is 800 and 0, respectively.

Table 12: Asset and liability cash flows (Example #1(a), no default)

	Y0	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Beginning balance	0	1000	1000	1000	1000	1000	1000	1000	1000
Investment	1000								
Credit loss									
Capital repayments									−1000
Ending balance	1000	1000	1000	1000	1000	1000	1000	1000	0
Portfolio net yield		8%	8%	8%	8%	8%	8%	8%	8%
Net Income		80	80	80	80	80	80	80	80
Dividend credited to investors									
Senior tranche		45	45	45	45	45	45	45	45
Junior tranche		35	35	35	35	35	35	35	35
Capital repayment to investors									
Senior tranche									900
Junior tranche									100
Investments									
Senior tranche	−900								
Junior tranche	−100								
Total cash flows									
Senior tranche	−900	45	45	45	45	45	45	45	945
Junior tranche	−100	35	35	35	35	35	35	35	135
Portfolio	−1000	80	80	80	80	80	80	80	1080

Table 13: Asset and liability cash flows (Example #1(b), two defaults)

	Y0	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Beginning balance	0	1000	1000	900	900	900	900	800	800
Investment	700	300							
Cash	300								
Credit loss			100				100		
Capital repayments									−800
Ending balance	1000	1000	900	900	900	900	800	800	0
Portfolio net yield		8%	8%	8%	8%	8%	8%	8%	8%
Net Income		65	80	72	72	72	72	64	64
Dividend credited to investors									
Senior tranche		45	45	45	45	45	45	40	40
Junior tranche		20	35	27	27	27	27	24	24
Capital repayment to investors									
Senior tranche									800
Junior tranche									0
Investments									
Senior tranche	−900								
Junior tranche	−100								
Total cash flows									
Senior tranche	−900	45	45	45	45	45	45	40	840
Junior tranche	−100	20	35	27	27	27	27	24	24
Portfolio	−1000	65	80	72	72	72	72	64	864

The previous example highlights an important feature of a blended finance fund compared to a traditional collateralized debt obligation. In a CDO with junior and senior tranches, investors in the junior tranche stop receiving the premium leg when the senior tranche is impacted. However, this is not necessarily the case in a SBF fund. In the previous example, the second default causes the principal of the senior tranche to decrease from 900 to 800 at time $t > 6$. Since a 5% coupon has been promised, investors in the senior tranche receive $800 \times 5\% = 40$ in years $t = 7$ and $t = 8$. However, the entire portfolio has now an investment of 800, which earns a return of 8%, generating an annual income of 64. As a result, investors in the junior tranche continue to receive an annual coupon of $64 - 40 = 24$. This is because the portfolio is assumed to have the capacity to generate an annual return in excess of the coupon promised to the senior tranche. However, if this is not the case, and if permitted by the fund prospectus, the junior tranche can support the senior tranche by selling a portion of the portfolio to cover the promised coupon, provided it is not fully depleted. Of course, alternative cash flow structures can be implemented, including scenarios where junior tranche investors receive no payments if their tranche is fully exhausted.

3.3 Monte Carlo simulation

The previous example shows that the return of the investors in the senior and junior tranches is highly dependent on the credit risk scenario. Since we don't know which credit risk scenario will occur, it follows that the cash flow stream, credit loss and internal rate of return of each tranche can be considered stochastic. Therefore, it is important to estimate the probability distribution of these statistics. To do this, we use the classical Monte Carlo method described in Algorithm 1. For each scenario s, we can compute the cash flow stream $\boldsymbol{\mathcal{CF}}_{k,s}(t)$, the credit loss $\mathcal{L}oss_{k,s}(t)$, and the internal rate of return $\mathrm{IRR}_{k,s}$. The probability density function of each statistic is then estimated using the empirical distribution estimator.

Let us look at the previous example (Example #1). We recall that the maturity is 8 years, the capital is 1000, the equity tranche is 100, and the portfolio yield is 8% per year. At time $t = 0$, we invest in 10 bonds, each with a nominal of 100. We assume that for each issuer, the default time follows an exponential distribution ($\boldsymbol{\tau}_i \sim \mathcal{E}(\lambda_i)$), and the recovery rate is constant at 30%. The joint distribution of defaults is modeled using a Gaussian copula with a parameter matrix $\mathbb{C}_{10}(25\%)$. We consider several values of λ_i, simulate the cash flow streams, and compute the internal rate of return for the portfolio and the two tranches (equity and senior). Figures 13 and 14 show the empirical probability distribution[15] of IRR_k based on one million Monte Carlo simulations[16] when the default intensity λ_i is equal to 100 and 500 basis points, respectively. We note that the shape of the distribution differs significantly between these scenarios, implying that the risk exposure for junior and senior investors varies substantially. In particular, a higher default intensity increases the likelihood of extreme losses for equity tranche holders while shifting the risk-return profile for senior investors. This illustrates how changes in default intensity impact overall portfolio dynamics and investor risk profiles. Table 14 shows the summary statistics of the internal rate of return with respect to the default intensity. The first two statistics are the mean:

$$\hat{\mu}_k := \mathbb{E}[\mathrm{IRR}_k] = \frac{1}{n_S}\sum_{s=1}^{n_S} \mathrm{IRR}_{k,s}$$

[15] For each bin, we compute the probability $\Pr\{x_- \leq \mathrm{IRR}_k < x_+\}$, where x_- and x_+ are the left and right edges of the bin, respectively. The first bin of the histogram plot also collects all values below the left edge, so it corresponds to the probability $\Pr\{-\infty \leq \mathrm{IRR}_k < x_+\}$.

[16] The detailed algorithm is described in Algorithm 2 on page 32.

Algorithm 1 Monte Carlo simulation of a generic blended finance fund

Compute the simulated cash flow stream $\boldsymbol{\mathcal{CF}}_{k,s}(t)$, credit loss $\mathcal{L}oss_{k,s}(t)$, and internal rate of return $\mathrm{IRR}_{k,s}$

Initialize the number of simulations to n_S

for $s = 1 : n_s$ **do**

  Simulate the correlated default times $\left(\boldsymbol{\tau}_{1,s}, \ldots, \boldsymbol{\tau}_{n,s}\right)$ of the n issuers

  Simulate the recovery rates $\left(\boldsymbol{\mathcal{R}}_{1,s}, \ldots, \boldsymbol{\mathcal{R}}_{n,s}\right)$

  for $t = 1 : T$ **do**

    Simulate the cumulated loss of the portfolio:

$$\mathcal{L}oss_s(t) = \sum_{i=1}^{n} \left(1 - \boldsymbol{\mathcal{R}}_{i,s}\right) \cdot N_i \cdot \mathbb{1}\left\{\boldsymbol{\tau}_{i,s} \leq t\right\}$$

    Deduce the cumulated loss of tranche k:

$$\mathcal{L}oss_{k,s}(t) = \min\Big(D_k - A_k, \max\left(\mathcal{L}oss_s(t) - A_k, 0\right)\Big)$$

    Simulate the cash flow stream of tranche k:

$$\boldsymbol{\mathcal{CF}}_{k,s}(t) = \mathrm{Div}_{k,s}(t) + \mathrm{RePay}_{k,s}(t) - \mathrm{Inv}_k(t)$$

  end for

  Compute the internal rate of return for the s^{th} simulation:

$$\sum_{t=0}^{T} \frac{\boldsymbol{\mathcal{CF}}_{k,s}(t)}{\left(1 + \mathrm{IRR}_{k,s}\right)^t} = 0$$

end for

return the values of $\boldsymbol{\mathcal{CF}}_{k,s}(t)$, $\mathcal{L}oss_{k,s}(t)$ and $\mathrm{IRR}_{k,s}$

and the standard deviation:

$$\hat{\sigma}(\mathrm{IRR}_k) := \mathbb{E}^{1/2}\left[\left(\mathrm{IRR}_k - \mathbb{E}[\mathrm{IRR}_k]\right)^2\right] = \sqrt{\frac{1}{n_S - 1}\sum_{s=1}^{n_S}\left(\mathrm{IRR}_{k,s} - \hat{\mu}(\mathrm{IRR}_k)\right)^2}$$

We also report the downside (or zero-below) probability value, which is the probability that the internal rate of return is negative:

$$\hat{p}_k^0 := \Pr\{\mathrm{IRR}_k \leq 0\} = \frac{1}{n_S}\sum_{s=1}^{n_S} \mathbb{1}\left\{\mathrm{IRR}_{k,s} \leq 0\right\}$$

As expected, the average IRR decreases with default intensity, while the standard deviation and the zero-below probability increase. However, we note that the downside probability of the senior tranche is higher than that of the junior tranche, which is equal to zero (Figure 15). This clearly indicates that the structuring of this blended finance portfolio is not optimal. We have deliberately chosen this example to illustrate that the design of a blended finance structure — including the capital stack, the waterfall mechanism and the revenue sharing model — is not straightforward and is more complex than commonly assumed. A poorly

Algorithm 2 Monte Carlo simulation (Example #1)

Compute the internal rate of return $\mathrm{IRR}_{k,s}$
Initialize the number of simulations to n_S
for $s = 1 : n_s$ **do**
  Simulate $X \leftarrow \mathcal{N}(0,1)$, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ where $\varepsilon_i \leftarrow \mathcal{N}(0,1)$, $Z \leftarrow \sqrt{\rho} X + \sqrt{1-\rho}\varepsilon$, and $u \leftarrow \Phi(Z)$
  Simulate $\boldsymbol{\tau}_i \leftarrow -\ln(1-u_i)/\lambda_i$
  $\boldsymbol{\mathcal{CF}}_{\mathrm{senior}}(0) \leftarrow D_{\mathrm{senior}} - A_{\mathrm{senior}}$, $\boldsymbol{\mathcal{CF}}_{\mathrm{junior}}(0) \leftarrow D_{\mathrm{junior}} - A_{\mathrm{junior}}$ and $\boldsymbol{\mathcal{CF}}_{\mathrm{portfolio}}(0) = \boldsymbol{\mathcal{CF}}_{\mathrm{senior}}(0) + \boldsymbol{\mathcal{CF}}_{\mathrm{junior}}(0)$
  Compute the ending balances $\mathrm{EB}_{\mathrm{portfolio}} = D_{\mathrm{senior}}$, $\mathrm{EB}_{\mathrm{senior}} = D_{\mathrm{senior}} - A_{\mathrm{senior}}$ and $\mathrm{EB}_{\mathrm{junior}} \leftarrow D_{\mathrm{junior}} - A_{\mathrm{junior}}$
  for $t = 1 : T$ **do**
    Update the beginning balances: $\mathrm{BB}_{\mathrm{portfolio}} \leftarrow \mathrm{EB}_{\mathrm{portfolio}}$, $\mathrm{BB}_{\mathrm{senior}} \leftarrow \mathrm{EB}_{\mathrm{senior}}$ and $\mathrm{BB}_{\mathrm{junior}} \leftarrow \mathrm{EB}_{\mathrm{junior}}$
    Simulate the credit loss of the portfolio:

$$\mathcal{L}oss_{\mathrm{portfolio}}(t) = \sum_{i=1}^{n} (1 - \boldsymbol{\mathcal{R}}_i) \cdot N_i \cdot \mathbb{1}\{\boldsymbol{\tau}_i \le t\}$$

    Compute $\mathrm{Div}_{\mathrm{portfolio}} = y \cdot \mathrm{BB}_{\mathrm{portfolio}}$ ▷ y is the portfolio net yield
    $\mathrm{Div}_{\mathrm{senior}} \leftarrow \min(c_{\mathrm{senior}}\, \mathrm{BB}_{\mathrm{senior}}, \mathrm{Div}_{\mathrm{portfolio}})$ ▷ c_{senior} is the senior coupon
    $\mathrm{Div}_{\mathrm{junior}} \leftarrow \mathrm{Div}_{\mathrm{portfolio}} - \mathrm{Div}_{\mathrm{senior}}$
    Deduce the credit loss for each tranche:

$$\begin{cases} \mathcal{L}oss_{\mathrm{junior}}(t) = \min(\mathrm{BB}_{\mathrm{junior}}, \mathcal{L}oss_{\mathrm{portfolio}}(t)) \\ \mathcal{L}oss_{\mathrm{senior}}(t) = \mathcal{L}oss_{\mathrm{portfolio}}(t) - \mathcal{L}oss_{\mathrm{junior}}(t) \end{cases}$$

    $\mathrm{EB}_{\mathrm{senior}} \leftarrow \mathrm{EB}_{\mathrm{senior}} - \mathcal{L}oss_{\mathrm{senior}}(t)$ and $\mathrm{EB}_{\mathrm{junior}} \leftarrow \mathrm{EB}_{\mathrm{junior}} - \mathcal{L}oss_{\mathrm{junior}}(t)$
    if $t = T$ **then**
      $\mathrm{RePay}_{\mathrm{portfolio}} \leftarrow \mathrm{BB}_{\mathrm{portfolio}} - \mathcal{L}oss_{\mathrm{portfolio}}(t)$
      $\mathrm{RePay}_{\mathrm{senior}} \leftarrow \min\left(\mathrm{RePay}_{\mathrm{portfolio}}, \mathrm{BB}_{\mathrm{senior}}\right)$
      $\mathrm{RePay}_{\mathrm{junior}} \leftarrow \mathrm{RePay}_{\mathrm{portfolio}} - \mathrm{RePay}_{\mathrm{senior}}$
    else
      $\mathrm{RePay}_{\mathrm{portfolio}} \leftarrow 0$, $\mathrm{RePay}_{\mathrm{senior}} \leftarrow 0$ and $\mathrm{RePay}_{\mathrm{junior}} \leftarrow 0$
    end if
    Update the ending balances: $\mathrm{EB}_{\mathrm{senior}} \leftarrow \mathrm{EB}_{\mathrm{senior}} - \mathrm{RePay}_{\mathrm{senior}}$, $\mathrm{EB}_{\mathrm{junior}} \leftarrow \mathrm{EB}_{\mathrm{junior}} - \mathrm{RePay}_{\mathrm{junior}}$ and $\mathrm{EB}_{\mathrm{portfolio}} \leftarrow \mathrm{EB}_{\mathrm{senior}} + \mathrm{EB}_{\mathrm{junior}}$
    Calculate the cash flow streams:

$$\boldsymbol{\mathcal{CF}}_{k,s}(t) = \mathrm{Div}_k + \mathrm{RePay}_k$$

    where k = portfolio, senior and junior
  end for
  Compute the internal rate of return for the s^{th} simulation:

$$\sum_{t=0}^{T} \frac{\boldsymbol{\mathcal{CF}}_{k,s}(t)}{\left(1 + \mathrm{IRR}_{k,s}\right)^t} = 0$$

  where k = portfolio, senior and junior
end for
return the values of $\mathrm{IRR}_{k,s}$

Figure 13: Distribution of the internal rate of return — Example #1, $\lambda_i = 100\,\text{bps}$

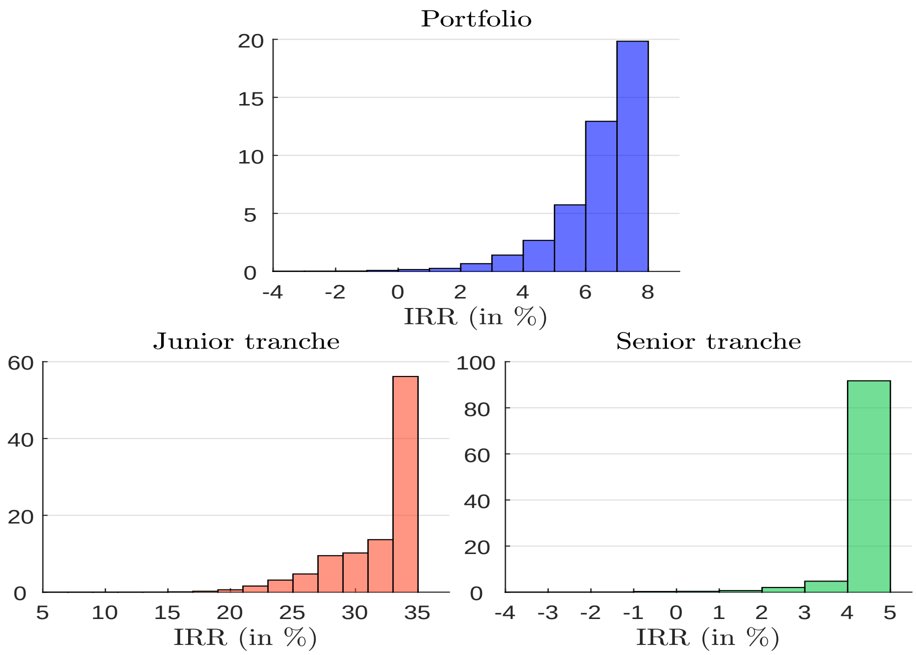


Figure 14: Distribution of the internal rate of return — Example #1, $\lambda_i = 500\,\text{bps}$

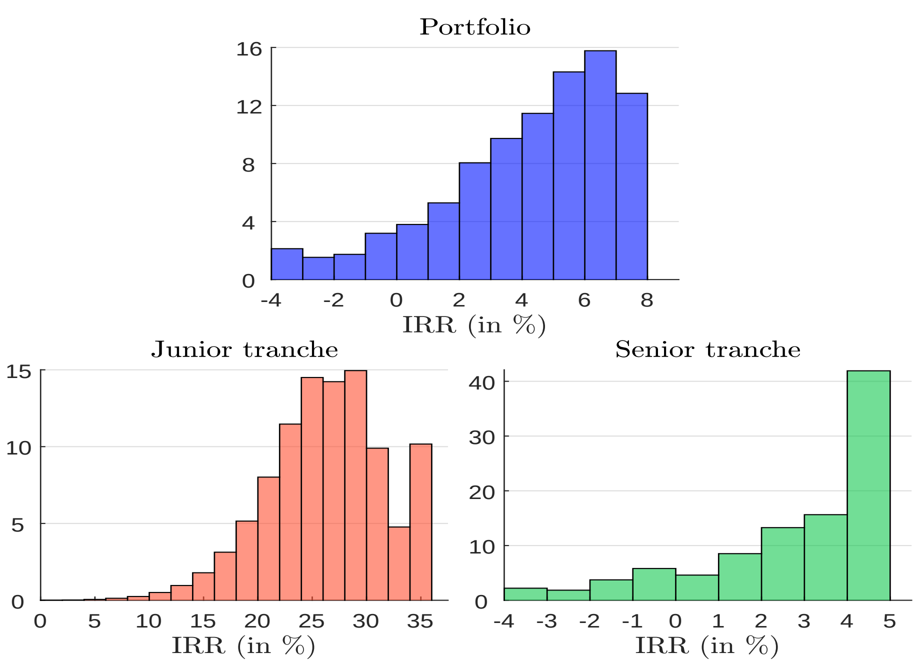
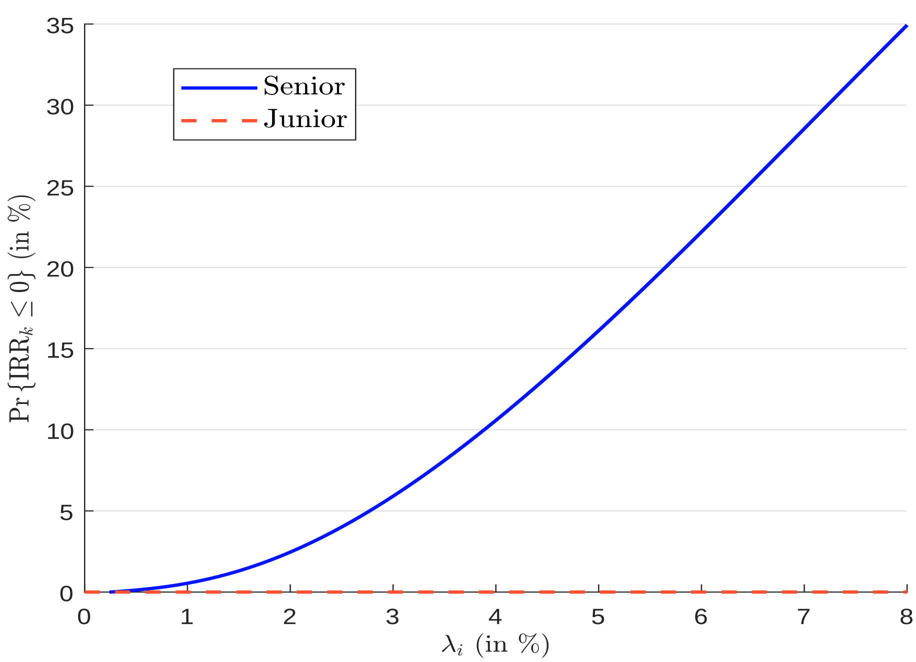


Table 14: Mean, standard deviation and zero-probability of the internal rate of return (in %) — Example #1

λ_i (in %)	Portfolio $\hat{\mu}_k$	Portfolio $\hat{\sigma}_k$	Portfolio $\hat{p}_k^0$	Junior $\hat{\mu}_k$	Junior $\hat{\sigma}_k$	Junior $\hat{p}_k^0$	Senior $\hat{\mu}_k$	Senior $\hat{\sigma}_k$	Senior $\hat{p}_k^0$
0.00	8.00	0.00	0.00	35.00	0.00	0.00	5.00	0.00	0.00
0.10	7.93	0.30	0.00	34.67	1.35	0.00	4.99	0.09	0.00
0.25	7.82	0.51	0.00	34.20	2.10	0.00	4.97	0.21	0.02
0.50	7.64	0.77	0.03	33.50	2.86	0.00	4.91	0.40	0.09
0.75	7.45	0.99	0.08	32.86	3.37	0.00	4.84	0.59	0.24
1.00	7.27	1.18	0.17	32.28	3.74	0.00	4.75	0.76	0.48
2.00	6.55	1.78	0.97	30.31	4.58	0.00	4.31	1.36	2.44
3.00	5.85	2.24	2.65	28.75	4.98	0.00	3.79	1.86	5.91
4.00	5.17	2.60	5.22	27.44	5.20	0.00	3.23	2.27	10.59
5.00	4.51	2.89	8.59	26.32	5.36	0.00	2.67	2.60	16.11
6.00	3.88	3.13	12.59	25.33	5.49	0.00	2.10	2.87	22.19
7.00	3.27	3.32	17.06	24.44	5.59	0.00	1.54	3.09	28.55
8.00	2.69	3.47	21.92	23.63	5.69	0.00	0.99	3.26	34.93
9.00	2.13	3.59	26.97	22.87	5.79	0.00	0.47	3.40	41.19
10.00	1.60	3.68	32.11	22.16	5.87	0.00	−0.04	3.50	47.15

structured portfolio can result in unintended risk allocations, potentially exposing senior investors to greater downside risk than junior investors, which is contrary to the intended purpose of risk tranching. This highlights the importance of carefully calibrating financial structures to ensure a fair and efficient allocation of risk and return.

Figure 15: Downside probability of the junior and senior tranches — Example #1



The previous example is not entirely realistic because the senior tranche is not adequately protected by the junior tranche. For example, despite a high number of defaults when the default intensity is set to 10%, the senior tranche has a higher probability of a zero-below return and a lower average IRR. This result is primarily due to the structure of the blended finance fund, in particular the thickness of the junior tranche, but other factors also play a role. For a more realistic illustration, we consider a second portfolio with similar characteristics. The maturity T of the blended fund is 8 years and the capital is 1000. At time $t = 0$, we invest in 10 bonds, each with a nominal of 100. We assume that $\boldsymbol{\tau}_i \sim \mathcal{E}(\lambda_i)$, the recovery rate is constant at 30%, and the default copula is Gaussian with a uniform parameter ρ. The promised coupon c_{senior} for the senior tranche is set to 5%. We assume an extra risk premium π of 1% at the portfolio level. The net yield of the portfolio is then the sum of the senior coupon, the extra risk premium and the average credit spread of the portfolio:

$$y = c_{\text{senior}} + \pi + s$$

where:

$$s = \frac{1}{\sum_{i=1}^{n} N_i} \sum_{i=1}^{n} N_i \left(1 - \boldsymbol{\mathcal{R}}_i\right) \lambda_i$$

The net yield is not constant and increases with the credit risk of the portfolio. Similarly, it is important to relate the thickness of the junior tranche to the exposure to credit risk. Using a Gaussian approximation, we compute the credit value-at-risk at the confidence interval α as follows:

$$\mathrm{VaR}_{\alpha}\left(\mathcal{L}oss\left(t\right)\right) \approx \mathbb{E}\left[\mathcal{L}oss\left(t\right)\right] + \Phi^{-1}\left(\alpha\right)\sqrt{\operatorname{var}\left(\mathcal{L}oss\left(t\right)\right)}$$

where:

$$\mathbb{E}\left[\mathcal{L}oss\left(t\right)\right] = \sum_{i=1}^{n} N_i \left(1 - \boldsymbol{\mathcal{R}}_i\right)\left(1 - e^{-\lambda_i t}\right)$$

and:

$$\begin{aligned}
\operatorname{var}\left(\mathcal{L}oss\left(t\right)\right) &= \sum_{i=1}^{n} N_i^2 \left(1 - \boldsymbol{\mathcal{R}}_i\right)^2 \left(1 - e^{-\lambda_i t}\right) e^{-\lambda_i t} + \\
&\quad \sum_{i \neq j} N_i N_j \left(1 - \boldsymbol{\mathcal{R}}_i\right)\left(1 - \boldsymbol{\mathcal{R}}_j\right) \cdot \\
&\quad \left(\mathbf{C}\left(1 - e^{-\lambda_i t}, 1 - e^{-\lambda_j t}; \rho\right) - \left(1 - e^{-\lambda_i t}\right)\left(1 - e^{-\lambda_j t}\right)\right)
\end{aligned}$$

where $\mathbf{C}\left(u_1, u_2; \rho\right)$ is the bivariate Gaussian copula. The detachment point of the junior tranche is set to an initial buffer and the credit value-at-risk at maturity:

$$D_{\text{junior}} = 100 + \mathrm{VaR}_{\alpha}\left(\mathcal{L}oss\left(T\right)\right)$$

We also have $A_{\text{junior}} = 0$, $A_{\text{senior}} = D_{\text{junior}}$, and $D_{\text{senior}} = \sum_{i=1}^{n} N_i$. Table 15 presents the results for $\rho = 25\%$. We observe that the senior tranche is well protected by the junior tranche, and the downside probability is lower for the senior tranche. Figure 16 shows the size of the junior and senior tranches with respect to the default intensity. As expected, the size of the junior tranche increases with default intensity, while the size of the senior tranche decreases. Additionally, we notice that the detachment point D_{junior} depends on the default correlation (Figure 17). When the correlation is high, a thicker junior tranche is required to provide adequate protection to the senior tranche.

Remark 5. *Protecting the senior tranche requires defining an optimal value for the detachment point* D_{junior}*. Analyzing the average IRR* $\hat{\mu}_{\text{senior}}$*, the standard deviation* $\hat{\sigma}_{\text{senior}}$ *and the zero-below probability* $\hat{p}^0_{\text{senior}}$ *is not enough. First, a solution to fully protect the senior tranche is to use a very high value of* D_{junior} *— the extreme case is* $D_{\text{junior}} = 100\%$*. However,*

Table 15: Mean, standard deviation and zero-probability of the internal rate of return (in %) — Example #2, $\rho = 25\%$, $\alpha = 95\%$

λ_i	Portfolio			Junior			Senior		
(in %)	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$
0.00	6.00	0.00	0.00	15.00	0.00	0.00	5.00	0.00	0.00
0.10	6.00	0.30	0.00	12.01	2.26	0.89	5.00	0.06	0.00
0.25	5.99	0.50	0.01	10.65	3.13	2.54	4.99	0.11	0.00
0.50	5.99	0.77	0.08	9.42	4.08	3.04	4.99	0.18	0.02
0.75	5.98	0.98	0.21	8.71	4.52	5.24	4.98	0.23	0.02
1.00	5.97	1.17	0.39	8.18	4.95	6.96	4.97	0.26	0.05
2.00	5.95	1.77	1.26	7.12	5.58	9.58	4.96	0.34	0.11
3.00	5.95	2.23	2.56	6.64	5.89	11.22	4.96	0.35	0.08
4.00	5.97	2.61	3.59	6.42	5.99	11.89	4.96	0.31	0.06
5.00	6.01	2.92	4.48	6.35	6.00	12.82	4.97	0.25	0.00
6.00	6.08	3.17	5.49	6.34	5.99	13.11	4.98	0.18	0.00
7.00	6.17	3.39	6.09	6.42	5.90	13.05	4.99	0.08	0.00
8.00	6.28	3.58	6.37	6.54	5.80	13.04	5.00	0.00	0.00
9.00	6.41	3.73	6.51	6.71	5.66	12.69	5.00	0.00	0.00
10.00	6.56	3.87	6.53	6.90	5.57	12.43	5.00	0.00	0.00

Figure 16: Tranching of the blended finance fund — Example #2, $\alpha = 95\%$

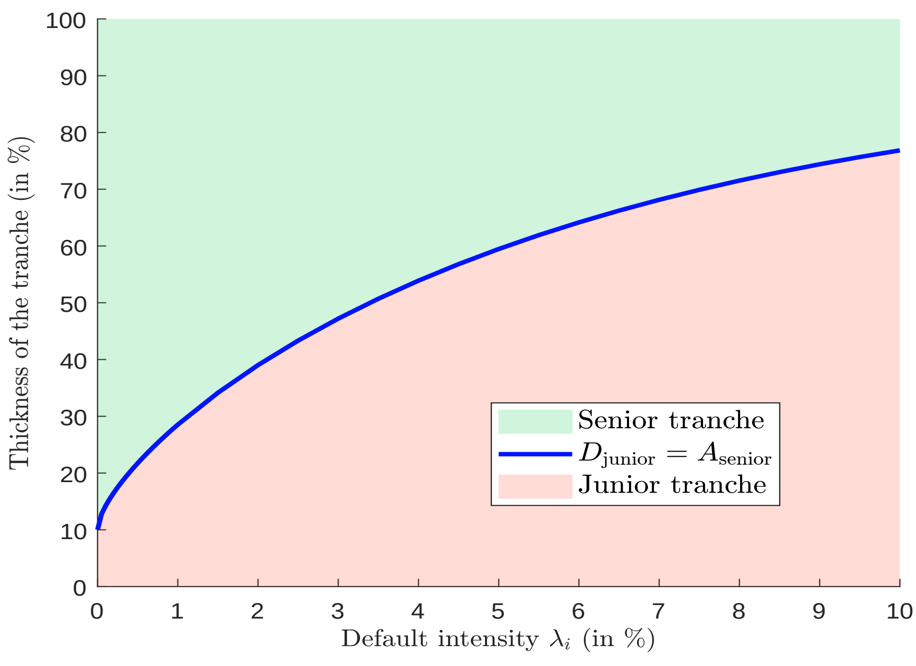
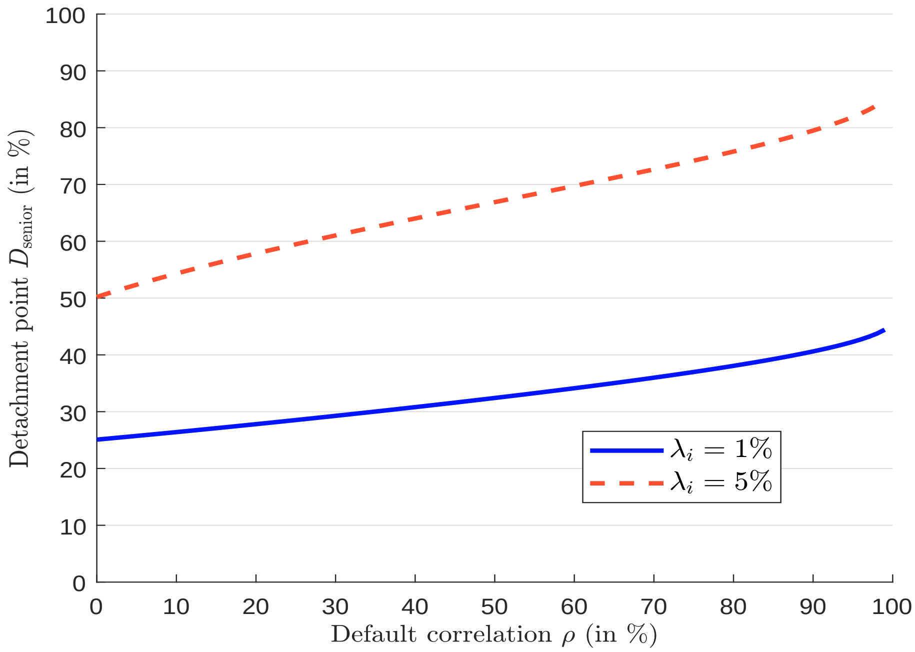


Figure 17: Detachment point of the junior tranche — Example #2, $\alpha = 95\%$



this reduces the leverage and it is not the purpose of a structured blended finance fund to have only one tranche. Second, the zero-below probability is not an exhaustive statistic for measuring the risk of the senior tranche. The cumulative distribution function $\Pr\{\text{IRR}_{\text{senior}} \leq \theta\}$ *for some thresholds is shown below when the intensity default is set to* 1%:

Threshold θ	0.00%	1.00%	2.00%	3.00%	4.00%	4.50%	5.00%
Buffer & $\alpha = 95\%$	0.05%	0.09%	0.19%	0.55%	1.10%	1.44%	100.00%
No buffer & $\alpha = 90\%$	0.20%	0.53%	1.05%	1.67%	3.53%	8.27%	100.00%

For instance, the zero-below probability is 0.05% *for Example #2. However, we notice that* $\Pr\left\{\text{IRR}_{\text{senior}} \leq 4\%\right\} = 1.10\%$. *If we assume that* $D_{\text{junior}} = \text{VaR}_{90\%}\left(\mathcal{L}oss\,(T)\right)$, *this probability becomes* 3.53%, *which is relatively large, while the zero-below probability is only* 0.20%. *We may wonder if this distribution probability of* $\text{IRR}_{\text{senior}}$ *is compatible with a senior AAA-rated tranche*[17].

3.4 Calibration of the asset-liability structure

The previous example illustrates the importance of a well-calibrated asset-liability structure for a blended finance fund. We need to distinguish between the asset side and the liability side of the SBF fund. Understanding the asset side means assessing the risk and return of the asset portfolio, while understanding the liability side means analyzing the risk and return of the different tranches. Typically, the asset portfolio is modeled first, and then the liability structure is defined based on the asset characteristics. However, we could imagine an alternative approach where the liability structure — in particular the leverage ratio — is specified first and then the asset portfolio is constructed to match this liability structure. In practice, the liability-to-asset calibration procedure is quite rare, while the asset-to-liability

[17] See Table 26 on page 80 for the results when there is no initial buffer and α is set to 90%.

calibration procedure is common. Of course, this is a theoretical perspective, as we generally observe adjustments to the asset portfolio after the liability structure is defined, aimed at improving the asset-liability match.

3.4.1 What factors influence the asset-liability structure?

Below is a list of the various factors that affect the asset and liability side of an SBF fund:

Asset side

- Duration of the assets
- Return of the assets
- Interest rate risk of the assets
- Credit risk of the assets
- Currency risk of the assets
- Liquidity risk of the assets

Liability side

- Maturity of the fund
- Mezzanine or two-tranche
- Fixed or float coupon
- Fund size and the leverage ratio
- Concessionality
- Senior protection mechanisms

The duration, return, and risk characteristics of the assets — particularly interest rate risk, credit risk, currency risk, and liquidity risk — are fundamental in shaping the overall risk-return profile of the asset portfolio. This profile, in turn, plays a critical role in determining the structure and feasibility of the fund's liabilities. For example, if the asset portfolio has an average credit rating of A or BBB+, the inclusion of a subordinated equity tranche would not be justified because there is insufficient risk or excess return to compensate junior and equity investors. In practice, however, the average credit rating of a blended finance portfolio is typically in the BBB to BB range. This level of credit quality provides sufficient risk and return dispersion to support a tiered capital structure, including mezzanine and equity tranches, and to attract different types of investors with varying risk appetites. While the risk-return profile of the asset portfolio is the primary driver of the thickness of the equity tranche[18], the inclusion of a mezzanine tranche often depends on the specific investor base and their appetite for intermediary risk-return positions. Some investors actively seek mezzanine exposure as a way to enhance returns without fully absorbing equity risk. Another important consideration is the relationship between the average duration of the asset portfolio and the overall maturity of the fund. This duration mismatch influences whether the coupon promised to senior investors can be fixed or must remain variable, possibly linked to the net asset value of the senior tranche. If the average duration of the assets closely matches the maturity of the fund, there is little uncertainty about future cash flows, particularly with respect to interest rate risk. Conversely, if the duration of the assets is significantly shorter than the maturity of the fund, reinvestment will be required and the liability structure may be exposed to increased interest rate risk. This reinvestment risk can affect the predictability of returns for senior investors and complicate the design of the senior tranche. In the asset-to-liability approach, the leverage ratio is a consequence of the risk-return profile of the asset portfolio. However, this leverage ratio can be influenced by the presence or absence of a mezzanine tranche, the degree of concessionality, and the introduction of specific senior projection mechanisms. Finally, it is essential that concessionality is clearly defined

[18] This discussion does not consider guarantees associated with specific assets — such as those provided by export credit agencies — which can significantly influence the asset-liability structure.

and accurately measured. This includes quantifying the economic value of the concessional features and ensuring alignment with the strategic objectives of the blended finance vehicle.

3.4.2 Which metrics to monitor and how to benchmark them?

To calibrate the asset-liability structure, it is important to define the metrics that are essential for assessing the risk-return profile of each tranche and to compare the statistical measures with some benchmarks. In fact, the following three metrics can be used for each tranche and the portfolio:

- Internal rate of return
- Credit loss
- Coupon

For each metric, we can calculate the traditional statistics (mean, median, standard deviation, 95% and 99% value-at-risk), and some statistics such as the number of coupons not paid, etc.

Table 16: Average yield spread of US corporate bonds over treasuries (in bps)

Period	AAA	AA	A	BBB	BB	B	CCC
1997–1999	52	66	84	123	229	416	895
2000–2009	102	117	152	225	425	638	1 324
2010–2019	66	88	117	184	343	519	988
1997	36	49	61	82	165	321	684
2008	219	282	335	394	691	1 019	1 635
2020	75	87	114	196	393	589	1 257
2021	47	55	69	113	248	394	672
2022	64	82	113	175	314	494	1 003
2023	52	70	112	163	292	460	1 006
2024	37	51	79	117	213	331	879
1997–2024	76	93	122	187	351	539	1 093

Source: ICE BofA US Corporate Indices & Authors' calculations.

The choice of the benchmarks is also critical because investors have certain expectations depending on the risk they are taking. Typically, a senior tranche corresponds to a rating of AAA (or AA+), a mezzanine tranche aligns with a BBB or BB rating, and an equity tranche is associated with a B or CCC rating. Table 16 shows the average yield spreads of US corporate bonds over treasuries for the different credit ratings. Between 1997 and 2024, the excess yields averaged 76, 187 and 1 093 bps for AAA, BBB and CCC rated bonds, respectively. However, these spreads have fluctuated over time, reaching a low in 1997 and a high in 2008. For instance, the difference in yield spreads between 2008 and 1997 was 183 bps for AAA corporate bonds, 312 bps for BBB corporate bonds, and nearly 1 000 bps for CCC corporate bonds. These fluctuations underscore the influence of economic conditions on credit spreads, but also the risk aversion of investors.

In structured finance, an accurate assessment of the credit risk of each tranche is paramount for investors. Therefore, another important benchmark is the expected loss of each tranche, as it determines the rating assigned to the senior, mezzanine and senior tranches. In Table 17, we report the expected loss[19] by credit rating for different maturities (1, 3, 5, 7 and

[19]The expected loss for credit rating i is calculated as follows:

$$\mathrm{EL}_i(t) = (1 - \boldsymbol{\mathcal{R}}_i) \cdot \big(1 - \mathbf{S}_i(t)\big)$$

10 years). We use data from two credit rating agencies: Moody's and S&P Global Ratings. By construction, expected loss increases with both lower credit ratings and longer maturities. For instance, the five-year expected loss for a BBB-rated issuer is 87 bps according to Moody's and 78 bps according to S&P, respectively. At the ten-year horizon, the expected loss rises to 9.30% and 9.27%, respectively. Overall, the figures calculated using Moody's and S&P data are very similar for most rating categories, with the main exception being the CCC rating category. This discrepancy arises because Moody's figures refer specifically to CCC-rated issuers, while S&P combines CCC and lower ratings together into the CCC category.

Table 17: Relationship between credit ratings and expected loss (in %)

Rating	Moody's					S&P Global Ratings				
	1Y	3Y	5Y	7Y	10Y	1Y	3Y	5Y	7Y	10Y
AAA	0.00	0.00	0.02	0.04	0.11	0.00	0.03	0.08	0.15	0.29
AA	0.01	0.05	0.11	0.20	0.40	0.01	0.06	0.13	0.22	0.41
A	0.03	0.13	0.27	0.48	0.92	0.03	0.12	0.24	0.42	0.78
BBB	0.10	0.41	0.87	1.47	2.61	0.09	0.37	0.78	1.32	2.36
BB	0.56	2.04	3.89	5.98	9.30	0.37	1.68	3.56	5.76	9.27
B	2.02	6.43	10.89	15.10	20.69	2.01	7.69	13.26	18.04	23.80
CCC	5.89	15.27	22.32	27.75	33.82	18.36	34.14	39.95	42.88	45.58

Source: Moody's (2023), S&P Global Ratings (2025) & Authors' calculations.

Table 18: 95% value-at-risk (in %) of a homogeneous fine-grained loss portfolio assuming 40% recovery rate and 20% default correlation

Rating	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
AAA	0.00	0.01	0.02	0.04	0.08	0.12	0.18	0.26	0.35	0.46
AA	0.05	0.13	0.22	0.33	0.46	0.62	0.81	1.02	1.27	1.54
A	0.14	0.31	0.52	0.78	1.09	1.44	1.84	2.29	2.78	3.32
BBB	0.42	0.96	1.60	2.34	3.17	4.08	5.05	6.08	7.16	8.27
BB	2.12	4.35	6.70	9.10	11.51	13.89	16.21	18.45	20.60	22.63
B	6.63	12.22	17.15	21.52	25.37	28.77	31.77	34.41	36.74	38.80
CCC	16.01	25.43	32.01	36.87	40.58	43.48	45.80	47.68	49.22	50.50

Source: Moody's (2023) & Authors' calculations.

Benchmarking credit value-at-risk is more complex because it depends on the credit risk model. If we assume a single bond or loan, the credit VaR is defined as[20]:

$$\mathbf{F}^{-1}(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq 1 - p \\ \mathbf{G}^{-1}\left(\dfrac{\alpha + p - 1}{p}\right) & \text{if } \alpha > 1 - p \end{cases}$$

where $\mathbf{G}(x)$ is the cumulative distribution function of the loss given default. The previous approach can be generalized to a portfolio of bonds, but there is no general analytical formula in this case, so benchmarking is not relevant. Another approach is to use a canonical model,

where $\boldsymbol{\mathcal{R}}_i$ is set to 40%, $\mathbf{S}_i(t) = 1 - \mathbf{e}_i^\top \exp(t\Lambda)\,\mathbf{e}_K$ and Λ is the estimated Markov generator of the credit migration matrix.

[20]Because we have:

$$\mathbf{F}(x) = (1 - p) + \mathbf{1}\{x > 0\} \cdot p\mathbf{G}(x)$$

such as the Basel II credit risk model. Assuming a homogeneous fine-grained portfolio, the credit VaR is equal to[21]:

$$\mathbf{F}^{-1}(\alpha) = (1 - \boldsymbol{\mathcal{R}}) \cdot \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)$$

where p is the probability of default, ρ is the standard default correlation and α is the confidence level of the value-at-risk. Assuming a standard asset correlation of 20% and a recovery rate of 40%, we obtain the results shown in Table 18. The 95% value-at-risk is significant in absolute terms for high yield portfolios. However, it becomes even more important in relative terms for investment grade portfolios. That is, the ratio between the value-at-risk and the expected loss is higher for better credit ratings than for lower ratings — highlighting the greater sensitivity of high-quality portfolios to rare default events. The main reason for this is the shape of the probability distribution, which is completely different for IG and HY ratings. In Figure 18 we show the probability density function of credit losses, which has the following expression:

$$\begin{cases} f(x) = \dfrac{1}{(1-\boldsymbol{\mathcal{R}})} g\left(\dfrac{x}{1-\boldsymbol{\mathcal{R}}}\right) \\ g(y) = \sqrt{\dfrac{1-\rho}{\rho}} \exp\left(\frac{1}{2}\left(\Phi^{-1}(y)\right)^2 - \dfrac{1}{2\rho}\left(\sqrt{1-\rho}\Phi^{-1}(y) - \Phi^{-1}(p)\right)^2\right) \end{cases}$$

where $g(y)$ is the density function of credit losses when the recovery rate is set to zero. We see that IG-rated credit portfolios have an L-shaped distribution (with high skewness and kurtosis), while HY-rated credit portfolios have a bell-shaped distribution.

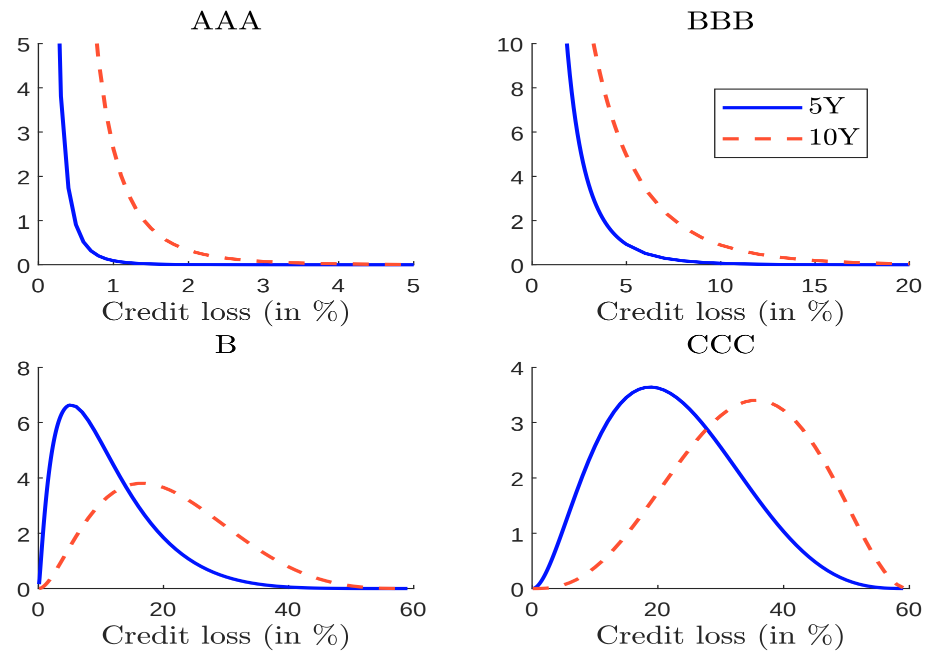
Remark 6. *The case of BB-rated portfolios is more complex. In fact, they exhibit an L-shaped distribution for short maturities (e.g., less than 3 years) and a bell-shaped distribution for long maturities (e.g., more than 5 years), i.e., they shift from an IG profile in the short term to a HY profile in the long term (Figure 36 on page 81). Portfolio loss density is also sensitive to default correlation, especially for portfolios with low credit quality. For example, Figure 37 on page 81 shows the probability density function of portfolio losses at a default correlation of 40%. For high credit ratings, we observe a translation of the density. For low credit ratings, the bell shape is less pronounced and can turn into a U-shaped curve when the default correlation is very high. These results have important implications for the management of a blended finance fund. For example, the structure of a well-diversified SBF fund may not be the same as the structure of a country-specific SBF fund.*

3.4.3 What are the calibration parameters?

There are many degrees of freedom in calibrating the asset-liability structure of a blended finance fund. To reduce the dimensionality of the calibration problem and simplify the structuring process, we explicitly assume that the asset portfolio is exogenously given. This is a realistic assumption in most cases, as the sponsor or originator typically has a clear mandate or impact-oriented objective that drives the composition of the asset portfolio. Consequently, the liability structure must adapt to the characteristics of this asset portfolio rather than the reverse, implying that the optimization problem focuses on calibrating the liability structure. The first key parameter to calibrate is the maturity profile of the fund's liabilities. A common and intuitive approach is to match the maturity of the fund to the average maturity of the underlying assets, ensuring a duration match that simplifies liquidity

[21] The proof is given in Appendix A.2 on page 77.

Figure 18: Probability density function of portfolio loss by credit rating ($\boldsymbol{\mathcal{R}} = 40\%$ and $\rho = 20\%$)



and reinvestment considerations. This duration-matching approach has the advantage of minimizing duration and performance risk. However, in some cases, the maturity of the fund may exceed the weighted average maturity of the asset pool. In the extended liability approach, the reinvestment strategy must be defined, either through a rolling mechanism or an explicit reinvestment policy, which introduces additional sources of risk such as market and liquidity risk. The optimal choice depends on the fund's risk management framework and the expected market conditions over the investment horizon. Empirical evidence suggests that duration-matched structures exhibit lower NAV volatility, particularly in steepening yield curve environments. In all cases, these risk dynamics must be quantitatively modeled to assess their impact on tranche performance and investor exposure[22].

Once the maturity of the fund is determined, a fundamental decision concerns the optimal number of tranches in the capital structure. The key question is whether a two-tranche structure (junior-senior) is sufficient or whether a three-tranche structure (junior-mezzanine-senior) provides superior risk-return characteristics. Let $\mathcal{L}oss\,(t)$ be the credit loss of the portfolio at time t. In a three-tranche structure, this loss can be decomposed as:

$$\mathcal{L}oss\,(t) = \mathcal{L}oss_{\text{junior}}\,(t) + \mathcal{L}oss_{\text{mezzanine}}\,(t) + \mathcal{L}oss_{\text{senior}}\,(t)$$

The introduction of a mezzanine tranche is optimal if each component in this decomposition has a distinct statistical profile with minimal overlap in their probability distributions. This decision depends primarily on four factors: (i) the loss distribution characteristics of the asset portfolio, (ii) the correlation structure of the assets (both within and across sectors and countries), (iii) the risk appetite and return expectations of the investors, and (iv) the time horizon of the fund. For example, if the loss distribution is multimodal rather than

[22] We can also consider differentiated maturities, particularly shorter ones for senior tranches, to better manage the funding of the blended finance structure.

bell-shaped, a mezzanine tranche becomes particularly valuable. This is also the case if the asset portfolio exhibits default clustering. Similarly, statistical analysis shows that longer maturity horizons typically warrant more granular tranche structures due to the widening confidence intervals in the loss distributions over time. Indeed, the relevance of introducing a mezzanine tranche increases with longer fund maturities. This is because the probability distribution of cumulative losses becomes more diffuse over time, with increased uncertainty due to compounding credit risk, macroeconomic shifts and evolving market conditions. In such environments, a three-tier tranche structure allows for a more granular allocation of risk, allowing investors to more precisely select exposure according to their risk tolerance and capital constraints.

Another critical structural parameter is the detachment point of each tranche. Calibrating these thresholds requires an explicit formulation of the profit and loss allocation mechanism that governs how cash flows, losses, and recoveries are distributed among tranches under different scenarios. Once a preliminary tranche structure is calibrated, we can iterate by testing various enhancement mechanisms, including first-loss guarantees, over-collateralization, or reserve accounts that increase the protection of the senior tranche. The evaluation of these mechanisms also requires an assessment of the concessionality of the SBF fund. Ultimately, the optimal structure of an SBF fund should balance the objectives of capital preservation for senior investors, adequate risk-adjusted return for mezzanine investors, and impact maximization for junior investors. This requires a systematic and iterative calibration process and sensitivity analysis to key economic assumptions.

4 Investor preferences and optimal structuring

Investor preferences play a critical role in determining the optimal structure of blended finance funds. A key distinction is between the preferences of the junior tranche investors and those of the senior tranche investors. The sponsor — typically the junior tranche investor — has a dual objective. First, driven by a responsible investment mandate, the sponsor seeks to maximize the leverage ratio, thereby mobilizing as much private capital as possible. Second, given the higher risk exposure, the sponsor also seeks a minimum return as compensation. In contrast, senior tranche investors do not necessarily have explicit ESG objectives, although many blended finance participants do incorporate responsible investment principles. These investors are primarily motivated by the opportunity to earn a higher return than a similarly rated bond due to the nature of the junior capital. This risk premium also compensates for the illiquidity of the underlying assets and the closed-end structure of most blended finance vehicles. As a result, senior investors may demand an additional risk premium. The asset manager's role is to structure the fund to ensure fairness between junior and senior investors. Ethically and commercially, asset managers have no interest in favoring one group over the other, as both are typically their clients. Nevertheless, finding the optimal and equitable structure is complex because there is no one-size-fits-all solution. In this context, we propose a set of tools to help asset managers achieve a fair balance between investor interests.

4.1 Relationship between risk aversion and risk premium

To understand the rationale behind a structured blended finance fund, it is important to first analyze the risk premia required by different types of investors. Typically, the junior tranche carries credit risk equivalent to a CCC rating, while the senior tranche is structured to achieve a AAA rating. As a result, these two tranches are at opposite ends of the risk spectrum. In the first part, we examine how investors price the risk and return of these tranches. In the second part, we explore the economic rationale behind the use of tranching structures in blended finance.

4.1.1 Investor-required risk premium

Following Hull *et al.* (2005), we compute the risk-neutral default intensity by:

$$\lambda_i^{(\text{risk-neutral})} = \frac{y_i - r}{1 - \boldsymbol{\mathcal{R}}}$$

where y_i is the bond yield for the rating $\boldsymbol{R}_i$ and r is the risk-free return, while the historical default intensity is computed using the migration matrix P:

$$\lambda_i^{(\text{historical})} = \frac{-\ln\left(1 - \mathbf{e}_i P^{\tau} \mathbf{e}_K\right)}{\tau}$$

where τ is a standard maturity. In the following analysis, we set τ to 10 years. The ratio of the risk-neutral default intensity to the historical default intensity is called the risk premium multiplier and is denoted by α_i. Let EL be the expected loss. Assuming that the risk-neutral loss given default is equal to the historical loss given default, we obtain:

$$\alpha_i := \frac{\lambda_i^{(\text{risk-neutral})}}{\lambda_i^{(\text{historical})}} \approx \frac{\text{EL}_i^{(\text{risk-neutral})}}{\text{EL}_i^{(\text{historical})}}$$

This leads to the multiplicative risk premium model:

$$\pi_i^{(\text{risk-neutral})} = \alpha_i \pi_i^{(\text{historical})} \tag{10}$$

Similarly, we define the additive risk premium model as:

$$\pi_i^{(\text{risk-neutral})} = \pi_i^{(\text{historical})} + \Delta\pi_i \tag{11}$$

where $\Delta\pi_i$ denotes the additional risk premium.

Table 19: Real world and risk neutral default intensities ($\boldsymbol{\mathcal{R}} = 40\%$, 1997–2024)

Rating	$\lambda_i^{(\text{historical})}$	$\lambda_i^{(\text{risk-neutral})}$	$\Delta\pi_i$	α_i
AAA	2 bps	126 bps	75 bps	71.2
AA	6 bps	155 bps	90 bps	24.0
A	15 bps	204 bps	113 bps	13.4
BBB	45 bps	311 bps	160 bps	7.0
BB	168 bps	585 bps	250 bps	3.5
B	423 bps	898 bps	285 bps	2.1
CCC	830 bps	1 822 bps	596 bps	2.2

In Table 19, we report the values of $\lambda_i^{(\text{historical})}$, $\lambda_i^{(\text{risk-neutral})}$, $\Delta\pi_i$ and α_i. We confirm that $\lambda_i^{(\text{risk-neutral})} > \lambda_i^{(\text{historical})}$, indicating the presence of a positive additional risk premium ($\Delta\pi_i > 0$) and a risk premium multiplier greater than one ($\alpha_i > 1$), regardless of the rating category. As noted by Hull *et al.* (2005), the ratio α_i decreases and the difference $\Delta\pi_i$ increases as credit ratings decline. This suggests that investors demand an additional risk premium for lower-rated assets. In the additive model, this additional premium increases with the credit risk of the asset. For example, investors demand more compensation for investing in CCC-rated bonds than in BBB-rated bonds. This additional compensation cannot be attributed to default risk alone, as this is already captured by the historical risk premium $\pi_i^{(\text{historical})}$. It must therefore reflect other sources of risk. Hull *et al.* (2005) identifies two such factors: liquidity risk and non-diversifiable risk. Liquidity risk is a well-documented driver of expected corporate bond returns (Lin *et al.*, 2011), while the role of non-diversifiable risk is less clearly established. The key idea is that defaults become more difficult to diversify during periods of market stress. In addition, Hull suggests that the probability measures used by bond traders and managers may deviate from long-term historical averages. They may overweight adverse historical scenarios, effectively skewing the distribution. As a result, these risk factors help explain the gap between historical and risk-neutral spreads after accounting for default risk. As both liquidity and non-diversifiable risk tend to be positively correlated with credit risk, we observe a positive relationship between the level of credit risk and the additional compensation — above and beyond default risk — that investors demand. If we focus on the multiplicative model, a slightly different picture emerges. While the absolute additional risk premium increases as credit ratings decline, the multiplicative (relative) risk premium actually decreases with lower credit ratings. In relative terms, investors in AAA-rated corporate bonds demand a higher risk premium multiplier than investors in CCC-rated bonds. There are two main reasons for this. First, investors in AAA bonds tend to be more risk averse than investors in CCC bonds. Second, the skewness risk — the risk of rare and extreme losses — is significantly higher for investment grade bonds than for high yield bonds. This helps to explain why the risk premium multiplier tends to be higher for well-rated bonds (often above 10), as the subjective perception of risk is much higher than the objective risk. Conversely, the multiplier for lower-rated bonds tends to be lower (often below 5) because the subjective and objective risks are more closely aligned.

The previous figures were calibrated between 1997 and 2024 using daily bond yields. These models can be extended to include the dynamics of spreads: $\pi_i^{(\text{risk-neutral})}(t) = \alpha_i(t)\,\pi_i^{(\text{historical})}$ and $\pi_i^{(\text{risk-neutral})}(t) = \pi_i^{(\text{historical})} + \Delta\pi_i(t)$. The two models are equivalent since the historical risk premium is assumed to be constant[23] and we have the relationship[24]: $\Delta\pi_i(t) \propto \alpha_i(t)$. In what follows, we adopt the multiplicative model because it is both more tractable and easier to calibrate. Additionally, it also better captures the stark contrast in risk perception between investors in junior and senior tranches in the blended finance market. Junior tranche investors typically have a deliberately low financial risk aversion, which is effectively reflected in the multiplicative framework. Following Gregory (2014), we assume that:

$$\alpha = \left(\frac{\gamma}{\text{EL}}\right)^{\beta} \tag{12}$$

We calibrate this function[25] using the previous figures in Table 19 and obtain the α curve shown in Figure 19. The estimated values are $\hat{\gamma} = 0.110$ and $\hat{\beta} = 0.574$. We also present the calibrated curves for the years 2024 and 2008. The parameter γ can be interpreted as a measure of overall risk aversion in the credit market, governing the general level of risk sensitivity. The parameter β, on the other hand, captures the discrepancy in risk attitudes

[23]See Figure 38 on page 82, which compares the histogram of $\alpha_i(t)$ with the histogram of $\Delta\pi_i(t)$ for the two extreme ratings (AAA and CCC).

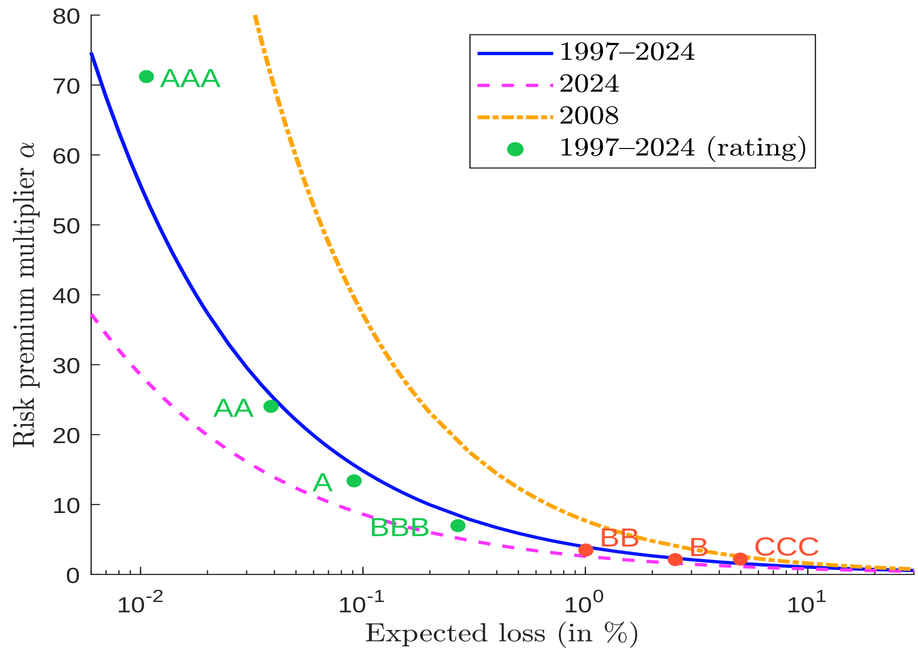
[24]The exact formula is $\Delta\pi_i(t) = \left(\alpha_i(t) - 1\right)\pi_i^{(\text{historical})}$.

[25]The parameters β and γ are estimated using the following linear regression:

$$\ln \alpha_i = \beta \ln \gamma - \beta \ln\left((1-R)\,\lambda_i^{(\text{historical})}\right) + \varepsilon_i$$

between risk-averse investors and risk-seeking investors[26]. It is clear that risk aversion (both in terms of level and slope) is higher in 2024 than in 2008. When designing a blended finance fund, it is essential to account for the dynamics of the risk premia demanded by investors[27]. For instance, an excessively high value of β may be prohibitive, as it implies a disproportionately high relative risk premium required by senior investors.

Figure 19: Risk premium multiplier α



Source: ICE BOFA US Corporate Bonds, Moody's & Authors' calculations.

4.1.2 Economic rationale behind tranching structures

The previous framework can be used to understand the economic rationale for tranching. The expected loss $\mathrm{EL}_{\text{portfolio}}$ of the asset portfolio can be decomposed as a weighted average of the expected losses EL_k of the different tranches:

$$\mathrm{EL}_{\text{portfolio}} = \sum_{k=1}^{m} \frac{(D_k - A_k)}{(D_m - A_1)} \mathrm{EL}_k = \sum_{k=1}^{m} \omega_k \, \mathrm{EL}_k$$

where m is the number of tranches, A_k and D_k are the attachment and detachment points of the k^{th} tranche, and ω_k is the weight in % of the k^{th} tranche. Let α_k be the relative return required by investor for taking the risk of the k^{th} tranche. It follows that:

$$\pi_{\text{portfolio}} = \alpha \, \mathrm{EL}_{\text{portfolio}} > \sum_{k=1}^{m} \alpha_k \omega_k \, \mathrm{EL}_k = \sum_{k=1}^{m} \omega_k \pi_k = \pi_{\text{tranching}} \tag{13}$$

where $\pi_{\text{portfolio}}$, π_k and $\pi_{\text{tranching}}$ are the risk premia of the asset portfolio, the k^{th} tranche and the tranching structure. According to Gregory (2014), $\pi_{\text{portfolio}} = \alpha \, \mathrm{EL}_{\text{portfolio}}$ represents "*the compensation for taking the credit risk of the asset portfolio*", while $\pi_{\text{tranching}} =$

[26] The impacts of the parameters β and γ are illustrated in Figure 39 on page 82.
[27] The time series of $\hat{\beta}$ and $\hat{\gamma}$ are shown in Figure 40 on page 83.

$\sum_{k=1}^{m} \omega_k \pi_k$ is "*the total compensation that must be paid to the various investors.*" Consider a typical investor who is faced with the choice of buying the entire portfolio of assets or investing in the individual tranches. In the first case, the investor will demand higher compensation than in the second. At first glance, this may seem counterintuitive, since buying all the tranches is economically equivalent to buying the entire portfolio. However, this result occurs because investors focus on the credit risk associated with each tranche. They tend to demand higher multiplicative risk premiums for safer (lower credit risk) tranches than for riskier tranches. It is important to note that this example is somewhat stylized and does not fully reflect real-world behavior. In practice, an investor who buys the senior tranche is unlikely to also invest in the junior tranche. As a result, inequality $\pi_{\text{portfolio}} > \pi_{\text{tranching}}$ holds due to the heterogeneity of investors and their different levels of risk aversion. Combining Equations (12) and (13), we finally obtain:

$$\pi_{\text{portfolio}} > \pi_{\text{tranching}} \Leftrightarrow \mathrm{EL}_{\text{portfolio}} > \left(\sum_{k=1}^{m} \omega_k \, \mathrm{EL}_k^{1-\beta}\right)^{\dfrac{1}{1-\beta}} \tag{14}$$

This property holds[28] if $\beta > 0$ and $\beta \neq 1$, which is the case in the credit market because of the risk aversion function of investors. If investors' risk aversion does not follow the hyperbolic pattern described by Equation (12), it is possible that $\pi_{\text{portfolio}} < \pi_{\text{structuring}}$, implying that the structure is not economically viable. In such cases, there may be insufficient demand for investments in the various tranches, resulting in insufficient participation. Put differently, investors may be reluctant to engage in this form of structured blended finance.

Let us consider an example[29] to illustrate the previous property. We use the input parameters shown in Table 20. For each rating, we report the five-year probability of default (in %), the annual spread (in basis points), and the α multiplier. Two sets of α parameters are used: set (a) satisfies the hyperbolic property, while set (b) does not, instead exhibiting a more parabolic shape. The portfolio has a maturity of five years. We invest in 100 green bonds, each with a nominal value of \$1 and a five-year probability of default of 2% (equivalent to a BBB rating). We assume a recovery rate of 40% and a homogeneous default correlation of 20%. Without loss of generality, we do not model cash flows. Alternatively, we consider a tranche structure with junior, mezzanine and senior layers. The detachment point of the junior tranche is set at \$5, and that of the mezzanine tranche at \$10.

Table 20: Input parameters (Example #3)

Rating	5Y PD (in %)	s (in bps)		α	
		(a)	(b)	(a)	(b)
AAA	0.15	25	50	20.00	10.00
AA	0.25	40	100	15.00	8.00
A	0.60	75	150	10.00	5.00
BBB	2.25	150	200	5.00	4.00
BB	11.50	300	700	2.00	4.00
B	32.00	600	1000	1.50	5.00
CCC	60.00	1 200	2 500	1.25	6.00

Results are shown in Table 21. The second column indicates the weight ω_k of the tranche in the junior-mezzanine-senior structure. For example, the junior tranche has a

[28] Using Appendix A.3 on page 78, we deduce that the inequality holds if $p = 1 - \beta < 1$ or $\beta > 0$.
[29] This example is inspired by Gregory (2014).

Table 21: Calculation of the economic excess spread generated by the tranching structure (Example #3, set (a) and $\rho = 20\%$)

Tranche	ω_k (in %)	EL_k	$\mathrm{EL}_k^\star$ (in %)	$\boldsymbol{R}_k$	α_k	π_k (in %)	s_k (in bps)
Junior	5	1.102	22.034	CCC	1.2	27.542	1 200
Mezzanine	5	0.088	1.755	BB	2.0	3.509	300
Senior	90	0.015	0.017	AAA	20.0	0.344	25
Tranching	100	1.205	1.205	BBB	1.5	1.862	98
Portfolio	100	1.205	1.205	BBB	5.0	6.024	150
Difference	0	0.000	0.000		3.5	4.163	52

weight of 5%. The third column is the expected loss EL_k of each tranche[30]. The fourth column is the normalized expected loss $\mathrm{EL}_k^\star = \mathrm{EL}_k / \omega_k$, the fifth column is the rating of the tranche[31], while the sixth column is the alpha multiplier α_k. Then we report the risk premium $\pi_k = \alpha_k \mathrm{EL}_k^\star$ in the seventh column and the spread in the eighth column. The five-year risk premia required by junior, mezzanine and senior investors are 27.5%, 3.51% and 34 bps, respectively[32]. Calculating the risk premium for the junior-mezzanine-senior structure[33] gives $\pi_{\text{tranching}} = 1.862\%$. It is higher than the portfolio risk premium ($\pi_{\text{portfolio}} = 6.024\%$). Similarly, we obtain $s_{\text{tranching}} = 98\,\text{bps} < s_{\text{portfolio}} = 150\,\text{bps}$. This implies that this structure may reward investors better than what is currently priced by the credit market. This is due to the fact that the structure has significantly reduced the risk premium multiplier from 5.0 to 1.5. Indeed, the structure has rebalanced and concentrated the risk on the junior tranche, which comprises the less risk-averse investors.

Table 22: Calculation of the economic excess spread generated by the tranching structure (Example #3, set (b) and $\rho = 20\%$)

Tranche	ω_k (in %)	EL_k	$\mathrm{EL}_k^\star$ (in %)	$\boldsymbol{R}_k$	α_k	π_k (in %)	s_k (in bps)
Junior	5	1.102	22.034	CCC	6.0	132.201	2 500
Mezzanine	5	0.088	1.755	BB	4.0	7.018	700
Senior	90	0.015	0.017	AAA	10.0	0.172	50
Tranching	100	1.205	1.205	BBB	5.9	7.116	205
Portfolio	100	1.205	1.205	BBB	4.0	4.819	200
Difference	0	0.000	0.000		−1.9	−2.296	− 5

Now we consider the set (b) of parameters. Since EL_k and $\mathrm{EL}_k^\star$ do not depend on α_k and s_k, we get the same expected losses, but the values of $\pi_{\text{tranching}}$ and $s_{\text{tranching}}$ are different.

[30] These figures were obtained using the Monte Carlo method with 10^6 simulations. We have:

$$\mathrm{EL}_k = \frac{1}{n_S} \sum_{s=1}^{n_S} \min \left(D_k - A_k, \max \left(\sum_{i=1}^{n} N_i \cdot (1 - \boldsymbol{\mathcal{R}}_i) \cdot \mathbb{1}\left\{ \boldsymbol{\tau}_{i,s} \leq T \right\} - A_k, 0 \right) \right)$$

where n_S is the number of simulations, $N_i = \$1$, $Recovery_i = 40\%$, $\boldsymbol{\tau}_{i,s}$ is the simulated default time of bond i for simulation s, $T = 5$ years, A_k and D_k are the attachment and detachment points of the k^{th} tranche (*e.g.*, $A_{\text{mezzanine}} = \$5$ and $D_{\text{mezzanine}} = \$10$).

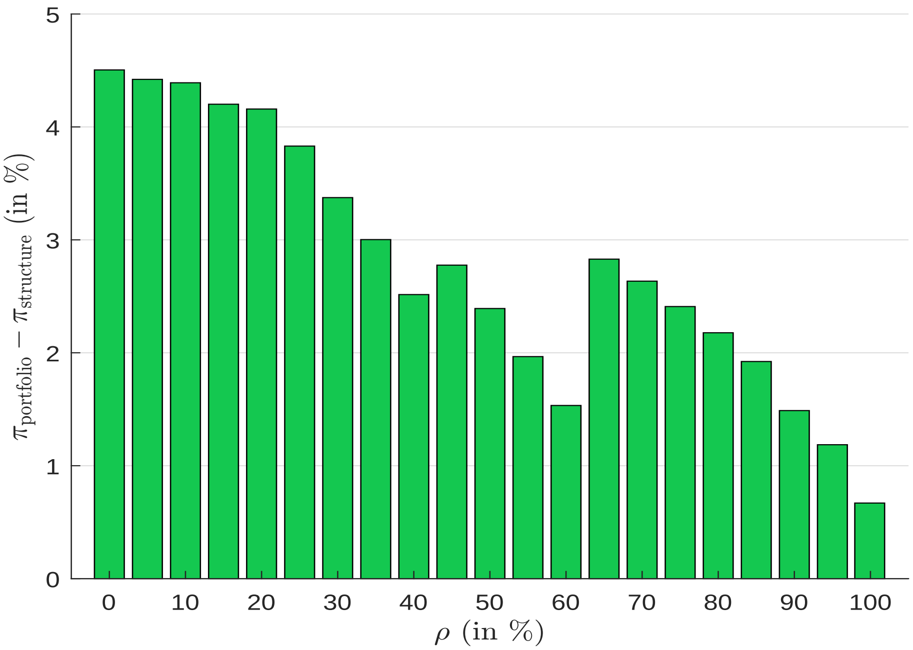
[31] We calculate the 5-year probability of default $\mathrm{PD}_k(5Y) = \mathrm{EL}_k^\star / (1 - \boldsymbol{\mathcal{R}})$ and assign the rating $r^\star = \sup \left\{ r : \mathrm{PD}_k(5Y) \leq \mathrm{PD}_r(5Y) \right\}$ where $\mathrm{PD}_r(5Y)$ is the 5-year probability of default for rating r.

[32] Note that the AAA risk premium is very low, as the five-year expected loss on the AAA tranche is about 1.7 bps. The senior tranche is more of a super senior tranche, which explains the 34 bps.

[33] We have $\pi_{\text{tranching}} = \omega_{\text{junior}}\pi_{\text{junior}} + \omega_{\text{mezzanine}}\pi_{\text{mezzanine}} + \omega_{\text{senior}}\pi_{\text{senior}} = 5\% \times 27.542\% + 5\% \times 3.509\% \times 90\% \times 0.344\% = 1.862\%$.

More importantly, we have $\pi_{\text{tranching}} > \pi_{\text{portfolio}}$ and $\mathcal{s}_{\text{tranching}} > \mathcal{s}_{\text{portfolio}}$. We are not able to structure the portfolio to offer a better risk-return profile than the market. In this context, this junior-mezzanine-senior structure is not viable due to the risk aversion function of the investors in the credit market.

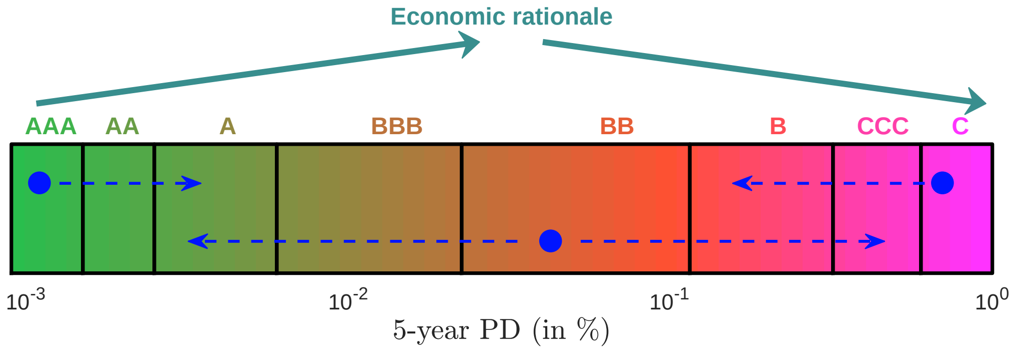
Figure 20: Extra risk premium (Example #3, set (a))



The previous analysis may give the impression that the value added by the structure depends only on risk aversion patterns. In fact, it also depends strongly on the assets. First, we can show that there is more value added when the default correlation is low than when the default correlation is high. The reason is that there is a real difference between the three layers, and risk clustering is really effective when the default correlation is low. In contrast, when the default correlation is high, it becomes difficult to differentiate the risk/return profiles across tranches, thereby reducing the effectiveness of the structure. This observation has important implications for the design of an SBF fund. In particular, it highlights the importance of building a diversified portfolio. A diversified fund, composed of assets from different countries and sectors, is better suited for effective structuring than a fund concentrated in a single sector or geographic area. Figure 41 on page 83 illustrates how asset default probabilities affect the value of the junior-mezzanine-senior structure. If all assets have a uniform probability of default of 0.5%, the economic justification for tranche-based structuring is significantly weakened. However, if the portfolio is composed of 80% assets with a 0.5% probability of default and 20% with an 8.0% probability of default, the value proposition of the structure improves significantly.

More generally, the economic rationale and value added of the junior-mezzanine-senior structure tends to increase with the average default probability of the portfolio up to a certain threshold. This peak typically occurs near the crossover point between BBB and BB ratings (Figure 21). Beyond this point, the value added begins to decline. As a result, structuring offers limited benefit for portfolios at the extremes of the credit spectrum, such as those rated AAA or CCC. Consider the case of a fine-grained credit portfolio with homogeneous

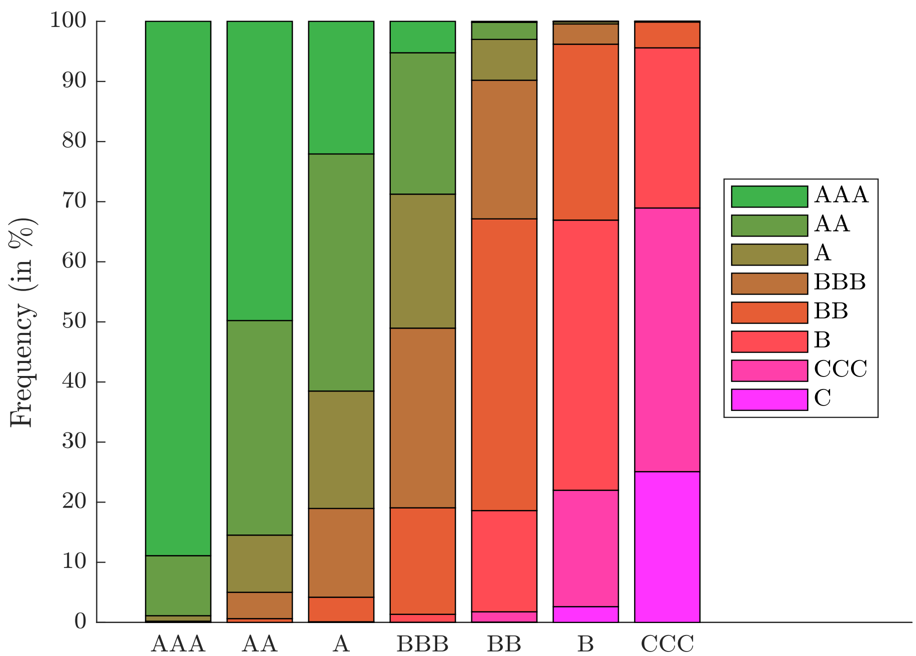
Figure 21: Economic rationale and credit quality of the asset portfolio



assets, each having a probability of default p. We calculate the frequency with which the portfolio loss falls into the j^{th} rating category[34]:

$$f_j(p) = \Pr\left\{Loss(t) \in \boldsymbol{R}_j\right\}$$

Figure 22: Frequency $f_j(p)$ ($t = 5$ years, Moody's, $\rho = 20\%$)



Source: Moody's & Authors' calculations.

Table 23 gives the values of the frequency $f_j(p)$ for different levels of default correlation ρ. We consider three asset portfolios rated AAA, BBB, and CCC, using the midpoint value

[34]The analytical expression is:

$$f_j(p) = \Phi\left(\sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}\left(p_j^+\right) - \sqrt{\frac{1}{\rho}}\Phi^{-1}(p)\right) - \Phi\left(\sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}\left(p_j^-\right) - \sqrt{\frac{1}{\rho}}\Phi^{-1}(p)\right)$$

where $\left[p_j^-, p_j^+\right]$ is the range of default probabilities corresponding to the rating $\boldsymbol{R}_j$.

of the probability of default p for each rating band. We observe that when the defaults are independent ($\rho = 0\%$), the default frequency $f_j(p)$ remains concentrated within the asset's own rating band. On the contrary, as the default correlation increases, the default frequency tends to spread beyond its original rating classification. In particular, even at a high default correlation of 80%, the default frequency for the AAA-rated portfolio remains largely within the investment grade category. This is not the case for the BBB-rated portfolio, which exhibits a greater dispersion of default frequency across the upper and lower rating bands. Graphically, we obtain the frequency dispersion shown in Figure 22. We verify that the dispersion is more important for the BBB- and BB-rated asset portfolios.

Table 23: Frequency $f_j(p)$ of three asset portfolios rated AAA, BBB and CCC ($t = 5$ years)

	$\boldsymbol{R}_j$	Example #3				Moody's			
		$\rho = 0\%$	20%	50%	80%	$\rho = 0\%$	20%	50%	80%
Portfolio AAA	AAA	100.00	87.77	93.60	98.06	100.00	88.89	95.37	99.00
	AA	0.00	5.35	1.78	0.35	0.00	9.99	3.29	0.52
	A	0.00	4.98	2.22	0.50	0.00	0.91	0.74	0.17
	BBB	0.00	1.80	1.75	0.55	0.00	0.20	0.44	0.15
	BB	0.00	0.10	0.60	0.38	0.00	0.01	0.15	0.11
	B	0.00	0.00	0.05	0.11	0.00	0.00	0.01	0.03
	CCC	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.01
	C	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01
Portfolio BBB	AAA	0.00	14.97	55.17	83.27	0.00	5.23	45.32	81.71
	AA	0.00	8.72	6.26	1.93	0.00	23.52	20.73	6.19
	A	0.00	21.28	10.66	3.15	0.00	22.29	9.98	2.70
	BBB	100.00	36.32	14.19	4.25	100.00	29.89	11.09	3.08
	BB	0.00	18.09	10.83	4.18	0.00	17.72	9.29	3.19
	B	0.00	0.62	2.46	1.89	0.00	1.32	2.78	1.61
	CCC	0.00	0.00	0.39	0.84	0.00	0.03	0.67	0.82
	C	0.00	0.00	0.04	0.50	0.00	0.00	0.14	0.70
Portfolio CCC	AAA	0.00	0.00	0.24	8.51	0.00	0.00	0.47	14.60
	AA	0.00	0.00	0.15	1.32	0.00	0.00	1.48	6.93
	A	0.00	0.00	0.50	2.80	0.00	0.00	1.88	4.53
	BBB	0.00	0.01	2.24	6.04	0.00	0.11	5.09	7.33
	BB	0.00	1.47	11.37	12.61	0.00	4.29	15.99	12.82
	B	0.00	22.38	22.74	13.88	0.00	26.64	22.22	12.01
	CCC	100.00	52.91	28.14	14.28	100.00	43.86	22.40	10.89
	C	0.00	23.23	34.63	40.56	0.00	25.09	30.46	30.88

Source: Moody's & Authors' calculations.

4.2 The junior-senior structure

While both blended finance and bank securitization use structured finance techniques, they serve fundamentally different purposes and are motivated by different goals. In traditional bank securitizations — such as collateralized loan obligations (CLOs) or collateralized debt obligations (CDOs) — the primary objective is to reduce balance sheet risk and optimize regulatory capital requirements. These structures are deeply rooted in regulatory arbitrage and capital efficiency, allowing banks to transfer credit risk and improve their capital ratios by selling loans or debt instruments they already own. Blended finance, on the other hand, operates under a very different logic. The underlying assets are typically off the balance sheet of the sponsor. In the case of the AP EGO fund, for example, there is a seven-year ramp-up period during which the portfolio manager (Amundi) actively invests in newly issued emerging market green bonds (Bolton *et al.*, 2020). These assets are not legacy holdings of the sponsor, in this case the International Finance Corporation (IFC), part of the World Bank Group. The purpose is therefore not to offload risk, but to initiate new investments. In this sense, while bank securitization can be seen as a form of disinvestment by the sponsor, blended finance is better understood as a sponsor-led investment strategy. A second key difference is the role and motivation of the sponsor. In blended finance structures, the sponsor typically invests in the equity tranche and takes a first loss position. This is a strategic choice aimed at leveraging public or concessional capital to attract private investment in the senior tranches. While blended finance is not always formally classified as impact investing, many of its projects pursue impact-oriented outcomes. The sponsor's objective is often to maximize the development or environmental impact of the overall investment by leveraging as much additional capital as possible. In contrast, in a bank securitization, the sponsor generally avoids holding the subordinated tranche unless it is necessary — typically only when external risk-seeking investors, such as hedge funds, do not fully subscribe. In such cases, the sponsor's participation is more of a residual necessity than a strategic commitment. In summary, although both approaches use structured finance and tranching, their underlying goals are very different: bank securitization prioritizes capital relief and risk transfer, while blended finance focuses on catalytic investment and mobilizing capital for impact. Therefore, the traditional literature on the optimal design of securities[35] based on information asymmetries and distribution costs cannot help us understand the structuring of a blended finance fund.

4.2.1 Comparison of sponsor and investor risk aversion and preferences

We consider a mean-variance framework and the quadratic utility function:

$$\mathcal{U}(x) = \big(\mu(x) - r\big) - \frac{\phi}{2}\sigma^2(x)$$

where x is the portfolio, $\mu(x)$ is the expected return, $\sigma(x)$ is the portfolio volatility and r is the risk-free rate. Here, ϕ is the risk aversion coefficient of the quadratic utility. Maximizing the utility function is equivalent to minimizing the γ-problem:

$$x^\star = \arg\max \frac{1}{2} x^\top \Sigma x - \gamma x^\top (\mu - r\mathbf{1}_n)$$

where $\gamma = \phi^{-1}$ is the investor's risk tolerance. Roncalli (2013) shows that the optimal portfolio is given by:

$$x^\star = \gamma \Sigma^{-1}(\mu - r\mathbf{1}_n)$$

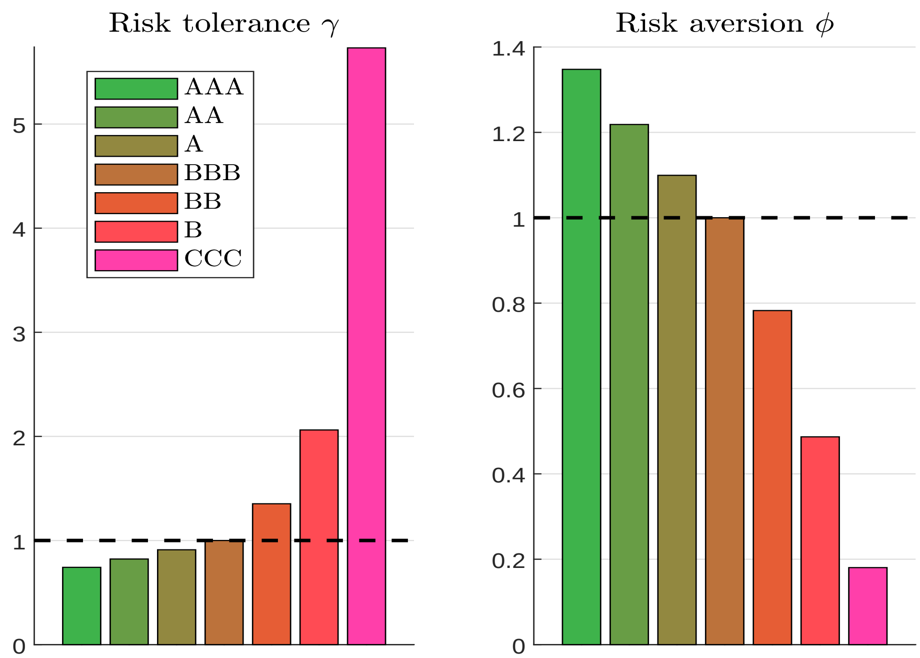
where:

$$\gamma = \frac{1}{\mathbf{1}_n^\top \Sigma^{-1}(\mu - r\mathbf{1}_n)}$$

Using historical returns from the ICE BOFA US Corporate Bonds database, we estimate the risk tolerance coefficient associated with the j^{th} rating using the empirical estimator $\hat{\gamma}_j = \hat{\sigma}_j^2/\hat{\pi}_j$ where $\hat{\pi}_j$ and $\hat{\sigma}_j$ are the long-term historical risk premium and volatility of corporate bonds rated $\boldsymbol{R}_j$. Results are given in Figure 23, where all values have been normalized such that the risk aversion of the BBB investor is set to one. As expected, risk tolerance increases as credit ratings decline, reflecting the well-known relationship between credit quality and investor behavior. Interestingly, the BBB investor appears more closely

[35] See for instance Boot and Thakor (1993), Plantin (2004) and DeMarzo (2005).

Figure 23: Investor's risk aversion and tolerance (1997–2024)



Source: ICE BOFA US Corporate Bonds& Authors' calculations.

aligned in risk tolerance with the A investor than with the BB investor. This suggests a nonlinear pattern: the change in risk aversion within the investment-grade segment is more gradual compared to the more pronounced shift observed in the high-yield segment. For instance, when the senior tranche is rated AAA and the junior tranche is rated CCC, we observe a risk aversion factor of 7. This implies that the sponsor — associated with the junior tranche — has a risk tolerance seven times higher than that of the institutional investors holding the senior tranche.

Institutional investors in the senior tranche and the sponsor of the junior tranche are both considered responsible investors. However, they play fundamentally different roles within the structure of a blended finance vehicle. Indeed, it is unusual for an investor without an established ESG policy or involvement in sustainable finance to hold a portion of the senior debt. In practice, participation in the senior tranche is often viewed as an intentional, values-driven investment, a commitment to sustainable principles and an expression of the investor's ESG stewardship strategy. However, despite their shared identity as responsible investors, sponsors and institutional investors diverge significantly in their approach to risk. This divergence stems from the structural difference in risk exposure between the senior and junior tranches. The senior tranche is typically rated AAA, which places it in the investment grade category. In contrast, the junior tranche — typically unrated — is often in the crossover or high yield categories. Its implied credit quality is typically below BBB and often ends up in the B range, although it can vary between CCC+ and BB+. As shown in Figure 23, the gap in risk appetite between, say, BBB investors and B investors is not trivial. Therefore, the sponsor of the junior tranche must have a significantly higher risk tolerance and a fundamentally different risk aversion profile than traditional senior investors. This difference in risk appetite is one of the main barriers to scaling blended finance initiatives. Among development finance institutions (DFIs), not all organizations have the capacity to take on such elevated levels of risk. In addition to risk tolerance constraints, many lack the specialized infrastructure and expertise required to effectively monitor and manage these risk exposures. As a result, the junior-senior structure requires not only sophisticated knowledge of financial structuring approaches, but also the participation of two categories of responsible investors with diametrically opposed risk profiles.

4.2.2 Fixed vs. optimized structure

Consider a 10-year bullet credit portfolio consisting of 100 bonds, each rated BB+. Tranche ratings are assigned by mapping the expected loss to the corresponding credit rating. Cumulative average default rates as published by S&P Global Corporate in 2025 are used and linearly interpolated to estimate values for intermediate rating notches (*e.g.*, BB+). A fixed recovery rate of 40% is assumed. Credit risk dependence is modeled using the Gaussian copula approach with correlation parameter ρ. For each tranche structure, we compute both the tranche size and the historical credit spread based on historical data from 7–10 year portfolios sourced from the ICE BofA US Corporate index. This data is also interpolated to improve granularity. Rating and spread data are summarized in Table 24.

Table 24: 10Y PD and historical spread data

Rating	PD	Spread	Rating	PD	Spread
AAA	0.12%	77 bps	BBB−	7.51%	242
AA+	0.41%	90 bps	BB+	11.61%	287
AA	0.70%	104 bps	BB	15.70%	332
AA−	1.11%	114 bps	BB−	22.02%	382
A+	1.51%	125 bps	B+	28.34%	431
A	1.91%	135 bps	B	34.66%	481
A−	2.41%	155 bps	B−	39.18%	628
BBB+	2.92%	176 bps	CCC+	43.70%	776
BBB	3.42%	197 bps	CCC	48.22%	923

Following the methodology outlined in Gregory (2014), we examine how changes in default correlation ρ affect the excess spread under both fixed and optimized tranche structures. We consider three fixed structures: 10/90, 30/70, and 50/50, where the first number denotes the size of the junior tranche and the second denotes the size of the senior tranche. In the case of the optimized structure, we maximize the leverage ratio subject to a rating constraint on the senior tranche:

$$\begin{aligned} D^{\star}_{\text{junior}} &= \arg\max \boldsymbol{\mathcal{LR}} = \frac{D_{\text{senior}} - A_{\text{senior}}}{D_{\text{junior}} - A_{\text{junior}}} \\ & \text{u.c.} \quad \left\{ \begin{array}{l} \boldsymbol{R}_{\text{senior}} \succeq \boldsymbol{R}^{\min}_{\text{senior}} \\ D_{\text{junior}} = A_{\text{senior}} \end{array} \right. \end{aligned} \tag{15}$$

where $\boldsymbol{\mathcal{LR}}$ is the leverage ratio, $\boldsymbol{R}_{\text{senior}}$ is the rating of the senior tranche, and $\boldsymbol{R}^{\min}_{\text{senior}}$ is the minimum acceptable rating. The objective function reflects the sponsor's preference to maximize capital mobilization, while the constraint captures the requirements of institutional investors seeking limited exposure to the credit risk of the portfolio.

When using a fixed tranche structure, controlling the rating of the senior tranche becomes a challenge. For example, in a 30/70 structure, the senior tranche only achieves a AAA rating if the default correlation is below 30%. If the correlation exceeds this threshold, the rating is downgraded from AA+ to BBB− (see Table 27 on page 84 and Figure 24). Structures with a

Figure 24: Fixed 30/70 structure

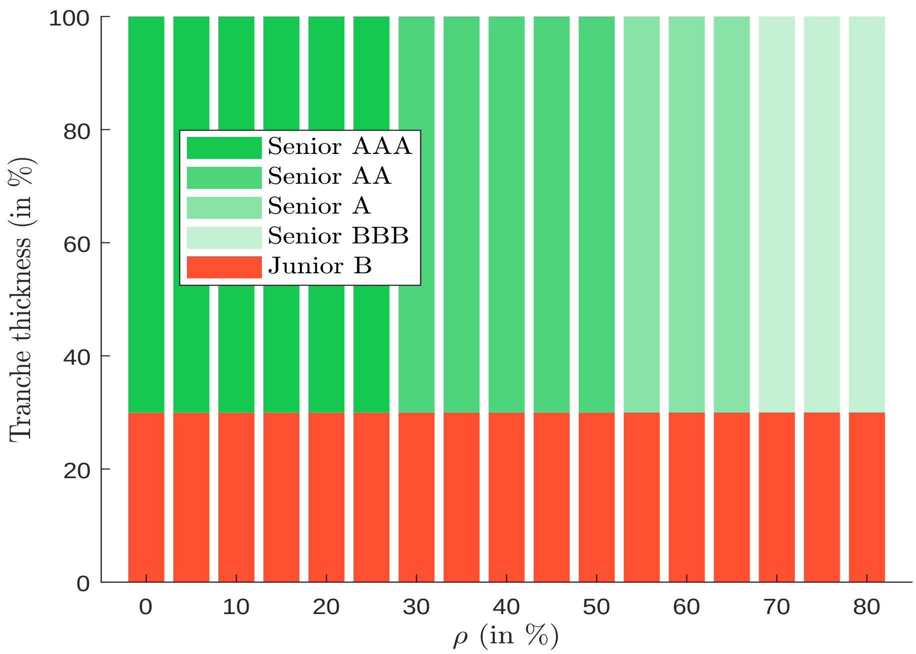


Figure 25: Optimized AAA structure

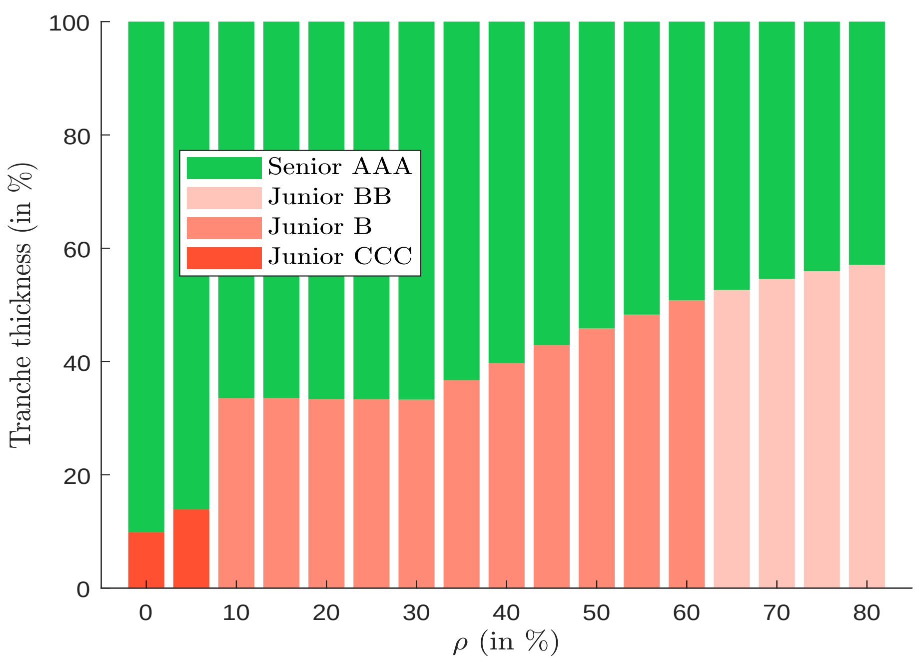


Figure 26: Leverage ratio of fixed and optimized structures

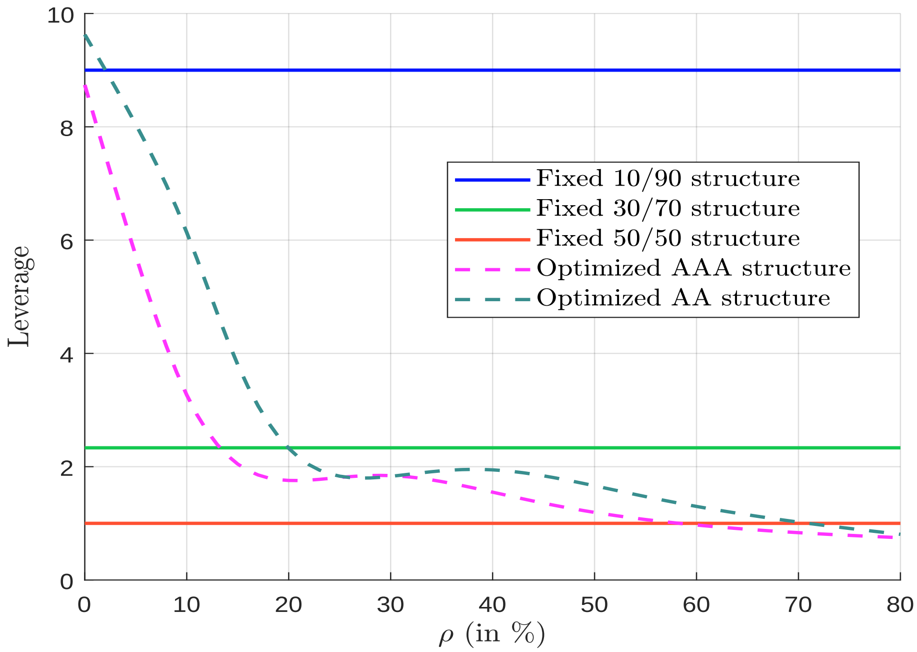
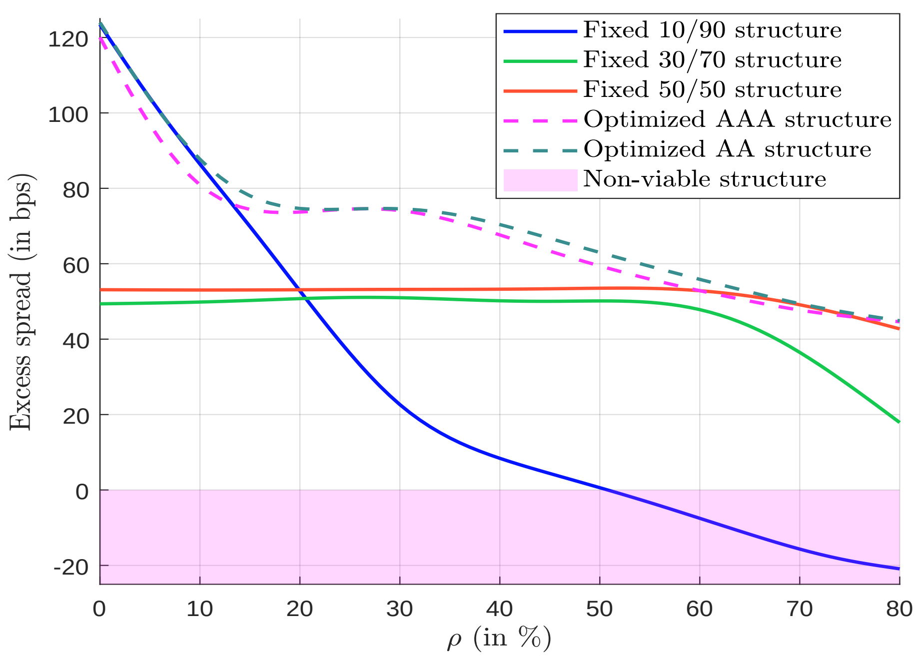


Figure 27: Excess risk premium in bps of fixed and optimized structures



thinner junior tranche show greater rating volatility, especially when the default correlation is between 0% and 30%. This poses a critical risk since the senior tranche may be exposed to credit losses as the default correlation increases. Conversely, increasing the thickness of the junior tranche helps stabilize the rating of the senior tranche. However, this comes at the cost of a significant reduction in the leverage ratio, which may limit capital mobilization[36]. Figure 25 shows the results of the optimization problem when we fix the rating of the senior tranche to AAA. In this case, the optimal size of the junior tranche depends on the default correlation. It is relatively low (less than 10%) when $\rho = 0\%$ and increases to 57.1% when ρ reaches 80%. We observe a jump in the range $\rho \in \left[5\%, 10\%\right]$ as the size of the junior tranche increases by about 20%. If we consider a lower rating constraint for the senior tranche ($\boldsymbol{R}_{\text{senior}} \succ \text{AA}-$), we obtain very similar results[37].

The leverage ratios corresponding to the different structures are shown in Figure 26. As expected, for fixed structures the leverage ratio remains constant by definition. However, the results for optimized structures are more informative. As expected, relaxing the rating constraint on the senior tranche leads to higher leverage ratios — but only when the default correlation is relatively low (below 20%). Figure 27 shows the corresponding excess risk premium. As expected, some fixed structures can become non-viable at high default correlations. For example, the 10/90 structure fails to generate a positive excess spread when $\rho \geq 50\%$. At more realistic levels of default correlation, optimized structures consistently outperform fixed structures. This implies that the portfolio manager has greater flexibility in allocating the excess risk premium between junior and senior tranches, thus achieving a better balance that can satisfy both groups of investors.

Remark 7. *The results on the excess spread are not a coincidence, as there is an equivalence between maximizing the leverage ratio and maximizing the excess risk premium*[38].

4.3 The case of the mezzanine tranche

We now introduce a mezzanine tranche into the capital structure. Starting with a fixed junior-senior structure, denoted $\omega_{\text{junior}}/\omega_{\text{senior}}$, two common approaches can be used to transform it into a three-tier structure: $\omega'_{\text{junior}}/\omega'_{\text{mezzanine}}/\omega'_{\text{senior}}$. The first is the split approach, where the original junior tranche is divided into junior and mezzanine tranches: $\omega_{\text{junior}} = \omega'_{\text{junior}} + \omega'_{\text{mezzanine}}$. The underlying rationale is to spread the risk across two different classes of investors. The second is the complement approach, where a portion of the senior tranche is reallocated to create a mezzanine tranche: $\omega_{\text{senior}} - \omega'_{\text{senior}} = \omega'_{\text{mezzanine}}$. The idea here is to introduce an intermediate layer that enhances the protection of the senior tranche. In the following analysis, we adopt simple illustrative rules. In the split approach, the junior tranche is split into two equal parts, while in the complement approach, the mezzanine tranche is fixed at 5% of the total structure. The corresponding results are shown in Tables 28–30 on page 86. The performance of the split approach is not always intuitive. In some cases, such as the 5/5/90 structure, it does not clearly distinguish between the junior and mezzanine tranches. In others, such as the 25/25/50 structure, it can result in a mezzanine tranche that behaves more like the senior tranche, undermining its intended purpose. In contrast, the complement approach tends to produce more desirable results. For example, in the 10/5/85 structure, the inclusion of a mezzanine tranche improves the credit quality and protection of the senior tranche. However, this approach may be redundant

[36]Results for fixed 10/90 and 50/50 structures are shown in Table 27 on page 84 and Figures 43 and 44 on page 85.

[37]See Figure 45 on page 86

[38]See the proof in Appendix A.4 on page 79.

when the mezzanine rating is close to that of the senior tranche, as observed in the 50/5/45 structure.

Since the previous results are difficult to understand, we consider the optimized approach to better understand when the mezzanine tranche is useful. A natural extension of the optimization program (15) is given by:

$$\begin{aligned}\left\{D^{\star}_{\text{junior}}, D^{\star}_{\text{mezzanine}}\right\} &= \arg\max \boldsymbol{\mathcal{LR}} = \frac{D_{\text{senior}} - A_{\text{mezzanine}}}{D_{\text{junior}} - A_{\text{junior}}} \\ & \text{u.c.} \quad \begin{cases} \boldsymbol{R}_{\text{senior}} \succeq \boldsymbol{R}^{\min}_{\text{senior}} \\ \boldsymbol{R}_{\text{junior}} \preceq \boldsymbol{R}^{\max}_{\text{junior}} \\ D_{\text{junior}} = A_{\text{mezzanine}} \\ D_{\text{mezzanine}} = A_{\text{senior}} \end{cases}\end{aligned} \tag{16}$$

where $\boldsymbol{R}^{\min}_{\text{senior}}$ is the minimum acceptable rating for the senior tranche, and $\boldsymbol{R}^{\max}_{\text{junior}}$ is the maximum acceptable rating for the senior tranche. However, this optimization problem is ill-posed. The solution effectively replaces the junior tranche with the mezzanine tranche, causing the junior tranche to disappear and the leverage ratio to diverge to infinity. This result illustrates that maximizing leverage in a three-tier structure is not an appropriate objective, as the respective roles of the junior and mezzanine tranches become ambiguous. Furthermore, the formulation assumes that the sponsor only invests in the junior tranche, whereas in practice the sponsor often participates in the mezzanine tranche of a blended finance fund. Therefore, an alternative optimization framework is needed. In the two-tier case, we observed that maximizing leverage is equivalent to maximizing the excess risk premium. We now generalize this insight to the three-tier structure as follows:

$$\begin{aligned}\left\{D^{\star}_{\text{junior}}, D^{\star}_{\text{mezzanine}}\right\} &= \arg\max \Delta\pi \\ & \text{u.c.} \quad \begin{cases} \boldsymbol{R}_{\text{senior}} \succeq \boldsymbol{R}^{\min}_{\text{senior}} \\ \boldsymbol{R}_{\text{junior}} \succeq \boldsymbol{R}^{\min}_{\text{junior}} \\ D_{\text{junior}} = A_{\text{mezzanine}} \\ D_{\text{mezzanine}} = A_{\text{senior}} \end{cases}\end{aligned} \tag{17}$$

where:

$$\begin{cases} \Delta\pi = \pi_{\text{portfolio}} - \omega_{\text{junior}}\pi_{\text{junior}} - \omega_{\text{mezzanine}}\pi_{\text{mezzanine}} - \omega_{\text{senior}}\pi_{\text{senior}} \\ \omega_{\text{junior}} = \dfrac{D_{\text{junior}} - A_{\text{junior}}}{D_{\text{senior}} - A_{\text{junior}}} \\ \omega_{\text{senior}} = \dfrac{D_{\text{senior}} - A_{\text{senior}}}{D_{\text{senior}} - A_{\text{junior}}} \\ \omega_{\text{mezzanine}} = 1 - \omega_{\text{junior}} - \omega_{\text{senior}} \end{cases}$$

We impose a minimum acceptable rating $\boldsymbol{R}^{\min}_{\text{junior}}$ on the junior tranche — typically a non-investment grade rating. This constraint is critical because it indirectly imposes a limit on the leverage ratio. The constraint $\boldsymbol{R}_{\text{senior}} \succeq \boldsymbol{R}^{\min}_{\text{senior}}$ determines the required credit quality of the senior tranche. In most cases, $\boldsymbol{R}^{\min}_{\text{senior}}$ is set to AAA. Within this framework, we aim to optimize the excess risk premium, ideally ensuring that the mezzanine tranche achieves an investment grade rating. This is particularly important as blended finance structures with two non-investment grade tranches are generally difficult to market to investors.

Consider the previous example with $\boldsymbol{R}^{\min}_{\text{junior}} = \text{CCC}$ and $\boldsymbol{R}^{\min}_{\text{senior}} = \text{AAA}$. The results are shown in Figure 28. We observe that the mezzanine tranche appears only when the default

Figure 28: Optimized junior-mezzanine-senior CCC/AAA structure

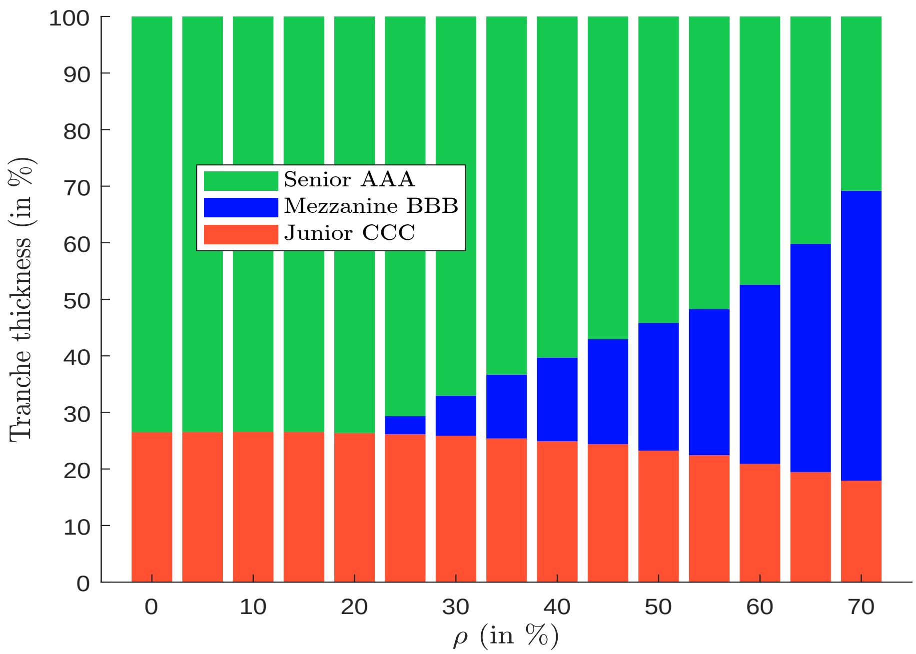


Figure 29: Leverage ratio of optimized structures

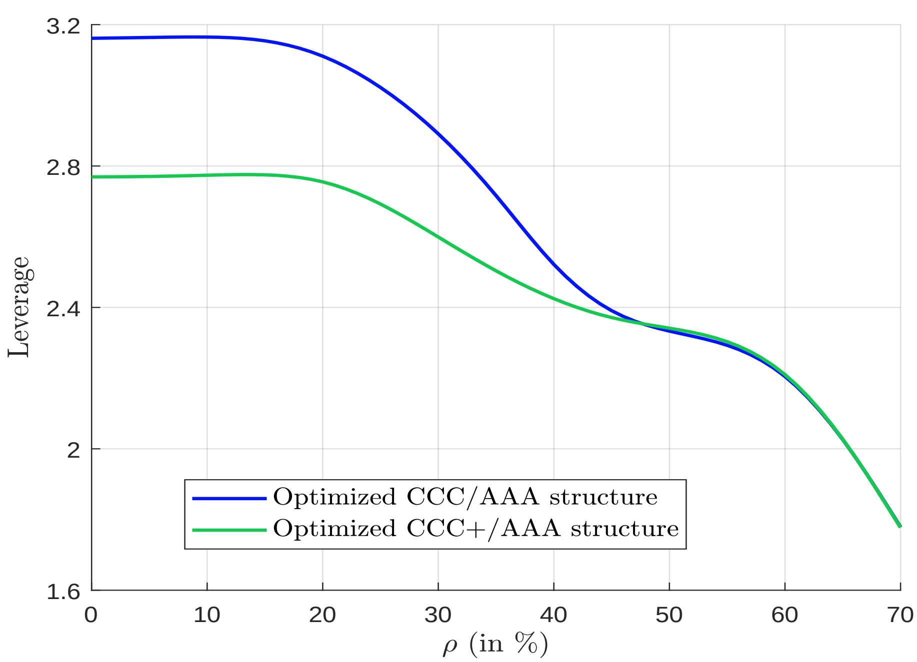
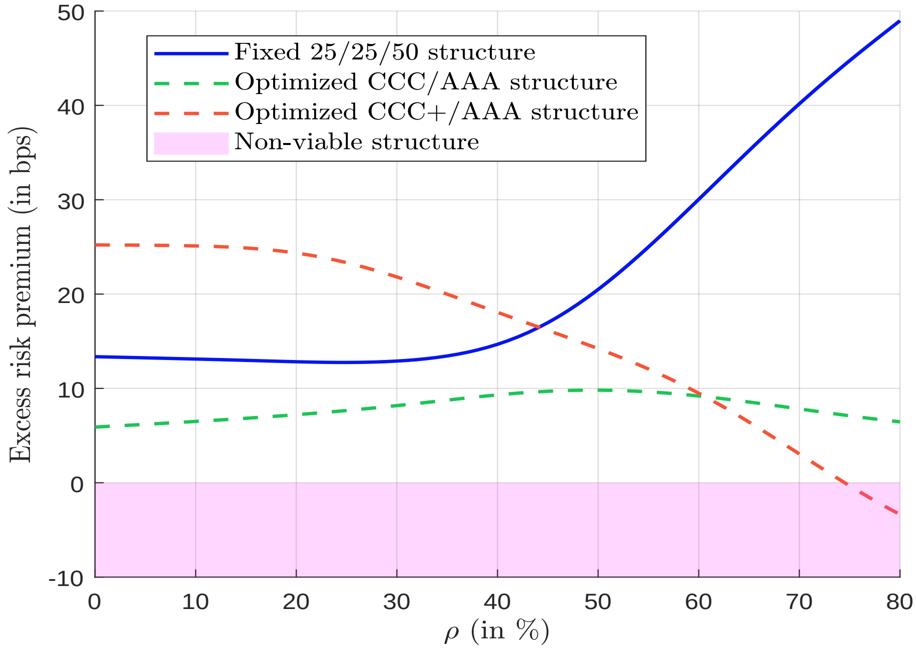


Figure 30: Excess risk premium in bps of fixed and optimized structures



correlation is greater than or equal to 20%. Below this threshold, the optimized structure consists solely of senior and junior tranches. The justification for including a mezzanine tranche depends on the level of default correlation. A mezzanine tranche is justified when there is a significant risk of systemic default. Conversely, if the credit risk of the portfolio is well diversified, the addition of a mezzanine tranche is unnecessary, as the risk can be efficiently allocated between the senior and junior tranches. However, we find that even modest increases in default correlation quickly necessitate the inclusion of a mezzanine tranche. The resulting three-tier structure proves advantageous for managing the ratings of the senior and junior tranches — something that is less easily achieved with only two tiers. Figures 29 and 30 illustrate the leverage and excess risk premium of the optimized solution. One might ask whether the rating of the junior tranche can be improved. To explore this, we solve the optimization problem again with $\boldsymbol{R}_{\text{junior}}^{\min} = \text{CCC+}$ and $\boldsymbol{R}_{\text{senior}}^{\min} = \text{AAA}$. The results remain largely the same[39]. Obviously, increasing the minimum acceptable rating of the junior tranche results in lower leverage (Figure 29). However, it also results in a higher excess risk premium (Figure 30). This underscores a key trade-off in structuring: maximizing leverage versus maximizing excess risk premium in a junior-mezzanine-senior configuration. Finally, these findings highlight that the role of the mezzanine tranche is non-trivial and closely linked to the underlying default correlation.

4.4 Senior protection mechanisms

In addition to well-established payment mechanisms such as the waterfall or capital stack, three additional tools can be used to further protect the capital and dividend payments of the senior tranche. These include loss carryforward, dividend sponsorship and cash reserves.

- Loss carryforward is a financial strategy used to manage and mitigate losses, particu-

[39]See Table 31 and Figure 46 on page 88.

Algorithm 3 Loss carryforward mechanism without sponsoring

Initialize the number of simulations to n_S

for $s = 1 : n_s$ **do**

Simulate the correlated default times $\left(\boldsymbol{\tau}_{1,s}, \ldots, \boldsymbol{\tau}_{n,s}\right)$ of the n issuers

Set $\mathrm{EB}_{\mathrm{junior},s}(0) \leftarrow \mathrm{Inv}_{\mathrm{junior}}$ and $\mathrm{EB}_{\mathrm{senior},s} \leftarrow \mathrm{Inv}_{\mathrm{senior}}$

Initialize the reweight ratio $\mathrm{RW}_s(0)$ to the scalar 1

Set the cumulative carried loss $\mathrm{CCL}_s(0) \leftarrow 0$

for $t = 1 : T$ **do**

Update the beginning balances: $\mathrm{BB}_{\mathrm{junior},s}(t) \leftarrow \mathrm{EB}_{\mathrm{junior},s}(t-1)$ and $\mathrm{BB}_{\mathrm{senior},s}(t) \leftarrow \mathrm{EB}_{\mathrm{senior},s}(t-1)$

Simulate the credit loss and the recovery of the portfolio:

$$\left\{\begin{array}{l} \mathcal{L}oss_s(t-1,t) = \sum_{i=1}^n (1-\boldsymbol{\mathcal{R}}_i) \cdot N_i \cdot \mathrm{RW}_s(t-1) \cdot \mathbb{1}\left\{t-1 < \boldsymbol{\tau}_{i,s} \leq t\right\} \\ \boldsymbol{\mathcal{R}}_s(t) = \sum_{i=1}^n \boldsymbol{\mathcal{R}}_i \cdot N_i \cdot \mathrm{RW}_s(t-1) \cdot \mathbb{1}\left\{t-1 < \boldsymbol{\tau}_{i,s} \leq t\right\} \end{array}\right.$$

Compute the dividends of the portfolio and the senior tranche:

$$\left\{\begin{array}{l} \mathrm{Div}_{\mathrm{portfolio},s}(t) = \sum_{i=1}^n N_i \cdot c_i \cdot \mathrm{RW}_s(t-1) \cdot \mathbb{1}\left\{\boldsymbol{\tau}_{i,s} > t\right\} \\ \mathrm{Div}_{\mathrm{senior},s}(t) = \min\left(\mathrm{Div}_{\mathrm{portfolio},s}(t), c_{\mathrm{senior}} \cdot \mathrm{BB}_{\mathrm{senior},s}(t)\right) \end{array}\right.$$

where c_i is the coupon of Bond i and c_{senior} is the senior coupon

Compute the dividends of the junior tranche without and with netting:

$$\left\{\begin{array}{l} \mathrm{Div}_{\mathrm{junior},s}^{\mathrm{w/o}}(t) = \mathrm{Div}_{\mathrm{portfolio},s}(t) - \mathrm{Div}_{\mathrm{senior},s}(t) \\ \mathrm{Div}_{\mathrm{junior},s}(t) = \max\left(0, \mathrm{Div}_{\mathrm{junior},s}^{\mathrm{w/o}}(t) - \mathcal{L}oss_s(t-1,t) + \mathrm{CCL}_s(t-1)\right) \end{array}\right.$$

Update the cumulative carried loss:

$$\mathrm{CCL}_s(t) \leftarrow \mathrm{CCL}_s(t-1) + \mathrm{Div}_{\mathrm{junior},s}^{\mathrm{w/o}}(t) - \mathcal{L}oss_s(t-1,t) - \mathrm{Div}_{\mathrm{junior},s}(t)$$

Compute the amount to reinvest in the portfolio when $t < T$:

$$\mathrm{ReInv}_s(t) = \mathrm{Div}_{\mathrm{junior},s}^{\mathrm{w/o}}(t) - \mathrm{Div}_{\mathrm{junior},s}(t) + \boldsymbol{\mathcal{R}}_s(t)$$

Update the reweight ratio:

$$\mathrm{RW}_s(t) \leftarrow \mathrm{RW}_s(t-1) \cdot \left(1 + \frac{\mathrm{ReInv}_s(t)}{\sum_{j=1}^n \mathrm{RW}_s(t-1) \cdot \mathbb{1}\left\{\boldsymbol{\tau}_{j,s} > t\right\}}\right)$$

Compute the repayment ($\mathrm{RePay}_s(t) = 0$ if $t < T$):

$$\mathrm{RePay}_s(t) = \mathbb{1}\{t = T\} \cdot \left(\sum_{i=1}^n N_i \cdot \mathrm{RW}_s(t-1) \cdot \mathbb{1}\left\{\boldsymbol{\tau}_{i,s} > t\right\} + \mathrm{ReInv}_s(t)\right)$$

Allocate the loss to the junior and senior tranches:

$$\left\{\begin{array}{l} \mathcal{L}oss_{\mathrm{junior},s}(t) = \min\left(\max\left(D_{\mathrm{junior}} - \sum_{u=1}^{t-1} \mathcal{L}oss_s(u-1,u), 0\right), \mathcal{L}oss_s(t-1,t)\right) \\ \mathcal{L}oss_{\mathrm{senior},s}(t) = \mathcal{L}oss_s(t) - \mathcal{L}oss_{\mathrm{junior},s}(t) \end{array}\right.$$

Compute the repayments:

$$\left\{\begin{array}{l} \mathrm{RePay}_{\mathrm{senior},s}(t) = \min\left(\mathrm{RePay}_s(t), \mathrm{BB}_{\mathrm{senior},s}(t)\right) \\ \mathrm{RePay}_{\mathrm{junior},s}(t) = \mathrm{RePay}_s(t) - \mathrm{RePay}_{\mathrm{senior},s}(t) \end{array}\right.$$

Update the ending balances: $\mathrm{EB}_{\mathrm{junior},s}(t) = \mathrm{BB}_{\mathrm{junior},s}(t) - \mathcal{L}oss_{\mathrm{junior},s}(t) - \mathrm{RePay}_{\mathrm{junior},s}(t)$ and $\mathrm{EB}_{\mathrm{senior},s}(t) = \mathrm{BB}_{\mathrm{senior},s}(t) - \mathcal{L}oss_{\mathrm{senior},s}(t) - \mathrm{RePay}_{\mathrm{senior},s}(t)$

end for

end for

larly in the early stages of a project. Rather than allowing these losses to immediately affect the financial returns of the fund, they are deferred and carried into future periods. Typically, these losses are absorbed by concessional capital or junior investors, thereby protecting the expected returns of senior investors. When the project starts generating profits in later periods, those profits are first used to offset the accumulated losses before any distributions are made to investors. In some cases, junior investors may choose to absorb a portion of the losses in the year they occur by voluntarily reducing or waiving their dividends.

- Dividend sponsorship is a mechanism whereby first loss capital is used to make up any shortfall in expected dividend payments to senior investors, particularly if the fund's income or performance falls below target levels. In the case of an open-ended structure, the sponsoring entity may inject additional capital. Alternatively, a portion of the concessional capital equal to the dividend shortfall may be reallocated to the senior tranche. For example, consider a SBF vehicle with a capital structure of 10% junior and 90% senior tranches. If the fund returns 4% in a given year, but the promised coupon to senior investors is 5%, there is a shortfall of 1%. To cover this shortfall, 0.5% of the value of the fund must be liquidated (*i.e.*, $4\% - 90\% \times 5\% = 0.5\%$), with the impact borne by the junior capital.

- Finally, a SBF vehicle may include a dedicated cash reserve account as a buffer to ensure that debt service obligations — including interest and principal payments to senior investors — are met even during periods of temporary cash flow shortfalls. These reserves are typically funded at the inception of the project or periodically as part of the financial structure. Once the reserve account reaches a predefined threshold — either in value or after a specified number of years — additional contributions can be discontinued. For example, in the Amundi Planet EGO Fund, a dedicated first loss buffer has been established to absorb realized trading credit losses, impairments and capital losses resulting from credit events affecting issuers of debt securities, up to the outstanding balance of the buffer (Amundi, 2021, Section 7.1, page 80).

To illustrate the impact of senior protection mechanisms, we analyze the same 10-year bullet portfolio used previously, consisting of 100 BB+ rated green bonds. The assumptions regarding default correlation and recovery rates remain unchanged. Each bond pays an annual coupon of 8% and is purchased at par. We evaluate the effect of the loss carryforward mechanism[40] within a junior-senior structure[41], where the senior tranche offers a fixed 6% coupon. Results are shown in Tables 32–34 on page 89. The findings indicate that the loss carryforward mechanism provides enhanced protection to the senior tranche compared to the traditional no-offset approach. Specifically, the senior tranche exhibits lower expected losses and a reduced 95% value-at-risk under the loss carryforward setup. We also report the repayment rate of principal[42] (RPR), which shows an improvement, as shown in Figure

[40] It is described in Algorithm 3 on page 61. We assume that, at the end of each period, we reinvest any recovery amounts and the undistributed portion of junior dividends in remaining bonds.

[41] Under the loss carryforward mechanism, default losses can be offset by dividends that the junior tranche might receive in the current or future years.

[42] The repayment rate is the ratio of expected principal repayment to initial investment for tranche k:

$$\mathrm{RPR}_k = \frac{1}{n_s} \sum_{s=1}^{n_s} \frac{\mathrm{RePay}_{k,s}\left(T\right)}{\mathrm{Inv}_k}$$

Figure 31: Percentage difference in repayment rate between loss carryforward and no-loss offset approaches (senior tranche)

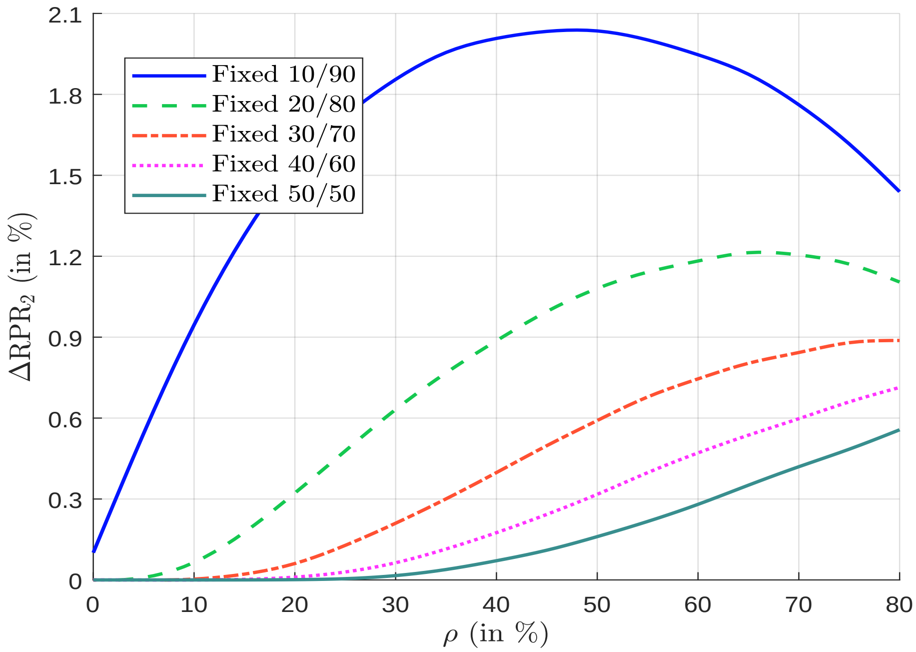


Figure 32: Percentage difference in cumulative impairment rate between loss carryforward and no-loss offset approaches (senior tranche)

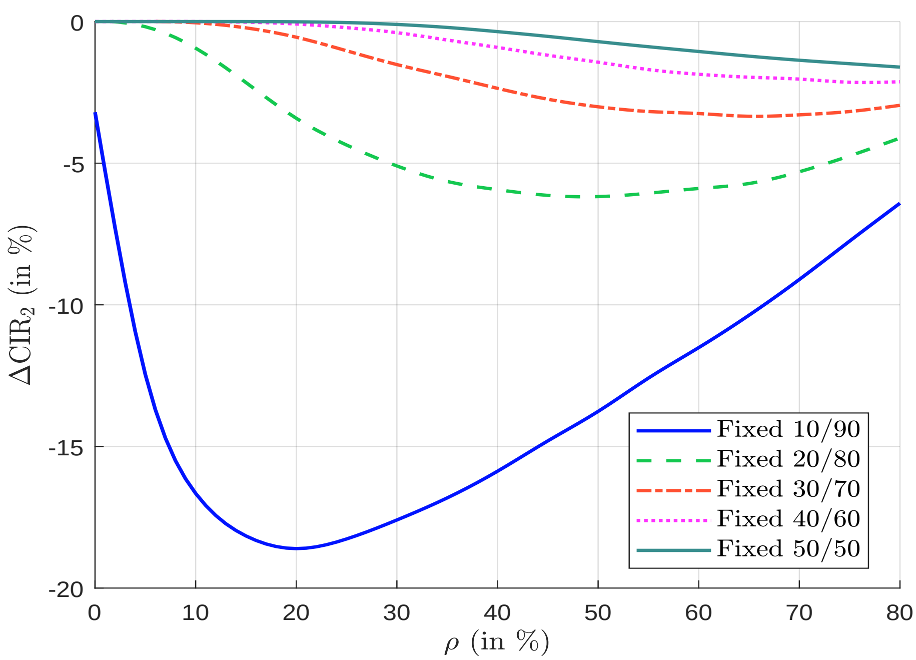


Figure 33: Percentage difference in skipped dividend frequency between loss carryforward and no-loss offset approaches (junior tranche)

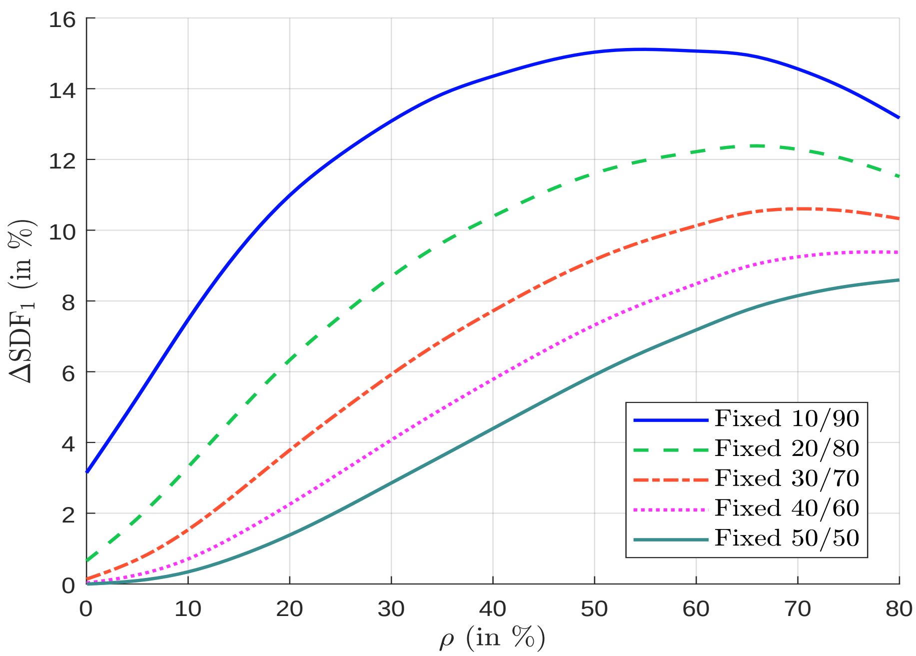
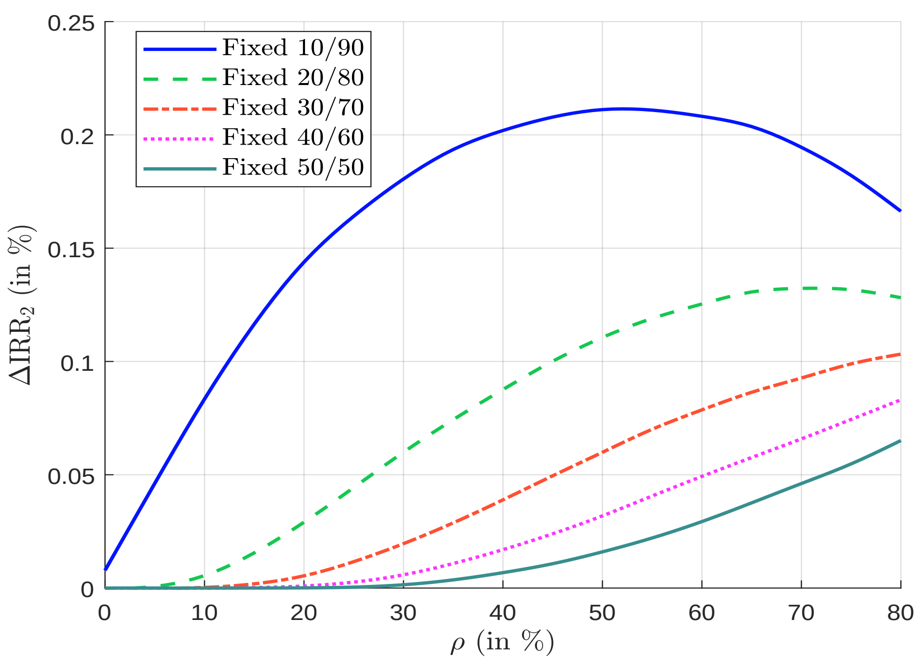


Figure 34: Percentage difference in internal rate of return between loss carryforward and no-loss offset approaches (senior tranche)



31. A reduction in the cumulative impairment rate[43] (CIR) is observed, indicating a lower probability of losses hitting the senior tranche due to the application of the loss carryforward mechanism (Figure 32). This increased protection is due to the withholding of some dividend payments to the junior tranche. As shown in Figure 33, the number of skipped dividend payments to the junior tranche increases when the loss carryforward mechanism is implemented[44]. Finally, we obtain a better IRR for the senior tranche (Figure 34). In this example, we can gain between 0 and 20 bps depending on the fixed structure[45].

4.5 Concessionality

4.5.1 Definition of the concessionality premium

The concessionality premium is defined as the difference between the market risk premium of the portfolio and the financing cost provided by the blended finance fund. Assuming that the financing cost is equal to the return to the investor, we define the concessionality premium as:

$$\pi_{\text{concessionality}} = \pi_{\text{portfolio}}^{\text{market}} - \pi_{\text{portfolio}}^{\text{blended}}$$

Accordingly, the concessionality rate is given by:

$$\mathcal{CR} = \frac{\pi_{\text{concessionality}}}{\pi_{\text{portfolio}}^{\text{market}}}$$

In a junior-senior structure, we found that:

$$\begin{cases} \pi_{\text{portfolio}}^{\text{blended}} = \pi_{\text{tranching}}^{\text{blended}} + \pi_{\text{tranching}}^{\text{extra}} \\ \pi_{\text{tranching}}^{\text{blended}} = \omega_{\text{junior}} \pi_{\text{junior}}^{\text{market}} + \omega_{\text{senior}} \pi_{\text{senior}}^{\text{market}} \end{cases}$$

The blended portfolio risk premium thus consists of the market risk premium derived from the tranching structure and an additional risk premium, which can be decomposed as:

$$\pi_{\text{tranching}}^{\text{extra}} = \omega_{\text{junior}} \pi_{\text{junior}}^{\text{extra}} + \omega_{\text{senior}} \pi_{\text{senior}}^{\text{extra}}$$

where $\pi_{\text{junior}}^{\text{extra}}$ and $\pi_{\text{senior}}^{\text{extra}}$ are the additional risk premia assigned to the junior and senior tranches, respectively. We deduce that:

$$\begin{aligned} \pi_{\text{portfolio}}^{\text{market}} &= \pi_{\text{portfolio}}^{\text{blended}} + \pi_{\text{concessionality}} \\ &= \pi_{\text{tranching}}^{\text{market}} + \pi_{\text{tranching}}^{\text{extra}} + \pi_{\text{concessionality}} \\ &= \omega_{\text{junior}} \left(\pi_{\text{junior}}^{\text{market}} + \pi_{\text{junior}}^{\text{extra}} \right) + \omega_{\text{senior}} \left(\pi_{\text{senior}}^{\text{market}} + \pi_{\text{senior}}^{\text{extra}} \right) + \pi_{\text{concessionality}} \end{aligned}$$

[43] The cumulative impairment rate measures the frequency with which tranche k will experience at least one loss over the life of the fund:

$$\text{CIR}_k = \frac{1}{n_s} \sum_{s=1}^{n_s} \mathbb{1} \left\{ \sum_{t=1}^{T} \mathcal{L}oss_{k,s} \left(t-1, t \right) > 0 \right\}$$

[44] The skipped dividend frequency measures the probability that tranche k will not receive a dividend in a given year:

$$\text{SDF}_k = \frac{1}{n_s} \sum_{s=1}^{n_s} \left(\frac{1}{T} \sum_{t=1}^{T} \mathbb{1} \left\{ \text{Div}_{k,s} \left(t \right) = 0 \right\} \right)$$

[45] The IRR statistics for the junior tranche should be interpreted with caution, as it is not possible to calculate the IRR for the scenarios in which the junior tranche receives no cash flows.

We conclude that:

$$
\begin{aligned}
\pi_{\text{concessionality}} &= \pi_{\text{portfolio}}^{\text{market}} - \omega_{\text{junior}} \left(\pi_{\text{junior}}^{\text{market}} + \pi_{\text{junior}}^{\text{extra}} \right) - \omega_{\text{senior}} \left(\pi_{\text{senior}}^{\text{market}} + \pi_{\text{senior}}^{\text{extra}} \right) \\
&= \Delta\pi_{\text{tranching}}^{\text{market}} - \omega_{\text{junior}} \pi_{\text{junior}}^{\text{extra}} - \omega_{\text{senior}} \pi_{\text{senior}}^{\text{extra}}
\end{aligned}
$$

The concessionality premium is then the difference between the additional market risk premium generated by the tranching structure and the additional risk premia attributed to the investors.

4.5.2 An example

We assume the following market credit spreads: $\pi_{\text{AAA}} = 70$ bps, $\pi_{\text{BB}} = 300$ bps, and $\pi_{\text{CCC}} = 1\,000$ bps. Consider a blended finance fund with the following structure: $\boldsymbol{R}_{\text{portfolio}} = \text{BB}$, $\omega_{\text{junior}} = 20\%$, $\boldsymbol{R}_{\text{junior}} = \text{CCC}$, $\omega_{\text{senior}} = 80\%$, $\boldsymbol{R}_{\text{senior}} = \text{AAA}$. We deduce that $\pi_{\text{portfolio}}^{\text{market}} = 300$ bps, $\pi_{\text{junior}}^{\text{market}} = 1\,000$ bps and $\pi_{\text{AAA}} = 70$ bps. The tranching premium — the spread difference attributable to the portfolio structure — is calculated as:

$$
\begin{aligned}
\Delta\pi_{\text{tranching}}^{\text{market}} &= \pi_{\text{portfolio}}^{\text{market}} - \omega_{\text{junior}} \pi_{\text{junior}}^{\text{market}} - \omega_{\text{senior}} \pi_{\text{senior}}^{\text{market}} \\
&= 300 - 0.2 \times 1\,000 - 0.8 \times 70 \\
&= 44 \text{ bps}
\end{aligned}
$$

We further assume the following additional risk premia:

$$
\begin{cases}
\pi_{\text{junior}}^{\text{extra}} = 60 \text{ bps} \\
\pi_{\text{senior}}^{\text{extra}} = 10 \text{ bps}
\end{cases}
$$

The concessionality premium is then:

$$
\begin{aligned}
\pi_{\text{concessionality}} &= \Delta\pi_{\text{tranching}}^{\text{market}} - \omega_{\text{junior}} \pi_{\text{junior}}^{\text{extra}} - \omega_{\text{senior}} \pi_{\text{senior}}^{\text{extra}} \\
&= 44 - 0.2 \times 60 - 0.8 \times 10 \\
&= 24 \text{ bps}
\end{aligned}
$$

This corresponds to a concessionality rate of 8%:

$$
\boldsymbol{\mathcal{CR}} = \frac{24}{300} = 8\%
$$

As a result, the projects will be financed with a blended spread of 276 bps instead of the market spread of 300 bps:

$$
\pi_{\text{portfolio}}^{\text{blended}} = \pi_{\text{portfolio}}^{\text{market}} - \pi_{\text{concessionality}} = 300 - 24 = 276 \text{ bps}
$$

Finally, we have:

$$
\begin{cases}
\pi_{\text{junior}} = \pi_{\text{junior}}^{\text{market}} + \pi_{\text{junior}}^{\text{extra}} = 1\,000 + 60 = 1\,060 \text{ bps} \\
\pi_{\text{senior}}^{\text{extra}} = \pi_{\text{senior}}^{\text{market}} + \pi_{\text{senior}}^{\text{extra}} = 70 + 10 = 80 \text{ bps}
\end{cases}
$$

Since $\Delta\pi_{\text{tranching}}^{\text{market}} > 0$, all parties benefit from this structure:

- Project issuer receives cheaper financing (276 bps vs. 300 bps);
- Junior sponsor earns an additional expected premium of 60 bps;

Table 25: Decomposition of the additional risk premium

Solution set	#1	#2	#3	#4	#5	#6	#7	#8
$\pi_{\text{junior}}^{\text{extra}}$	0.0	50.0	100.0	0.0	220.0	210.0	-5.0	-60.0
$\pi_{\text{senior}}^{\text{extra}}$	0.0	5.0	10.0	55.0	0.0	40.0	0.0	-5.0
$\pi_{\text{concessionality}}$	44.0	30.0	16.0	0.0	0.0	-30.0	45.0	60.0
$\mathcal{CR}$ (in %)	14.7	10.0	5.3	0.0	0.0	-10.0	15.0	20.0

- Senior investor receives a 10 bps enhanced yield.

As discussed above, the room for maneuver in a blended finance structure depends significantly on the additional risk premium generated by the tranching. For example, if $\Delta\pi_{\text{tranching}}^{\text{market}} < 0$, then at least one party will be disadvantaged. More generally, once the structure of the blended finance instrument is defined, the additional risk premium is given. This premium must be allocated among the parties through a combination of $\pi_{\text{junior}}^{\text{extra}}$, $\pi_{\text{senior}}^{\text{extra}}$ and $\pi_{\text{concessionality}}$:

$$\omega_{\text{junior}}\pi_{\text{junior}}^{\text{extra}} + \omega_{\text{senior}}\pi_{\text{senior}}^{\text{extra}} + \pi_{\text{concessionality}} = \Delta\pi_{\text{tranching}}^{\text{market}} \tag{18}$$

This implies that there is a trade-off between the level of concessionality and the additional compensation offered to junior and senior investors. In Table 25, we present several alternative decompositions for the example discussed earlier. In each case, the total additional premium remains 44 bps. In Solution #1, the additional risk premium is fully allocated to the concessionality premium. All parties win in Solutions #2 and #3, while only one party wins in Solutions #4 and #5. At least one party loses in Solutions #6, #7 and #8.

4.5.3 Properties

Since we have:

$$\mathcal{LR} = \frac{\omega_{\text{senior}}}{\omega_{\text{junior}}} = \frac{1-\omega_{\text{junior}}}{\omega_{\text{junior}}}$$

it follows that the junior thickness is related to the leverage ratio as:

$$\omega_{\text{junior}} = \frac{1}{\mathcal{LR}+1}$$

From Equation (18), we derive the following relationship:

$$\pi_{\text{junior}}^{\text{extra}} + \mathcal{LR}\pi_{\text{senior}}^{\text{extra}} = (\mathcal{LR}+1)\left(\Delta\pi_{\text{tranching}}^{\text{market}} - \pi_{\text{concessionality}}\right)$$

We distinguish several cases:

1. No additional risk premium for investors
   If neither the junior nor the senior investor receives an additional risk premium, the entire excess premium from the tranching is absorbed as concessionality:
   $$\begin{cases} \pi_{\text{junior}}^{\text{extra}} = 0 \\ \pi_{\text{senior}}^{\text{extra}} = 0 \end{cases} \Longrightarrow \pi_{\text{concessionality}} = \Delta\pi_{\text{tranching}}^{\text{market}}$$

2. No excess risk premium from tranching
   If tranching does not generates an excess risk premium and the senior investor does not receive extra compensation, the concessionality premium $\pi_{\text{concessionality}} > 0$ should be financed by the sponsor. Indeed, we have:
   $$\begin{cases} \Delta\pi_{\text{tranching}}^{\text{market}} = 0 \\ \pi_{\text{senior}}^{\text{extra}} = 0 \end{cases} \Longrightarrow \pi_{\text{junior}}^{\text{extra}} = -\left(\mathcal{LR}+1\right)\pi_{\text{concessionality}} < 0$$

3. Senior investor is not compensated
If $\pi^{\text{extra}}_{\text{senior}} = 0$, the junior investor can earn an extra premium if the concessionality premium is less than the excess risk premium generated by the tranching. In fact, we have:

$$\begin{cases} \Delta\pi^{\text{market}}_{\text{tranching}} > 0 \\ \pi^{\text{extra}}_{\text{senior}} = 0 \end{cases} \implies \pi^{\text{extra}}_{\text{junior}} = (\boldsymbol{\mathcal{LR}} + 1)\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi_{\text{concessionality}}\right)$$

We deduce that:

$$\pi^{\text{extra}}_{\text{junior}} > 0 \Leftrightarrow \Delta\pi^{\text{market}}_{\text{tranching}} > \pi_{\text{concessionality}}$$

4. Junior investor is not compensated
If $\pi^{\text{extra}}_{\text{junior}} = 0$, the senior investor can earn an additional premium if the concessionality premium is less than the excess risk premium generated by the tranching. Indeed, we have:

$$\begin{cases} \Delta\pi^{\text{market}}_{\text{tranching}} > 0 \\ \pi^{\text{extra}}_{\text{junior}} = 0 \end{cases} \implies \pi^{\text{extra}}_{\text{senior}} = \left(1 + \boldsymbol{\mathcal{LR}}^{-1}\right)\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi_{\text{concessionality}}\right)$$

We deduce that:

$$\pi^{\text{extra}}_{\text{senior}} > 0 \Leftrightarrow \Delta\pi^{\text{market}}_{\text{tranching}} > \pi_{\text{concessionality}}$$

5. General case
In the most general case, the additional premium for the junior investor is:

$$\pi^{\text{extra}}_{\text{junior}} = (\boldsymbol{\mathcal{LR}} + 1)\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi_{\text{concessionality}}\right) - \boldsymbol{\mathcal{LR}}\pi^{\text{extra}}_{\text{senior}}$$

If the leverage ratio is high ($\boldsymbol{\mathcal{LR}} \gg 1$), this simplifies approximately to:

$$\pi^{\text{extra}}_{\text{junior}} \approx \boldsymbol{\mathcal{LR}}\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi_{\text{concessionality}} - \pi^{\text{extra}}_{\text{senior}}\right)$$

This shows the sensitivity of the junior tranche to the leverage ratio and the allocation of the tranching premium.

Remark 8. *Another interesting property is the decomposition of the concessionality premium, which can be expressed as:*

$$\pi_{\text{concessionality}} = \frac{1}{(\boldsymbol{\mathcal{LR}} + 1)}\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi^{\text{extra}}_{\text{junior}}\right) + \frac{\boldsymbol{\mathcal{LR}}}{(\boldsymbol{\mathcal{LR}} + 1)}\left(\Delta\pi^{\text{market}}_{\text{tranching}} - \pi^{\text{extra}}_{\text{senior}}\right)$$

This formula shows that the concessionality premium is a weighted average of the difference between the total tranching premium and the additional premia allocated to junior and senior investors. The weights are derived from the leverage ratio and reflect the relative size of the senior and junior tranches. By construction, this decomposition gives more weight to the senior tranche. For instance, in a fixed 20/80 structure, the leverage ratio is 4, meaning that the junior and senior components are weighted 20% and 80%, respectively. This implies that an additional basis point given to the senior investor reduces the concessionality premium four times more than an additional basis point given to the junior investor[46]. In other words, concessionality is more sensitive to adjustments in the senior tranche due to its larger share in the portfolio.

[46] The trade-off relationship is as follows:

$$\frac{1}{(\boldsymbol{\mathcal{LR}} + 1)}\Delta\pi^{\text{extra}}_{\text{junior}} + \frac{\boldsymbol{\mathcal{LR}}}{(\boldsymbol{\mathcal{LR}} + 1)}\Delta\pi^{\text{extra}}_{\text{senior}} = 0$$

4.5.4 Comparison of pay-through structures and direct investments

We now compare the concessionality premium of a direct investment with a blended finance structure. For a direct investment, the financing cost is given by:

$$c_{\text{financing}}^{\text{direct}} = r + \pi_{\text{portfolio}}^{\text{market}} - \pi_{\text{concessionality}}^{\text{direct}}$$

where r is the free rate. In a blended finance structure, the senior investor does not contribute to the concessionality premium. The financing cost becomes:

$$c_{\text{financing}}^{\text{blended}} = r + \pi_{\text{tranching}}^{\text{blended}} + \pi_{\text{distributed}}^{\text{extra}} - \frac{1}{\mathcal{LR} + 1} \pi_{\text{concessionality}}^{\text{sponsor}}$$

where $\pi_{\text{concessionality}}^{\text{sponsor}}$ is the concessionality premium funded by the sponsor (the junior investor), $\pi_{\text{distributed}}^{\text{extra}} = \varphi_{\text{distributed}}^{\text{extra}} \pi^{\text{extra}} = \varphi_{\text{distributed}}^{\text{extra}} \left(\pi_{\text{portfolio}}^{\text{market}} - \pi_{\text{tranching}}^{\text{blended}} \right)$ is the portion of the extra premium distributed to investors, and $\varphi_{\text{distributed}}^{\text{extra}} \in [0, 1]$ is the distribution factor. Using this analytical framework, three cases are obtained:

1. If $\pi_{\text{tranching}}^{\text{blended}} < \pi_{\text{portfolio}}^{\text{market}}$, the structure is not viable because $\pi^{\text{extra}} < 0$ and it fails to generate a sufficient risk premium to cover the cost of risk-sharing;

2. If $\pi_{\text{tranching}}^{\text{blended}} = \pi_{\text{portfolio}}^{\text{market}}$, the structure is viable because $\pi^{\text{extra}} = 0$. However, no surplus premium is available for distribution. The financing cost difference becomes:

$$c_{\text{financing}}^{\text{blended}} - c_{\text{financing}}^{\text{direct}} = \pi_{\text{concessionality}}^{\text{direct}} - \frac{1}{\mathcal{LR} + 1} \pi_{\text{concessionality}}^{\text{sponsor}}$$

This implies:

$$c_{\text{financing}}^{\text{blended}} \leq c_{\text{financing}}^{\text{direct}} \Leftrightarrow \pi_{\text{concessionality}}^{\text{sponsor}} \geq (\mathcal{LR} + 1) \, \pi_{\text{concessionality}}^{\text{direct}}$$

This means that the blended instrument can offer lower financing costs if the sponsor provides sufficient concessionality. In effect, the sponsor funds both the junior and senior tranches.

3. If $\pi_{\text{tranching}}^{\text{blended}} > \pi_{\text{portfolio}}^{\text{market}}$, the structure is viable ($\pi^{\text{extra}} > 0$) and we obtain:

$$\begin{aligned} c_{\text{financing}}^{\text{blended}} - c_{\text{financing}}^{\text{direct}} &= \pi_{\text{concessionality}}^{\text{direct}} - \frac{\pi_{\text{concessionality}}^{\text{sponsor}}}{\mathcal{LR} + 1} + \pi_{\text{distributed}}^{\text{extra}} - \pi^{\text{extra}} \\ &= \pi_{\text{concessionality}}^{\text{direct}} - \left(\frac{\pi_{\text{concessionality}}^{\text{sponsor}}}{\mathcal{LR} + 1} + \left(1 - \varphi_{\text{distributed}}^{\text{extra}}\right) \pi^{\text{extra}} \right) \end{aligned}$$

It follows that:

$$c_{\text{financing}}^{\text{blended}} \leq c_{\text{financing}}^{\text{direct}} \Leftrightarrow \pi_{\text{concessionality}}^{\text{blended}} \geq \pi_{\text{concessionality}}^{\text{direct}}$$

where:

$$\pi_{\text{concessionality}}^{\text{blended}} = \frac{1}{\mathcal{LR} + 1} \pi_{\text{concessionality}}^{\text{sponsor}} + \underbrace{\left(1 - \varphi_{\text{distributed}}^{\text{extra}}\right) \pi^{\text{extra}}}_{\pi_{\text{concessionality}}^{\text{extra}}}$$

The concessionality premium of the blended finance instrument can be enhanced by the concessionality component arising from the non-distributed portion of the risk premium. If we assume the standard hypothesis, this implies that:

$$c_{\text{financing}}^{\text{blended}} \leq c_{\text{financing}}^{\text{direct}} \Leftrightarrow \pi_{\text{concessionality}}^{\text{extra}} \geq \frac{\mathcal{LR}}{\mathcal{LR} + 1} \pi_{\text{concessionality}}^{\text{direct}}$$

5 Conclusion

This paper presents a comprehensive framework for analyzing structured blended finance (SBF) funds, with a focus on the design, structuring, risk modeling, and portfolio management. The paper addresses the growing need for financial mechanisms that align private capital with the sustainable development goals (SDGs), particularly in emerging and frontier markets. As a key mobilization strategy, blended finance aims to catalyze private-sector participation in high-impact development projects by using concessional capital to mitigate risks and enhance the financial viability of projects that would otherwise struggle to attract investment. Our work emphasizes the crucial role of structuring in reconciling the frequently conflicting objectives of public and private investors, especially when financial returns and social impact must coexist.

We begin by distinguishing blended finance from other adjacent concepts, such as public-private partnerships (PPPs) and impact investing. While there are areas of overlaps, blended finance is best understood as a structuring approach rather than a fixed investment strategy. It is unique in its reliance on concessional finance and tiered capital structures. This structuring capability is one of blended finance's core strengths. It allows blended finance to create asymmetric risk-return profiles and align diverse investor expectations. However, this strength also presents a downside. The complexity of blended finance structures can make them difficult for both public and private investors to understand and navigate (British International Investment, 2025). As a result, despite its relatively strong growth, blended finance remains a niche within the broader field of sustainable finance. Most blended finance funds are small, highly specialized, and tailored to experienced or mission-aligned investors. While there are a few flagship funds, they remain limited in number. Moreover, a small group of development finance institutions (DFIs) is responsible for the majority of large-scale blended finance transactions. This concentration can be attributed to the technical complexity in structuring such vehicles. On the private side, participation is usually confined to a narrow circle of institutional investors, suggesting that capital mobilization through blended finance remains modest compared to its potential.

Structuring a blended finance fund requires robust modeling of the credit risk associated with the underlying asset portfolio. This paper outlines the current state of credit risk modeling for blended finance portfolios and presents a variety of techniques, ranging from survival functions and transition probability matrices to copula-based models. These methodologies are critical for analyzing tranche performance under stress — particularly the senior tranches, which are designed to attract institutional investors who are subject to credit rating constraints. We also examine the impact of default correlation on the portfolio's loss distribution and demonstrate that the performance of senior tranches is especially sensitive to tail dependence. To capture regional and sectoral risk dynamics more accurately, we incorporate multi-factor models, which allow for more granular risk decomposition across SBF portfolios. Beyond credit risk, cash flow analysis is a second critical pillar in SBF fund modeling. While modeling cash flows on the asset side is relatively straightforward, modeling cash flows on the liability side is more complex due to the variety of structuring choices. These choices include differences in tranche payment rules, waterfall structures, tranche maturities, performance triggers, but also protection mechanisms for the senior tranche. All of these factors significantly affect the risk-return profile for investors.

By focusing on the junior-senior structure, we can better illustrate the economic rationale behind a structured blended finance fund. The relationship between risk-neutral and historical probability measures of credit risk is neither fixed nor linear. Rather, it varies significantly with the investor's degree of risk aversion (Gregory, 2014). Specifically, a highly risk-averse investor will require a greater compensation for bearing credit risk than a less risk-averse one. This leads to a nonlinear hyperbolic relationship between historical default intensities and risk-neutral default intensities. As a result, the required risk premium becomes a multiple of the expected loss — typically with a higher multiplier for senior investors and a lower one for junior investors. A key insight is that a junior-senior (or pay-through) structure generates an additional risk premium compared to the asset portfolio or a pass-through structure, assuming that the fund is properly calibrated. Put differently, the pool of investors in a structured blended finance fund may require a lower overall risk premium than individual investors in a direct investment, even when the aggregate risk exposure is the same. This result stems from the interplay between the hyperbolic risk relationship and the mathematical properties of the weighted power mean. This extra risk premium benefits the asset manager, who can tailor risk-return profiles for different investor groups more precisely, as well as the sponsor, whose extra-financial objectives are supported. Specifically, the additional premium can finance the concessionality component of the structure. Without this additional risk premium, direct investments generally offer a better concessionality premium than blended finance structures due to leverage effects. However, historical data demonstrates that in certain cases, blended finance structures can effectively compete with direct investments, particularly when the senior tranche carries an AAA rating and the junior tranche is rated CCC+ or below.

The Monte Carlo simulation results further highlight the sensitivity of tranche-level performance to key structural parameters, such as default correlations and cash flow design. These results demonstrate that poorly calibrated structures can allocate excessive downside risk to senior investors, which undermines the rationale behind risk tranching. Our analysis emphasizes the importance of meticulous calibration via scenario analysis and robust benchmarking. Additionally, the simulations show that the resilience of the junior-senior structure depends on the diversification of the underlying asset portfolio. A well-diversified portfolio allows for the creation of an asymmetric risk-return profile that aligns more closely with the distinct utility functions of junior and senior investors. Therefore, it is more economically rational for a structured blended finance fund to have a portfolio manager who spreads investments across multiple countries, sectors, and asset types rather than concentrating exposure in a single region or industry.

If default correlation among bond or project issuers is high or diversification is poor[47], it becomes necessary to reinforce the traditional junior-senior tranche structure[48]. Several strategies can be considered. The simplest method is to reduce the leverage ratio and raise the junior tranche's detachment point. Typically, the senior tranche's detachment point is below 30%, corresponding to a leverage ratio above 2. However, under conditions of concentration risk or high default correlation, the senior tranche may be limited to less than 60% of the capital structure, implying a leverage ratio below 1.5. A second approach involves implementing specific protection mechanisms, such as loss carryforward techniques or dividend sponsorship. A third method introduces a mezzanine tranche between the junior and senior tranches. However, a junior-mezzanine-senior structure tends to be less transparent

[47] This typically occurs when the asset portfolio is concentrated in a few large projects

[48] Another challenge associated with a poorly diversified asset portfolio is recovery risk. In market practice, it is common to assume a fixed, constant recovery rate. This simplification is supported by standard credit risk modeling. This approach is theoretically justified under the assumption of a finely grained portfolio where idiosyncratic risk is diversified away, and random loss given default (LGD) can be reasonably approximated by its expected value. However, when this assumption does not hold, such as in highly concentrated portfolios, it becomes necessary to model recovery rates as stochastic variables. In these cases, loss risk cannot be reduced to default risk alone because fluctuations in recovery also become a significant component of total credit risk. This is particularly evident in the context of a blended finance fund concentrated in a single project. In that scenario, relying on an expected recovery rate is inadequate for assessing the project's overall risk. The actual recovery value becomes highly uncertain and must be explicitly modeled to capture the full scope of potential outcomes.

than a standard junior-senior configuration, and the utility function of mezzanine investors is not always straightforward to define. Determining the optimal layering in a three-tranche structure is challenging. Often, the mezzanine tranche behaves similarly to either the junior or the senior tranche, because it is difficult to precisely calibrate its risk-return positioning. Nevertheless, adding a mezzanine tranche can be valuable when creating two asymmetric risk-return profiles is not feasible. This often occurs when the asset portfolio's rating distribution is wide or skewed toward lower ratings. The average rating of emerging market (EM) bonds is typically BBB−, which is the lowest investment grade category[49]. When ratings are distributed above the crossover band, transforming the asset portfolio into two tranches with distinct risk profiles is relatively straightforward. The same applies when the portfolio contains assets within the crossover category. However, a particular challenge emerges when the asset portfolio contains both crossover bonds and pure high-yield bonds, which is relatively common in the emerging market debt universe[50]. In such cases, the portfolio's systematic risk may be relatively large compared to its idiosyncratic risks, reducing the effectiveness of tranching based solely on diversification. A mezzanine tranche can play a valuable role here by serving as a buffer between junior and senior tranches and improving overall risk allocation.

All these findings highlight that the key issue in structuring a blended finance vehicle is the determination of the junior tranche's detachment point. This parameter directly defines the level of credit enhancement and the leverage ratio. Indirectly, it influences the additional risk premium and affects the level of concessionality needed. This is why a significant portion of the discussions among stakeholders in blended finance centers around setting the appropriate detachment point. However, this focus can be misleading — you can't see the forest for the trees. The detachment point is not an exogenous input, but rather an endogenous outcome of the overall fund structure. Specifically, it depends on the characteristics of the asset portfolio (*e.g.*, credit quality, duration, diversification) as well as the liability structure, including the maturity and risk appetite of the different tranches. Furthermore, choosing the detachment point highlights the significant trade-off in blended finance between achieving a higher leverage ratio and providing adequate downside protection to private investors. Meeting these goals requires the expertise of all key stakeholders — the sponsor, asset manager, and structurer — to ensure that the fund meets its financial and development objectives.

Beyond this initial publication on blended finance fund structuring, several critical questions and research areas require further exploration. First, while credit and default risk modeling methodologies are well-established, currency risk remains a fundamental challenge. Currently, currency risk poses a persistent barrier to expanding blended finance in emerging markets. Most structured blended finance deals are denominated in hard currencies, creating substantial currency mismatch risk for local projects that generate revenues in local currencies. Although solutions such as currency hedging facilities exist, they are not widely available and are often costly. Therefore, there is an urgent need for new research and innovation to develop more robust, scalable, and cost-effective frameworks for structuring blended finance in local currencies. Addressing this challenge is essential for improving the sustainability and impact of blended finance in developing economies and reducing vulnerabilities in cross-currency financing structures. Second, more attention must be given to improving transparency and governance in blended finance structures. These structures often involve a complex network of stakeholders, including sponsors, structurers,

[49] This is another big difference from banks' CLOs.

[50] As of the end of December 2024, the composition of the JPM EMBI index was as follows: 9.0% in AA, 19.5% in A, 27.7% in BBB, 23.9% in BB, 11.7% in B, and 7.1% in CCC. The remaining 1.2% consisted of securities rated below CCC− or unrated instruments.

and investors, each with their own distinct roles, priorities, and incentives. Without clarity in these relationships, misaligned expectations or suboptimal capital allocation may occur. Future research should explore how improved transparency, standardization, and disclosure practices can enhance governance. Specifically, a better understanding of stakeholders' respective utility functions is important for aligning incentives across parties, especially when combining public investors, who prioritize extra-financial returns, with private investors, who focus on risk-adjusted financial returns. Third, market scalability remains a major obstacle. Blended finance instruments tend to be illiquid and tailored to specific transactions, which limits their replicability and appeal to mainstream investors. This illiquidity also limits fund scalability and impacts dynamic asset-liability management. Furthermore, blended finance usually requires long investment horizons to achieve developmental outcomes. However, many institutional investors operate on shorter cycles and face pressure to generate annual returns. The discrepancy between the timeframe of development goals and financial expectations creates a structural impediment. Additionally, regulatory frameworks such as Basel III and Solvency II often impose high capital charges on investments in blended finance portfolios. Consequently, there are currently few incentives for private investors to participate on a large scale. Addressing this issue will require regulatory dialogue and potentially the creation of preferential treatments or guarantees that recognize the de-risking role of public capital and development finance institutions.

In summary, this paper has sought to bridge the theoretical and practical dimensions of structuring blended finance funds. While significant progress has been made in understanding the mechanics of risk modeling, tranche design, and portfolio diversification, important challenges remain. These include managing currency and governance risks, enhancing transparency, improving investor alignment, and enabling market growth through better liquidity and regulatory frameworks. Meeting these challenges will require the coordinated efforts of development finance institutions, asset managers, financial engineers, investors, and regulators. Ultimately, the promise of blended finance lies not only in its ability to mobilize capital at scale, but also in its potential to redefine the boundaries between financial and extra-financial value creation, supporting both economic development and sustainable investment paradigms.

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A Mathematical results

A.1 Mathematical expectation and variance of the portfolio loss

We define the portfolio loss as follows:

$$\mathcal{L}oss(t) = \sum_{i=1}^{n} N_i \, \mathrm{LGD}_i \, D_i(t)$$

where $\mathrm{LGD}_i = 1 - \boldsymbol{\mathcal{R}}_i$ is the loss given default, and $D_i(t) = \mathbb{1}\{\boldsymbol{\tau}_i \leq t\} \sim \mathcal{B}\left(p_i(t)\right)$ is the default indicator function. We assume that $\mathrm{LGD}_i \perp \mathrm{LGD}_j$ and $\mathrm{LGD}_i \perp \boldsymbol{\tau}_i$. We deduce that the expected loss is given by:

$$\mathbb{E}\left[\mathcal{L}oss(t)\right] = \sum_{i=1}^{n} N_i \mathbb{E}\left[\mathrm{LGD}_i\right] p_i(t) \tag{19}$$

where $p_i(t)$ is the default probability of issuer i at the time horizon t. To compute the variance of the loss, we first calculate $\mathbb{E}\left[\mathcal{L}oss(t)^2\right]$:

$$\begin{aligned}
\mathbb{E}\left[\mathcal{L}oss(t)^2\right] &= \mathbb{E}\left[\left(\sum_{i=1}^{n} N_i \, \mathrm{LGD}_i \, D_i(t)\right)^2\right] \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} N_i N_j \mathbb{E}\left[\mathrm{LGD}_i \, \mathrm{LGD}_j\right] \mathbb{E}\left[D_i(t) \, D_j(t)\right]
\end{aligned}$$

If $i = j$, we have $\mathbb{E}\left[\mathrm{LGD}_i^2\right] = \sigma^2\left(\mathrm{LGD}_i\right) + \mathbb{E}^2\left[\mathrm{LGD}_i\right]$ and $\mathbb{E}\left[D_i(t)^2\right] = p_i(t)$. For $i \neq j$, we obtain $\mathbb{E}\left[\mathrm{LGD}_i \, \mathrm{LGD}_j\right] = \mathbb{E}\left[\mathrm{LGD}_i\right]\mathbb{E}\left[\mathrm{LGD}_j\right]$ and $\mathbb{E}\left[D_i(t) \, D_j(t)\right] = \mathbf{C}\left(p_i(t), p_j(t)\right)$ where $\mathbf{C}$ is the bivariate copula between the default times $\boldsymbol{\tau}_i$ and $\boldsymbol{\tau}_j$. We also compute:

$$\mathbb{E}^2\left[\mathcal{L}oss(t)\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} N_i N_j \mathbb{E}\left[\mathrm{LGD}_i\right] \mathbb{E}\left[\mathrm{LGD}_j\right] p_i(t) \, p_j(t)$$

Thus, the variance of the portfolio loss is given by:

$$\begin{aligned}
\mathrm{var}\left(\mathcal{L}oss(t)\right) &= \mathbb{E}\left[\mathcal{L}oss(t)^2\right] - \mathbb{E}^2\left[\mathcal{L}oss(t)\right] \\
&= \sum_{i=1}^{n} N_i^2 \mathbb{E}\left[\mathrm{LGD}_i^2\right] \mathbb{E}\left[D_i^2(t)\right] - \sum_{i=1}^{n} N_i^2 \mathbb{E}^2\left[\mathrm{LGD}_i\right] p_i^2(t) \\
&\quad \sum_{i \neq j} N_i N_j \mathbb{E}\left[\mathrm{LGD}_i \, \mathrm{LGD}_j\right] \left(\mathbb{E}\left[D_i(t) \, D_j(t)\right] - p_i(t) \, p_j(t)\right) \\
&= \sum_{i=1}^{n} N_i^2 \mathbb{E}^2\left[\mathrm{LGD}_i\right] p_i(t) \left(1 - p_i(t)\right) + \sum_{i=1}^{n} N_i^2 \sigma^2\left(\mathrm{LGD}_i\right) p_i(t) + \\
&\quad \sum_{i \neq j} N_i N_j \mathbb{E}\left[\mathrm{LGD}_i\right] \mathbb{E}\left[\mathrm{LGD}_j\right] \left(\mathbf{C}\left(p_i(t), p_j(t)\right) - p_i(t) \, p_j(t)\right)
\end{aligned} \tag{20}$$

In the case where default times are independent and recovery rates are constant, the variance of the loss simplifies to:

$$\mathrm{var}\left(\mathcal{L}oss(t)\right) = \sum_{i=1}^{n} N_i^2 \, \mathrm{LGD}_i^2 \, p_i(t) \left(1 - p_i(t)\right)$$

A.2 Limiting probability distribution of the portfolio loss

We follow the analysis of Vasicek (1991, 2002). Merton (1974) assumes that the firm i defaults if the value of assets $A_i(t)$ falls below a threshold D_i, which is the default barrier related to the contractual value of the debt. The dynamics of the assets follow a geometric brownian motion:

$$\mathrm{d}A_i(t) = \mu_i A_i(t)\,\mathrm{d}t + \sigma_i A_i(t)\,\mathrm{d}W_i(t)$$

where $A_i(0) = A_i$. We deduce that:

$$A_i(t) = A_i \exp\left(\left(\mu_i - \frac{1}{2}\sigma_i^2\right)t + \sigma_i W_i(t)\right)$$

Since the default occurs when $A_i(t) \le D_i$, we deduce that:

$$\begin{aligned} p_i(t) &= \Pr\left\{D_i(t) = 1\right\} \\ &= \Pr\left\{A_i(t) \le D_i\right\} \\ &= \Phi\left(\frac{\ln D_i - \ln A_i - \left(\mu_i - \frac{1}{2}\sigma_i^2\right)t}{\sigma_i\sqrt{t}}\right) \end{aligned}$$

Using a traditional one-factor model $Z_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$, we get[51]:

$$\begin{aligned} p_i(t, X) &= \Pr\left\{D_i(t) = 1 \mid X\right\} \\ &= \Pr\left\{Z_i \le B_i \mid X\right\} \\ &= \Phi\left(\frac{B_i - \sqrt{\rho}X}{\sqrt{1-\rho}}\right) \end{aligned}$$

We also deduce that $p_i(t) = \Phi(B_i)$, which implies that $B_i = \Phi^{-1}\left(p_i(t)\right)$.

A.2.1 Cumulative distribution function

Vasicek (2002) considers a homogeneous portfolio of n loans with the same notional N, a 100% loss given default, the same probability of default p and the same maturity t:

$$\mathcal{L}oss_n(t) = \frac{\sum_{i=1}^n N_i D_i(t)}{\sum_{i=1}^n N_i} = \frac{1}{n}\sum_{i=1}^n D_i(t)$$

We denote by $\mathcal{L}oss_\infty(t)$ the loss of the asymptotic portfolio when $n \to \infty$. The limiting distribution of $\mathcal{L}oss_\infty(t)$ has the following expression:

$$\begin{aligned} \mathbf{F}(x) &= \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) \\ &= \Phi\left(\sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}(x) - \sqrt{\frac{1}{\rho}}\Phi^{-1}(p)\right) \end{aligned}$$

[51] See Roncalli (2020, page 173) for the proof.

A.2.2 Probability density function

We have:

$$\begin{aligned} f(x) &= \frac{\partial\, \mathbf{F}(x)}{\partial\, x} \\ &= \sqrt{\frac{1-\rho}{\rho}}\phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right)\frac{\partial\, \Phi^{-1}(x)}{\partial\, x} \end{aligned}$$

We deduce that[52]:

$$\begin{aligned} f(x) &= \sqrt{\frac{1-\rho}{\rho}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right)^2\right)\sqrt{2\pi}\exp\left(\frac{1}{2}\left(\Phi^{-1}(x)\right)^2\right) \\ &= \sqrt{\frac{1-\rho}{\rho}}\exp\left(\frac{1}{2}\left(\Phi^{-1}(x)\right)^2 - \frac{1}{2\rho}\left(\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)\right)^2\right) \end{aligned}$$

A.2.3 Quantile function

We have:

$$\begin{aligned} \mathbf{F}(x) = \alpha &\Leftrightarrow \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) = \alpha \\ &\Leftrightarrow \Phi^{-1}(x) = \frac{\Phi^{-1}(p) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \\ &\Leftrightarrow x = \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right) \end{aligned}$$

We deduce that:

$$\mathbf{F}^{-1}(\alpha) = \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)$$

A.3 Property of the weighted power mean

We assume that $w_k \geq 0$, $\sum_{k=1}^{n} w_k = 1$, $x_k > 0$. We define the weighted power mean as:

$$M_p(x) = \left(\sum_{i=1}^{n} w_i x_i^p\right)^{1/p}$$

where $x = (x_1, \ldots, x_n)$. A fundamental property of the weighted power mean is:

$$\text{If } q > p, \text{ then } M_q(x) \geq M_p(x)$$

and the equality holds if and only if $x_1 = x_2 = \ldots = x_n$. We deduce that:

$$\sum_{i=1}^{n} w_i x_i > \left(\sum_{i=1}^{n} w_i x_i^p\right)^{1/p} \Leftrightarrow M_1(x) > M_p(x) \Rightarrow p < 1$$

[52] Following Roncalli (2020, Section A.2.2.3), we have:

$$\frac{\partial\, \Phi^{-1}(x)}{\partial\, x} = \frac{1}{\phi\left(\Phi^{-1}(x)\right)} = \frac{\sqrt{2\pi}}{\exp\left(-\frac{1}{2}\left(\Phi^{-1}(x)\right)^2\right)} = \sqrt{2\pi}\exp\left(\frac{1}{2}\left(\Phi^{-1}(x)\right)^2\right)$$

A.4 Equivalence between maximizing the leverage ratio and maximizing the excess risk premium in a junior-senior structure

We have:

$$\begin{aligned}\pi_{\text{tranching}} &= \omega_{\text{junior}}\pi_{\text{junior}} + \omega_{\text{senior}}\pi_{\text{senior}} \\ &= \left(1-\omega_{\text{senior}}\right)\pi_{\text{junior}} + \omega_{\text{senior}}\pi_{\text{senior}} \\ &= \pi_{\text{junior}} + \omega_{\text{senior}}\left(\pi_{\text{senior}} - \pi_{\text{junior}}\right)\end{aligned}$$

The excess risk premium due to tranching is given by:

$$\begin{aligned}\Delta\pi &= \pi_{\text{portfolio}} - \pi_{\text{tranching}} \\ &= \pi_{\text{portfolio}} - \pi_{\text{junior}} + \omega_{\text{senior}}\left(\pi_{\text{junior}} - \pi_{\text{senior}}\right)\end{aligned}$$

Since $\pi_{\text{junior}} - \pi_{\text{senior}} > 0$, we observe that maximizing the excess risk premium is equivalent to maximizing the size of the senior tranche:

$$\max \Delta\pi := \max \omega_{\text{senior}}$$

Now, recall that the leverage ratio is defined as:

$$\boldsymbol{\mathcal{LR}} = \frac{\omega_{\text{senior}}}{\omega_{\text{junior}}} = \omega_{\text{junior}}^{-1} - 1$$

Therefore:

$$\max \boldsymbol{\mathcal{LR}} = \max \omega_{\text{junior}}^{-1} = \min \omega_{\text{junior}} = \max \omega_{\text{senior}}$$

We conclude that:

$$\max \Delta\pi \Leftrightarrow \max \boldsymbol{\mathcal{LR}}$$

B Additional results

Figure 35: Distribution of the internal rate of return — Example #1, $\lambda_i = 1\,000$ bps

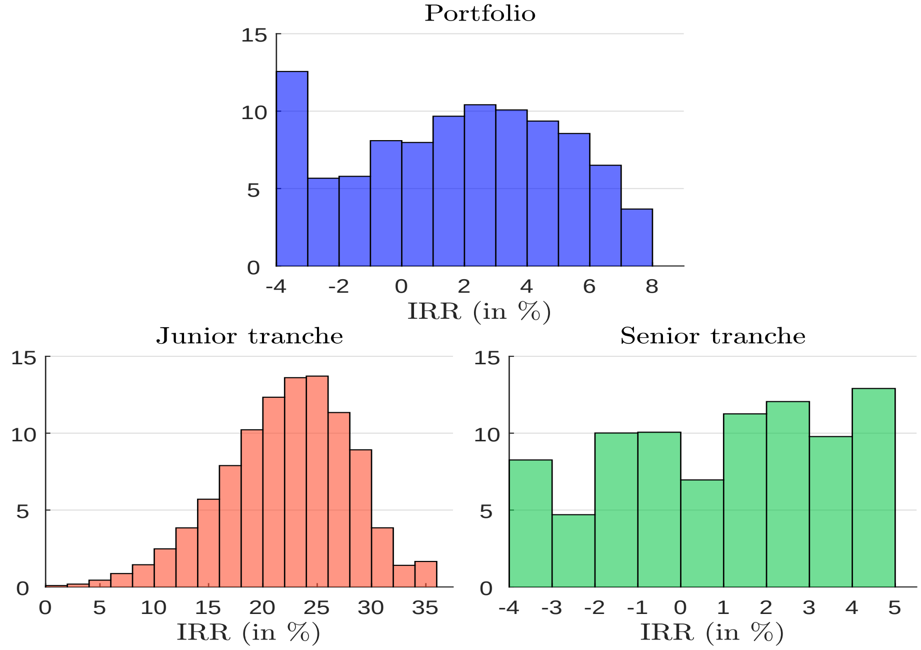

Table 26: Mean, standard deviation and zero-probability of the internal rate of return (in %) — Example #2, $\rho = 25\%$, $\alpha = 90\%$, no initial buffer

λ_i	Portfolio			Junior			Senior		
(in %)	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$	$\hat{\mu}_k$	$\hat{\sigma}_k$	$\hat{p}_k^0$
0.00	6.00	0.00	0.00		0.00	0.00	5.00	0.00	0.00
0.10	6.00	0.30	0.00	36.02	1.73	0.00	4.96	0.20	0.00
0.50	5.99	0.77	0.09	16.25	4.75	0.54	4.91	0.41	0.09
1.00	5.97	1.17	0.40	11.87	5.88	6.58	4.87	0.57	0.20
2.00	5.95	1.77	1.25	9.08	6.66	12.47	4.81	0.75	0.45
3.00	5.95	2.23	2.54	7.99	7.02	15.09	4.78	0.85	0.76
4.00	5.97	2.61	3.58	7.45	7.19	17.16	4.76	0.89	1.00
5.00	6.01	2.92	4.48	7.17	7.31	17.49	4.75	0.90	0.74
6.00	6.08	3.17	5.47	7.03	7.39	17.88	4.76	0.87	0.82
7.00	6.16	3.39	6.08	7.05	7.36	18.09	4.76	0.83	1.05
8.00	6.28	3.58	6.36	7.08	7.42	18.10	4.79	0.77	0.17
9.00	6.41	3.73	6.50	7.23	7.35	17.82	4.80	0.71	0.00
10.00	6.56	3.86	6.53	7.41	7.33	17.02	4.82	0.62	0.00

Figure 36: Probability density function of BB-rated credit portfolio loss ($\mathcal{R} = 40\%$ and $\rho = 20\%$)

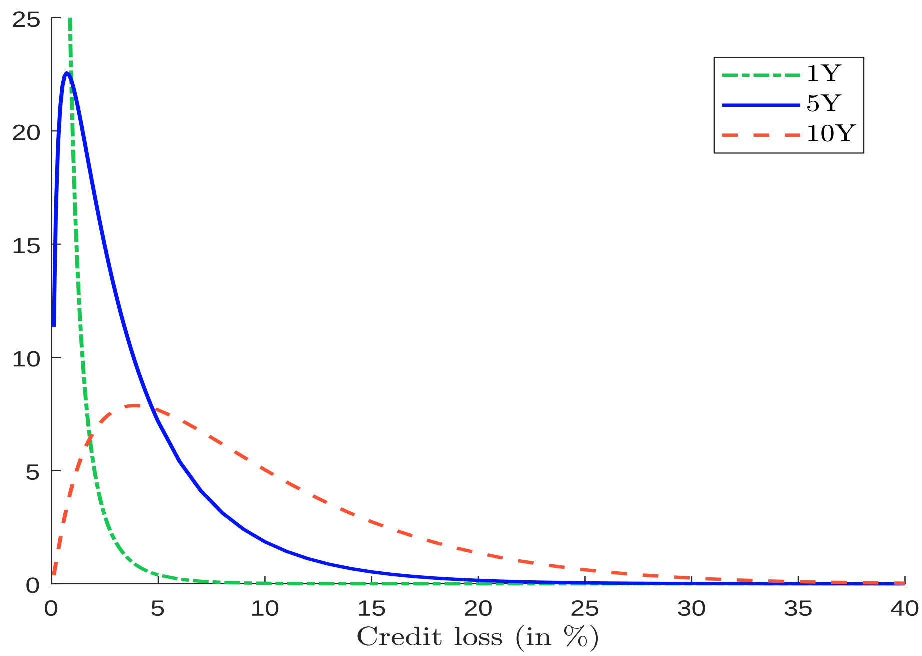

Figure 37: Probability density function of portfolio loss by credit rating ($\mathcal{R} = 40\%$ and $\rho = 40\%$)

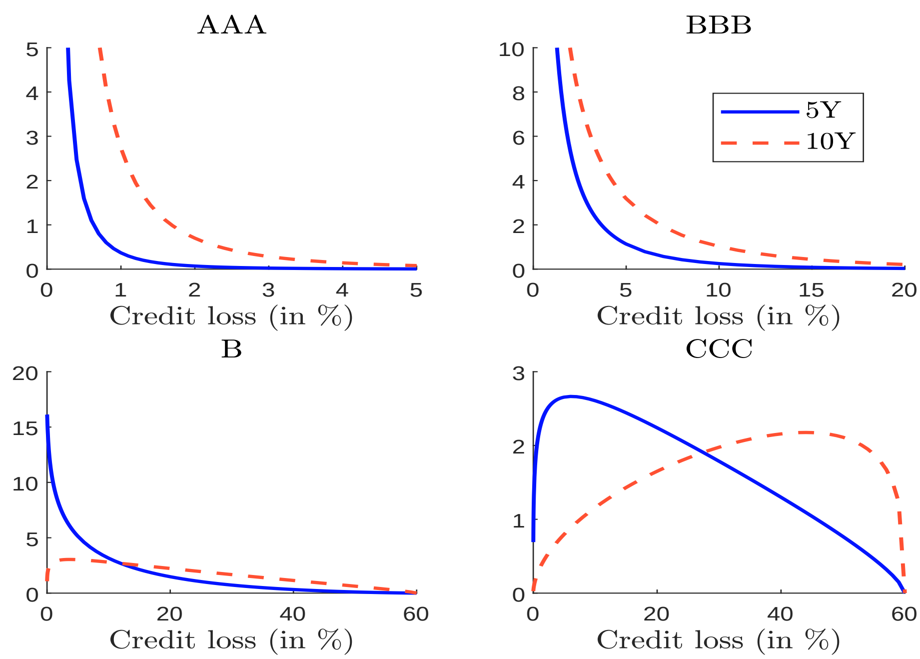

Figure 38: Histogram of α_i and $\Delta\pi_i$ (1997–2024, AAA and CCC ratings)

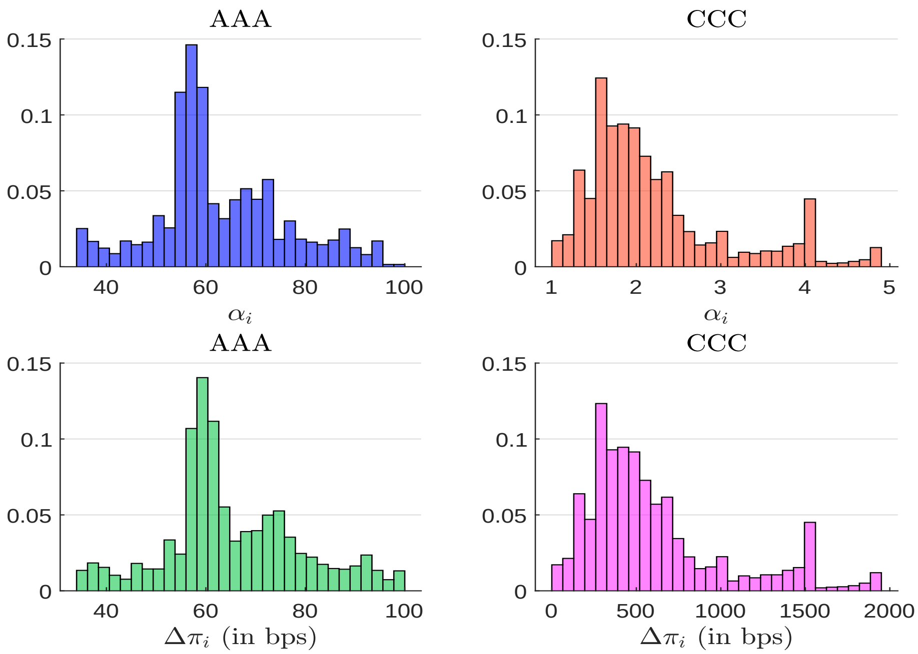

Figure 39: Impact of β and γ on the risk premium multiplier α

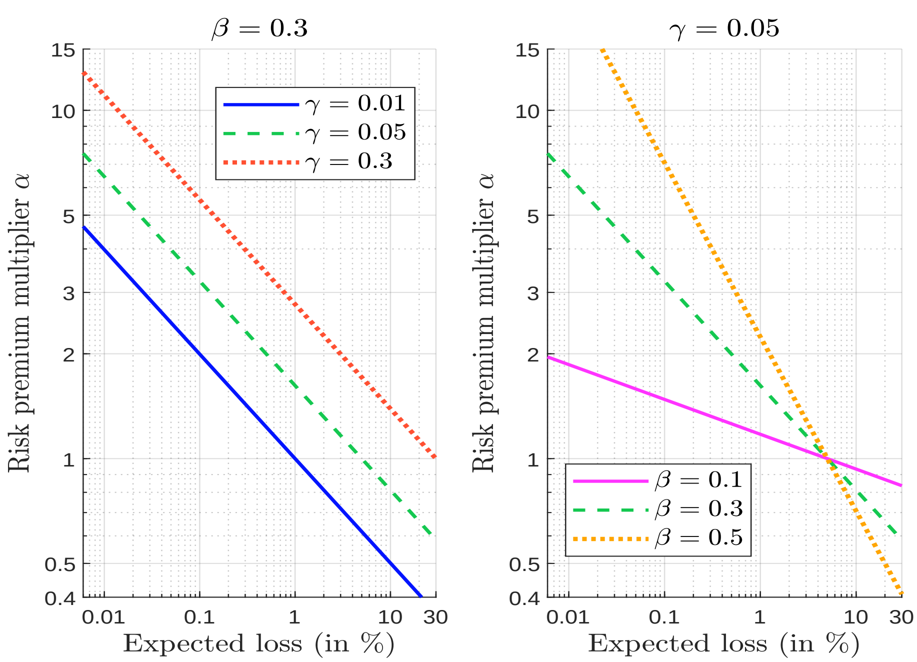

Figure 40: Rolling one-year estimates of β and γ

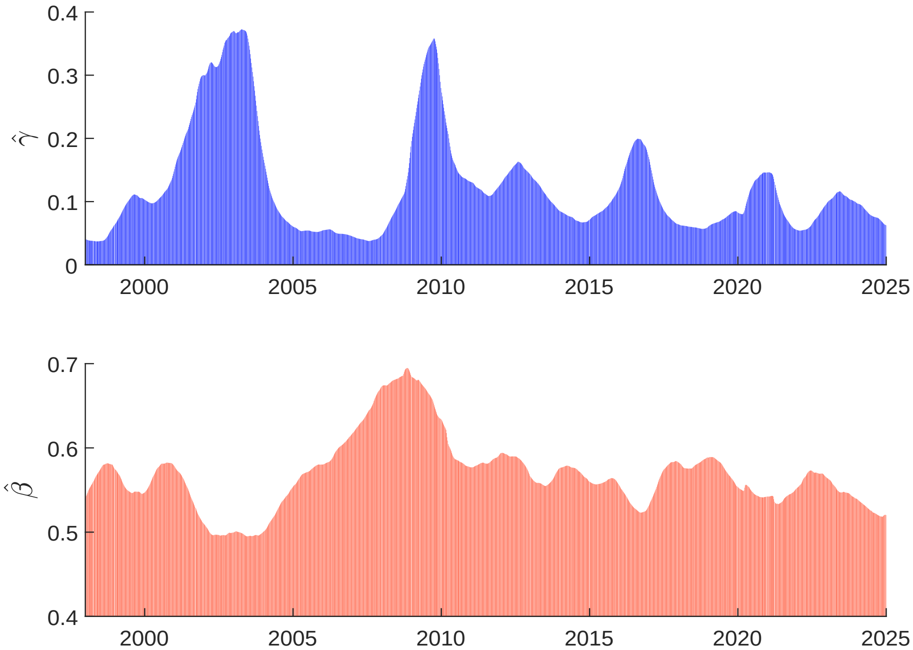

Figure 41: Extra risk premium (Example #3, set (a))

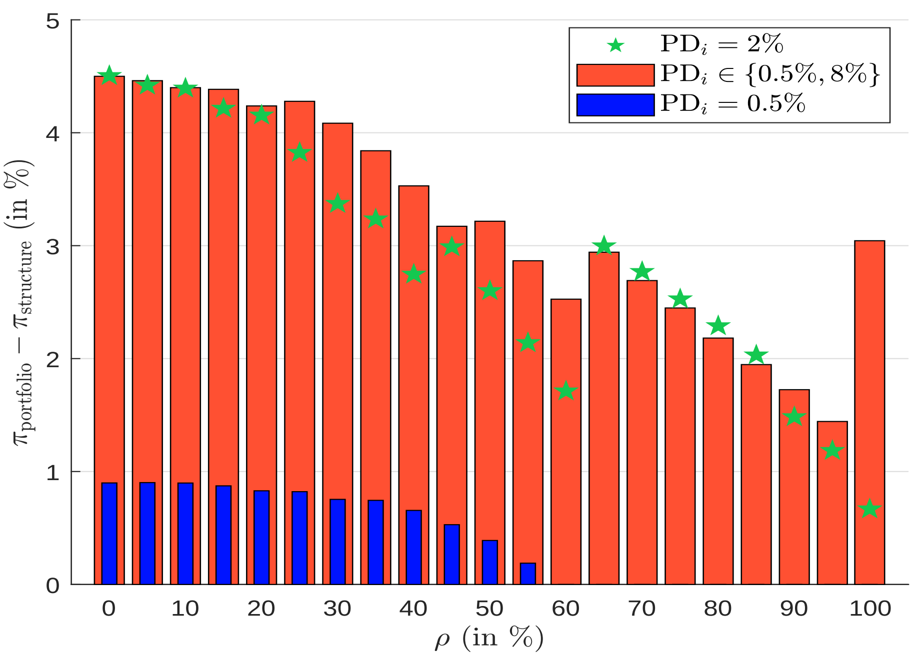

Figure 42: Frequency $f_j(p)$ ($t = 5$ years, Example #3, $\rho = 20\%$)

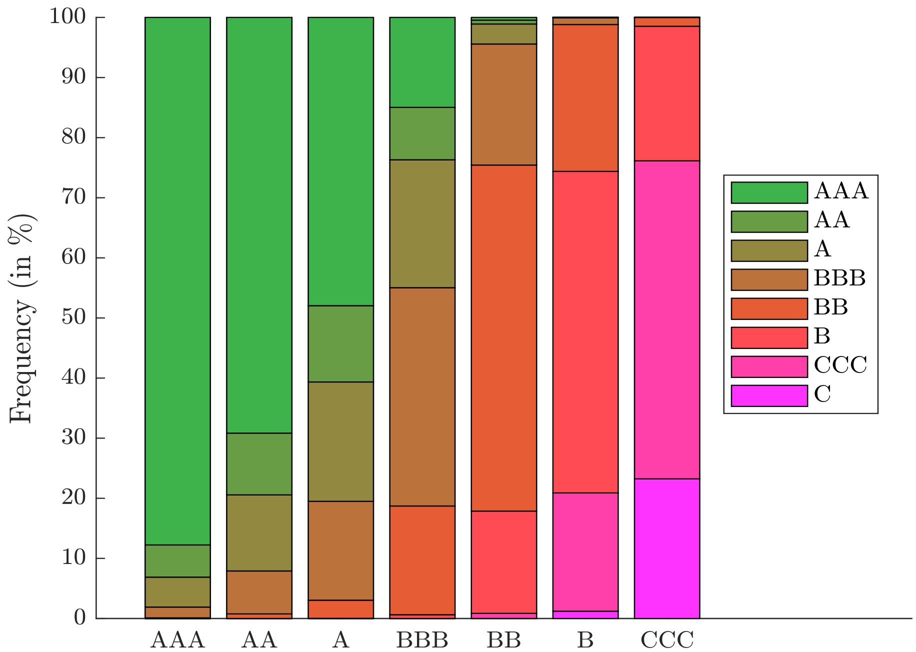

Table 27: Credit rating of the junior and senior tranches

ρ (in %)	10/90		30/70		50/50		opt-AAA		opt-AA	
	Junior	Senior	Junior	Senior	Junior	Senior	Junior	Senior	Junior	Senior
0	CCC	AAA	B−	AAA	B+	AAA	CCC	AAA	CCC	AA+
5	CCC	AA−	B−	AAA	B+	AAA	CCC	AAA	CCC	AA+
10	CCC	A+	B−	AAA	B+	AAA	B	AAA	CCC	AA-
15	CCC	A−	B−	AAA	B+	AAA	B	AAA	B	AAA
20	CCC	BBB+	B−	AAA	B+	AAA	B	AAA	B	AAA
25	CCC	BBB	B−	AAA	B+	AAA	B	AAA	B	AAA
30	CCC	BBB	B−	AA+	B+	AAA	B	AAA	B	AAA
35	CCC	BBB−	B−	AA+	B+	AAA	B	AAA	B	AA+
40	CCC	BBB−	B−	AA	B+	AAA	B	AAA	B	AA+
45	CCC	BBB−	B−	AA−	B+	AAA	B+	AAA	B	AA+
50	CCC	BBB−	B−	AA−	B+	AAA	B+	AAA	B	AA
55	CCC	BBB−	B−	A+	B+	AAA	B+	AAA	B+	AA
60	CCC	BBB−	B−	A	B+	AA+	B+	AAA	B+	AA+
65	CCC	BBB−	B	A−	B+	AA+	BB−	AAA	B+	AA
70	CCC	BBB−	B	BBB+	B+	AA+	BB−	AAA	B+	AA+
75	CCC	BBB−	B	BBB+	B+	AA	BB−	AAA	BB−	AA+
80	CCC	BB+	B	BBB−	B+	AA−	BB−	AAA	BB−	AA+

Figure 43: Fixed 10/90 structure

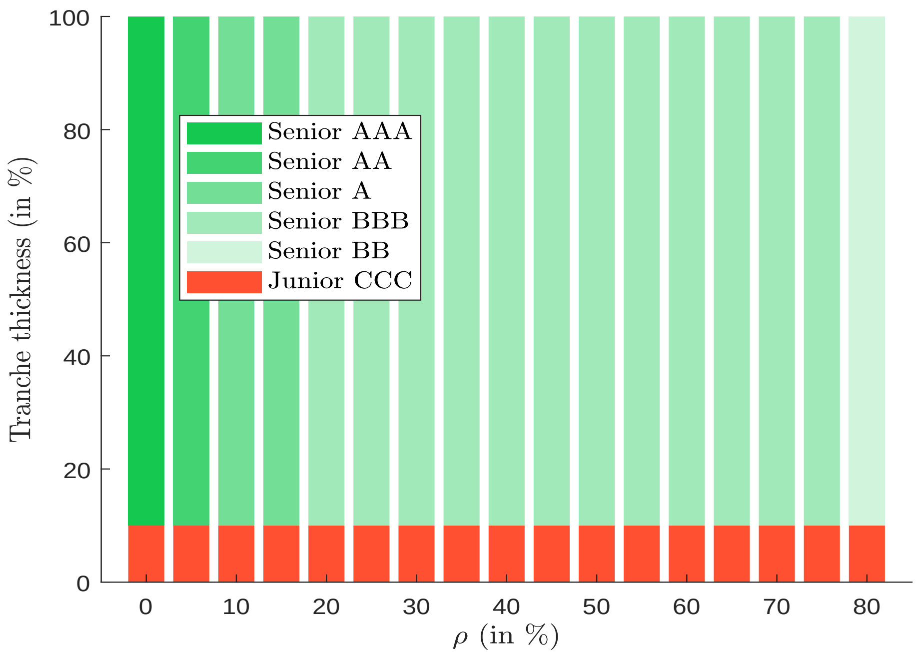

Figure 44: Fixed 50/50 structure

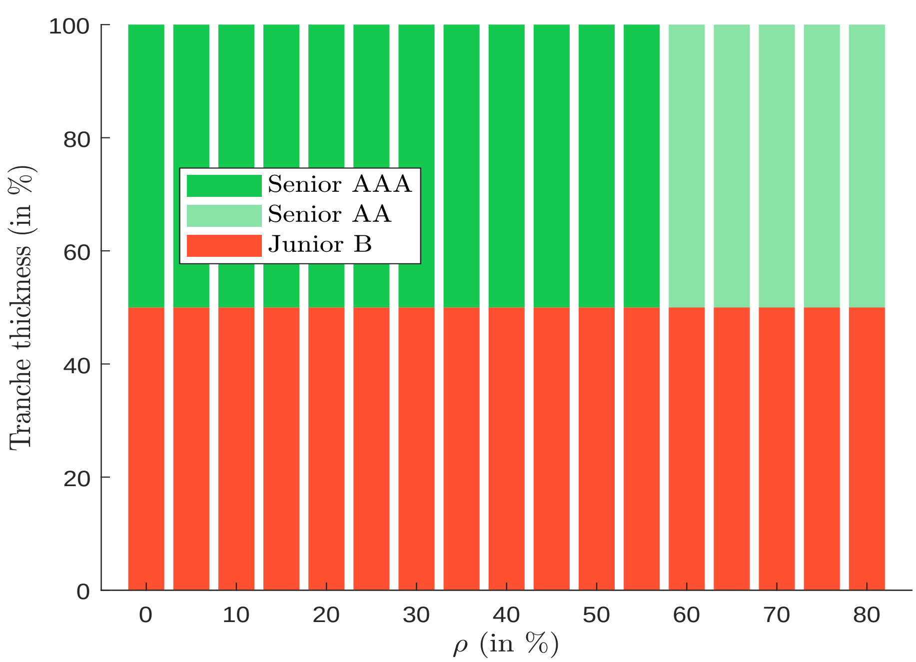

Figure 45: Optimized AA structure

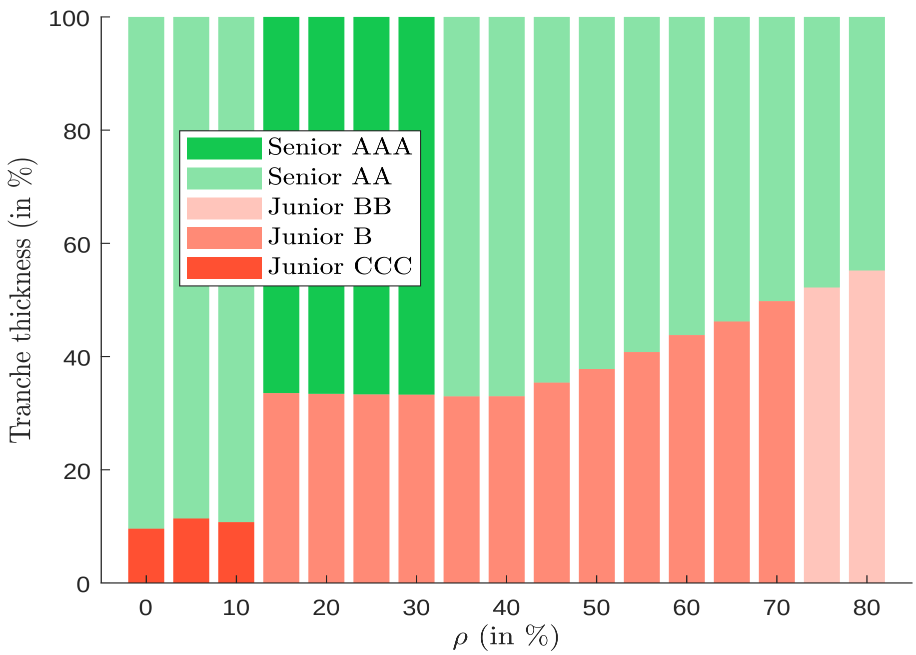

Table 28: Credit rating of the junior, mezzanine and senior tranches (10/90 vs. 5/5/90 vs. 10/5/85)

ρ (in %)	10/90		5/5/90			10/5/85		
	Junior	Senior	Junior	Mezzanine	Senior	Junior	Mezzanine	Senior
0	CCC	AAA	CCC	CCC	AAA	CCC	A−	AAA
5	CCC	AA−	CCC	CCC	AA−	CCC	BB	AAA
10	CCC	A+	CCC	CCC	A+	CCC	BB−	AA+
15	CCC	A−	CCC	CCC	A−	CCC	B+	AA
20	CCC	BBB+	CCC	CCC	BBB+	CCC	B+	AA−
25	CCC	BBB	CCC	CCC	BBB	CCC	B	A+
30	CCC	BBB	CCC	CCC	BBB	CCC	B	A
35	CCC	BBB−	CCC	CCC	BBB−	CCC	B	A−
40	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB+
45	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB
50	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB−
55	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB−
60	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB−
65	CCC	BBB−	CCC	CCC	BBB−	CCC	B	BBB−
70	CCC	BBB−	CCC	CCC+	BBB−	CCC	B	BBB−
75	CCC	BBB−	CCC	CCC+	BBB−	CCC	B	BBB−
80	CCC	BB+	CCC	B−	BB+	CCC	B	BBB−

Table 29: Credit rating of the junior, mezzanine and senior tranches (30/70 vs. 15/15/70 vs. 30/5/65)

ρ (in %)	30/70		15/15/70			30/5/65		
	Junior	Senior	Junior	Mezzanine	Senior	Junior	Mezzanine	Senior
0	B−	AAA	CCC	AAA	AAA	B−	AAA	AAA
5	B−	AAA	CCC	AA+	AAA	B−	AAA	AAA
10	B−	AAA	CCC	A	AAA	B−	AAA	AAA
15	B−	AAA	CCC	BBB−	AAA	B−	AA+	AAA
20	B−	AAA	CCC	BBB−	AAA	B−	AA	AAA
25	B−	AAA	CCC	BBB−	AAA	B−	AA−	AAA
30	B−	AA+	CCC	BB+	AA+	B−	A	AAA
35	B−	AA+	CCC	BB+	AA+	B−	BBB+	AA+
40	B−	AA	CCC	BB	AA	B−	BBB−	AA+
45	B−	AA−	CCC	BB	AA−	B−	BBB−	AA
50	B−	AA−	CCC	BB	AA−	B−	BBB−	AA
55	B−	A+	CCC	BB−	A+	B−	BBB−	AA−
60	B−	A	CCC	BB−	A	B−	BB+	AA−
65	B	A−	CCC	BB−	A−	B	BB+	A+
70	B	BBB+	CCC	BB−	BBB+	B	BB+	A
75	B	BBB+	CCC	BB−	BBB+	B	BB	A−
80	B	BBB−	CCC+	BB−	BBB−	B	BB	BBB+

Table 30: Credit rating of the junior, mezzanine and senior tranches (50/50 vs. 25/25/50 vs. 50/5/45)

ρ (in %)	50/50		25/25/50			50/5/45		
	Junior	Senior	Junior	Mezzanine	Senior	Junior	Mezzanine	Senior
0	B+	AAA	CCC	AAA	AAA	B+	AAA	AAA
5	B+	AAA	CCC	AAA	AAA	B+	AAA	AAA
10	B+	AAA	CCC	AAA	AAA	B+	AAA	AAA
15	B+	AAA	CCC	AA+	AAA	B+	AAA	AAA
20	B+	AAA	CCC	AA	AAA	B+	AAA	AAA
25	B+	AAA	CCC	AA−	AAA	B+	AAA	AAA
30	B+	AAA	CCC	A+	AAA	B+	AAA	AAA
35	B+	AAA	CCC	A−	AAA	B+	AA+	AAA
40	B+	AAA	CCC+	BBB+	AAA	B+	AA+	AAA
45	B+	AAA	CCC+	BBB−	AAA	B+	AA−	AAA
50	B+	AAA	CCC+	BBB−	AAA	B+	A+	AAA
55	B+	AAA	CCC+	BBB−	AAA	B+	A	AAA
60	B+	AA+	CCC+	BBB−	AA+	B+	A−	AA+
65	B+	AA+	B−	BB+	AA+	B+	BBB	AA+
70	B+	AA+	B−	BB+	AA+	B+	BBB−	AA+
75	B+	AA	B−	BB+	AA	B+	BBB−	AA
80	B+	AA−	B	BB+	AA−	B+	BBB−	AA−

Table 31: Credit rating of the junior, mezzanine and senior tranches (AAA vs. CCC/AAA vs. CCC+/AAA optimized solutions)

ρ (in %)	opt-AAA		opt-CCC/AAA			opt-CCC+/AAA		
	Junior	Senior	Junior	Mezzanine	Senior	Junior	Mezzanine	Senior
0	CCC	AAA	CCC		AAA	CCC+		AAA
5	CCC	AAA	CCC		AAA	CCC+		AAA
10	B	AAA	CCC		AAA	CCC+		AAA
15	B	AAA	CCC		AAA	CCC+		AAA
20	B	AAA	CCC	BBB+	AAA	CCC+		AAA
25	B	AAA	CCC	BBB	AAA	CCC+	BBB+	AAA
30	B	AAA	CCC	BBB−	AAA	CCC+	BBB	AAA
35	B	AAA	CCC	BBB−	AAA	CCC+	BBB−	AAA
40	B	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
45	B+	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
50	B+	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
55	B+	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
60	B+	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
65	BB−	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA
70	BB−	AAA	CCC+	BBB−	AAA	CCC+	BBB−	AAA

Figure 46: Optimized junior-mezzanine-senior CCC+/AAA structure

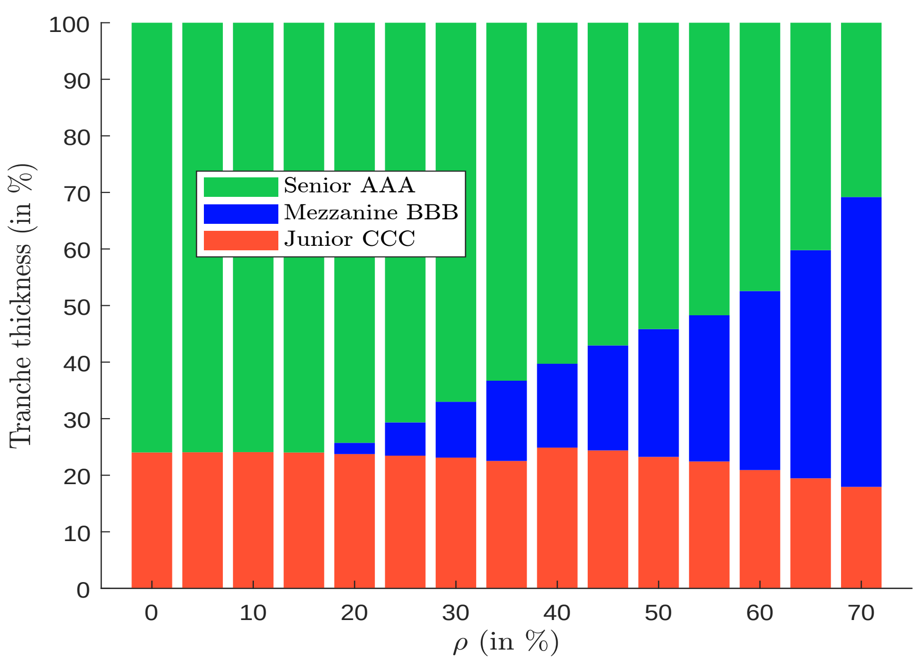

Table 32: Impact (in %) of the loss carryforward mechanism on the fixed 90/10 junior-senior structure

ρ	No-offset loss					Loss carryforward					Junior tranche				$\Delta\mathrm{IRR}_k$	
	EL_2	VaR_2	RPR_2	CIR_2	IRR_2	EL_2^*	VaR_2^*	RPR_2^*	CIR_2^*	IRR_2^*	SDF_1	IRR_1	SDF_1^*	IRR_1^*	Senior	Junior
0%	0.04	0.00	99.90	3.14	5.99	0.00	0.00	100.00	0.00	6.00	0.00	20.55	3.15	17.71	0.01	−2.84
5%	0.35	2.72	99.46	12.57	5.95	0.00	0.00	100.00	0.01	6.00	0.00	20.60	5.22	17.69	0.05	−2.91
10%	0.71	4.91	99.04	16.94	5.91	0.01	0.00	99.99	0.31	6.00	0.00	20.69	7.50	17.71	0.08	−2.98
15%	1.04	7.12	98.65	19.16	5.88	0.07	0.00	99.93	1.01	5.99	0.00	20.80	9.43	17.86	0.12	−2.95
20%	1.37	9.36	98.28	20.53	5.84	0.16	0.00	99.83	1.91	5.98	0.00	20.89	11.01	18.05	0.14	−2.85
25%	1.67	11.59	97.96	21.19	5.80	0.30	0.00	99.67	2.92	5.96	0.00	20.98	12.12	18.33	0.16	−2.65
30%	1.97	13.18	97.62	21.59	5.76	0.48	0.00	99.48	4.00	5.94	0.01	21.06	13.08	18.59	0.18	−2.47
35%	2.29	15.50	97.28	21.73	5.72	0.70	0.00	99.24	4.92	5.91	0.01	21.13	13.89	18.86	0.19	−2.27
40%	2.59	17.86	96.96	21.74	5.68	0.95	2.98	98.96	5.86	5.88	0.03	21.21	14.35	19.11	0.20	−2.10
45%	2.90	20.26	96.62	21.55	5.63	1.23	6.25	98.65	6.75	5.84	0.05	21.29	14.82	19.28	0.21	−2.02
50%	3.20	22.67	96.28	21.35	5.58	1.53	9.36	98.32	7.57	5.79	0.09	21.36	15.13	19.30	0.21	−2.06
55%	3.52	25.22	95.95	20.88	5.53	1.87	12.89	97.95	8.30	5.74	0.13	21.44	15.25	19.27	0.21	−2.16
60%	3.84	27.81	95.60	20.47	5.46	2.24	16.41	97.54	8.95	5.67	0.23	21.48	15.25	19.01	0.21	−2.47
65%	4.22	30.52	95.19	20.08	5.39	2.66	20.32	97.07	9.71	5.59	0.36	21.49	15.37	18.55	0.20	−2.94
70%	4.59	34.08	94.80	19.46	5.29	3.13	24.67	96.56	10.35	5.48	0.58	21.52	15.12	18.19	0.19	−3.33
75%	4.95	37.87	94.41	18.64	5.19	3.60	29.68	96.03	10.90	5.37	0.81	21.53	14.79	17.88	0.18	−3.66
80%	5.46	43.69	93.89	17.86	5.02	4.27	36.96	95.32	11.44	5.18	1.22	21.47	14.38	17.73	0.17	−3.74

EL_k, VaR_k, RPR_k, CIR_k, SDF_k, and IRR_k are the expected loss, 95% value-at-risk, repayment rate, cumulative impairment rate, skipped dividend frequency, and internal rate of return of tranche k, respectively, where $k = 1$ corresponds to the junior tranche and $k = 2$ to the senior tranche. When the loss carry forward mechanism is applied, these statistics are denoted with an asterisk (*e.g.*, EL_k^*). The performance difference between the no-offset loss approach and the loss carryforward mechanism is calculated as $\Delta\mathrm{IRR}_k = \mathrm{IRR}_k^* - \mathrm{IRR}_k$.

Table 33: Impact (in %) of the loss carryforward mechanism on the fixed 70/30 junior-senior structure

ρ	No-offset loss					Loss carryforward					Junior tranche				$\Delta\mathrm{IRR}_k$	
	EL_2	VaR_2	RPR_2	CIR_2	IRR_2	EL_2^*	VaR_2^*	RPR_2^*	CIR_2^*	IRR_2^*	SDF_1	IRR_1	SDF_1^*	IRR_1^*	Senior	Junior
0%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	10.13	0.16	9.89	0.00	−0.24
5%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	10.07	0.66	9.87	0.00	−0.20
10%	0.00	0.00	100.00	0.03	6.00	0.00	0.00	100.00	0.00	6.00	0.00	10.00	1.53	9.85	0.00	−0.15
15%	0.01	0.00	99.98	0.24	6.00	0.00	0.00	100.00	0.01	6.00	0.00	9.91	2.62	9.83	0.00	−0.08
20%	0.04	0.00	99.93	0.59	5.99	0.01	0.00	99.99	0.06	6.00	0.00	9.81	3.80	9.80	0.01	−0.01
25%	0.11	0.00	99.85	1.20	5.99	0.02	0.00	99.98	0.16	6.00	0.00	9.70	4.87	9.77	0.01	0.07
30%	0.20	0.00	99.73	1.90	5.97	0.05	0.00	99.94	0.36	5.99	0.00	9.59	5.93	9.75	0.02	0.16
35%	0.34	0.00	99.56	2.61	5.96	0.12	0.00	99.86	0.69	5.99	0.00	9.48	6.90	9.77	0.03	0.29
40%	0.51	0.00	99.36	3.43	5.93	0.21	0.00	99.75	1.08	5.97	0.01	9.39	7.72	9.79	0.04	0.41
45%	0.71	0.00	99.12	4.24	5.91	0.33	0.00	99.62	1.49	5.96	0.03	9.29	8.53	9.85	0.05	0.56
50%	0.94	0.57	98.85	5.00	5.87	0.49	0.00	99.44	2.00	5.93	0.05	9.20	9.22	9.92	0.06	0.72
55%	1.22	3.85	98.53	5.78	5.83	0.69	0.00	99.22	2.58	5.90	0.09	9.12	9.81	10.01	0.07	0.90
60%	1.54	7.19	98.17	6.38	5.78	0.96	0.00	98.92	3.18	5.85	0.18	9.03	10.27	10.09	0.08	1.06
65%	1.91	10.66	97.76	7.15	5.71	1.28	0.00	98.57	3.76	5.80	0.29	8.90	10.82	10.06	0.09	1.15
70%	2.34	15.25	97.29	7.68	5.61	1.68	0.00	98.13	4.42	5.71	0.49	8.81	11.08	9.96	0.09	1.15
75%	2.78	20.12	96.81	8.29	5.52	2.07	1.18	97.70	5.09	5.62	0.70	8.70	11.25	9.39	0.10	0.70
80%	3.41	27.60	96.15	8.92	5.35	2.70	12.30	97.03	5.98	5.45	1.09	8.52	11.40	8.45	0.10	−0.07

EL_k, VaR_k, RPR_k, CIR_k, SDF_k, and IRR_k are the expected loss, 95% value-at-risk, repayment rate, cumulative impairment rate, skipped dividend frequency, and internal rate of return of tranche k, respectively, where $k = 1$ corresponds to the junior tranche and $k = 2$ to the senior tranche. When the loss carry forward mechanism is applied, these statistics are denoted with an asterisk (*e.g.*, EL_k^*). The performance difference between the no-offset loss approach and the loss carryforward mechanism is calculated as $\Delta\mathrm{IRR}_k = \mathrm{IRR}_k^* - \mathrm{IRR}_k$.

Table 34: Impact (in %) of the loss carryforward mechanism on the fixed 50/50 junior-senior structure

ρ	No-offset loss					Loss carryforward					Junior tranche				ΔIRR_k	
	EL_2	VaR_2	RPR_2	CIR_2	IRR_2	EL_2^*	VaR_2^*	RPR_2^*	CIR_2^*	IRR_2^*	SDF_1	IRR_1	SDF_1^*	IRR_1^*	Senior	Junior
0%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.39	0.01	8.33	0.00	−0.06
5%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.37	0.09	8.32	0.00	−0.05
10%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.34	0.33	8.31	0.00	−0.03
15%	0.00	0.00	100.00	0.00	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.31	0.80	8.30	0.00	−0.02
20%	0.00	0.00	100.00	0.01	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.27	1.38	8.28	0.00	0.01
25%	0.00	0.00	99.99	0.04	6.00	0.00	0.00	100.00	0.00	6.00	0.00	8.22	2.09	8.25	0.00	0.03
30%	0.01	0.00	99.98	0.12	6.00	0.00	0.00	100.00	0.02	6.00	0.00	8.15	2.87	8.22	0.00	0.07
35%	0.04	0.00	99.95	0.28	5.99	0.01	0.00	99.98	0.07	6.00	0.00	8.07	3.62	8.18	0.00	0.12
40%	0.08	0.00	99.89	0.53	5.99	0.03	0.00	99.96	0.17	6.00	0.01	7.97	4.40	8.16	0.01	0.18
45%	0.15	0.00	99.81	0.81	5.98	0.07	0.00	99.92	0.29	5.99	0.01	7.87	5.18	8.13	0.01	0.26
50%	0.24	0.00	99.69	1.20	5.97	0.13	0.00	99.85	0.49	5.98	0.03	7.75	5.94	8.12	0.02	0.36
55%	0.37	0.00	99.53	1.64	5.95	0.21	0.00	99.75	0.75	5.97	0.05	7.63	6.65	8.11	0.02	0.49
60%	0.57	0.00	99.30	2.16	5.91	0.37	0.00	99.58	1.10	5.94	0.11	7.48	7.27	8.14	0.03	0.66
65%	0.83	0.00	99.01	2.76	5.87	0.56	0.00	99.36	1.54	5.91	0.20	7.31	7.97	8.16	0.04	0.85
70%	1.17	0.00	98.63	3.44	5.80	0.86	0.00	99.05	2.08	5.85	0.35	7.12	8.49	8.22	0.05	1.10
75%	1.51	0.00	98.25	4.03	5.73	1.15	0.00	98.73	2.54	5.78	0.53	6.95	8.97	8.23	0.05	1.27
80%	2.09	0.00	97.64	4.89	5.59	1.67	0.00	98.20	3.28	5.65	0.87	6.68	9.45	7.98	0.07	1.30

EL_k, VaR_k, RPR_k, CIR_k, SDF_k, and IRR_k are the expected loss, 95% value-at-risk, repayment rate, cumulative impairment rate, skipped dividend frequency, and internal rate of return of tranche k, respectively, where $k = 1$ corresponds to the junior tranche and $k = 2$ to the senior tranche. When the loss carry forward mechanism is applied, these statistics are denoted with an asterisk (*e.g.*, EL_k^*). The performance difference between the no-offset loss approach and the loss carryforward mechanism is calculated as $\Delta\text{IRR}_k = \text{IRR}_k^* - \text{IRR}_k$.