Liquidity Stress Testing in Asset Management

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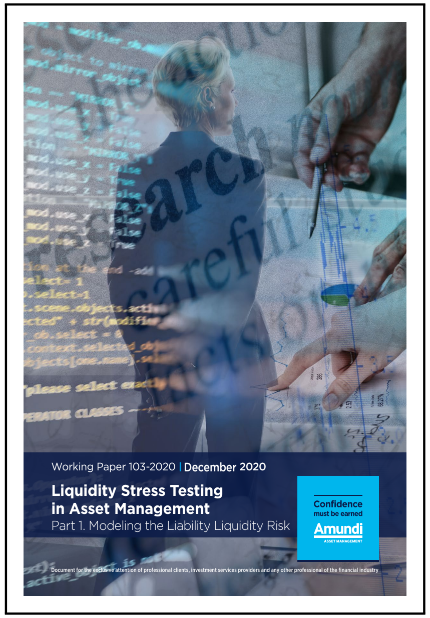
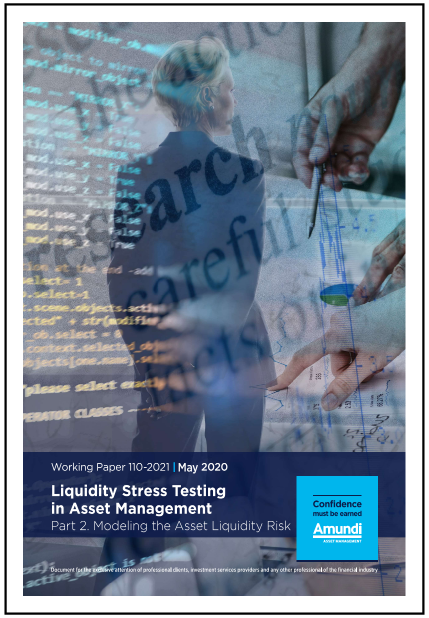
[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

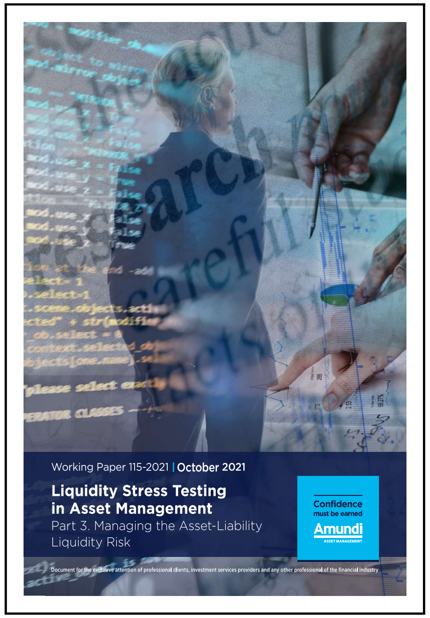
Amundi publications on liquidity stress testing



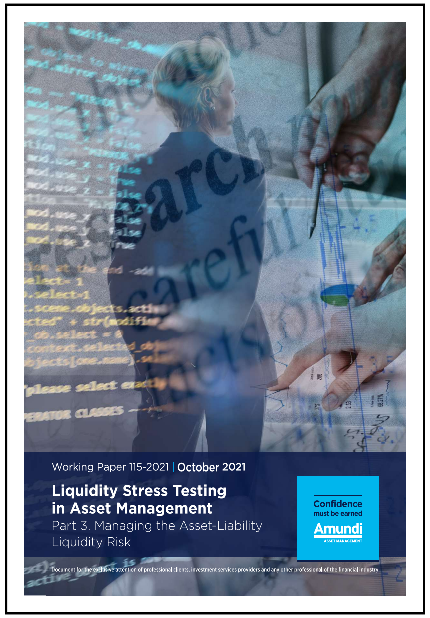
Liability Liquidity Risk



Asset Liquidity Risk



Asset-Liability Management



Step-by-step Practical Guide

Amundi Research Center: `https://research-center.amundi.com`

arXiv, ResearchGate & SSRN

What means liquidity risk

The liquidity management problem does not concern illiquid assets,

... but liquid assets

$\Rightarrow$ Issues to classify assets into liquid and illiquid instruments

What means liquidity risk

"[...] there is also broad belief among users of financial liquidity — traders, investors and central bankers — that the principal challenge is not the ***average level*** *of financial liquidity ... but its* ***variability*** *and uncertainty " (Persaud, 2003).*

What means liquidity risk

Two implicit concepts of liquidity risk:

- Liquidity (or illiquididy) risk premium (additional return of illiquid assets) $\Rightarrow$ Buy-and-hold strategies
- Liquidity risk run (or cycle) $\Rightarrow$ Trading strategies
  - Liquid assets $\rightarrow$ illiquid assets[2] (supply/demand imbalance)
  - Liquid assets $\rightarrow$ liquid assets (spillover effects)

Predictable (illiquidity premium) $\neq$ **Unpredictable** (liquidity ~~risk~~ run)

$\Rightarrow$ Systemic risk concerns assets that are supposed to be liquid, but can become illiquid (systemic risk $\neq$ systematic risk)

[2] The concept of transaction cost is not valid for illiquid assets (no distinction between price variation and trading cost)

The liquidity-adjusted CAPM

L-CAPM (Acharya and Pedersen, 2005)

We note L_i the relative (stochastic) illiquidity cost of Asset i. At the equilibrium, we have:

$$\mathbb{E}[R_i - L_i] - R_f = \tilde{\beta}_i (\mathbb{E}[R_M - L_M] - R_f)$$

where:

$$\tilde{\beta}_i = \frac{\operatorname{cov}(R_i - L_i, R_M - L_M)}{\operatorname{var}(R_M - L_M)}$$

CAPM in the frictionless economy

$\Downarrow$

CAPM in net returns (including illiquidity costs)

The liquidity-adjusted CAPM

The liquidity-adjusted beta can be decomposed into four beta(s):

- $\beta_i = \beta\left(R_{i,} R_M\right)$ is the standard market beta
- $\beta\left(L_{i,}, L_M\right)$ is the beta associated to the commonality in liquidity with the market liquidity
- $\beta\left(R_{i,}, L_M\right)$ is the beta associated to the return sensitivity to market liquidity
- $\beta\left(L_{i,}, R_M\right)$ is the beta associated to the liquidity sensitivity to market returns

Spillover effect between $\beta\left(L_{i,}, L_M\right)$, $\beta\left(R_{i,}, L_M\right)$ and $\beta\left(L_{i,}, R_M\right)$

The (il)liquidity risk premium

Acharya and Pedersen (2005)

If assets face some liquidity costs, the relationship between the risk premium and the beta of asset i becomes:

$$\mathbb{E}[R_i] - R_f = \alpha_i + \beta_i (\mathbb{E}[R_M] - R_f)$$

where α_i is a function of the relative liquidity of Asset i with respect to the market portfolio and the liquidity beta(s):

$$\begin{aligned} \alpha_i &= \left(\mathbb{E}[L_i] - \tilde{\beta}_i \mathbb{E}[L_M]\right) + \\ & \quad \beta(L_{i,}, L_M)\,\pi_M - \beta(R_{i,}, L_M)\,\pi_M - \beta(L_{i,}, R_M)\,\pi_M \end{aligned}$$

Three liquidity risks

In fact, we have:

$$\alpha_i = \text{illiquidity level} + \text{illiquidity covariance risks}$$

1. $\beta\left(L_{i,}, L_M\right)$
   - An asset that becomes illiquid when the market becomes illiquid should have a higher risk premium
   - Substitution effects when the market becomes illiquid
2. $\beta\left(R_{i,}, L_M\right)$
   - Assets that perform well in times of market illiquidity should have a lower risk premium
   - Relationship with solvency constraints
3. $\beta\left(L_{i,}, R_M\right)$
   - Investors accept a lower risk premium on assets that are liquid in a bear market
   - Selling markets $\neq$ buying markets

Impact of the liquidity on the stock market

The dot-com crisis (2000-2003)

If we consider the S&P 500 index, we obtain:

- 55% of stocks post a negative performance

$$\approx 75\% \text{ of MC}$$

- 45% of stocks post a positive performance

$$\text{Maximum drawdown} = 49\ \%$$

Small caps stocks $\nearrow$
Value stocks $\nearrow$

Systematic risk crisis

The GFC crisis (2008)

If we consider the S&P 500 index, we obtain:

- 95% of stocks post a negative performance

$$\approx 97\% \text{ of MC}$$

- 5% of stocks post a positive performance

$$\text{Maximum drawdown} = 56\ \%$$

Small caps stocks $\searrow$
Value stocks $\searrow$

Systemic risk crisis

The specific status of the stock market

The interconnectedness nature of illiquid assets and liquid assets: the example of the Global Financial Crisis

- Subprime crisis $\Leftrightarrow$ banks (credit risk)
- Banks $\Leftrightarrow$ asset management, e.g. hedge funds (funding & leverage risk)
- Asset management $\Leftrightarrow$ equity market (liquidity risk)
- Equity market $\Leftrightarrow$ banks (asset-price & collateral risk)

The equity market is the ultimate liquidity provider:
GFC $\gg$ internet bubble

Remark

$^1/_3$ *of the losses in the stock market is explained by the liquidity supply*

Relationship between diversification & liquidity

During good times

- Medium correlation between liquid assets
- Illiquid assets have low impact on liquid assets
- Low substitution effects

During bad times

- High correlation between liquid assets
- Illiquid assets have a high impact on liquid assets
- High substitution effects

Main effect:

$$\mathbb{E}[L_i]$$

Main effects:

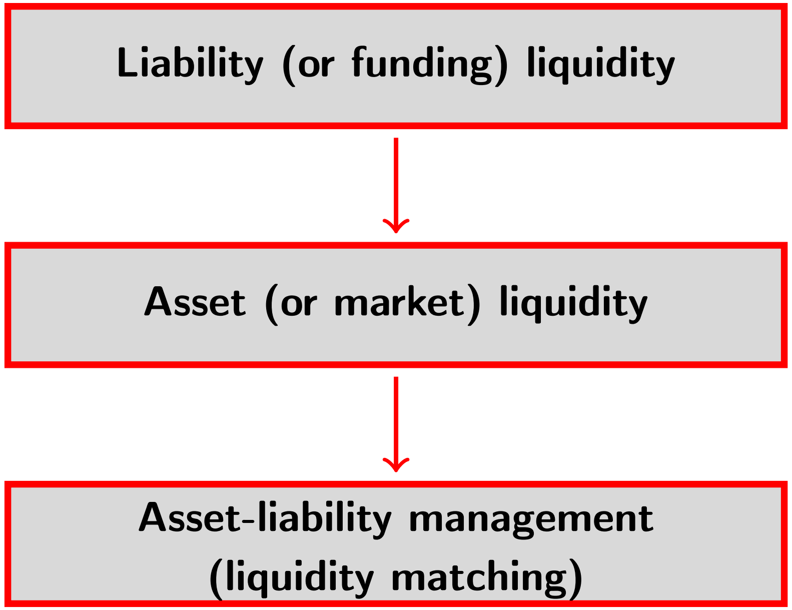
$$\beta(L_i, R_M) \text{ and } \beta(R_i, L_M)$$

Regulation of liquidity risk in asset management

- Basel III Accord and Liquidity Coverage Ratio (BCBS, 2010, 2013)
- Systemic risk of non-bank non-insurer SIFIs (FSB, 2010, 2015)
- Liquidity management of US asset managers (SEC, 2015-2018): Rule 22e-4, IC-32316, etc.
- Liquidity stress testing (LST) in Europe (ESMA, 2019,2020)

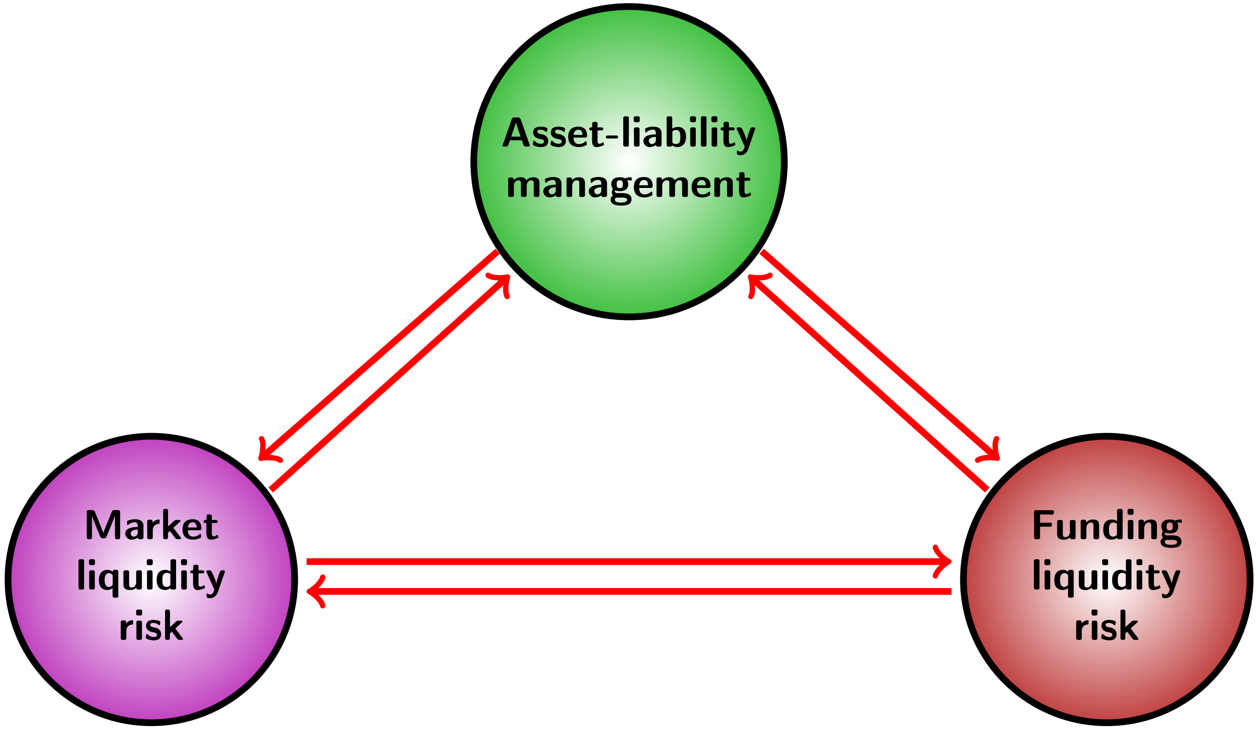
Liquidity stress testing

Figure: The sequential approach of liquidity stress testing



Liquidity stress testing

Figure: The network approach of liquidity stress testing



Liquidity stress testing in asset management

Liability liquidity risk measurement

Defining a redemption shock scenario

Two measurement approaches:

1. Historical approach $\Rightarrow$ non-parametric risk measures: historical VaR and CVaR
2. Frequency-severity modeling approach $\Rightarrow$ parametric risk measures: analytical VaR, CVaR and stress scenarios
   - Zero-inflated statistical model (population-based model)
   - Behaviorial model (individual-based model)
   - Factor-based model

Liquidity stress testing in asset management

Asset liquidity risk measurement

Liquidating a redemption portfolio and measuring its trading cost

Two measurement approaches:

- Liquidity risk profile
  - Liquidation ratio
  - Time to liquidation
  - Liquidation shortfall
- Liquidity cost
  - Transaction cost & market impact
  - Implementation shortfall and effective cost

Liquidity stress testing in asset management

Asset-liability liquidity risk management

Managing the liquidity risk (asset-liability matching)

Three families of tools:

- Liquidity measurement tools
  - Redemption coverage ratio (RCR)
  - Liquidation policy (vertical vs waterfall slicing)
  - Reverse stress testing
- Liquidity management tools
  - Liquidity buffer and cash hoarding
  - Redemption suspension (a-LMT): redemption suspension, side pockets, gates
  - Swing pricing and anti-dilution levies (ADL)
- Liquidity monitoring tools
  - Macro-approach of liquidity monitoring
  - Micro-approach of liquidity monitoring

Number of observations (without mandates)

AUM $\geq$ €5 mn

- 1 159 415 observations from January, 1st 2019 to August, 19th 2020
- Breakdown by fund and investor categories:

Total number n of observations	Balanced	Bond	Enhanced Treasury	Equity	Money Market	Other	Structured	Total
Auto-consumption	16 147	43 189	3 783	43 737	6 008	13 793	0	126 657
Central bank	1 281	580	0	476	0	0	0	2 337
Corporate	1 862	6 542	2 305	5 468	7 812	4 235	0	28 224
Corporate pension fund	2 344	8 650	427	9 031	2 670	1 277	0	24 399
Employee savings plan	9 894	4 240	1 349	19 145	3 232	0	5 279	43 139
Institutional	6 858	36 792	3 716	41 104	8 329	16 029	0	112 828
Insurance	3 436	13 011	3 303	21 832	8 543	5 750	0	55 875
Other	7 577	12 751	5 428	4 155	9 333	11 788	0	51 032
Retail	115 394	77 879	6 692	95 393	14 798	27 834	83 118	421 108
Sovereign	2 969	2 261	854	3 405	2 853	1 746	0	14 088
Third-party distributor	55 696	75 591	4 929	114 171	10 732	13 483	5 126	279 728
Total	223 458	281 486	32 786	357 917	74 310	95 935	93 523	1 159 415

- Most representative investors = retail, third-party distributor, auto-consumption and institutional
- Most representative fund categories = equity, bond and balanced

Number of redemptions (without mandates)

AUM $\geq$ €5 mn

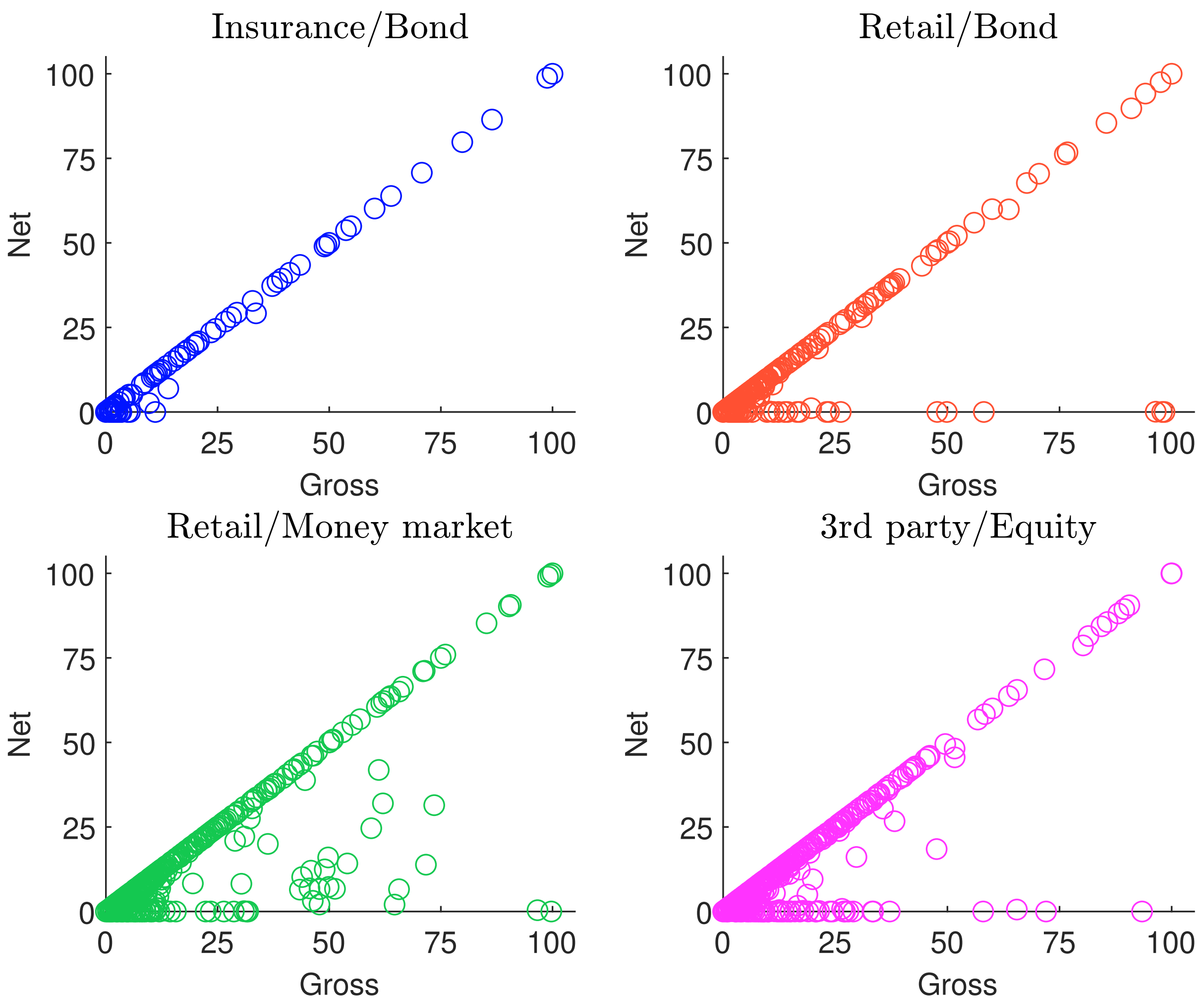
- 360 670 redemptions from January, 1st 2019 to August, 19th 2020
- Breakdown by liquidity class and client:

Total number n_1 of redemptions	Balanced	Bond	Enhanced Treasury	Equity	Money Market	Other	Structured	Total
Auto-consumption	3 492	8 385	1 135	11 137	3 040	881	0	28 070
Central bank	2	2	0	7	0	0	0	11
Corporate	280	405	144	157	3 110	9	0	4 105
Corporate pension fund	190	292	17	304	202	0	0	1 005
Employee savings plan	264	120	40	519	74	0	145	1 162
Institutional	1 328	2 312	73	2 677	2 734	166	0	9 290
Insurance	419	874	114	1 576	2 385	60	0	5 428
Other	733	493	200	804	2 008	262	0	4 500
Retail	51 454	35 079	3 932	67 250	6 770	4 875	22 707	192 067
Sovereign	484	72	9	343	520	1	0	1 429
Third-party distributor	18 808	28 242	2 266	52 445	7 077	4 431	334	113 603
Total	77 454	76 276	7 930	137 219	27 920	10 685	23 186	360 670

- Not all the cells can be calibrated: the example of central banks!

Gross or net redemption rates?

Figure: Gross versus net redemption rates



Risk measures

Let $\mathbf{F}$ be the probability distribution of the redemption rate

- Mean:
$$\mathbb{M} = \int_0^1 x \, \mathrm{d}\mathbf{F}(x)$$
- Quantile (or the value-at-risk) at the confidence level α:
$$\mathbb{Q}(\alpha) = \mathbf{F}^{-1}(\alpha)$$
- Average beyond the quantile (or the conditional value-at-risk):
$$\mathbb{C}(\alpha) = \mathbb{E}\left[\mathcal{R} \mid \mathcal{R} \geq \mathbf{F}^{-1}(\alpha)\right]$$

$\mathbb{M}$-statistic (in %)

- $\mathbb{M}$ corresponds to the daily mean
- The average redemption rate is equal to 22 bps if we consider all fund categories, 1.06% if we consider the money market funds, 6 bps if we consider the equity/employee savings plan cell, etc.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	0.27	0.36	0.65	0.30	1.58	0.18		0.38
Central bank	0.01	0.06		0.11				0.04
Corporate	0.08	0.15	0.27	0.25	1.52	0.07		0.54
Corporate pension fund	0.17	0.05	0.10	0.10	0.55	0.00		0.13
Employee savings plan	0.03	0.05	0.13	0.06	0.06		0.08	0.06
Institutional	0.13	0.16	0.64	0.18	1.47	0.06		0.27
Insurance	0.17	0.15	0.12	0.16	0.90	0.08		0.26
Other	0.08	0.10	0.33	0.21	0.76	0.02		0.23
Retail	0.15	0.14	0.26	0.16	0.91	0.07	0.04	0.15
Sovereign	0.01	0.01	0.16	0.19	1.91	0.06		0.45
Third-party distributor	0.12	0.24	0.67	0.19	0.92	0.28	0.08	0.23
Total	0.14	0.20	0.40	0.18	1.06	0.11	0.04	0.22

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

$\mathbb{Q}$-statistic (in %)

- $\mathbb{Q}$ corresponds to the daily value-at-risk at the 99% confidence level
- The average value-at-risk is equal to 3.50% if we consider all fund categories

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	2.93	7.57	12.62	5.46	25.98	3.23		7.44
Central bank	0.00	0.00		0.12				0.00
Corporate	0.30	1.58	4.90	3.88	24.14	0.00		12.71
Corporate pension fund	0.39	0.05	1.30	0.03	13.09	0.00		0.50
Employee savings plan	1.06	1.70	2.35	1.08	2.51		0.25	1.13
Institutional	0.84	1.94	8.68	3.10	34.82	0.00		5.11
Insurance	0.32	0.21	3.87	0.50	18.39	0.00		5.25
Other	0.73	0.56	2.40	2.20	14.75	0.05		3.41
Retail	2.01	1.50	4.72	1.65	18.36	1.17	0.45	1.92
Sovereign	0.11	0.14	7.98	0.22	66.36	0.00		8.28
Third-party distributor	1.32	4.59	11.13	3.38	14.66	3.96	1.11	3.90
Total	1.77	3.18	6.30	2.68	21.76	1.19	0.45	3.50

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

$\mathbb{C}$-statistic (in %)

- $\mathbb{C}$ corresponds to the daily conditional value-at-risk at the 99% confidence level
- The average conditional value-at-risk is equal to 15.79% if we consider all fund categories

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	21.08	23.37	40.73	21.24	54.96	15.50		24.86
Central bank	1.28	6.05		10.11				4.38
Corporate	7.31	14.98	22.80	22.48	38.37	6.52		28.21
Corporate pension fund	17.22	5.14	9.24	9.58	32.33	0.00		13.06
Employee savings plan	2.48	3.16	10.60	4.91	4.97		7.91	4.86
Institutional	10.99	15.40	62.30	16.27	58.10	6.26		22.79
Insurance	16.35	14.65	10.59	15.32	37.28	7.62		21.24
Other	7.45	9.84	32.56	18.61	46.88	2.17		20.22
Retail	7.02	8.34	15.99	8.95	44.38	5.03	3.03	9.18
Sovereign	0.39	1.35	15.20	17.97	86.47	5.73		39.85
Third-party distributor	6.69	14.53	42.24	11.22	32.68	20.16	6.85	13.72
Total	8.14	14.23	31.15	12.94	46.13	9.32	3.52	15.79

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

Confidence level in estimated values

Confidence level (with respect to the number of observations)

$\circ\circ\circ$ $0 - 10$, $\circ\circ$ $11 - 50$, $\circ$ $51 - 200$, $\bullet$ $201 - 1\,000$, $\bullet\bullet$ $1\,001 - 10\,000$, $\bullet\bullet\bullet$ $+10\,000$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Auto-consumption	●●●	●●●	●●	●●●	●●	●●●	○○○
Central bank	●●	●	○○○	●	○○○	○○○	○○○
Corporate	●●	●●	●●	●●	●●	●●	○○○
Corporate pension fund	●●	●●	●	●●	●●	●●	○○○
Employee savings plan	●●	●●	●●	●●●	●●	○○○	●●
Institutional	●●	●●●	●●	●●●	●●	●●●	○○○
Insurance	●●	●●●	●●	●●●	●●	●●	○○○
Other	●●	●●●	●●	●●	●●	●●●	○○○
Retail	●●●	●●●	●●	●●●	●●●	●●●	●●●
Sovereign	●●	●●	●	●●	●●	●●	○○○
Third-party distributor	●●●	●●●	●●	●●●	●●●	●●●	●●

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured

From historical stress scenarios to expert stress scenarios

Historical figures $\Rightarrow$ expert figures

For instance, we may assume that:

- The redemption shocks from central banks are generally lower than those from institutional, sovereign and corporate investors
- The redemption shocks from wage savings and retail clients are generally lower than those from third-party distributors
- The redemption shocks coming from auto-consumption are the highest because of the all-or-nothing approach
- Bond and equity funds have similar redemption shocks, whereas the redemption shocks experienced by balanced funds are lower
- Enhanced treasury funds must have higher redemption shocks than bond funds, whereas the highest redemption shocks are observed for money market funds

Statistical issues

- Let $\mathbb{R}(\alpha) = \mathbb{C}(\alpha) / \mathbb{Q}(\alpha)$ be the ratio between the conditional value-at-risk and the value-at-risk
- In the case of a Gaussian distribution, we have:

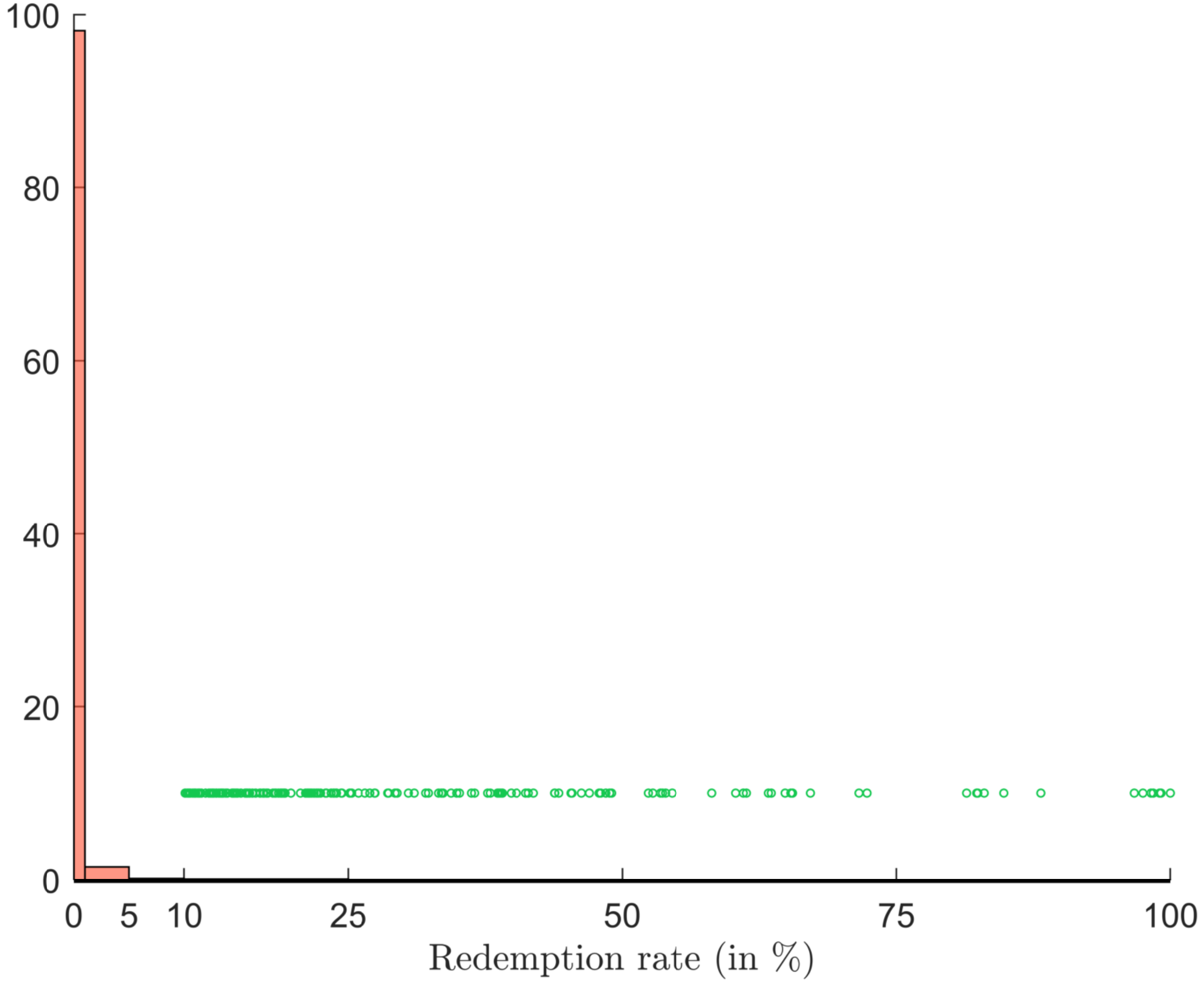
$$\mathbb{R}(\alpha) = \frac{\phi\left(\Phi^{-1}(\alpha)\right)}{(1-\alpha)\,\Phi^{-1}(\alpha)}$$

  This ratio is equal to 1.37 and 1.15 when $\alpha = 90\%$ and $\alpha = 99\%$
- $\mathbb{R}(\alpha) \ll 2$ in the case of market and credit risks
- $\mathbb{R}(\alpha) \gg 2$ in the case of the operational risk

On average, we obtain $\mathbb{R}(99\%) = 4.5$ for the redemption risk!

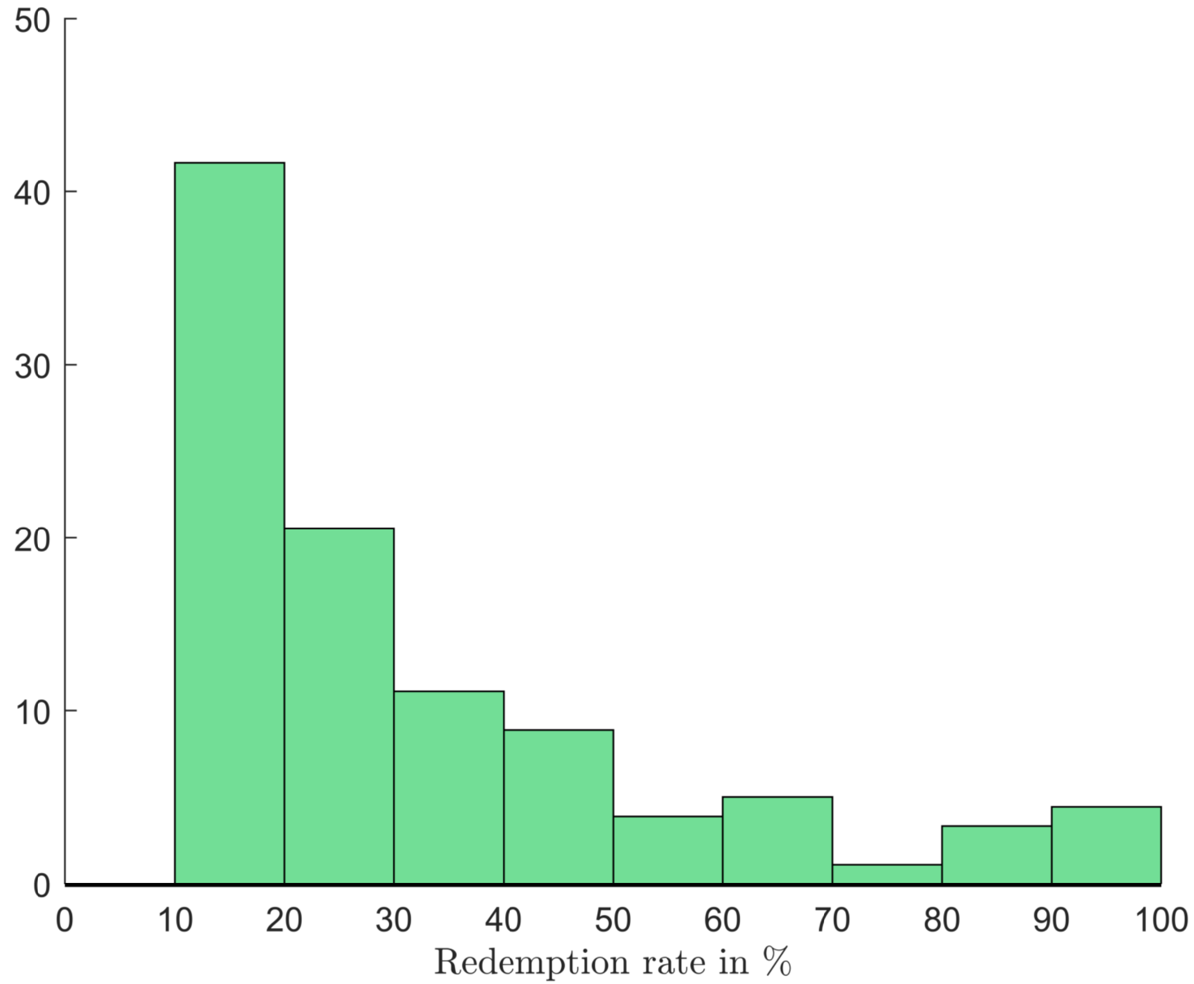
Statistical issues

Figure: Histogram of redemption rates (retail/equity)



Statistical issues

Figure: Histogram of redemption rates larger than 10% (retail/equity)



Statistical issues

Table: Frequencies of redemption rates

Range	Frequency: Retail Equity	Sovereign Bond	Corporate Money Market	Institutional Balanced
0%	29.502%	96.816%	60.189%	80.636%
]0, 0.01%]	13.704%	0.487%	2.074%	3.514%
]0.01%, 0.05%]	26.744%	0.796%	3.008%	5.862%
]0.05%, 0.1%]	13.138%	0.796%	2.048%	3.587%
]0.1%, 0.5%]	12.938%	0.708%	7.258%	4.812%
]0.1%, 1%]	2.101%	0.310%	3.571%	0.714%
]1%, 10%]	1.684%	0.044%	17.844%	0.656%
]10%, 20%]	0.079%	0.000%	2.650%	0.058%
]20%, 50%]	0.077%	0.044%	1.165%	0.102%
]50%, 100%]	0.031%	0.000%	0.192%	0.058%

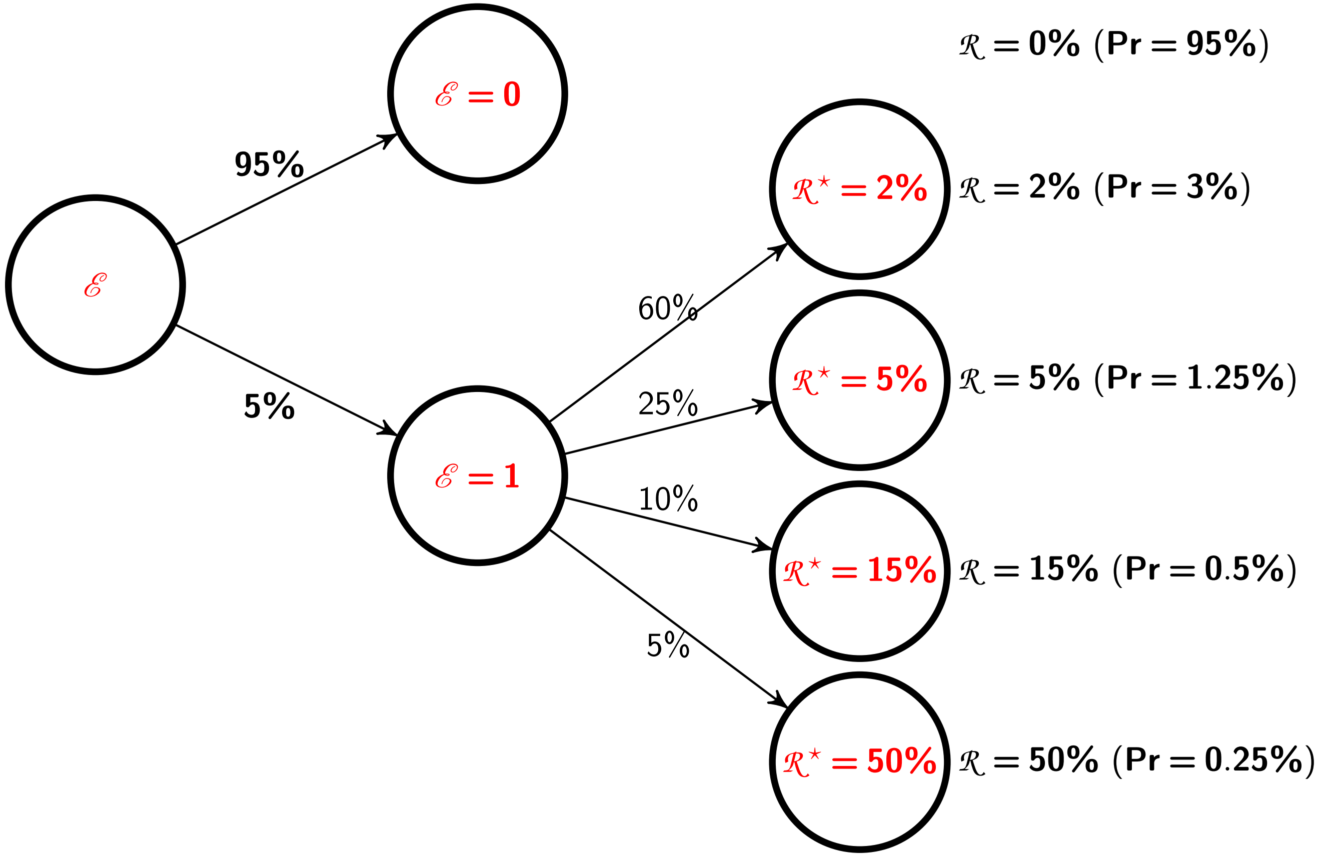
The frequency-severity approach

In this approach, we assume that the redemption is driven by two risk factors:

1. The redemption frequency, which measures the occurrence $\mathcal{E}$ of the redemption
2. The redemption severity $\mathcal{R}^{\star}$, which measures the amount of the redemption

The frequency-severity approach

Figure: Zero-inflated modeling of the redemption risk



The frequency-severity approach

In this approach, the model is specified by two probability distributions:

1. The redemption event $\mathcal{E}$ follows a bernoulli distribution $\mathcal{B}(p)$:

$$\Pr\{\mathcal{E} = 1\} = \Pr\{\mathcal{R} > 0\} = p$$

2. The redemption severity $\mathcal{R}^{\star}$ follows a continuous probability distribution **G**:

$$\Pr\{\mathcal{R} \leq x \mid \mathcal{E} = 1\} = \mathbf{G}(x)$$

The unconditional probability distribution of the redemption rate is:

$$\mathbf{F}(x) = \mathbb{1}\{x \geq 0\} \cdot (1 - p) + \mathbb{1}\{x > 0\} \cdot p \cdot \mathbf{G}(x)$$

whereas its density probability function is:

$$f(x) = \begin{cases} 1 - p & \text{if } x = 0 \\ p \cdot g(x) & \text{otherwise} \end{cases}$$

The frequency-severity approach

The redemption rate is the product of the redemption event and the redemption severity:

$$\mathcal{R} = \mathcal{E} \cdot \mathcal{R}^{\star}$$

Statistical moments

- Mean

$$\mathbb{E}\left[\mathcal{R}\right] = p\mathbb{E}\left[\mathcal{R}^{\star}\right]$$

- Variance

$$\sigma^2\left(\mathcal{R}\right) = p\sigma^2\left(\mathcal{R}^{\star}\right) + p\left(1-p\right)\mathbb{E}^2\left[\mathcal{R}^{\star}\right]$$

- Skewness

$$\gamma_1\left(\mathcal{R}\right) = \ldots$$

- Excess kurtosis

$$\gamma_2\left(\mathcal{R}\right) = \ldots$$

The frequency-severity approach

Statistical risk measures of the zero-inflated model

- The formula of the value-at-risk is equal to:

$$\mathbb{Q}(\alpha) = \mathbb{1}\{p > 1-\alpha\} \cdot \mathbf{G}^{-1}\left(\frac{\alpha + p - 1}{p}\right)$$

- In the case $p > 1 - \alpha$, we have:

$$\mathbb{C}(\alpha) = \frac{p}{1-\alpha}\int_{\mathbb{Q}(\alpha)}^{1} x g(x)\,\mathrm{d}x$$

- The parametric stress scenario is equal to:

$$\mathbb{S}(\mathcal{T}) = \mathbb{1}\{p > \mathcal{T}^{-1}\} \cdot \mathbf{G}^{-1}\left(1 - \frac{1}{p\mathcal{T}}\right)$$

where $\mathcal{T}$ is the return time of the scenario

The frequency-severity approach

- The choice of the severity distribution is an important issue
- $\mathcal{R}^{\star}$ is a random variable between 0 and 1
- It is natural to use the two-parameter beta distribution $\mathscr{B}(a,b)$
- We have:

$$\mathbf{G}(x) = \mathfrak{B}(x;a,b)$$

where $\mathfrak{B}(x;a,b)$ is the incomplete beta function

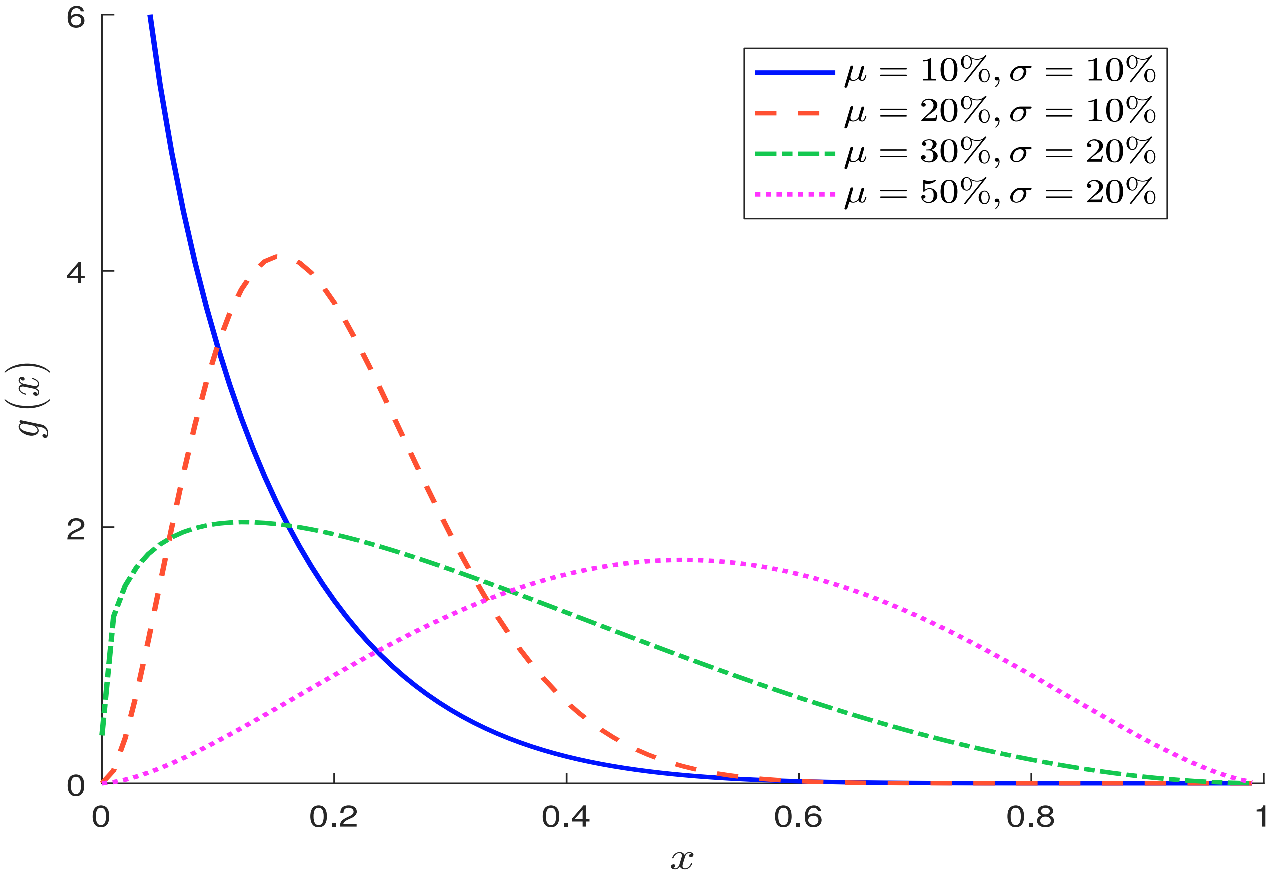
$p-\mu-\sigma$ parameterization

The zero-inflated model is defined by:

1. The probability p of the redemption frequency
2. The mean μ of the redemption severity
3. The standard deviation σ of the redemption severity

The frequency-severity approach

Figure: Density function of the beta distribution



The frequency-severity approach

Statistical moments

- Mean

$$\mathbb{E}\left[\mathcal{R}\right] = p\frac{a}{a+b}$$

- Variance

$$\sigma^2\left(\mathcal{R}\right) = p\frac{ab+\left(1-p\right)a^2\left(a+b+1\right)}{\left(a+b\right)^2\left(a+b+1\right)}$$

- Skewness

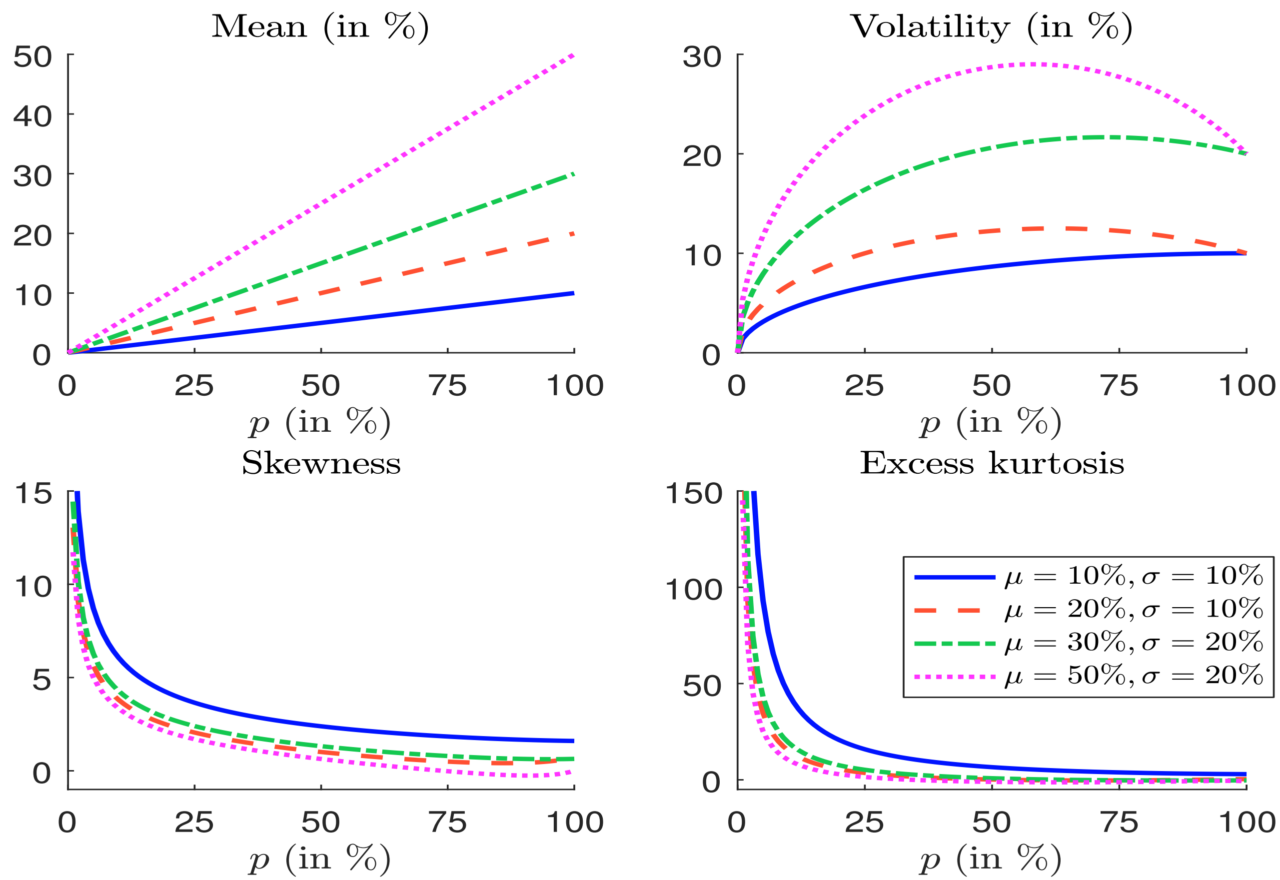
$$\gamma_1\left(\mathcal{R}\right) = \frac{2\left(b-a\right)\sqrt{a+b+1}}{\left(a+b+2\right)\sqrt{ab}}$$

- Excess kurtosis

$$\gamma_2\left(\mathcal{R}\right) = \frac{6\left(a-b\right)^2\left(a+b+1\right)}{ab\left(a+b+2\right)\left(a+b+3\right)} - \frac{6}{\left(a+b+3\right)}$$

The frequency-severity approach

Figure: Statistical moments of the zero-inflated beta distribution



The frequency-severity approach

Two approaches for estimating the zero-inflated model:

1. Method of maximum likelihood
2. Method of moments

We obtain the following solution:

- For the parameter p, we have:

$$\hat{p} = \frac{n_1}{n_0 + n_1}$$

  where $n_0 = \sum_{i=1}^{n} \mathbb{1}\{\mathcal{R}_i = 0\}$ and $n_1 = \sum_{i=1}^{n} \mathbb{1}\{\mathcal{R}_i > 0\}$
- The estimates $\hat{a}$ and $\hat{b}$ correspond to the ML or MM estimates of the redemption severity data $\mathcal{R}_i^{\star}$

Estimation of p (in %)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	21.63	19.41	30.00	25.46	50.60	6.39		22.16
Central bank	0.16	0.34		1.47				0.47
Corporate	15.04	6.19	6.25	2.87	39.81	0.21		14.54
Corporate pension fund	8.11	3.38	3.98	3.37	7.57	0.00		4.12
Employee savings plan	2.67	2.83	2.97	2.71	2.29		2.75	2.69
Institutional	19.36	6.28	1.96	6.51	32.83	1.04		8.23
Insurance	12.19	6.72	3.45	7.22	27.92	1.04		9.71
Other	9.67	3.87	3.68	19.35	21.52	2.22		8.82
Retail	44.59	45.04	58.76	70.50	45.75	17.51	27.32	45.61
Sovereign	16.30	3.18	1.05	10.07	18.23	0.06		10.14
Third-party distributor	33.77	37.36	45.97	45.94	65.94	32.86	6.52	40.61
Total	34.66	27.10	24.19	38.34	37.57	11.14	24.79	31.11

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

Estimation of μ (in %)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	1.24	1.88	2.15	1.19	3.11	2.81		1.70
Central bank								
Corporate	0.55	2.50			3.82			3.73
Corporate pension fund		1.54		2.84	7.26			3.21
Employee savings plan	1.29			2.08				2.10
Institutional	0.67	2.62		2.80	4.46			3.23
Insurance	1.36	2.20		2.19	3.21			2.66
Other	0.87	2.60		1.10	3.51	0.99		2.65
Retail	0.34	0.31	0.44	0.23	1.98	0.43	0.15	0.33
Sovereign	0.06			1.84	10.48			4.46
Third-party distributor	0.35	0.64	1.45	0.42	1.40	0.86	1.21	0.56
Total	0.40	0.73	1.64	0.48	2.82	0.98	0.18	0.72

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

Estimation of σ (in %)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Auto-consumption	7.38	6.86	9.73	5.98	8.80	9.10		7.09
Central bank								
Corporate	5.55	9.57			7.49			8.70
Corporate pension fund		10.36		13.51	13.14			12.09
Employee savings plan	3.26			8.40				8.61
Institutional	5.46	9.99		9.23	11.46			10.86
Insurance	8.66	10.56		10.11	8.13			9.35
Other	3.61	9.36		7.27	11.88	6.70		10.68
Retail	2.80	2.58	3.32	2.10	7.52	3.22	2.64	2.88
Sovereign	0.25			9.90	21.63			14.94
Third-party distributor	2.68	3.48	7.63	2.58	4.71	5.84	6.98	3.37
Total	3.31	4.35	8.93	3.50	8.66	6.08	3.03	4.55

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured, (8) = total

Confidence level in the estimation of p

Confidence level (with respect to the number of observations)

$\circ\circ\circ$ $0-10$, $\circ\circ$ $11-50$, $\circ$ $51-200$, $\bullet$ $201-1\,000$, $\bullet\bullet$ $1\,001-10\,000$, $\bullet\bullet\bullet$ $+10\,000$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Auto-consumption	•••	•••	••	•••	••	•••	∘∘∘
Central bank	••	•	∘∘∘	•	∘∘∘	∘∘∘	∘∘∘
Corporate	••	••	••	••	••	••	∘∘∘
Corporate pension fund	••	••	•	••	••	••	∘∘∘
Employee savings plan	••	••	••	•••	••	∘∘∘	••
Institutional	••	•••	••	•••	••	•••	∘∘∘
Insurance	••	•••	••	•••	••	••	∘∘∘
Other	••	•••	••	••	••	•••	∘∘∘
Retail	•••	•••	••	•••	•••	•••	•••
Sovereign	••	••	•	••	••	••	∘∘∘
Third-party distributor	•••	•••	••	•••	•••	•••	••

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured

Confidence level in the estimation of μ and σ

Confidence level (with respect to the number of redemptions)

$\circ\circ\circ$ $0-10$, $\circ\circ$ $11-50$, $\circ$ $51-200$, $\bullet$ $201-1\,000$, $\bullet\bullet$ $1\,001-10\,000$, $\bullet\bullet\bullet$ $+10\,000$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Auto-consumption	$\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet$	$\bullet$	$\circ\circ\circ$
Central bank	$\circ\circ\circ$	$\circ\circ\circ$	$\circ\circ\circ$	$\circ\circ\circ$	$\circ\circ\circ$	$\circ\circ\circ$	$\circ\circ\circ$
Corporate	$\bullet$	$\bullet$	$\circ$	$\circ$	$\bullet\bullet$	$\circ\circ\circ$	$\circ\circ\circ$
Corporate pension fund	$\circ$	$\bullet$	$\circ\circ$	$\bullet$	$\bullet$	$\circ\circ\circ$	$\circ\circ\circ$
Employee savings plan	$\bullet$	$\circ$	$\circ\circ$	$\bullet$	$\circ$	$\circ\circ\circ$	$\circ$
Institutional	$\bullet\bullet$	$\bullet\bullet$	$\circ$	$\bullet\bullet$	$\bullet\bullet$	$\circ$	$\circ\circ\circ$
Insurance	$\bullet$	$\bullet$	$\circ$	$\bullet\bullet$	$\bullet\bullet$	$\circ$	$\circ\circ\circ$
Other	$\bullet$	$\bullet$	$\circ$	$\bullet$	$\bullet\bullet$	$\bullet$	$\circ\circ\circ$
Retail	$\bullet\bullet\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet\bullet$
Sovereign	$\bullet$	$\circ$	$\circ\circ\circ$	$\bullet$	$\bullet$	$\circ\circ\circ$	$\circ\circ\circ$
Third-party distributor	$\bullet\bullet\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet$	$\bullet\bullet$	$\bullet$

(1) = balanced, (2) = bond, (3) = enhanced treasury, (4) = equity, (5) = money market, (6) = other, (7) = structured

The frequency-severity approach

From historical stress scenarios to expert stress scenarios

Historical figures $\Rightarrow$ expert figures

Experts define the triplet (p, μ, σ) for each matrix cell and the stress scenarios are computed using the following formulas:

$$\mathbb{S}\left(\mathscr{T}; p, \mu, \sigma\right) = \mathscr{B}^{-1}\left(1 - \frac{1}{p\mathscr{T}}; \frac{\mu^2\left(1-\mu\right)}{\sigma^2} - \mu, \frac{\mu\left(1-\mu\right)^2}{\sigma^2} - (1-\mu)\right)$$

where $\mathscr{B}^{-1}\left(\alpha; a, b\right)$ is the α-quantile of the beta distribution with parameters a and b

The individual-based model

- The frequency-severity approach is an aggregate population model:

$$\mathcal{R} = \mathcal{E} \cdot \mathcal{R}^{\star}$$

  where $\mathcal{E}$ indicates if there is a redemption or not (frequency) and $\mathcal{R}^{\star}$ is the redemption amount in % (severity)

- The frequency-severity approach is a reduced form of the individual-based model:

$$\mathcal{R} = \sum_{i=1}^{n} \omega_i \cdot \mathcal{E}_i \cdot \mathcal{R}_i^{\star}$$

  where n is the number of unitholders, $\omega = (\omega_1, \ldots, \omega_n)$ is the fund liability structure, $\mathcal{E}_i$ is the redemption decision of investor i and $\mathcal{R}_i^{\star}$ is the redemption amount in % of investor i

The individual-based model

- The population-based model is characterize by the 2-tuple:

$$\mathbf{ZI}\left(p,\mathbf{G}\right) \longrightarrow \mathbf{ZI}\left(p,a,b\right) \text{ or } \mathbf{ZI}\left(p,\mu,\sigma\right)$$

- The individual-based model is characterized by the 4-tuple:

$$\mathbf{IM}\left(n,\omega,\tilde{p},\tilde{\mathbf{G}}\right) \longrightarrow \mathbf{IM}\left(n,\omega,\tilde{p},\tilde{a},\tilde{b}\right) \text{ or } \mathbf{IM}\left(n,\omega,\tilde{p},\tilde{\mu},\tilde{\sigma}\right)$$

- Under some assumptions, the individual-based model may be characterized by the 3-tuple:

$$\mathbf{IM}\left(n,\omega,\tilde{p},\tilde{\mathbf{G}}\right) \longrightarrow \mathbf{IM}\left(\mathscr{H}\left(\omega\right),\tilde{p},\tilde{\mathbf{G}}\right)$$

where $\mathscr{H}\left(\omega\right)$ is the Herdfindahl index of the liability structure $\omega = \left(\omega_1,\ldots,\omega_n\right)$

The individual-based model

Why?

We have:

$$\mathbb{E}\left[\mathcal{R}\right] = \underbrace{p\mu}_{\textbf{ZI}} = \underbrace{\tilde{p}\tilde{\mu}}_{\textbf{IM}}$$

and:

$$\sigma^2\left(\mathcal{R}\right) = \underbrace{p\sigma^2 + p\left(1-p\right)\mu^2}_{\textbf{ZI}} = \underbrace{\tilde{p}\left(\tilde{\sigma}^2 + \left(1-\tilde{p}\right)\tilde{\mu}^2\right)\overbrace{\sum_{i=1}^{n}\omega_i^2}^{\mathcal{H}(\omega)}}_{\textbf{IM}}$$

On the importance of the fund liability structure

Measuring the fund liability concentration

The redemption profiles strongly depend on the fund liability structure and its Herfindahl index:

$$\mathcal{H}(\omega) = \sum_{i=1}^{n} \omega_i^2$$

To better understand $\mathcal{H}(\omega)$, we introduce the effective number of unitholders:

$$\mathcal{N}(\omega) = \frac{1}{\mathcal{H}(\omega)}$$

We have:

$$\underbrace{1}_{\textit{Concentration}} \leq \mathcal{N}(\omega) \leq \underbrace{n}_{\textit{Diversification}}$$

On the importance of the fund liability structure

Example

- If $\mathscr{H}(\omega) = 0.20$, $\mathscr{N}(\omega) = 5$ and the fund liability structure is equivalent to a fund with 5 effective (or equally-weighted) unitholders
- If $\mathscr{H}(\omega) = 0.01$, $\mathscr{N}(\omega) = 100$ and the fund liability structure is equivalent to a fund with 100 effective (or equally-weighted) unitholders

Impact of the liability concentration on the redemption rate

Illustration

We consider an investment fund with highly active managers:

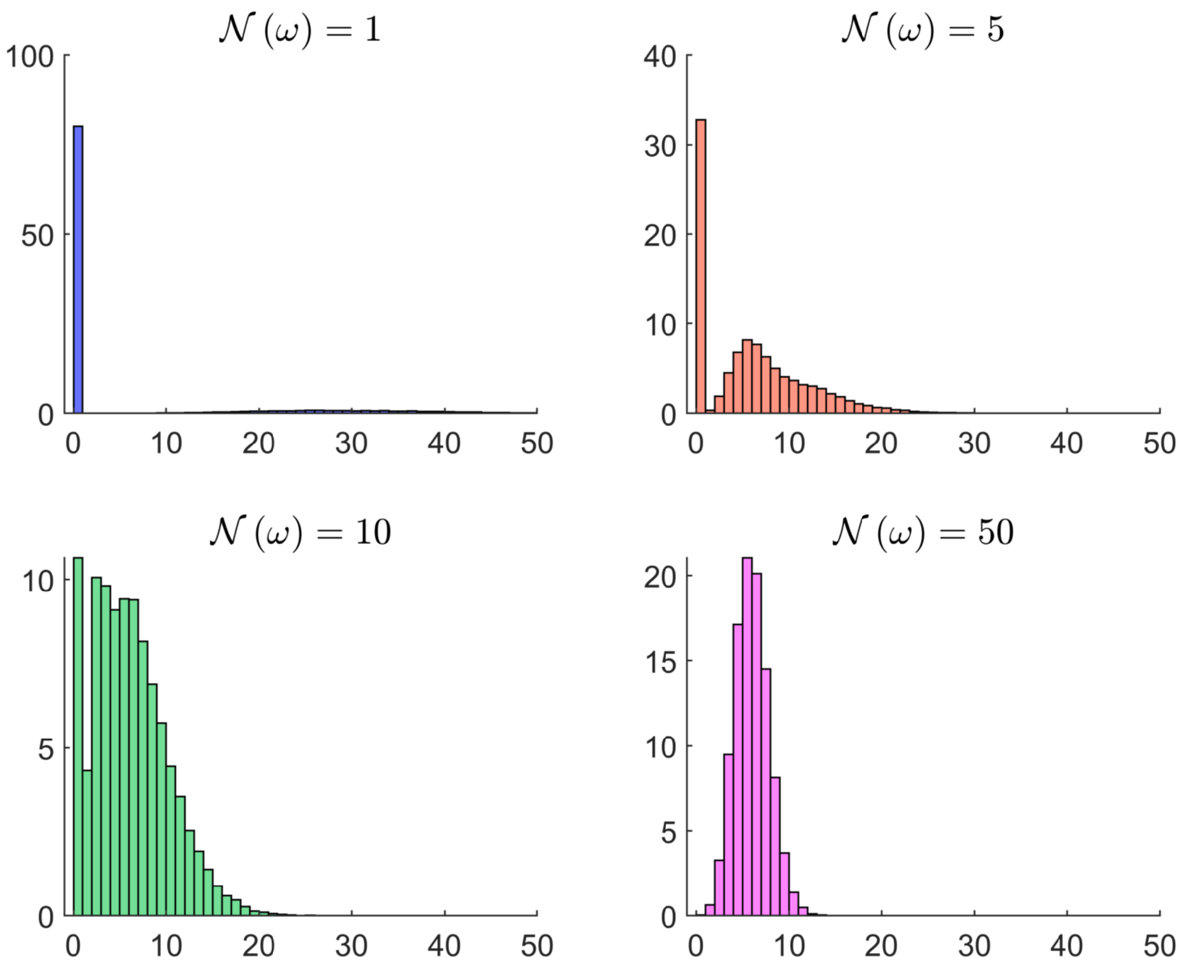
- the redemption frequency $\tilde{p}$ is equal to 20%, meaning that the investor redeems once a week on average;
- the average redemption severity $\tilde{\mu}$ is equal to 30%;
- the uncertainty $\tilde{\sigma}$ on the redemption severity is equal to 10%;

We obtain the following results:

Liability structure $\mathcal{N}(\omega)$	Value-at-risk $\mathbb{Q}(99\%)$	Conditional VaR $\mathbb{C}(99\%)$
1	47.1%	52.2%
5	22.2%	25.1%
10	17.2%	19.1%
50	10.6%	11.3%

Impact of the liability concentration on the redemption rate

Figure: Histogram of the redemption rate in %



Correlation risk

We have:

$$\mathcal{R} = \sum_{i=1}^{n} \omega_i \cdot \mathcal{E}_i \cdot \mathcal{R}_i^\star$$

where the random variables $(\mathcal{E}_1, \ldots, \mathcal{E}_n, \mathcal{R}_1^\star, \ldots, \mathcal{R}_n^\star)$ are not necessarily independent. We can consider three different correlation patterns:

1. The redemption events $\mathcal{E}_i$ and $\mathcal{E}_j$ are correlated
2. The redemption severities $\mathcal{R}_i^\star$ and $\mathcal{R}_j^\star$ are correlated
3. $\mathcal{E}_i$ and $\mathcal{R}_i^\star$ are correlated

$\Rightarrow$ The first correlation pattern is the most relevant

We assume that the dependence function of $(\mathcal{E}_1, \ldots, \mathcal{E}_n)$ is given by the copula function $\mathbf{C}$

Correlation risk

What is the impact of correlation risk?

- If redemptions are independent, we have:

$$\mathbb{E}\left[\mathcal{R}\right] = \tilde{p}\tilde{\mu}$$

and:

$$\sigma^2\left(\mathcal{R}\right) = \tilde{p}\left(\tilde{\sigma}^2 + (1-\tilde{p})\,\tilde{\mu}^2\right)\mathscr{H}\left(\omega\right)$$

- If the redemptions are correlated, we have:

$$\mathbb{E}\left[\mathcal{R}\right] = \tilde{p}\tilde{\mu}$$

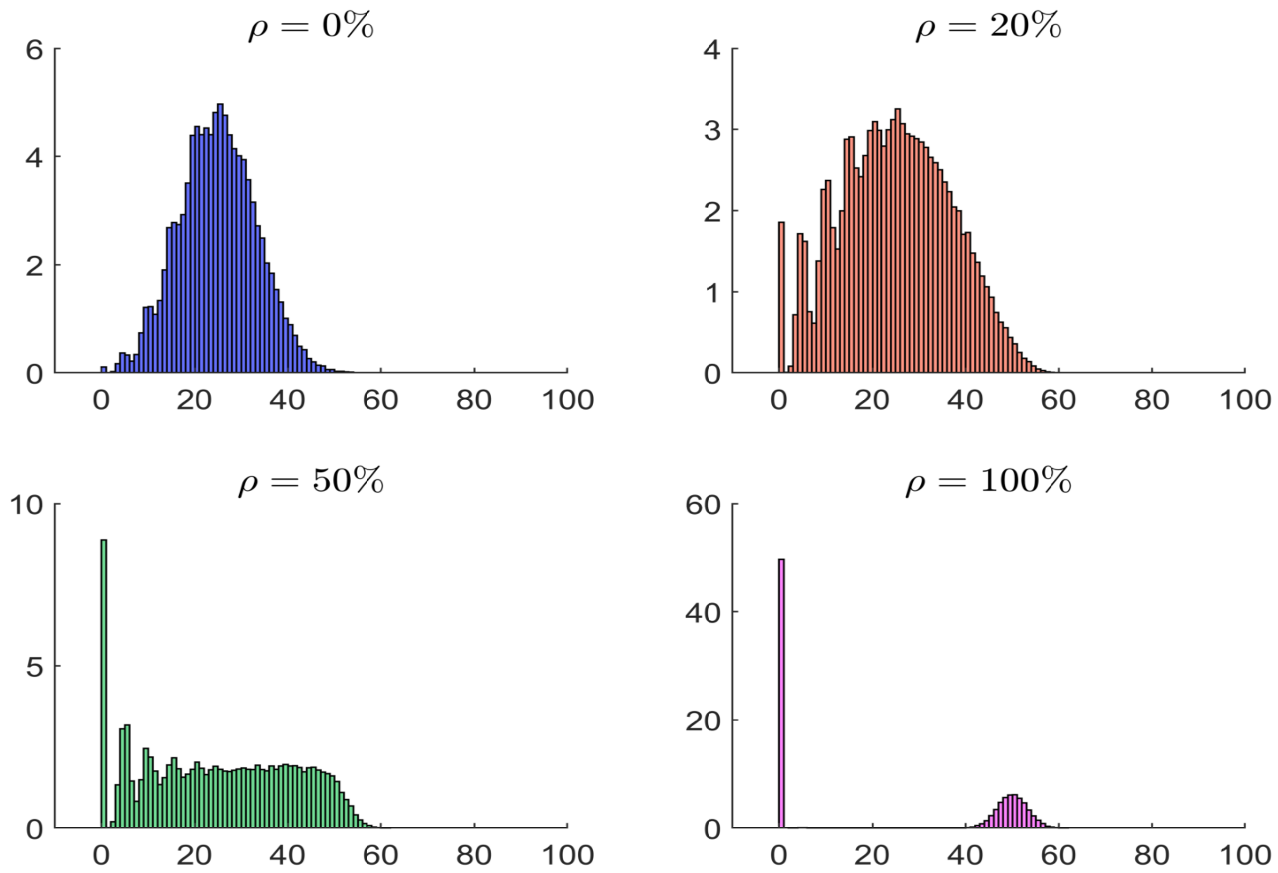
and:

$$\sigma^2\left(\mathcal{R}\right) = \left(\tilde{p}\tilde{\sigma}^2 + \left(\tilde{p} - \breve{\mathbf{C}}\left(\tilde{p},\tilde{p}\right)\right)\tilde{\mu}^2\right)\mathscr{H}\left(\omega\right) + \left(\breve{\mathbf{C}}\left(\tilde{p},\tilde{p}\right) - \tilde{p}^2\right)\tilde{\mu}^2$$

where $\breve{\mathbf{C}}$ is the survival copula associated to $\mathbf{C}$

Correlation risk

Figure: Histogram of the redemption rate in % with respect to the frequency correlation ($\tilde{p} = 50\%, \tilde{\mu} = 50\%, \tilde{\sigma} = 10\%, n = 10$)



Evidence of the cross-correlation risk

Table: Inter-class Spearman correlation

Category #1	Category #2	Balanced	Bond	Equity	Money Market
Retail	Third-party distributor	**53.0%**	**52.9%**	**52.1%**	**3.3%**
Retail	Institutional	10.4%	23.2%	22.0%	**−6.5%**
Retail	Insurance	**3.0%**	18.8%	31.6%	**−12.3%**
Third-party distributor	Institutional	13.5%	**48.0%**	**54.1%**	24.0%
Third-party distributor	Insurance	23.1%	21.5%	22.8%	39.2%
Institutional	Insurance	**2.5%**	16.2%	16.4%	29.8%
Average		17.6%	30.1%	33.2%	12.9%

Evidence of the time-correlation risk

- Auto-correlation risk

Table: Autocorrelation of the redemption rate in %

	Balanced	Bond	Equity	Money market
Institutional	**25.9****	−2.2	−1.5	**24.2****
Insurance	−1.5	9.9	5.4	17.8**
Retail	1.9	−2.1	9.8	11.7**
Third party distributor	2.7	7.4	5.5	**23.2****

- Sell-herding and spillover risk

$\Rightarrow$ Development of a Monte Carlo method to take into account the auto-correlation risk, but spillover risk is too complicated to be integrated

Where does the stress come from?

Three models

- Frequency model

$$\mathcal{R}(t) = \beta_0 + \beta_1 \mathcal{F}(t) + u(t)$$

- Severity model

$$\mathcal{R}(t) = \beta_0 + \beta_1 \mathcal{R}^{\star}(t) + u(t)$$

- Frequency/severity model

$$\mathcal{R}(t) = \beta_0 + \beta_1 \mathcal{F}(t) + \beta_2 \mathcal{R}^{\star}(t) + u(t)$$

Where does the stress come from?

Table: Coefficient of determination $\mathfrak{R}_c^2$ in % — Frequency model

	Balanced	Bond	Equity	Money market
Institutional	2.4	36.2	53.4	17.2
Insurance	0.9	11.6	10.8	17.8
Retail	37.2	34.5	14.7	18.4
Third party distributor	11.5	31.6	17.7	11.5

Where does the stress come from?

Table: Coefficient of determination $\mathfrak{R}_c^2$ in % — Severity model

	Balanced	Bond	Equity	Money market
Institutional	87.2	74.8	44.5	87.5
Insurance	99.2	84.0	83.3	90.1
Retail	77.6	88.4	98.1	80.8
Third party distributor	93.1	91.5	92.1	95.0

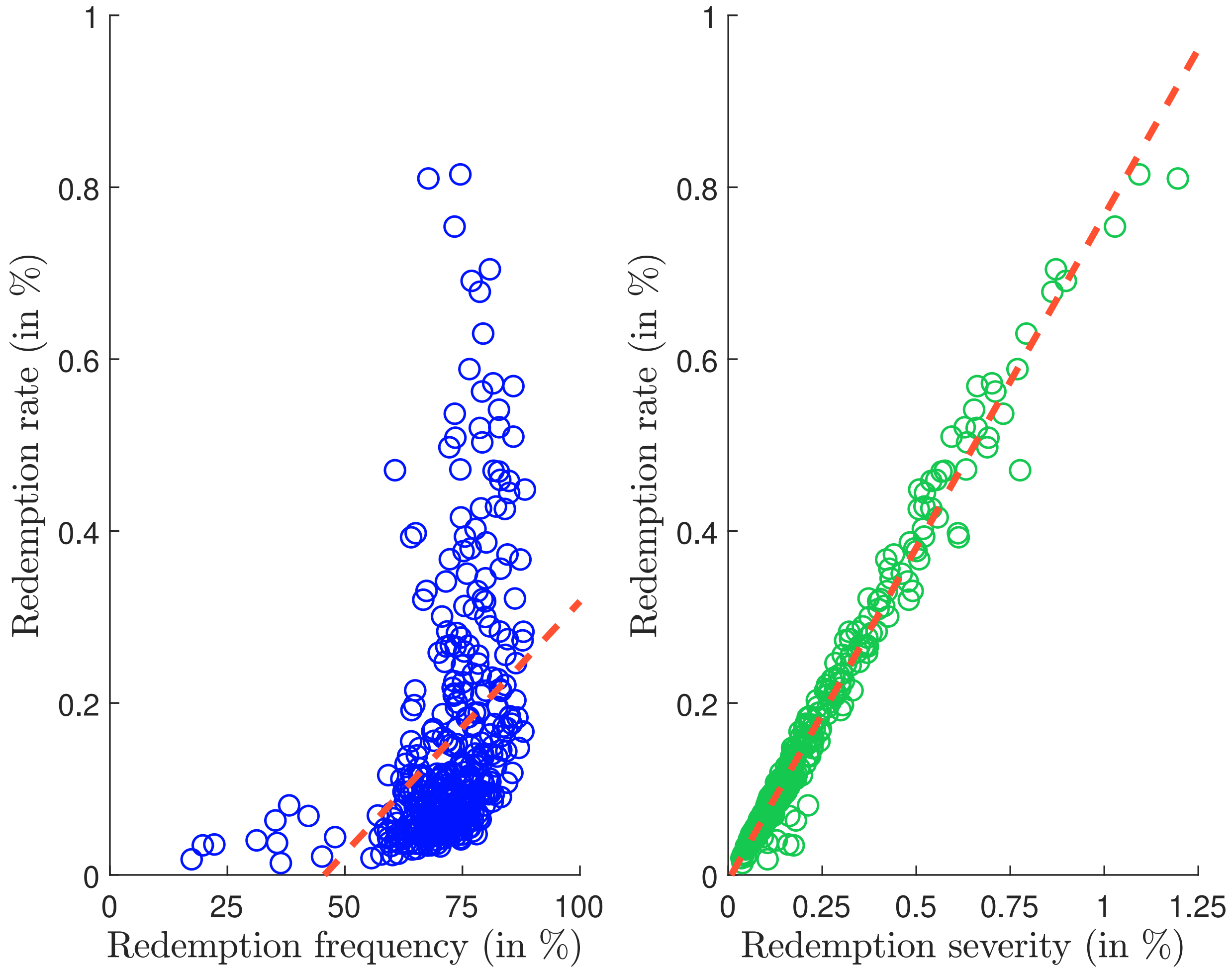
Where does the stress come from?

Table: Coefficient of determination $\mathfrak{R}_c^2$ in % — Frequency/severity model

	Balanced	Bond	Equity	Money market
Institutional	88.2	84.7	81.7	93.3
Insurance	99.3	86.2	86.4	94.9
Retail	92.5	95.4	99.3	92.3
Third party distributor	97.0	96.3	95.7	97.3

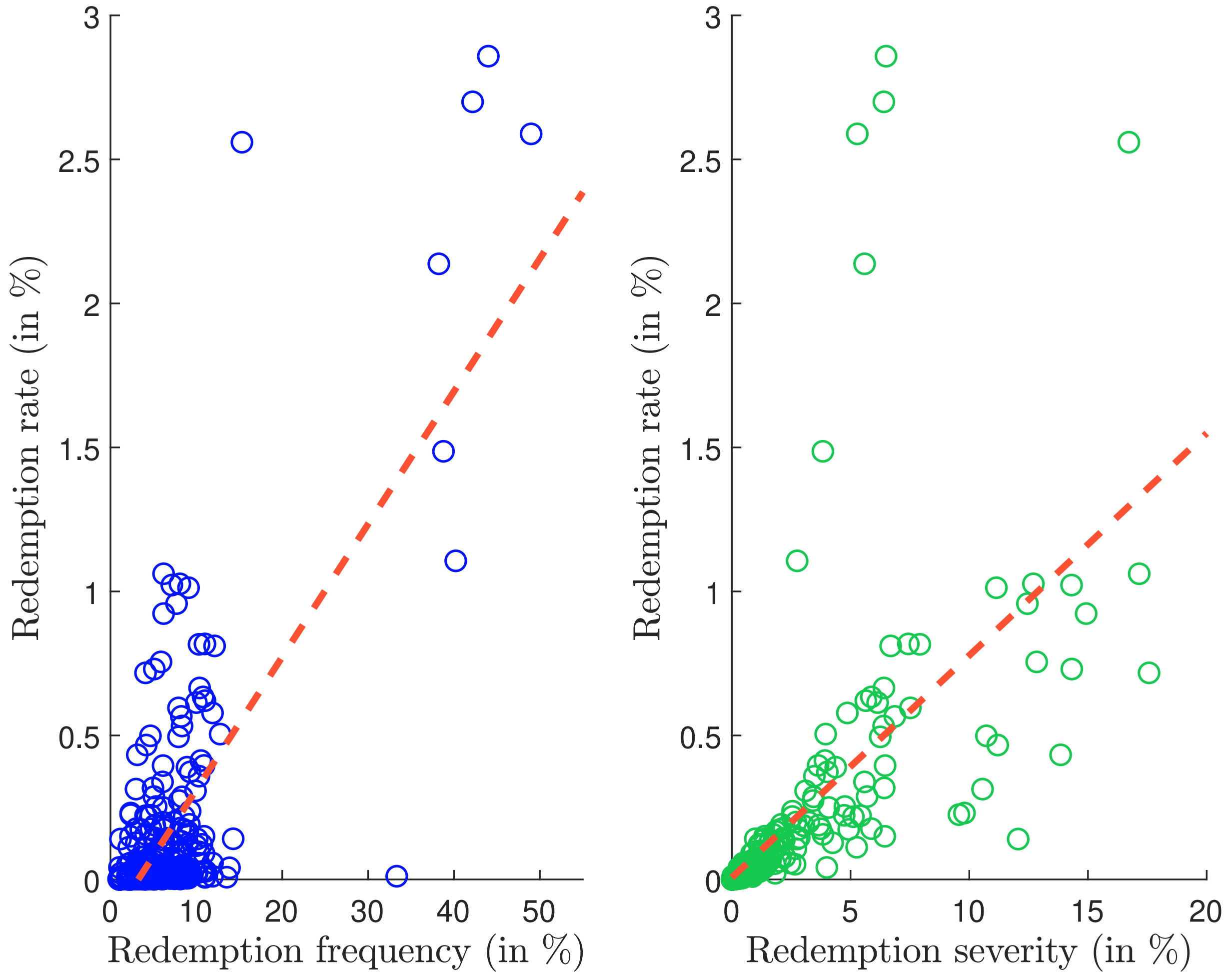
Where does the stress come from?

Figure: Relationship between $\mathcal{R}(t)$, $\mathcal{F}(t)$ and $\mathcal{R}^{\star}(t)$ (retail/equity)



Where does the stress come from?

Figure: Relationship between $\mathcal{R}(t)$, $\mathcal{F}(t)$ and $\mathcal{R}^{\star}(t)$ (institutional/equity)



The flow-performance relationship

- "*Inflows are related only to relative performance*"
- "*Outflows are related only to absolute fund performance*"

Relative vs absolute performance

We can test the following model:

$$\left\{ \begin{array}{l} R_f(t) = \alpha_f(t) + \beta_f(t) R_{\mathrm{mkt}}(t) + \varepsilon(t) \\ \mathcal{R}_f(t) = \gamma_f + \delta_f \alpha_f(t-1) + \varphi_f R_f(t-1) + \eta(t) \end{array} \right.$$

where $R_f(t)$ is the return of the fund f, $R_{\mathrm{mkt}}(t)$ is the return of the market risk factor and $\mathcal{R}_f(t)$ is the redemption rate of the fund f

$\Rightarrow$ Extension to dynamic models:

$$\mathcal{R}_f(t) = \gamma_f + \sum_{h=1}^{p} \left(\phi_f^{(h)} \mathcal{R}_f(t-h) + \delta_f^{(h)} \alpha_f(t-h) + \varphi_f^{(h)} R_f(t-h) \right) + \eta(t)$$

The macro stress-testing approach

We consider three market risk factors:

- The performance of the bond market
- The performance of the stock market
- The market volatility

We assume that there is a linear relationship between the redemption rate and these factors:

$$\mathcal{R}(t) = \beta_0 + \beta_1 \mathscr{F}_{\text{bond}}(t) + \beta_2 \mathscr{F}_{\text{stock}}(t) + \beta_3 \mathscr{F}_{\text{vol}}(t) + u(t)$$

The macro stress-testing approach

Table: Coefficient of determination $\mathfrak{R}_c^2$ in % — one-day time horizon

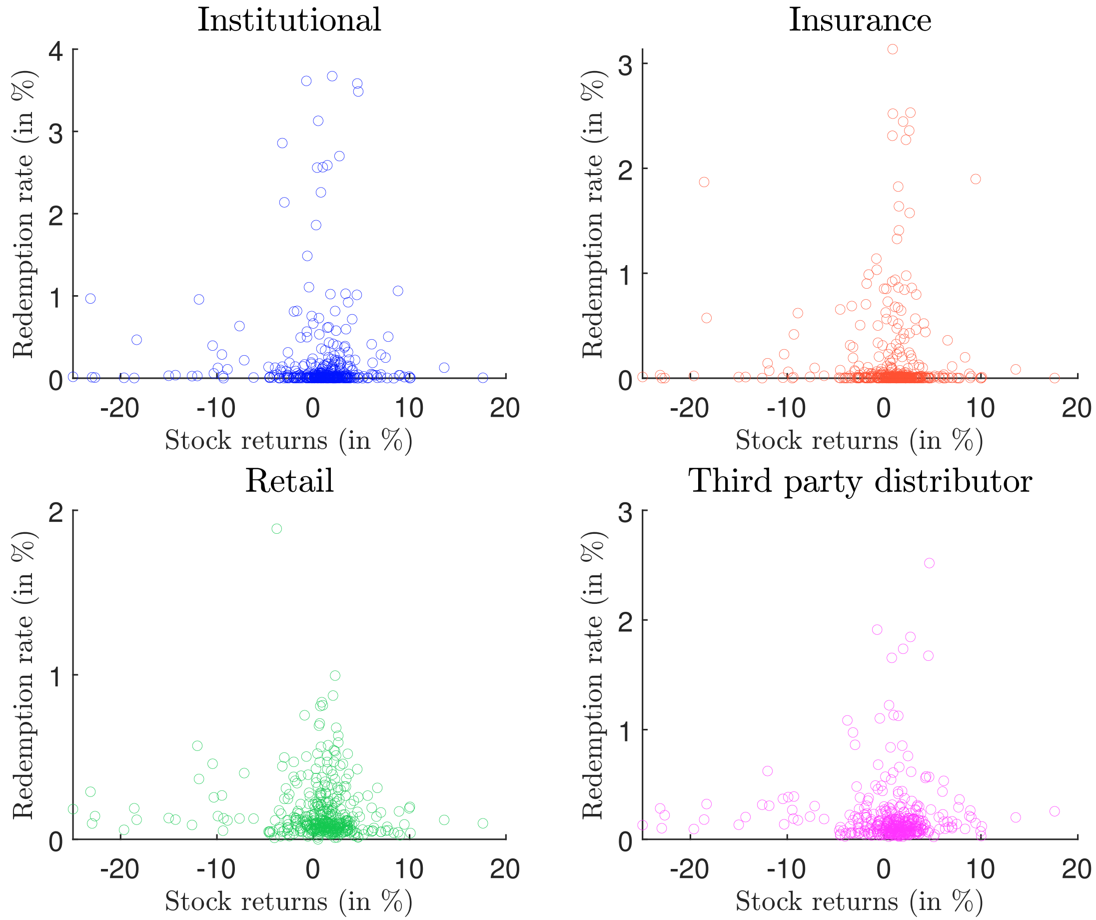
	Balanced	Bond	Equity	Money market
Institutional	0.3	0.8	1.6	1.9
Insurance	0.1	0.1	0.6	0.8
Retail	0.5	3.1	1.4	0.6
Third party distributor	0.7	1.5	1.3	4.4

Table: Coefficient of determination $\mathfrak{R}_c^2$ in % — two-week time horizon

	Balanced	Bond	Equity	Money market
Institutional	1.3	0.7	2.8	2.8
Insurance	0.1	0.3	1.5	5.1
Retail	2.3	2.0	0.8	0.9
Third party distributor	1.1	2.1	1.5	3.7

The macro stress-testing approach

Figure: Relationship between redemption rate and two-week stock returns (equity category)



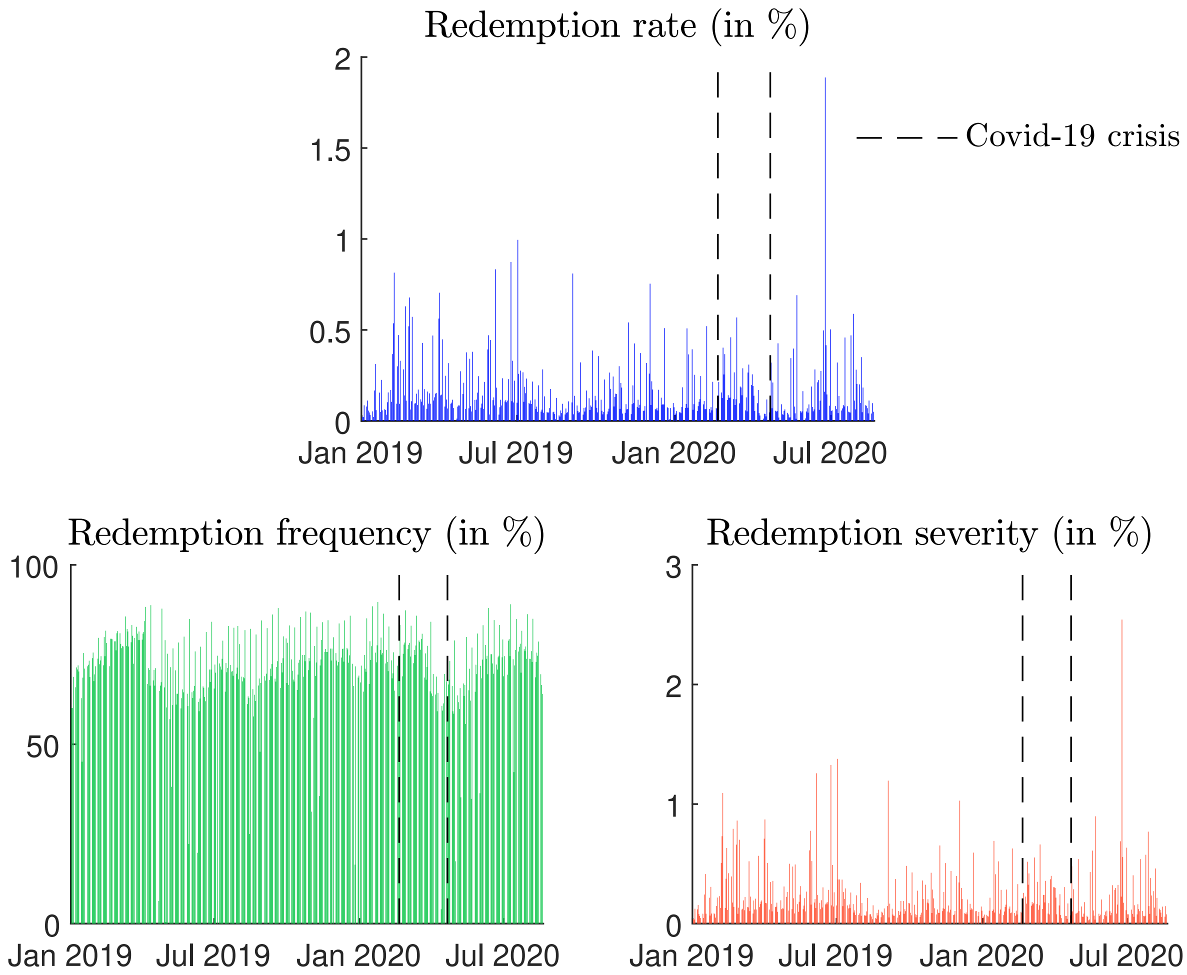
The macro stress-testing approach

Table: Relative variation of the redemption rate $\mathcal{R}(t)$ when $\mathrm{VIX} \geq 30$

	Balanced	Bond	Equity	Money market
Institutional	$+17.3\%$	$+54.7\%$	$+74.3\%$	$+64.7\%$
Insurance	-63.4%	-1.1%	-14.2%	$+75.7\%$
Retail	$+6.1\%$	$+21.5\%$	$+13.8\%$	-4.5%
Third party distributor	$+37.6\%$	$+43.6\%$	$+49.5\%$	$+22.7\%$

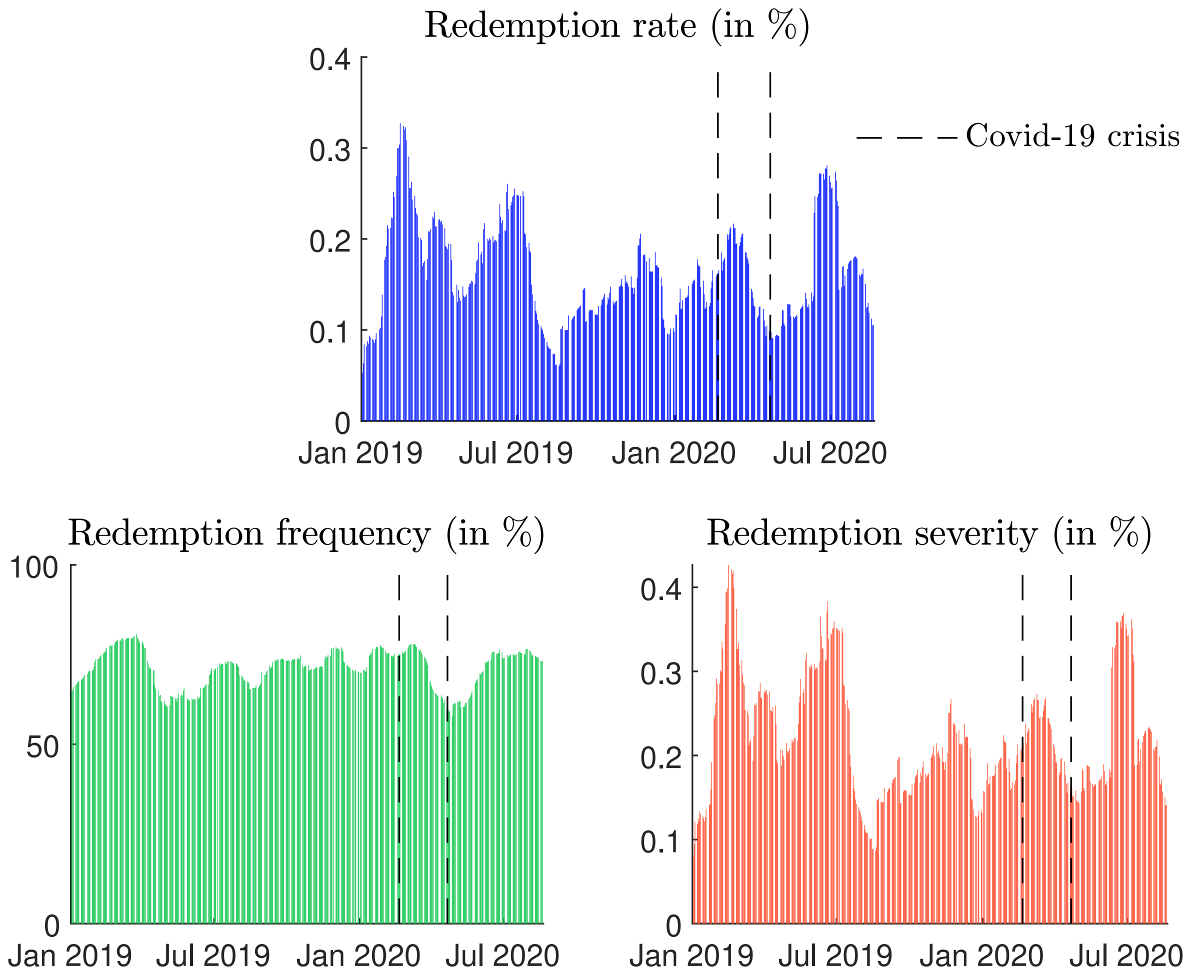
Dynamic analysis

Figure: Time-series of $\mathcal{R}(t)$, $\mathcal{F}(t)$ and $\mathcal{R}^{\star}(t)$ (retail/equity)



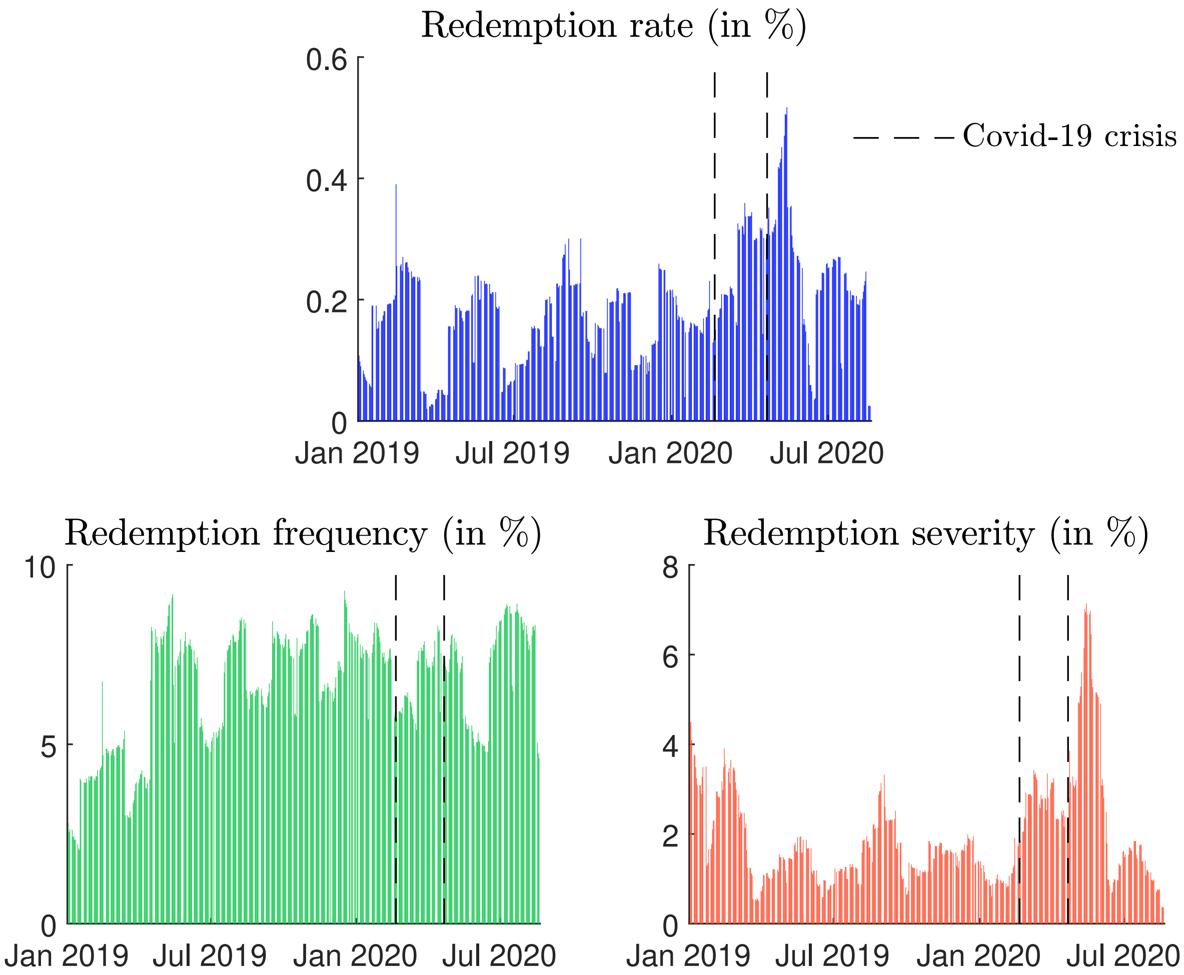
Dynamic analysis

Figure: One-month moving average (retail/equity)



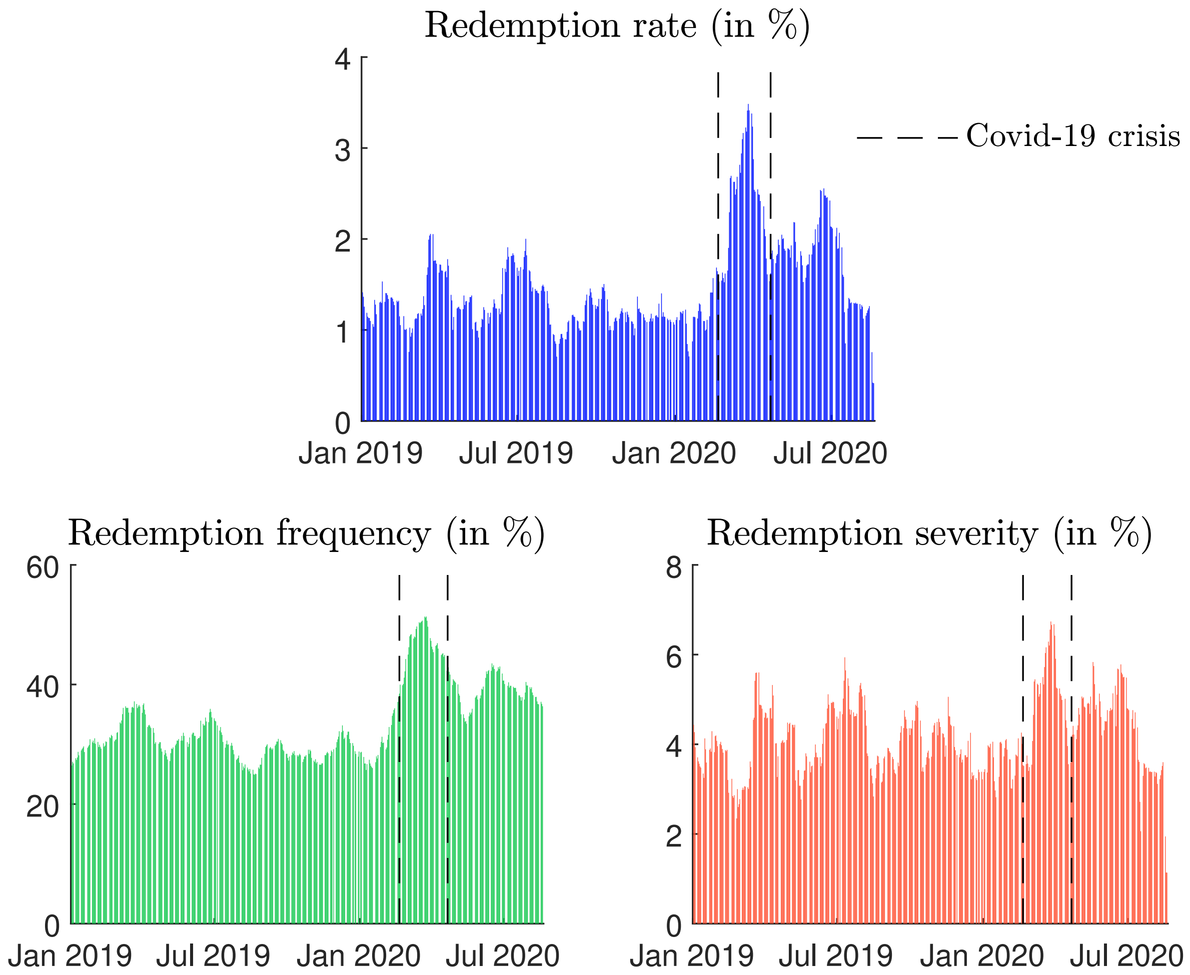
Dynamic analysis

Figure: One-month moving average (institutional/equity)



Dynamic analysis

Figure: One-month moving average (institutional/money market)



Which model for a stress testing program?

Several approaches for modeling the transaction cost function:

- **Machine learning model** $\Rightarrow$ Pre-trade analytics?
- **Statistical risk model** $\Rightarrow$ Risk management?
- **Benchmark analytical model** $\Rightarrow$ LST?

$\Rightarrow$ **In a stress testing exercise, the number of unknowns is greater than the number of knowns**

Definition

We decompose the unit transaction cost by two parts:

$$\boldsymbol{c}(x) = s + \boldsymbol{\pi}(x)$$

where:

1. s does not depend on the trade size and represents the half bid-ask spread of the security
2. $\boldsymbol{\pi}(x)$ depends on the trade size x and represents the price impact of the trade

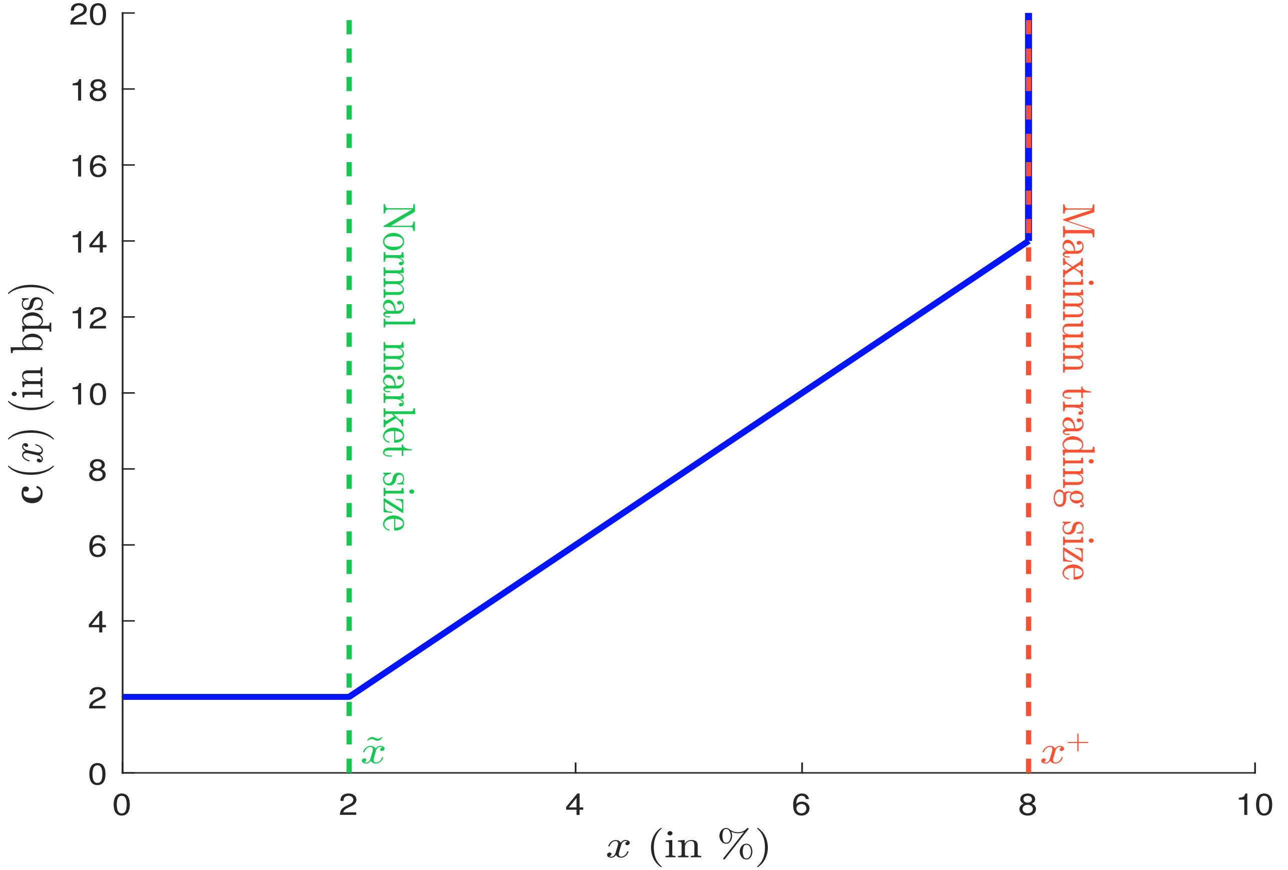
Participation rate

The trade size (or the participation rate) x is the ratio between the number of shares q that is traded (sold or purchased) and the daily trading volume v:

$$x = \frac{q}{v} = \frac{q \cdot P}{v \cdot P} = \frac{Q}{V}$$

Toy model

Figure: Simple modeling of unitary transaction costs



Toy model

- Normal regime (moderate price impact)
- Second regime (medium to high price impact)
- Trading limit (unacceptable price impact)

$\Rightarrow$ **The trading limit is part of the trading policy.** It is an important factor for a LST model (liquidation policy) and is the main parameter to define the liquidation ratio, time to liquidation and liquidation shortfall.

$\Rightarrow$ Distinction between **two regimes**: **small** and **big** sizes

The theory

The power-law property of price impact

Under no-arbitrage condition, Jusselin and Rosenbaum (2020) show that the market impact is power-law:

$$\boldsymbol{\pi}(x) := \boldsymbol{\pi}(x;\gamma) = \varphi_\gamma \boldsymbol{\sigma} x^\gamma$$

where $\gamma > 0$ is a scalar, $\boldsymbol{\sigma}$ is the daily volatility (or the market risk) of the security and φ_γ is a scaling factor

The theory

We retrieve most of academic findings:

- Seminal paper of Kyle (1985): $\gamma = 1$
- Empirical studies showed that $\gamma \in [0.3, 0.7]$:
  - Loeb (1983) and Torre (1997) estimated that $\gamma = 0.5$ (Barra)
  - Almgren *et al.* (2005) found that $\gamma = 3/5$ (Citi/BECS)
  - Engle *et al.* (2012) found that $\gamma \approx 0.43$ for NYSE stocks and $\gamma \approx 0.37$ for NASDAQ stocks (Morgan Stanley)
  - Frazzini *et al.* (2018) estimated that the average exponent is equal to 0.35 for developed equity markets (AQR)
  - Bacry *et al.* found that $\gamma \in [0.5, 0.8]$ (CFM)
- The statistical issue is the database, which is biased towards small size orders ($x \ll 1\%$)

The theory

The model of linear price impact

- When $\gamma = 1$,we have:

$$\boldsymbol{c}(x_i) = s_i + \boldsymbol{\pi}(x_i) = s_i + \varphi\,\sigma_i\,x_i$$

- The total cost for the portfolio $x = (x_1, \ldots, x_n)$ is equal to:

$$\begin{aligned} \mathbf{TC}(x) &= \sum_{i=1}^{n} x_i \cdot \boldsymbol{c}(x_i) \\ &= \sum_{i=1}^{n} x_i \cdot s_i + \varphi \sum_{i=1}^{n} \sigma_i\,x_i^2 \\ &= x^\top s + \varphi x^\top S x \end{aligned}$$

  where $s = (s_1, \ldots, s_n)$ and $S = \operatorname{diag}(\sigma_1, \ldots, \sigma_n)$

- Model of linear price impact = model of quadratic transaction costs

The theory

The square-root model

- When $\gamma = 0.5$,we have:

$$\boldsymbol{c}(x_i) = s_i + \boldsymbol{\pi}(x_i) = s_i + \varphi\,\sigma_i\sqrt{x_i}$$

- The total cost for the portfolio $x = (x_1, \ldots, x_n)$ is equal to:

$$\begin{aligned} \mathbf{TC}(x) &= \sum_{i=1}^{n} x_i \cdot \boldsymbol{c}(x_i) \\ &= \sum_{i=1}^{n} x_i \cdot s_i + \varphi \sum_{i=1}^{n} \sigma_i x_i \sqrt{x_i} \\ &= x^\top s + \varphi \sigma^\top \odot x^{1.5} \end{aligned}$$

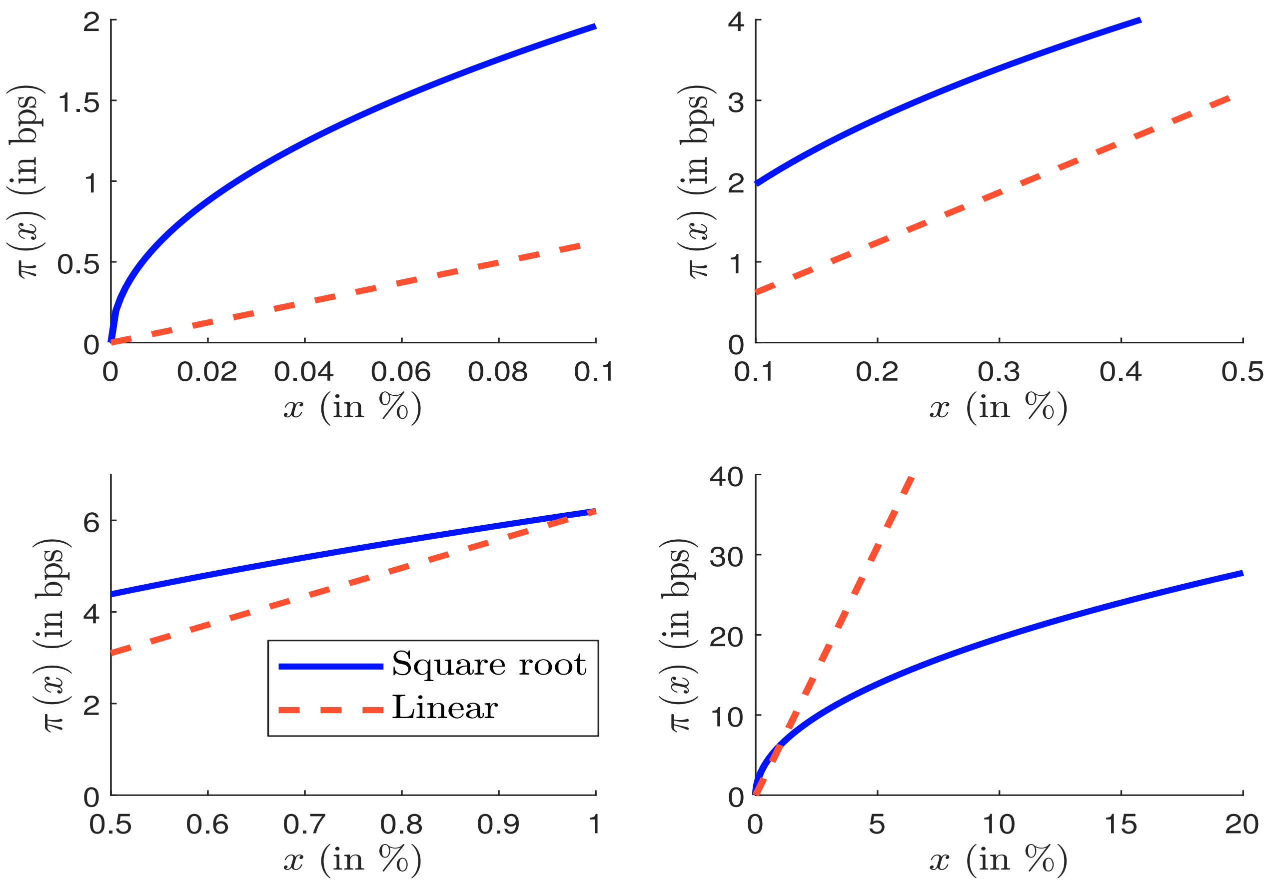
where $s = (s_1, \ldots, s_n)$ and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$

The theory

"Empirically, both a linear model and a square root model explain transaction costs well. A square-root model explains transaction costs for orders in the 90th to 99th percentiles better than a linear model; a linear model explains transaction costs for the largest 1% of orders slightly better than the square-root model" (Kyle and Obizhaeva, 2016, page 1347).

The theory

Figure: Square-root model versus linear model ($\sigma = 10\%$)



General formula of the two-regime model

We have:

$$\boldsymbol{\pi}(x) = \begin{cases} \varphi_1 \sigma x^{\gamma_1} & \text{if } x \leq \tilde{x} \\ \varphi_2 \sigma x^{\gamma_2} & \text{if } \tilde{x} \leq x \leq x^+ \\ +\infty & \text{if } x > x^+ \end{cases}$$

where γ_1 and γ_2 are the exponents of the two market impact regimes

- We must have $\gamma_2 > \gamma_1$ (convex $\Longrightarrow$ concave[3])
- Since the cost function $\boldsymbol{\pi}(x)$ is continuous, this implies that

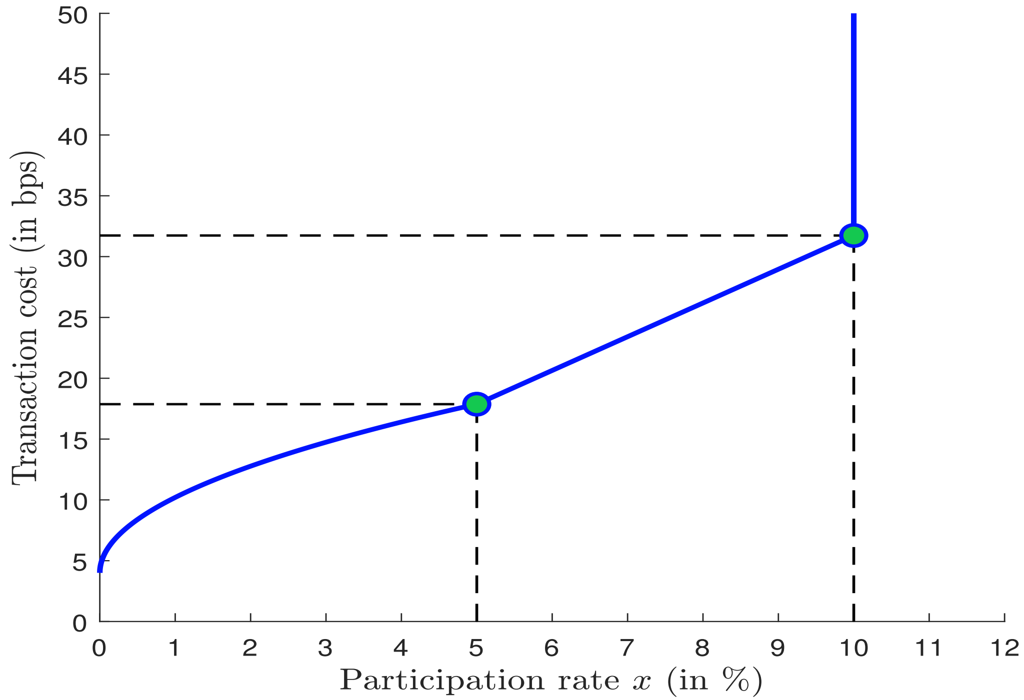
$$\varphi_2 = \varphi_1 \frac{\tilde{x}^{\gamma_1}}{\tilde{x}^{\gamma_2}}$$

- The price impact model is defined by the 5-tuple $(\varphi_1, \gamma_1, \gamma_2, \tilde{x}, x^+)$
- In fact, only φ_1 and γ_1 are calibrated whereas the other parameters are set by the experts

[3] It is better to impose $\gamma_2 \geq \mathbf{1.0} > \gamma_1$

Concavity versus convexity

Figure: Two-regime model



- The security-specific parameters are: $s = 4$ bps, $\sigma = 10\%$ and $v = 1\,000\,000$
- The liquidation policy is set to $x^+ = 10\%$.
- The estimated model parameters are $\varphi_1 = 1$ and $\gamma_1 = 0.5$
- The expert parameters are $\gamma_2 = 1$ and $\tilde{x} = x^+/2$

Stress testing in the two-regime model

How does stress testing impact transaction costs?

- We assume that the model is invariant
- This means that the model parameters are constant
- The stress scenario impacts only the security-specific parameters:
  - Bid-ask spread
  - Volatility
  - Volume

$\Rightarrow$ we retrieve the three components of the liquidity risk:

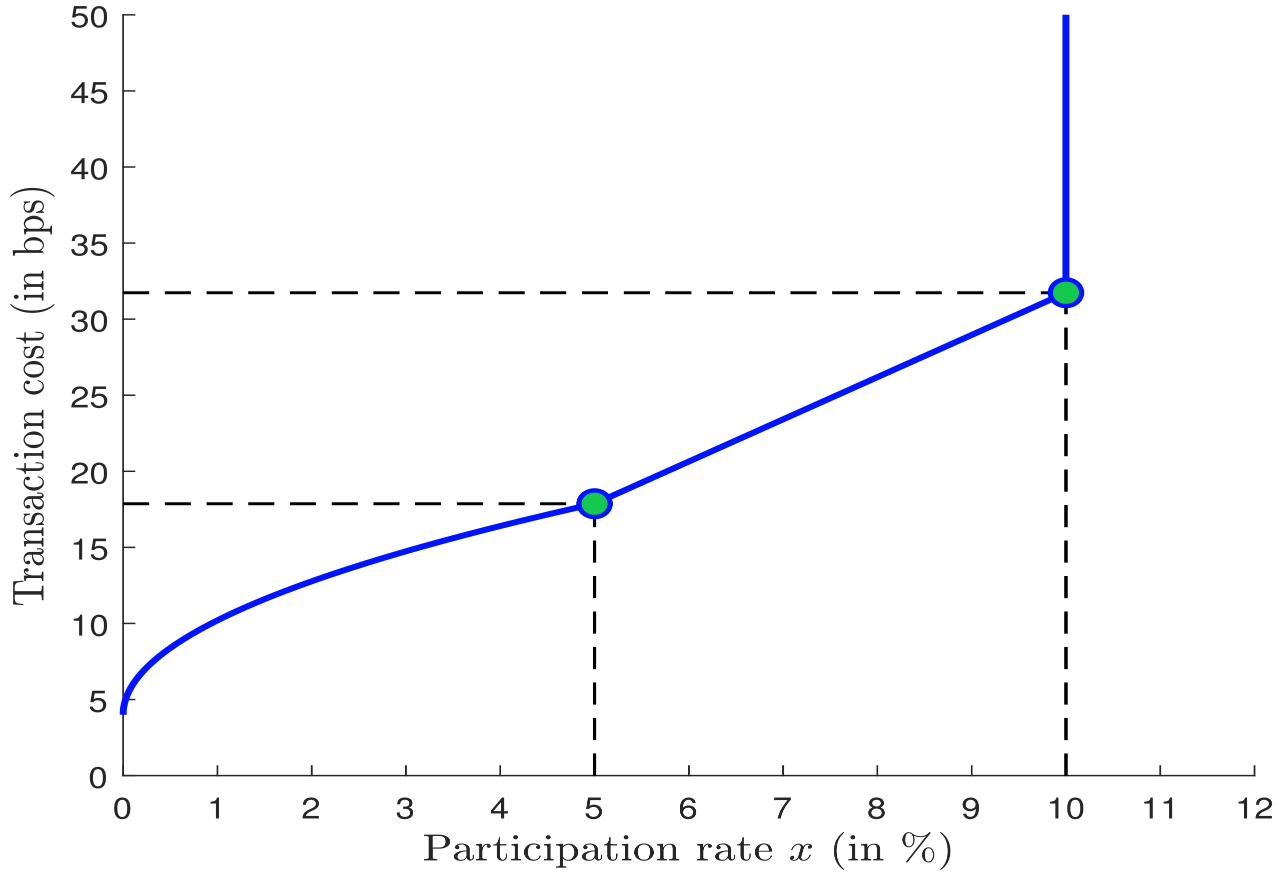
1. Spread risk
2. Market risk
3. Depth risk

Stress testing in the two-regime model

x-**invariance** $\not\Rightarrow$ q-**invariance**

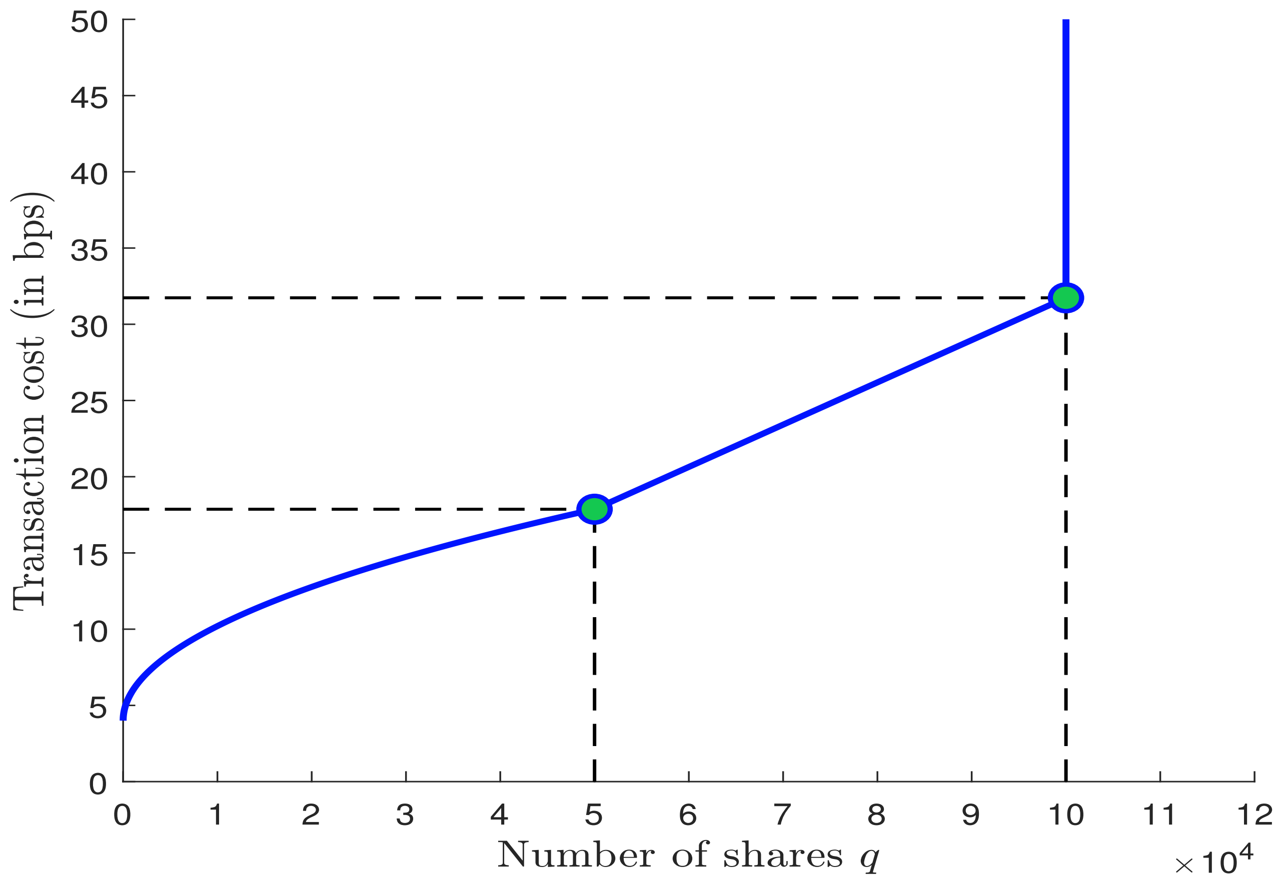
Stress testing in the two-regime model

Figure: The x-approach of the unit transaction cost



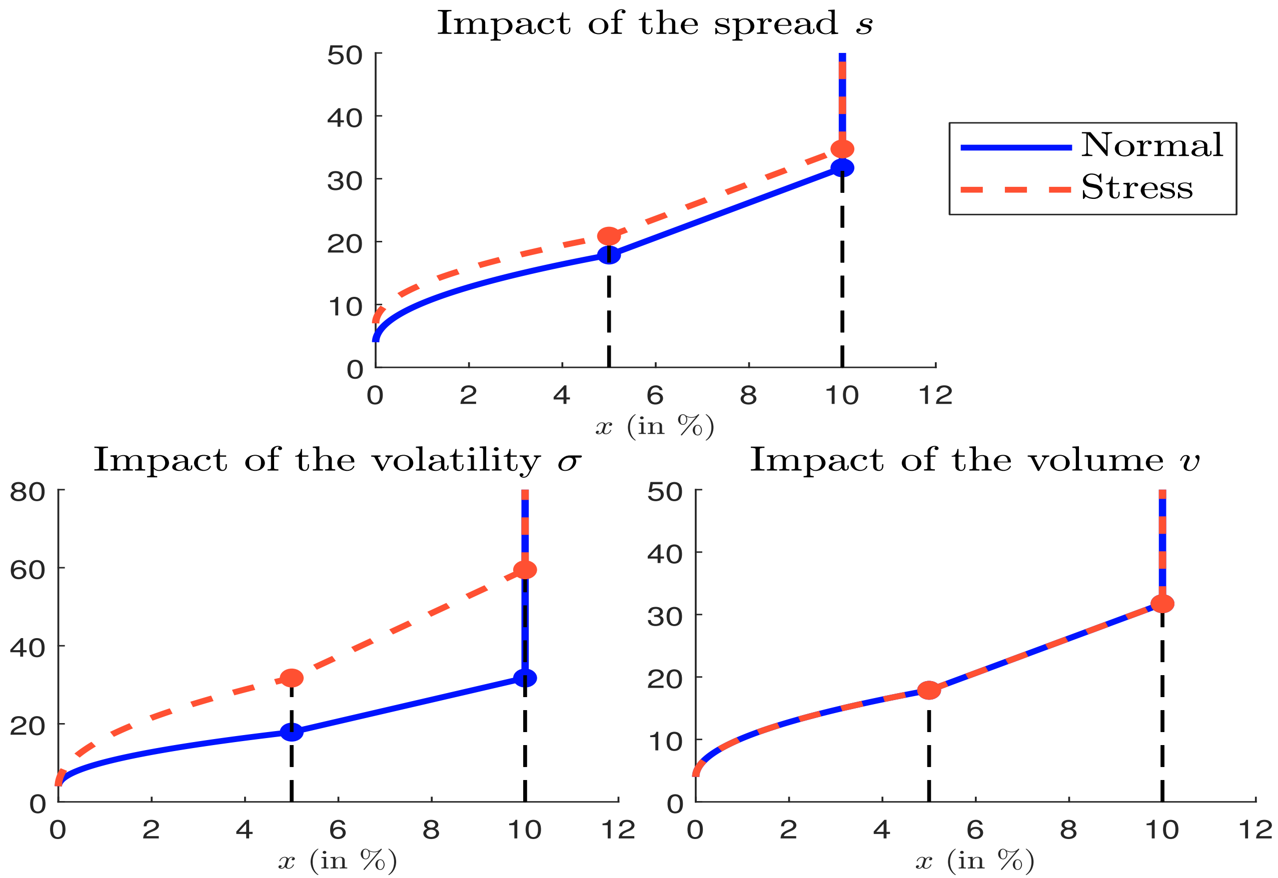
Stress testing in the two-regime model

Figure: The q-approach of the unit transaction cost



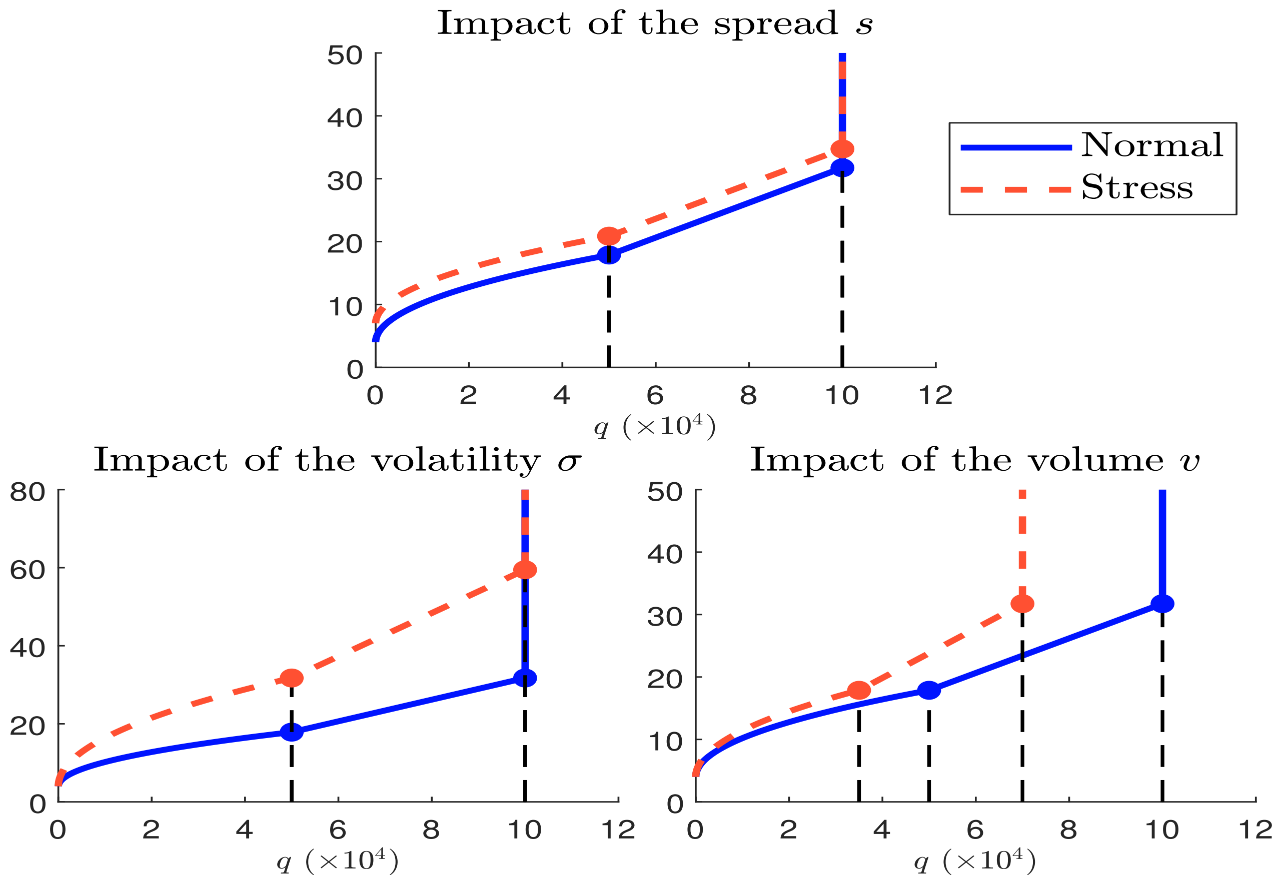
Stress testing in the two-regime model

Figure: Impact of security-specific parameters in the x-approach ($s \nearrow +75\%$,, $\sigma \nearrow \times 2$, $v \searrow 30\%$)



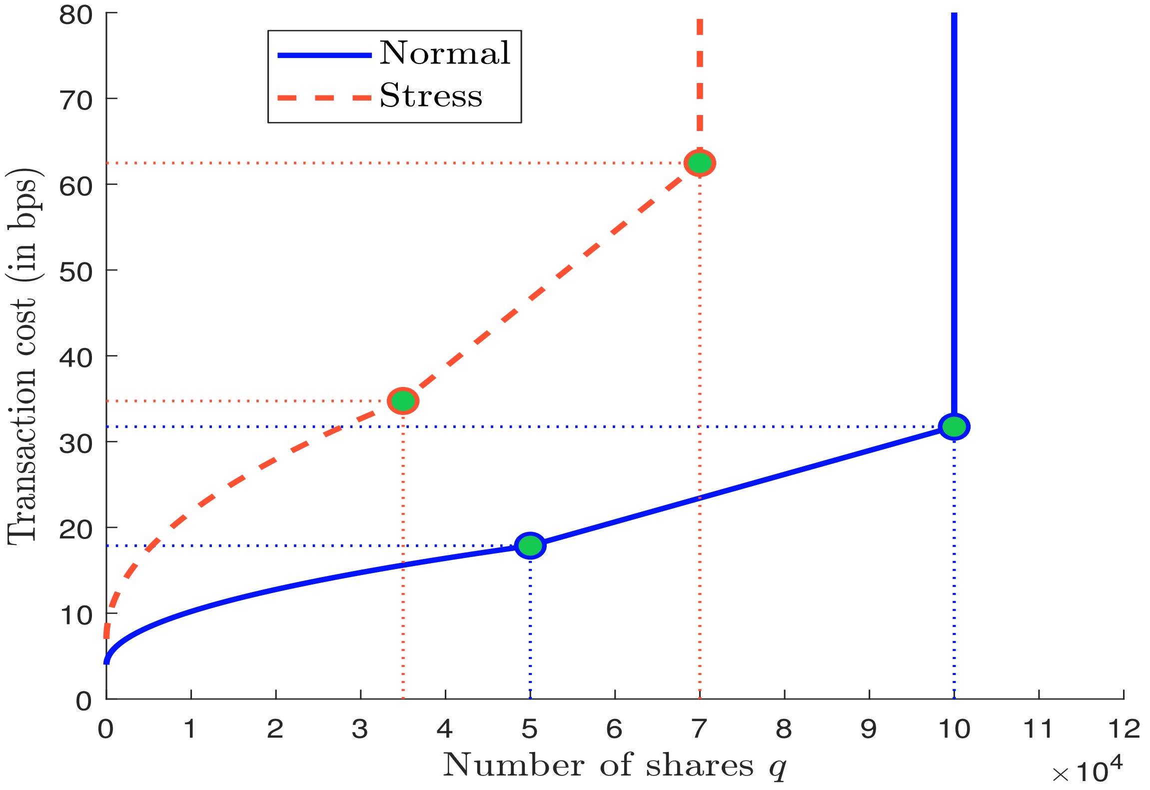
Stress testing in the two-regime model

Figure: Impact of security-specific parameters in the q-approach ($s \nearrow +75\%,$, $\sigma \nearrow \times 2$, $v \searrow 30\%$)



Stress testing in the two-regime model

Figure: Comparing the unit transaction cost in the normal and stress periods ($s \nearrow +75\%$,, $\sigma \nearrow \times 2$, $v \searrow 30\%$)



In a normal period, we can buy/sell $100\,000$ **shares (cost** $= 31.7$ **bps)**

In a stress period, we can buy/sell $70\,000$ **shares (cost** $= 62.5$ **bps)**

Asset liquidity measures

Several measures:

- Liquidity risk profile
  - Liquidation ratio $\mathbf{LR}(q;h)$
  - Liquidation time $\mathbf{LT}(q;p)$
  - Liquidation shortfall $\mathbf{LS}(q)$
- Liquidity cost
  - Transaction cost $\mathbf{TC}(q)$
  - Implementation shortfall/effective cost $\mathbf{IS}(q)$

$\Rightarrow$ The liquidity risk profile only depends on the liquidation policy x_i^+ or q_i^+ whereas the liquidity cost depends on the unit transaction cost (including the liquidation policy)

Liquidation ratio

- The number of shares $q_i(h)$ liquidated after h trading days is defined as follows:

$$q_i(h) = \min\left(\left(q_i - \sum_{k=0}^{h-1} q_i(k)\right)^+, q_i^+\right)$$

  where $q_i(0) = 0$ and q_i^+ is the maximum number of shares that can be sold during a trading day for the asset i
- The liquidation ratio $\mathbf{LR}(q; h)$ is the proportion of the redemption scenario q that is liquidated after h trading days:

$$\mathbf{LR}(q; h) = \frac{\sum_{i=1}^{n} \sum_{k=1}^{h} q_i(k) \cdot P_i}{\sum_{i=1}^{n} q_i \cdot P_i}$$

$\Rightarrow$ $\mathbf{LR}(q; 5) = 80\%$ means that we can fulfill 80% of the redemption after five trading days

Time to liquidation

The liquidation time is the inverse function of the liquidation ratio:

$$\mathbf{LT}(q;p) = \mathbf{LR}^{-1}(q;p) = \inf\{h : \mathbf{LR}(q;h) \geq p\}$$

For instance, $\mathbf{LT}(q;75\%) = 8$ means that we need 8 trading days to fulfill 75% of the redemption

Remark

The liquidation period $h^+ = \mathbf{LT}(q;1) = \inf\{h : \mathbf{LR}(q;h) = 1\}$ *indicates how many trading days we need to liquidate the redemption scenario* q

Liquidation shortfall

The liquidation shortfall $\mathbf{LS}(q)$ is defined as the remaining redemption that cannot be fulfilled after one trading day:

$$\mathbf{LS}(q) = 1 - \mathbf{LR}(q; 1)$$

Transaction cost

The transaction cost of the redemption scenario $q = (q_1, \ldots, q_n)$ is equal to:

$$\mathbf{TC}(q) = \sum_{i=1}^{n} \sum_{h=1}^{h^+} \mathbb{1}\{q_i(h) > 0\} \cdot q_i(h) \cdot P_i \cdot \boldsymbol{c}_i\left(\frac{q_i(h)}{v_i}\right)$$

Implementation shortfall and effective cost

- The current value of the redemption scenario is equal to:

$$\mathbb{V}^{\mathrm{mid}}(q) = \sum_{i=1}^{n} q_i(t) \cdot P_i^{\mathrm{mid}}(t)$$

where $q_i(t)$ and $P_i^{\mathrm{mid}}(t)$ are the number of shares to sell and the mid-price for the security i at the current time t

- The value of the liquidated portfolio is equal to:

$$\mathbb{V}^{\mathrm{liquidated}}(q) = \sum_{i=1}^{n} \sum_{t_k \geq t} q_i(t_k) \cdot P_i^{\mathrm{bid}}(t_k)$$

where $q_i(t_k)$ and $P_i^{\mathrm{bid}}(t_k)$ are the number of shares that were sold and the bid price for the security i at the execution time t_k

- The effective cost is then the difference between $V^{\mathrm{mid}}(t)$ and $V^{\mathrm{liquidated}}(t)$:

$$\mathbf{IS}(q) = \max\left(\mathbb{V}^{\mathrm{mid}}(q) - \mathbb{V}^{\mathrm{liquidated}}(q), 0\right)$$

Asset liquidity measures

Several ways to break down these measures: decomposition by trading day, decomposition by asset, decomposition bid-ask spread/price impact, etc.

Data

- Universe of MSCI USA & MSCI Europe indices
- Large caps (1 053) & small caps (2 622)
- We exclude trades of index portfolios
- 149 896 observations

Calibration procedure

- For each observation i, we have the transaction cost $\boldsymbol{c}_i$, the (end-of-day) bid-ask spread s_i, the participation rate x_i and the daily volatility σ_i
- We consider the following regression:

$$\boldsymbol{c}_i = \alpha + \beta^{(s)} s_i + \beta^{(\boldsymbol{\pi})} \sigma_i x_i^{\gamma_1} + \varepsilon_i$$

  where ε_i is a residual
- We estimate the parameters α, $\beta^{(s)}$, $\beta^{(\boldsymbol{\pi})}$ and γ_1 by NLS (single stocks) and panel analysis (multiple stocks)
- 3 models:
  1. $\beta^{(s)} = 1$, $\beta^{(\boldsymbol{\pi})} = 1$ and $\gamma_1 = 0.5$
  2. $\gamma_1 = 0.5$
  3. No constraints on the parameters

Multiple stocks analysis

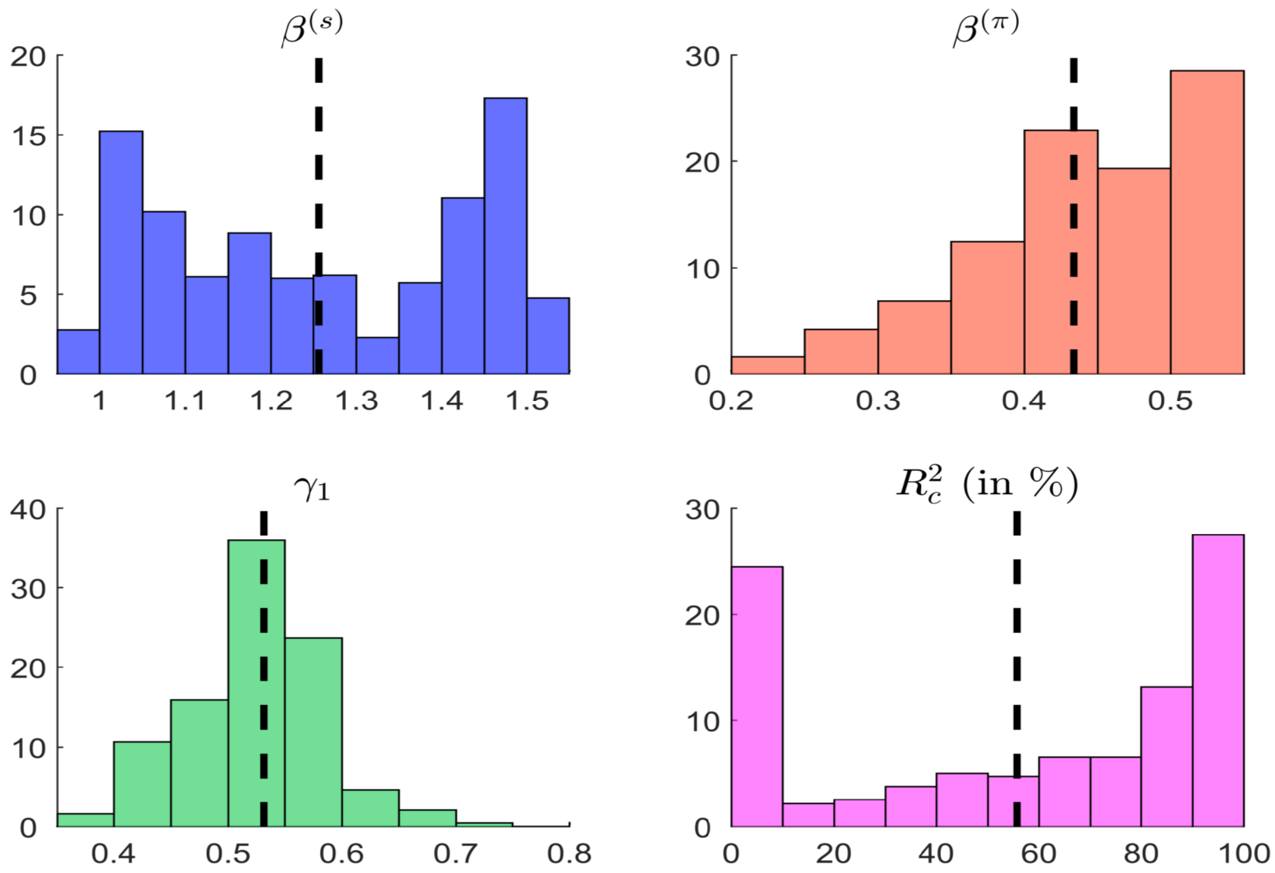
Table: Non-linear least squares estimation (large cap stocks)

Parameter	Model (1)	Model (2)	Model (3)	$x > 0.5\%$
α	0.00	0.00	0.00	0.00
$\beta^{(s)}$	1.00	1.45	1.45	1.47
$\beta^{(\pi)}$	1.00	0.19	0.30	**0.70**
γ_1	0.50	0.50	**0.58**	**0.84**
R_c^2	**15.87%**	**97.12%**	**97.84%**	87.58%

$\Rightarrow$ End-of-day bid-ask spread $\ll$ intra-day bid-ask spread?

Single stock analysis

Figure: Histogram of estimated parameters (large cap stocks)



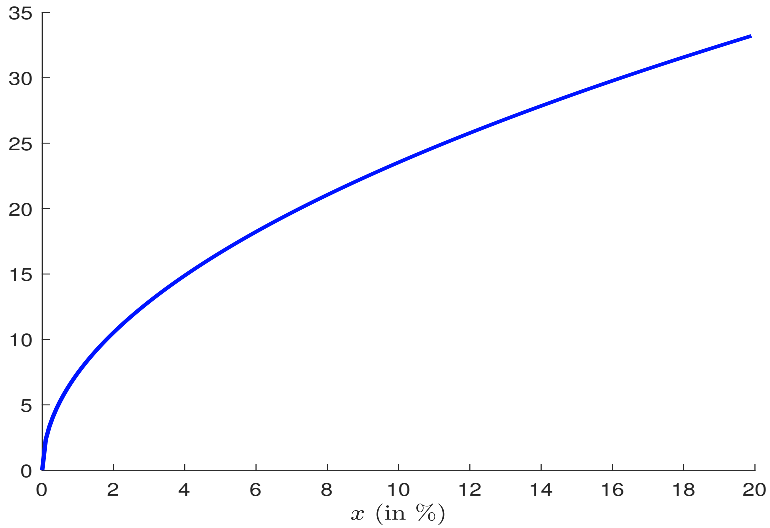
Benchmark model

Benchmark formula for large cap stocks

We propose the following benchmark formula for the transaction cost model:

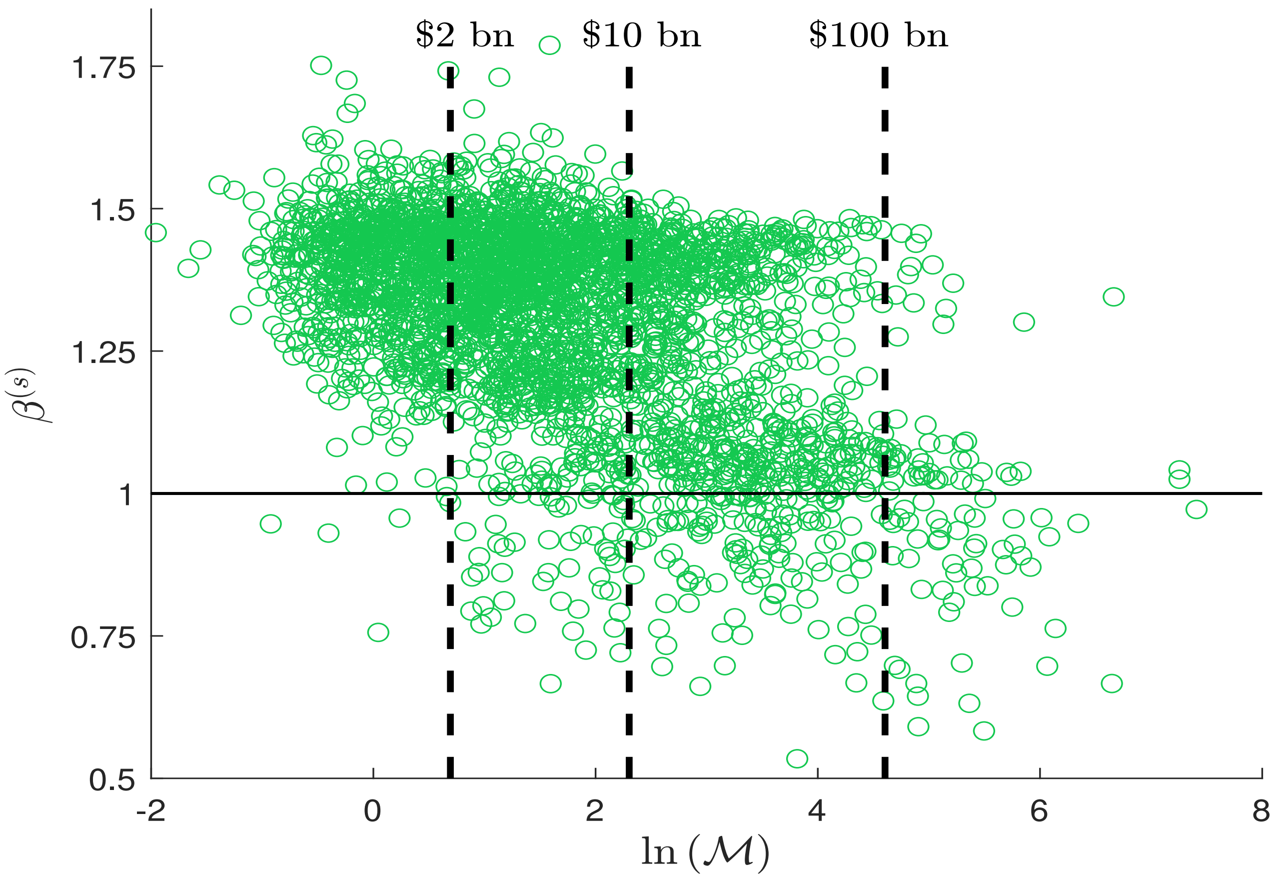
$$\boldsymbol{c}_i \left(q_i; s_{i,t}, \sigma_{i,t}, v_{i,t}\right) = 1.25 \cdot s_{i,t} + 0.40 \cdot \sigma_{i,t} \sqrt{x_{i,t}}$$

Figure: Estimated price impact (in bps) when σ is equal to 30%



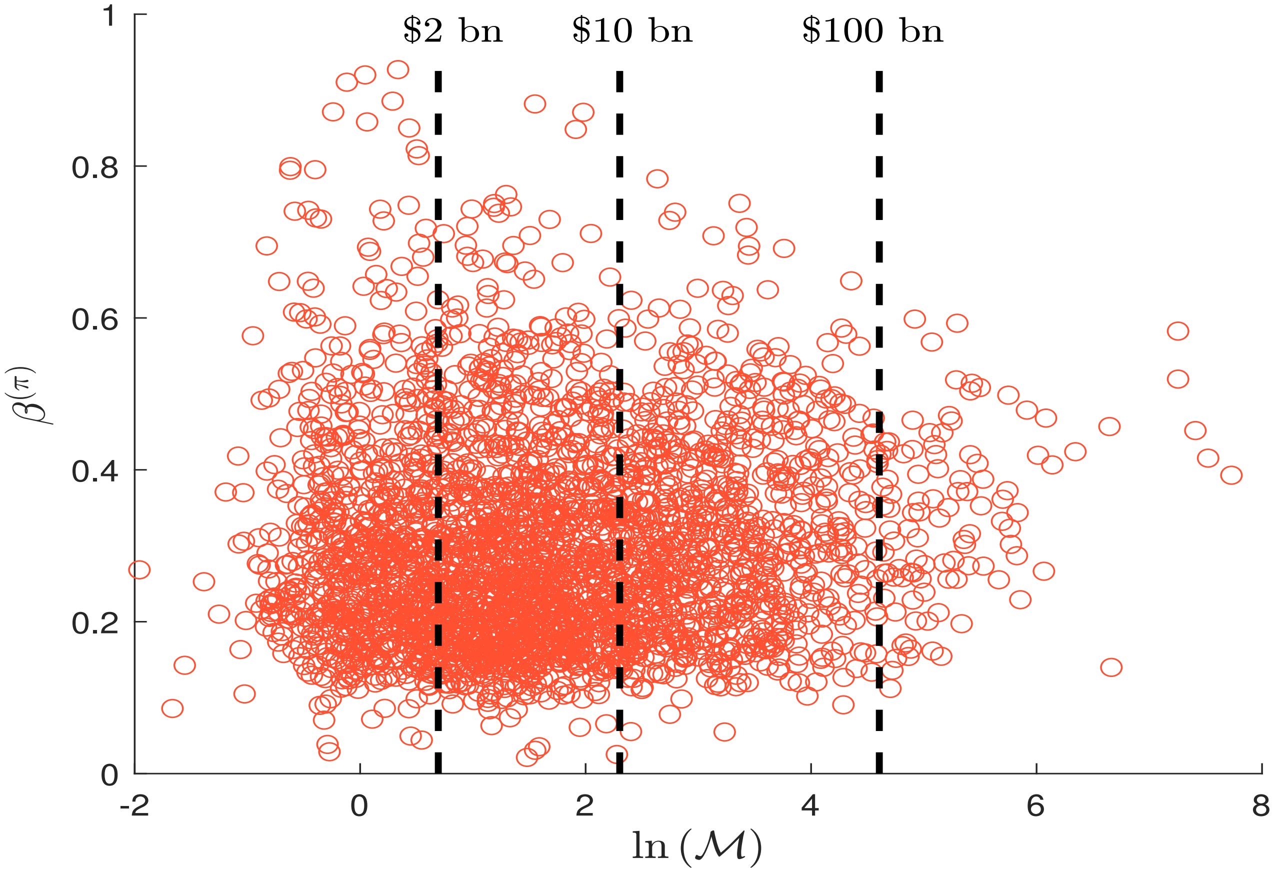
The case of small caps

Figure: Relationship between the market capitalization $\mathcal{M}$ and the parameter $\beta^{(s)}$



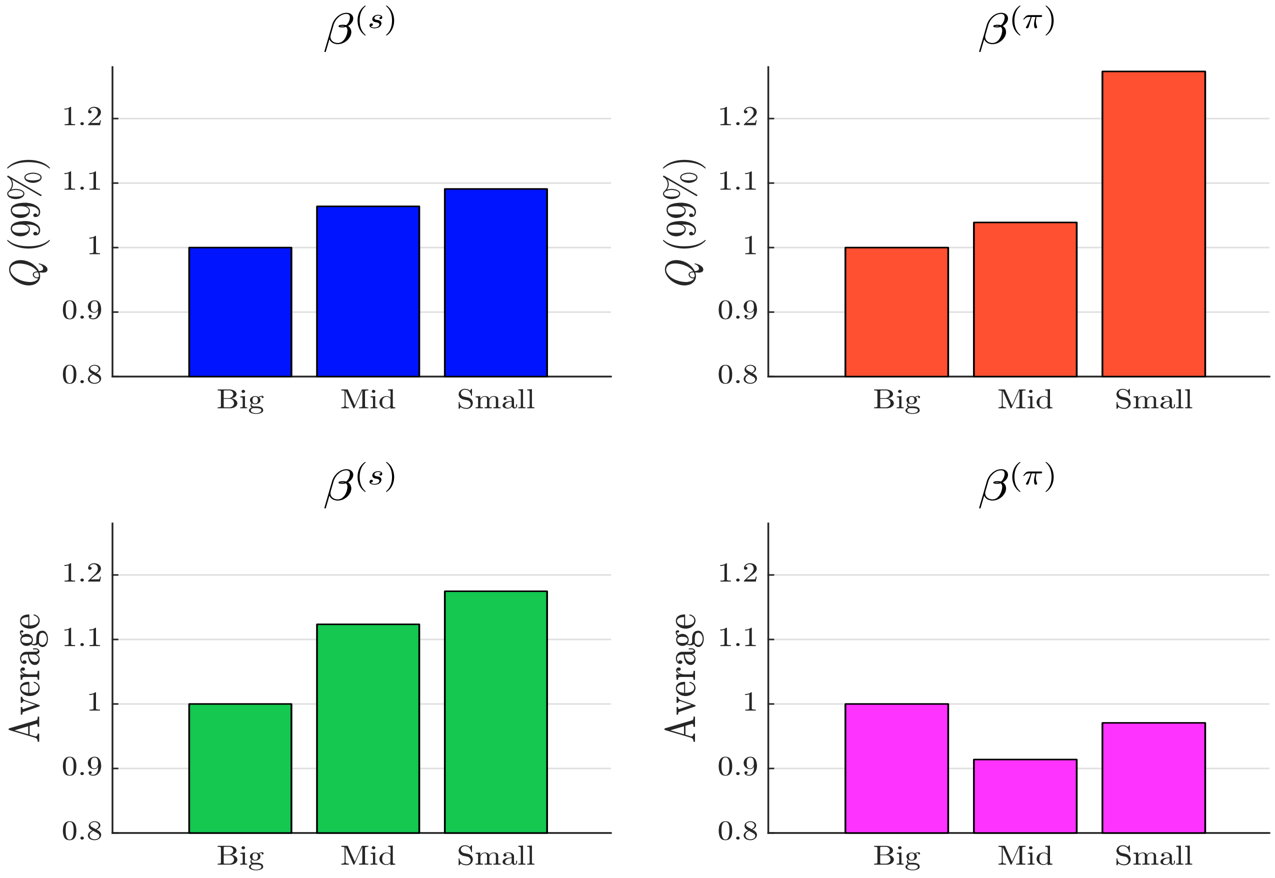
The case of small caps

Figure: Relationship between the market capitalization $\mathcal{M}$ and the parameter $\beta^{(\pi)}$



The case of small caps

Figure: Ratio of the parameters $\beta^{(s)}$ and $\beta^{(\pi)}$ with respect to the values of the big cap class



Benchmark model

Benchmark formula for large cap stocks

The benchmark formula for large cap stocks is:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, v_{i,t}\right) = \mathbf{1.25} \cdot s_{i,t} + \mathbf{0.40} \cdot \sigma_{i,t}\sqrt{x_{i,t}}$$

Benchmark formula for small cap stocks

We propose the following benchmark formula for the transaction cost model:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, v_{i,t}\right) = \mathbf{1.40} \cdot s_{i,t} + \mathbf{0.50} \cdot \sigma_{i,t}\sqrt{x_{i,t}}$$

$\Rightarrow$ The price impact is 25% higher for small caps when we consider the **same** participation rate

Benchmark model

Table: Price impact (in bps) for large cap stocks

σ (in %)	x (in %) 0.01	0.05	0.10	0.50	1	5	10	20	30
10	0.2	0.6	0.8	1.8	2	6	8	11	14
20	0.5	1.1	1.6	3.5	5	11	16	22	27
30	0.7	1.7	2.4	5.3	7	17	24	33	41
40	1.0	2.2	3.1	7.0	10	22	31	44	54
50	1.2	2.8	3.9	8.8	12	28	39	55	68
60	1.5	3.3	4.7	10.5	15	33	47	67	82

Table: Price impact (in bps) for small cap stocks

σ (in %)	x (in %) 0.01	0.05	0.10	0.50	1	5	10	20	30
10	0.3	0.7	1.0	2.2	3	7	10	14	17
20	0.6	1.4	2.0	4.4	6	14	20	28	34
30	0.9	2.1	2.9	6.6	9	21	29	42	51
40	1.2	2.8	3.9	8.8	12	28	39	55	68
50	1.6	3.5	4.9	11.0	16	35	49	69	85
60	1.9	4.2	5.9	13.2	19	42	59	83	102

How to define the participation rate in the case of bonds?

Definition

The (volume-based) participation rate is equal to:

$$x = \frac{q}{v} = \frac{Q}{V}$$

where q is the number of shares to trade and v is the daily trading volume

The participation rate can also be expressed with the nominal values Q and V

How to define the participation rate in the case of bonds?

- This is the key variable of the transaction cost formula
- It is not observed in case of bonds
- It is not always relevant because the trade of some bonds occurs infrequently (zero-trading days):
  - **79.4% of US IG bonds and 84.1% of US HY bonds are not traded on a monthly basis** between January 1995 to December 1999 (Hotchkiss and Jostova, 2017)
  - **The median number of zero-trading days was equal to 60.7% on a quarterly basis** from Q4 2004 to Q2 2009 in the US corporate bond market (Dick-Nielsen *et al.*, 2012)

How to define the participation rate in the case of bonds?

The turnover is a related measure to the trading volume:

$$\tau = \frac{V}{\mathcal{M}} = \frac{v}{n}$$

where V is the trading volume, $\mathcal{M}$ is the market capitalisation of the security and n is the number of issued shares

Definition

The outstanding-based participation rate is equal to:

$$y = \frac{q}{n}$$

Relationship between x and y

Since we have $V = \tau\mathcal{M}$, we deduce that:

$$x = \frac{Q}{\tau\mathcal{M}} = \frac{y}{\tau} \qquad \text{and} \qquad y = \tau x$$

Figures about the turnover

According to SIFMA (2021), the daily turnover ratio is:

- 0.36% for US corporate bonds in 2019
- higher for HY bonds (0.65% for US HY bonds) than for IG bonds (0.27% for US IG bonds)
- relatively stable (in the range 0.30% - 0.36% between 2005 and 2019)
- 0.44% in 2002
- 4.6% for bills, 1.2% for TIPS and 3.5% for notes and bonds (US treasury securities)

According to AFME (2020), the daily turnover ratio for sovereign bonds is:

- above 1% and close to 1.5% for Germany, Spain and UK
- between 0.5% and 1.0% for Belgium, France, Ireland, Italy, Netherlands, and Portugal
- lower than 0.5% for Denmark and Greece

Figures about the turnover

Remark

- The average yearly turnover of the US stock market is 150% for the last 20 years
- The highest turnover has been reached in 2008 ($\approx$ 400%)
- The turnover is larger than 100% since 1998
- For US corporate bonds, the turnover is lower than 100% since 2003
- The turnover of the European stock market $\approx$ half the turnover of the US stock market
- The turnover of the stocks that belong to the Eurostoxx 50 Index is lower than 75% since the beginning of the year

Magnitude order of the outstanding-based participation rate

Table: Outstanding-based participation rate (in bps) with respect to x and τ

τ	x (in %)								
(in %)	0.01	0.05	0.10	0.50	1	5	10	20	30
0.5	0.005	0.025	0.05	0.25	0.5	2.5	5	10	15
1.0	0.010	0.050	0.10	0.50	1.0	5.0	10	20	30
2.0	0.020	0.100	0.20	1.00	2.0	10.0	20	40	60
4.0	0.040	0.200	0.40	2.00	4.0	20.0	40	80	120

- Volume-based participation rates are expressed in %
- Outstanding-based participation rates are expressed in bps

The single-regime transaction cost function

Transaction cost function

We have:

$$\begin{aligned} \boldsymbol{c}(q; s_t, \sigma_t, v_t) &= \beta^{(s)} s_t + \beta^{(\pi)} \sigma_t \left(\frac{q}{\tau n}\right)^{\gamma_1} \\ &= \beta^{(s)} s_t + \tilde{\beta}^{(\pi)} \sigma_t y^{\gamma_1} \end{aligned}$$

where:

- q is the number of shares to trade, v_t the daily trading volume, s_t is the bid-ask spread, σ_t is the volatility
- $y = n^{-1} q$ is the outstanding-based participation rate
- $\beta^{(s)}$ is the scaling factor for the spread
- $\tilde{\beta}^{(\pi)}$ is the scaling factor of the price impact: $\tilde{\beta}^{(\pi)} = \tau^{-\gamma_1} \beta^{(\pi)}$

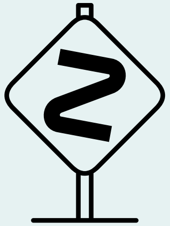
We have 3 parameters to estimate: $\beta^{(s)}$, $\tilde{\beta}^{(\pi)}$ and γ_1

The case of sovereign bonds

Data

- 196 286 trades from January 2018 to December 2020
- We use Bloomberg data prices

Data warning



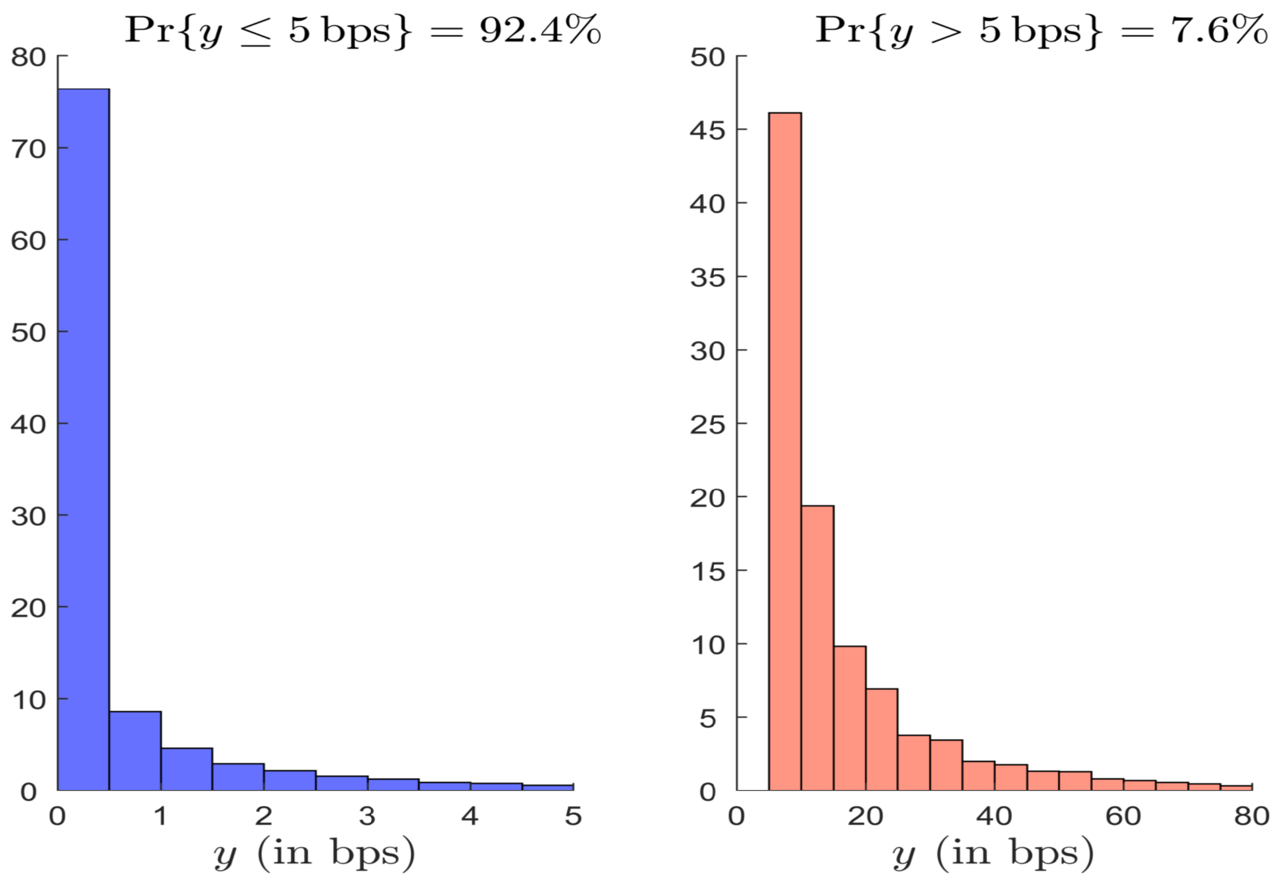
- Accuracy of data recording
- Representative sample?
- Trades with negative costs (**opportunistic trades**)
- Rejected inference (**rejected trades** because high/prohibitive costs)

$\Rightarrow$ Big issue for a liquidity stress testing program!

The case of sovereign bonds

Data

Figure: Histogram of the outstanding-based participation rate (sovereign bonds)



The case of sovereign bonds

Model estimation

For each observation i, we have the transaction cost $\boldsymbol{c}_i$, the spread s_i, the outstanding-based participation rate y_i and the daily volatility $\boldsymbol{\sigma}_i$. We run a two-stage regression model:

$$\begin{cases} \ln\left(\boldsymbol{c}_i - s_i\right) - \ln\sigma_i = c_\gamma + \gamma_1 \ln y_i + u_i & \text{if } \boldsymbol{c}_i > s_i \\ \boldsymbol{c}_i = c_\beta + \mathscr{D}_i^{(s)} \beta^{(s)} s_i + \mathscr{D}_i^{(\boldsymbol{\pi})} \tilde{\beta}^{(\boldsymbol{\pi})} \sigma_i y_i^{\gamma_1} + v_i & \end{cases}$$

where c_γ and c_β are two intercepts, u_i and v_i are two residuals. The dummy variables are $\mathscr{D}_i^{(s)} = \mathbb{1}\left\{\boldsymbol{c}_i > 0\right\}$ and $\mathscr{D}_i^{(\boldsymbol{\pi})} = \mathbb{1}\left\{\boldsymbol{c}_i > s_i\right\}$

The case of sovereign bonds

Model estimation

Table: Two-stage estimation of the sovereign bond transaction cost model

Parameter	Estimate	Stderr	t-student	p-value
c_γ	0.3004	0.0500	6.0096	0.0000
γ_1	**0.2037**	0.0046	44.6050	0.0000
c_β	0.0002	0.0000	15.7270	0.0000
$\beta^{(s)}$	**0.9099**	0.0109	83.3412	0.0000
$\tilde{\beta}^{(\pi)}$	**2.1521**	0.0153	140.6059	0.0000

$R^2 = 39.87\%$ $\qquad$ $R_c^2 = 28.94\%$

The case of sovereign bonds

Improvement of the model estimation

- The previous model can be improved by considering more liquidity buckets: issuers, currencies
- We can propose a parameterization of $\tilde{\beta}^{(\boldsymbol{\pi})}$:

$$\tilde{\beta}^{(\boldsymbol{\pi})} = f\left(\mathscr{F}_1, \ldots, \mathscr{F}_m\right)$$

where $\{\mathscr{F}_1, \ldots, \mathscr{F}_m\}$ are a set of bond characteristics (Ben Slimane and de Jong, 2017). If we assume that the parameters γ_1 and $\beta^{(s)}$ are the same for all the bonds, we observe that $\tilde{\beta}^{(\boldsymbol{\pi})}$ is an increasing function of the credit spread, the duration and the issue date (or the age of the bond).

The case of sovereign bonds

Improvement of the model estimation

Table: Estimation of the sovereign bond transaction cost model by issuer

Issuer	γ_1	c_β	$\beta^{(s)}$	$\tilde{\beta}^{(\pi)}$	R^2 (in %)	R_c^2 (in %)
Austria	0.2255	−0.0002	0.8599	3.1385	54.1	48.4
Belgium	0.2482	−0.0000	0.8097	3.3974	44.0	32.5
EM	**0.0519**	0.0010	0.6828	**0.4473**	74.9	47.4
Finland	0.2894	0.0000	0.7002	4.0287	46.3	31.8
France	0.2138	0.0000	0.8794	3.0087	40.1	29.7
Germany	0.2415	0.0001	0.9811	2.7007	51.6	38.7
Ireland	0.2098	0.0001	0.5403	2.4097	43.9	26.7
Italy	0.1744	−0.0004	**2.7385**	1.9030	31.3	**22.3**
Japan	**0.0657**	0.0001	0.4700	**0.6407**	79.5	56.4
Netherlands	0.2320	−0.0000	0.7640	3.7709	46.9	34.2
Portugal	0.2318	0.0001	0.9250	3.0248	49.6	33.0
Spain	0.2185	0.0000	1.2547	2.0758	40.9	26.7
United Kingdom	0.2194	0.0003	0.6837	2.3367	51.2	30.3
USA	**0.1252**	0.0001	1.0626	**1.2866**	53.8	40.9

The case of sovereign bonds

Improvement of the model estimation

Table: Estimation of the sovereign bond transaction cost model by currency

Currency	γ_1	c_β	$\beta^{(s)}$	$\tilde{\beta}^{(\pi)}$	R^2 (in %)	R_c^2 (in %)
EUR	0.2262	0.0000	1.0233	2.9122	35.2	25.7
GBP	0.2117	0.0002	1.3602	2.0878	48.8	30.2
JPY	**0.0834**	0.0001	**0.4811**	**0.8553**	75.6	50.9
USD	0.1408	0.0004	0.8430	**1.0121**	61.5	46.9

The case of sovereign bonds

Benchmark model

Benchmark formula for sovereign bonds

We propose the following benchmark formula for the transaction cost model:

$$\boldsymbol{c}_i \left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.25 \cdot s_{i,t} + 3.00 \cdot \sigma_{i,t} y_i^{0.25}$$

Benchmark formula for large cap stocks

The benchmark formula for large cap stocks is:

$$\boldsymbol{c}_i \left(q_i; s_{i,t}, \sigma_{i,t}, v_{i,t}\right) = 1.25 \cdot s_{i,t} + 0.40 \cdot \sigma_{i,t} x_{i,t}^{0.50}$$

The case of sovereign bonds

Benchmark model

Table: Price impact (in bps) for sovereign bonds

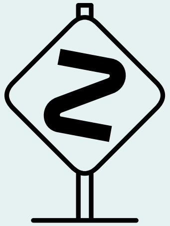
σ (in %)	y (in bps) 0.01	0.10	1	2.5	5	10	20	50	100	
1	0.6	1.0	1.9	2.3	2.8	3	4	5	6	
2	1.2	2.1	3.7	4.7	5.6	7	8	10	12	
3	1.8	3.1	5.6	7.0	8.3	10	12	15	18	
5	2.9	5.2	9.3	11.7	13.9	17	20	25	29	
10	5.9	10.5	18.6	23.4	27.8	33	39	49	59	
15	8.8	15.7	27.9	35.1	41.7	50	59	74	88	
30	17.7	31.4	55.8	70.2	83.5	99	118	148	177	
30	35.2	55.8	88.5	106.3	122.1	140	161	193	222	($\gamma = \mathbf{0.20}$)
30	17.7	31.4	55.8	70.2	83.5	99	118	148	177	($\gamma = \mathbf{0.25}$)
30	4.4	9.9	22.2	30.6	39.0	50	63	87	111	($\gamma = \mathbf{0.35}$)
30	0.6	1.8	5.6	8.8	12.5	18	25	39	56	($\gamma = \mathbf{0.50}$)

The case of corporate bonds

Data

- 258 153 trades from January 2018 to December 2020
- We use Bloomberg data prices

Data warning



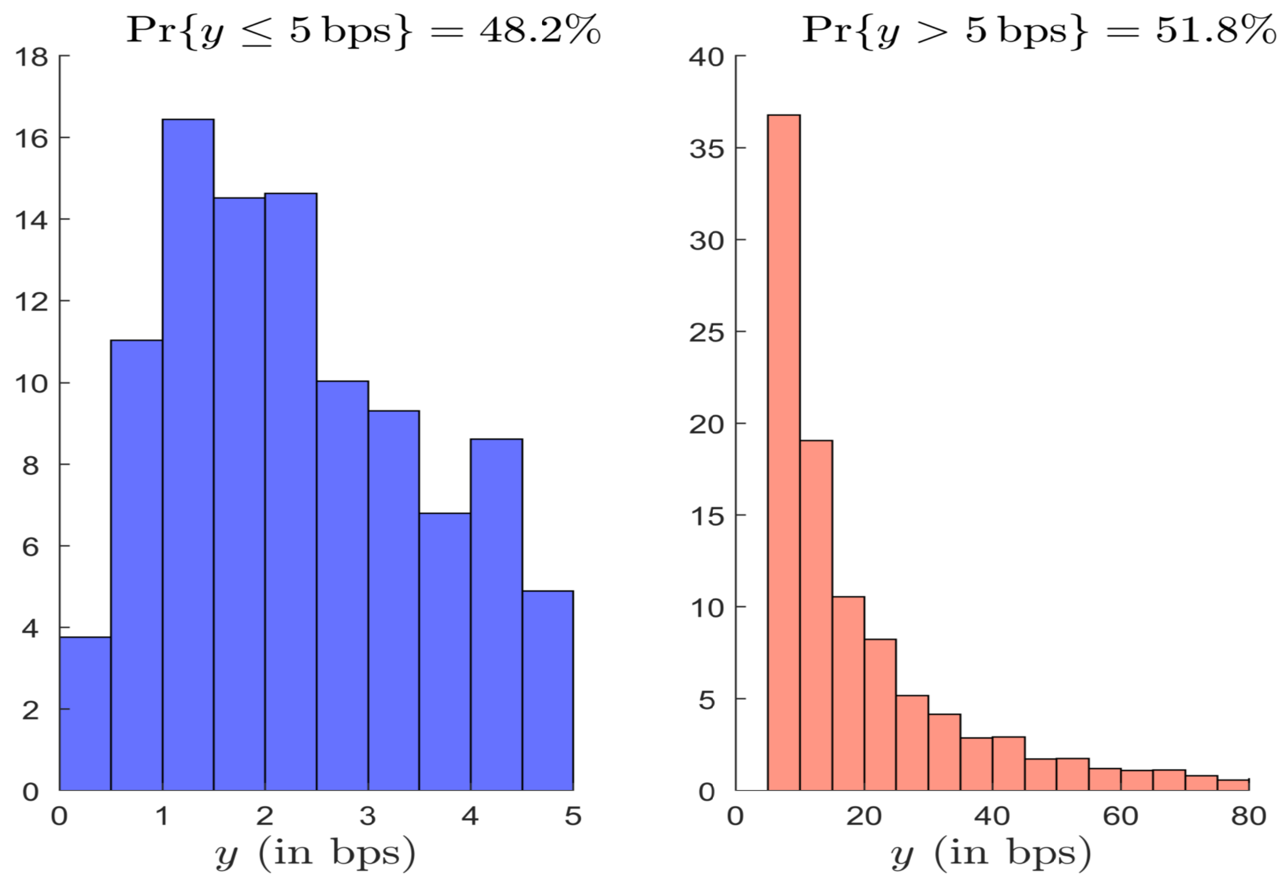
- Accuracy of data recording
- Representative sample?
- Trades with negative costs (**opportunistic trades**)
- Rejected inference (**rejected trades** because high/prohibitive costs)

$\Rightarrow$ Big issue for a liquidity stress testing program!

The case of corporate bonds

Data

Figure: Histogram of the outstanding-based participation rate (corporate bonds)



The case of corporate bonds

Model estimation

For each observation i, we have the transaction cost $\boldsymbol{c}_i$, the spread $\mathscr{s}_i$, the outstanding-based participation rate y_i and the daily volatility σ_i. We run the two-stage regression model:

$$\begin{cases} \ln\left(\boldsymbol{c}_i - \mathscr{s}_i\right) - \ln\sigma_i = c_\gamma + \gamma_1 \ln y_i + u_i & \text{if } \boldsymbol{c}_i > \mathscr{s}_i \\ \boldsymbol{c}_i = c_\beta + \mathscr{D}_i^{(\mathscr{s})}\beta^{(\mathscr{s})}\mathscr{s}_i + \mathscr{D}_i^{(\boldsymbol{\pi})}\tilde{\beta}^{(\boldsymbol{\pi})}\sigma_i y_i^{\gamma_1} + v_i & \end{cases}$$

The case of corporate bonds

Model estimation

Table: Two-stage estimation of the corporate bond transaction cost model with the volatility risk measure

Parameter	Estimate	Stderr	t-student	p-value
c_γ	0.3652	0.0338	10.8119	0.0000
γ_1	**0.1168**	0.0045	26.1322	0.0000
c_β	0.0008	0.0000	77.4368	0.0000
$\beta^{(s)}$	0.7623	0.0042	183.1617	0.0000
$\tilde{\beta}^{(\pi)}$	0.9770	0.0044	224.1741	0.0000
	$R^2 = 64.77\%$		$R_c^2 = 41.66\%$	

- The explanatory power is relatively high
- The model is misspecified if we focus on short-term corporate bonds when the time-to-maturity is less than two years $\Rightarrow R_c^2 = 18.86\%$
- The historical volatility is not available for 20.95% of observations

The case of corporate bonds

Asset risk measure

Volatility = asset risk measure when high ratio of zero-trading days?

- The asset risk is measured by the daily volatility σ_i in the model
- Price volatility is not a good measure for measuring the risk of a bond
- This is particularly true when the bonds are traded at a very low frequency, implying a low turnover

New formulation of the transaction cost function

We propose to use the following transaction cost function:

$$\boldsymbol{c}_i \left(q_i ; s_{i,t}, \sigma_{i,t}, n_{i,t} \right) = \beta^{(s)} s_{i,t} + \beta^{(\boldsymbol{\pi})} \mathscr{R}_{i,t} y_{i,t}^{\gamma_1}$$

where $\mathscr{R}_{i,t}$ is a better risk measure than the bond volatility

The case of corporate bonds

Asset risk measure

We have:

$$\sigma^2 (\mathrm{d} \ln B_i (t, D_i)) = D_i^2 \sigma^2 (\mathrm{d} r (t)) + D_i^2 (\sigma_i^{\mathfrak{s}})^2 \mathfrak{s}_i^2 (t) \,\mathrm{d} t$$

If the credit risk component is sufficient large with respect to the interest rate component, we obtain:

$$\begin{aligned} \sigma (\mathrm{d} \ln B_i (t, D_i)) &\approx \sigma_i^{\mathfrak{s}} \cdot D_i \cdot \mathfrak{s}_i (t) \\ &= \sigma_i^{\mathfrak{s}} \cdot \mathrm{DTS}_i (t) \end{aligned}$$

where $\mathrm{DTS}_i (t)$ is the duration-times-spread (or DTS) measure

We propose to replace the daily volatility by the DTS measure

The case of corporate bonds

Model estimation

Table: Two-stage estimation of the corporate bond transaction cost model with the DTS risk measure

Parameter	Estimate	Stderr	t-student	p-value
c_γ	-3.4023	0.0309	-109.9488	0.0000
γ_1	**0.0796**	0.0041	19.5020	0.0000
c_β	0.0005	0.0000	55.7256	0.0000
$\beta^{(s)}$	0.7153	0.0034	207.4743	0.0000
$\tilde{\beta}^{(\pi)}$	0.0356	0.0001	300.5100	0.0000

$R^2 = 68.64\%$ $\quad R_c^2 = 46.45\%$

The case of corporate bonds

Model estimation

- Bond traders may be very active
- They may decide to not sell or buy the bond if the transaction cost is high
- They generally execute a sell or buy order of a bond with a high participation rate only if the trading impact is limited
- Big trades are opportunistic and not systematic $\neq$ small and medium trades

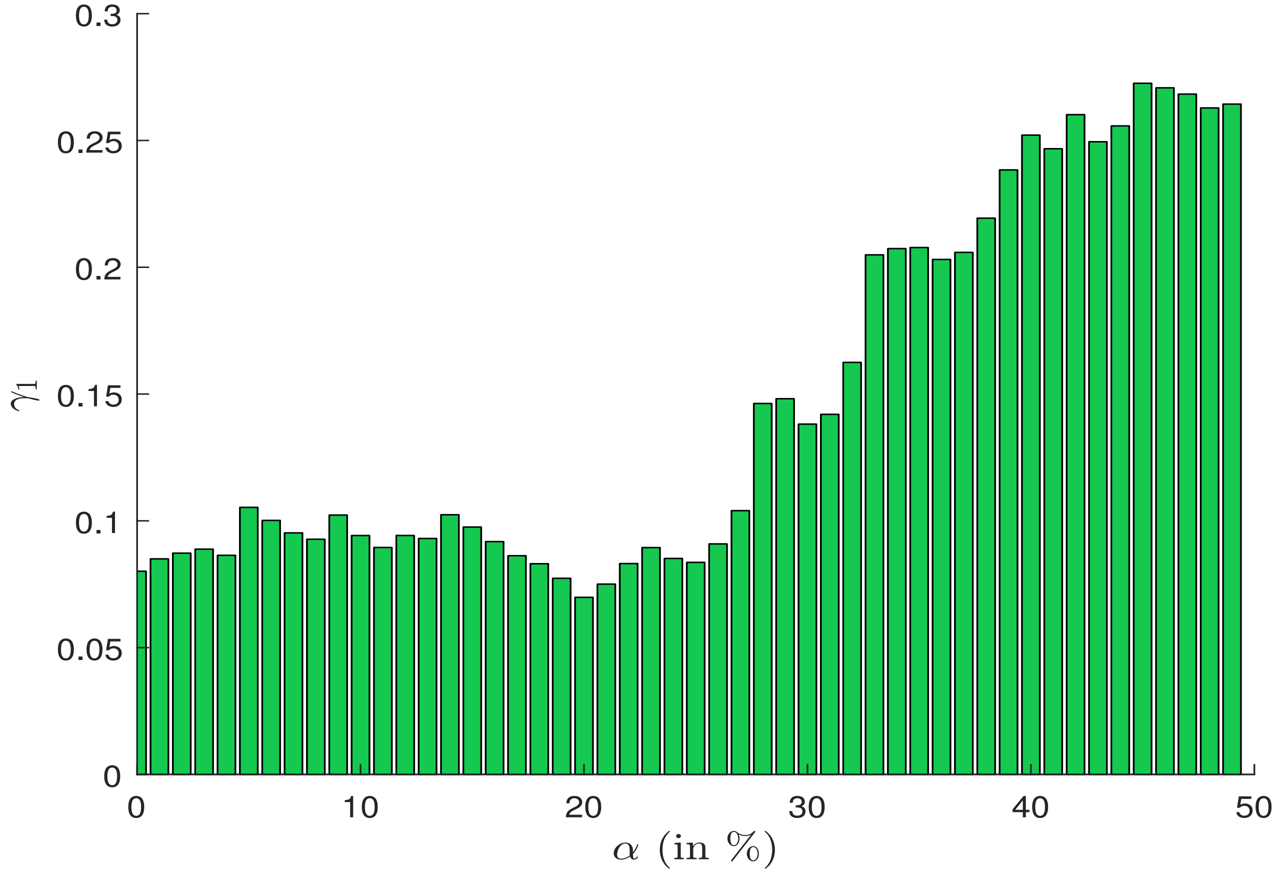
Systematic trading $\neq$ opportunistic trading

$\Rightarrow$ We remove the proportion α of the trades with the biggest participation rate

The case of corporate bonds

Model estimation

Figure: Estimated value of γ_1 with respect to the proportion α



The case of corporate bonds

Model estimation

Table: Estimation of the corporate bond transaction cost model when γ_1 is set to 0.25

Parameter	Estimate	Stderr	t-student	p-value
γ_1	0.2500			
$\beta^{(s)}$	0.8979	0.0028	323.2676	0.0000
$\tilde{\beta}^{(\boldsymbol{\pi})}$	0.1131	0.0004	293.5226	0.0000

$R^2 = 66.24\%$ $\qquad$ $R_c^2 = 42.35\%$

The case of corporate bonds

Benchmark model

Benchmark formula for corporate bonds

We propose the following benchmark formula for the transaction cost model:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.50 \cdot s_{i,t} + 0.125 \cdot \mathrm{DTS}_{i,t}\, y_i^{0.25}$$

Benchmark formula for sovereign bonds

The benchmark formula for the transaction cost model is:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.25 \cdot s_{i,t} + 3.00 \cdot \sigma_{i,t}\, y_i^{0.25}$$

The case of corporate bonds

Benchmark model

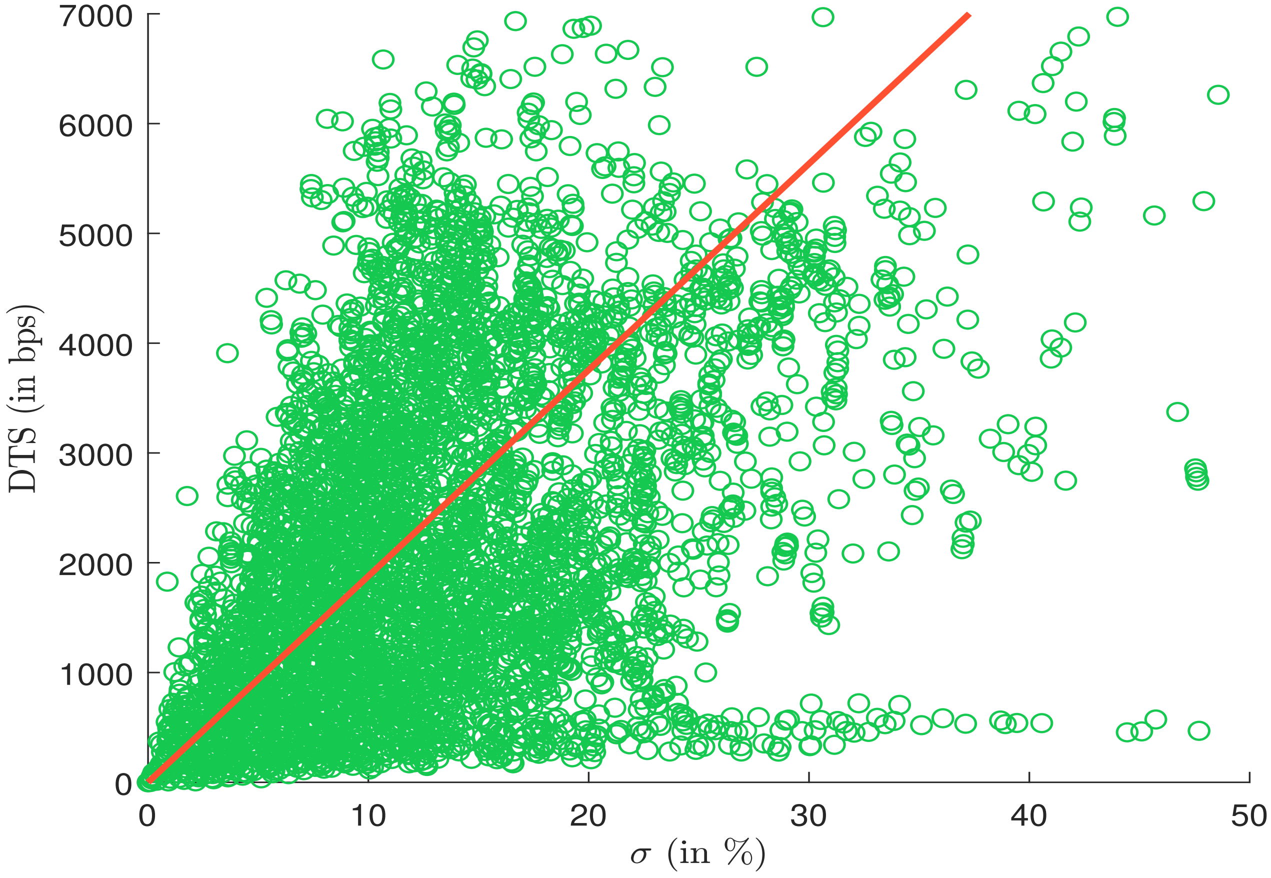
Table: Price impact (in bps) for corporate bonds

DTS (in bps)	y (in bps) 0.01	0.10	1	2.5	5	10	20	50	100	500
20	0.1	0.1	0.3	0.3	0.4	0	1	1	1	1
50	0.2	0.4	0.6	0.8	0.9	1	1	2	2	3
100	0.4	0.7	1.3	1.6	1.9	2	3	3	4	6
250	1.0	1.8	3.1	3.9	4.7	6	7	8	10	15
500	2.0	3.5	6.3	7.9	9.3	11	13	17	20	30
1 000	4.0	7.0	12.5	15.7	18.7	22	26	33	40	59
2 000	7.9	14.1	25.0	31.4	37.4	44	53	66	79	118
5 000	19.8	35.1	62.5	78.6	93.5	111	132	166	198	296
10 000	39.5	70.3	125.0	157.2	186.9	222	264	332	395	591

Back to sovereign bonds

Benchmark model

Figure: Relationship between volatility and duration-times-spread



Back to sovereign bonds
Benchmark model

Equivalent formula for sovereign bonds

Previously, we had:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.25 \cdot s_{i,t} + 3.00 \cdot \sigma_{i,t} y_i^{0.25}$$

Using the average relationship between volatility and DTS, we obtain:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.25 \cdot s_{i,t} + 0.10 \cdot \mathrm{DTS}_{i,t}\, y_i^{0.25}$$

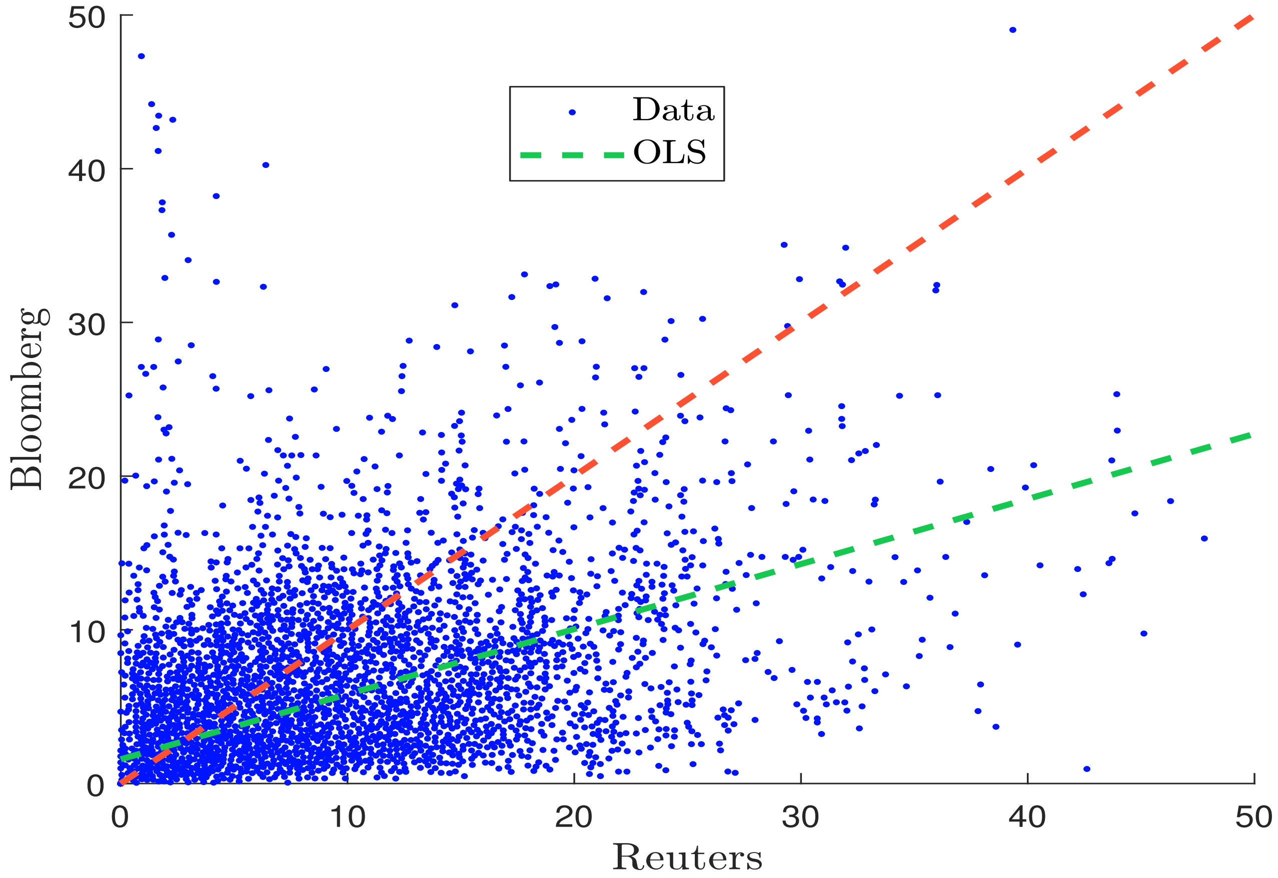
Benchmark formula for corporate bonds

We recall that the benchmark formula for the transaction cost model is:

$$\boldsymbol{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, n_i\right) = 1.50 \cdot s_{i,t} + 0.125 \cdot \mathrm{DTS}_{i,t}\, y_i^{0.25}$$

Why $1.25 \cdot s_{i,t}$ and $1.50 \cdot s_{i,t}$?

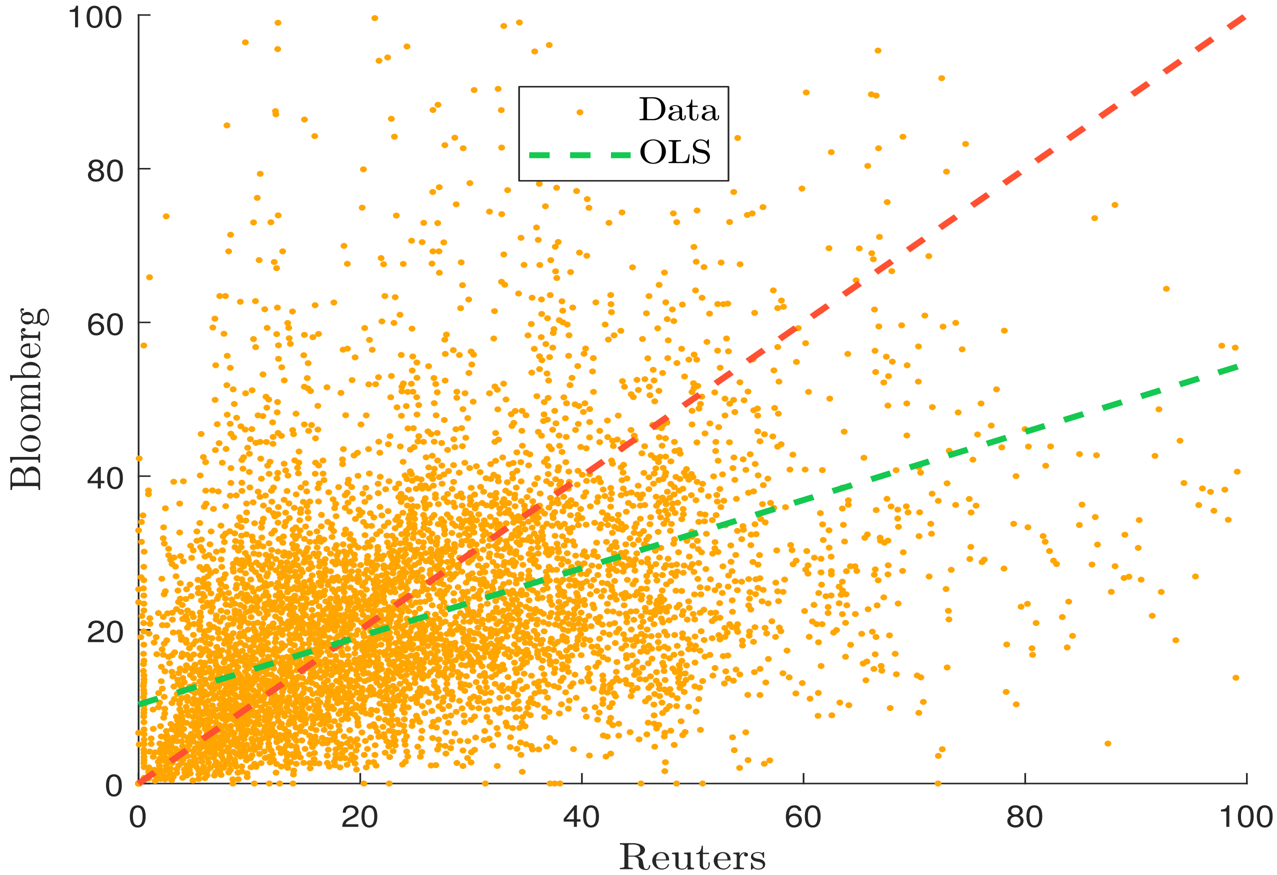
Figure: Scatter plot of Reuters and Bloomberg bid-ask spreads (sovereign bonds)



$$s_{\text{Bloomberg}} = 1.6 + 0.42 \cdot s_{\text{Reuters}} + \varepsilon$$

Why $1.25 \cdot s_{i,t}$ and $1.50 \cdot s_{i,t}$?

Figure: Scatter plot of Reuters and Bloomberg bid-ask spreads (corporate bonds)



$$s_{\text{Bloomberg}} = 10.3 + 0.44 \cdot s_{\text{Reuters}} + \varepsilon$$

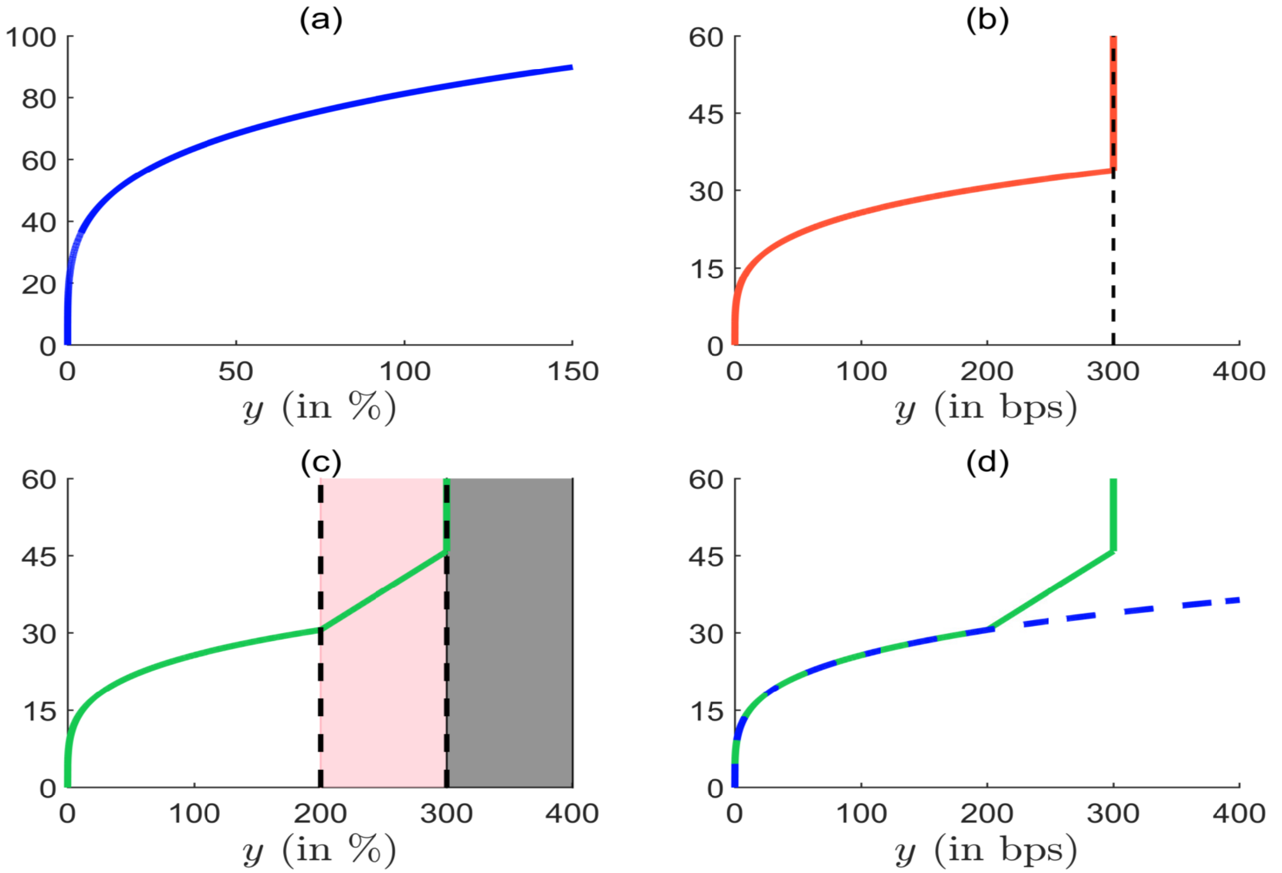
Understanding the size effect

Table: Impact of size on the transaction cost

Size	Spread			Price impact (stocks)			Price impact (bonds)		
	Unit cost	Total cost	Average cost	Unit cost	Total cost	Average cost	Unit cost	Total cost	Average cost
$\times 1$	$\times 1$	$\times 1$	$+0\%$	$\times 1.0$	$\times 1.0$	$+0\%$	$\times 1.0$	$\times 1.0$	$+0\%$
$\times 2$	$\times 1$	$\times 2$	$+0\%$	$\times 1.4$	$\times 2.8$	$+41\%$	$\times 1.2$	$\times 2.4$	$+19\%$
$\times 3$	$\times 1$	$\times 3$	$+0\%$	$\times 1.7$	$\times 5.2$	$+73\%$	$\times 1.3$	$\times 3.9$	$+32\%$
$\times 4$	$\times 1$	$\times 4$	$+0\%$	$\times 2.0$	$\times 8.0$	$+100\%$	$\times 1.4$	$\times 5.7$	$+41\%$
$\times 5$	$\times 1$	$\times 5$	$+0\%$	$\times 2.2$	$\times 11$	$+124\%$	$\times 1.5$	$\times 7.5$	$+50\%$
$\times 10$	$\times 1$	$\times 10$	$+0\%$	$\times 3.2$	$\times 32$	$+216\%$	$\times 1.8$	$\times 18$	$+78\%$

Extension to the two-regime model

Figure: From the single-regime model to the two-regime model (corporate bonds, DTS = 650 bps)



Extension to the two-regime model

The second regime is fixed by the experts. It is a 3-step approach:

- Choose the trading limit x^{+} (or y^{+}) of the liquidity policy, e.g.,

$$x^{+} = 10\%$$

- Set the inflexion point $\tilde{x}$ with respect to trading limit, e.g.,s

$$\tilde{x} = \frac{2}{3} x^{+}$$

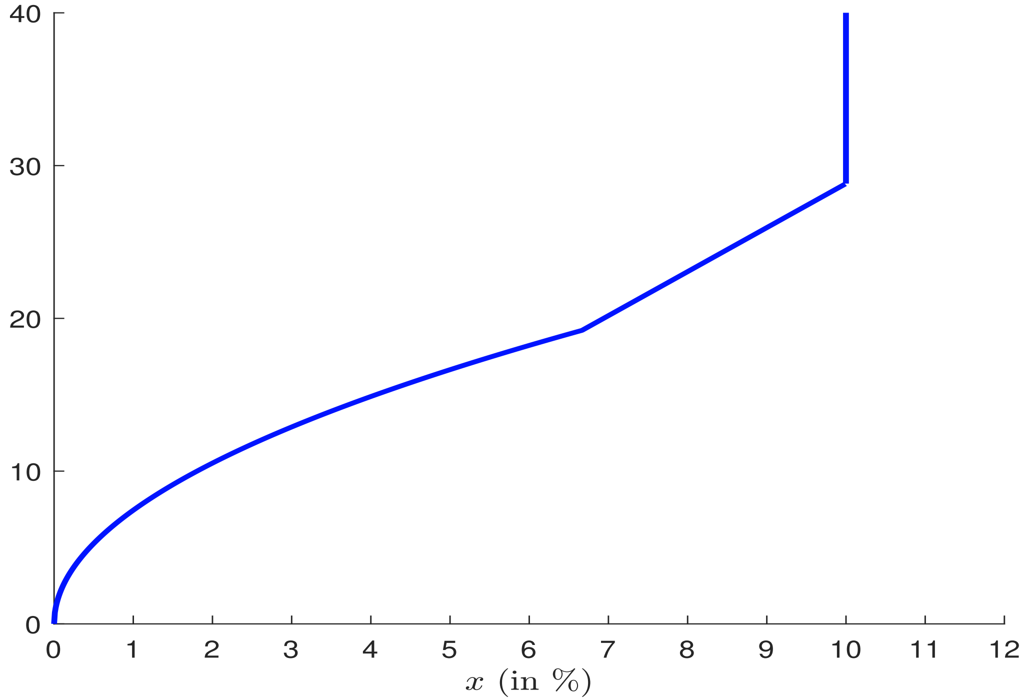
- Choose the convexity exponent for the second regime, e.g.,

$$\gamma_2 = 1$$

Extension to the two-regime model

Example with large cap stocks

$$\mathbf{c}_i\left(q_i; s_{i,t}, \sigma_{i,t}, v_{i,t}\right) = \begin{cases} 1.25 \cdot s_{i,t} + 0.50 \cdot \sigma_{i,t}\sqrt{x_{i,t}} & \text{if } x \leq 6.66\% \\ 1.25 \cdot s_{i,t} + 1.55 \cdot \sigma_{i,t} x_{i,t} & \text{if } 6.66\% \leq x \leq 10\% \\ +\infty & \text{if } x > 10\% \end{cases}$$



Estimated price impact (in bps) when σ is equal to 30%

Definition

A stress scenario is defined by two time dimensions:

1. The stress period h (or the period between the current date and the stress date)
2. The return time $\mathscr{T}$ of the stress scenario

$\Rightarrow$ We obtain $\mathscr{S}(\mathscr{T}, h)$ stress scenarios, where $\mathscr{T}$ measures the severity of the stress scenario and h measures the time horizon of the stress scenario

The stress scenario is an increasing function of both h and $\mathscr{T}$

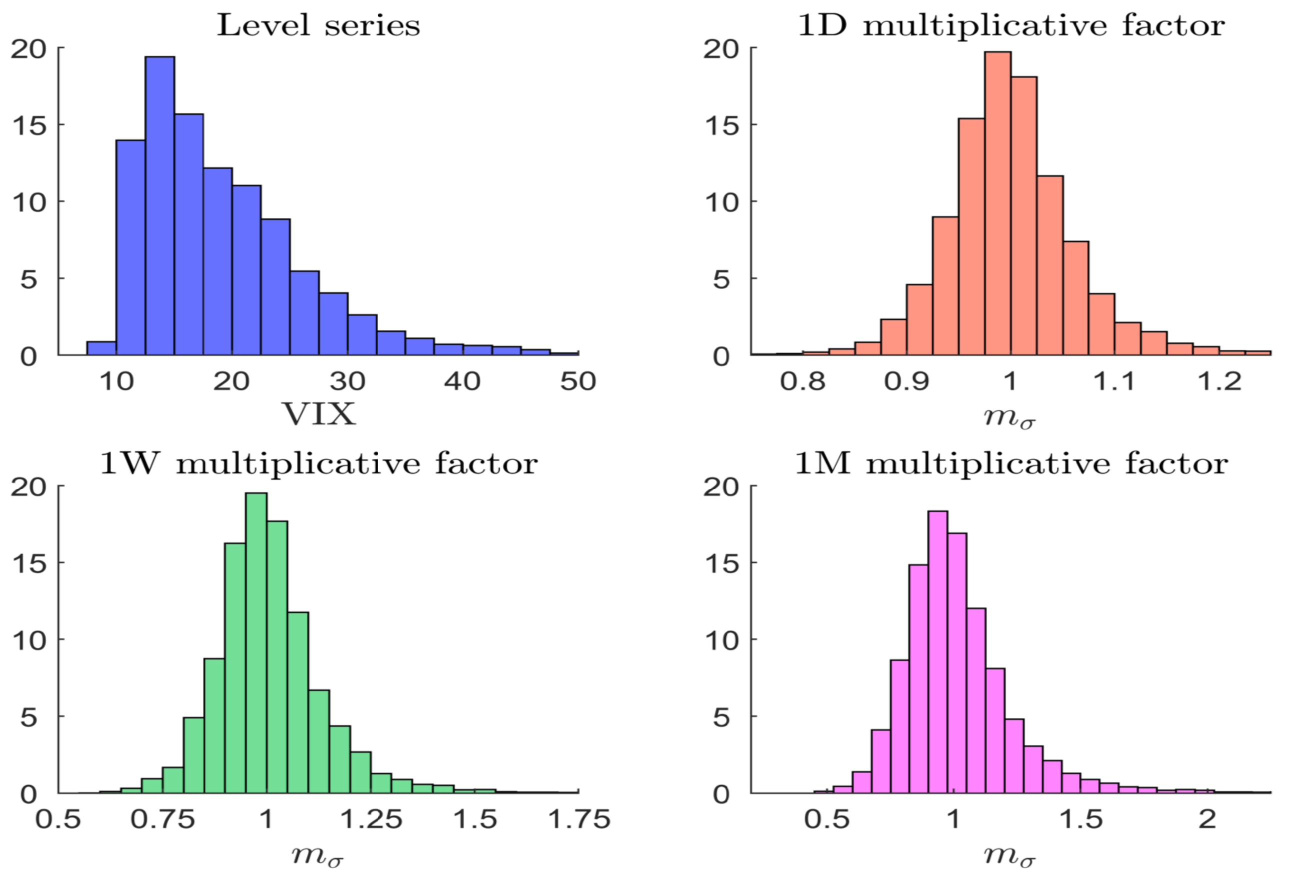
$\Rightarrow$ **The risk is to obtain $\mathscr{S}(\infty, \infty) = \infty$**

Methodology

- Additive or multiplicative stress scenarios?
- Historical stress scenarios
- Extreme value theory (EVT) scenarios
  - Block Maxima based on the Generalized Extreme Value distribution (BM/GEV)
  - Peak Over Threshold based on the Generalized Pareto distribution (POT/GPD)

Stress scenarios of the volatility (equity)

Figure: Empirical distribution of the multiplicative factor m_σ



Stress scenarios of the volatility (equity)

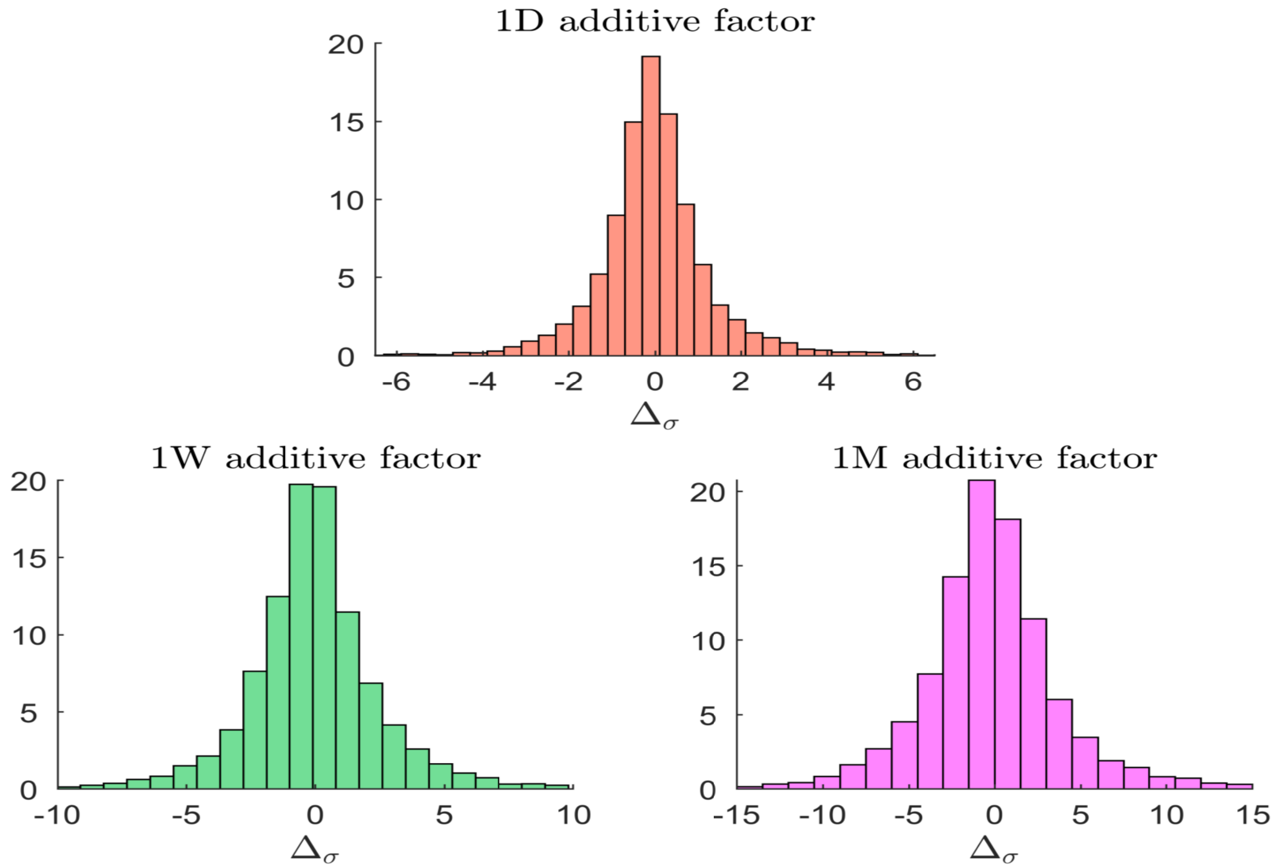
Table: Multiplicative stress scenarios of the volatility

	$\mathscr{T}$ (in years)	0.385	1/2	1	2	5	10	50
	α (in %)	99.00	99.23	99.62	99.81	99.92	99.96	99.99
1D	Historical	1.23	1.25	1.32	1.43	1.50	1.57	
	BM/GEV	1.20	1.22	1.29	1.37	1.51	1.65	2.09
	POT/GPD	1.23	1.25	1.33	1.41	1.52	1.62	1.90
1W	Historical	1.46	1.51	1.70	1.89	2.26	2.56	
	BM/GEV	1.34	1.38	1.50	1.64	1.86	2.06	2.66
	POT/GPD	1.46	1.51	1.68	1.87	2.18	2.47	3.35
1M	Historical	1.96	2.05	2.44	2.99	4.23	5.08	
	BM/GEV	1.47	1.55	1.78	2.04	2.46	2.83	3.99
	POT/GPD	1.96	2.08	2.45	2.96	3.88	4.86	8.53

$\Rightarrow$ **The 2Y weekly multiplicative stress scenario is equal to** $\times 1.80$

Stress scenarios of the volatility (equity)

Figure: Empirical distribution of the additive factor Δ_σ



Stress scenarios of the volatility (equity)

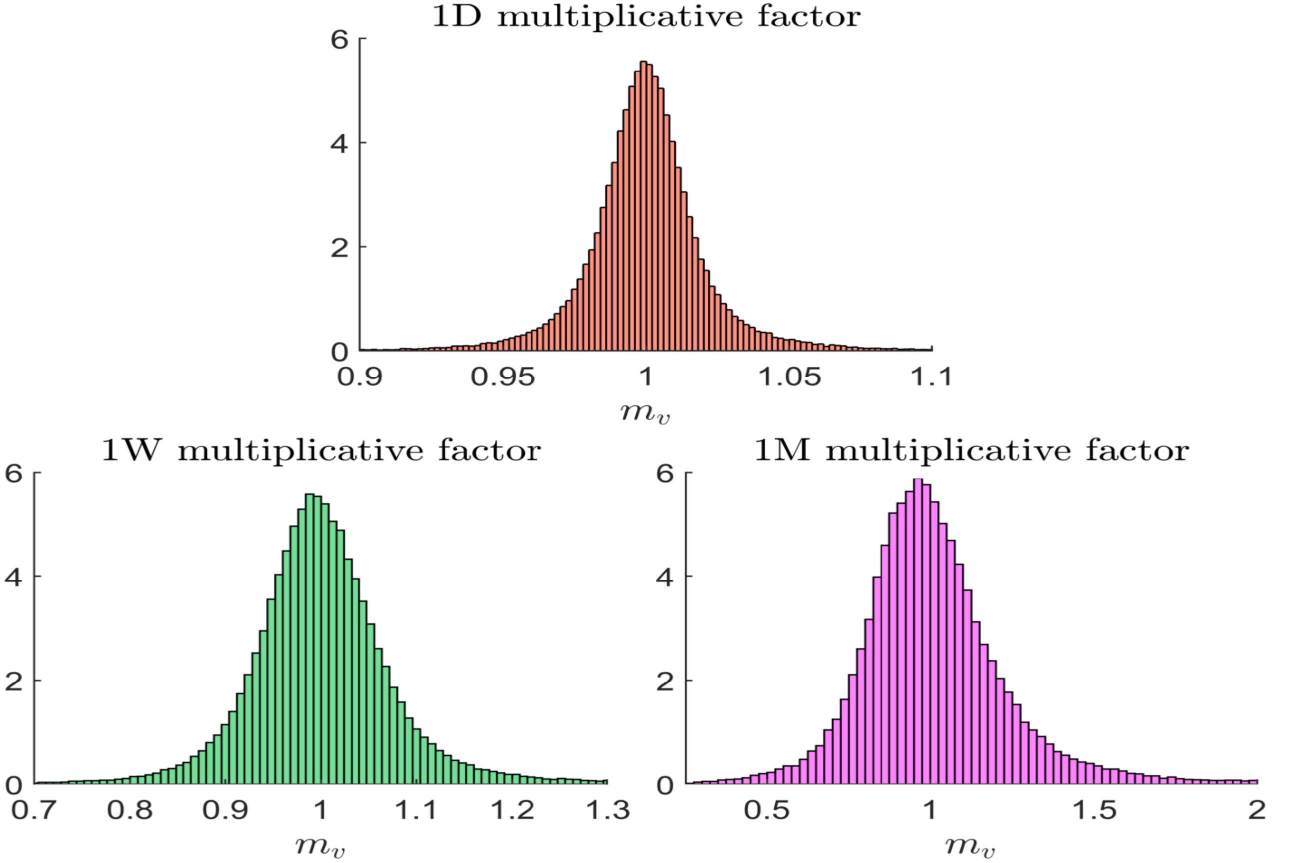
Table: Additive stress scenarios of the volatility

	$\mathscr{T}$ (in years)	0.385	1/2	1	2	5	10	50
	α (in %)	99.00	99.23	99.62	99.81	99.92	99.96	99.99
1D	Historical	4.94	5.50	7.72	10.77	14.15	18.22	
	BM/GEV	3.91	4.51	6.42	8.94	13.59	18.50	37.34
	POT/GPD	4.93	5.58	7.12	8.42	9.85	10.74	12.31
1W	Historical	9.49	10.88	14.50	20.43	24.56	27.97	
	BM/GEV	6.08	6.97	9.66	12.95	18.53	23.96	42.34
	POT/GPD	9.57	10.65	13.92	17.92	24.61	30.99	51.86
1M	Historical	16.83	19.04	27.22	35.62	46.59	61.40	
	BM/GEV	7.84	9.13	12.74	16.80	23.03	28.54	44.68
	POT/GPD	16.64	19.67	27.70	35.77	46.51	54.70	73.88

$\Rightarrow$ **The 2Y weekly additive stress scenario is equal to $+17\%$**

Stress scenarios of the trading volume (equity)

Figure: Empirical distribution of the multiplicative factor m_v



Stress scenarios of the trading volume (equity)

Table: Stress scenarios of the trading volume

	$\mathscr{T}$ (in years)		0.385	1/2	1	2	5	10	50
	α (in %)		99.00	99.23	99.62	99.81	99.92	99.96	99.99
	Historical		0.93	0.93	0.91	0.88	0.84	0.80	0.71
	BM/GEV	Pooling	0.94	0.94	0.92	0.90	0.87	0.85	0.80
1W	POT/GPD	Pooling	0.95	0.94	0.91	0.88	0.84	0.80	0.70
	BM/GEV	Averaging	0.94	0.94	0.93	0.92	0.91	0.90	0.89
	POT/GPD	Averaging	0.93	0.92	0.92	0.91	0.91	0.90	0.89
	Historical		0.79	0.77	0.72	0.67	0.61	0.55	0.48
	BM/GEV	Pooling	0.86	0.85	0.81	0.78	0.74	0.71	0.65
1W	POT/GPD	Pooling	0.87	0.83	0.75	0.68	0.61	0.56	0.47
	BM/GEV	Averaging	0.87	0.86	0.84	0.82	0.79	0.77	0.73
	POT/GPD	Averaging	0.82	0.81	0.79	0.78	0.76	0.75	0.72
	Historical		0.50	0.48	0.41	0.36	0.31	0.29	0.26
	BM/GEV	Pooling	0.72	0.69	0.62	0.56	0.50	0.46	0.39
1M	POT/GPD	Pooling	0.40	0.38	0.36	0.33	0.31	0.29	0.26
	BM/GEV	Averaging	0.75	0.73	0.68	0.63	0.58	0.55	0.49
	POT/GPD	Averaging	0.62	0.60	0.57	0.54	0.50	0.48	0.42

$\Rightarrow$ **The 2Y weekly multiplicative stress scenario is equal to** $\times 0.75$

Stress scenarios of the bid-ask spread (equity)

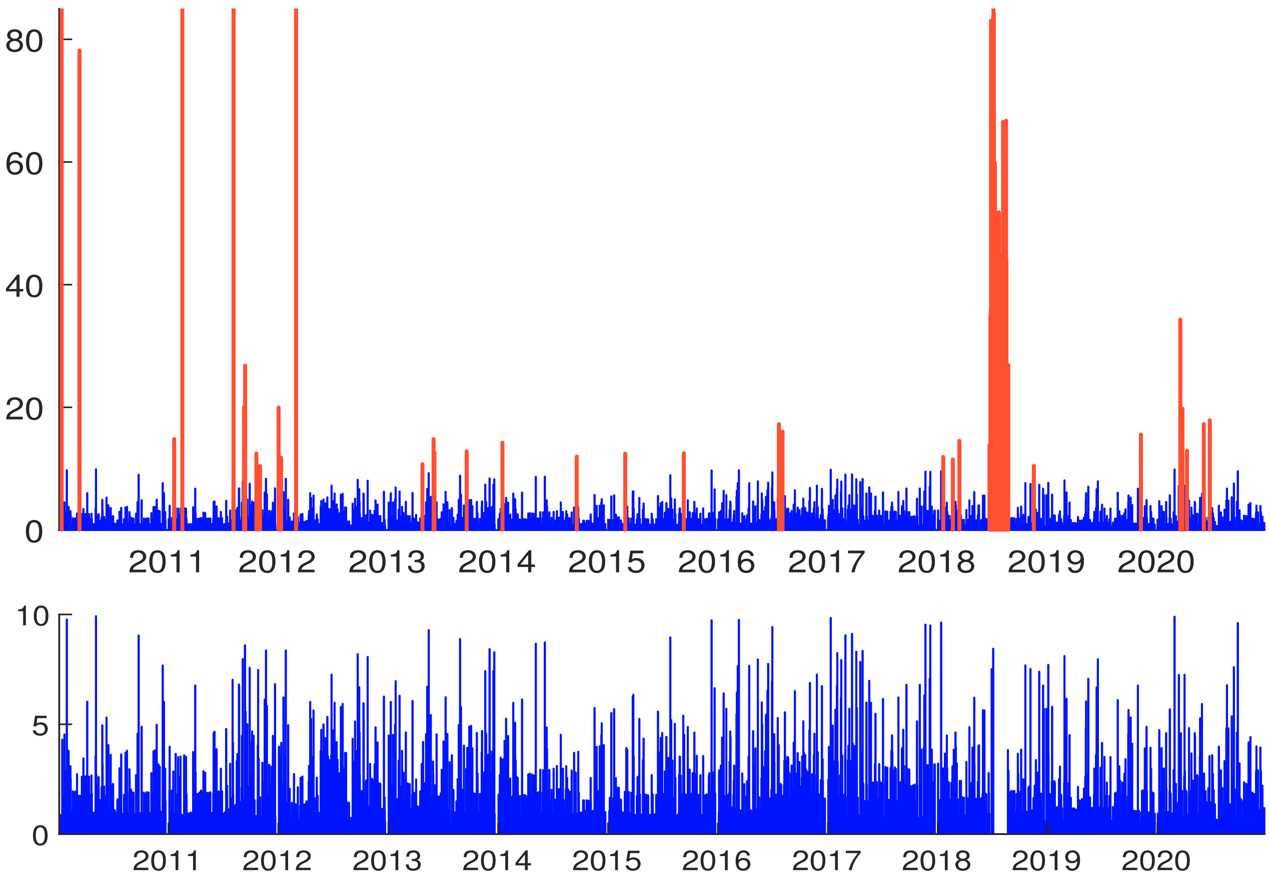
No available robust data

Some figures with Factset/Bloomberg for the Eurostoxx 50 stocks (2010 – 2020) — daily multiplicative and additive factors:

Frequency	Factset	Bloomberg
$\Pr\{s < 0\}$	0.01%	0.24%
$\Pr\{m_s > 10\}$	0.77%	0.62%
$\Pr\{m_s > 5\}$	3.49%	3.12%
$\Pr\{\lvert\Delta_s\rvert > 100 \text{ bps}\}$	0.63%	0.44%
$\Pr\{\lvert\Delta_s\rvert > 25 \text{ bps}\}$	4.52%	3.05%

Stress scenarios of the bid-ask spread (equity)

Figure: Historical bid-ask spread (in bps) of BNP Paribas (median spread = 1.22)



Stress scenarios of the bid-ask spread

Table: Multiplicative stress scenarios of the bid-ask spread

	$\mathscr{T}$ (in years)	0.385	1/2	1	2	5	10	50
	α (in %)	99.00	99.23	99.62	99.81	99.92	99.96	99.99
1D	Historical	1.66	1.73	1.93	2.40	2.75	7.11	
	BM/GEV	1.63	1.70	1.92	2.19	2.64	3.08	4.56
	POT/GPD	1.65	1.71	1.94	2.32	3.24	4.49	11.70
1W	Historical	1.74	1.88	2.58	3.49	6.78	9.76	
	BM/GEV	1.67	1.76	2.05	2.41	3.07	3.75	6.27
	POT/GPD	1.81	1.93	2.41	3.22	5.20	7.92	23.78
1M	Historical	2.54	2.92	5.12	6.65	9.62	9.98	
	BM/GEV	1.75	1.86	2.18	2.58	3.25	3.90	6.12
	POT/GPD	2.40	2.64	3.52	4.85	7.72	11.21	27.90

$\Rightarrow$ **The 2Y weekly multiplicative stress scenario is equal to $\times 3\%$**

Stress scenarios of the bid-ask spread

Table: Additive stress scenarios of the bid-ask spread

	$\mathscr{T}$ (in years)	0.385	1/2	1	2	5	10	50
	α (in %)	99.00	99.23	99.62	99.81	99.92	99.96	99.99
1D	Historical	1.67	1.82	2.93	6.46	10.94	18.14	
	BM/GEV	1.42	1.63	2.28	3.14	4.71	6.37	12.70
	POT/GPD	1.77	2.04	3.07	4.76	8.78	14.17	44.13
1W	Historical	1.98	2.37	5.19	10.10	12.36	19.11	
	BM/GEV	1.48	1.70	2.40	3.33	5.08	6.94	14.17
	POT/GPD	2.19	2.57	3.91	6.00	10.63	16.43	45.46
1M	Historical	3.36	3.98	7.90	10.60	16.04	21.36	
	BM/GEV	1.51	1.77	2.62	3.82	6.20	8.91	20.46
	POT/GPD	2.99	3.57	5.73	9.23	17.33	27.95	84.86

$\Rightarrow$ **The 2Y weekly additive stress scenario is equal to $+6.5$ bps**

The example of large cap stocks

Inputs

We recall that the transaction cost function is:

$$\boldsymbol{c}_i\left(q_i; s_i, \sigma_i, v_i\right) = \begin{cases} 1.25 \cdot s_i + 0.40 \cdot \sigma_i \sqrt{x_i} & \text{if } x_i \leq 6.66\% \\ 1.25 \cdot s_i + 1.55 \cdot \sigma_i x_i & \text{if } 6.66\% \leq x_i \leq 10\% \\ +\infty & \text{if } x_i > 10\% \end{cases}$$

We use the following parameters:

- $s = 4$ bps
- The 2Y weekly stress scenario is:
  - $\Delta_s = 8$ bps
  - $\Delta_\sigma = 20\%$
  - $m_v = 0.75$
- **N** = **normal** transaction cost
- **S** = **stressed** transaction cost

The example of large cap stocks

The normal transaction cost function is:

$$\boldsymbol{c}_i = \begin{cases} 1.25 \cdot s_i + 0.40 \cdot \sigma_i \sqrt{x_i} & \text{if } x_i \leq 6.66\% \\ 1.25 \cdot s_i + 1.55 \cdot \sigma_i x_i & \text{if } 6.66\% \leq x_i \leq 10\% \\ +\infty & \text{if } x_i > 10\% \end{cases}$$

The stressed transaction cost function becomes:

$$\boldsymbol{c}_i = \begin{cases} 1.25 \cdot (s_i + 8 \text{ bps}) + 0.40 \cdot \left(\sigma_i + \dfrac{20\%}{\sqrt{260}}\right) \sqrt{x_i} & \text{if } x_i \leq 6.66\% \\ 1.25 \cdot (s_i + 8 \text{ bps}) + 1.55 \cdot \left(\sigma_i + \dfrac{20\%}{\sqrt{260}}\right) x_i & \text{if } 6.66\% \leq x_i \leq 10\% \\ +\infty & \text{if } x_i > 10\% \end{cases}$$

where:

$$x_i = \frac{q_i}{0.75 \cdot v_i}$$

The example of large cap stocks

Table: Stress testing computation

x	Case	Annualized volatility 10%	15%	20%	25%	30%	35%	40%	Liquidation LT	LS one-day	LS two-day
		$\boldsymbol{c}(q; s, \sigma, v)$ (in bps)									
0.00%	N	5.0	5.0	5.0	5.0	5.0	5.0	5.0	1	0%	0%
	S	15.0	15.0	15.0	15.0	15.0	15.0	15.0	1	0%	0%
0.01%	N	5.2	5.4	5.5	5.6	5.7	5.9	6.0	1	0%	0%
	S	15.9	16.0	16.1	16.3	16.4	16.6	16.7	1	0%	0%
0.10%	N	5.8	6.2	6.6	7.0	7.4	7.7	8.1	1	0%	0%
	S	17.7	18.2	18.6	19.1	19.5	20.0	20.4	1	0%	0%
0.50%	N	6.8	7.6	8.5	9.4	10.3	11.1	12.0	1	0%	0%
	S	21.1	22.1	23.1	24.1	25.1	26.1	27.2	1	0%	0%
1.00%	N	7.5	8.7	10.0	11.2	12.4	13.7	14.9	1	0%	0%
	S	23.6	25.0	26.5	27.9	29.3	30.8	32.2	1	0%	0%
5.00%	N	10.5	13.3	16.1	18.9	21.6	24.4	27.2	1	0%	0%
	S	34.2	37.4	40.6	43.8	47.0	50.2	53.4	1	0%	0%
7.50%	N	12.2	15.8	19.4	23.0	26.6	30.2	33.8	1	0%	0%
	S	43.8	48.6	53.4	58.2	63.0	67.8	72.6	1	0%	0%
10.00%	N	14.6	19.4	24.2	29.0	33.8	38.6	43.4	1	0%	0%
	S	40.0	44.2	48.4	52.5	56.7	60.9	65.0	2	2.5%	0%
20.00%	N	14.6	19.4	24.2	29.0	33.8	38.6	43.4	2	10%	0%
	S	41.4	45.8	50.2	54.6	59.0	63.4	67.8	3	12.5%	5%

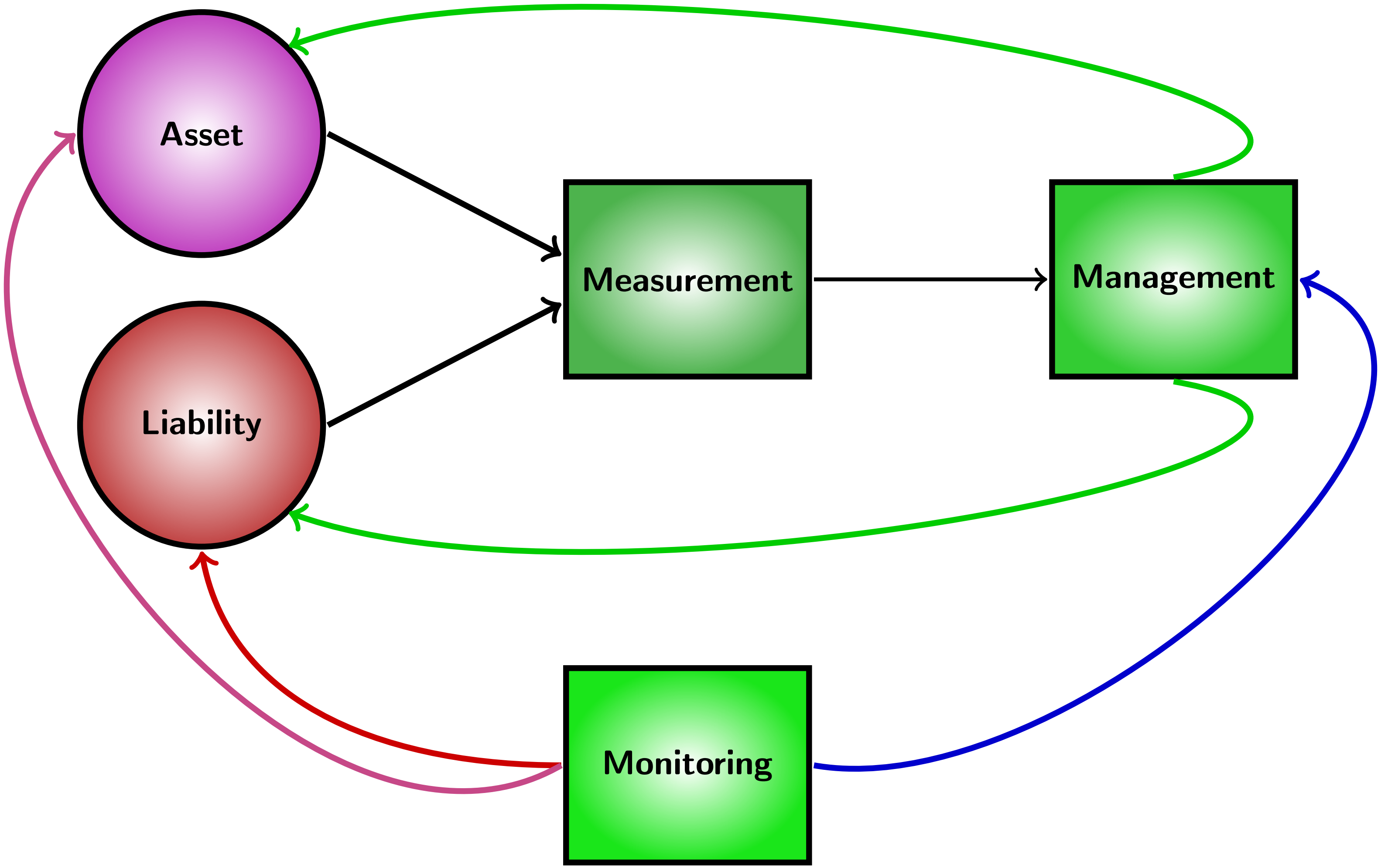
Framework

The three Ms of the Basel III Accord:

1. Measurement
2. Management
3. Monitoring

Framework

Figure: The Three Ms and the ALM of liquidity risk



Redemption coverage ratio

Definition

The redemption coverage ratio (RCR) is "a measurement of the ability of a fund's assets to meet funding obligations arising from the liabilities side of the balance sheet, such as a redemption shock" (ESMA, 2020).

The redemption coverage ratio was introduced by Bouveret (2017), who defines it as follows:

$$\mathrm{RCR} = \frac{\text{Liquid assets}}{\text{Net outflows}}$$

where net outflows and liquid assets correspond respectively to redemption shocks and the amount of the portfolio that can be liquidated over a given time horizon

$\Rightarrow$ Two possible cases: $\mathrm{RCR} \geq \mathbf{1}$ and $\mathrm{RCR} < \mathbf{1}$

Redemption coverage ratio

Liquidity shortfall

The liquidity shortfall is the amount of additional assets to be sold in order to the fulfil the redemption request:

$$\mathrm{LS} = \frac{\max\left(0, \text{Net outflows} - \text{Liquid assets}\right)}{\mathrm{TNA}}$$

ALM vocabulary

- Funding ratio, liquidity ratio, liquidity coverage ratio, redemption coverage ratio
- Funding gap, liquidity gap, liquidity shortfall

Redemption coverage ratio

Time to liquidation approach

We have:

$$\mathrm{RCR}(h) = \frac{\mathcal{A}(h)}{\mathcal{R}} = \frac{\mathbb{A}(h)}{\mathbb{R}}$$

where $\mathcal{A}$ is the ratio of liquid assets in the fund, $\mathcal{R}$ is the redemption shock expressed in % and $\mathbb{A}(h)$ and $\mathbb{R}$ correspond to the dollar value of $\mathcal{A}(h)$ and $\mathcal{R}$

The expression of the liquidity shortfall is:

$$\mathrm{LS}(h) = \frac{\max(0, \mathbb{R} - \mathbb{A}(h))}{\mathrm{TNA}} = \max(0, \mathcal{R} - \mathcal{A}(h))$$

Redemption coverage ratio

Time to liquidation approach

- The total net assets are equal to $\mathrm{TNA} = \sum_{i=1}^{n} \omega_i \cdot P_i$ where $\omega = (\omega_1, \ldots, \omega_n)$ is the asset portfolio of the fund
- The redemption shock expressed in dollars is $\mathbb{R} = \mathcal{R} \cdot \mathrm{TNA}$
- $q = (q_1, \ldots, q_n)$ is the redemption portfolio
- $q_i(h)$ is the number of shares liquidated during the h^{th} trading day:

$$q_i(h) = \min\left(\left(q_i - \sum_{k=0}^{h-1} q_i(k)\right)^{+}, q_i^{+}\right)$$

  where $q_i(0) = 0$ and q_i^{+} is the maximum number of shares that can be sold during a trading day for the asset i
- The amount of liquid assets is equal to the amount of assets that can be sold:

$$\mathbb{A}(h) = \sum_{i=1}^{n} \sum_{k=1}^{h} q_i(k) \cdot P_i$$

- The ratio of liquid assets is defined as $\mathcal{A}(h) = \dfrac{\mathbb{A}(h)}{\mathrm{TNA}}$

Redemption coverage ratio

Time to liquidation approach

Relationship with the liquidation ratio

- The redemption coverage ratio is equal to:

$$\mathrm{RCR}(h) = \frac{\mathbb{V}(q)}{\mathbb{R}} \cdot \mathbf{LR}(q; h)$$

where $\mathbb{V}(q) = \sum_{i=1}^{n} q_i \cdot P_i$ is the value function of the portfolio q

- The liquidity shortfall $\mathbf{LS}(h)$ is equal to

$$\mathbf{LS}(h) = \mathcal{R} \cdot \max\left(0, 1 - \frac{\mathbb{V}(q)}{\mathbb{R}} \cdot \mathbf{LR}(q; h)\right)$$

Redemption coverage ratio

Liquidation methods

1. Naive pro-rata liquidation (vertical slicing)

$$q = \mathcal{R} \cdot \omega$$

We have:

$$\mathrm{RCR}(h) = \mathbf{LR}(q; h) \leq 1$$

2. Optimal pro-rata liquidation (proportional slicing)

$$q = \varphi(h) \cdot \omega$$

where $\varphi(h) = \inf_{i=1,\ldots,n} \min\left(h \cdot \frac{q_i^+}{\omega_i}, 1\right) \geq \mathcal{R}$ and $\varphi(\infty) \geq \mathcal{R}$. We have:

$$\mathrm{RCR}(h) = \frac{\varphi(h)}{\mathcal{R}}$$

3. Waterfall liquidation (horizontal slicing)

$$q = \omega$$

Redemption coverage ratio

Example #1

Table: Large cap equity portfolio (Eurostoxx 50 index, $\mathrm{TNA} = €1$ bn)

i	ω_i	P_i	P_i^{bid}	P_i^{ask}	$\sigma_i^{\mathrm{yearly}}$	v_i
1	59 106	284.050	281.750	281.800	25.69	514 842
2	8 883	2630.500	2567.500	2568.500	31.14	56 255
3	150 027	143.600	142.880	142.920	13.75	629 509
4	184 310	112.000	111.640	111.660	26.42	1 316 600
5	130 520	200.950	200.150	200.200	21.75	750 684
6	268 123	54.460	53.280	53.290	27.08	1 736 372
7	131 520	698.400	690.100	690.300	31.82	754 901
8	651 421	24.415	24.760	24.765	18.72	4 358 304
9	3 192 430	5.641	5.988	5.990	33.74	54 130 721
10	5 544 072	3.2805	3.2465	3.2480	29.85	72 371 040
⋮						
50	163 680	54.060	52.680	52.700	19.20	976 446

The portfolio holding ω_i and the daily volume v_i are measured in number of shares, the yearly volatility $\sigma_i^{\mathrm{yearly}}$ is expressed in %, whereas the prices (P_i, P_i^{bid} and P_i^{ask}) are in euros.

Redemption coverage ratio

Example #1

We have:

$$q_i^+ = x_i^+ \cdot v_i$$

where $x_i^+ = 10\%$

Table: Redemption coverage ratio (large cap equity portfolio, $\mathrm{TNA} =$ €1 bn)

Redemption rate $\mathcal{R}$		5%	10%	25%	50%	75%	90%
Vertical slicing	$\mathrm{RCR}\,(1)$	1.00	1.00	1.00	0.96	0.81	0.72
	$\mathrm{RCR}\,(2)$	1.00	1.00	1.00	1.00	1.00	0.98
	$\mathrm{RCR}\,(3)$	1.00	1.00	1.00	1.00	1.00	1.00
Waterfall liquidation	$\mathrm{RCR}\,(1)$	13.38	6.69	2.68	1.34	0.89	0.74
	$\mathrm{RCR}\,(2)$	19.29	9.64	3.86	1.93	1.29	1.07
	$\mathrm{RCR}\,(3)$	20.00	10.00	4.00	2.00	1.33	1.11

Redemption coverage ratio

Example #1

Table: Redemption coverage ratio in a normal period (large cap equity portfolio, $\mathcal{R} = 5\%$, waterfall liquidation)

TNA	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	13.38	3.02	1.51	0.75
$h = 2$	19.29	6.04	3.02	1.51
$h = 5$	20.00	13.38	7.49	3.77

Asset-liability stress scenario

- Liability stress scenario: $\mathcal{R} \nearrow$
- Asset stress scenario: $q_i^+ \searrow$

$$q_i^+ = m_v \cdot x_i^+ \cdot v_i$$

where $m_v < 1$

Redemption coverage ratio

Example #1

Table: Stress testing of the redemption coverage ratio (large cap equity portfolio, $\mathcal{R}^{\text{stress}} = 20\%$, waterfall liquidation)

	$m_v = 1.00$				$m_v = 0.75$			
TNA	€1 bn	€5 bn	€10 bn	€20 bn	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	3.35	0.75	0.38	0.19	2.67	0.57	0.28	0.14
$h = 2$	4.82	1.51	0.75	0.38	4.33	1.13	0.57	0.28
$h = 5$	5.00	3.35	1.87	0.94	5.00	2.67	1.41	0.71
	$m_v = 0.50$				$m_v = 0.10$			
TNA	€1 bn	€5 bn	€10 bn	€20 bn	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	1.87	0.38	0.19	0.09	0.38	0.08	0.04	0.02
$h = 2$	3.35	0.75	0.38	0.19	0.75	0.15	0.08	0.04
$h = 5$	4.97	1.87	0.94	0.47	1.87	0.38	0.19	0.09

Redemption coverage ratio

Example #2

Table: Small cap equity portfolio (Eurostoxx index), $\mathrm{TNA} = €1$ bn)

i	ω_i	P_i	P_i^{bid}	P_i^{ask}	$\sigma_i^{\mathrm{yearly}}$	v_i
1	505 766	98.860	98.840	98.860	17.68	432 593
2	645 495	77.460	77.460	77.480	34.36	725 019
3	1 830 496	27.315	27.310	27.320	41.55	5 513 446
4	436 110	114.650	114.650	114.750	16.88	221 468
5	592 839	84.340	84.300	84.320	27.48	535 423
6	447 628	111.700	111.650	111.750	21.57	291 008
7	309 311	161.650	161.600	161.700	20.60	92 108
8	473 261	105.650	105.600	105.700	17.23	134 879
9	531 915	94.000	93.950	94.050	21.40	201 095
10	597 944	83.620	83.600	83.660	26.02	298 374
$\vdots$						
20	537 923	92.950	92.850	93.000	24.35	41 414

The portfolio holding ω_i and the daily volume v_i are measured in number of shares, the yearly volatility $\sigma_i^{\mathrm{yearly}}$ is expressed in %, whereas the prices (P_i, P_i^{bid} and P_i^{ask}) are in euros.

Redemption coverage ratio

Example #2

Table: Redemption coverage ratio in a normal period (small cap equity portfolio, $\mathcal{R} = 5\%$, waterfall liquidation)

TNA	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	1.28	0.64	0.43	0.32
$h = 2$	2.56	1.28	0.85	0.64
$h = 5$	5.89	3.20	2.13	1.60

Redemption coverage ratio

Example #2

Table: Stress testing of the redemption coverage ratio (small cap equity portfolio, $\mathcal{R}^{\text{stress}} = 20\%$, waterfall liquidation)

	$m_v = 1.00$				$m_v = 0.75$			
TNA	€1 bn	€2 bn	€3 bn	€4 bn	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	0.32	0.06	0.03	0.02	0.24	0.05	0.02	0.01
$h = 2$	0.64	0.13	0.06	0.03	0.48	0.10	0.05	0.02
$h = 5$	1.47	0.32	0.16	0.08	1.17	0.24	0.12	0.06
	$m_v = 0.50$				$m_v = 0.10$			
TNA	€1 bn	€2 bn	€3 bn	€4 bn	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	0.16	0.03	0.02	0.01	0.03	0.01	0.00	0.00
$h = 2$	0.32	0.06	0.03	0.02	0.06	0.01	0.01	0.00
$h = 5$	0.80	0.16	0.08	0.04	0.16	0.03	0.02	0.01

Redemption coverage ratio

HQLA approach

The redemption coverage ratio is defined as:

$$\mathrm{RCR}(h) = \frac{\sum_{k=1}^{m} w_k \cdot \mathrm{CCF}_k(h)}{\mathcal{R}}$$

where w_k is the weight of the k^{th} HQLA class and CCF_k is the cash conversion factor (CCF) of the k^{th} HQLA class

Two methods

1. Basel III framework
2. Risk sensitive framework

Redemption coverage ratio

HQLA approach (risk sensitive framework)

We have:

$$\mathrm{CCF}_{k,j}(h) = \mathrm{LF}_k(h) \cdot \left(1 - \mathrm{DF}_k\left(\frac{h}{2}\right)\right) \cdot \left(1 - \mathrm{SF}_k\left(\mathrm{TNA}_j, \mathscr{H}_j\right)\right)$$

where $\mathrm{LF}_k(h) \in [0,1]$ is the liquidity factor, $\mathrm{DF}_k(\tau_h) \in [0, \mathrm{MDD}_k]$ is the drawdown factor and $\mathrm{SF}_k \in [0,1]$ is the specific risk factor of the fund j:

$$\begin{cases} \mathrm{LF}_k(h) = \min\left(1.0, \lambda_k \cdot h\right) \\ \mathrm{DF}_k(h) = \min\left(\mathrm{MDD}_k, \eta_k \cdot \sqrt{h}\right) \\ \mathrm{SF}_k\left(\mathrm{TNA}_j, \mathscr{H}_j\right) = \min\left(\xi_k^{\mathrm{size}}\left(\frac{\mathrm{TNA}_j}{\mathrm{TNA}^\star} - 1\right)^+ + \xi_k^{\mathrm{concentration}}\left(\sqrt{\frac{\mathscr{H}_j}{\mathscr{H}^\star}} - 1\right)^+, \mathrm{SF}^+\right) \end{cases}$$

where λ_k is the selling intensity, MDD_k is the maximum drawdown, η_k is the loss intensity, TNA_j is the total net assets and $\mathscr{H}_j$ is the Herfindahl index of the fund

Redemption coverage ratio

Example

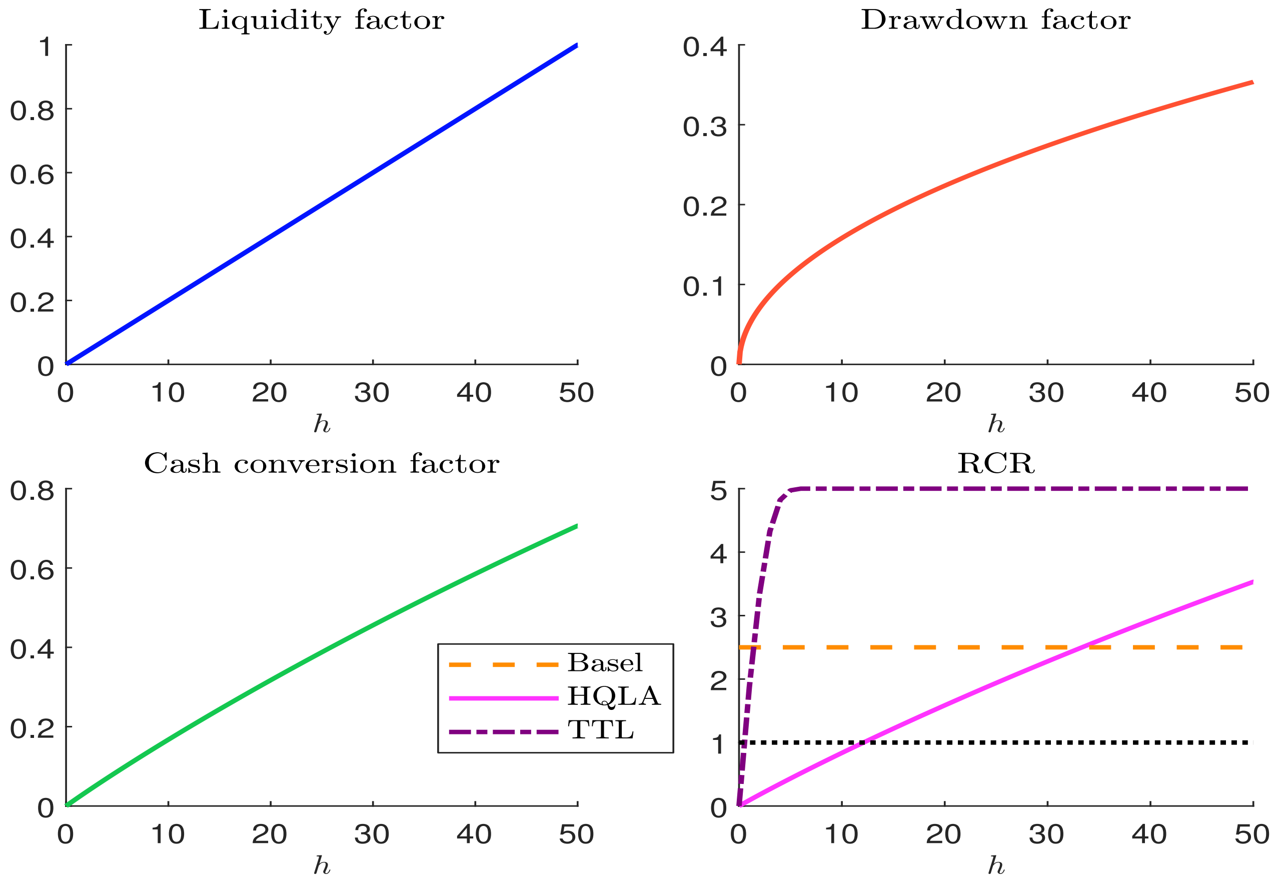
Parameters for equity portfolios:

- In the Basel framework, the value of $\mathrm{CCF}_k(h)$ is set to 50%
- Concerning the risk sensitive framework, we assume that $\lambda_k = 2\%$, $\eta_k = 5\%$, $\mathrm{MDD}_k = 50\%$, $\xi_k^{\mathrm{size}} = 10\%$, $\xi_k^{\mathrm{concentration}} = 25\%$, $\mathrm{TNA}^\star = 1$ bn, $\mathscr{H}^\star = 2\%$ and $\mathrm{SF}^+ = 0.80$

Redemption coverage ratio

Example

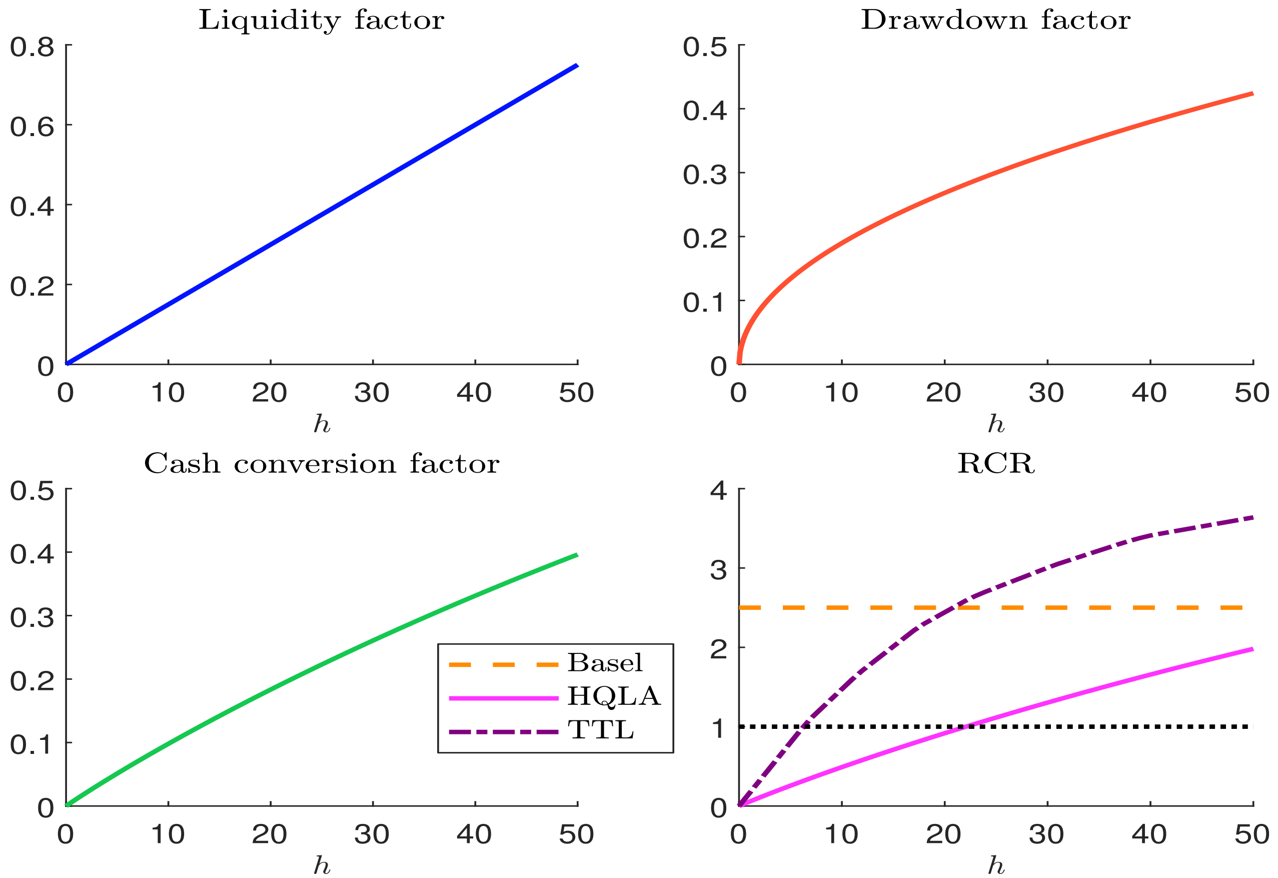
Figure: Redemption coverage ratio of the large cap portfolio ($\mathrm{TNA} =$ €1 bn, $\mathcal{R}^{\mathrm{stress}} = 20\%$ and $m_v = 50\%$)



Redemption coverage ratio

Example

Figure: Redemption coverage ratio of the small cap portfolio ($\mathrm{TNA} = €1$ bn, $\mathcal{R}^{\mathrm{stress}} = 20\%$ and $m_v = 50\%$)



Reverse stress testing

Definition

Reverse stress testing consists in finding the liquidity scenario such that $\mathrm{RCR}(h) = \mathrm{RCR}^-$ where RCR^- is the minimum acceptable level of the redemption coverage ratio

A standard value of RCR^- is 50%.

Reverse stress testing

Liability RST scenario

We have:

$$\mathrm{RCR}(h) \leq \mathrm{RCR}^{-} \Leftrightarrow \mathcal{R} \geq \mathcal{R}^{\mathrm{RST}} := \left\{ \mathcal{R} \in [0,1] : \mathrm{RCR}(h) = \mathrm{RCR}^{-} \right\}$$

Table: Liability reverse stress testing scenario $\mathcal{R}^{\mathrm{RST}}$ in % ($\mathrm{TNA} = €1$ bn, pro-rata liquidation, $\mathrm{RCR}^{-} = 50\%$)

	Large cap portfolio				Small cap portfolio			
m_v	1.00	0.75	0.50	0.10	1.00	0.75	0.50	0.10
$h = 1$	144.1	108.1	72.1	14.5	9.7	7.3	4.9	1.0
$h = 2$	288.2	216.2	144.1	28.9	19.3	14.5	9.7	2.0
$h = 3$	432.3	324.2	216.2	43.3	28.9	21.7	14.5	2.9
$h = 4$	576.3	432.3	288.2	57.7	38.5	28.9	19.3	3.9
$h = 5$	720.4	540.3	360.2	72.1	48.1	36.1	24.1	4.9

Reverse stress testing

Asset RST scenario

We have:

$$\mathrm{RCR}(h) \leq \mathrm{RCR}^{-} \Leftrightarrow m_v \leq m_v^{\mathrm{RST}} := \left\{ m_v \in [0,1] : \mathrm{RCR}(h) = \mathrm{RCR}^{-} \right\}$$

Table: Asset reverse stress testing scenario m_v^{RST} ($\mathrm{TNA} =$ €1 bn, pro-rata liquidation, $\mathrm{RCR}^{-} = 50\%$)

	Large cap portfolio				Small cap portfolio			
$\mathcal{R}$	5%	10%	20%	50%	5%	10%	20%	50%
$h = 1$	0.04	0.07	0.14	0.35	0.52	1.04	2.08	5.20
$h = 2$	0.02	0.04	0.07	0.17	0.26	0.52	1.04	2.60
$h = 3$	0.01	0.02	0.05	0.12	0.17	0.35	0.69	1.73
$h = 4$	0.01	0.02	0.04	0.09	0.13	0.26	0.52	1.30
$h = 5$	0.01	0.01	0.03	0.07	0.11	0.21	0.42	1.04

Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

- The participation rate $x_i(h)$ at the trading day h is equal to:

$$x_i(h) = \frac{q_i(h)}{v_i}$$

where v_i is the daily volume

- The unit transaction cost function is equal to:

$$\boldsymbol{c}_i(x_i(h)) = \begin{cases} 1.25\, s_i + 0.40\, \sigma_i \sqrt{x_i(h)} & \text{if } x_i(h) \leq \tilde{x}_i \\ 1.25\, s_i + \dfrac{0.40}{\sqrt{\tilde{x}_i}} \sigma_i x_i & \text{if } \tilde{x}_i \leq x_i(h) \leq x_i^+ \\ +\infty & \text{if } x_i(h) > x_i^+ \end{cases}$$

where s_i is the bid-ask spread, σ_i is the daily volatility, $x_i^+ = 10\%$ and $\tilde{x}_i = \frac{2}{3} x_i^+$

Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

- The transaction cost of the trade associated to $q_i(h)$ is equal to:

$$\mathbf{TC}_i(q_i(h)) = Q_i(h) \cdot \boldsymbol{c}_i(x_i(h))$$

where $Q_i(h) = q_i(h) \cdot P_i$ is the nominal value $q_i(h)$

- The total transaction cost of the redemption portfolio is then equal to:

$$\mathbf{TC}(q) = \sum_{h=1}^{h^+} \sum_{i=1}^{n} \mathbf{TC}_i(q_i(h))$$

Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

- The redemption unit cost is equal to:

$$\boldsymbol{c}(q) = \frac{\mathbf{TC}(q)}{\sum_{i=1}^{n} q_i \cdot P_i}$$

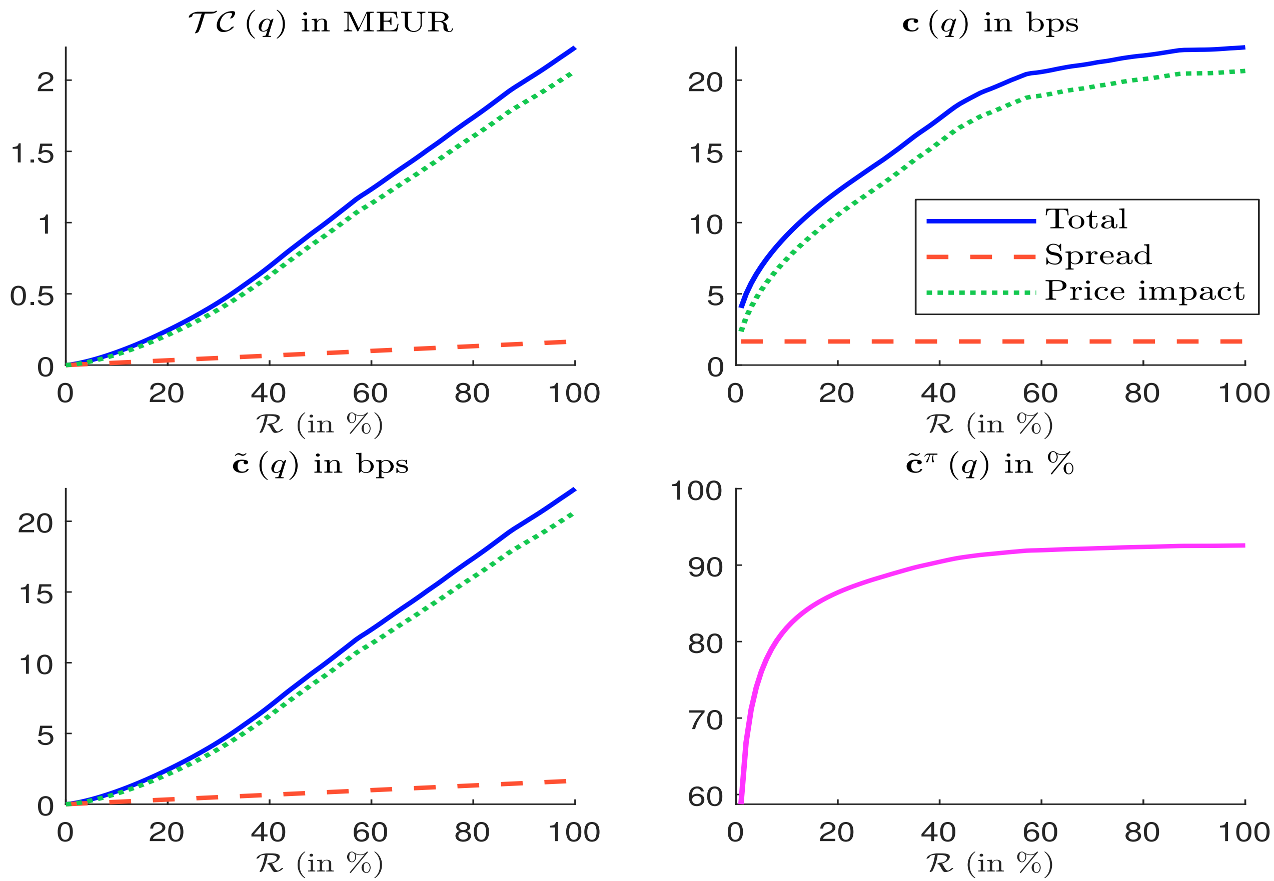
- The investor unit cost is equal to:

$$\tilde{\boldsymbol{c}}(q) = \frac{\mathbf{TC}(q)}{\mathrm{TNA}} = \frac{\mathbf{TC}(q)}{\sum_{i=1}^{n} \omega_i \cdot P_i} = \mathcal{R} \cdot \boldsymbol{c}(q)$$

Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

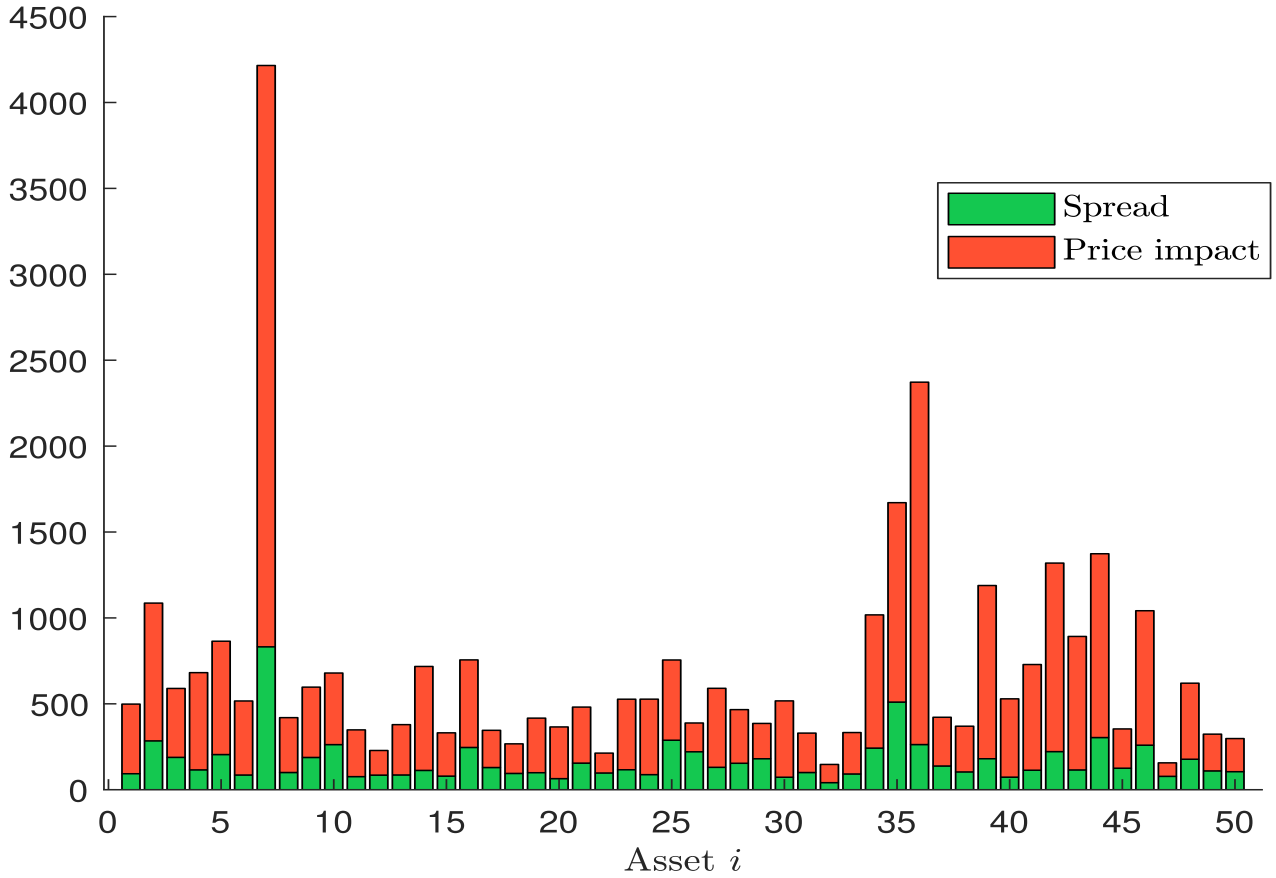
Figure: Transaction cost of the large cap portfolio (TNA = €1 bn)



Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

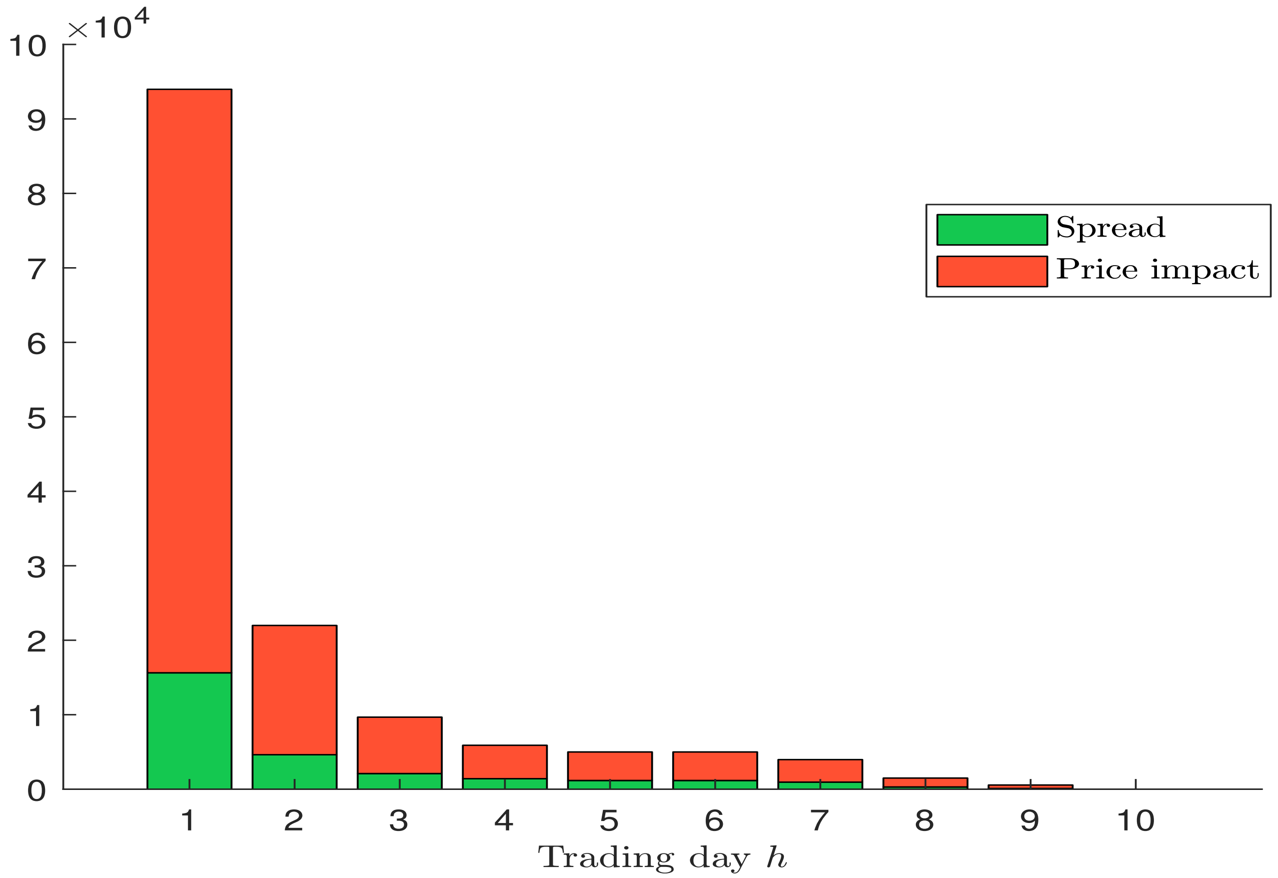
Figure: Breakdown per asset in € (large cap portfolio, TNA = €1 bn, $\mathcal{R} = 5\%$)



Transaction cost analysis

Analytics of the transaction cost function (equity portfolios)

Figure: Breakdown per trading day in € (small cap portfolio, $\mathrm{TNA} =$ €1 bn, $\mathcal{R} = 5\%$)



Transaction cost analysis

Stress testing (equity portfolios)

- The asset stress scenario is defined by the triplet $(\Delta_s, \Delta_\sigma, m_v)$
- The liquidation policy becomes:

$$q_i^+ = x_i^+ \cdot (m_v \cdot v_i)$$

- The transaction cost function remains the same:

$$\boldsymbol{c}_i(x_i(h)) = \begin{cases} 1.25\,(s_i + \Delta_s) + 0.40\left(\sigma_i + \dfrac{\Delta_\sigma}{\sqrt{260}}\right)\sqrt{x_i} & \text{if } x_i(h) \leq \tilde{x}_i \\ 1.25\,(s_i + \Delta_s) + \dfrac{0.40}{\sqrt{\tilde{x}_i}}\left(\sigma_i + \dfrac{\Delta_\sigma}{\sqrt{260}}\right)x_i & \text{if } \tilde{x}_i \leq x_i(h) \leq x_i^+ \\ +\infty & \text{if } x_i(h) > x_i^+ \end{cases}$$

where:

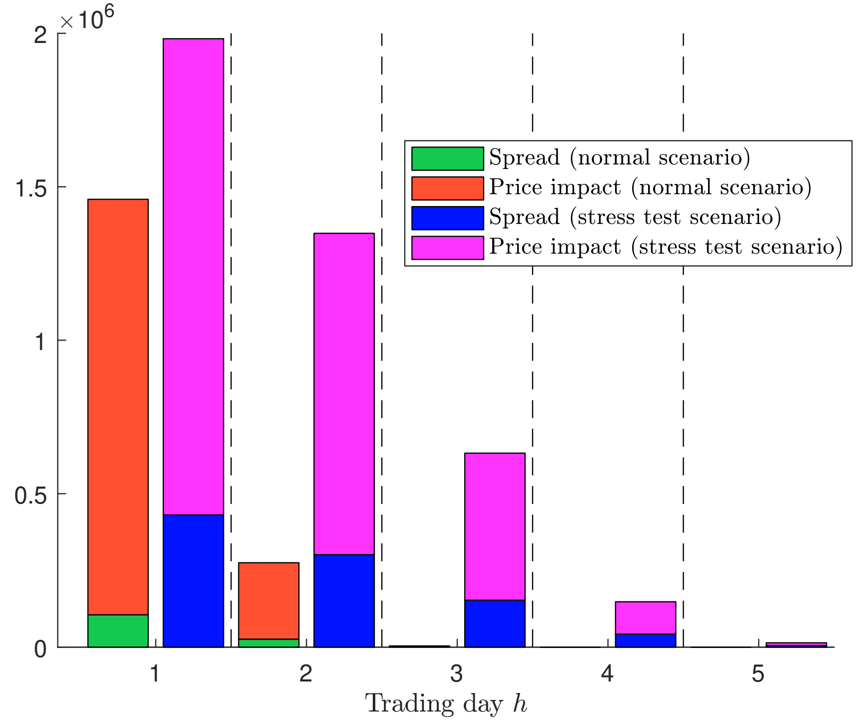
$$x_i(h) = \frac{q_i(h)}{m_v \cdot v_i}$$

Transaction cost analysis

Stress testing (equity portfolios)

- We use the following stress scenario: $\Delta_s = 8$ bps, $\Delta_\sigma = 20\%$ and $m_v = 0.50$
- We obtain $\mathbf{TC}^{\text{stress}}(q) = 4\,124\,811$ euros, $\mathbf{c}^{\text{stress}}(q) = 51.56$ bps and $\tilde{\mathbf{c}}^{\text{stress}}(q) = 41.25$ bps
- Previously, we had $\mathbf{TC}^{\text{normal}}(q) = 1\,738\,156$ euros, $\mathbf{c}^{\text{normal}}(q) = 21.73$ bps and $\tilde{\mathbf{c}}^{\text{normal}}(q) = 17.38$ bps

Figure: Breakdown per trading day in € (large cap portfolio, $\mathrm{TNA} = €1$ bn, $\mathcal{R} = 80\%$)



The case of bond portfolios

The case of bond portfolios is similar to the case of equity portfolios, but there are three main differences

The case of bond portfolios

Differences with equity portfolios

Computing the liquidation portfolio

- We consider nominal values $Q_i = q_i \cdot P_i$ instead of real values q_i
- The redemption portfolio is then defined by $Q = (Q_1, \ldots, Q_n)$
- The liquidation process becomes:

$$Q_i(h) = \min\left(\left(Q_i - \sum_{k=0}^{h-1} Q_i(k)\right)^+, Q_i^+\right)$$

where $Q_i(0) = 0$

- Q_i^+ is the maximum trading limit that can be sold during a trading day for the bond i

In real life, the liquidation process is more complex since it deals with minimum tradable amounts and lot sizes. Nevertheless, the previous approach remains valid for performing LST

The case of bond portfolios

Differences with equity portfolios

Asset-liability stress scenario for computing the RCR

- Liability stress scenario: $\mathcal{R} \nearrow$
- Asset stress scenario: $Q_i^+ \searrow$

$$Q_i^+ \leftarrow m_{Q^+} \cdot Q_i^+$$

where $m_{Q^+} < 1$

The case of bond portfolios

Differences with equity portfolios

Computing the transaction cost

- The participation rate is defined with respect to the outstanding amount $\mathcal{M}_i$ and not the daily volume v_i:

$$y_i(h) = \frac{Q_i(h)}{\mathcal{M}_i}$$

- For sovereign bonds, the unit transaction cost function is equal to:

$$\boldsymbol{c}_i(y_i(h)) = \begin{cases} 1.25\, s_i + 3.00\, \sigma_i y_i(h)^{0.25} & \text{if } y_i(h) \leq \tilde{y}_i \\ 1.25\, s_i + \dfrac{3.00}{\tilde{y}_i^{0.75}} \sigma_i y_i(h) & \text{if } \tilde{y}_i \leq y_i(h) \leq y_i^+ \end{cases}$$

where $\tilde{y}_i = \frac{2}{3} y_i^+$ and $y_i^+ = \frac{Q_i^+}{\mathcal{M}_i}$

The case of bond portfolios

Differences with equity portfolios

Computing the transaction cost

- For corporate bonds, the formula becomes:

$$\boldsymbol{c}_i\left(y_i\left(h\right)\right)=\left\{\begin{array}{ll}1.50\,s_i+0.125\,\mathrm{DTS}_i\,y_i\left(h\right)^{0.25} & \text{if } y_i\left(h\right)\leq\tilde{y}_i\\ 1.50\,s_i+\dfrac{0.125}{\tilde{y}_i^{0.75}}\,\mathrm{DTS}_i\,y_i\left(h\right) & \text{if } \tilde{y}_i\leq y_i\left(h\right)\leq y_i^{+}\end{array}\right.$$

where DTS_i is the duration-times-spread measure of the bond i

The case of bond portfolios

Example #3

Table: Bond portfolio (USD-denominated, $\mathrm{TNA} = \$1$ bn)

i	Isin	ω_i	P_i	s_i	σ_i^{yearly}	DTS_i	$\mathcal{M}_i$	Q_i^+
1	US912828TY62	529 725	102.288	1.33	0.16		121 993	50
2	US91282CCC38	427 182	99.243	1.56	0.99		88 769	50
3	US91282CCT62	403 263	99.334	1.21	1.46		83 876	50
⋮								
10	US912810SX72	412 836	110.482	2.86	15.15		95 481	50
11	US912810SZ21	880 939	101.289	3.10	17.88		91 407	50
12	US037833CG39	120 000	105.636	3.43	0.92	43	1 750	6
⋮								
25	US06051GGC78	170 000	112.456	10.85	2.74	392	2 000	3
⋮								
45	US716743AR02	87 000	125.184	57.14	12.43	2336	2 750	6
46	US225433AF86	58 600	129.155	29.29	10.24	1962	1 925	6
47	US87938WAU71	77 700	126.664	27.90	10.78	2635	2 500	6

The portfolio holding ω_i is measured in number of shares, the price P_i is in US dollars, the yearly volatility σ_i^{yearly} is expressed in %, the (half) bid-ask spread s_i and the duration-times-spread DTS_i are in bps, whereas the outstanding amount $\mathcal{M}_i$ and the daily trading limit Q_i^+ are expressed in \$ mn.

The case of bond portfolios

Example #3

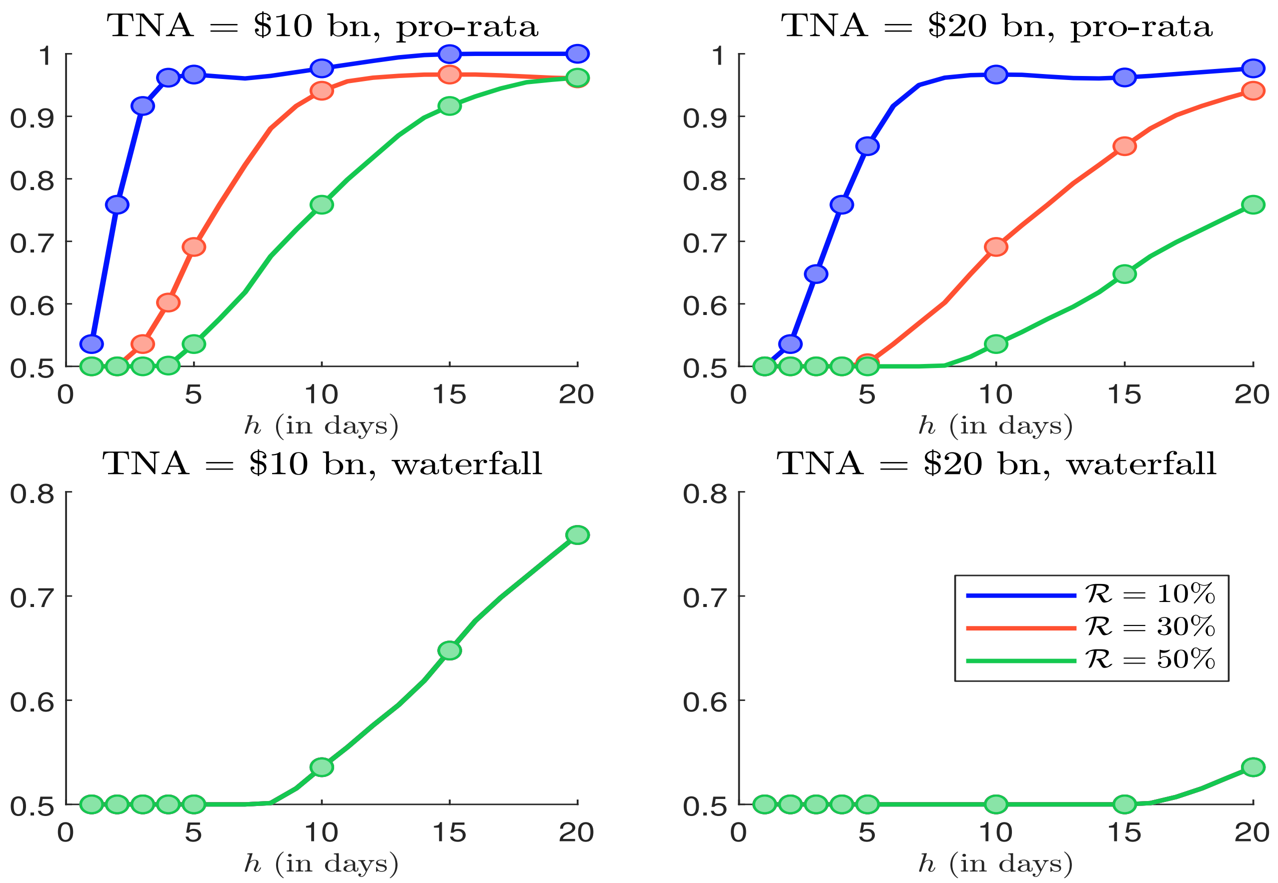
Table: Redemption coverage ratio RCR (h) (bond portfolio, $\mathcal{R} = 30\%$)

		Normal period				Stress period			
		Pro-rata		Waterfall		Pro-rata		Waterfall	
	TNA	10	20	10	20	10	20	10	20
h	1	0.251	0.126	0.251	0.126	0.126	0.063	0.126	0.063
	2	0.503	0.251	0.503	0.251	0.251	0.126	0.251	0.126
	3	0.704	0.377	0.754	0.377	0.377	0.188	0.377	0.188
	4	0.835	0.503	1.005	0.503	0.503	0.251	0.503	0.251
	5	0.900	0.622	1.257	0.628	0.622	0.314	0.628	0.314
	6	0.928	0.704	1.508	0.754	0.704	0.377	0.754	0.377
	7	0.940	0.773	1.759	0.880	0.773	0.440	0.880	0.440
	8	0.948	0.835	2.006	1.005	0.835	0.503	1.005	0.503
	9	0.953	0.873	2.195	1.131	0.873	0.565	1.131	0.565
	10	0.957	0.900	2.346	1.257	0.900	0.622	1.257	0.628

The case of bond portfolios

Example #3

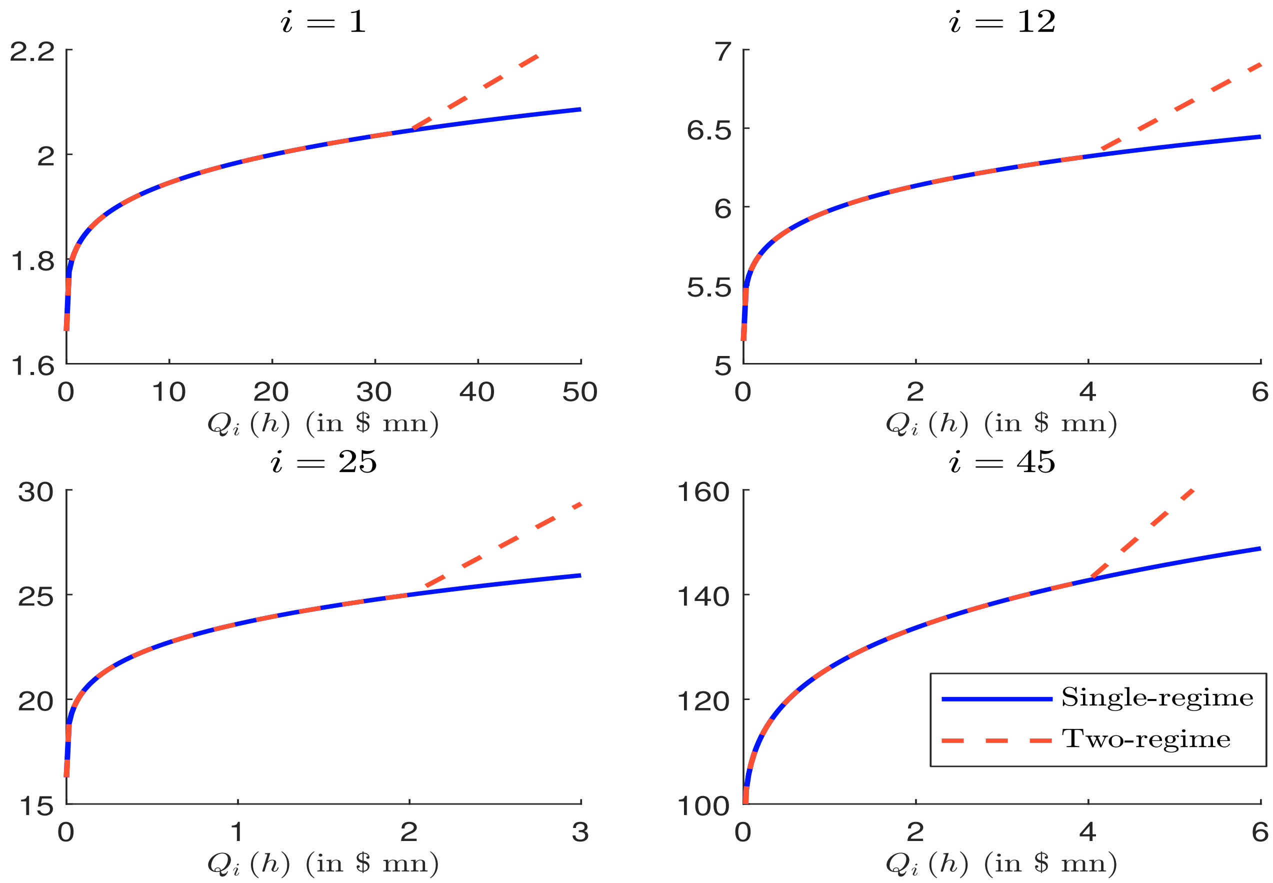
Figure: Multiplier stress factor $m_{\mathrm{RCR}}(h)$ ($m_{Q^+} = 50\%$)



The case of bond portfolios

Example #3

Figure: Unit transaction cost function



The case of bond portfolios

Example #3

- Stress scenario: $\mathcal{R} = 30\%$, $\Delta_s = 3$ bps, $\Delta_{\sigma^{\text{yearly}}} = 2\%$, $\Delta_{\text{DTS}} = 100$ bps and $m_{Q^+} = 0.50$
- **What is the issue?**

$$y_i^{\text{stress}}(h) = \frac{Q_i(h)}{\mathcal{M}_i} \quad \text{or} \quad y_i^{\text{stress}^*}(h) = \frac{Q_i(h)}{m_{Q^+} \cdot \mathcal{M}_i}?$$

Table: Spread and price impact components in bps (bond portfolio, $\text{TNA} = \$10$ bn, $\mathcal{R} = 30\%$)

Scenario	$c^s(q)$	$c^\pi(q)$	$c(q)$	$\tilde{c}^s(q)$	$\tilde{c}^\pi(q)$	$\tilde{c}(q)$
Normal	11.07	24.53	35.60	3.32	7.36	10.68
Stress	15.12	25.84	40.96	4.54	7.75	12.29
Stress*	15.12	30.73	45.85	4.54	9.22	13.75

Liquidity management tools

Table: LMTs available to European corporate debt funds (June 2020)

		AIF	UCITS
Short-term borrowing		78%	91%
Special arrangements	Gates	23%	73%
	Side pockets	10%	10%
	In-kind redemptions	34%	77%
Swing pricing		7%	57%
Anti-dilution levies		11%	17%

Source: ESMA (2020).

Liquidity buffer and cash holding

Definition

A liquidity buffer refers to the stock of cash instruments held by the fund manager in order to manage the future redemptions of investors

List of instruments:

- Cash (cash at hand, deposits)
- Cash equivalents (repo, money market funds, short-term debt securities)
- Liquid assets (government bonds, etc.)

Liquidity buffer and cash holding

Cost-benefit analysis

- We note w_{cash} as the cash-to-assets ratio:

$$w_{\text{cash}} = \frac{\text{cash}}{\text{TNA}}$$

- Let R be the random return of the fund. We have:

$$R = (1 - w_{\text{cash}}) R_{\text{asset}} + w_{\text{cash}} R_{\text{cash}}$$

- We assume that there is no more than one liquidity stress scenario per year

Liquidity buffer and cash holding

Cost-benefit analysis

Mean-variance analysis

- The expected return of the fund is equal to:

$$\mathbb{E}[R] = \mu_{\mathrm{asset}} - w_{\mathrm{cash}} \left(\mu_{\mathrm{asset}} - \mu_{\mathrm{cash}}\right)$$

- The volatility of the fund is equal to:

$$\begin{aligned}\sigma(R) &= \sqrt{w_{\mathrm{cash}}^2 \sigma_{\mathrm{cash}}^2 + w_{\mathrm{asset}}^2 \sigma_{\mathrm{asset}}^2 + 2 w_{\mathrm{cash}} w_{\mathrm{asset}} \rho_{\mathrm{cash,asset}} \sigma_{\mathrm{cash}} \sigma_{\mathrm{asset}}} \\ &\approx \left(1 - w_{\mathrm{cash}}\right) \sigma_{\mathrm{asset}}\end{aligned}$$

- We have:

$$\mathrm{SR}(R) \approx \mathrm{SR}\left(R_{\mathrm{asset}}\right)$$

and:

$$\beta\left(R \mid R_{\mathrm{asset}}\right) \approx 1 - w_{\mathrm{cash}} \leq 1$$

Liquidity buffer and cash holding

Cost-benefit analysis

Tracking error analysis

- The expected excess return is equal to:

$$\mathbb{E}\left[R \mid R_{\text{asset}}\right] = -w_{\text{cash}}\left(\mu_{\text{asset}} - \mu_{\text{cash}}\right)$$

- The tracking error volatility $\sigma\left(R \mid R_{\text{asset}}\right)$ is equal to:

$$\begin{aligned}
\sigma\left(R \mid R_{\text{asset}}\right) &= w_{\text{cash}}\sqrt{\sigma_{\text{asset}}^2 + \sigma_{\text{cash}}^2 - \rho_{\text{cash,asset}}\sigma_{\text{cash}}\sigma_{\text{asset}}} \\
&\approx w_{\text{cash}}\sigma_{\text{asset}}
\end{aligned}$$

- The information ratio is the opposite of the Sharpe ratio of the assets:

$$\mathrm{IR}\left(R \mid R_{\text{asset}}\right) \approx -\mathrm{SR}\left(R_{\text{asset}}\right)$$

Liquidity buffer and cash holding

Cost-benefit analysis

The liquidation gain is the difference of the transaction costs without and with the cash buffer:

$$\mathbf{LG}\left(w_{\mathrm{cash}}\right) = \mathbf{TC}_{\mathrm{without}} - \mathbf{TC}_{\mathrm{with}}$$

Let $\mathbf{TC}_{\mathrm{asset}}\left(\mathcal{R}\right)$ and $\mathbf{TC}_{\mathrm{cash}}\left(\mathcal{R}\right)$ be the asset and cash transaction cost functions. Implementing a cash buffer has two main effects on the liquidity gain:

- First, we sell cash instead of the assets if the redemption shock is lower than the cash buffer and we have:

$$\mathbf{TC}_{\mathrm{asset}}\left(\mathcal{R}\right) \gg \mathbf{TC}_{\mathrm{cash}}\left(\mathcal{R}\right)$$

- Second, we sell a lower proportion of risky assets if the redemption rate is greater than the cash-to-assets ratio and we have:

$$\mathbf{TC}_{\mathrm{asset}}\left(\mathcal{R}\right) \gg \mathbf{TC}_{\mathrm{asset}}\left(\mathcal{R} - w_{\mathrm{cash}}\right)$$

Liquidity buffer and cash holding

Cost-benefit analysis

We deduce that:

$$\begin{aligned} \mathbf{LG}(w_{\text{cash}}) &= \mathbf{TC}_{\text{asset}}(\mathcal{R}) - \mathbf{TC}_{\text{cash}}(\mathcal{R}) \cdot \mathbb{1}\{\mathcal{R} < w_{\text{cash}}\} - \\ & \quad \mathbf{TC}_{\text{asset}}((\mathcal{R} - w_{\text{cash}})) \cdot \mathbb{1}\{\mathcal{R} \geq w_{\text{cash}}\} \end{aligned}$$

and:

$$\begin{aligned} \mathbb{E}[\mathbf{LG}(w_{\text{cash}})] &= \int_0^{w_{\text{cash}}} (\mathbf{TC}_{\text{asset}}(\mathcal{R}) - \mathbf{TC}_{\text{cash}}(\mathcal{R}))\,\mathrm{d}\mathbf{F}(\mathcal{R}) + \\ & \quad \int_{w_{\text{cash}}}^1 (\mathbf{TC}_{\text{asset}}(\mathcal{R}) - \mathbf{TC}_{\text{asset}}(\mathcal{R} - w_{\text{cash}}))\,\mathrm{d}\mathbf{F}(\mathcal{R}) \end{aligned}$$

where $\mathbf{F}(x)$ is the distribution function of the redemption rate $\mathcal{R}$

Liquidity buffer and cash holding

Cost-benefit analysis

Expected liquidation gain

Under some assumptions, we obtain:

$$\mathbb{E}\left[\mathbf{LG}\left(w_{\text{cash}}\right)\right] \approx \int_0^{w_{\text{cash}}} \mathbf{TC}_{\text{asset}}\left(\mathcal{R}\right) \mathrm{d}\mathbf{F}\left(\mathcal{R}\right) + \mathbf{TC}_{\text{asset}}\left(w_{\text{cash}}\right) \cdot \left(1 - \mathbf{F}\left(w_{\text{cash}}\right)\right)$$

1. The first term corresponds to the expected transaction cost of liquidating the risky assets when the redemption rate is lower than the cash-to-assets ratio
2. The second term is the transaction cost of liquidating the asset amount equivalent to the cash buffer times the probability of observing a redemption shock greater than the cash buffer

Liquidity buffer and cash holding

Optimal cash buffer

- The objective of the fund manager is to minimize the expected cost of the buffer $\mathbf{BC}(w_{\text{cash}})$ and maximize its expected gain $\mathbf{BG}(w_{\text{cash}})$:

$$w_{\text{cash}}^{\star} = \arg\min_{w \in [0,1]} \underbrace{\mathbf{BC}(w_{\text{cash}}) - \mathbf{BG}(w_{\text{cash}})}_{\text{Net buffer cost } \mathbf{NBC}(w_{\text{cash}})}$$

- The minimum of $\mathbf{BC}(w_{\text{cash}})$ is reached at $w_{\text{cash}}^{\star} = 0$
- The maximum of $\mathbf{BG}(w_{\text{cash}})$ is obtained for $w_{\text{cash}}^{\star} = 1$

$\Rightarrow$ **Trade-off between these two functions**

Liquidity buffer and cash holding

Optimal cash buffer

- The net buffer cost $\mathbf{NBC}(w_{\text{cash}})$ is equal to:

$$
\begin{aligned}
\mathbf{NBC}(w_{\text{cash}}) &= -\mathbb{E}[R \mid R_{\text{asset}}] + \frac{\lambda}{2}\sigma^2(R \mid R_{\text{asset}}) - \mathbb{E}[\mathbf{LG}(w_{\text{cash}})] \\
&= w_{\text{cash}}(\mu_{\text{asset}} - \mu_{\text{cash}}) - \mathbb{E}[\mathbf{LG}(w_{\text{cash}})] + \\
&\quad \frac{\lambda}{2} w_{\text{cash}}^2 \left(\sigma_{\text{cash}}^2 + \sigma_{\text{asset}}^2 - 2\rho_{\text{cash,asset}}\sigma_{\text{cash}}\sigma_{\text{asset}}\right)
\end{aligned}
$$

  where $\lambda \geq 0$ represents the aversion parameter to the tracking error risk

- The solution to the optimization problem satisfies:

$$
\begin{aligned}
\underbrace{\mu_{\text{asset}} - \mu_{\text{cash}} - \frac{\partial\,\mathbb{E}[\mathbf{LG}(w_{\text{cash}})]}{\partial\, w_{\text{cash}}}}_{\text{negative/positive}} + & \\
\lambda \cdot \underbrace{w_{\text{cash}}\left(\sigma_{\text{cash}}^2 + \sigma_{\text{asset}}^2 - 2\rho_{\text{cash,asset}}\sigma_{\text{cash}}\sigma_{\text{asset}}\right)}_{\text{positive}} &= 0
\end{aligned}
$$

Liquidity buffer and cash holding

Example

- We use the square-root model:

$$\mathbf{TC}_{\text{asset}}(x) = x \cdot \left(s + \beta_{\pi} \sigma \sqrt{x}\right)$$

where s is the bid-ask spread, σ is the daily volatility and β_{π} is the price impact coefficient

- The cash is liquidated at a fixed rate $c \ll s$:

$$\mathbf{TC}_{\text{cash}}(x) = x \cdot c$$

- The redemption rate follows a power-law distribution ($\eta > 0$):

$$\mathbf{F}(x) = x^{\eta}$$

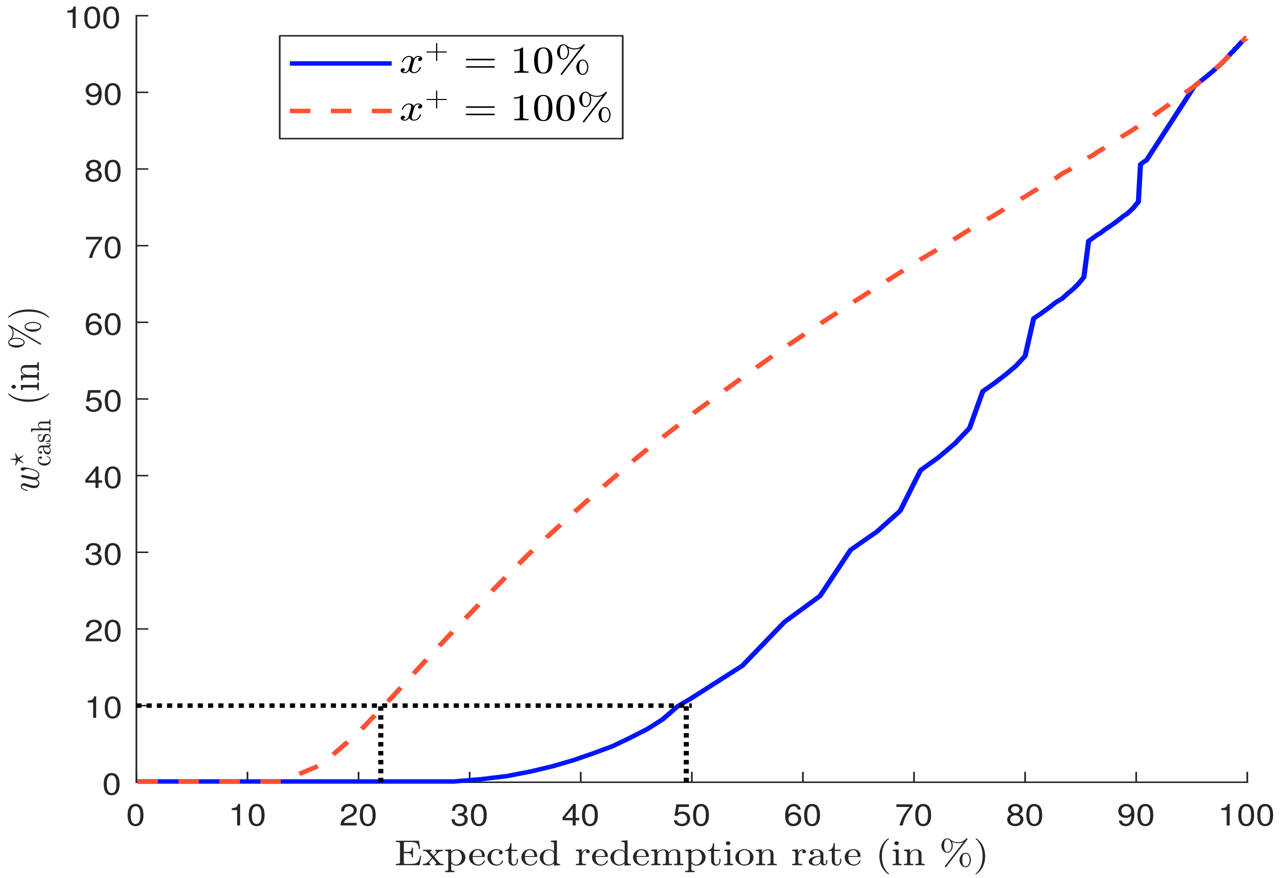
- We impose a daily trading limit x^{+}

$\Rightarrow$ We obtain closed-form formulas (otherwise we use numerical computation)

Liquidity buffer and cash holding

Optimal cash buffer

Figure: Optimal cash buffer ($\mu_{\text{asset}} - \mu_{\text{cash}} = 1\%$ and $\lambda = 0$)



$^{(*)}s = 50$ bps, $c = 1$ bp, $\sigma^{\text{yearly}} = 80\%$ and $\beta_\pi = 0.40$

Liquidity buffer and cash holding

Break-even risk premium

Definition

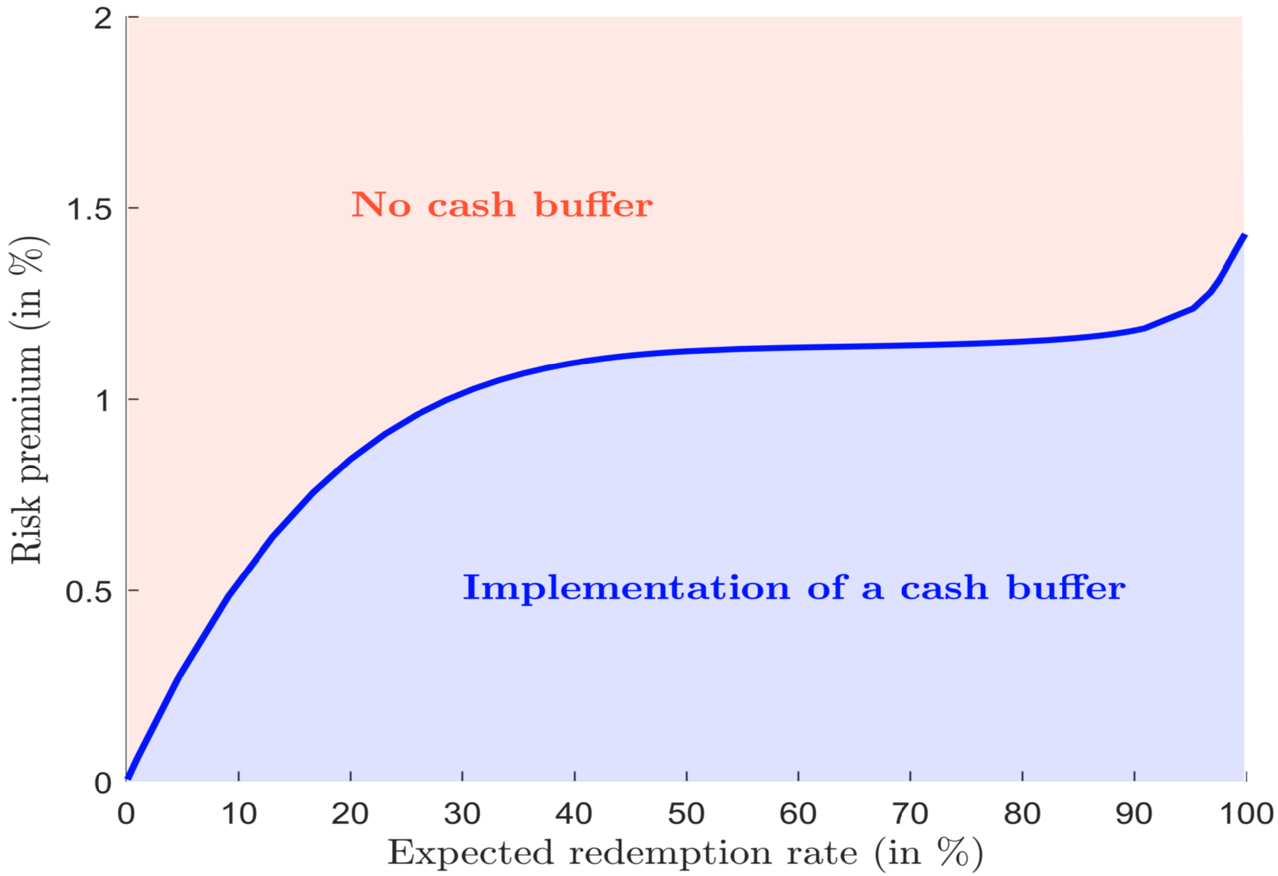
Given w_{cash}, we define the break-even risk premium $\rho\left(w_{\text{cash}}\right)$ as the value of $\mu_{\text{asset}} - \mu_{\text{cash}}$ such that the net cost function is minimum:

$$\rho\left(w_{\text{cash}}\right) = \frac{\partial\,\mathbb{E}\left[\mathbf{LG}\left(w_{\text{cash}}\right)\right]}{\partial\,w_{\text{cash}}} - \lambda\,w_{\text{cash}}\left(\sigma_{\text{cash}}^2 + \sigma_{\text{asset}}^2 - 2\rho_{\text{cash,asset}}\sigma_{\text{cash}}\sigma_{\text{asset}}\right)$$

Liquidity buffer and cash holding

Break-even risk premium

Figure: Implementation of a cash buffer when $x^{+} = 10\%$ ($\lambda = 0$)

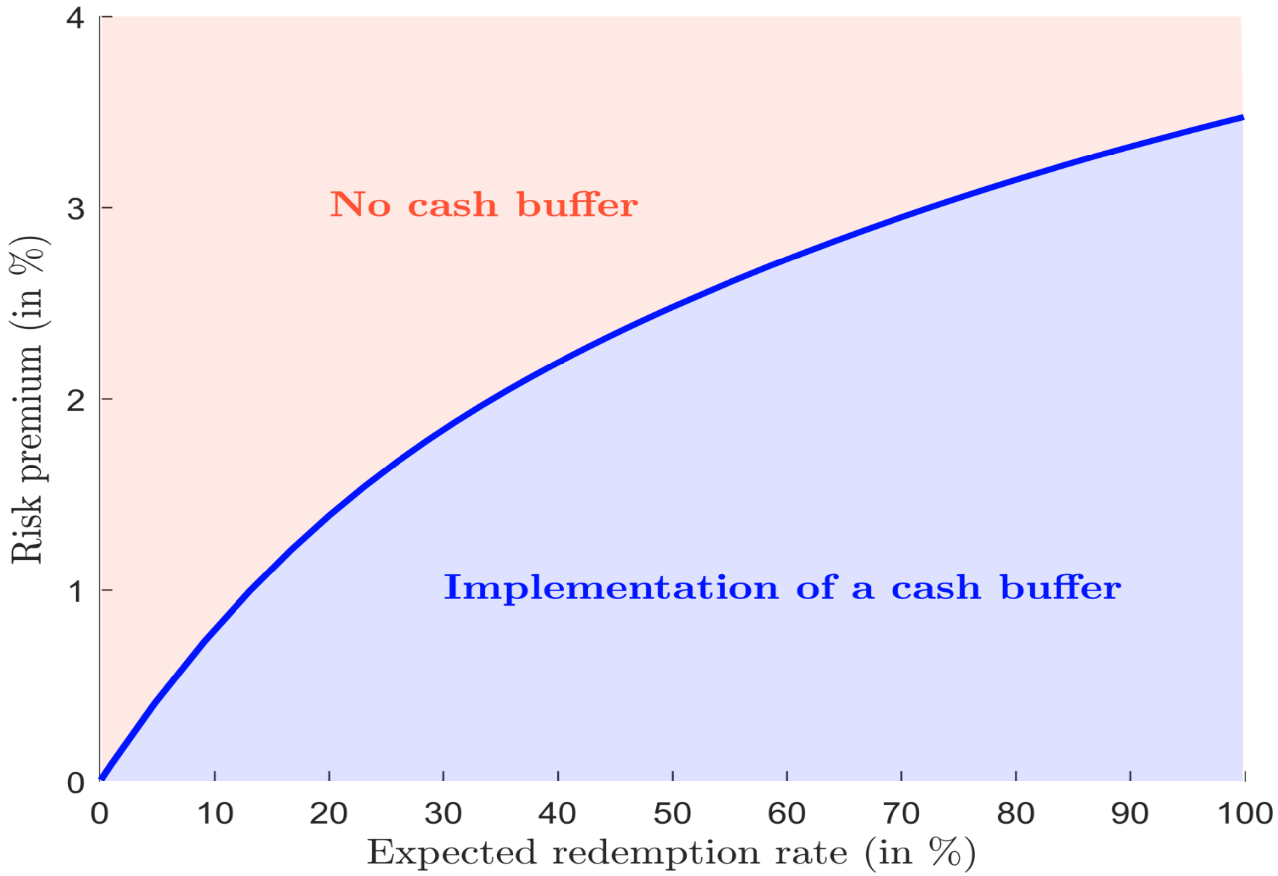


$^{(*)}$ $s = 50$ bps, $c = 1$ bp, $\sigma^{\text{yearly}} = 80\%$ and $\beta_{\pi} = 0.40$

Liquidity buffer and cash holding

Break-even risk premium

Figure: Implementation of a cash buffer when $x^{+} = 100\%$ ($\lambda = 0$)



$^{(*)}$ $s = 50$ bps, $c = 1$ bp, $\sigma^{\text{yearly}} = 80\%$ and $\beta_{\pi} = 0.40$

Liquidity buffer and cash holding

The debate on cash hoarding

Cash holding

- The fund manager implements the cash buffer before the redemption occurs
- The fund manager uses the cash buffer during the liquidity stress period

Fire sales may be stabilized

Static view of the asset risk premium

Cash hoarding

- The fund manager does not liquidate the cash buffer during the liquidity stress period
- He preserves the liquidity of the portfolio or even increases the proportion of cash during the stress period

Fire sales are amplified

Dynamic view of the risk premium

Special arrangements

- Redemption suspension and gate
- Side pocket
- In-kind redemption

Swing pricing

- Investor dilution
- Full vs partial vs dual swing pricing
- Swing threshold and swing factor
- Anti-dilution levies (ADL) (exit and entry fees)

$\Rightarrow$ Anti-dilution levies are more optimal than swing pricing

Liquidity monitoring tools

- Macro-economic approach of liquidity monitoring
- Micro-economic approach of liquidity monitoring

Conclusion

Liability liquidity risk measurement

Defining a redemption shock scenario

Two measurement approaches:

1. Historical approach $\Rightarrow$ non-parametric risk measures: historical VaR and CVaR
2. Frequency-severity modeling approach $\Rightarrow$ parametric risk measures: analytical VaR, CVaR and stress scenarios
    - Zero-inflated statistical model (population-based model)
    - Behaviorial model (individual-based model)
    - Factor-based model

Conclusion

Asset liquidity risk measurement

Liquidating a redemption portfolio and measuring its trading cost

Two measurement approaches:

- Liquidity risk profile
  - Liquidation ratio
  - Time to liquidation
  - Liquidation shortfall
- Liquidity cost
  - Transaction cost & market impact
  - Implementation shortfall and effective cost

Conclusion

Asset-liability liquidity risk management

Managing the liquidity risk (asset-liability matching)

Three families of tools:

- Liquidity measurement tools
  - Redemption coverage ratio (RCR)
  - Liquidation policy (vertical vs waterfall slicing)
  - Reverse stress testing
- Liquidity management tools
  - Liquidity buffer and cash hoarding
  - Redemption suspension (a-LMT): redemption suspension, side pockets, gates
  - Swing pricing and anti-dilution levies (ADL)
- Liquidity monitoring tools
  - Macro-approach of liquidity monitoring
  - Micro-approach of liquidity monitoring

Conclusion

What's next

- ESMA guidelines has boosted research on LST in asset management
- Good news: many models, many choices, many options
- Bad news: too many models, too many choices, too many options

On the of practicability of regulation

- Regulation means a minimum common framework ($\neq$ too many disparate elements)
- The example of operational risk in 2001!

Conclusion

What's next

$\Rightarrow$ ~~We~~ ESMA needs to standardize models and parameters:

- The time horizon h
- The redemption rate $\mathcal{R}$
- The asset liquidity multiplier m_v

whereas the asset manager must focus on the internal liquidity processes:

- The liquidation policy
- The trading limits
- Etc.

ESMA Guidelines $\Rightarrow$ Effective Regulatory Framework?

Disclaimer

Amundi, French joint stock company ("Société Anonyme") with a registered capital of € 1 086 262 605 and approved by the French Securities Regulator (Autorité des Marchés Financiers-AMF) under number GP 04000036 as a portfolio management company,

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