Net Zero Carbon Metrics*

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Abstract

This research project is both an update of the analysis on carbon emissions trajectories proposed by Le Guenedal *et al.* (2020) and a companion study of the climate risk measures defined by Le Guenedal and Roncalli (2022). While Le Guenedal *et al.* (2020) use carbon intensities, we extend the track-record projection approach by considering absolute carbon emissions. In particular, we propose a carbon budget approach that incorporates novel metrics for measuring the carbon emissions reduction targets and the relative positioning with respect to the net zero emissions (NZE) scenario. Indeed, current carbon emissions data are not sufficient to build portfolio alignment. The purpose of this paper is then to define net zero carbon metrics, which are necessary to enhance the disclosure and the debate on corporates' emissions (Créhalet, 2021; Le Meaux *et al.*, 2021).

These carbon metrics can be divided into two families. The static measures are NZE duration, NZE gap, NZE slope and NZE budget. They can be computed using a target scenario or the linear trend model. The dynamic NZE measures incorporate the past trajectory and the future scenarios of carbon emissions. For instance, we break down the carbon budget by error and revision time contributions. We also propose a velocity measure of the carbon emissions trend and two main dynamic NZE measures that are necessary to assess the performance of an issuer compared to the NZE scenario: the zero-velocity scenario and the burn-out scenario. These different measures can then be used to define the $\mathcal{PAC}$ framework, that analyzes the participation, ambition and credibility of issuers' NZE policies. Finally, we apply this framework to the CDP database. Empirical results show that net zero carbon emissions are challenging for many issuers for two reasons. The first is that some issuers have a lack of ambition concerning their NZE scenario. The second is that some targets are not compatible with past trends.

Keywords: Climate change, net zero emissions, reduction scenario, carbon budget, carbon trend, carbon reduction target, participation, ambition, credibility, portfolio alignment, decarbonization.

JEL Classification: G11, Q5.

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1 Introduction

Following the last Intergovernmental Panel on Climate Change report on the climate emergency (IPCC, 2021), countries, companies and investors are increasingly acknowledging the importance of reducing global greenhouse gas (GHG) emissions. It was already the case in December 2015 during the COP 21, where 196 countries signed the Paris Agreement to limit global warming to well below 2°C compared to pre-industrial levels. More recently, the IPCC underlined that achieving this goal would require reaching net zero of CO_2e emissions[1] around 2050 (IPCC, 2018). The same year, Carbone 4 developed the "*net zero initiative*" project to encourage organizations to reduce CO_2e emissions and increase CO_2e removals at a global level. While the COP26 was delayed by one year due to the Covid-19 crisis, the UN Framework Convention on Climate Change (UNFCCC) launched the "*race to zero campaign*" which coordinates multiple initiatives such as the UN-convened Net Zero Asset Owner Alliance and the Net Zero Asset Managers Initiative. The International Energy Agency (IEA) has produced a comprehensive roadmap to reach a net zero energy system by 2050 (IEA, 2021). This report received a lot of attention from asset managers (Créhalet, 2021; Le Meaux *et al.*, 2021). To achieve this ambitious goal, governments are implementing soft regulations to control the efforts of companies and the disclosure of their carbon emissions data. For instance, the EU disclosure regulation 2019/2088 and the EU taxonomy regulation 2020/852 allow investors to access scope 1 emissions and compare the levels in an increasingly standardized reporting. The key challenge of these regulations will be the provision of relevant data, in terms of frequency, quality and coverage.

Venturini (2022) conducts an extensive literature review of the relationship between climate change and market risk. The author highlights the recent debate on the pricing of climate change risks for equities. Pastor *et al.* (2021) put together the puzzle between the performance of green assets, climate news and ESG flows. They show that the recent good performance of green assets is not due to a higher risk premium. Unlike the daily media climate change concerns index introduced by Ardia *et al.* (2021), Taleb *et al.* (2020) use news volume information at the stock granularity level. They found that filtering broad equity universes with ESG news volume improves the return of best-in-class vs. worst-in-class portfolios that are built to measure the performance of ESG analysis (Bennani *et al.*, 2018; Drei *et al.*, 2019).

However, if we take a step back from the pricing of climate risk and focus on climate data, the academic literature also highlights several issues with estimated values. Indeed, Kalesnik *et al.* (2020) noticed that data providers estimate carbon emissions[2] when the values are not self-reported by companies to increase the coverage. They found that the quality of these estimates is poor:

> "*[...] much of the emissions data are estimated by data providers. As we evaluate the forward-looking carbon scores from several popular data providers, we find no evidence that these scores predict future changes in emissions. Further, we find that data on estimated emissions are at least 2.4 times less effective than self-reported data and provide little information to identify green companies in brown sectors*" (Kalesnik *et al.*, 2020, page 1).

[1] Carbon dioxide equivalent (CO_2e) is a term for describing different GHGs in a common unit. In this framework, a quantity of GHG is expressed as CO_2e by multiplying the GHG amount by its global warming potential (GWP). The GWP of a gas is the amount of CO_2 that would warm the earth equally. For instance, the 4$^{\text{th}}$ Assessment Report of the IPCC uses the following rules: 1 kg of methane corresponds to 25 kg of CO_2, and 1 kg of nitrous oxide corresponds to 298 kg of CO_2.

[2] For instance, by using carbon targets or trends.

These results call for substantial development of international regulations and standards with mandatory reporting of GHG data. Recent innovations in climate data concern temperature scores of companies or asset portfolios. For instance, CDP and WWF (2020) have developed a method of "*temperature ratings*", which provides a framework to translate corporate GHG emissions reduction targets into temperature scores. For each type of targets (absolute and intensity), they evaluate temperatures between 1°C and 5°C, with short-, medium- and long-term trends. Using the IPCC Special Report on 1.5°C scenario database, they use regression models to assign a temperature to each target. This framework can help investors to measure the quality of a company's ambition or to quantify the temperature of asset portfolios and financial indices (SBTi, 2021). These data show that the majority of G7 stock market indices are above 2°C and are not aligned with the trajectories recommended by the Paris Agreement. Portfolio alignment assessment is important for the financial sector's involvement in the energy transition. Nevertheless, Raynaud *et al.* (2021) indicate implementation issues when estimating temperature alignment by highlighting the discrepancies in the underlying methodologies.

In this research project, we propose several net zero carbon metrics, which can be used by asset owners and managers to perform portfolio alignment. As noticed by Le Guenedal and Roncalli (2022), portfolio alignment cannot be reduced to a portfolio decarbonization exercise with a carbon reduction trajectory. Portfolio alignment requires new carbon risk measures. For that, we address emissions data within a carbon budget approach to build budget-level reduction targets. We follow Le Guenedal *et al.* (2020) and focus on the emissions track-record of corporates. Indeed, we assume that investors account for corporates' past efforts to reduce their emissions. All things being equal, if two issuers A and B are similar in all points but issuer A has achieved a carbon emissions reduction in the past while issuer B has not reduced its carbon footprint or is far from its reduction target, we consider that investors must give more credibility to issuer A. Therefore, we propose a scenario-based budgeting approach, where we explicitly compare the pathway projected from the emissions track-record to the pathway expected from a net zero emissions (NZE) scenario. In this regard, carbon intensity measures, as described in IPCC (2014), have gained traction with investors. In Le Guenedal *et al.* (2020), we used intensities following two methodologies: the Sectoral Decarbonization Approach (SDA) established by SBTi (2015) and greenhouse gas emissions per unit of value added (GEVA). Intensities and absolute emissions differ by two main characteristics. On the one hand, the distribution of carbon intensities is less skewed making this metric more comparable between issuers and more constraining in a portfolio decarbonization exercise (Le Guenedal and Roncalli, 2022). On the other hand, carbon intensities are not always additive since they depend on the normalization factor. Therefore, working with absolute carbon emissions is more consistent when budgeting the reduction of CO_2 emissions. This also frees us from the conversion of the global scenario into sectorial intensity trajectories, and avoids data manipulation, e.g., M&A tricks where carbon intensity can improve while having no impact on global emissions (Créhalet, 2021).

This paper is organized as follows. Section Two covers the methodology for net zero carbon metrics. We introduce the carbon budget approach, which is an extension of the carbon emissions measure, and define the concepts of reduction target and trend. Section Three is dedicated to NZE measurement. In particular, using a scenario-based budgeting approach, we propose several static trajectory metrics: NZE duration, NZE gap, NZE slope and NZE budget. We also analyze the dynamic breakdown of the carbon budget and introduce the error and revision contributions. We also define the important concept of NZE velocity that allows us to compute the zero-velocity and burn-out scenarios. In Section Four, we apply this framework to the CDP database and discuss the application of the NZE metrics for portfolio management. Finally, Section 5 offers some concluding remarks.

2 Net zero emissions framework

In this section, we present the basics for building net zero carbon metrics. The three main tools are the carbon budget, the reduction target and the carbon trend. We note $\boldsymbol{\mathcal{CE}}_{i,j}(t)$ the absolute carbon emissions of issuer i for the scope j at time t. $\boldsymbol{\mathcal{CE}}_{i,j}(t)$ is measured in tCO_2e. The time frame is generally annual, implying that t takes the values 2020, 2021, 2022, etc. Nevertheless, we consider that $t \in \mathbb{R}^+$ because this simplifies the computation in particular when we manipulate areas that are calculated using mathematical integrals. To simplify the notation, we omit the subscript j when possible.

2.1 Carbon budget

The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period $[t_0, t]$. From a mathematical point of view, it corresponds to the signed area of the region bounded by the function $\boldsymbol{\mathcal{CE}}_i(t)$:

$$\boldsymbol{\mathcal{CB}}_i(t_0, t) = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}_i(s)\, \mathrm{d}s \tag{1}$$

Most of the time, issuer i has an objective to keep its GHG emissions under a certain acceptable level $\boldsymbol{\mathcal{CE}}_i^\star$. In this case, we can define the carbon budget as[3]:

$$\boldsymbol{\mathcal{CB}}_i(t_0, t) = \int_{t_0}^{t} \left(\boldsymbol{\mathcal{CE}}_i(s) - \boldsymbol{\mathcal{CE}}_i^\star\right) \mathrm{d}s = -(t - t_0) \cdot \boldsymbol{\mathcal{CE}}_i^\star + \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}_i(s)\, \mathrm{d}s \tag{3}$$

Therefore, Equation (1) defines the gross carbon budget whereas Equation (3) measures the net carbon budget. In this last case, the objective of the entity is that carbon emissions fluctuate as long as the convergence toward the objective is guaranteed at the target date $t^\star$: $\boldsymbol{\mathcal{CE}}_i(t^\star) \approx \boldsymbol{\mathcal{CE}}_i^\star$. Once this first objective has been met, the goal of the entity is to maintain a carbon budget around zero: $\boldsymbol{\mathcal{CB}}_i(t^\star, t) \approx 0$ when $t > t^\star$.

Table 1: Carbon emissions in $MtCO_2e$ (Example 1)

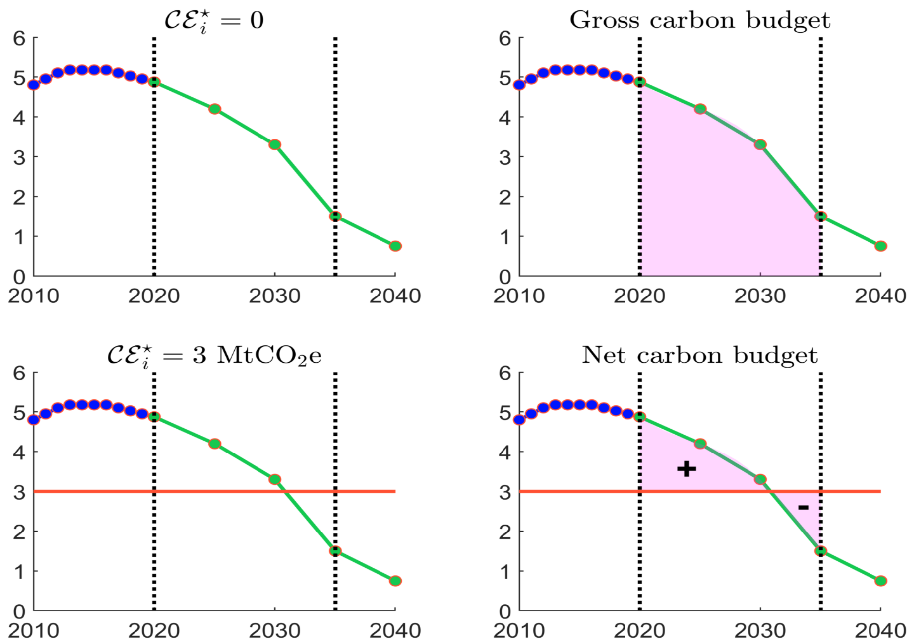
Year	2010	2011	2012	2013	2014	2015	2016	2017
$\boldsymbol{\mathcal{CE}}_i$	4.800	4.950	5.100	5.175	5.175	5.175	5.175	5.100
Year	2018	2019	2020*	2025*	2030*	2035*	2040*	2050*
$\boldsymbol{\mathcal{CE}}_i$	5.025	4.950	4.875	4.200	3.300	1.500	0.750	0.150

We consider Example 1 given in Table 1. Carbon emissions are reported in $MtCO_2e$. The data corresponds to observed values before 2019, and estimated values after this date. After 2020, we assume that the carbon emissions are linear between two dates. The first top-left panel in Figure 1 shows the carbon emissions pathway of this issuer. The gross carbon budget $\boldsymbol{\mathcal{CB}}_i(2020, 2035)$ between 2020 and 2035 is represented by the violet area in the second top-right panel. If we assume that the acceptable level $\boldsymbol{\mathcal{CE}}_i^\star$ is equal to 3 $MtCO_2e$ in 2035 (bottom-left panel), we notice that the net carbon budget is the difference between two areas (bottom-right panel). From January 2020 to October 2030, the carbon emissions are greater than $\boldsymbol{\mathcal{CE}}_i^\star$ and this period has a positive contribution to the carbon budget. On the contrary, the period from November 2030 to December 2035 has a negative contribution.

[3] If the objective is time-varying, we deduce that:

$$\boldsymbol{\mathcal{CB}}_i(t_0, t) = \int_{t_0}^{t} \left(\boldsymbol{\mathcal{CE}}_i(s) - \boldsymbol{\mathcal{CE}}_i^\star(s)\right) \mathrm{d}s = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}_i(s)\, \mathrm{d}s - \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}_i^\star(s)\, \mathrm{d}s \tag{2}$$

Figure 1: Computation of the carbon budget



Source: Authors' calculations.

Remark 1. *From a computational point of view, we can calculate the carbon budget using standard numerical integration. Otherwise, we can approximate the integral using the right Riemann sum with an annual step[4]:*

$$\mathcal{CB}_i(t_0, t) \approx \sum_{s=t_0+1}^{t} \left(\mathcal{CE}_i(s) - \mathcal{CE}_i^\star\right) \tag{4}$$

If we consider the previous example, the exact value of the gross carbon budget $\mathcal{CB}_i(2020, 2035)$ is equal to 53.4375 MtCO$_2$e whereas the approximated value is equal to 51.75 MtCO$_2$e. For the net carbon budget, we obtain respectively 8.4375 and 6.75 MtCO$_2$e. Therefore, we slightly underestimate the carbon budget using the right Riemann sum. We can verify that the figures calculated with the left Riemann sum (55.1250 and 10.1250) are slightly overestimated, whereas we obtain the exact values if we consider the mid-point rule.

2.2 Carbon reduction

Let $t_{\mathcal{L}ast}$ be the last reporting date. This implies that the carbon emissions $\mathcal{CE}_i(t)$ of issuer i are observable only when $t \leq t_{\mathcal{L}ast}$. For $t > t_{\mathcal{L}ast}$, we generally define the (estimated) carbon emissions as:

$$\mathcal{CE}_i(t) := \widehat{\mathcal{CE}}_i(t) = \left(1 - \mathcal{R}_i(t_{\mathcal{L}ast}, t)\right) \cdot \mathcal{CE}_i(t_{\mathcal{L}ast}) \tag{5}$$

where $\mathcal{R}_i(t_{\mathcal{L}ast}, t)$ is the carbon reduction between $t_{\mathcal{L}ast}$ and t. If $t_{\mathcal{L}ast} \in [t_0, t]$, we deduce that the carbon budget has the following expression:

$$\begin{aligned}\mathcal{CB}_i(t_0, t) &= (t - t_{\mathcal{L}ast})\left(\mathcal{CE}_i(t_{\mathcal{L}ast}) - \mathcal{CE}_i^\star\right) - (t_{\mathcal{L}ast} - t_0) \cdot \mathcal{CE}_i^\star + \\ &\quad \int_{t_0}^{t_{\mathcal{L}ast}} \mathcal{CE}_i(s)\,\mathrm{d}s - \mathcal{CE}_i(t_{\mathcal{L}ast}) \int_{t_{\mathcal{L}ast}}^{t} \mathcal{R}_i(t_{\mathcal{L}ast}, s)\,\mathrm{d}s \end{aligned} \tag{6}$$

[4]See Appendix A.3.1 on page 55 for the different approximation formulas.

Table 2: IEA NZE scenario (in $GtCO_2e$)

Year	2019	2025	2030	2035	2040	2045	2050
Gross emissions	35.90	30.30	21.50	13.70	7.77	4.30	1.94
CCS	0.00	−0.06	−0.32	−0.96	−1.46	−1.80	−1.94
Net emissions	35.90	30.24	21.18	12.74	6.31	2.50	0.00
Reduction (in %)	0.00	15.60	40.11	61.84	78.36	88.02	94.60

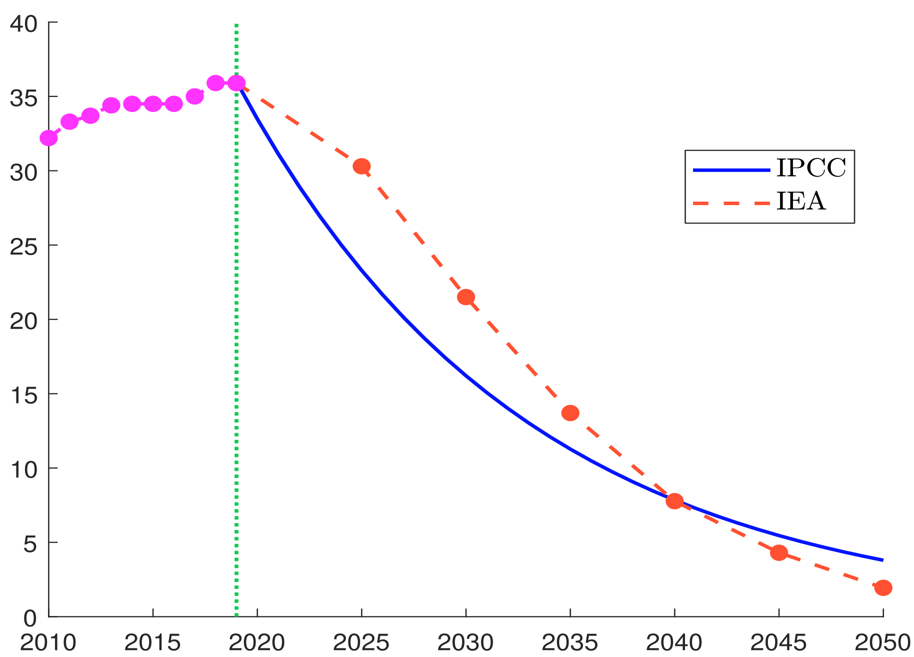
Source: IEA (2021, Chapter 2, Figure 2.3, page 55).

Remark 2. *If we calculate the carbon budget from the last reporting date ($t_0 = t_{\mathcal{L}ast}$), Equation (6) reduces to:*

$$\mathcal{CB}_i \left(t_{\mathcal{L}ast}, t\right) = \left(t - t_{\mathcal{L}ast}\right) \left(\mathcal{CE}_i \left(t_{\mathcal{L}ast}\right) - \mathcal{CE}_i^{\star}\right) - \mathcal{CE}_i \left(t_{\mathcal{L}ast}\right) \int_{t_{\mathcal{L}ast}}^{t} \mathcal{R}_i \left(t_{\mathcal{L}ast}, s\right) \mathrm{d}s \qquad (7)$$

The issue of this modeling is the availability of $\mathcal{R}_i \left(t_{\mathcal{L}ast}, t\right)$ for all the different issuers. One practical solution is to consider a benchmark reduction pathway. For instance, we can use a global carbon reduction scenario. Following IPCC (2021), we need to reduce total emissions by at least 7% every year between 2019 and 2050 if we want to achieve net zero emissions by 2050. IEA (2021) has also published its net zero emissions (NZE) scenario (see Table 2). It implies a reduction of 40.11% of carbon emissions in 2030 and 61.84% in 2035. In 2050, the gross emissions would be 1.94 $GtCO_2e$ compensated by the carbon capture and storage (CCS) technology. These two scenarios are reported in Figure 2.

Figure 2: Two net zero emissions scenarios



Source: IEA (2021) & Authors' calculations.

In the global approach, the reduction for issuer i is equal to the reduction calculated for the global scenario:

$$\mathcal{R}_i \left(t_{\mathcal{L}ast}, t\right) = \mathcal{R}_{\mathcal{G}lobal} \left(t_{\mathcal{L}ast}, t\right) \qquad (8)$$

It is obvious that this solution is not optimal since there is no impact difference between all the issuers. Another solution consists in using carbon reduction scenario at the country level. Indeed, governments are encouraged to set their national determined contributions (NDC) and carbon reduction mechanisms following the carbon budget approach. Therefore, we can use their carbon reduction pathways as a benchmark for the companies:

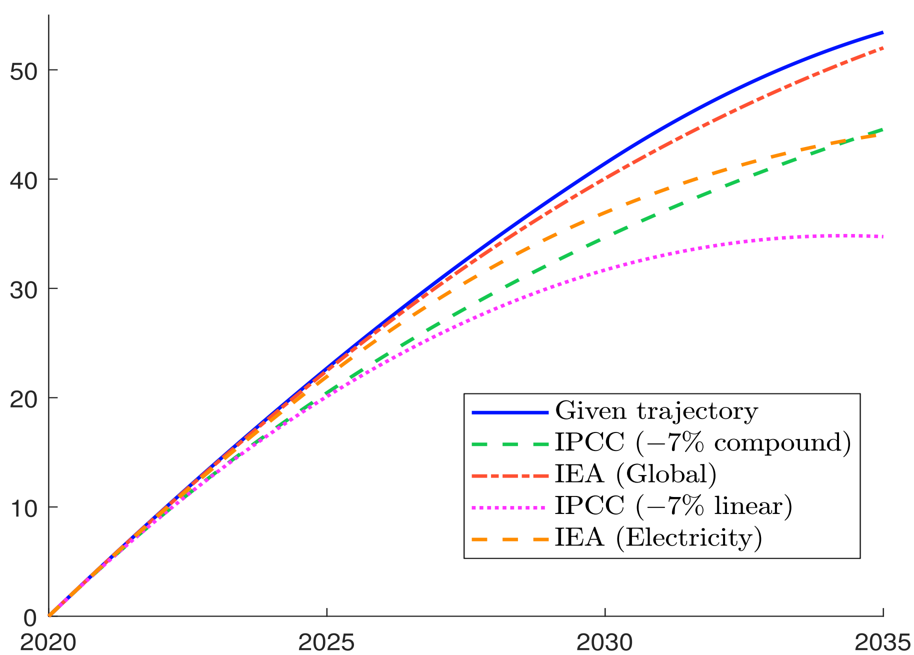
$$\boldsymbol{\mathcal{R}}_i \left(t_{\mathcal{L}ast}, t\right) = \boldsymbol{\mathcal{R}}_{\mathcal{C}ountry(c)} \left(t_{\mathcal{L}ast}, t\right) \qquad \text{if } i \in \mathcal{C}ountry\left(c\right) \tag{9}$$

Nevertheless, the pitfall of this method is that scope 3 emissions are not necessarily located in the country of the company. Instead of countries, we can also use sectors:

$$\boldsymbol{\mathcal{R}}_i \left(t_{\mathcal{L}ast}, t\right) = \boldsymbol{\mathcal{R}}_{\mathcal{S}ector(s)} \left(t_{\mathcal{L}ast}, t\right) \qquad \text{if } i \in \mathcal{S}ector\left(s\right) \tag{10}$$

Again, this approach may be puzzling especially in the context of non-homogeneous sector mapping[5]. Moreover, all these benchmark solutions ignore the idiosyncratic aspect of carbon reduction. This is why, below, we suggest working at the corporate level by considering two methods. The first one is based on reductions targets whereas the second one uses the concept of carbon trend.

Figure 3: Comparison of gross carbon budget with different scenarios (in $MtCO_2e$)



Source: IEA (2021); IPCC (2018) & Authors' calculations.

Let us consider the previous example. In Figure 3, we compare the carbon budget $\boldsymbol{\mathcal{CB}}(2020, t)$ for different reduction scenarios: given trajectory (Example 1), IPCC (−7% compound reduction), IEA (global scenario), IPCC (−7% linear reduction) and IEA (electricity sector scenario). We notice that the scenario of Example 1 is very close to the global IEA scenario.

[5]See Appendix A.4.1 on page 62.

2.3 Carbon reduction targets

Carbon reduction targets are defined by companies at a scope emissions level with different horizons. For instance, the issuer can commit to reduce its scope 1 emissions by 50% over a period of 20 years and its scope 3 emissions by 30% over a period of 10 years. Even if the time frame of carbon reduction targets goes to 60 years, most of reduction targets concern the next 20 years[6]. Moreover, we observe that most targets are underway or new. A large proportion of companies set targets to reduce emissions by less than 50% from their base year, but the proportion corresponding to a reduction of 100% is also significant. We also notice that some targets are reported over multiple scopes[7] and we can have multiple release dates. In this case, we can face overlapping targets that may be not consistent. For instance, a company can release a carbon reduction target in 2018 and another target in 2020 on the same emissions scope. Therefore, we must decide if we replace the first target with the second target or if we combine the two targets. In what follows, we present the basic methodology to compute annual reduction rates that are implied by carbon reduction targets, and then discuss how to deal with overlapping data.

The carbon reduction target setting is defined from the following space:

$$\mathcal{T} = \left\{ k \in [1, m] : \left(i, j, t_1^k, t_2^k, \boldsymbol{\mathcal{R}}_{i,j}\left(t_1^k, t_2^k\right) \right) \right\} \tag{11}$$

where k is the target index, m is the number of historical targets, i is the issuer, j is the scope, t_1^k is the beginning of the target period, t_2^k is the end of the target period, and $\boldsymbol{\mathcal{R}}_{i,j}\left(t_1^k, t_2^k\right)$ is the carbon reduction between t_1^k and t_2^k for the scope j announced by issuer i. The linear annual reduction rate for scope j and target k at time t is then given by:

$$\boldsymbol{\mathcal{R}}_{i,j}^k(t) = \mathbb{1}\left\{ t \in \left[t_1^k, t_2^k\right] \right\} \cdot \frac{\boldsymbol{\mathcal{R}}_{i,j}\left(t_1^k, t_2^k\right)}{t_2^k - t_1^k} \tag{12}$$

Then, we aggregate the different targets to obtain the linear annual reduction rate for scope j:

$$\boldsymbol{\mathcal{R}}_{i,j}(t) = \sum_{k=1}^{m} \boldsymbol{\mathcal{R}}_{i,j}^k(t) \tag{13}$$

The budget approach consists in converting these reported targets into absolute emissions reduction as follows:

$$\boldsymbol{\mathcal{R}}_i(t) = \underbrace{\frac{1}{\sum_{j=1}^{3} \boldsymbol{\mathcal{CE}}_{i,j}(t_0)}}_{\text{Total emissions}} \cdot \underbrace{\sum_{j=1}^{3} \boldsymbol{\mathcal{CE}}_{i,j}(t_0) \cdot \boldsymbol{\mathcal{R}}_{i,j}(t)}_{\text{Scope targeted reductions}} \tag{14}$$

Therefore, the carbon reduction $\boldsymbol{\mathcal{R}}_i(t)$ no longer depends on the scope and the target period. Once the reduction is established along the time horizon, the implied trajectory of the company emissions follows:

$$\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}arget}(t) := \widehat{\boldsymbol{\mathcal{CE}}}_i(t) = \left(1 - \boldsymbol{\mathcal{R}}_i\left(t_{\mathcal{L}ast}, t\right)\right) \cdot \boldsymbol{\mathcal{CE}}_i\left(t_{\mathcal{L}ast}\right) \tag{15}$$

where:

$$\boldsymbol{\mathcal{R}}_i\left(t_{\mathcal{L}ast}, t\right) = \sum_{s=t_{\mathcal{L}ast}+1}^{t} \boldsymbol{\mathcal{R}}_i(s) \tag{16}$$

[6] See Appendix A.2 on page 53 for the descriptive statistics of reduction targets.
[7] For instance, the target can concern only one scope, scope $1+2$ or all scopes.

We can then compute the carbon budget according to the carbon targets declared by the issuer.

Remark 3. *We notice that Equations (5) and (15) are very similar. In fact, the linear annual reduction method implies that the reduction* $\boldsymbol{\mathcal{R}}_i(t_{\mathcal{L}ast}, t)$ *is the sum of the annual reductions* $\boldsymbol{\mathcal{R}}_i(t)$ *between* $t_{\mathcal{L}ast}$ *and* t.

Table 3: Carbon reduction targets (Example 2)

k	Release Date	Scope	t_1^k	t_2^k	$\boldsymbol{\mathcal{R}}\left(t_1^k, t_2^k\right)$
1	01/08/2013	$\boldsymbol{\mathcal{SC}}_1$	2015	2030	45%
2	01/10/2019	$\boldsymbol{\mathcal{SC}}_2$	2020	2040	40%
3	01/01/2019	$\boldsymbol{\mathcal{SC}}_3$	2025	2050	25%

We consider the reduction targets given in Table 3. The dates t_1^k and t_2^k correspond to 1st January. For example, in August 2013, the company announced its willingness to reduce its carbon emissions by 45% between January 2015 and January 2030. Moreover, we assume that $\boldsymbol{\mathcal{CE}}_{i,1}(2020) = 10.33$, $\boldsymbol{\mathcal{CE}}_{i,2}(2020) = 7.72$ and $\boldsymbol{\mathcal{CE}}_{i,3}(2020) = 21.86$. For the first target, we deduce that:

$$\boldsymbol{\mathcal{R}}_{i,1}^1(t) = \begin{cases} 3\% & \text{if } t \in [2015, 2030[\\ 0\% & \text{otherwise} \end{cases}$$

and $\boldsymbol{\mathcal{R}}_{i,2}^1(t) = \boldsymbol{\mathcal{R}}_{i,3}^1(t) = 0$. For the second target, we have:

$$\boldsymbol{\mathcal{R}}_{i,2}^2(t) = \begin{cases} 2\% & \text{if } t \in [2020, 2040[\\ 0\% & \text{otherwise} \end{cases}$$

and $\boldsymbol{\mathcal{R}}_{i,1}^2(t) = \boldsymbol{\mathcal{R}}_{i,3}^2(t) = 0$. Finally, for the last target, we obtain:

$$\boldsymbol{\mathcal{R}}_{i,3}^3(t) = \begin{cases} 1\% & \text{if } t \in [2025, 2050[\\ 0\% & \text{otherwise} \end{cases}$$

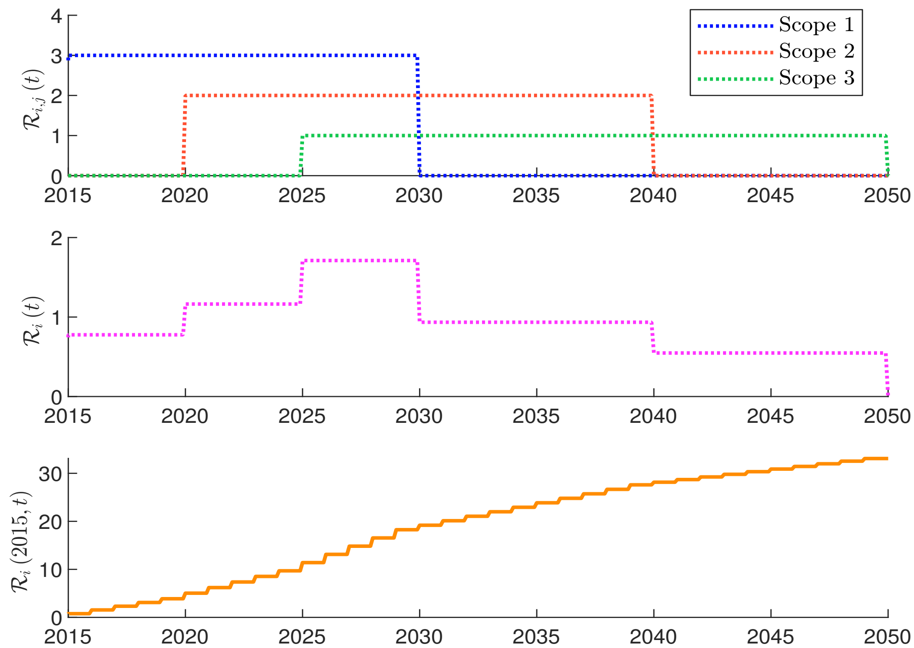
and $\boldsymbol{\mathcal{R}}_{i,1}^3(t) = \boldsymbol{\mathcal{R}}_{i,2}^3(t) = 0$. In the first panel in Figure 4, we have represented the carbon reductions $\boldsymbol{\mathcal{R}}_{i,1}(t)$, $\boldsymbol{\mathcal{R}}_{i,2}(t)$ and $\boldsymbol{\mathcal{R}}_{i,3}(t)$ for the three emissions scopes. Then, we compute the weighted average and obtain the global carbon reduction $\boldsymbol{\mathcal{R}}_i(t)$ given in the second panel. Finally, we have computed the cumulative reduction rate $\boldsymbol{\mathcal{R}}_i(2015, t)$.

Table 4: Carbon reduction targets (Example 3)

k	Release Date	Scope	t_1^k	t_2^k	$\boldsymbol{\mathcal{R}}\left(t_1^k, t_2^k\right)$
1	01/08/2013	$\boldsymbol{\mathcal{SC}}_1$	2015	2030	45%
2	01/03/2016	$\boldsymbol{\mathcal{SC}}_1$	2017	2032	60%
3	01/10/2018	$\boldsymbol{\mathcal{SC}}_2$	2019	2039	40%
4	01/11/2019	$\boldsymbol{\mathcal{SC}}_{1+2+3}$	2020	2050	75%

Let us now consider the case of overlapping data. In Table 4, we consider the carbon targets of an issuer that updates its expectations and changes its reduction policy between 2013 and 2019. First, it announced in 2013 a 45% reduction for the scope 1 emissions between 2015 and 2030. But, in 2016, it revised its target and proposed a more ambitious target for scope 1 emissions (60% for the next 15 years instead of 45%). In 2018, it decided to complete its carbon reduction policy by imposing a 40% reduction for the scope 2 emissions.

Figure 4: Reduction of the carbon emissions deduced from the three targets (Example 2)



Source: Authors' calculations.

Finally, it announced in 2019 a simplification of the targets. It will use a uniform reduction of 75% across the three scopes for the next 30 years. In Appendix A.4.2 on page 63, we propose an algorithm to fix this problem of overlapping targets. Results are given in Table 5. In this example, the first target is replaced by the second target in 2017. Then, the third target is added to the second target in 2019. Finally, the second and third targets are replaced by the fourth target in 2020.

Table 5: Computation of linear annual reduction rates (Example 3)

Scope	$\mathcal{R}_{i,j}(t)$	2015	2016	2017	2018	2019	2020 – 2050
$\mathcal{SC}_1$	$\mathcal{R}_{i,1}(t)$	3%	3%	4%	4%	4%	2.5%
$\mathcal{SC}_2$	$\mathcal{R}_{i,2}(t)$	0%	0%	0%	0%	2%	2.5%
$\mathcal{SC}_3$	$\mathcal{R}_{i,3}(t)$	0%	0%	0%	0%	0%	2.5%

Remark 4. *If we prefer to use the compound approach instead of the linear approach, we have to replace Equations (12) and (16) by:*

$$\mathcal{R}_{i,j}^{k}(t) = \mathbb{1}\left\{t \in \left[t_1^k, t_2^k\right]\right\} \cdot \left(1 - \left(1 - \mathcal{R}_{i,j}\left(t_1^k, t_2^k\right)\right)^{\frac{1}{t_2^k - t_1^k}}\right) \tag{17}$$

and:

$$\mathcal{R}_i\left(t_{\mathcal{L}ast}, t\right) = 1 - \prod_{s=t_{\mathcal{L}ast+1}}^{t} \left(1 - \mathcal{R}_i(s)\right) \tag{18}$$

2.4 Carbon trend

We define the carbon trend by considering a linear constant trend model. The associated linear regression model is:

$$\mathcal{CE}_i(t) = \beta_{i,0} + \beta_{i,1} t + u_i(t) \tag{19}$$

where $t \in [t_{\mathcal{F}irst}, t_{\mathcal{L}ast}]$. Using the least squares method, we can estimate the parameters $\beta_{i,0}$ and $\beta_{i,1}$. Then, we can build the carbon trajectory implied by the current trend by applying the projection:

$$\mathcal{CE}_i^{\mathcal{T}rend}(t) := \widehat{\mathcal{CE}}_i(t) = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} t \tag{20}$$

for $t > t_{\mathcal{L}ast}$. This model is very simple. The underlying idea is to extrapolate the past trajectory. Nevertheless, we can derive several net zero metrics that are useful to compare the existing track record of the issuer with its willingness to really reduce its carbon emissions.

Remark 5. *Let t_p be a pivot date*[8]*. If we consider the following parameterization:*

$$\mathcal{CE}_i(t) = \beta'_{i,0} + \beta'_{i,1}\left(t - t_p\right) + u_i(t) \tag{21}$$

we have the relationships: $\beta_{i,0} = \beta'_{i,0} - \beta'_{i,1} t_p$ and $\beta_{i,1} = \beta'_{i,1}$. We deduce that:

$$\left\{ \begin{array}{l} \beta'_{i,0} = \beta_{i,0} + \beta_{i,1} t_p \\ \beta'_{i,1} = \beta_{i,1} \end{array} \right. \tag{22}$$

If the pivot date is the current date — $t_p = t_0$, we have $\widehat{\mathcal{CE}}_i(t) = \beta'_{i,0} + \beta'_{i,1}(t - t_0)$ and $\widehat{\mathcal{CE}}_i(t_0) = \beta'_{i,0}$. If we would like to rescale the trend such that $\widehat{\mathcal{CE}}_i(t_0) = \mathcal{CE}_i(t_0)$, we obtain $\beta'_{i,0} = \mathcal{CE}_i(t_0)$, implying that we must change the intercept of the trend model. It is now equal to $\tilde{\beta}_{i,0} = \mathcal{CE}_i(t_0) - \hat{\beta}_{i,1} t_0$.

Table 6: Carbon emissions in $MtCO_2e$ (Example 4)

Year	2007	2008	2009	2010	2011	2012	2013
$\mathcal{CE}_i(t)$	57.82	58.36	57.70	55.03	51.73	46.44	47.19
Year	2014	2015	2016	2017	2018	2019	2020
$\mathcal{CE}_i(t)$	46.18	45.37	40.75	39.40	36.16	38.71	39.91

We consider the past carbon emissions given in Table 6. We obtain the following estimates: $\hat{\beta}_{i,0} = 3\,637.73$ and $\hat{\beta}_{i,1} = -1.7832$. We deduce that:

$$\begin{aligned} \mathcal{CE}_i^{\mathcal{T}rend}(t) &= 3\,637.73 - 1.7832 \cdot t \\ &= 35.61 - 1.7822 \cdot (t - 2020) \end{aligned} \tag{23}$$

If we use the pivot year $t_p = 2020$, the results are more intuitive. Indeed, we have $\mathcal{CE}_i^{\mathcal{T}\mathrm{rend}}(2020) = 35.61$. Since we have $\mathcal{CE}_i(2020) = 39.91$, we deduce that the current carbon emissions are greater than the figure given by the trend. This implies that the issuer has made less effort in recent years compared to the past history. We also notice that the carbon emissions are reduced by 1.7822 $MtCO_2e$ every year on average. If we would like to rescale the trend such that $\mathcal{CE}_i^{\mathcal{T}rend}(2020) = \mathcal{CE}_i(2020)$, we obtain:

$$\mathcal{CE}_i^{\mathcal{T}rend}(t) = 39.91 - 1.7822 \cdot (t - 2020) \tag{24}$$

[8] It is generally the current year.

3 Net zero emissions metrics

3.1 Static NZE measures

We consider a static approach where $t^{\star}$ is the target horizon. We note $\mathcal{CE}_i^{\mathrm{nze}}(t^{\star})$ the net zero emissions scenario for issuer i. Let t_0 be the current date. $\mathcal{CE}_i^{\mathrm{nze}}(t^{\star})$ can be computed using the issuer's targets or using a market-based consensus scenario:

$$\mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) = \left(1 - \mathcal{R}^{\star}(t_0, t^{\star})\right) \cdot \mathcal{CE}_i(t_0) \tag{25}$$

where $\mathcal{R}^{\star}(t_0, t^{\star})$ is the carbon reduction between t_0 and $t^{\star}$ expected by the market for this issuer. For instance, $\mathcal{R}^{\star}(t_0, t^{\star})$ can be equal to the expected reduction for the sector/industry of the issuer in order to achieve an NZE scenario.

3.1.1 The NZE duration

We use the generic notation $\widehat{\mathcal{CE}}_i(t)$ to name $\mathcal{CE}_i^{\mathcal{T}arget}(t)$ and $\mathcal{CE}_i^{\mathcal{T}rend}(t)$. The time to reach the NZE scenario (or NZE duration) is defined as follows:

$$\boldsymbol{\tau}_i = \left\{\inf t : \widehat{\mathcal{CE}}_i(t) \leq \mathcal{CE}_i^{\mathrm{nze}}(t^{\star})\right\} \tag{26}$$

If $\widehat{\mathcal{CE}}_i(t) = \mathcal{CE}_i^{\mathcal{T}arget}(t)$, we obtain the NZE duration $\boldsymbol{\tau}_i^{\mathcal{T}arget}$. This measure indicates if the carbon targets announced by the company are in line with the consensus scenario $\mathcal{CE}_i^{\mathrm{nze}}(t^{\star})$. If $\widehat{\mathcal{CE}}_i(t) = \mathcal{CE}_i^{\mathcal{T}rend}(t)$, we obtain the NZE duration $\boldsymbol{\tau}_i^{\mathcal{T}rend}$. In this case, this measure indicates if the issuer's track record is in line with its targets or the consensus scenario. We reiterate that:

$$\mathcal{CE}_i^{\mathcal{T}rend}(t) = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} t \tag{27}$$

We distinguish two cases:

1. If the slope $\hat{\beta}_{i,1}$ is positive, $\mathcal{CE}_i^{\mathcal{T}rend}(t)$ is an increasing function. There is a solution only if the current carbon emissions $\mathcal{CE}_i(t_0)$ are less than the NZE scenario:

$$\boldsymbol{\tau}_i^{\mathcal{T}rend} = \begin{cases} t_0 & \text{if } \mathcal{CE}_i(t_0) \leq \mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) \\ +\infty & \text{otherwise} \end{cases} \tag{28}$$

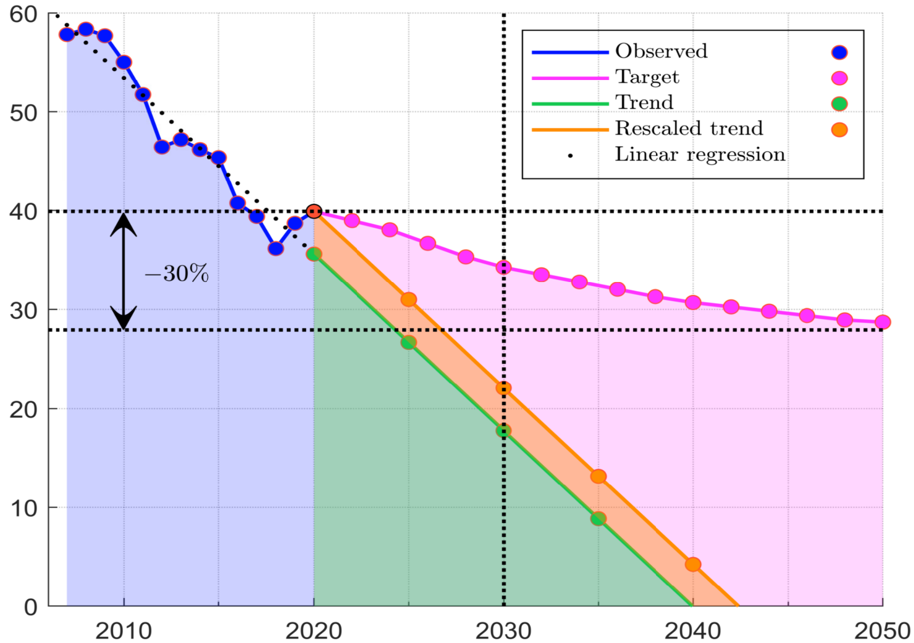
2. If the slope $\hat{\beta}_{i,1}$ is negative, $\mathcal{CE}_i^{\mathcal{T}rend}(t)$ is a decreasing function and we have:

$$\begin{aligned} \mathcal{CE}_i^{\mathcal{T}rend}(t) \leq \mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) & \Leftrightarrow \hat{\beta}_{i,0} + \hat{\beta}_{i,1} t \leq \mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) \\ & \Leftrightarrow t \geq \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) - \hat{\beta}_{i,0}}{\hat{\beta}_{i,1}} \\ & \Leftrightarrow t \geq t_0 + \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) - \left(\hat{\beta}_{i,0} + \hat{\beta}_{i,1} t_0\right)}{\hat{\beta}_{i,1}} \\ & \Leftrightarrow t \geq t_0 + \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) - \hat{\beta}'_{i,0}}{\hat{\beta}_{i,1}} \end{aligned} \tag{29}$$

where $\hat{\beta}'_{i,0} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} t_0$ is the intercept of the trend model when we use t_0 as the pivot date[9]. We deduce that:

$$\boldsymbol{\tau}_i^{\mathcal{T}rend} = t_0 + \left(\frac{\mathcal{CE}_i^{\mathrm{nze}}(t^{\star}) - \hat{\beta}'_{i,0}}{\hat{\beta}_{i,1}}\right) \tag{30}$$

Figure 5: Comparison of targets and trends (Example 5)



Source: Authors' calculations.

We consider Example 5, which corresponds to the trajectory given in Example 4 (Table 6 on page 11) and the targets defined in Example 2 (Table 3 on page 9). In Example 4, we have $\mathcal{CE}_{i,1}(2020) = 10.33$, $\mathcal{CE}_{i,2}(2020) = 7.72$ and $\mathcal{CE}_{i,3}(2020) = 21.86$, which implies that the carbon emissions for the three scopes are equal to $10.33 + 7.72 + 21.86 = 39.91$. This is exactly the value $\mathcal{CE}_i(2020)$ that is used in Example 4. We assume that the market-based NZE scenario for 2030 is a reduction of carbon emissions by 30%:

$$\mathcal{CE}_i^{\text{nze}}(2030) = 39.91 \times (1 - 30\%) = 27.94 \text{ MtCO}_2\text{e}$$

In Figure 5, we have represented the three carbon emissions trajectories obtained from the observations, the targets and the trend. We notice that it is not possible to reach the NZE scenario $\mathcal{CE}_i^{\text{nze}}(2030)$ if we consider the targets released by the company: $\boldsymbol{\tau}_i^{\mathcal{T}arget} = +\infty$. On the contrary, the past trend indicates that the NZE scenario can be reached quickly[10]: $\boldsymbol{\tau}_i^{\mathcal{T}rend} = 2024.3$. As such, there is a huge gap between the past efforts made by the company and its cautious goals as shown in Figure 25 on page 66. For instance, the reduction target for 2050 should be achieved before 2027 if the carbon emissions of the company follow the past trend observed between 2007 and 2020.

Remark 6. *A special NZE scenario case is to set $\mathcal{CE}_i^{\text{nze}} = 0$. In this case, $\boldsymbol{\tau}_i$ corresponds to the date when the company is expected to emit zero carbon emissions. For the rescaled trend, if $\hat{\beta}_{i,1} < 0$, we have $\boldsymbol{\tau}_i^{\mathcal{T}rend} = t_0 - \dfrac{\mathcal{CE}_i(t_0)}{\hat{\beta}_{i,1}}$. For instance, we obtain $\boldsymbol{\tau}_i^{\mathcal{T}rend} = 2042.38$, meaning that the company can reach carbon neutrality by 2043 if it continues the same effort to reduce its carbon emissions as observed during the period 2007–2020.*

[9] We can use the rescaled trend model where $\hat{\beta}'_{i,0} = \widehat{\mathcal{CE}}_i(t_0)$.

[10] If we use the rescaled trend such that $\mathcal{CE}_i^{\mathcal{T}rend}(2020) = \mathcal{CE}_i(2020) = 39.91$, we obtain $\boldsymbol{\tau}_i^{\mathcal{T}rend} = 2026.7$.

3.1.2 The NZE gap

The NZE gap is the expected distance between the estimated carbon emissions and the NZE scenario:

$$\mathcal{G}ap_i(t^\star) = \widehat{\mathcal{CE}}_i(t^\star) - \mathcal{CE}_i^{\mathrm{nze}}(t^\star) \tag{31}$$

Again, we can use the target scenario:

$$\mathcal{G}ap_i^{\mathcal{T}arget}(t^\star) = \mathcal{CE}_i^{\mathcal{T}arget}(t^\star) - \mathcal{CE}_i^{\mathrm{nze}}(t^\star) \tag{32}$$

or the trend model:

$$\mathcal{G}ap_i^{\mathcal{T}rend}(t^\star) = \mathcal{CE}_i^{\mathcal{T}rend}(t^\star) - \mathcal{CE}_i^{\mathrm{nze}}(t^\star) \tag{33}$$

In this last case, the NZE gap is decreasing with respect to the target date $t^\star$ if the slope of the trend is negative.

If we consider the previous example, we have $\mathcal{CE}_i^{\mathrm{nze}}(2030) = 27.94$, $\mathcal{CE}_i^{\mathcal{T}arget}(2030) = 34.27$ and $\mathcal{CE}_i^{\mathcal{T}rend}(2030) = 22.08$ for the rescaled trend model. We deduce that the NZE gaps are $\mathcal{G}ap_i^{\mathcal{T}arget}(2030) = 6.33$ and $\mathcal{G}ap_i^{\mathcal{T}rend}(2030) = -5.86$. If we define the NZE scenario by the target scenario $\mathcal{CE}_i^{\mathrm{nze}}(2030) = \mathcal{CE}_i^{\mathcal{T}arget}(2030) = 34.27$, we obtain $\mathcal{G}ap_i^{\mathcal{T}rend}(2030) = -12.19$.

3.1.3 The NZE slope

The NZE slope is the value of $\hat{\beta}_{i,1}$ such that the NZE gap is closed, meaning that $\mathcal{G}ap_i^{\mathcal{T}rend}(t^\star) = 0$. We have:

$$\begin{aligned}\mathcal{G}ap_i^{\mathcal{T}rend}(t^\star) = 0 \quad &\Leftrightarrow \quad \hat{\beta}_{i,0} + \hat{\beta}_{i,1} t^\star - \mathcal{CE}_i^{\mathrm{nze}}(t^\star) = 0 \\ &\Leftrightarrow \quad \hat{\beta}_{i,1} = \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^\star) - \hat{\beta}_{i,0}}{t^\star}\end{aligned} \tag{34}$$

We notice that $\hat{\beta}_{i,1}$ depends on the intercept of the trend model. This solution is not acceptable. Therefore, we assume that $\widehat{\mathcal{CE}}_i(t_0) = \mathcal{CE}_i(t_0)$ and we use the current date t_0 as the pivot date. It follows that:

$$\begin{aligned}\mathcal{G}ap_i^{\mathcal{T}rend}(t^\star) = 0 \quad &\Leftrightarrow \quad \hat{\beta}'_{i,0} + \hat{\beta}_{i,1}(t^\star - t_0) - \mathcal{CE}_i^{\mathrm{nze}}(t^\star) = 0 \\ &\Leftrightarrow \quad \hat{\beta}_{i,1} = \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^\star) - \hat{\beta}'_{i,0}}{t^\star - t_0}\end{aligned} \tag{35}$$

Since we have $\hat{\beta}'_{i,0} = \widehat{\mathcal{CE}}_i(t_0)$ and $\widehat{\mathcal{CE}}_i(t_0) = \mathcal{CE}_i(t_0)$, we deduce that the slope to close the gap is equal to:

$$\mathcal{S}lope_i(t^\star) = \frac{\mathcal{CE}_i^{\mathrm{nze}}(t^\star) - \mathcal{CE}_i(t_0)}{t^\star - t_0} \tag{36}$$

The slope is generally negative because the gap is negative if the NZE scenario has not already been reached. Moreover, the slope is a decreasing function of the gap. The higher the gap, the steeper the slope.

Remark 7. *We can normalize the slope metric using the current carbon emissions:*

$$\overline{\mathcal{S}lope}_i(t^\star) = \frac{\mathcal{S}lope_i(t^\star)}{\mathcal{CE}_i(t_0)} \tag{37}$$

Another normalization consists in using the current slope $\hat{\beta}_{i,1}$ of the trend model. In this case, we obtain the slope multiplier:

$$m_i^{\mathcal{S}lope} = \frac{\mathcal{S}lope_i(t^\star)}{\hat{\beta}_{i,1}} \tag{38}$$

If we consider Example 5, we obtain:

$$\mathcal{S}lope_i(2030) = \frac{27.94 - 39.91}{2030 - 2020} = -1.1973$$

We reiterate that the current value of $\hat{\beta}_{i,1}$ is equal to -1.7832. In order to achieve the NZE scenario by 2030, the company must reduce its carbon emissions by 1.1973 $MtCO_2e$ per year. This represents a reduction of 3% of current emissions per year. These efforts are less substantial than what the company has done in the past. Indeed, the slope multiplier is equal to 67.14%.

3.1.4 The NZE budget

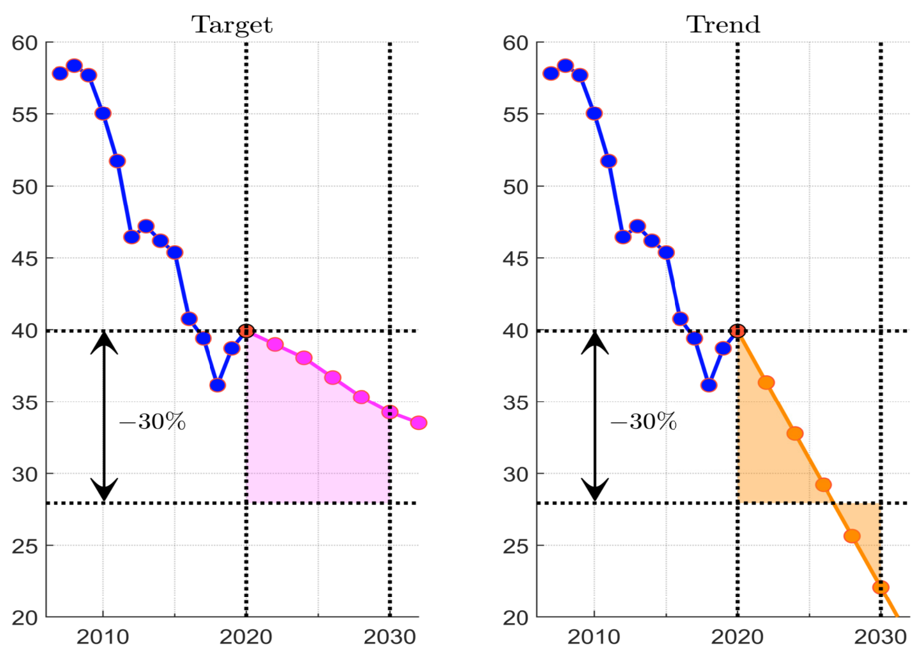
The NZE budget corresponds to the carbon budget between the date t_0 and the NZE date $t^\star$:

$$\mathcal{CB}_i(t_0, t^\star) = \int_{t_0}^{t^\star} \left(\widehat{\mathcal{CE}}_i(s) - \mathcal{CE}_i^{\mathrm{nze}}(t^\star)\right) \mathrm{d}s \tag{39}$$

As previously, we can compute the carbon budget with respect to the target trajectory or the trend. We note them respectively by $\mathcal{CB}_i^{\mathcal{T}arget}(t_0, t^\star)$ and $\mathcal{CB}_i^{\mathcal{T}rend}(t_0, t^\star)$. In Appendix A.3.1, we give the closed-form formulas of $\mathcal{CB}_i(t_0, t^\star)$ in the two cases (see Equations (107) and (112) on pages 56 and 57).

In Figure 6, we have reported the carbon budgets for Example 5. Using the closed-form formulas, we obtain $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) = 92.735$ $MtCO_2e$ and $\mathcal{CB}_i^{\mathcal{T}rend}(2020, 2030) = 30.568$ $MtCO_2e$. The difference between the two measures comes from the fact that the slope of the trend is steeper than the slope of the targets. Moreover, we notice that the last 4 years (2026–2030) have a negative contribution to $\mathcal{CB}_i^{\mathcal{T}rend}(2030)$ because the expected carbon emissions are below the NZE scenario.

Figure 6: Visualisation of $\mathcal{CB}_i^{\mathcal{T}arget}(2030)$ and $\mathcal{CB}_i^{\mathcal{T}rend}(2030)$ (Example 5)



Source: Authors' calculations.

Remark 8. *In Section 3.1.1 on page 12, we have defined the NZE duration with respect to the NZE gap. Similarly, we can define the duration with respect to the carbon budget:*

$$\boldsymbol{\tau}_i = \inf \left\{ t : \mathcal{CB}_i \left(t_0, t \right) \leq 0 \right\} \tag{40}$$

Therefore, $\boldsymbol{\tau}_i$ indicates the time required to obtain a zero carbon budget since the current date t_0. In the case of the trend model, we have:

$$\begin{aligned} \boldsymbol{\tau}_i^{\mathcal{T}rend} &= -t_0 - 2 \frac{\hat{\beta}_{i,0} - \mathcal{CE}_i^{\star}}{\hat{\beta}_{i,1}} \\ &= t_0 + 2 \frac{\mathcal{CE}_i^{\star} - \hat{\beta}'_{i,0}}{\hat{\beta}_{i,1}} \end{aligned} \tag{41}$$

For instance, using the rescaled trend model of Example 5, we obtain $\boldsymbol{\tau}_i^{\mathcal{T}rend} = 2033.43$ when $\mathcal{CE}_i^{\star} = 27.94$ $MtCO_2e$.

3.2 Dynamic NZE measures

In the previous section, we have defined static NZE metrics for a target date $t^{\star}$ given the track record of the company before the current date t_0. Most of the time, these metrics only need the current emissions $\mathcal{CE}_i \left(t_0 \right)$, and not necessarily all the historical data. In this section, we consider dynamic metrics, meaning that these metrics also depends on a future reporting date that is different from the starting date or the current date.

3.2.1 Dynamic analysis of the track record

Time contribution Let $t_1 > t_0$ be a future reporting date. We have:

$$\mathcal{CB}_i \left(t_0, t^{\star} \right) = \int_{t_0}^{t_1} \left(\widehat{\mathcal{CE}}_i \left(s \right) - \mathcal{CE}_i^{\text{nze}} \left(t^{\star} \right) \right) \mathrm{d}s + \int_{t_1}^{t^{\star}} \left(\widehat{\mathcal{CE}}_i \left(s \right) - \mathcal{CE}_i^{\text{nze}} \left(t^{\star} \right) \right) \mathrm{d}s \tag{42}$$

When the current date becomes t_1, we obtain:

$$\mathcal{CB}_i \left(t_0, t^{\star} \right) = \underbrace{\mathcal{CB}_i \left(t_0, t_1 \right)}_{\text{observed}} + \underbrace{\mathcal{CB}_i \left(t_1, t^{\star} \right)}_{\text{estimated}} \tag{43}$$

where the first component $\mathcal{CB}_i \left(t_0, t_1 \right)$ is based on realized carbon emissions and the second component $\mathcal{CB}_i \left(t_1, t^{\star} \right)$ depends on the mathematical expectation of future carbon emissions. A new reported value $\mathcal{CE}_i \left(t_1 \right)$ of carbon emissions can change the expectations, meaning that:

$$\mathbb{E} \left[\mathcal{CE}_i \left(t \right) \middle| \mathcal{F}_{t_0} \right] \neq \mathbb{E} \left[\mathcal{CE}_i \left(t \right) \middle| \mathcal{F}_{t_1} \right] \qquad \text{for } t \geq t_1 \tag{44}$$

For instance, $\mathcal{CE}_i \left(t_1 \right)$ can change the carbon trend $\mathcal{CE}_i^{\mathcal{T}rend} \left(t \right)$ because of the new estimated intercept or slope. Similarly, if the issuer announces new carbon targets at time t_1, the estimates $\mathcal{CE}_i^{\mathcal{T}arget} \left(t \right)$ will be different from before.

In order to perform a dynamic analysis, we introduce a new notation. Let $\mathcal{CB}_i \left(t_0, t_1, t^{\star} \right)$ be the carbon budget between the starting date t_0 and the target date $t^{\star}$, which is evaluated at the current date t_1. Equation (43) can be written as:

$$\mathcal{CB}_i \left(t_0, t_1, t^{\star} \right) = \mathcal{CB}_i \left(t_0, t_1, t_1 \right) + \mathcal{CB}_i \left(t_1, t_1, t^{\star} \right) \tag{45}$$

The contribution $\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right)$ of the new information observed at the date t_1 satisfies:

$$\mathcal{CB}_i\left(t_0, t_1, t^\star\right) = \mathcal{CB}_i\left(t_0, t_0, t^\star\right) + \mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right) \tag{46}$$

If $\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right) \leq 0$, the new published information allows the carbon budget to be reduced. Otherwise, it has a positive contribution and increases the carbon budget. $\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right)$ is called the time contribution of year t_1. We have:

$$\begin{aligned}
\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right) &= \mathcal{CB}_i\left(t_0, t_1, t^\star\right) - \mathcal{CB}_i\left(t_0, t_0, t^\star\right) \\
&= \int_{t_0}^{t^\star} \left(\mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_1}\right] - \mathcal{CE}_i^{\text{nze}}\left(t^\star\right)\right) \mathrm{d}s - \\
&\quad \int_{t_0}^{t^\star} \left(\mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_0}\right] - \mathcal{CE}_i^{\text{nze}}\left(t^\star\right)\right) \mathrm{d}s \\
&= \int_{t_0}^{t^\star} \left(\mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_1}\right] - \mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_0}\right]\right) \mathrm{d}s
\end{aligned} \tag{47}$$

We deduce that the time contribution is made up of two components:

$$\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right) = \mathcal{TC}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right) + \mathcal{TC}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right) \tag{48}$$

where $\mathcal{TC}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right)$ measures the forecast error between the observed trajectory and the estimate made at time t_0:

$$\mathcal{TC}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right) = \int_{t_0}^{t_1} \left(\mathcal{CE}_i(s) - \mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_0}\right]\right) \mathrm{d}s \tag{49}$$

and $\mathcal{TC}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right)$ measures the forecast revision:

$$\mathcal{TC}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right) = \int_{t_1}^{t^\star} \left(\mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_1}\right] - \mathbb{E}\left[\mathcal{CE}_i(s) \middle| \mathcal{F}_{t_0}\right]\right) \mathrm{d}s \tag{50}$$

By construction, we have:

- $\mathcal{TC}_i^{\text{error}}\left(t_0 \mid t_0, t^\star\right) = 0$ and $\mathcal{TC}_i^{\text{revision}}\left(t_0 \mid t_0, t^\star\right) = \mathcal{TC}_i\left(t_0 \mid t_0, t^\star\right)$ at the starting date;
- $\mathcal{TC}_i^{\text{error}}\left(t^\star \mid t_0, t^\star\right) = \mathcal{TC}_i\left(t^\star \mid t_0, t^\star\right)$ and $\mathcal{TC}_i^{\text{revision}}\left(t^\star \mid t_0, t^\star\right) = 0$ at the target date.

Remark 9. *We can normalize the previous quantities by current carbon emissions and the corresponding time period:*

$$\left\{\begin{array}{l}
\overline{\mathcal{TC}}_i\left(t_1 \mid t_0, t^\star\right) = \dfrac{\mathcal{TC}_i\left(t_1 \mid t_0, t^\star\right)}{\left(t^\star - t_0\right) \cdot \mathcal{CE}_i\left(t_0\right)} \\
\overline{\mathcal{TC}}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right) = \dfrac{\mathcal{TC}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right)}{\left(t_1 - t_0\right) \cdot \mathcal{CE}_i\left(t_0\right)} \\
\overline{\mathcal{TC}}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right) = \dfrac{\mathcal{TC}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right)}{\left(t^\star - t_1\right) \cdot \mathcal{CE}_i\left(t_0\right)}
\end{array}\right. \tag{51}$$

We obtain the following breakdown:

$$\overline{\mathcal{TC}}_i\left(t_1 \mid t_0, t^\star\right) = \varpi_0 \cdot \overline{\mathcal{TC}}_i^{\text{error}}\left(t_1 \mid t_0, t^\star\right) + \varpi_1 \cdot \overline{\mathcal{TC}}_i^{\text{revision}}\left(t_1 \mid t_0, t^\star\right) \tag{52}$$

where $\varpi_0 = \dfrac{t_1 - t_0}{t^\star - t_0}$, *and* $\varpi_1 = 1 - \varpi_0 = \dfrac{t^\star - t_1}{t^\star - t_0}$.

Application to the trend model Let $\hat{\beta}_{i,0}(t)$ and $\hat{\beta}_{i,1}(t)$ be the intercept and the slope coefficient of the trend model that is estimated at time t. We have:

$$\begin{aligned}\mathcal{TC}_i^{\text{error}}(t_1 \mid t_0, t^\star) &= \int_{t_0}^{t_1} \left(\mathcal{CE}_i(s) - \left(\hat{\beta}_0(t_0) + \hat{\beta}_{i,1}(t_0)\, s\right)\right) \mathrm{d}s \\ &= -\frac{1}{2}\hat{\beta}_{i,1}(t_0)\left(t_1^2 - t_0^2\right) - \hat{\beta}_{i,0}(t_0)(t_1 - t_0) + \int_{t_0}^{t_1} \mathcal{CE}_i(s)\, \mathrm{d}s \quad (53)\end{aligned}$$

and:

$$\begin{aligned}\mathcal{TC}_i^{\text{revision}}(t_1 \mid t_0, t^\star) &= \int_{t_1}^{t^\star} \left(\left(\hat{\beta}_{i,0}(t_1) + \hat{\beta}_{i,1}(t_1)\, s\right) - \left(\hat{\beta}_{i,0}(t_0) + \hat{\beta}_{i,1}(t_0)\, s\right)\right) \mathrm{d}s \\ &= \frac{1}{2}\left(\hat{\beta}_{i,1}(t_1) - \hat{\beta}_1(t_0)\right)\left(t^{\star 2} - t_1^2\right) + \left(\hat{\beta}_{i,0}(t_1) - \hat{\beta}_{i,0}(t_0)\right)(t^\star - t_1)\end{aligned} \quad (54)$$

Remark 10. *We notice that:*

$$\overline{\mathcal{TC}}_i^{\text{revision}}(t_1 \mid t_0, t^\star) = \frac{\left(\hat{\beta}_{i,0}(t_1) - \hat{\beta}_0(t_0)\right) + \left(\hat{\beta}_{i,1}(t_1) - \hat{\beta}_{i,1}(t_0)\right)\left(\dfrac{t^\star + t_1}{2}\right)}{\mathcal{CE}_i(t_0)} \quad (55)$$

The normalized forecast revision component is proportional to the intercept difference $\hat{\beta}_{i,0}(t_1) - \hat{\beta}_{i,0}(t_0)$ and the slope difference $\hat{\beta}_{i,1}(t_1) - \hat{\beta}_{i,1}(t_0)$ times the midpoint between t_1 and $t^\star$.

Example 6 is a slight modification of Example 5. We notice that the company has reduced its carbon emissions from 2007 to 2018 by 37.5%. However, it has also increased them in the last two years by 10.4%. Therefore, we can consider two carbon emissions scenarios for 2021. In the case of the black scenario, the company continues to increase its carbon emissions and we have $\mathcal{CE}_i(2021) = 41$ $MtCO_2e$. The green scenario is more optimistic since we assume that the company will continue its past efforts and we have $\mathcal{CE}_i(2021) = 36$ $MtCO_2e$.

Table 7: Carbon emissions in $MtCO_2e$ (Example 6)

Year	2007	2008	2009	2010	2011	2012	2013	2014
$\mathcal{CE}_i(t)$	57.82	58.36	57.70	55.03	51.73	46.44	47.19	46.18
Year	2015	2016	2017	2018	2019	2020	2021	
Scenario							Black	Green
$\mathcal{CE}_i(t)$	45.37	40.75	39.40	36.16	38.71	39.91	41	36

Table 8: Estimation of the rescaled trend model (Example 6)

Scenario	$\hat{\beta}_{i,0}$	$\hat{\beta}_{i,1}$	t_p	$\mathcal{CE}_i(t_p)$	$\tilde{\beta}_{i,0}$
2020	3 637.7316	−1.7832	2020	39.91	3 642.0362
Black	3 276.8078	−1.6038	2021	41.00	3 282.2509
Green	3 578.5078	−1.7538	2021	35.00	3 579.4009

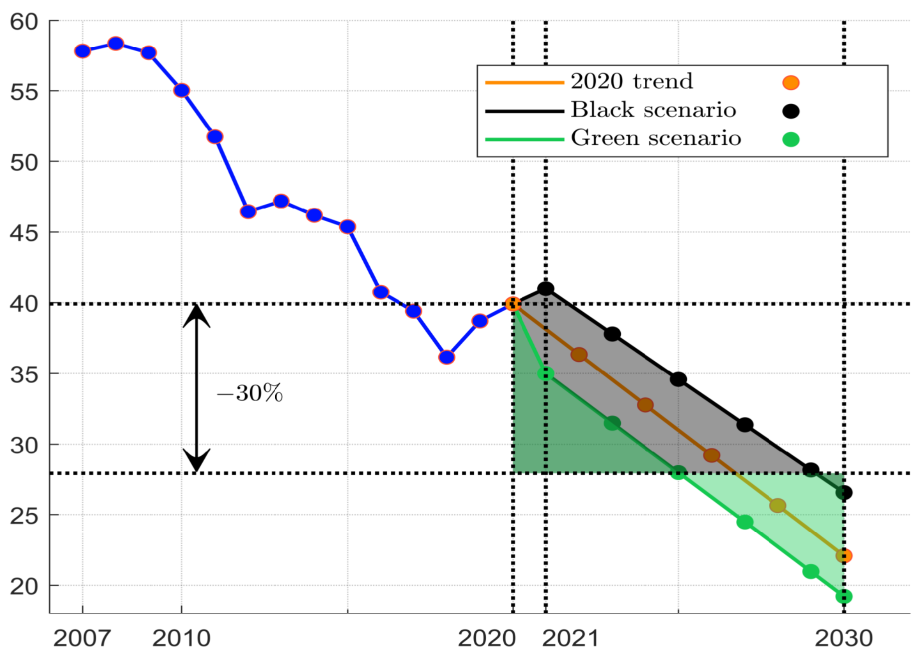
Using the carbon emissions data in Table 7, we estimate the trend model by considering the period 2007–2020 (2020 scenario). Then, we add each 2021 scenario and estimate the new trend model by considering the period 2007–2021 (black and green scenarios). Table 8 shows the estimates $\hat{\beta}_{i,0}$ and $\hat{\beta}_{i,1}$. Since we use the rescaled trend model, we also

Table 9: Time contribution of 2021 black and green scenarios in $MtCO_2e$ (Example 6)

Scenario	$\mathcal{CB}_i(t_0, t_1, t^\star)$	$\mathcal{CB}_i(t_0, t_1, t_1)$	$\mathcal{CB}_i(t_1, t_1, t^\star)$	$\mathcal{CB}_i(t_0, t_0, t^\star)$
2020	30.568	11.081	19.487	30.568
Black	65.132	12.518	52.614	30.568
Green	2.057	9.518	−7.461	30.568
Scenario	$\mathcal{CB}_i(t_0, t_1, t^\star)$	$\mathcal{TC}_i(t_1 \mid t_0, t^\star)$	$\mathcal{TC}_i^{\text{error}}(t_1 \mid t_0, t^\star)$	$\mathcal{TC}_i^{\text{revision}}(t_1 \mid t_0, t^\star)$
Black	65.132	34.563	1.437	33.127
Green	2.057	−28.512	−1.563	−26.948

report the pivot date t_p and the carbon emissions $\mathcal{CE}_i(t_p)$ that are applied to compute the intercept $\tilde{\beta}_{i,0} = \mathcal{CE}_i(t_p) - \hat{\beta}_{i,1} t_p$. The three trend models are represented in Figure 7. Since the black and green scenarios do not coincide with the current 2020 scenario, $\mathcal{CB}_i(2020, 2021, 2030)$ is different from $\mathcal{CB}_i(2020, 2020, 2030)$ as showed in Table 9. Indeed, the current carbon budget $\mathcal{CB}_i(2020, 2020, 2030)$ is equal to 30.568 $MtCO_2e$ whereas the new carbon budget $\mathcal{CB}_i(2020, 2021, 2030)$ is equal to 65.132 $MtCO_2e$ for the 2021 black scenario and 2.057 $MtCO_2e$ for the 2021 green scenario. This makes a big difference. In Table 9, we have also reported the time contribution $\mathcal{TC}_i(2021 \mid 2020, 2030)$ and the two components $\mathcal{TC}_i^{\text{error}}(2021 \mid 2020, 2030)$ and $\mathcal{TC}_i^{\text{revision}}(2021 \mid 2020, 2030)$. As expected, the black scenario has a positive contribution of 34.563 $MtCO_2e$. On the contrary, the time contribution of green scenario is negative and equal to −28.512 $MtCO_2e$, implying that the green scenario dramatically reduces the carbon budget over the period 2020–2030.

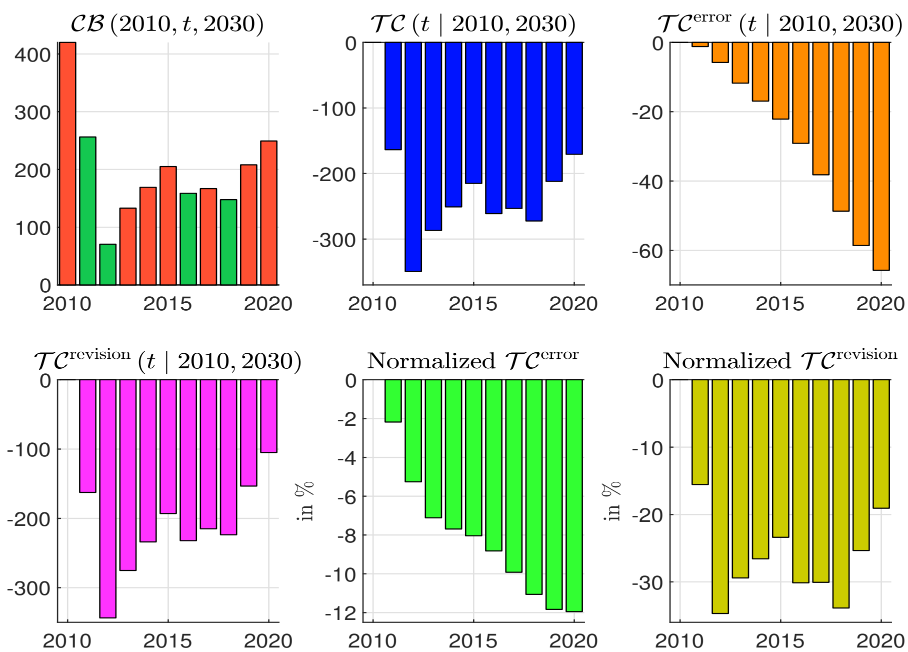
Figure 7: Impact of 2021 scenarios on the carbon budget (Example 6)



Source: Authors' calculations.

In the previous application, we consider a dynamic analysis between the current year t_0 and the next year $t_1 = t_0 + 1$. We can generalize this approach by considering any date between the starting date t_0 and the target date $t^\star$. In Figure 8, we report the carbon budget $\boldsymbol{\mathcal{CB}}_i(2010, t, 2030)$ by assuming that $2010 \leq t \leq 2020$ and $\boldsymbol{\mathcal{CE}}_i^\star(2030) = 25$ MtCO_2e. In this case, we see how adding a new observation changes the carbon budget. We have also reported the time contributions $\boldsymbol{\mathcal{TC}}_i(t \mid 2010, 2030)$, $\boldsymbol{\mathcal{TC}}_i^{\text{error}}(t \mid 2010, 2030)$ and $\boldsymbol{\mathcal{TC}}_i^{\text{revision}}(t \mid 2010, 2030)$. Since we observe a downward trend and we have $\boldsymbol{\mathcal{CE}}_i(t) > \boldsymbol{\mathcal{CE}}_i^\star(2030)$, the three contribution statistics are negative.

Figure 8: Dynamic analysis of the carbon budget $\boldsymbol{\mathcal{CB}}_i(2010, t, 2030)$ (Example 6)



Source: Authors' calculations.

Remark 11. *We observe that the revision component is the main contributor compared to the error component. As noticed previously, its dynamics depend on the dynamics of $\hat{\beta}_i(t)$, in particular the slope $\hat{\beta}_{i,1}(t)$ of the trend (see Table 23 on page 64).*

3.2.2 NZE velocity

The previous remark highlights the importance of the slope change $\Delta\hat{\beta}_{i,1}(t_1, t_2) = \hat{\beta}_{i,1}(t_2) - \hat{\beta}_{i,1}(t_1)$ between two dates t_1 and t_2. In this section, we analyze $\Delta\hat{\beta}_{i,1}(t_1, t_2)$ and define the mathematical notion of NZE velocity. Moreover, this analysis is necessary to define the NZE burn-out scenario, which is an important NZE metric.

Definition The NZE velocity $\boldsymbol{v}_i(t_1, t_2)$ is defined as:

$$\boldsymbol{v}_i(t_1, t_2) := \frac{\Delta\hat{\beta}_{i,1}(t_1, t_2)}{t_2 - t_1} \tag{56}$$

This measure is expressed in MtCO_2e. We also use the notation: $\boldsymbol{v}_i^{(h)}(t) = \boldsymbol{v}_i(t - h, t)$. This corresponds to the h-step velocity. In particular, the one-step velocity measures the change of the slope by adding a new observation: $\boldsymbol{v}_i^{(1)}(t) = \hat{\beta}_{i,1}(t) - \hat{\beta}_{i,1}(t-1)$. More generally, the NZE velocity measures the unit variation of the trend slope. A net zero emissions commitment implies a negative trend: $\hat{\beta}_{i,1}(t) < 0$. Nevertheless, it can take many years for a company to change the sign of the trend slope if it has a bad track record. Therefore, we can use the velocity to verify that the company is making significant efforts. In this case, we must have $\boldsymbol{v}_i^{(h)}(t) < 0$.

Table 10: Computation of the h-step velocity (Example 5)

t	$\hat{\beta}_{i,1}(t)$	$\boldsymbol{v}_i^{(1)}(t)$	$\boldsymbol{v}_i^{(2)}(t)$	$\boldsymbol{v}_i^{(5)}(t)$
2010	−0.903			
2011	−1.551	−0.648		
2012	−2.270	−0.719	−0.684	
2013	−2.204	0.067	−0.326	
2014	−2.076	0.127	0.097	
2015	−1.932	0.144	0.136	−0.206
2016	−2.006	−0.073	0.035	−0.091
2017	−2.016	−0.010	−0.042	0.051
2018	−2.069	−0.053	−0.032	0.027
2019	−1.949	0.120	0.034	0.026
2020	−1.783	0.166	0.143	0.030

Let us consider Example 5. The h-step velocity is reported in Table 10. Even if the slope is negative, the velocity has been positive over recent years. This indicates that the recent period is marked by a positive local carbon emissions trend. If we consider Example 7 given in Table 11, we face an opposite situation. The global trend is positive, but we observe efforts from the issuer to reduce carbon emissions in the recent period (see Table 12).

Table 11: Carbon emissions in $MtCO_2e$ (Example 7)

Year	2007	2008	2009	2010	2011	2012	2013
$\boldsymbol{\mathcal{CE}}_i(t)$	35.10	36.36	36.17	37.12	38.43	37.89	38.59
Year	2014	2015	2016	2017	2018	2019	2020
$\boldsymbol{\mathcal{CE}}_i(t)$	37.20	38.85	38.75	38.45	37.12	36.54	33.11

Table 12: Computation of the h-step velocity (Example 7)

t	$\hat{\beta}_{i,1}(t)$	$\boldsymbol{v}_i^{(1)}(t)$	$\boldsymbol{v}_i^{(2)}(t)$	$\boldsymbol{v}_i^{(5)}(t)$
2010	0.687			
2011	0.842	0.155		
2012	0.689	−0.153	0.001	
2013	0.635	−0.054	−0.103	
2014	0.444	−0.191	−0.122	
2015	0.435	−0.009	−0.100	−0.050
2016	0.402	−0.034	−0.021	−0.088
2017	0.352	−0.050	−0.042	−0.067
2018	0.258	−0.094	−0.072	−0.075
2019	0.174	−0.084	−0.089	−0.054
2020	0.018	−0.156	−0.120	−0.084

The zero-velocity scenario The velocity $\boldsymbol{v}_i^{(h)}(t)$ can be calculated by using the brute force method[11] or implementing the state space model[12] that is described in Appendix A.3.2 on page 59. In particular, we show that:

$$\begin{aligned}\boldsymbol{v}_i^{(1)}(t+1) &= \hat{\beta}_{i,1}(t+1) - \hat{\beta}_{i,1}(t) \\ &= \varphi(n)\Big(12(n+2)\cdot\overline{\mathcal{CE}}_i(t) - 18(n+1)\cdot\widetilde{\mathcal{CE}}_i(t) + 6(n-1)\cdot\mathcal{CE}_i(t+1)\Big)\end{aligned} \quad (57)$$

where n is the number of available observations until the date t and:

$$\varphi(n) = \frac{1}{(n-1)(n+1)(n+2)} \quad (58)$$

We deduce that:

$$\boldsymbol{v}_i^{(1)}(t+1) \leq 0 \Leftrightarrow \mathcal{CE}_i(t+1) \leq \mathcal{ZV}_i^{(1)}(t+1) \quad (59)$$

where $\mathcal{ZV}_i^{(1)}(t+1)$ is the value of carbon emissions to obtain a zero velocity:

$$\mathcal{ZV}_i^{(1)}(t+1) = \frac{18(n+1)\cdot\widetilde{\mathcal{CE}}_i(t) - 12(n+2)\cdot\overline{\mathcal{CE}}_i(t)}{6(n-1)} \quad (60)$$

If $\mathcal{CE}_i(s) = \mathcal{CE}_i$ for all $s \leq t$, we have $\widetilde{\mathcal{CE}}_i(t) = \overline{\mathcal{CE}}_i(t) = \mathcal{CE}_i$ and $\mathcal{ZV}_i^{(1)}(t+1) = \mathcal{CE}_i$.

Table 13: Computation of the zero-velocity scenario $\mathcal{ZV}_i^{(h)}(2021)$

h	Example 5		Example 7	
	$\mathcal{ZV}_i^{(h)}(2021)$	$\mathcal{R}_i(2020, 2021)$	$\mathcal{ZV}_i^{(h)}(2021)$	$\mathcal{R}_i(2020, 2021)$
1	33.82	15.25%	37.18	−12.30%
2	27.20	31.85%	43.42	−31.13%
3	22.39	43.90%	46.79	−41.31%
4	24.51	38.59%	50.53	−52.63%
5	24.92	37.57%	52.55	−58.70%

Let us consider Example 5. The date $t = 2020$ corresponds to $n = 14$. Moreover, we have $\overline{\mathcal{CE}}_i(2020) = 47.1964$ and $\widetilde{\mathcal{CE}}_i(2020) = 43.3328$. We deduce that $\mathcal{ZV}_i^{(1)}(2021) = 33.8222$ MtCO_2e. In order to observe a negative velocity $\boldsymbol{v}_i^{(1)}(2021)$, the company must reduce its carbon emissions by at least 15.25%. For instance, if $\mathcal{CE}_i(2021) = 30$, we obtain $\boldsymbol{v}_i^{(1)}(2021) = -0.0956$. The previous analysis can be generalized to the h-step velocity. The break-even level $\mathcal{ZV}_i^{(h)}(t)$ is the value $\mathcal{CE}_i(t)$ such that the h-step velocity is equal to zero. Therefore, we obtain $\mathcal{ZV}_i^{(h)}(t)$ by numerically solving the equation $\boldsymbol{v}_i^{(h)}(t) = 0$. In Table 13, we report the solution $\mathcal{ZV}_i^{(h)}(2021)$ for different values of h and the corresponding reduction rate $\mathcal{R}_i(2020, 2021) = 1 - \mathcal{ZV}_i^{(h)}(2021)/\mathcal{CE}_i(2020)$. In the case of Example 5, this analysis confirms that the company must dramatically reduce its carbon emissions in 2021 if it wants to stay on track to maintain the past carbon emissions reduction trends. For instance, we have $\mathcal{ZV}_i^{(3)}(2021) = 22.39$ MtCO_2e, implying a reduction of 43.90% in order to maintain a zero velocity between 2018 and 2021. On the contrary, in the case of Example 7, the company can increase its carbon emissions in 2021 without dramatically modifying the slope of the trend. This is because the trend is clearly identified in Example 7, which is not the case in Example 5.

[11] It consists in performing the rolling least squares by adding a new observation at each step.

[12] See Equations (150) and (151) on page 61.

A stochastic interpretation of the velocity Following Roncalli (2020), the linear trend (LT) model can be written as:

$$\left\{ \begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1 \end{array} \right. \tag{61}$$

where $u(t) \sim \mathcal{N}(0, \sigma_u^2)$. In this case, we have $y(t) = \beta_0 + \beta_1 t + u(t)$ where $\beta_0 = \mu(t_0) - \beta_1 t_0$. A way to introduce a stochastic trend is to add a noise $\eta(t)$ in the trend equation: $\mu(t) = \mu(t-1) + \beta_1 + \eta(t)$ where $\eta(t) \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$. Let us now assume that the slope of the trend is also stochastic:

$$\left\{ \begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1(t-1) + \eta(t) \\ \beta_1(t) = \beta_1(t-1) + \zeta(t) \end{array} \right. \tag{62}$$

where $\zeta(t) \sim \mathcal{N}\left(0, \sigma_\zeta^2\right)$. This model is called the local linear trend (LLT) model (Roncalli, 2020, page 653). Using the Kalman filter, we can estimate both the stochastic trend $\mu(t)$ and the stochastic slope $\beta_1(t)$. We notice that the one-step velocity is equal to the innovation of the slope:

$$\boldsymbol{v}^{(1)}(t) = \hat{\beta}_1(t) - \hat{\beta}_1(t-1) = \hat{\zeta}(t) \tag{63}$$

Let us come back to Example 5. For each issuer, we have estimated the parameters $(\sigma_u, \sigma_\eta, \sigma_\zeta)$ by maximizing the Whittle log-likelihood function[13]. Then, we have run the Kalman filter to obtain $\hat{\mu}_i(t)$, $\hat{\beta}_{i,1}(t)$, $\hat{u}_i(t)$, $\hat{\eta}_i(t)$ and $\hat{\zeta}_i(t)$ for the issuer i. In Table 14, we report the values of $\boldsymbol{v}_i^{(1)}(t)$ and compare them with the previous ones that were estimated using the rolling least squares (RLS). We notice that the magnitude of the velocity (in absolute value) is greater with the KF/LLT model than with the RLS/LT model. This is normal since the rolling least squares estimate a global slope from the beginning of the sample to time t whereas the Kalman filter estimates a local slope for the period $[t-1, t]$. Therefore, $\hat{\beta}_{i,1}(t)$ is an estimator of the average slope in the case of the RLS/LT model and an estimator of the marginal slope in the case of the KF/LLT model.

Table 14: Computation of the 1-step velocity using the rolling least squares and the Kalman filter (Example 5)

	RLS/LT		KF/LLT	
t	$\hat{\beta}_{i,1}(t)$	$\boldsymbol{v}_i^{(1)}(t)$	$\hat{\beta}_{i,1}(t)$	$\boldsymbol{v}_i^{(1)}(t)$
2010	−0.903		−0.257	−0.234
2011	−1.551	−0.648	−0.599	−0.343
2012	−2.270	−0.719	−1.199	−0.600
2013	−2.204	0.067	−0.968	0.231
2014	−2.076	0.127	−0.962	0.006
2015	−1.932	0.144	−0.939	0.023
2016	−2.006	−0.073	−1.506	−0.567
2017	−2.016	−0.010	−1.509	−0.003
2018	−2.069	−0.053	−1.785	−0.276
2019	−1.949	0.120	−1.101	0.684
2020	−1.783	0.166	−0.696	0.405

[13] The spectral generating function associated with Model (62) is given in Roncalli (2020, pages 679-680) whereas the Whittle log-likelihood function is given in Roncalli (2020, pages 686-687).

Remark 12. *Since the local linear trend model is more robust when estimating the velocity, we continue to use a linear trend because it is more tractable and easier to understand.*

3.2.3 The NZE burn-out scenario

The burn-out scenario refers to a sudden and violent reduction of carbon emissions in order to satisfy the NZE trajectory. We reiterate that the NZE gap is the expected distance between the estimated carbon emissions and the NZE scenario. The NZE burn-out scenario is then the value of the carbon emissions next year such that the gap is equal to zero, meaning that the NZE scenario will be achieved on average. The burn-out scenario is denoted by $\mathcal{BO}_i\left(t+1, \mathcal{CE}_i^{\text{nze}}(t^\star)\right)$ where t is the last reporting date, $\mathcal{CE}_i^{\text{nze}}(t^\star)$ is the NZE scenario and $t^\star$ is the NZE date.

Definition Let $\mathcal{R}_i^{\mathcal{T}arget}(t+1, t^\star)$ be the reduction rate between the date $t+1$ and the NZE date $t^\star$ when we consider the issuer's carbon targets. We have:

$$\widehat{\mathcal{CE}}_i(t^\star) = \left(1 - \mathcal{R}_i^{\mathcal{T}arget}(t+1, t^\star)\right) \cdot \mathcal{CE}_i(t+1) \tag{64}$$

Since the burn-out scenario satisfies the equation $\widehat{\mathcal{CE}}_i(t^\star) = \mathcal{CE}_i^{\text{nze}}(t^\star)$, we deduce that:

$$\mathcal{BO}_i^{\mathcal{T}arget}\left(t+1, \mathcal{CE}_i^{\text{nze}}(t^\star)\right) = \frac{\mathcal{CE}_i^{\text{nze}}(t^\star)}{1 - \mathcal{R}_i^{\mathcal{T}arget}(t+1, t^\star)} \tag{65}$$

If we consider the linear trend model, we have:

$$\widehat{\mathcal{CE}}_i(t^\star) = \hat{\beta}_{i,0}(t+1) + \hat{\beta}_{i,1}(t+1) \cdot t^\star \tag{66}$$

where $\hat{\beta}_{i,0}(t+1) = \hat{\beta}_{i,0}(t) + \Delta\hat{\beta}_{i,0}(t, t+1)$, $\hat{\beta}_{i,1}(t+1) = \hat{\beta}_{i,1}(t) + \Delta\hat{\beta}_{i,1}(t, t+1)$, and $\Delta\hat{\beta}_{i,0}(t, t+1)$ and $\Delta\hat{\beta}_{i,1}(t, t+1)$ are the variations of the intercept and the slope due to the new carbon emissions $\mathcal{CE}_i(t+1)$. We deduce that:

$$\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1, \mathcal{CE}_i^{\text{nze}}(t^\star)\right) = \left\{\mathcal{CE}_i(t+1) : \hat{\beta}_{i,0}(t+1) + \hat{\beta}_{i,1}(t+1) \cdot t^\star = \mathcal{CE}_i^{\text{nze}}(t^\star)\right\} \tag{67}$$

Table 15: Computation of the burn-out scenario $\mathcal{BO}_i\left(2021, \mathcal{CE}_i^{\text{nze}}(2030)\right)$ (Example 5)

$\mathcal{CE}_i^{\text{nze}}(2030)$	Target		Trend	
	$\mathcal{BO}_i^{\mathcal{T}arget}(2021)$	$\mathcal{R}_i(2020, 2021)$	$\mathcal{BO}_i^{\mathcal{T}rend}(2021)$	$\mathcal{R}_i(2020, 2021)$
5.00	5.76	85.58%	6.45	83.84%
10.00	11.51	71.16%	17.17	56.99%
15.00	17.27	56.73%	27.88	30.14%
20.00	23.02	42.31%	38.59	3.30%
25.00	28.78	27.89%	49.31	−23.55%
−30%	32.16	19.42%	55.60	−39.32%

We consider again Example 5. Previously, we have calculated $\mathcal{CE}_i^{\mathcal{T}arget}(2021) = 39.4457$ $MtCO_2e$ and $\mathcal{CE}_i^{\mathcal{T}arget}(2030) = 34.2653$ $MtCO_2e$. We deduce that $\mathcal{R}_i^{\mathcal{T}arget}(2021, 2030) = 13.13\%$. In Table 15, we report the value of the target burn-out scenario for different NZE scenarios. For instance, if $\mathcal{CE}_i^{\text{nze}}(2030) = 5$ $MtCO_2e$, $\mathcal{BO}_i^{\mathcal{T}arget}\left(2021, \mathcal{CE}_i^{\text{nze}}(2030)\right) = 5.76$ $MtCO_2e$. We have also reported the burn-out scenario for the linear trend model. We observe a discrepancy between target and trend burn-out scenarios that confirms that the carbon targets are less ambitious than the company's past efforts.

Mathematical analysis In Appendix A.3.2 on page 61, we show that:

$$\begin{aligned}\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1,\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)\right) &= \varphi_1\left(n,n^{\star}\right)\left(\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)-\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)\right)- \\ & \quad \varphi_2\left(n,n^{\star}\right)\overline{\mathcal{CE}}_i\left(t\right)+\varphi_3\left(n,n^{\star}\right)\widetilde{\mathcal{CE}}_i\left(t\right)\end{aligned} \tag{68}$$

where n and $n^{\star}$ are the numbers of observations until the dates t and $t^{\star}$, and:

$$\left\{\begin{array}{l}\varphi_1\left(n,n^{\star}\right)=\dfrac{\left(n+1\right)\left(n+2\right)}{\left(6n^{\star}-2n-4\right)} \\ \varphi_2\left(n,n^{\star}\right)=\dfrac{\left(n+2\right)\left(12n^{\star}-4n-8\right)}{\left(n-1\right)\left(6n^{\star}-2n-4\right)} \\ \varphi_3\left(n,n^{\star}\right)=\dfrac{\left(n+1\right)\left(18n^{\star}-6n-12\right)}{\left(n-1\right)\left(6n^{\star}-2n-4\right)}\end{array}\right. \tag{69}$$

Since $n^{\star}>n$, we have $\varphi_1\left(n,n^{\star}\right)>0$, $\varphi_2\left(n,n^{\star}\right)>0$ and $\varphi_3\left(n,n^{\star}\right)>0$. We notice that $\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1,\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)\right)$ is a decreasing function of the NZE gap $\mathcal{G}ap_i\left(t^{\star}\right)=\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)-\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)$ and the mean $\overline{\mathcal{CE}}_i\left(t\right)$, but an increasing function of $\widetilde{\mathcal{CE}}_i\left(t\right)$. Nevertheless, $\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)$, $\overline{\mathcal{CE}}_i\left(t\right)$ and $\widetilde{\mathcal{CE}}_i\left(t\right)$ are not independent. Indeed, we have:

$$\begin{aligned}\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right) &= \hat{\beta}_{i,0}\left(t\right)+\hat{\beta}_{i,1}\left(t\right)n^{\star} \\ &= \frac{4n+2-6n^{\star}}{n-1}\overline{\mathcal{CE}}_i\left(t\right)+\frac{6n^{\star}-3n-3}{n-1}\widetilde{\mathcal{CE}}_i\left(t\right)\end{aligned} \tag{70}$$

It follows that the relationship between $\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1,\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)\right)$, $\overline{\mathcal{CE}}_i\left(t\right)$ and $\widetilde{\mathcal{CE}}_i\left(t\right)$ is more complex. However, we can show that $\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1,\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)\right)$ is generally a decreasing function of the difference $\overline{\mathcal{CE}}_i\left(t\right)-\widetilde{\mathcal{CE}}_i\left(t\right)$. Moreover, we have:

$$\lim_{t^{\star}\rightarrow\infty}\mathcal{BO}_i^{\mathcal{T}rend}\left(t+1,\mathcal{CE}_i^{\mathrm{nze}}\left(t^{\star}\right)\right)=3\frac{n+1}{n-1}\widetilde{\mathcal{CE}}_i\left(t\right)-2\frac{n+2}{n-1}\overline{\mathcal{CE}}_i\left(t\right) \tag{71}$$

In Figure 26 on page 67, we have reported the burn-out scenario $\mathcal{BO}_i^{\mathcal{T}rend}\left(t,20\right)$ for 4 parameter sets: #1 ($n=10$, $\overline{\mathcal{CE}}_i\left(t\right)=100$, $\widetilde{\mathcal{CE}}_i\left(t\right)=100$, $\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)=80$), #2 ($n=10$, $\overline{\mathcal{CE}}_i\left(t\right)=100$, $\widetilde{\mathcal{CE}}_i\left(t\right)=80$, $\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)=60$), #3 ($n=10$, $\overline{\mathcal{CE}}_i\left(t\right)=100$, $\widetilde{\mathcal{CE}}_i\left(t\right)=80$, $\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)=40$) and #4 ($n=20$, $\overline{\mathcal{CE}}_i\left(t\right)=100$, $\widetilde{\mathcal{CE}}_i\left(t\right)=80$, $\widehat{\mathcal{CE}}_i\left(t^{\star}\mid\mathcal{F}_t\right)=40$). It is interesting to observe the impact of n and $n^{\star}$. The company has to make more effort when n is large or $n^{\star}-n$ is small, because changing the trend implies dramatically changing the current carbon emissions.

3.3 Participation, ambition and credibility for an effective NZE alignment strategy

In this section, we define the three pillars that help to evaluate a company's NZE alignment strategy. These three pillars are participation, ambition and credibility. They form the $\mathcal{PAC}$ framework, and they can be quantified using the NZE metrics.

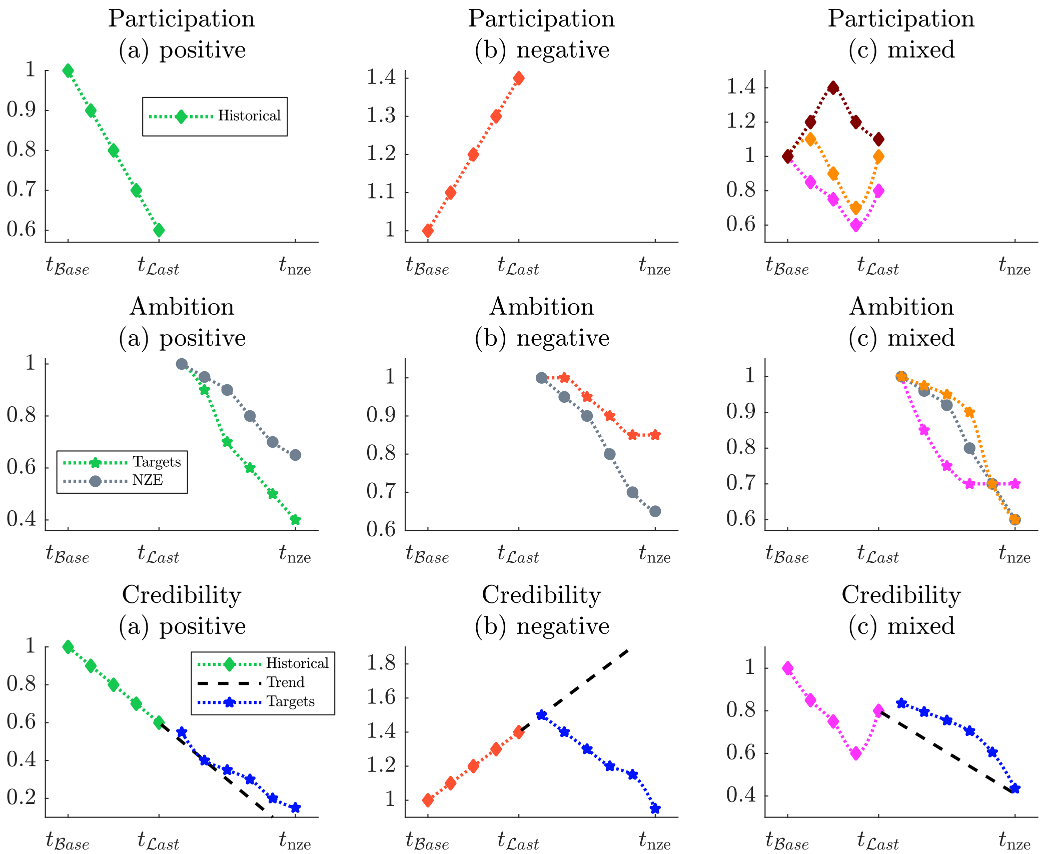
3.3.1 Definition of the $\mathcal{PAC}$ framework

Based on these new quantitative metrics, we can answer several operational questions. First, is the trend of the issuer in line with the net zero emissions scenario? For this purpose, most of the measures proposed in the previous section provide some elements of response, in particular the gap and budget metrics. In addition, to our knowledge, the dynamic NZE measures are the first quantitative indicators that allow us to track the trend of issuers' carbon emissions with respect to their historical track record. However, in the context of the current climate emergency, trajectory analysis alone is no longer sufficient. Issuers are encouraged to implement disruptive efforts at the individual level to reverse the global warming trend. These future intentions are generally expressed through their reduction targets. Although the current coverage of investment universes in terms of targets is limited (see Appendix A.2 on page 53), a more restrictive normative environment on GHG emissions is likely to increase investor interest in this type of information. Therefore, a second question arises. Is the commitment of the issuer to fight climate change ambitious? In particular, we would like to know if the target trajectory is above, below or in line with the NZE consensus scenario, which is appropriate for the sector of the issuer. This is an important topic, because achieving the net zero emissions scenario can only be possible if there are no free riders. Finally, a third question is critical and certainly the most important issue. Is the target setting of this issuer relevant and robust? Indeed, we may wonder if the target trajectory is a too ambitious promise and a form of green washing or, on the contrary, a plausible scenario. Therefore, the assessment of the issuer's targets has three dimensions or pillars: (historical) participation, ambition and credibility. They form the $\mathcal{PAC}$ framework.

3.3.2 The assessment of the $\mathcal{PAC}$ pillars

These three pillars depend on the carbon trajectories $\boldsymbol{\mathcal{CE}}_i(t)$, $\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}rend}(t)$, $\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}arget}(t)$ and $\boldsymbol{\mathcal{CE}}_i^{\mathrm{nze}}(t)$, where $\boldsymbol{\mathcal{CE}}_i(t)$ is the time series of historical carbon emissions, $\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}rend}(t)$ and $\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}arget}(t)$ are the estimated carbon emissions deduced from the trend model and the targets, and $\boldsymbol{\mathcal{CE}}_i^{\mathrm{nze}}(t)$ is the market-based NZE scenario. Generally, the participation only depends on the past observations and corresponds to the track record analysis of historical carbon emissions. The ambition compares the target trajectory on one side and the NZE scenario or the trend on the other side. Indeed, we measure to what extent companies are willing to reverse their current carbon emissions and have objectives that match the NZE scenario. Finally, we can measure the credibility of the targets by comparing the current trend of carbon emissions and the reduction targets or by analyzing the recent dynamics of the track record. We note $t_{\mathcal{B}ase}$ as the base date, $t_{\mathcal{L}ast}$ as the last reporting date and t_{nze} as the target date of the NZE scenario. In Figure 9, we illustrate the underlying ideas of the $\mathcal{PAC}$ pillars. Let us consider the first three panels. They show the historical carbon emissions of different companies. It is obvious that the company in the top-left panel has a positive participation to slow global warming, whereas the participation of the company in the top-center panel is negative. In the top-right panel, we give three examples that are mixed. In this case, we do not observe a clear pattern: downward or upward trend of carbon emissions. Therefore, the company's participation can be measured by the metrics that are related to the carbon trend. The next three panels in Figure 9 illustrate the ambition pillar. In this case, we directly compare the carbon targets of the company and the market-based NZE scenario. The companies belong to the same sector, implying that the NZE scenario is the same for the middle-left, middle-center and middle-right panels. The middle-left panel shows an ambitious company since its carbon targets are lower than the NZE scenario. In other words, the company has announced that it will make a greater effort than is expected by the market. On the contrary, the company in the middle-center panel is less ambitious, because it plans to reduce its carbon emissions at a slower pace. Finally, the middle-right panel presents two mixed situations. The first one concerns a company that has high ambitions at the beginning of the period $[t_{\mathcal{L}ast}, t_{\mathrm{nze}}]$ but it has not disclosed its ambitions for the end of the period. The company's ambition in the short term is then

Figure 9: Illustration of the participation, ambition and credibility pillars



Source: Authors' calculations.

counterweighted by the absence of ambition in the long run. The second example is about a company that concentrates its ambition in the long run. These two examples question the true willingness of these companies to substantially reduce their carbon emissions. Finally, the credibility pillar is illustrated in the last three panels in Figure 9. In this case, we compare the carbon emissions trend and the targets communicated by the company. The bottom-left panel corresponds to a credible company, since it has announced more or less a reduction trajectory that is in line with what it has done in the past. This is not the case of the company in the bottom-center panel. Clearly, it has announced a reduction of its carbon emissions, but it has continuously increased them in the past. Again, the bottom-right panel presents a mixed situation. The company has announced a reduction trajectory that is not very far from the past trend, but there are two issues. The first one is that it has increased its carbon emissions in the short term, implying that we can have some doubts about the downward trend. The second issue is that it accelerates its objective of carbon emissions reduction at the end of the period $[t_{\mathcal{L}ast}, t_{\rm nze}]$ in order to meet the requirements of the NZE scenario, but its efforts are not very substantial in the short term.

In Table 16, we report the different indicators that can been used to assess each pillar. For the participation dimension, we find metrics that are related to the carbon trend such as the slope[14] $\hat{\beta}_{i,1}$ or the estimated carbon emissions $\boldsymbol{\mathcal{CE}}_i^{\mathcal{T}rend}(t)$. We can also use

[14] The quality of the slope estimation is characterized by the coefficient of determination $\Re^2$ of the linear

the current NZE gap $\mathcal{G}ap_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}) = \mathcal{CE}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}) - \mathcal{CE}_i^{\mathrm{nze}}(t_{\mathcal{L}ast})$ or the time contribution $\mathcal{TC}_i(t_{\mathcal{L}ast}+1 \mid t_{\mathcal{L}ast}, t_{\mathrm{nze}})$. The ambition can be assessed using the NZE gap $\mathcal{G}ap_i^{\mathcal{T}arget}(t_{\mathrm{nze}}) = \mathcal{CE}_i^{\mathcal{T}arget}(t_{\mathrm{nze}}) - \mathcal{CE}_i^{\mathrm{nze}}(t_{\mathrm{nze}})$ or the NZE duration $\boldsymbol{\tau}_i^{\mathcal{T}arget}$. A more sophisticated measure is to compare the normalized[15] carbon budget $\overline{\mathcal{CB}}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$ of the company and the normalized carbon budget $\overline{\mathcal{CB}}_{\mathcal{S}ector}^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$ of the corresponding sector. Concerning the credibility, we have several measures that depend on the carbon trend: $\boldsymbol{\tau}_i^{\mathcal{T}rend}$, $\mathcal{G}ap_i^{\mathcal{T}rend}(t_{\mathrm{nze}}) = \mathcal{CE}_i^{\mathcal{T}rend}(t_{\mathrm{nze}}) - \mathcal{CE}_i^{\mathrm{nze}}(t_{\mathrm{nze}})$ and $m_i^{\mathcal{S}lope}$. We can also compare the implied slope of the company $\overline{\mathcal{S}lope}_i(t_{\mathrm{nze}})$ and the implied slope of the sector $\overline{\mathcal{S}lope}_{\mathcal{S}ector}(t_{\mathrm{nze}})$. The velocity $\boldsymbol{v}_i^{(1)}(t_{\mathcal{L}ast})$ can also be used to measure the short-term credibility of the company. The scenarios $\mathcal{ZV}_i^{(1)}(t_{\mathcal{L}ast}+1)$ and $\mathcal{BO}_i(t_{\mathcal{L}ast}+1, \mathcal{CE}_i^{\mathrm{nze}}(t^{\mathrm{nze}}))$ can also be compared to the current carbon emissions $\mathcal{CE}_i(t_{\mathcal{L}ast})$. If the scenarios are above the thresholds[16] $\varphi_{\mathcal{ZV}} \cdot \mathcal{CE}_i(t_{\mathcal{L}ast})$ and $\varphi_{\mathcal{ZV}} \cdot \mathcal{CE}_i(t_{\mathcal{L}ast})$, this indicates that the scenarios are credible. In Table 16, we indicate for each metric the mathematical conditions to satisfy the company's participation, ambition and credibility. If they are not verified, it does not necessarily mean that the company does not participate, its ambition is weak or its targets are not credible. In this case, extra-financial analysis must be conducted to understand these figures before drawing any conclusion on the company's NZE status.

Table 16: The three pillars of an effective NZE strategy

Pillar	Metric	Condition
Participation	Gap	$\mathcal{G}ap_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}) \leq 0$
	Reduction	$\mathcal{R}_i(t_{\mathcal{B}ase}, t_{\mathcal{L}ast}) < 0$
	Time contribution	$\mathcal{TC}_i(t_{\mathcal{L}ast}+1 \mid t_{\mathcal{L}ast}, t_{\mathrm{nze}}) < 0$
	Trend	$\hat{\beta}_{i,1} < 0$ and $\mathfrak{R}_i^2 > 50\%$
	Trend	$\mathcal{CE}_i^{\mathcal{T}rend}(t)$ for $t > t_{\mathcal{L}ast}$
	Velocity	$\boldsymbol{v}_i^{(1)}(t_{\mathcal{L}ast}) \leq 0$
Ambition	Budget	$\overline{\mathcal{CB}}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}}) \leq \overline{\mathcal{CB}}_{\mathcal{S}ector}^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$
	Budget	$\mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}}) \leq \mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$
	Duration	$\boldsymbol{\tau}_i^{\mathcal{T}arget} \leq t_{\mathrm{nze}}$
	Gap	$\mathcal{G}ap_i^{\mathcal{T}arget}(t_{\mathrm{nze}}) \leq 0$
Credibility	Budget	$\mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}}) > \mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$
	Burn-out Scenario	$\mathcal{BO}_i(t_{\mathcal{L}ast}+1, \mathcal{CE}_i^{\mathrm{nze}}(t^{\mathrm{nze}})) \geq \varphi_{\mathcal{BO}} \cdot \mathcal{CE}_i(t_{\mathcal{L}ast})$
	Duration	$\boldsymbol{\tau}_i^{\mathcal{T}rend} \leq t_{\mathrm{nze}}$
	Gap	$\mathcal{G}ap_i^{\mathcal{T}rend}(t_{\mathrm{nze}}) \leq 0$
	Gap	$\mathcal{G}ap_i^{\mathcal{T}rend}(t_{\mathrm{nze}}) \leq \mathcal{G}ap_i^{\mathcal{T}arget}(t_{\mathrm{nze}})$
	Slope	$\overline{\mathcal{S}lope}_i(t_{\mathrm{nze}}) \geq \overline{\mathcal{S}lope}_{\mathcal{S}ector}(t_{\mathrm{nze}})$
	Slope	$m_i^{\mathcal{S}lope} \ll 1$
	Trend	$\mathfrak{R}_i^2 > 50\%$
	Zero-velocity	$\mathcal{ZV}_i^{(1)}(t_{\mathcal{L}ast}+1) \geq \varphi_{\mathcal{ZV}} \cdot \mathcal{CE}_i(t_{\mathcal{L}ast})$

Remark 13. *If we compare the carbon budget $\mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$ using the targets and the carbon budget $\mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\mathrm{nze}})$ using the trend model, we can face two extreme situations:*

regression model. In particular, we notice that the examples on Panel (c) in Figure 9 probably have low $\mathfrak{R}^2$. This information can be used as a complementary metric for both participation and credibility.

[15] We can normalize the carbon budget with the current carbon emissions $\mathcal{CE}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast})$ or the carbon emissions of the base year $\mathcal{CE}_i^{\mathcal{T}rend}(t_{\mathcal{B}ase})$.

[16] We assume that $\varphi_{\mathcal{ZV}} < 1$ and $\varphi_{\mathcal{BO}} < 1$.

1. *The company is ambitious but not credible if* $\mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\text{nze}}) \ll \mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\text{nze}})$;

2. *The company is credible but not ambitious if* $\mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\text{nze}}) \gg \mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\text{nze}})$.

We notice that the same criterion can then be used for the ambition and credibility pillars, meaning that the pillars can be (negatively) correlated[17].

3.3.3 A scoring system for the three NZE pillars

In order to analyze the $\mathcal{PAC}$ pillars, we can use a scoring system. The scores are built using the z-score technique:

$$z_i = \frac{s_i - \mu(s_i)}{\sigma(s_i)} \tag{72}$$

where s_i is the score of the issuer i, and $\mu(s_i)$ and $\sigma(s_i)$ are the cross-section mean and standard deviation. Then, the z-scores are transformed into q-scores:

$$q_i = \Phi(z_i) \tag{73}$$

In this case, we have $q_i \in [0,1]$. We assume that the pillar's measurement is an increasing function of the score s_i. The higher the score, the better the pillar. For instance, in the case of the participation score $\mathcal{PS}_i$, we can use the opposite of the normalized slope coefficient:

$$s_i = -\frac{\hat{\beta}_{i,1}}{\mathcal{CE}_i(t_{\mathcal{L}ast})} \tag{74}$$

For the ambition score $\mathcal{AS}_i$, we can consider the carbon budget metric:

$$s_i = \frac{\mathcal{CB}_i^{\mathcal{T}rend}(t_{\mathcal{L}ast}, t_{\text{nze}}) - \mathcal{CB}_i^{\mathcal{T}arget}(t_{\mathcal{L}ast}, t_{\text{nze}})}{\mathcal{CE}_i(t_{\mathcal{L}ast})} \tag{75}$$

The credibility score $\mathcal{CS}_i$ can be built by using the burn-out scenario:

$$s_i = \frac{\mathcal{BO}_i\left(t_{\mathcal{L}ast} + 1, \mathcal{CE}_i^{\text{nze}}(t^{\text{nze}})\right)}{\mathcal{CE}_i(t_{\mathcal{L}ast})} \tag{76}$$

We can also extend the single-score method to a multi-score approach[18]:

$$q_i = \frac{1}{m}\sum_{j=1}^{m} \Phi\left(\frac{s_{i,j} - \mu_j(s_{i,j})}{\sigma_j(s_{i,j})}\right) \tag{78}$$

where $s_{i,j}$ is the score of the issuer i for the metric j and $\mu_j(s_{i,j})$ and $\sigma_j(s_{i,j})$ are the cross-section mean and standard deviation of the metric j.

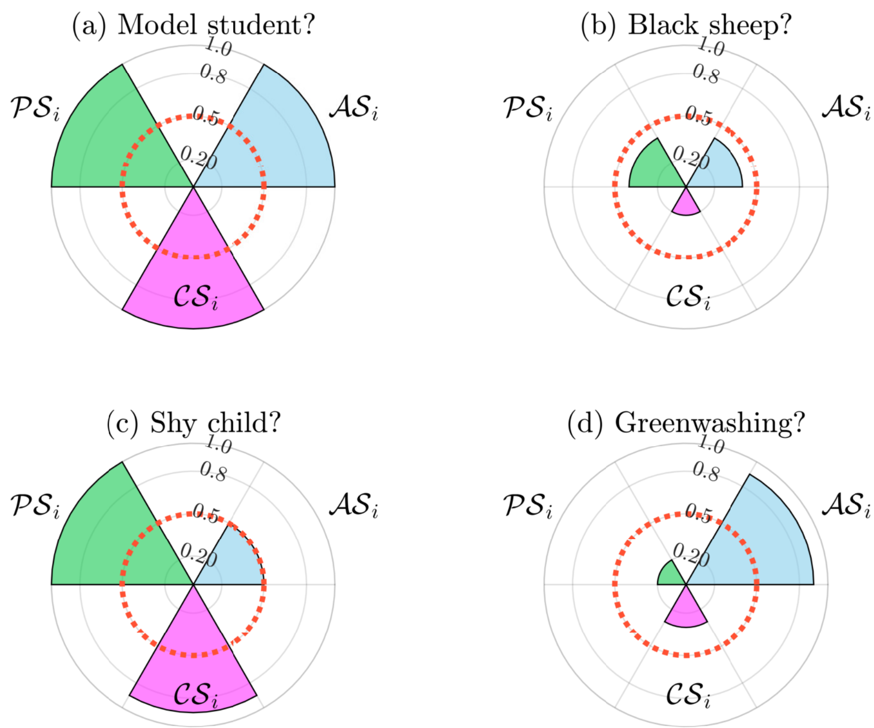
In Figure 10, we have represented several configurations of the $\mathcal{PAC}$ scoring system. If the three scores $\mathcal{PS}_i$, $\mathcal{AS}_i$ and $\mathcal{CS}_i$ are high and greater than 0.5 (which is the median value of the q-score), the company is both ambitious and credible and has already made some efforts to reduce its carbon emissions. On the contrary, in Panel (b), we have a company, whose three

[17] In Section 4.2.1 on page 37, we elaborate the correlation patterns between the different pillars.

[18] An alternative approach is to use the following aggregation method:

$$q_i = \Phi\left(\frac{1}{m}\sum_{j=1}^{m} \frac{s_{i,j} - \mu_j(s_{i,j})}{\sigma_j(s_{i,j})}\right) \tag{77}$$

Figure 10: The $\mathcal{PAC}$ scoring system



Source: Authors' calculations.

scores are below the median. These two extreme cases are very frequent. Nevertheless, we can also obtain a more balanced scoring. For instance, Panel (c) corresponds to a company that has substantially reduced its past emissions but has announced weak reduction targets. Therefore, its ambition score is low, but its credibility score is high. It may be a company that does not talk a lot about its climate change policy, but its track record has demonstrated that it is committed. Finally, Panel (d) represents the scoring of a company with very high ambition, but it has continuously increased its carbon emissions in the past. Therefore, we can suspect a type of greenwashing. These examples show that the three dimensions are correlated. For instance, we can assume a positive correlation between participation and credibility, and a negative correlation between ambition and credibility. Indeed, high credibility can only be obtained if participation is high or ambition is weak. Similarly, low credibility can be associated with excessively high ambition or weak participation. Therefore, the correlation between participation and credibility is unclear[19].

4 Empirical results and applications

In this section, we present the empirical results when we consider the previous framework to analyze the NZE policy of issuers and the scope 3 carbon emissions. First, we propose some narratives concerning the example of three companies. Then, we consider the CDP database and estimate the different net zero carbon metrics. We calculate the correlation matrix of these measures and define the most relevant metrics. We also apply the $\mathcal{PCA}$ framework to the MSCI World index. We can then conduct an analysis of this portfolio in terms of

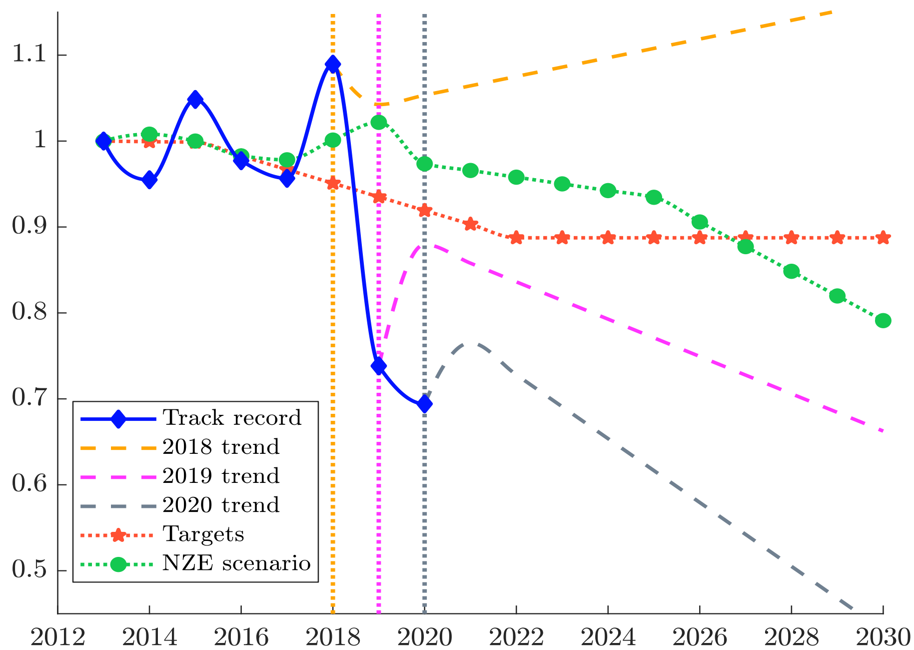
[19]This issue is discussed in Section 4.2.1 on page 37.

sector, country and time horizon. We also analyze the global dynamics of the MSCI World index these last years. Finally, we propose a new model to perform portfolio alignment with respect to an NZE scenario. First, we show that portfolio decarbonization and portfolio alignment are two different concepts. Second, we illustrate how net zero carbon metrics can be used in portfolio alignment.

4.1 A tale of three companies

For cross-sectional company comparisons, we introduce a common base year, which is for instance 2013. Then, we rebase our trajectories by the carbon emissions $\boldsymbol{\mathcal{CE}}_i\,(2013)$. In what follows, we consider three examples to illustrate the framework previously defined.

Figure 11: Carbon emissions, trends and targets and NZE scenario (Company A)

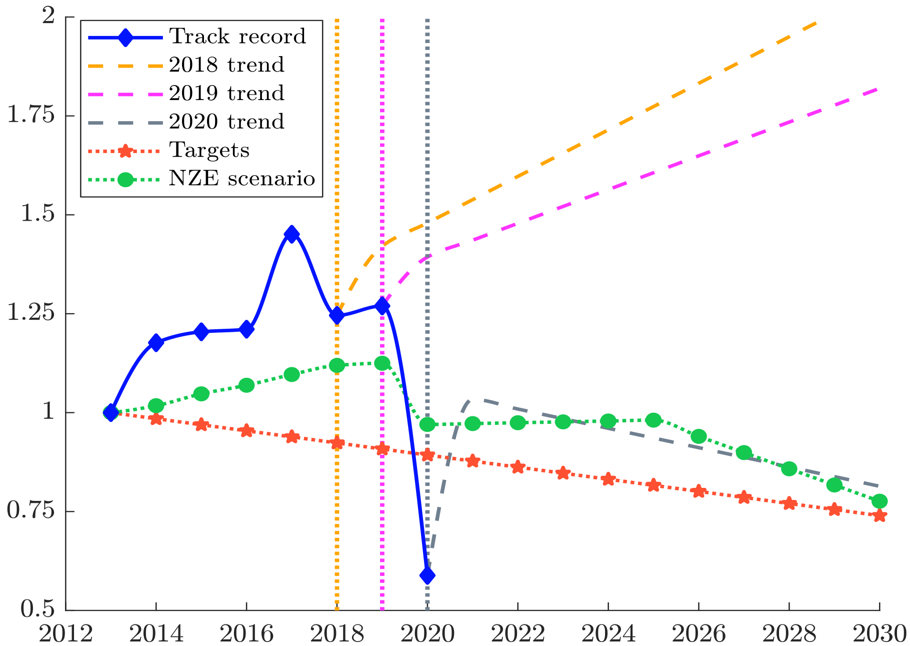


Source: CDP database (2021) & Authors' calculations.

Company A is a US based multinational technology conglomerate. We report its carbon emissions and its targets in Figure 11. Concerning the market-based scenario, we use the NZE IEA scenario. In our IEA sectorial mapping, we classify the company within the Industrials IEA sector, which implies a substantial reduction requirement as indicated in Appendix A.4.1 on page 62. Company A is a particularly relevant example of the expectations from investors in the new NZE context. Indeed, participation switched favorably after 2018 with a significant reduction in the scope 3 emissions sourced from the use of sold products. We observe the change of sign of the slope between the yellow dashed line (2018 trend) and the pink dashed line (2019 trend). The credibility is confirmed with the 2020 data point as the duration $\boldsymbol{\tau}^{\mathcal{T}rend}$ drops under the duration of the NZE scenario. Company A has communicated in September 2021 on their ambition. They committed to net zero GHG emissions by 2040. Our dataset[20] did not include these most recent targets for Company A.

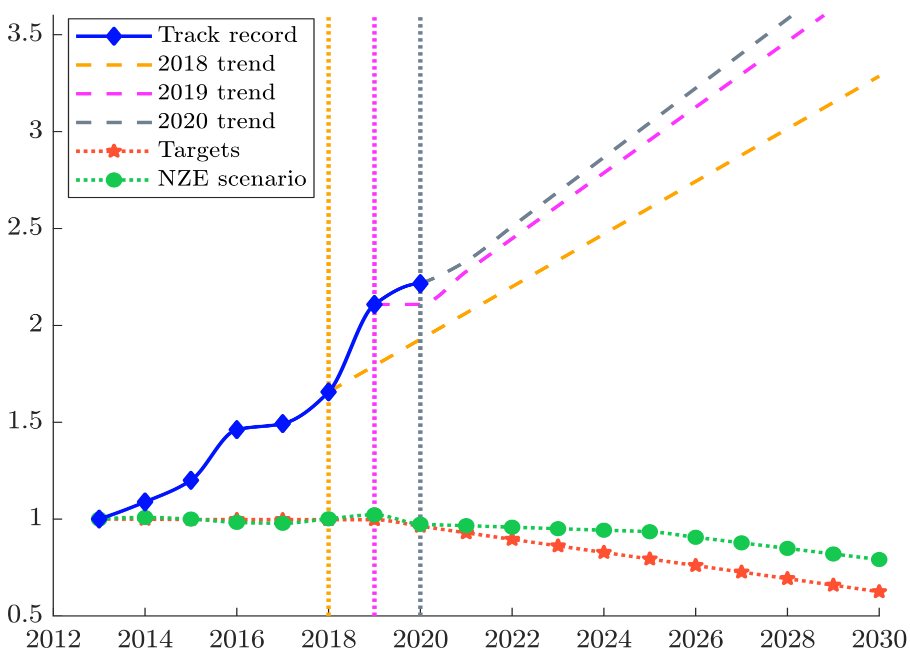
[20]Indeed, for now, we are using target data which are only updated annually.

Figure 12: Carbon emissions, trends and targets and NZE scenario (Company B)



Source: CDP database (2021) & Authors' calculations.

Figure 13: Carbon emissions, trends and targets and NZE scenario (Company C)



Source: CDP database (2021) & Authors' calculations.

Figure 12 illustrates Company B which is a major US airline. We classify Company B in the Transport IEA sector which also requires a substantial reduction. Its participation switched favorably in 2020 as the grey dashed line (2020 trend) changed sign compared to the yellow and pink dashed lines (2018 trend and 2019 trend). The credibility has not switched even with the significant drop in 2020, since the duration $\tau^{\mathcal{T}rend}$ remains larger than the NZE time horizon. Company B announced a carbon neutrality ambition in September 2021; however, their policy seems to require the purchase of carbon offsets. We have preferred to focus on emissions rather than offset strategies in our analysis. All-the-more so given that for company B, the reduction of $\mathcal{CE}_i\,(2020)$ sourced from both scope 1 and scope 3 emissions is related to the drop in activity due to the Covid-19 crisis. Since the company did not engage in a structural change, this does not bode well for the coming years.

Company C is a European multinational company which supplies industrial resources and services to various industries (Figure 13). As such, we classify it within the Industrials IEA sector for the NZE scenario. Again, it is required to display a sharp reduction as for Company A. The company has a clear ambition and has embraced the NZE context. However, our metrics indicate that in terms of participation, the trend has not been negative and has deteriorated in previous years. We stress here that although Company C pays attention to its carbon intensity policy, it has not been active on the absolute carbon emissions level. Company C positions its business on the climate change opportunity. For instance, it has stated that carbon capture projects are an opportunity to reduce its carbon taxation and to engage in clean technologies. However, the company has not yet reconciled the revenues deriving from such technologies aiming at mitigating climate change — the monetary aspect — with the importance of protecting the environment and biodiversity.

4.2 Empirical results

4.2.1 Results with the CDP database

We generalize the previous analysis to the CDP database. For that, we only consider the issuers having at least one reduction target between 2013 and 2030 and a full track record of carbon emissions between 2013 and 2020. We also remove some outliers and impose having an ISIN code for the issuer in order to match the CDP data with the IEA and GICS sector classification. Finally, we obtain a database of 751 issuers. In Table 24 on page 64, we give the breakdown per GICS and IEA sectors. We notice that Industry and Other are the two most represented IEA sectors, followed by Electricity and Transport[21]. If we focus on the GICS classification, Financials and Industrials dominate the other sector. In Table 25 on page 65, we find that 80% of issuers are located in developed markets (44% in Europe and 31% in North America). If we consider EM issuers, they are mainly located in Brazil, Korea and Taiwan since these three countries represent 52% of the EM issuers. Some EM countries are almost absent: one issuer in China, five issuers in India and one issuer in Russia.

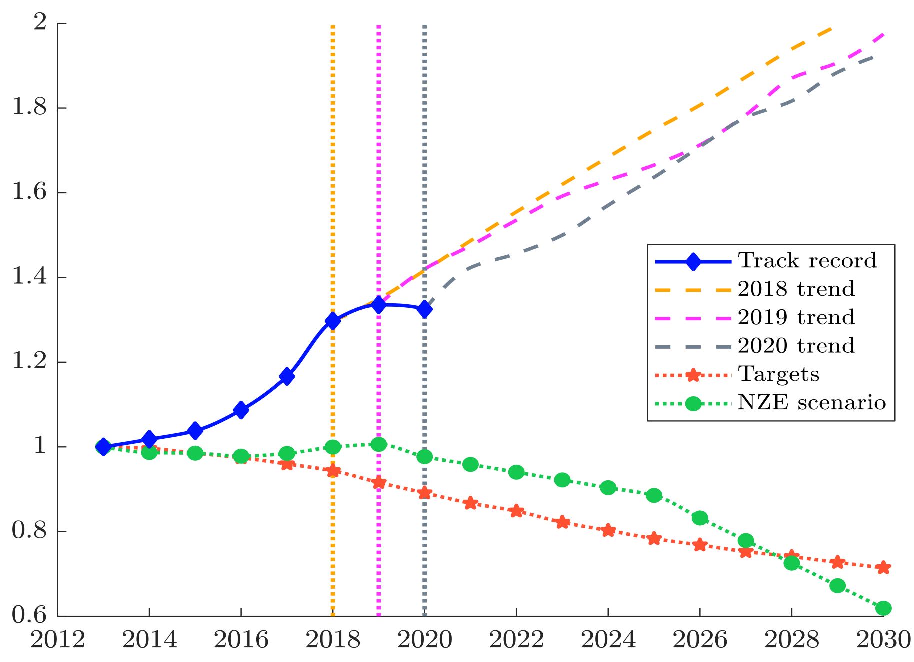
Remark 14. *Since there are very few comprehensive observations (*751 *issuers) and some strong representative bias in terms of country/sector levels, we have to be very careful to (over) interpret the below results. First, we note an obvious reporting bias using CDP database: questionnaire being aimed at climate-relevant industries mainly. The organization therefore targets sectors that matter the most. On the other hand, we observe an endogenous bias, because we only select the issuers that have announced reduction targets. Therefore, we can assume that these issuers are greener that the full universe. A third bias is related to the measurement of the scope 3 emissions. Indeed, we know that it is a challenge for a company to estimate both upstream and downstream carbon emissions. Actually, their measurement*

[21] The Buildings sector is not represented.

continues to be complicated and non-exhaustive. Therefore, the scope 3 emissions of some companies may increase just because they improve their measurement approach or use more exhaustive data.

Global analysis In Figure 14, we report the median value of carbon emissions $\mathcal{CE}_i(t)$, the median value of target trajectories $\mathcal{CE}_i^{\mathcal{T}arget}(t)$ and the average value of NZE scenarios $\mathcal{CE}_i^{\text{nze}}(t)$. We also show the rescaled carbon trends $\mathcal{CE}_i^{\mathcal{T}rend}(t \mid \mathcal{F}_{2018})$, $\mathcal{CE}_i^{\mathcal{T}rend}(t \mid \mathcal{F}_{2019})$ and $\mathcal{CE}_i^{\mathcal{T}rend}(t \mid \mathcal{F}_{2020})$. On a global basis, we observe an increase of carbon emissions between 2013 and 2019, and a plateau in 2020, which is probably due to emissions reduction related to the Covid-19 crisis. Carbon targets are in line with the NZE scenario until 2025. After this date, we clearly see that the targeted reduction rates are lower than the NZE required reduction rates[22]. Concerning the trend, we do not observe a large difference between the three years (2018, 2019 and 2020). We notice that the reduction targets are more or less in line with the NZE scenario. However, and more strikingly, there is a huge gap between the upward trend between 2013 and 2020 and what has been announced by the companies. Figure 14 perfectly illustrates the interest of the $\mathcal{PAC}$ framework, since we observe inconsistencies between the ambition of these issuers on one side, and their participation and credibility on another side.

Figure 14: Carbon emissions, trends and targets and NZE scenario (median analysis, global universe)



Source: CDP database (2021) & Authors' calculations.

For each issuer, we compute the slope of the trend $\hat{\beta}_{i,1}(t_{\mathcal{L}ast})$ for the last three years 2018, 2019 and 2020. In Table 17, we report the 25th percentile, the median and the 75th percentile of the slope by considering two normalization factors. The first one uses the

[22] This suggests that monitoring the companies' targets in the coming years is very important as they will be required to express more ambitious reduction for the long term.

carbon emissions $\mathcal{CE}_i(2013)$ of the base year whereas the second one considers the last reported carbon emissions $\mathcal{CE}_i(t_{\mathcal{L}ast})$, where $t_{\mathcal{L}ast}$ is respectively equal to 2018, 2019 and 2020. The median analysis confirms the previous results. Even if the slope has decreased in recent years, it is positive on average. For instance, the median value of $\hat{\beta}_{i,1}(2020)$ is equal to 3.66% of the 2020 carbon emissions. This implies an increase of carbon emissions of 36.6 $ktCO_2e$ per year if the current carbon emissions are equal to 1 $MtCO_2e$. In Table 17, we also observe the skewness of the slope since we have an asymmetry between the 25th and 75th percentiles. When we compare Figures 27 and 28 on page 67, the skewness is striking only when we normalize by the base year (2013). In fact, we face a problem when we normalize the metrics by the base year's carbon emissions. Indeed, if the slope of the issuer is highly negative, we underestimate the lower percentiles and overestimate the higher percentiles because of the compound effect. This is why it is better to use the last reported carbon emissions. Nevertheless, we continue to observe a relatively strong skewness. One of the reasons is that the carbon emissions of past years may be underestimated by the issuers, because the measuring methodology and tools were less mature to compute carbon emissions in an accurate way, in particular for the scope 3 emissions, which are still subject to much uncertainty today. In Table 17, we also report the frequency of negative slopes. On average, only 33% of issuers have a negative trend, implying that 67% of issuers have not reduced their carbon emissions since 2013.

Table 17: Statistics of the normalized slope and velocity (expressed in %)

Slope		$\frac{\hat{\beta}_{i,1}(t_{\mathcal{L}ast})}{\mathcal{CE}_i(2013)}$			$\frac{\hat{\beta}_{i,1}(t_{\mathcal{L}ast})}{\mathcal{CE}_i(t_{\mathcal{L}ast})}$			$\#\left\{\hat{\beta}_{i,1} < 0\right\}$
		$Q_{25\%}$	$Q_{50\%}$	$Q_{75\%}$	$Q_{25\%}$	$Q_{50\%}$	$Q_{75\%}$	
$t_{\mathcal{L}ast}$	2018	-2.44	6.06	41.29	-2.85	4.46	12.93	32.36
	2019	-2.13	6.38	44.23	-2.31	4.18	11.42	29.56
	2020	-2.97	6.16	52.01	-3.82	3.66	10.60	32.62
Velocity		$\frac{v_i^{(1)}(t_{\mathcal{L}ast})}{\mathcal{CE}_i(2013)}$			$\frac{v_i^{(1)}(t_{\mathcal{L}ast})}{\mathcal{CE}_i(t_{\mathcal{L}ast})}$			$\#\left\{v_i^{(1)}(t_{\mathcal{L}ast}) < 0\right\}$
		$Q_{25\%}$	$Q_{50\%}$	$Q_{75\%}$	$Q_{25\%}$	$Q_{50\%}$	$Q_{75\%}$	
$t_{\mathcal{L}ast}$	2019	-4.38	-0.09	2.62	-2.15	-0.37	1.99	51.27
	2020	-6.99	-1.53	1.15	-3.68	-0.99	1.11	65.11

Source: CDP database (2021) & Authors' calculations.

The velocity analysis demonstrates that the slope variation is negative on average both in 2019 and 2020. Moreover, we observe an acceleration of these dynamics. Indeed, the velocity was negative in 2019 in 51% of cases, whereas this figure is equal to 65% in 2020. In fact, all the statistics (median, 25th and 75th percentiles) are lower in 2020 than in 2019 whatever the normalization method.

Table 18: Statistics of the budget difference

$\Delta\mathcal{CB}_i(2020, 2030)$	$Q_{25\%}$	$Q_{50\%}$	$Q_{75\%}$	$\#\{<0\}$
$\mathcal{CB}_i^{\mathcal{T}rend}(2020,2030) - \mathcal{CB}_i^{\text{nze}}(2020,2030)$	-3.00	5.78	13.45	32.9%
$\mathcal{CB}_i^{\mathcal{T}arget}(2020,2030) - \mathcal{CB}_i^{\text{nze}}(2020,2030)$	-1.54	-0.18	0.54	59.9%
$\mathcal{CB}_i^{\mathcal{T}arget}(2020,2030) - \mathcal{CB}_i^{\mathcal{T}rend}(2020,2030)$	-14.48	-7.19	2.64	68.9%

Source: CDP database (2021) & Authors' calculations.

Let us now consider a budget analysis. We define the following budgets: the trend budget $\mathcal{CB}_i^{\mathcal{T}rend}(2020, 2030) = \int_{2020}^{2030} \mathcal{CE}_i^{\mathcal{T}rend}(t)\,\mathrm{d}t$, the target budget $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) = \int_{2020}^{2030} \mathcal{CE}_i^{\mathcal{T}arget}(t)\,\mathrm{d}t$ and the NZE budget $\mathcal{CB}_i^{\mathrm{nze}}(2020, 2030) = \int_{2020}^{2030} \mathcal{CE}_i^{\mathrm{nze}}(t)\,\mathrm{d}t$. In Table 18, we consider the carbon budget normalized by the current emissions and have reported the percentile statistics. Only 33% of companies have a negative carbon budget $\mathcal{CB}_i^{\mathcal{T}rend}(2020, 2030) \leq \mathcal{CB}_i^{\mathrm{nze}}(2020, 2030)$ between 2020 and 2030. This means that 67% of companies have a trend budget larger than the NZE budget. This contrasts with the figures on $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) - \mathcal{CB}_i^{\mathrm{nze}}(2020, 2030)$. Indeed, almost 60% of companies have carbon budget objectives larger than the NZE corresponding scenario. As a consequence, we verify that the trend budget is greater than the target budget on average. Again, we observe a high skew on the difference between the trend budget and the target budget, since some companies have a trend which is incompatible with their targets.

In Figure 15, we show the Spearman correlations between 12 net zero carbon metrics[23]: (1) the slope $\hat{\beta}_{i,1}$, (2) the velocity $\boldsymbol{v}_i^{(1)}(2020)$, (3) the current gap of the trend model $\mathcal{G}ap_i^{\mathcal{T}rend}(2020)$, (4) the 2030 gap of the carbon targets $\mathcal{G}ap_i^{\mathcal{T}arget}(2030)$, (5) the net budget of the carbon targets $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) - \mathcal{CB}_i^{\mathrm{nze}}(2020, 2030)$, (6) the budget difference $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) - \mathcal{CB}_i^{\mathcal{T}rend}(2020, 2030)$, (7) the trend duration $\boldsymbol{\tau}_i^{\mathcal{T}rend}$, (8) the 2030 gap of the trend model $\mathcal{G}ap_i^{\mathcal{T}rend}(2030)$, (9) the gap difference $\mathcal{G}ap_i^{\mathcal{T}rend}(2030) - \mathcal{G}ap_i^{\mathcal{T}arget}(2030)$, (10) the (non-normalized) slope multiplier $m_i^{\mathcal{S}lope}$, (11) the burn-out scenario $\mathcal{BO}_i\left(2021, \mathcal{CE}_i^{\mathrm{nze}}(2030)\right)$ and (12) the zero-velocity scenario $\mathcal{ZV}_i^{(1)}(2021)$. As expected, we observe a high positive correlation between:

- the slope $\hat{\beta}_{i,1}$, the current gap of the trend model $\mathcal{G}ap_i^{\mathcal{T}rend}(2020)$ and the trend duration $\boldsymbol{\tau}_i^{\mathcal{T}rend}$;
- the 2030 gap of the carbon targets $\mathcal{G}ap_i^{\mathcal{T}arget}(2030)$ and the net budget of the carbon targets $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) - \mathcal{CB}_i^{\mathrm{nze}}(2020, 2030)$;
- the trend duration $\boldsymbol{\tau}_i^{\mathcal{T}rend}$ and the slope multiplier $m_i^{\mathcal{S}lope}$;
- the 2030 gap of the trend model $\mathcal{G}ap_i^{\mathcal{T}rend}(2030)$ and the gap difference $\mathcal{G}ap_i^{\mathcal{T}rend}(2030) - \mathcal{G}ap_i^{\mathcal{T}arget}(2030)$;

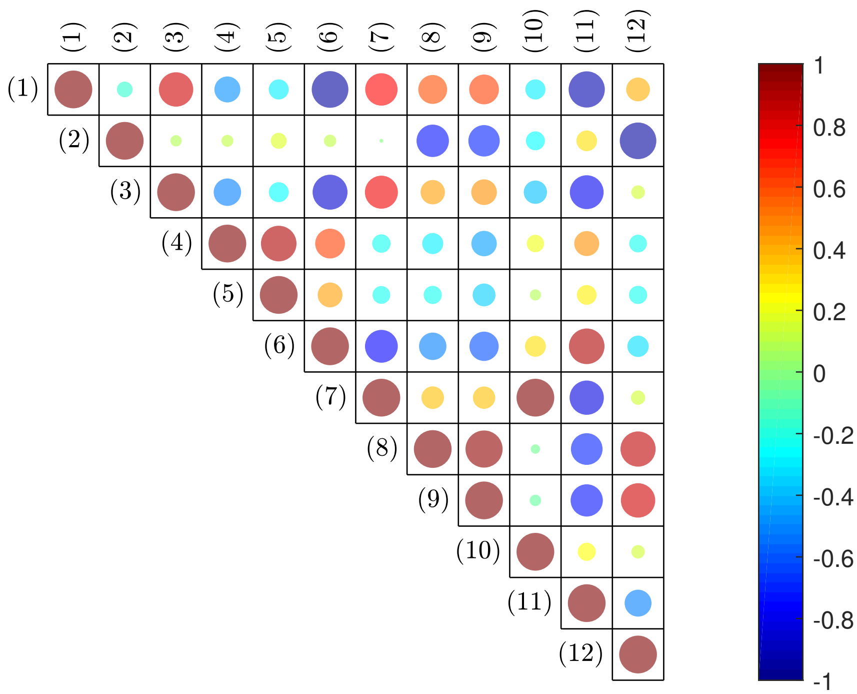
and a high negative correlation between:

- the slope $\hat{\beta}_{i,1}$ and the current gap of the trend model $\mathcal{G}ap_i^{\mathcal{T}rend}(2020)$ on the one hand, and the budget difference $\mathcal{CB}_i^{\mathcal{T}arget}(2020, 2030) - \mathcal{CB}_i^{\mathcal{T}rend}(2020, 2030)$ and the burn-out scenario $\mathcal{BO}_i\left(2021, \mathcal{CE}_i^{\mathrm{nze}}(2030)\right)$ on the other hand;
- the trend duration $\boldsymbol{\tau}_i^{\mathcal{T}rend}$ and the burn-out scenario $\mathcal{BO}_i\left(2021, \mathcal{CE}_i^{\mathrm{nze}}(2030)\right)$.

If we consider the $\mathcal{PAC}$ scoring system built with the single scores given by Equations (74), (75) and (76), we obtain the correlation matrix in Figure 16. We notice that the participation score is negatively correlated to the ambition score. This indicates that companies that have already made efforts to reduce their carbon emissions are less ambitious. On the contrary, we observe a positive correlation between the participation score and the credibility score, meaning that a company can only be credible if it has already provided evidence of engagement. Finally, ambition and credibility scores are negatively correlated. This seems obvious because unambitious scenarios of carbon reduction have a greater probability of occurring than highly ambitious scenarios and are much easier to achieve!

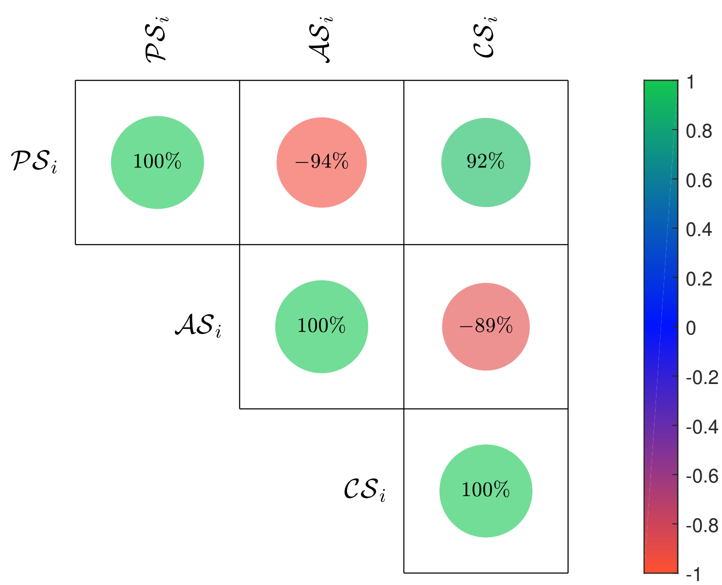
[23] The carbon metrics are normalized by the carbon emissions $\mathcal{CE}_i(2020)$.

Figure 15: Rank correlation matrix of the $\mathcal{PAC}$ metrics



Source: CDP database (2021) & Authors' calculations.

Figure 16: Rank correlation matrix of the $\mathcal{PAC}$ scoring system



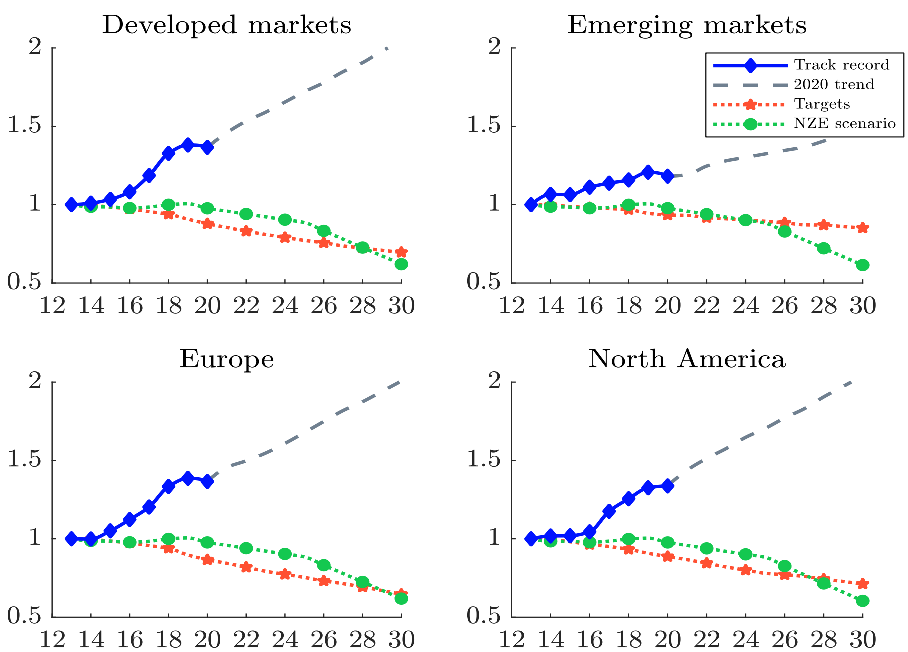
The scoring system is based on the q-scores with $s_i = -\hat{\beta}_{i,1} / \mathcal{CE}_i\,(2020)$ for the participation score $\mathcal{PS}_i$, $s_i = \left(\mathcal{G}ap_i^{\mathcal{T}arget}\,(t_{\text{nze}}) - \mathcal{G}ap_i^{\mathcal{T}rend}\,(t_{\text{nze}})\right) / \mathcal{CE}_i\,(2020)$ for the ambition score $\mathcal{AS}_i$, and $s_i = \mathcal{BO}_i\left(2021, \mathcal{CE}_i^{\text{nze}}\,(2030)\right)$ for the credibility score $\mathcal{CS}_i$.

Source: CDP database (2021) & Authors' calculations.

Remark 15. *The correlations between the three scores depend on the metrics used to build the $\mathcal{PAC}$ scoring system. For instance, if we use the gap difference*[24] *for the credibility score $\mathcal{CS}_i$ instead of the burnout scenario, the magnitude of correlations is lower but the sign is the same (see Figure 38 on page 73). Nevertheless, we can obtain a positive weak correlation between ambition and credibility if we use the slope multiplier ($s_i = 1 - m_i^{\mathcal{S}lope}$) to measure the credibility (see Figure 39 on page 73).*

Regional analysis We have done the same previous exercise by distinguishing developed and emerging markets. Results are given in Figures 29 and 30 on page 68. Although the significance of the results is mitigated by reporting bias, we notice two stylized facts. Reporting companies located in developed markets have higher carbon emissions and also higher reduction targets than companies located in emerging markets. If we compare European and American issuers, the carbon trajectories (carbon emissions, reduction targets and nze scenario) are very similar[25]. These different results are summarized in Figure 17. In fact, it is very difficult to extract robust information from this analysis, because the sample is small (751 issuers), and the sector representation is not the same across countries and regions. Moreover, there is a strong bias, because this analysis only concerns issuers that have announced some reduction targets. By construction, we can consider that these issuers are the most concerned by climate change. Nevertheless, we also observe that some issuers are among the most polluting companies. In this case, they may be motivated by showing to the world that they are taking action in the fight against climate change.

Figure 17: Carbon emissions, trends and targets and NZE scenario (median analysis, regional analysis)



Source: IEA (2021), CDP database (2021) & Authors' calculations.

[24]The score s_i is equal to $\left(\mathcal{G}ap_i^{\mathcal{T}arget}(2030) - \mathcal{G}ap_i^{\mathcal{T}rend}(2030)\right) / \mathcal{CE}_i(2020)$.

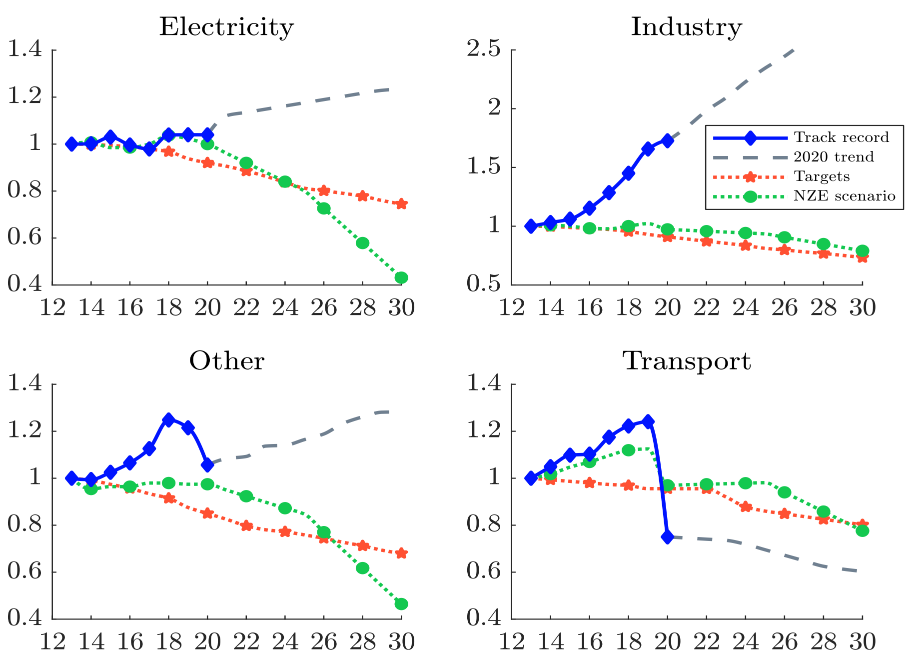
[25]See Figures 31 and 32 on page 69.

Table 19: Frequencies of targets lower or greater than the trend in 2030

	Lower	Greater
DM	79.21%	20.79%
EM	71.03%	28.97%
Europe	79.17%	20.83%
North America	76.92%	23.08%
EMU	79.90%	20.10%
Asia	67.61%	32.39%
Global	77.63%	22.37%

Remark 16. *We can split companies into two groups: those such that* $\mathcal{CE}_i^{\mathcal{T}arget}(2030) < \mathcal{CE}_i^{\mathcal{T}rend}(2030)$ *and the others. As reported in Table 19, our endogenous bias is reflected with an overall* 77% *of companies showing an over-estimation of their targets against their current trend. Asia is a recent entrant in the target setting exercise and we observe a figure of* 67%*, 12 points below Europe.*

Figure 18: Carbon emissions, trends and targets and NZE scenario (median analysis, sectorial analysis)



Source: IEA (2021), CDP database (2021) & Authors' calculations.

Sectorial analysis The sectorial analysis is more interesting, because we observe some large differences between the sectors[26] in Figure 18. First, the trajectory of carbon emissions is highly dependent on the IEA sector. Electricity is the sector that has been making the greatest effort whereas we observe a large increase in carbon emissions for the Industry sector. The impact of the Covid-19 pandemic on the transport sector is particularly striking. Second, there are small differences between the reduction targets, except for the Transport

[26] See also Figures 34–37 on pages 71 and 72.

sector that is slightly less ambitious. Third, the NZE scenarios are different, but they reflect the IEA approach of NZE policies. In particular, we observe two main families of NZE scenarios: Electricity and Other on one side and Industry and Transport on the other.

4.2.2 Application to the MSCI World index

We extend our analysis to a larger universe, considering the MSCI World index from 2013. For that, we use the end-of-year composition. First, we analyze the trend trajectory of the index portfolio. Second, we compare the track record with the NZE scenario.

Remark 17. *We do not compute the synthetic reduction targets since we have seen that only a small number of issuers have published carbon targets. Analyzing the targets of a portfolio is then too early at this stage of research.*

Portfolio analysis of carbon emissions Computing the carbon emissions of a portfolio $x = (x_1, \ldots, x_n)$ relatively to its dynamic weights and changes in carbon emissions raises more questions than computing its intensity[27] at a given date. Following Le Guenedal and Roncalli (2022), the carbon emissions contribution of a nominal exposure W_i to the stock i is equal to:

$$\boldsymbol{\mathcal{CEC}}_i (W_i) = \frac{W_i \cdot \mathcal{FP}_i}{\mathcal{MC}_i} \cdot \boldsymbol{\mathcal{CE}}_i \tag{80}$$

where $\mathcal{FP}_i$ is the float percentage associated with the stock i and $\mathcal{MC}_i$ is the free-float market capitalization. The quantity $\mathcal{FP}_i \cdot \boldsymbol{\mathcal{CE}}_i$ indicates which quantity of carbon emissions emitted by the issuer i must be attributed to the public investors. For instance, if $\mathcal{FP}_i = 75\%$, this means that only 75% of the number of shares can be traded. Therefore, we normalize the carbon emissions amount $\mathcal{FP}_i \cdot \boldsymbol{\mathcal{CE}}_i$ by the holding ratio $W_i / \mathcal{MC}_i$. For example, if we assume that $\mathcal{FP}_i = 90\%$, $\mathcal{MC}_i = \$20$ bn, $\boldsymbol{\mathcal{CE}}_i = 3\,116\,272$ tCO_2e, we obtain $\boldsymbol{\mathcal{CEC}}_i \left(\$100 \text{ mn}\right) = 14\,023.22$ tCO_2e.

Equation (80) can be rewritten as:

$$\boldsymbol{\mathcal{CEC}}_i (W_i) = \frac{W_i}{\mathcal{MV}_i} \cdot \boldsymbol{\mathcal{CE}}_i = \varpi_i \cdot \boldsymbol{\mathcal{CE}}_i \tag{81}$$

where $\mathcal{MV}_i = \mathcal{MC}_i / \mathcal{FP}_i$ is the market value of the issuer i and ϖ_i is the ownership ratio. Let W be the nominal value of the portfolio. We have $W_i = W \cdot x_i$. The carbon emissions of the portfolio are then the sum of the carbon emissions contributions:

$$\begin{aligned} \boldsymbol{\mathcal{CE}} (x; W) &= \sum_{i=1}^{n} \boldsymbol{\mathcal{CEC}}_i (W_i) \\ &= W \cdot \overline{\boldsymbol{\mathcal{CE}}} (x; W) \end{aligned} \tag{82}$$

where $\overline{\boldsymbol{\mathcal{CE}}} (x; W)$ is the normalized carbon emissions for an investment of \$1:

$$\overline{\boldsymbol{\mathcal{CE}}} (x; W) = \sum_{i=1}^{n} \frac{x_i}{\mathcal{MV}_i} \cdot \boldsymbol{\mathcal{CE}}_i \tag{83}$$

Generally, $\boldsymbol{\mathcal{CE}} (x; W)$ is expressed in tCO_2e per million dollars in investment.

[27] In this last case, we generally compute the weighted-average carbon intensity (WACI):

$$\boldsymbol{\mathcal{CI}} (x) = \sum_{i=1}^{n} x_i \cdot \boldsymbol{\mathcal{CI}}_i \tag{79}$$

Remark 18. *If we assume that $\mathcal{FP}_i = 100\%$, then we have:*

$$\overline{\mathcal{CE}}\left(x; W\right) = \sum_{i=1}^{n} \frac{x_i \cdot \mathcal{CE}_i}{\mathcal{MC}_i} \tag{84}$$

If the portfolio is an index, this means that $x_i \propto \mathcal{MC}_i$, and we have:

$$\overline{\mathcal{CE}}\left(x; W\right) = \frac{\sum_{i=1}^{n} \mathcal{FP}_i \cdot \mathcal{CE}_i}{\sum_{i=1}^{n} \mathcal{MC}_i} \tag{85}$$

If we both assume that $\mathcal{FP}_i = 100\%$ and the portfolio is an index, the carbon emissions of the portfolio is equal to the product between the sum of carbon emissions of all constituents and the ownership ratio of the index portfolio:

$$\mathcal{CE}\left(x; W\right) = \varpi\left(W\right) \sum_{i=1}^{n} \mathcal{CE}_i \tag{86}$$

where $\varpi\left(W\right) = W / \sum_{i=1}^{n} \mathcal{MC}_i$.

Carbon emissions of the MSCI World index Let us compute the scope $1+2+3$ carbon emissions of a \$1 mn investment. In practice, we cannot directly use Equation (82) because we do not observe the data at the same frequency. Therefore, we use the following formula:

$$\overline{\mathcal{CE}}\left(x\left(t\right); W\right) = \sum_{i=1}^{n} \frac{x_i\left(t\right) \cdot \mathcal{FP}_i\left(t\right)}{\mathcal{MC}_i\left(t\right)} \cdot \mathcal{CE}_i\left(t-h\right) \tag{87}$$

where h is the lag due to the availability of carbon emissions data. For instance, if h is set to one year, we obtain results given in Table 20. In the second column, we have indicated the percentage of the portfolio weights for which we do not have the information on the carbon emissions[28]. For instance, carbon data are missing for 4.30% of the portfolio at the end of 2020. Therefore, we have to adjust Equation (87). The first approach consists in removing the issuers for which we do not have the data and rescaling the weights in order to obtain 100%. In the second and third approaches, we fill in missing data by the sectorial average of carbon emissions. The second approach considers the arithmetic mean, whereas we use the cap-weighted mean in the third approach. We notice that the choice of the missing data imputation is not neutral. For instance, $\mathcal{CE}\left(x; \$1 \text{ mn}\right)$ is equal to 389.6 tCO_2e in 2013 if we rescale the weights. This figure becomes 401.0 and 451.7 tCO_2e if we use the second and third imputation methods. Therefore, we observe some significant differences, in particular between the first and third methods. We also notice that these differences are very small if we focus on the portfolio's carbon intensity. The reason is that carbon emissions are correlated to revenues, meaning that the dispersion of carbon intensities is smaller, and the choice of the missing data imputation method has less impact.

The results reported in Table 20 are disturbing. Indeed, the carbon emissions of a \$ 1 mn investment decreases by 61% using the second filling method between 2013 and 2021 whereas the carbon intensity only decreases by 24% during the same period. At first sight, the computation of carbon emissions seems to be more robust, because it does not depend

[28]The proportion of missing data is equal to 7.09% in 2021, which is a large number. This illustrates the lag of reporting data when we manipulate carbon emissions. For instance, if the lag h is set to two years, the proportion of missing data becomes 5.58% in 2021 (see Table 26 on page 65).

Table 20: Scope $1+2+3$ carbon emissions of the MSCI World portfolio in tCO_2e ($W = \$1$ mn, $h = 1$ year)

Year	Missing	$\mathcal{CE}(x;W)$			$\mathcal{CI}(x)$		
		(1)	(2)	(3)	(1)	(2)	(3)
2013	3.63%	389.6	401.0	451.7	346.3	346.3	346.1
2014	3.72%	372.7	383.0	426.5	343.0	341.3	341.2
2015	4.51%	371.4	381.3	417.7	325.9	324.5	324.4
2016	3.85%	325.3	337.9	364.6	340.5	341.4	341.4
2017	2.79%	272.6	277.6	295.1	355.9	352.9	352.9
2018	2.31%	330.4	337.4	359.2	351.4	348.7	348.6
2019	3.67%	267.1	268.4	282.5	315.6	313.8	313.6
2020	4.30%	206.7	210.9	225.2	275.1	272.9	272.6
2021	7.09%	138.1	154.6	181.7	259.9	262.1	262.4

(1), (2) and (3) indicates the data imputation method: removing and rescaling for (1), filling with the sectorial arithmetic mean for (2) and the sectorial cap-weighted mean for (3).

Source: MSCI (2021), Trucost reporting year (2021) & Authors' calculations.

on the normalization variable when we calculate the carbon intensity. Nevertheless, the portfolio's carbon emissions depend on the market value, and we have:

$$\mathcal{CE}(x;W) = W \cdot \sum_{i=1}^{n} \frac{x_i}{\mathcal{MV}_i} \cdot \mathcal{CE}_i \propto \sum_{i=1}^{n} x_i \cdot \mathcal{CI}_i^{\mathcal{MV}} \tag{88}$$

where $\mathcal{CI}_i^{\mathcal{MV}}$ is a carbon intensity measure normalized by the market value $\mathcal{MV}_i$ of the issuer:

$$\mathcal{CI}_i^{\mathcal{MV}} = \frac{\mathcal{CE}_i}{\mathcal{MV}_i} \tag{89}$$

Therefore, the computation of $\mathcal{CE}(x;W)$ is very sensitive to the market conditions. To understand this issue, we have reported in Table 21 the total market capitalization of the MSCI World index and the evolution of the carbon emissions. Between 2013 and 2021, the index's market capitalization has increased from \$31.9 tn to 62.4 tn, which represents a growth of 95%. During the same period, carbon emissions were reduced by 24%.

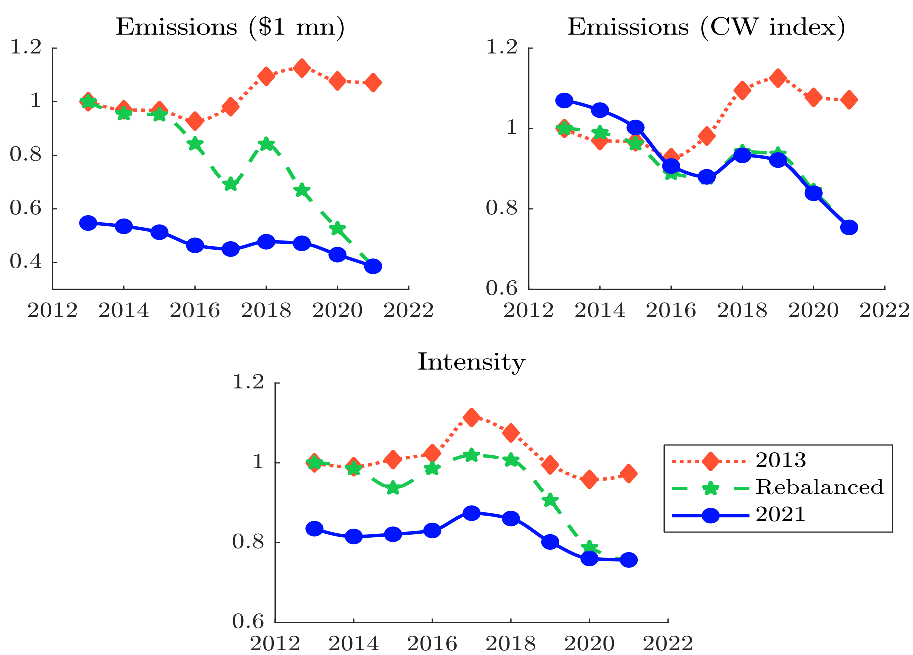
Table 21: Scope $1+2+3$ carbon emissions of the MSCI World index in $GtCO_2e$ ($h = 1$ year)

Year	$\sum_{i=1}^{n} \mathcal{MC}_i$ (in \$ tn)	$\sum_{i=1}^{n} \mathcal{CE}_i$		
		(1)	(2)	(3)
2013	31.9	12.8	12.8	14.4
2014	33.1	12.8	12.7	14.1
2015	32.3	12.4	12.3	13.5
2016	33.7	11.3	11.4	12.3
2017	40.4	11.4	11.2	11.9
2018	35.8	12.1	12.1	12.8
2019	44.7	12.3	12.0	12.6
2020	51.4	11.2	10.8	11.6
2021	62.4	9.5	9.7	11.3

Source: MSCI (2021), Trucost reporting year (2021) & Authors' calculations.

In Figure 19, we have reported the rebased carbon emissions and intensity of the \$1 mn portfolio and the index[29]. We notice that they both decrease over time. Nevertheless, the slope is higher for the portfolio's carbon emissions than for the carbon intensity or the index's carbon emissions. We have also reported the values when we consider the fixed composition of the MSCI World index in 2013 and 2021 instead of the rebalanced composition. We obtain opposite results. If the portfolio corresponds to the 2013 composition of the MSCI World index, we observe an increase in carbon emissions over time. If the portfolio corresponds to the 2021 composition of the MSCI World index, the decrease in the carbon emissions is lower than with the rebalanced composition in the case of the \$1 mn portfolio and similar to the index. These results are disturbing and are difficult to interpret. In fact, there are several factors that explain these curves. First, as we have already seen in Tables 20 and 21, the change in market capitalization has a big impact and explains the large decrease of the rebalanced portfolio. Nevertheless, this effect can not explain the differences between the 2013 and 2021 compositions, because the market capitalization values are fixed in both cases. This is why we can make the assumption that there is a shift from brown stocks to green stocks in cap-weighted indices. As a consequence, the carbon emissions of cap-weighted indices decrease faster than at the global level. Therefore, the composition of cap-weighted indices is another factor to explain these patterns. A deeper analysis shows that we have both an allocation effect and a selection effect. Indeed, we observe a decrease in the weight of polluting sectors[30]. We also notice that the weight of the less intensive carbon emitters within a polluting sector increases.

Figure 19: Scope $1+2+3$ carbon emissions and intensity (MSCI World index, $h = 1$ year)



Source: MSCI (2021), Trucost reporting year (2021) & Authors' calculations.

[29]We use the second approach to fill the missing data.
[30]30% on average during the last eight years.

Carbon trend of the MSCI World index We recall that:

$$\boldsymbol{\mathcal{CE}}\left(t,x;1\right)=\sum_{i=1}^{n}w_i\cdot\boldsymbol{\mathcal{CE}}_i\left(t\right) \tag{90}$$

where:

$$w_i=\frac{x_i\cdot\mathcal{FP}_i}{\mathcal{MC}_i} \tag{91}$$

Since we have $\boldsymbol{\mathcal{CE}}_i\left(t\right)=\beta_{i,0}+\beta_{i,1}t+u_i\left(t\right)$, we deduce that:

$$\boldsymbol{\mathcal{CE}}\left(t,x;1\right)=\underbrace{\left(\sum_{i=1}^{n}w_i\beta_{i,0}\right)}_{\beta_0(x)}+\underbrace{\left(\sum_{i=1}^{n}w_i\beta_{i,1}\right)}_{\beta_1(x)}t+\underbrace{\left(\sum_{i=1}^{n}w_iu_i\left(t\right)\right)}_{\varepsilon(t)} \tag{92}$$

or:

$$\boldsymbol{\mathcal{CE}}\left(t,x;1\right)=\beta_0\left(x\right)+\beta_1\left(x\right)\cdot t+\varepsilon\left(t\right) \tag{93}$$

For a given portfolio x, it follows that the slope $\beta_1\left(x\right)$ is the weighted average of the individual slopes:

$$\beta_1\left(x\right)=\sum_{i=1}^{n}w_i\beta_{i,1} \tag{94}$$

but the beta weights are not equal to the portfolio weights:

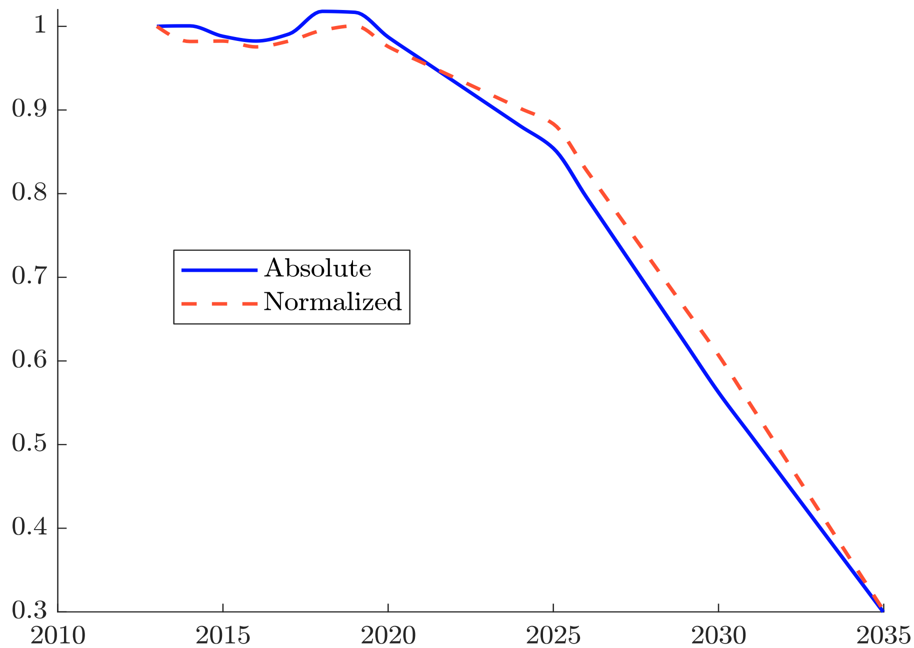
$$\beta_1\left(x\right)\neq\sum_{i=1}^{n}x_i\beta_{i,1} \tag{95}$$

If we consider the carbon emissions reported in Table 20 and estimate the linear regression, the slope is equal to -28.80 $tCO_2e/\$$ mn. If we compute the slope using Equation (94), its value is equal to -4.69 in 2019, -4.07 in 2020 and -3.24 in 2021. We observe a big difference between the two methods. These results confirm the impact of the market valuation and the sector composition in the carbon emissions reduction of the MSCI World index. On average, we estimate that only 25% of the reduction is solely explained by the efforts of issuers.

Remark 19. *We have to be very careful when defining the carbon trend of a portfolio because of aggregation issues. In Figure 41 on page 74, we have reported the histogram of the rescaled slope for all the issuers that were in the MSCI World index between December 2013 and December 2021. The red bars correspond to the frequencies when we use the base year 2013 for rescaling the slope. In this case, the distribution is right-skewed. If we use the last year 2021 to rescale the slope, we obtain the blue bars, and the distribution is left-skewed. However, by construction, the two distributions have the same number of positive (*56.3%*) and negative (*43.7%*) slopes.*

NZE scenario of the MSCI World index Concerning the computation of the NZE scenario, we can proceed in two ways. First, we can calculate the trajectory of the absolute carbon emissions for each issuer and aggregate all the trajectories using Equations (90) and (91). Second, we can directly aggregate the normalized NZE scenarios using the portfolio weights. Results are reported in Figure 20. The two approaches largely provide the same trajectories.

Figure 20: NZE scenario of the MSCI World index (2021)



Source: IEA (2021), MSCI (2021), Trucost reporting year (2021) & Authors' calculations.

4.3 Portfolio alignment with respect to an NZE scenario

The metrics proposed in this article question net zero alignment policies of asset owners and managers. Indeed, we may wonder what the objective function of a portfolio manager is when implementing net zero alignment. From the ESG analyst perspective, the materiality of net zero carbon metrics is clear since its objective is to analyze the participation, ambition and credibility of issuers. From the portfolio manager point of view, defining an NZE investment framework is more difficult since we can have contradictory objectives. We reiterate that the NZE goal is to transform the high-carbon economy into a net zero carbon economy. Therefore, we can assume that NZE investment policies correspond to a set of rules that supports and stimulates the transition.

Portfolio alignment is generally associated with portfolio decarbonization (Bolton *et al.*, 2021; Jondeau *et al.*, 2021; Le Guenedal and Roncalli, 2022). In fact, considering that NZE alignment consists in decarbonizing the portfolio is a very simplistic view. Let us consider a fund manager that is only invested in sectors such as Communication Services, Financials, Health Care or Information Technology. In this case, he has a low-carbon portfolio. Does it help to promote the transition and achieve a net zero emissions scenario by 2050? We reiterate that portfolio management is the modern term for capital allocation. If all investors cut their current exposures to sectors that have high carbon emissions (e.g. Energy, Industrials, Materials and Utilities), this implies that these sectors will experience a high capital shortfall. This is not the underlying idea of net zero. Indeed, the transition can occur – smoothly or not – only if the most polluting sectors are financed in order to improve the environmental efficiency of their processes. In fact, the capital required to transform these sectors is huge. This is why portfolio alignment goes beyond the concept of portfolio decarbonization. To summarize, asset owners and managers face the dilemma between reducing their exposure to issuers with a high carbon footprint (portfolio decarbonization) and continuing to finance the biggest polluters so that they can find green energy solutions.

A relatively simple approach when we consider portfolio optimization is to split the investment universe into two buckets (high climate impact sectors or HCIS and low climate impact sectors or LCIS), and to impose a minimum exposure constraint on high climate impact sectors as described in Le Guenedal and Roncalli (2022). This is the approach of Paris-aligned benchmarks (PAB). In this case, we may think that these HCIS sectors will continue to benefit from capital in order to undergo transformation. Nevertheless, when we implement portfolio optimization using a carbon reduction pathway and a HCIS constraint, we notice two important stylized factors. First, we observe some important sector allocation effects. In particular, portfolio optimization tends to reallocate the capital from Energy and Utilities to Industrials in the HCIS bucket and foster Financials in the LCIS bucket. Moreover, these macro-sector effects are amplified if we consider a granular industry classification, e.g. between Energy Equipment & Services and Oil, Gas & Consumable Fuels, Electric Utilities and Gas/Water Utilities, etc. The second stylized factor concerns stock selection effects. Within an industry, we observe a winner-takes-all solution, meaning that the allocation is concentrated on the less carbon-intensive issuer. Therefore, we observe a large discrepancy between a top-down analysis and a bottom-up analysis when we study the optimized portfolio solutions. While the top-down approach concludes that the HCIS constraint is satisfied and the level of decarbonization is achieved, the bottom-up approach reveals that the solution is not well diversified in terms of issuers and represented industries. Some issuers that are essential to the NZE transition are excluded, implying a shift of capital to the less polluting issuers.

Financing the transition to a low-carbon economy is the core objective of NZE investment policies. Therefore, we cannot reduce NZE portfolio alignment to portfolio decarbonization even if we add some tricks like the HCIS constraint. This is why engagement is key to supporting companies on the path towards a low-carbon economy. This is the first stage. Nevertheless, the coverage rate of the portfolios by asset owners and managers has been relatively low until now. The time has come to accelerate and adopt a more systematic approach of engagement and voting. In this case, the net zero carbon metrics are useful to systematically screen all portfolio issuers. We can then compare issuers to their peers and detect those that are not on the right track. Net zero carbon metrics, in particular those based on carbon trends, can then be introduced in portfolio optimization in order to anticipate future potential divestments. As said previously, if we reduce the NZE investment policy to a portfolio decarbonization exercise, the main risk is that the investment universe becomes small enough to cause the asset management industry to suffer.

Financing the transition, engagement and voting are now part of asset management's responsibilities and duties. Nevertheless, the ultimate goal of regulatory bodies remains investment portfolio decarbonization. Here, we face an issue, because the implementation may (and will certainly) be very difficult in particular if the decarbonization of the world economy does not occur or occurs to a lesser extend than expected. The risk for the investment universe takes its root in the decoupling risk between portfolio decarbonization and decarbonization of the economy. The political bet is that the decarbonization of finance will automatically lead to decarbonization of the economy. This might be the case. Nevertheless, if the lag between the two decarbonization pathways is too large, we may face the risk of no-feasible solution for NZE portfolio alignment[31].

[31] When we started writing this research paper, we planned to develop this section with empirical results and mathematical methods. Since the use of net zero carbon metrics in portfolio alignment is relatively complex, we have decided to develop this section in two forthcoming companion articles dedicated to those different issues (Barahhou *et al.*, 2022; Ben Slimane *et al.*, 2022).

5 Conclusion

The climate emergency requires a sharp reduction of GHG emissions in the next 10 years. For instance, the global emissions was equal to 36 $GtCO_2e$ in 2019, whereas "*using global mean surface air temperature [...] gives an estimate of the remaining carbon budget of* 580 *$GtCO_2e$ for a* 50% *probability of limiting warming to* 1.5°C, *and* 420 *$GtCO_2e$ for a* 66% *probability (medium confidence)*" (IPCC, 2018). The countdown has begun, and we do not have much time left for action. At this stage, individual issuers' trajectories are far from meeting the reduction requirements. Worse still, most of them even show an increase in carbon emissions in recent years. The regulatory and normative framework is therefore being tightened in order to force companies to do everything possible to limit their contribution to global warming. In response to these requirements, companies need to plan their low-carbon transition. For instance, they are increasingly communicating their intention to reduce carbon emissions in percentage terms compared to a reference year over a given period. These carbon emissions trajectories and reduction targets are in essence extra-financial information whose materiality has been more or less proven in the past. However, in the context of a world in transition to a greener economy, their financial materiality should be consolidated by (at least) two channels. First, this information should allow us to anticipate the effects of direct risks on issuers, such as additional costs linked to pollution (e.g., carbon taxes), effects on demand (e.g., boycotts and demand shifts), etc., which could reduce the profitability and the valuation of their securities and also increase business disruptions. Second, being able to identify companies whose alignment appears to be impossible allows us to protect our portfolios from the risk of indirect market effects, linked to an exclusion and/or underweighting of the associated securities in net zero carbon strategies. Beyond the financial aspects and the reduction of transition risks, it is necessary to establish a clear quantitative framework to compare actors' performance in order to build a coherent net zero investment strategy. Moreover, we reiterate that institutional investors are regrouping into the (UN-convened) net zero asset owner alliance with the objective to "*transition their portfolios to net zero GHG emissions by 2050*". Asset managers have also formed the net zero asset managers initiative which currently boasts 220 international asset managers and USD 57 trillion in assets under management. As all these asset owners and managers are committed to shift to net zero investment solutions in the coming years, there is a high demand for developing specific net zero carbon metrics and understanding the common basics of an NZE investment policy.

It is in this context that this research paper contributes to practitioners' intention. In particular, we have three main contributions. First, we formalize both carbon trajectory and target information, whose raw structure means that using these data is not easy. We therefore introduce a budget approach that allows us to harmonize and homogenize the information. Concretely, we extend the emissions trajectory approach of Le Guenedal *et al.* (2020) by introducing an explicit reference to the market-based net zero emissions scenario and the issuer-based reduction targets scenario. Then, we propose several metrics which are homogeneous to carbon emissions (in tCO_2e and tCO_2e/year), and normalize these metrics to provide comparability between companies. These static metrics measure the duration, the gap, and the slope to achieve an NZE scenario. Second, we develop the first measurements characterizing the alignment trends. In this case, we propose to analyze the time decomposition of carbon budgets, and the breakdown into error and revision contributions. Moreover, we define two NZE measures: the zero-velocity and burnout scenarios. The first scenario quantifies the efforts made by the issuer in the past and its short-term leeway. The second scenario is a stress scenario that computes the reduction of carbon emissions in the next year in order to change the trend pathway and satisfy the NZE scenario. Third, using the set of quantitative metrics introduced, we propose a novel methodology to assess the participation, ambition and credibility of issuers in the race towards a carbon neutrality. The participation shows the progress of the company in terms of transition. On the basis of this pillar, it is possible to distinguish the issuers whose historical trend is in line with the conditions of the NZE scenario. The ambition score is used to classify the consistency of companies' reduction targets with the NZE scenario. In this context, a positive ambition score characterizes appropriate consideration of the NZE constraints in the target setting. The construction of such a score in a consistent way is only possible thanks to the inter-scope aggregation that we propose in this paper. Finally, the credibility pillar measures, among other things, the gap between what is said (carbon targets) and what is done (historical carbon emissions). This information allows us to avoid green washing attempts when the trend shows that the reduction targets will not be reached. Taking all these pillars, we introduce a $\mathcal{PAC}$ framework for controlling the trajectory and targets of individual issuers, and those of investment portfolios by aggregation.

Empirical results show that achieving NZE is a long rocky road. Using a database of issuers that have already set carbon targets, the results are disappointing. On average, we observe that their carbon emissions have increased in recent years even though we observed a stabilization in 2020. Only one third of these issuers present a negative trend. Moreover, we observe an asymmetry between issuers that are reducing their carbon emissions and those that are increasing them. Indeed, the positive growth rate of some issuers' emissions is very large and out of control. Nevertheless, this bleak global picture is counterbalanced by the velocity figures in 2019 and 2020. For more than half of these issuers, we observe a decrease in the carbon emissions slope. If we focus on their carbon targets, they are in line with the 2030 NZE scenario on average. Nevertheless, we also observe that some targets are not credible if we compare them with their past participation to reduce carbon emissions. Therefore, we obtain a negative correlation between ambition and credibility. These results are interesting even if they only concern issuers that have already disclosed their ambition to reduce carbon emissions. This introduces an obvious bias in the analysis. CDP questionnaires targets specific industries and issuers, which lead us to make these two adverse observations: (i) responding issuers are showing "*good faith*" and therefore we can assume that they are the most virtuous, indicating that the picture is darker at the global level; (ii) or we could also assume that they are the most exposed to climate change (belonging to climate relevant sectors targeted by CDP) and thus, they face big challenges that cannot be solved in just a few years. In any case, these results advocate for a greater transparency of NZE metrics. In this context, asset owners and managers must accelerate their engagement policies if they do not want the let the gap widen between the – both economic and financial – effective decarbonization required and the effective emissions pathway pursued by individual companies. Too much mismatch between the financial decarbonization pathway and the economic decarbonization pathway is then a big issue for the asset management industry.

To our knowledge, our research is the first study to propose asset-level metrics that allow us to assess the performance of an issuer towards the NZE scenario. We believe that bringing such metrics to asset owners and managers will undoubtedly question their appraisal of issuers who generally separate their commitments to the climate action business development from their own emissions (including scope 3 emissions). The objective of these metrics is then to promote transparent and comparable information that will support communication between investors and corporates and a unified framework that will help asset owners and managers to define their engagement policies and NZE investment strategies.

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A Technical appendix

A.1 Notations

- $\beta_{i,0}$ is the intercept of the linear trend model:

$$\mathcal{CE}_i(t) = \beta_{i,0} + \beta_{i,1}t + u_i(t)$$

- $\hat{\beta}_{i,0}$ is the estimated value of $\beta_{i,0}$ using the track record of issuer i.
- $\beta'_{i,0}$ is the intercept of the rescaled linear trend model:

$$\mathcal{CE}_i(t) = \beta'_{i,0} + \beta_{i,1}\left(t - t_p\right) + u_i(t)$$

  where t_p is the pivot date. We have $\hat{\beta}'_{i,0} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1}t_p = \widehat{\mathcal{CE}}_i\left(t_p\right)$. In the case where the pivot date is the current date — $t_p = t_0$, we have $\hat{\beta}'_{i,0} = \widehat{\mathcal{CE}}_i(t_0) = \mathcal{CE}_i(t_0)$.

- $\tilde{\beta}_{i,0}$ is the intercept of the fitted model:

$$\widehat{\mathcal{CE}}_i(t) = \tilde{\beta}_{i,0} + \hat{\beta}_{i,1}t$$

  where $\tilde{\beta}_{i,0} = \mathcal{CE}_i\left(t_p\right) - \hat{\beta}_{i,1}t_p$.

- $\beta_{i,1}$ is the slope of the linear trend model.
- $\hat{\beta}_{i,1}$ is the estimated value of $\beta_{i,1}$ using the track record of issuer i.
- $\mathcal{BO}_i\left(t+1, \mathcal{CE}_i^{\text{nze}}(t^\star)\right)$ is the burn-out scenario such that $\widehat{\mathcal{CE}}_i(t^\star) = \mathcal{CE}_i^{\text{nze}}(t^\star)$.
- $\mathcal{CB}_i(t_1, t_2)$ is the carbon budget of issuer i over the time period $[t_1, t_2]$.
- $\mathcal{CB}_i(t_0, t_1, t^\star)$ is the carbon budget between the starting date t_0 and the target date $t^\star$, which is evaluated at the current date t_1.
- $\mathcal{CE}_i(t)$ corresponds to the absolute carbon emissions of issuer i at time t.
- $\mathcal{CE}_{i,j}(t)$ corresponds to scope j emissions ($j = 1, 2, 3$).
- $\widehat{\mathcal{CE}}_i(t)$ is the estimated value of $\mathcal{CE}_i(t)$. It is generally equal to the carbon reduction target $\mathcal{CE}_i^{\mathcal{T}arget}(t)$ or the carbon trend $\mathcal{CE}_i^{\mathcal{T}rend}(t)$.
- $\mathcal{CE}_i^\star(t)$ is the target value of carbon emissions at time t.
- $\mathcal{CE}_i^\star$ is the objective value of carbon emissions during a given period.
- $\mathcal{CE}_i^{\text{nze}}(t^\star)$ is the net zero emissions scenario for issuer i at time $t^\star$.
- $\mathcal{CE}_i^{\mathcal{T}arget}(t)$ is the target value of $\mathcal{CE}_i(t)$ announced by issuer i.
- $\mathcal{CE}_i^{\mathcal{T}rend}(t)$ is the trend value of $\mathcal{CE}_i(t)$ computed with the track record of issuer i:

$$\mathcal{CE}_i^{\mathcal{T}rend}(t) = \hat{\beta}_{i,0} + \hat{\beta}_{i,1}t$$

  If we use the rescaled trend model, we have:

$$\mathcal{CE}_i^{\mathcal{T}rend}(t) = \hat{\beta}'_{i,0} + \hat{\beta}_{i,1}\left(t - t_p\right)$$

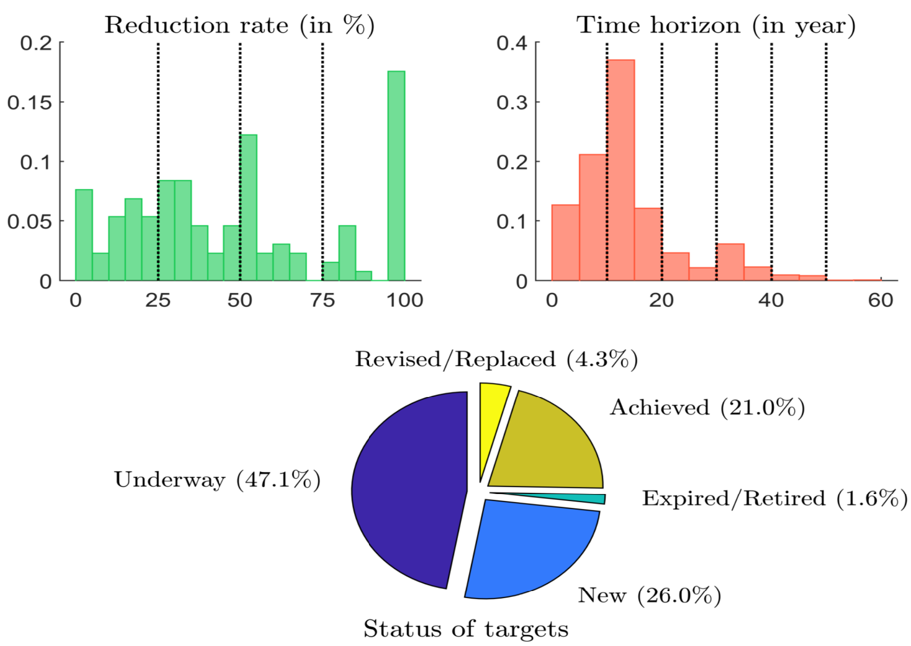
- $\mathcal{G}ap_i\left(t^{\star}\right)$ is the NZE gap at time $t^{\star}$. It is equal to the difference between the estimated carbon emissions and the NZE scenario:
$$\mathcal{G}ap_i\left(t^{\star}\right) = \widehat{\boldsymbol{\mathcal{CE}}}_i\left(t^{\star}\right) - \boldsymbol{\mathcal{CE}}_i^{\mathrm{nze}}\left(t^{\star}\right)$$
- $\mathcal{G}ap_i^{\mathcal{T}arget}\left(t^{\star}\right)$ is the NZE gap at time $t^{\star}$ when $\widehat{\boldsymbol{\mathcal{CE}}}_i\left(t^{\star}\right) = \boldsymbol{\mathcal{CE}}_i^{\mathcal{T}arget}\left(t\right)$.
- $\mathcal{G}ap_i^{\mathcal{T}rend}\left(t^{\star}\right)$ is the NZE gap at time $t^{\star}$ when $\widehat{\boldsymbol{\mathcal{CE}}}_i\left(t^{\star}\right) = \boldsymbol{\mathcal{CE}}_i^{\mathcal{T}rend}\left(t\right)$.
- $\boldsymbol{\mathcal{R}}_i\left(t\right)$ is the annual reduction of carbon emissions at time t.
- $\boldsymbol{\mathcal{R}}_i\left(t_1, t_2\right)$ is the total reduction of carbon emissions between t_1 and t_2.
- $\boldsymbol{\mathcal{SC}}_j$ corresponds to scope j emissions ($j = 1, 2, 3$).
- $\boldsymbol{\mathcal{SC}}_{1+2+3}$ corresponds to scope $1 + 2 + 3$ emissions.
- $\mathcal{S}lope_i\left(t^{\star}\right)$ is the NZE slope to close the gap at time $t^{\star}$.
- t is the generic date. It is generally measured in years: $t = 2020, 2021, \ldots$
- t_0 is the current date or the starting date. This notation is also used in place of the last reporting date $t_{\mathcal{L}ast}$ when there is no confusion. When t_0 is the starting date of the analysis period $[t_0, t]$, it may be a past or future date.
- $t^{\star}$ is the generic target date (it may be equal to $t^{\mathcal{T}arget}$ or t^{nze}). For example, $t^{\star}$ may be 2025, 2030 or 2050.
- $t_{\mathcal{B}ase}$ is the base year of the analysis. This date is used for normalizing the different measures.
- $t_{\mathcal{F}irst}$ is the starting date of the period $[t_{\mathcal{F}irst}, t_{\mathcal{L}ast}]$, which is used for estimating the trend model.
- $t_{\mathcal{L}ast}$ is the last reporting year of carbon emissions. It is generally equal to $t_0 - 1$ or $t_0 - 2$ where t_0 is the current date. For instance, in 2021, reporting data concern the year 2019 or the year 2020.
- t_p is the pivot date for the rescaled trend model.
- $\boldsymbol{\tau}_i$ is the NZE duration.
- $\boldsymbol{\mathcal{TC}}_i\left(t_1 \mid t_0, t^{\star}\right)$ is the time contribution of year t_1. It is defined for a carbon budget over the time period $[t_0, t^{\star}]$.
- $\boldsymbol{\mathcal{TC}}_i^{\mathrm{error}}\left(t_1 \mid t_0, t^{\star}\right)$ measures the forecast error between the observed trajectory and the estimate done at time t_0 for the time period $[t_0, t_1]$.
- $\boldsymbol{\mathcal{TC}}_i^{\mathrm{revision}}\left(t_1 \mid t_0, t^{\star}\right)$ measures the forecast revision for the time period $[t_1, t^{\star}]$ due to the information published between t_0 and t_1.
- $\boldsymbol{v}_i\left(t_1, t_2\right)$ is the NZE velocity between t_1 and t_2:
$$\boldsymbol{v}_i\left(t_1, t_2\right) = \frac{\Delta\hat{\beta}_{i,1}\left(t_1, t_2\right)}{t_2 - t_1}$$
- $\boldsymbol{v}_i^h\left(t\right) = \boldsymbol{v}_i\left(t - h, t\right)$ is the h-step velocity.
- $\boldsymbol{\mathcal{ZV}}_i^{(h)}\left(t + 1\right)$ is the zero-velocity scenario such that $\boldsymbol{v}_i^h\left(t + 1\right) = 0$.

A.2 Data

We use the data from CDP for both historical carbon emissions and reduction targets announced by companies. For carbon intensity and absolute historical records, Trucost also proposes a wider dataset with some estimated values. In terms of coverage, the carbon scopes have a relatively high figure in recent years. Nevertheless, the data quality and coverage are lower in the past.

If we consider the 2021 CDP database, we have 3 767 referenced absolute targets. In Figure 21, we report the frequencies of their scale, time horizon and status. We notice that the reduction target is exactly equal to 50% and 100% in 8.4% and 17.6% of cases, whereas the median reduction rate is equal to 41%. The time horizon of these targeted reduction rates goes from 0 to 60 years with a median value of 11 years, and 85% of carbon targets concern the next 20 years. In the bottom panel, we report the status of carbon targets. Almost 50% of them are underway, whereas 26% of targets are new. We also see that 21% of carbon targets are already achieved, meaning that they concern the period before 2021.

Figure 21: Status, time horizon and scale of reduction targets



Source: CDP database (2021) & Authors' calculations.

On the target side, the coverage of corporate respondents disclosing their targets is still too low at the global level. Table 22 indicates the number of companies that disclose their reduction targets for the MSCI EMU, North America and EM Asia indices[32]. They are split according to the climate relevant IEA sectors[33]. We distinguish issuers that have public and non-public reduction targets, and non-respondent companies. For instance, if we consider the MSCI EMU universe, we have 235 issuers. The breakdown is the following: 185 issuers

[32] At the end of October 2021, the number of stocks is respectively equal to 235 for the MSCI EMU index, 715 for the MSCI North America index and 1 153 for the MSCI EM Asia index.

[33] See Appendix A.4.1 on page 62.

with public targets, 22 issuers with non-public targets and 28 issuers that have no reduction targets or have not disclosed. The number of non-responder issuers is then 11.9% of the MSCI EMU index universe. For the North America and Asia universes, we obtain a smaller proportion of respondents. Indeed, we have respectively 24.8% and 71.5% of non-responder issuers[34].

Table 22: Coverage of CDP data for the MSCI index universes

IEA sector	All Issuers			EMU			North America			EM Asia		
	P	NP	NR	P	NP	NR	P	NP	NR	P	NP	NR
Electricity	227	57	381	27		3	51	6	21	18	12	85
Industry	1 136	262	1 237	85	7	11	202	29	51	120	40	390
Other	904	264	1 318	71	15	14	196	35	99	80	50	335
Transport	88	21	81	2			18	1	6	4	5	14
Total	2 355	604	3 017	185	22	28	467	71	177	222	107	824
# issuers		5 976			235			715			1 153	
Frequency (in %)	39.4	10.1	50.5	78.7	9.4	11.9	65.3	9.9	24.8	19.3	9.3	71.5

P = public, NP = non-public, NR = non-responder.

Source: CDP database (2021), MSCI indices & Authors' calculations.

[34] There are some differences between issuers' accounting and targets' accounting because some companies have multiple ISIN codes.

A.3 Mathematical results

A.3.1 Computation of the carbon budget

Numerical solution We consider the partition $\left\{[t_0, t_0 + \Delta t], \ldots, [t - \Delta t, t]\right\}$ of $[t_0, t]$. The right Riemann approximation is:

$$\begin{aligned} \boldsymbol{\mathcal{CB}}_i(t_0, t) &= \int_{t_0}^{t} \left(\boldsymbol{\mathcal{CE}}_i(s) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \mathrm{d}s \\ &\approx \sum_{k=1}^{m} \left(\boldsymbol{\mathcal{CE}}_i(t_0 + k\Delta t) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \cdot \Delta t \end{aligned} \tag{96}$$

where $m = \dfrac{t - t_0}{\Delta t}$. If we use the left Riemann sum, we obtain:

$$\boldsymbol{\mathcal{CB}}_i(t_0, t) \approx \sum_{k=0}^{m-1} \left(\boldsymbol{\mathcal{CE}}_i(t_0 + k\Delta t) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \cdot \Delta t \tag{97}$$

Finally, the midpoint rule is given by:

$$\boldsymbol{\mathcal{CB}}_i(t_0, t) \approx \sum_{k=1}^{m} \left(\boldsymbol{\mathcal{CE}}_i\left(t_0 + \frac{k}{2}\Delta t\right) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \cdot \Delta t \tag{98}$$

In the case of a yearly partition, the previous formulas are simplified since we have $\Delta t = 1$. For instance, the right Riemann sum becomes:

$$\begin{aligned} \boldsymbol{\mathcal{CB}}_i(t_0, t) &\approx \sum_{k=1}^{m} \left(\boldsymbol{\mathcal{CE}}_i(t_0 + k\Delta t) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \\ &= \sum_{s=t_0+1}^{t} \left(\boldsymbol{\mathcal{CE}}_i(s) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) \end{aligned} \tag{99}$$

Special cases

Constant linear reduction If we use a constant linear reduction rate:

$$\boldsymbol{\mathcal{R}}_i(t_{\mathcal{L}ast}, t) = \boldsymbol{\mathcal{R}}_i \cdot (t - t_{\mathcal{L}ast}) \tag{100}$$

we obtain the following semi-analytical expression:

$$\begin{aligned} \boldsymbol{\mathcal{CB}}_i(t_0, t) = {} & (t - t_{\mathcal{L}ast})\left(\boldsymbol{\mathcal{CE}}_i(t_{\mathcal{L}ast}) - \boldsymbol{\mathcal{CE}}_i^{\star}\right) - (t_{\mathcal{L}ast} - t_0)\,\boldsymbol{\mathcal{CE}}_i^{\star} + \\ & \int_{t_0}^{t_{\mathcal{L}ast}} \boldsymbol{\mathcal{CE}}_i(s)\,\mathrm{d}s - \boldsymbol{\mathcal{R}}_i \frac{(t - t_{\mathcal{L}ast})^2}{2} \boldsymbol{\mathcal{CE}}_i(t_{\mathcal{L}ast}) \end{aligned} \tag{101}$$

because we have:

$$\begin{aligned} \int_{t_{\mathcal{L}ast}}^{t} \boldsymbol{\mathcal{R}}_i(t_{\mathcal{L}ast}, s)\,\mathrm{d}s &= \boldsymbol{\mathcal{R}}_i \int_{t_{\mathcal{L}ast}}^{t} (s - t_{\mathcal{L}ast})\,\mathrm{d}s \\ &= \boldsymbol{\mathcal{R}}_i \frac{(t - t_{\mathcal{L}ast})^2}{2} \end{aligned} \tag{102}$$

Constant compound reduction If we use a constant compound reduction rate:

$$\mathcal{CE}_i(t) = (1 - \mathcal{R}_i)^{(t - t_{\mathcal{L}ast})} \cdot \mathcal{CE}_i(t_{\mathcal{L}ast}) \tag{103}$$

we deduce that:

$$\begin{aligned}\int_{t_{\mathcal{L}ast}}^{t} \mathcal{CE}_i(s)\,\mathrm{d}s &= \mathcal{CE}_i(t_{\mathcal{L}ast}) \int_{t_{\mathcal{L}ast}}^{t} (1 - \mathcal{R}_i)^{(s - t_{\mathcal{L}ast})}\,\mathrm{d}s \\ &= \mathcal{CE}_i(t_{\mathcal{L}ast}) \left[\frac{(1 - \mathcal{R}_i)^{(s - t_{\mathcal{L}ast})}}{\ln(1 - \mathcal{R}_i)}\right]_{t_{\mathcal{L}ast}}^{t} \\ &= \frac{(1 - \mathcal{R}_i)^{(t - t_{\mathcal{L}ast})} - 1}{\ln(1 - \mathcal{R}_i)} \mathcal{CE}_i(t_{\mathcal{L}ast}) \end{aligned} \tag{104}$$

It follows that:

$$\begin{aligned}\mathcal{CB}_i(t_0, t) &= -(t - t_0) \cdot \mathcal{CE}_i^\star + \int_{t_0}^{t} \mathcal{CE}_i(s)\,\mathrm{d}s \\ &= -(t - t_0) \cdot \mathcal{CE}_i^\star + \int_{t_0}^{t_{\mathcal{L}ast}} \mathcal{CE}_i(s)\,\mathrm{d}s + \left(\frac{(1 - \mathcal{R}_i)^{(t - t_{\mathcal{L}ast})} - 1}{\ln(1 - \mathcal{R}_i)}\right) \mathcal{CE}_i(t_{\mathcal{L}ast}) \end{aligned} \tag{105}$$

Linear function We assume that:

$$\mathcal{CE}_i(t) = \beta_{i,0} + \beta_{i,1} t \tag{106}$$

It follows that:

$$\begin{aligned}\mathcal{CB}_i(t_0, t) &= \int_{t_0}^{t} \left(\beta_{i,0} + \beta_{i,1} s - \mathcal{CE}_i^\star\right)\mathrm{d}s \\ &= \left[\frac{1}{2}\beta_{i,1} s^2 + \left(\beta_{i,0} - \mathcal{CE}_i^\star\right) s\right]_{t_0}^{t} \\ &= \frac{1}{2}\beta_{i,1}\left(t^2 - t_0^2\right) + \left(\beta_{i,0} - \mathcal{CE}_i^\star\right)(t - t_0) \end{aligned} \tag{107}$$

Piecewise linear function We assume that $\mathcal{CE}_i(t)$ is known for $t \in \{t_0, t_1, \ldots, t_m\}$ and $\mathcal{CE}_i(t)$ is linear between two consecutive dates:

$$\mathcal{CE}_i(t) = \mathcal{CE}_i(t_{k-1}) + \frac{\mathcal{CE}_i(t_k) - \mathcal{CE}_i(t_{k-1})}{t_k - t_{k-1}}(t - t_{k-1}) \qquad \text{if } t \in [t_{k-1}, t_k] \tag{108}$$

We deduce that:

$$\begin{aligned}\mathcal{CB}_i(t_0, t) = &\sum_{k=1}^{k(t)} \int_{t_{k-1}}^{t_k} \left(\mathcal{CE}_i(s) - \mathcal{CE}_i^\star\right)\mathrm{d}s + \\ &\int_{t_{k(t)}}^{t} \left(\mathcal{CE}_i(s) - \mathcal{CE}_i^\star\right)\mathrm{d}s \end{aligned} \tag{109}$$

where $k(t) = \{\max k : t_k \leq t\}$. We notice that Equation (108) can be written as:

$$\mathcal{CE}_i(t) = \underbrace{\frac{t_k}{t_k - t_{k-1}}\mathcal{CE}_i(t_{k-1}) - \frac{t_{k-1}}{t_k - t_{k-1}}\mathcal{CE}_i(t_k)}_{\beta_{i,0,k}} + \underbrace{\frac{\mathcal{CE}_i(t_k) - \mathcal{CE}_i(t_{k-1})}{t_k - t_{k-1}}}_{\beta_{i,1,k}} t \tag{110}$$

Using Equation (107), we conclude that:

$$\begin{aligned}\boldsymbol{\mathcal{CB}}_i(t_0,t) &= \frac{1}{2}\sum_{k=1}^{k(t)}\beta_{i,1,k}\left(t_k^2-t_{k-1}^2\right)+\sum_{k=1}^{k(t)}\left(\beta_{i,0,k}-\boldsymbol{\mathcal{CE}}_i^{\star}\right)(t_k-t_{k-1})+\\ &\quad \frac{1}{2}\beta_{i,1,k(t)+1}\left(t^2-t_{k(t)}^2\right)+\left(\beta_{i,0,k(t)+1}-\boldsymbol{\mathcal{CE}}_i^{\star}\right)\left(t-t_{k(t)}\right)\end{aligned} \tag{111}$$

We can simplify this expression as follows:

$$\begin{aligned}\boldsymbol{\mathcal{CB}}_i(t_0,t) &= \frac{1}{2}\sum_{k=1}^{k(t)}\left(\boldsymbol{\mathcal{CE}}_i(t_k)-\boldsymbol{\mathcal{CE}}_i(t_{k-1})\right)(t_k+t_{k-1})+\\ &\quad \sum_{k=1}^{k(t)}\left(\boldsymbol{\mathcal{CE}}_i(t_{k-1})-\boldsymbol{\mathcal{CE}}_i^{\star}\right)t_k-\sum_{k=1}^{k(t)}\left(\boldsymbol{\mathcal{CE}}_i(t_k)-\boldsymbol{\mathcal{CE}}_i^{\star}\right)t_{k-1}+\\ &\quad \frac{1}{2}\left(\boldsymbol{\mathcal{CE}}_i(t)-\boldsymbol{\mathcal{CE}}_i\left(t_{k(t)}\right)\right)\left(t+t_{k(t)}\right)+\\ &\quad \left(\boldsymbol{\mathcal{CE}}_i\left(t_{k(t)}\right)-\boldsymbol{\mathcal{CE}}_i^{\star}\right)t-\sum_{k=1}^{k(t)}\left(\boldsymbol{\mathcal{CE}}_i(t)-\boldsymbol{\mathcal{CE}}_i^{\star}\right)t_{k(t)}\end{aligned} \tag{112}$$

A.3.2 The linear trend model

The linear trend model is defined as:

$$y(t)=\operatorname{trend}(t)+u(t)\qquad \text{for } t=1,2,\ldots,n \tag{113}$$

where $y(t)$ is the dependent variable and the function $\operatorname{trend}(t)$ is equal to:

$$\operatorname{trend}(t)=\beta_0+\beta_1 t \tag{114}$$

Least squares estimation We can write the linear regression model (113)–(114) as:

$$y(t)=x(t)^{\top}\beta(n)+u(t) \tag{115}$$

We have:

$$\hat{\beta}(n)=\left(X^{\top}X\right)^{-1}X^{\top}Y \tag{116}$$

where $Y=\left(y(1),\ldots,y(n)\right)$, $X=\begin{pmatrix}\mathbf{1}_n & \mathbf{t}_n\end{pmatrix}$ and $\mathbf{t}_n=(1,2,\ldots,n)$. We deduce that:

$$\begin{aligned}X^{\top}X &= \begin{pmatrix} n & \sum_{t=1}^n t \\ \sum_{t=1}^n t & \sum_{t=1}^n t^2\end{pmatrix}\\ &= \frac{1}{6}\begin{pmatrix} 6n & 3n(n+1) \\ 3n(n+1) & n(n+1)(2n+1)\end{pmatrix}\end{aligned} \tag{117}$$

and:

$$X^{\top}Y=\begin{pmatrix}\sum_{t=1}^n y(t) \\ \sum_{t=1}^n t y(t)\end{pmatrix} \tag{118}$$

It follows that[35]:

$$\left(X^{\top}X\right)^{-1}=\frac{2}{n(n+1)(n-1)}\begin{pmatrix}(n+1)(2n+1) & -3(n+1) \\ -3(n+1) & 6\end{pmatrix} \tag{120}$$

[35] We have:

$$\det\left(X^{\top}X\right)=\frac{n^2(n+1)(n-1)}{12} \tag{119}$$

Finally, we obtain:

$$\hat{\beta}(n) = \frac{2}{n(n+1)(n-1)} \begin{pmatrix} (n+1)(2n+1)\sum_{t=1}^{n} y(t) - 3(n+1)\sum_{t=1}^{n} ty(t) \\ 6\sum_{t=1}^{n} ty(t) - 3(n+1)\sum_{t=1}^{n} y(t) \end{pmatrix} \tag{121}$$

Relationship between $\hat{\beta}_0$, $\hat{\beta}_1$, $\bar{y}_n$ and $\tilde{y}_n$ The arithmetic mean is equal to:

$$\bar{y}_n = \frac{1}{n}\sum_{t=1}^{n} y(t) \tag{122}$$

whereas the weighted average is defined as:

$$\tilde{y}_n = \frac{1}{\sum_{t=1}^{n} t}\sum_{t=1}^{n} ty(t) = \frac{2}{n(n+1)}\sum_{t=1}^{n} ty(t) \tag{123}$$

In this case, the t^{th} observation $y(t)$ is weighted by t. It follows that:

$$\sum_{t=1}^{n} y(t) = n\bar{y}_n \tag{124}$$

and:

$$\sum_{t=1}^{n} ty(t) = \frac{n(n+1)}{2}\tilde{y}_n \tag{125}$$

Using Equation (121), we conclude that:

$$\hat{\beta}_0(n) = \frac{2(2n+1)\bar{y}_n - 3(n+1)\tilde{y}_n}{n-1} \tag{126}$$

and:

$$\hat{\beta}_1(n) = \frac{6}{n-1}(\tilde{y}_n - \bar{y}_n) \tag{127}$$

We notice that $\hat{\beta}_0(n)$ is a weighted average of $\bar{y}_n$ and $\tilde{y}_n$ since we have[36]:

$$\hat{\beta}_0(n) = \omega_1\bar{y}_n - \omega_2\tilde{y}_n \tag{128}$$

where:

$$\omega_1 = 4 + \frac{6}{n-1} \tag{129}$$

and:

$$\omega_2 = 3 + \frac{6}{n-1} \tag{130}$$

Another interesting relationship is:

$$\hat{\beta}_0(n) + \hat{\beta}_1(n) = 4\tilde{y}_n - 3\bar{y}_n \tag{131}$$

[36] We verify that $\omega_1 - \omega_2 = 1$.

Computation of $\Delta\hat{\beta}_0(n,n+1)$ and $\Delta\hat{\beta}_1(n,n+1)$ We now consider the impact of a new observation $y(n+1)$. In this case, we have:

$$\left\{\begin{array}{l}\hat{\beta}_0(n+1)=\hat{\beta}_0(n)+\Delta\hat{\beta}_0(n,n+1)\\ \hat{\beta}_1(n+1)=\hat{\beta}_1(n)+\Delta\hat{\beta}_1(n,n+1)\end{array}\right. \tag{132}$$

In order to compute the adjustment factors $\Delta\hat{\beta}_0(n,n+1)$ and $\Delta\hat{\beta}_1(n,n+1)$, we use the following decomposition:

$$\begin{aligned}\bar{y}_{n+1} &= \frac{1}{n+1}\sum_{t=1}^{n+1} y(t)\\ &= \frac{n}{n+1}\bar{y}_n+\frac{1}{n+1}y(n+1)\end{aligned} \tag{133}$$

and:

$$\begin{aligned}\tilde{y}_{n+1} &= \frac{2}{(n+1)(n+2)}\sum_{t=1}^{n+1} ty(t)\\ &= \frac{n}{n+2}\tilde{y}_n+\frac{2}{n+2}y(n+1)\end{aligned} \tag{134}$$

We have:

$$\Delta\hat{\beta}_0(n,n+1)=-\frac{4n+8}{(n-1)(n+1)}\bar{y}_n+\frac{6}{n-1}\tilde{y}_n-\frac{2}{n+1}y(n+1) \tag{135}$$

and:

$$\Delta\hat{\beta}_1(n,n+1)=\frac{12}{(n-1)(n+1)}\bar{y}_n-\frac{18}{(n-1)(n+2)}\tilde{y}_n+\frac{6}{(n+1)(n+2)}y(n+1) \tag{136}$$

We notice that the adjustment $\Delta\hat{\beta}_0(n,n+1)$ or $\Delta\hat{\beta}_1(n,n+1)$ is a weighted average of $\bar{y}_n$, $\tilde{y}_n$ and $y(n+1)$ such that the sum of weights are equal to zero[37].

Dynamic analysis

Transition equation Equations (133)–(136) form a first-order Markov linear process:

$$z(n+1)=A(n)z(n)+B(n)y(n+1) \tag{139}$$

where:

$$z(n)=\begin{pmatrix}\hat{\beta}_0(n)\\ \hat{\beta}_1(n)\\ \bar{y}_n\\ \tilde{y}_n\end{pmatrix} \tag{140}$$

[37] We have:

$$-\frac{4n+8}{(n-1)(n+1)}+\frac{6}{n-1}-\frac{2}{n+1}=0 \tag{137}$$

and:

$$\frac{12}{(n-1)(n+1)}-\frac{18}{(n-1)(n+2)}+\frac{6}{(n+1)(n+2)}=0 \tag{138}$$

Indeed, we have:

$$\begin{pmatrix} \hat{\beta}_0(n+1) \\ \hat{\beta}_1(n+1) \\ \bar{y}_{n+1} \\ \tilde{y}_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & A_{1,3}(n) & A_{1,4}(n) \\ 0 & 1 & A_{2,3}(n) & A_{2,4}(n) \\ 0 & 0 & A_{3,3}(n) & 0 \\ 0 & 0 & 0 & A_{4,4}(n) \end{pmatrix} \begin{pmatrix} \hat{\beta}_0(n) \\ \hat{\beta}_1(n) \\ \bar{y}_n \\ \tilde{y}_n \end{pmatrix} + \begin{pmatrix} B_1(n) \\ B_2(n) \\ B_3(n) \\ B_4(n) \end{pmatrix} y(n+1) \tag{141}$$

where:

$$\left\{ \begin{array}{l} A_{1,3}(n) = -\dfrac{4n+8}{(n+1)(n-1)} \\ A_{1,4}(n) = \dfrac{6}{n-1} \\ A_{2,3}(n) = \dfrac{12}{(n-1)(n+1)} \\ A_{2,4}(n) = -\dfrac{18}{(n+2)(n-1)} \\ A_{3,3}(n) = \dfrac{n}{n+1} \\ A_{4,4}(n) = \dfrac{n}{n+2} \end{array} \right. \tag{142}$$

and:

$$\left\{ \begin{array}{l} B_1(n) = -\dfrac{2}{n+1} \\ B_2(n) = \dfrac{6}{(n+2)(n+1)} \\ B_3(n) = \dfrac{1}{n+1} \\ B_4(n) = \dfrac{2}{n+2} \end{array} \right. \tag{143}$$

Because we have a Markov process, we can easily compute the value of $z(n+h)$ where $h \geq 1$ given the current value $z(n)$. For instance, we have:

$$\begin{aligned} z(n+2) &= A(n+1)z(n+1) + B(n+1)y(n+2) \\ &= A(n+1)A(n)z(n) + A(n+1)B(n)y(n+1) + B(n+1)y(n+2) \end{aligned} \tag{144}$$

and:

$$\begin{aligned} z(n+3) &= A(n+2)z(n+2) + B(n+2)y(n+3) \\ &= A(n+2)A(n+1)A(n)z(n) + A(n+2)A(n+1)B(n)y(n+1) + \\ &\quad A(n+2)B(n+1)y(n+2) + B(n+2)y(n+3) \end{aligned} \tag{145}$$

More generally, we have:

$$z(n+h) = \left(\prod_{j=0}^{h-1} A(n+j)\right) z(n) + \sum_{j=0}^{h-1} \left(\prod_{k=j+1}^{h-1} A(n+k)\right) B(n+j)y(n+j+1) \tag{146}$$

with the convention:

$$\prod_{k=j+1}^{h-1} A(n+k) = I_4 \qquad \text{if } j+1 > h-1 \tag{147}$$

Measurement equation Using Equations (139) and (146), we can compute several quantities. For instance, we retrieve the time-varing estimates $\hat{\beta}_0(n+h)$ and $\hat{\beta}_1(n+h)$ by using the following measurement equation:

$$\begin{pmatrix} \hat{\beta}_0(n+h) \\ \hat{\beta}_1(n+h) \end{pmatrix} = Cz(n+h) \tag{148}$$

where:

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{149}$$

If we are interested in the variations $\Delta\hat{\beta}_0(n, n+h)$ and $\Delta\hat{\beta}_1(n, n+h)$, we have:

$$\begin{pmatrix} \Delta\hat{\beta}_0(n, n+h) \\ \Delta\hat{\beta}_1(n, n+h) \end{pmatrix} = C\left(z(n+h) - z(n)\right) \tag{150}$$

or:

$$\begin{aligned} \begin{pmatrix} \Delta\hat{\beta}_0(n, n+h) \\ \Delta\hat{\beta}_1(n, n+h) \end{pmatrix} &= C\left(\left(\prod_{j=0}^{h-1} A(n+j)\right) - I_4\right) z(n) + \\ &\quad C\sum_{j=0}^{h-1}\left(\prod_{k=j+1}^{h-1} A(n+k)\right) B(n+j)\, y(n+j+1) \end{aligned} \tag{151}$$

Computation of the burn-out scenario We would like to compute $y(n+1)$ such that $\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) = y^\star$. We have:

$$\begin{aligned} \hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) &= \hat{\beta}_0(n+1) + \hat{\beta}_1(n+1) \cdot n^\star \\ &= \left(\hat{\beta}_0(n) + \Delta\hat{\beta}_0(n, n+1)\right) + \\ &\quad \left(\hat{\beta}_1(n) + \Delta\hat{\beta}_1(n, n+1)\right) \cdot n^\star \\ &= \hat{y}\left(n^\star \mid \mathcal{F}_n\right) + \Delta\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) \end{aligned} \tag{152}$$

where $\Delta\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) = \Delta\hat{\beta}_0(n, n+1) + \Delta\hat{\beta}_1(n, n+1)\, n^\star$. Since we have:

$$\begin{aligned} \Delta\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) &= \frac{12n^\star - 4n - 8}{(n-1)(n+1)}\bar{y}_n + \\ &\quad \frac{6n + 12 - 18n^\star}{(n-1)(n+2)}\tilde{y}_n + \\ &\quad \frac{6n^\star - 2n - 4}{(n+1)(n+2)} y(n+1) \end{aligned} \tag{153}$$

we deduce that:

$$\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) = y^\star \Leftrightarrow \Delta\hat{y}\left(n^\star \mid \mathcal{F}_{n+1}\right) = y^\star - \hat{y}\left(n^\star \mid \mathcal{F}_n\right) \tag{154}$$

It follows that:

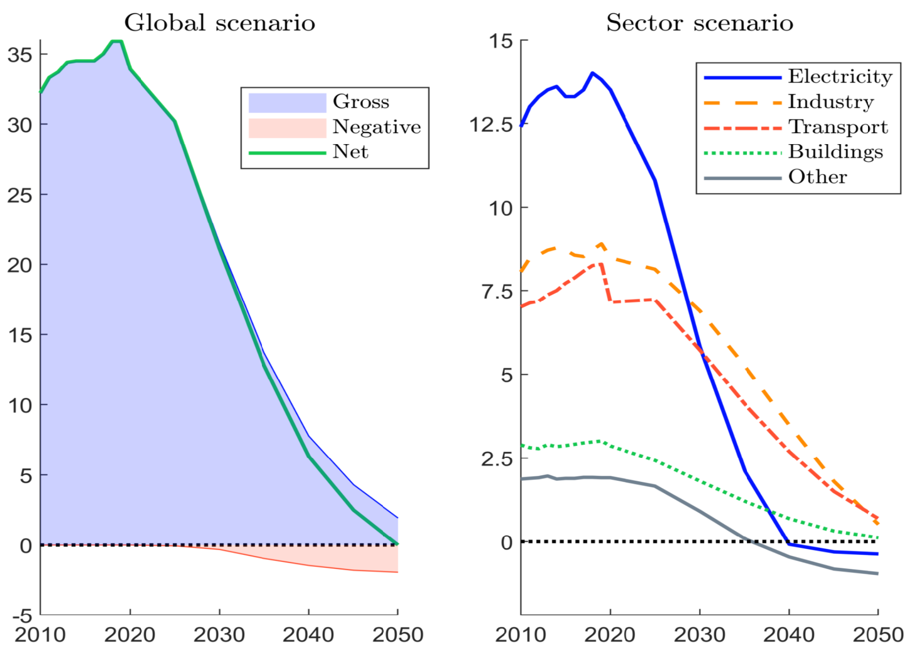
$$\begin{aligned} y(n+1) &= \frac{(n+1)(n+2)}{(6n^\star - 2n - 4)}\left(y^\star - \hat{y}\left(n^\star \mid \mathcal{F}_n\right)\right) - \\ &\quad \frac{(n+2)(12n^\star - 4n - 8)}{(n-1)(6n^\star - 2n - 4)}\bar{y}_n - \\ &\quad \frac{(n+1)(6n + 12 - 18n^\star)}{(n-1)(6n^\star - 2n - 4)}\tilde{y}_n \end{aligned} \tag{155}$$

A.4 Complementary materials

A.4.1 IEA NZE scenario

The International Energy Agency has produced a comprehensive roadmap for the net zero energy sector by 2050 (IEA, 2021). The first steps (up to 2030) depend on existing technologies but half of the later phase (2030–2050) relies on innovations. This 1.5°C scenario looks at the Energy sector and the three main consumption sectors (Buildings, Industry and Transport). These carbon pathways are reported in Figure 22.

Figure 22: CO_2 emissions by sector in the NZE scenario



Source: IEA (2021) & Authors' calculations.

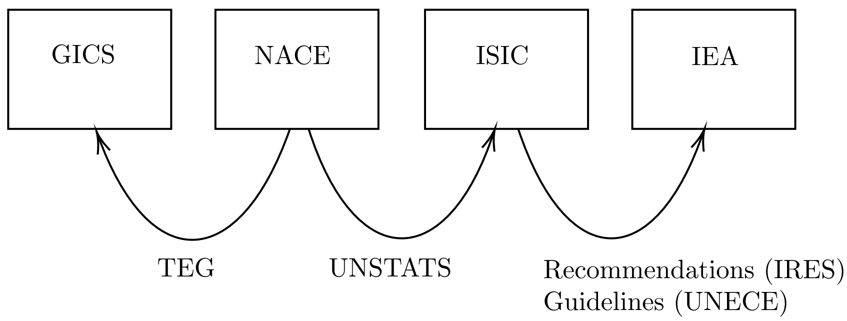
Since we analyze financial markets, we need to identify if an asset belongs to an IEA sector. However, asset owners and managers are more familiar with the Global Industry Classification System (GICS) (Chan *et al.*, 2007). The statistical classification of economic activities in the European Community (NACE) becomes a pivot classification on one hand[38], whereas the UN statistical division[39] matches the NACE categories to the International Standard Industrial Classification (ISIC) of all economic activities on the other hand. United Nations (2018) gives recommendations for the energy sector (IRES) with references to the relevant ISIC category. Following Carvalho and Guyon (2020) who map the industry sector of the IRES classification to the manufacturing, construction and non-fuel mining industries with the corresponding ISIC codes, we perform a similar mapping using the correspondence process given in Figure 23. In their guidelines for the application of environmental indicators, the United Nations Economic Commission for Europe (UNECE) specifies the ISIC divisions for Transport. Moreover, we do not separate Buildings and the residual Other sector since the NZE trajectories are relatively close between them. Using NACE as the

[38] The technical expert group on sustainable finance provides a NACE to GICS correspondence (TEG, 2020).

[39] See the website `unstats.un.org/unsd/classifications`.

pivot classification, we match the GICS sub-industry with the IEA sectors. For 600 GICS sub-industries out of 648, we have a dominant IEA sector where NACE divisions, groups and classes corresponding to a given GICS sub-industry are alloacted at 75% or more to one IEA sector. As a result, we have a perfect match between GICS sub-industries and IEA sectors in 72% of cases.

Figure 23: Sector classification correspondence

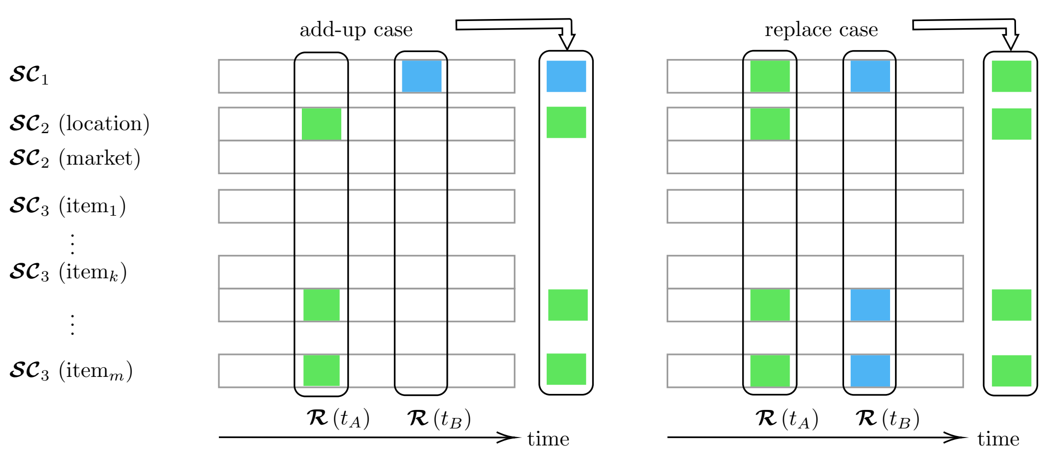


Source: Authors' calculations.

A.4.2 Algorithm for managing carbon targets

For each company, we combine the available targets. We translate each target into a vector of emissions reduction per year with the associated scopes ($\mathcal{SC}_1$, $\mathcal{SC}_2$ and $\mathcal{SC}_3$). We iterate from the most recent target $\mathcal{R}_i(t_B)$ to the oldest target $\mathcal{R}_i(t_A)$. At each step, we decide if we should bring the older target into the combined target or if we should replace the combined target with the older target. As showed in Figure 24, the trigger is the overlap between the scopes. Indeed, if the older target's scopes are complementary with the current combined target, we add the targets (add-up case), while if the older target has a better (overlapping) scope emissions coverage, we retain the older target (replace case).

Figure 24: Carbon target aggregation



Source: Authors' calculations.

B Additional results

B.1 Tables

Table 23: Estimation of the slope (Example 6)

t	$\hat{\beta}_{i,1}(t)$
2010	-0.9030
2011	-1.5510
2012	-2.2703
2013	-2.2036
2014	-2.0763
2015	-1.9325
2016	-2.0059
2017	-2.0161
2018	-2.0691
2019	-1.9488
2020	-1.7832

Source: Authors' calculations.

Table 24: Number of issuers by sector

GICS sector	IEA sector				Total
	Electricity	Industry	Other	Transport	
Communication Services			41		41
Consumer Discretionary		52	29		81
Consumer Staples		57	16		73
Energy	23		7	4	34
Financials			135		135
Health Care	4	29	7		40
Industrials	3	74	42	16	135
Information Technology		46	24		70
Materials		63			63
Real Estate		21	2		23
Utilities	56				56
Total	86	342	303	20	751

Source: CDP database (2021) & Authors' calculations.

Table 25: Number of issuers by region

Region		IEA sector				Total
		Electricity	Industry	Other	Transport	
DM		66	276	245	19	606
	EMU	31	88	71	4	194
	Europe-ex-EMU	7	67	60	4	138
	North America	26	93	106	9	234
	Other DM	2	28	8	2	40
EM		20	66	58	1	145
Total		86	342	303	20	751

Source: CDP database (2021) & Authors' calculations.

Table 26: Scope 1 + 2 + 3 carbon emissions of the MSCI World index in tCO_2e ($W = \$1$ mn, $h = 2$ years)

Year	Missing	$\mathcal{CE}(x;W)$			$\mathcal{CI}(x;W)$		
		(1)	(2)	(3)	(1)	(2)	(3)
2013	5.51%	390.3	408.8	475.2	353.7	353.0	352.8
2014	5.23%	378.4	395.6	457.9	349.6	346.3	346.3
2015	6.26%	381.7	399.3	453.9	319.5	316.9	316.5
2016	5.61%	352.4	378.2	428.9	338.9	339.5	339.5
2017	4.69%	267.8	284.9	316.5	330.7	330.4	330.4
2018	3.63%	306.4	320.3	353.3	360.5	357.3	357.1
2019	4.27%	264.5	274.3	301.5	340.7	337.1	336.9
2020	6.72%	227.0	232.6	256.4	292.3	289.2	288.5
2021	5.58%	166.7	172.1	188.9	267.3	263.3	263.1

Source: Trucost reporting year (2021), MSCI (2022) & Authors' calculations.

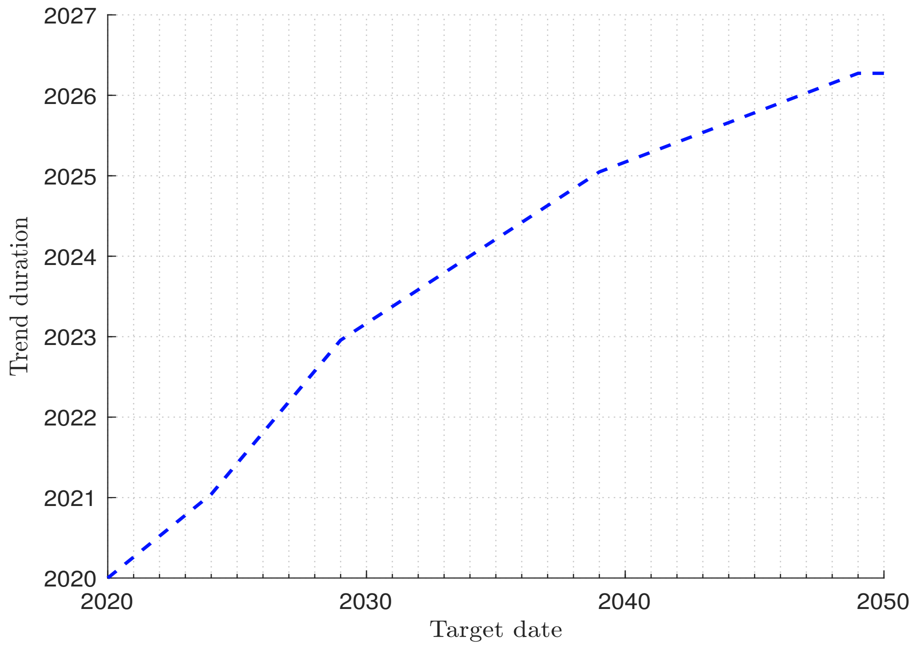
Table 27: Scope 1 + 2 carbon emissions of the MSCI World index in tCO_2e ($W = \$1$ mn, $h = 1$ year)

Year	Missing	$\mathcal{CE}(x;W)$			$\mathcal{CI}(x)$		
		(1)	(2)	(3)	(1)	(2)	(3)
2013	3.63%	209.1	213.8	234.9	192.1	191.6	191.3
2014	3.72%	201.7	205.3	223.0	194.8	193.0	192.9
2015	4.51%	202.7	207.4	222.2	183.6	182.4	182.4
2016	3.85%	188.0	195.8	208.7	200.1	200.7	200.9
2017	2.79%	149.3	151.5	158.7	198.7	196.7	196.6
2018	2.31%	170.3	172.5	180.5	190.4	188.6	188.6
2019	3.67%	135.0	135.4	140.0	168.0	166.9	167.1
2020	4.30%	106.6	108.1	112.5	144.0	142.5	142.4
2021	7.09%	68.5	77.5	89.1	131.5	134.1	134.4

Source: Trucost reporting year (2021), MSCI (2022) & Authors' calculations.

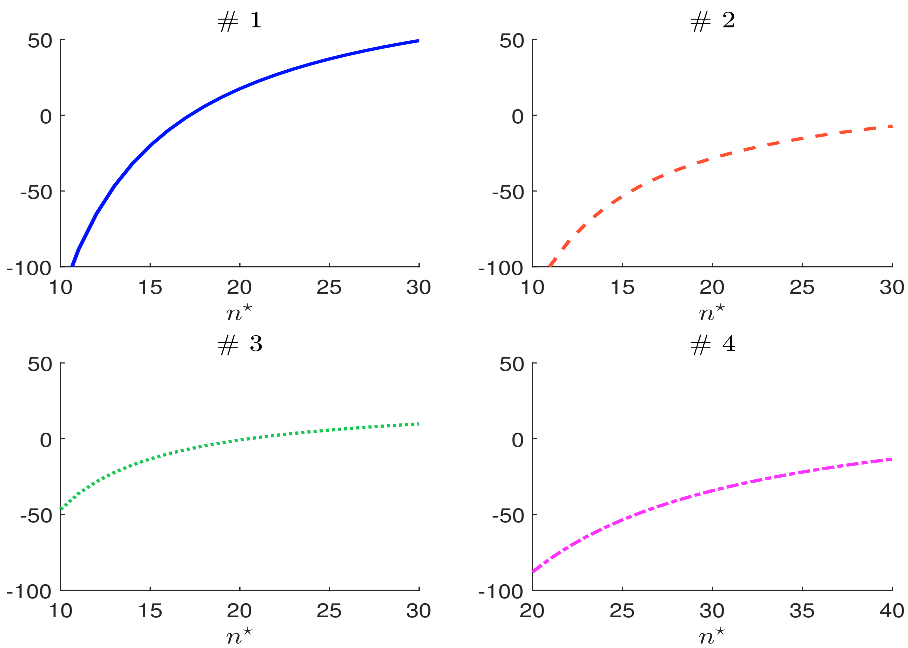
B.2 Figures

Figure 25: Relationship between target dates and rescaled trend durations (Example 5)



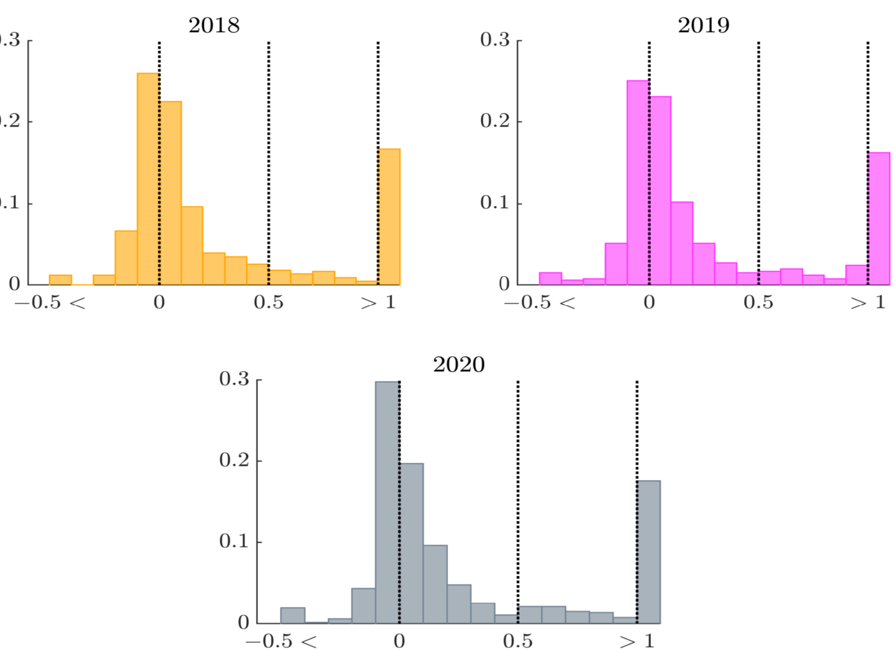
Source: Authors' calculations.

Figure 26: Impact of n and $n^\star$ on the burn-out scenario



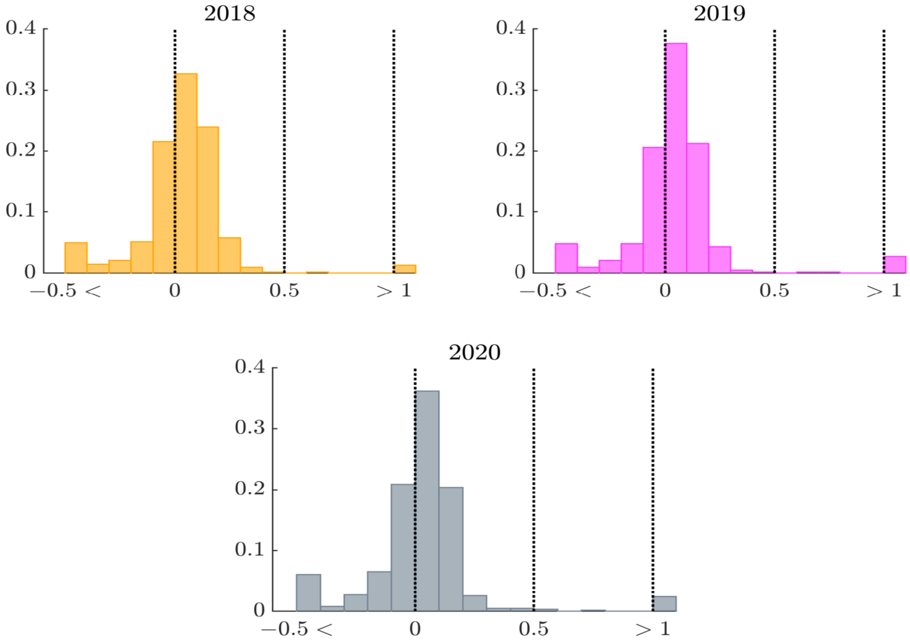
Source: Authors' calculations.

Figure 27: Histogram of the slope $\hat{\beta}_{i,1}(t_{\mathcal{L}ast})$ normalized by the carbon emissions $\boldsymbol{\mathcal{CE}}_i(t_0)$



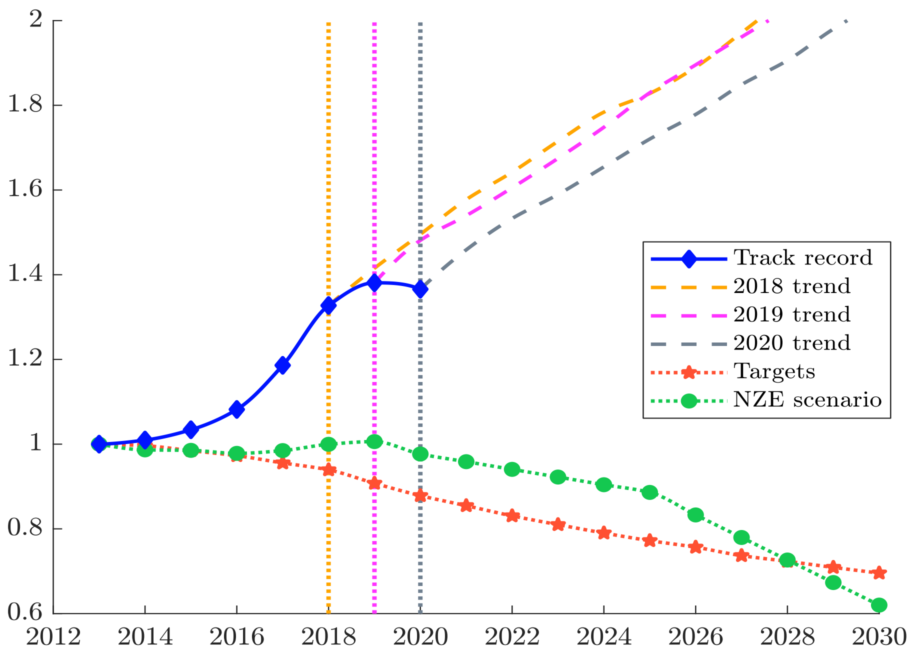
Source: CDP database (2021) & Authors' calculations.

Figure 28: Histogram of the slope $\hat{\beta}_{i,1}(t_{\mathcal{L}ast})$ normalized by the carbon emissions $\boldsymbol{\mathcal{CE}}_i(t_{\mathcal{L}ast})$



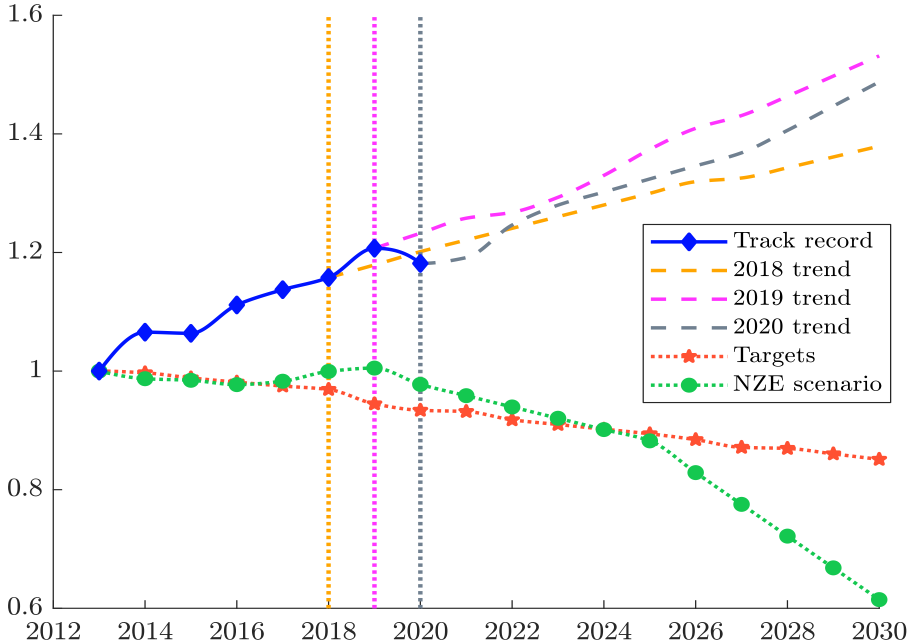
Source: CDP database (2021) & Authors' calculations.

Figure 29: Carbon emissions, trends and targets and NZE scenario (median analysis, developed markets)



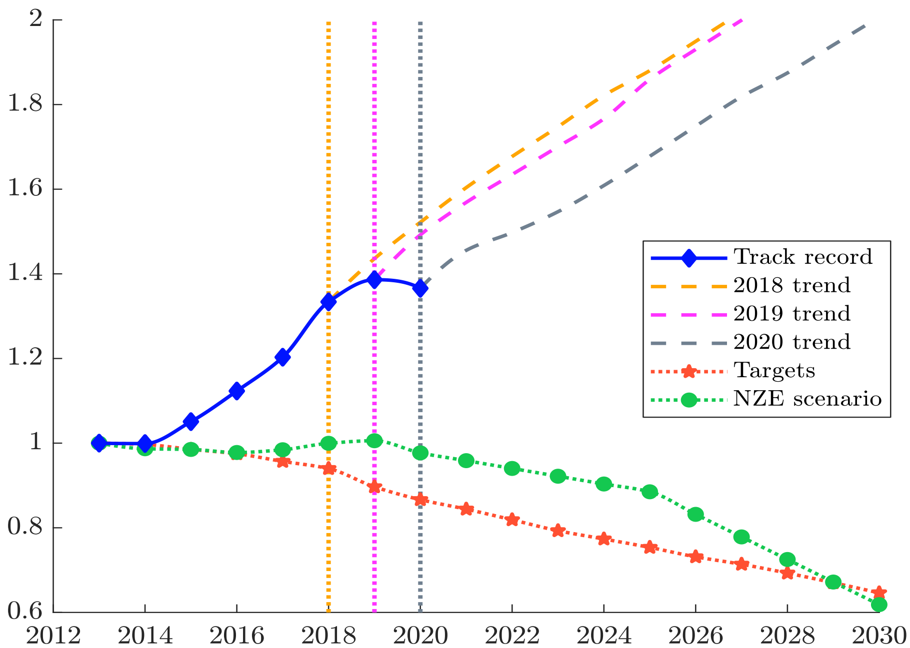
Source: CDP database (2021) & Authors' calculations.

Figure 30: Carbon emissions, trends and targets and NZE scenario (median analysis, emerging markets)



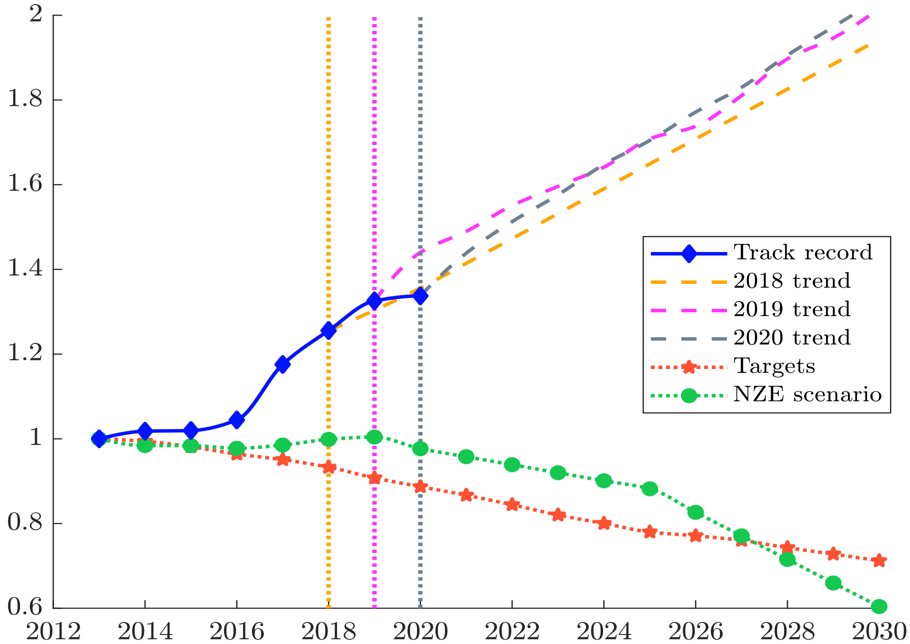
Source: CDP database (2021) & Authors' calculations.

Figure 31: Carbon emissions, trends and targets and NZE scenario (median analysis, Europe)



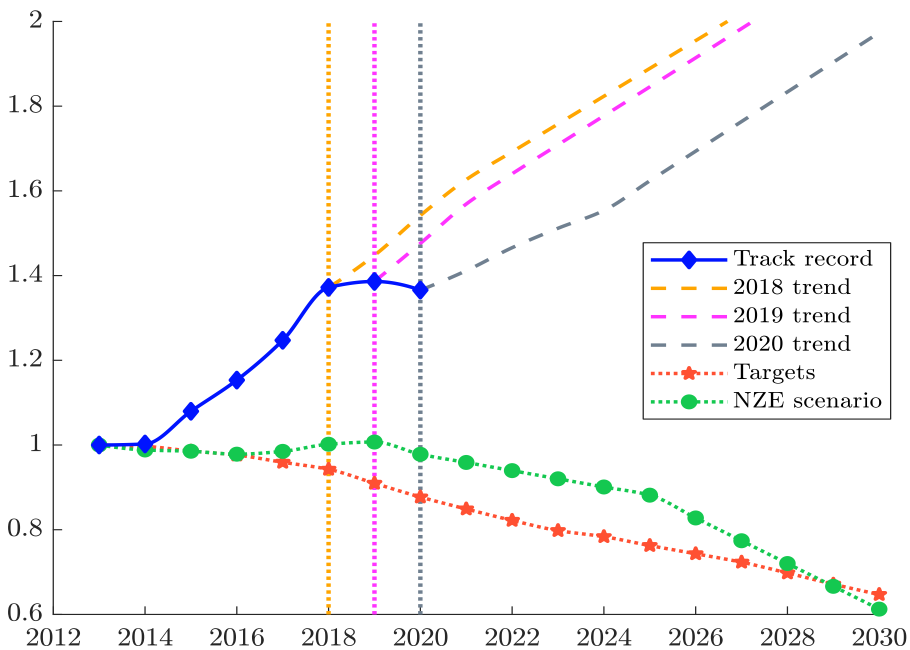
Source: CDP database (2021) & Authors' calculations.

Figure 32: Carbon emissions, trends and targets and NZE scenario (median analysis, North America)



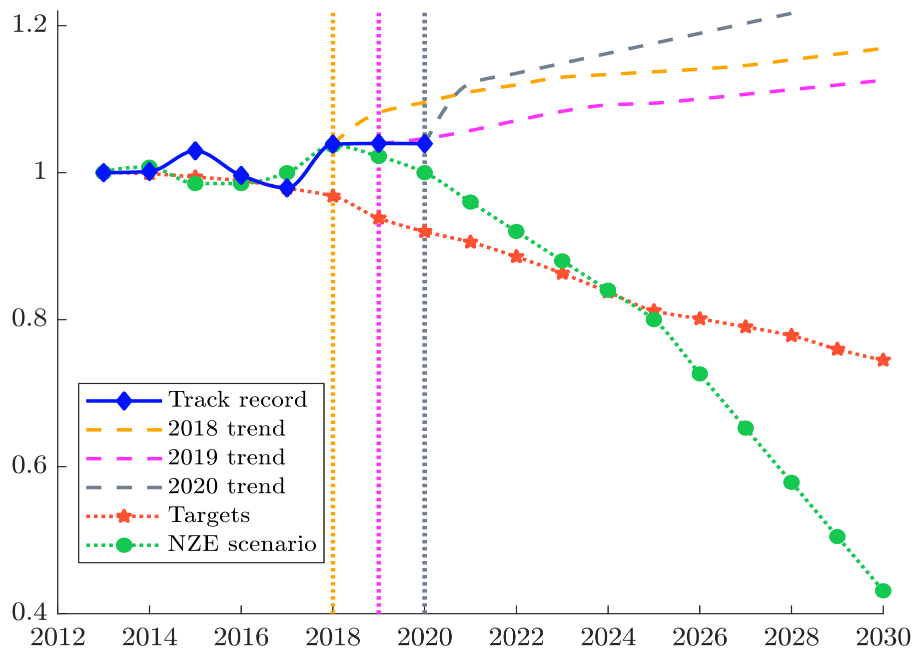
Source: CDP database (2021) & Authors' calculations.

Figure 33: Carbon emissions, trends and targets and NZE scenario (median analysis, EMU)



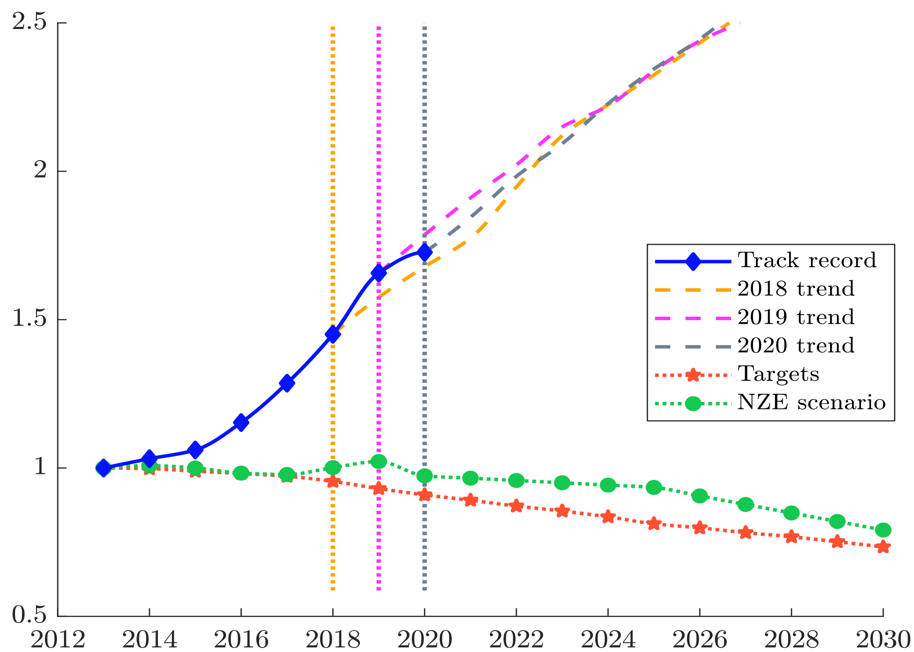
Source: CDP database (2021) & Authors' calculations.

Figure 34: Carbon emissions, trends and targets and NZE scenario (median analysis, Electricity IEA sector)



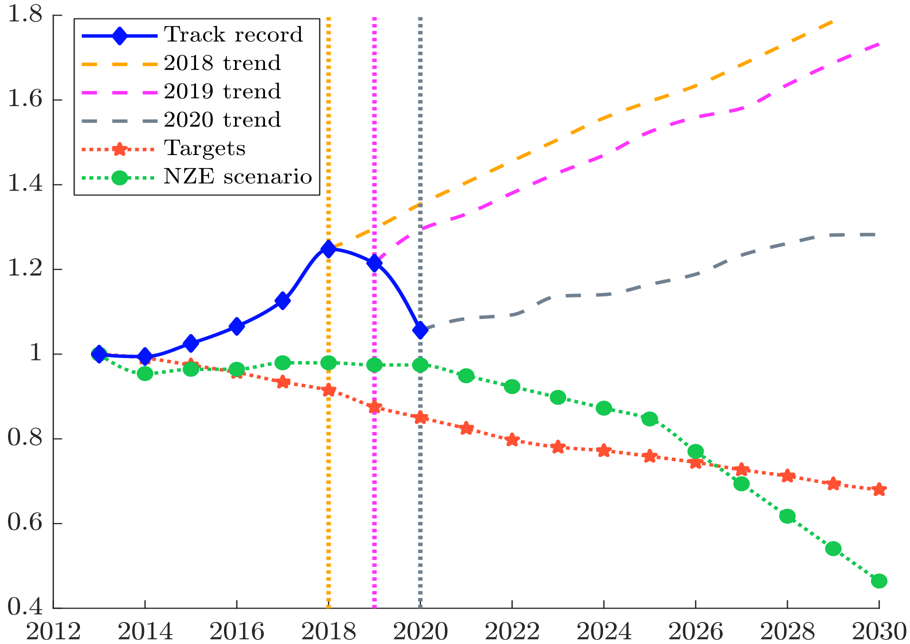
Source: CDP database (2021) & Authors' calculations.

Figure 35: Carbon emissions, trends and targets and NZE scenario (median analysis, Industry IEA sector)



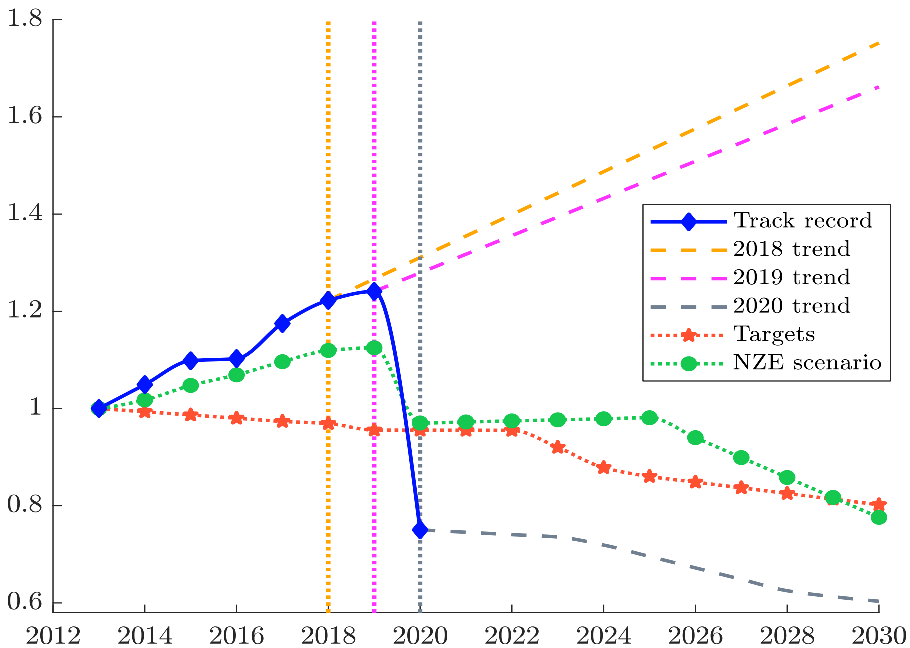
Source: CDP database (2021) & Authors' calculations.

Figure 36: Carbon emissions, trends and targets and NZE scenario (median analysis, Other IEA sector)



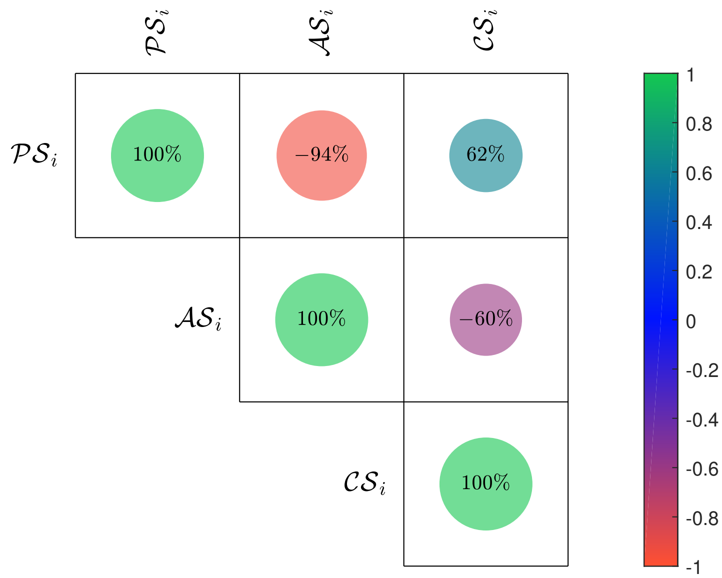
Source: CDP database (2021) & Authors' calculations.

Figure 37: Carbon emissions, trends and targets and NZE scenario (median analysis, Transport IEA sector)



Source: CDP database (2021) & Authors' calculations.

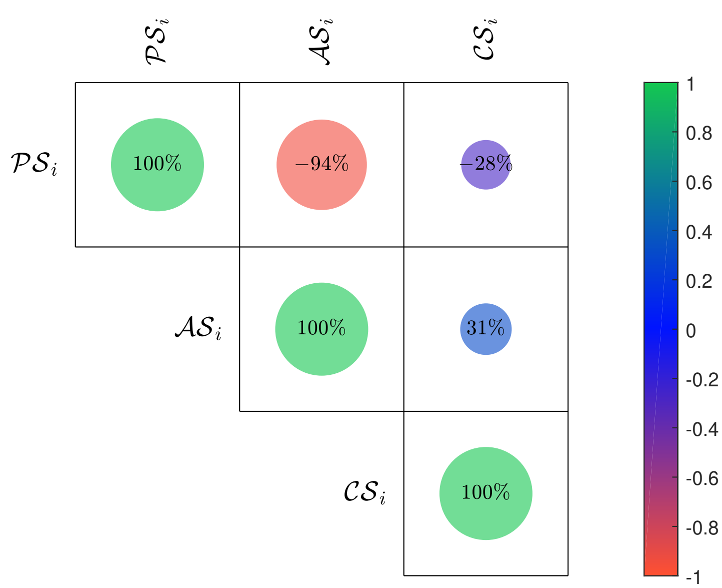
Figure 38: Rank correlation matrix of the $\mathcal{PAC}$ scoring system



The scoring system is based on the q-scores with $s_i = -\hat{\beta}_{i,1} / \boldsymbol{\mathcal{CE}}_i\,(2020)$ for the participation score $\mathcal{PS}_i$, $s_i = \left(\boldsymbol{\mathcal{CB}}_i^{\mathcal{T}rend}\,(2020, 2030) - \boldsymbol{\mathcal{CB}}_i^{\mathcal{T}arget}\,(2020, 2030)\right) / \boldsymbol{\mathcal{CE}}_i\,(2020)$ for the ambition score $\mathcal{AS}_i$, and $s_i = \left(\mathcal{G}ap_i^{\mathcal{T}arget}\,(2030) - \mathcal{G}ap_i^{\mathcal{T}rend}\,(2030)\right) / \boldsymbol{\mathcal{CE}}_i\,(2020)$ for the credibility score $\mathcal{CS}_i$.

Source: CDP database (2021) & Authors' calculations.

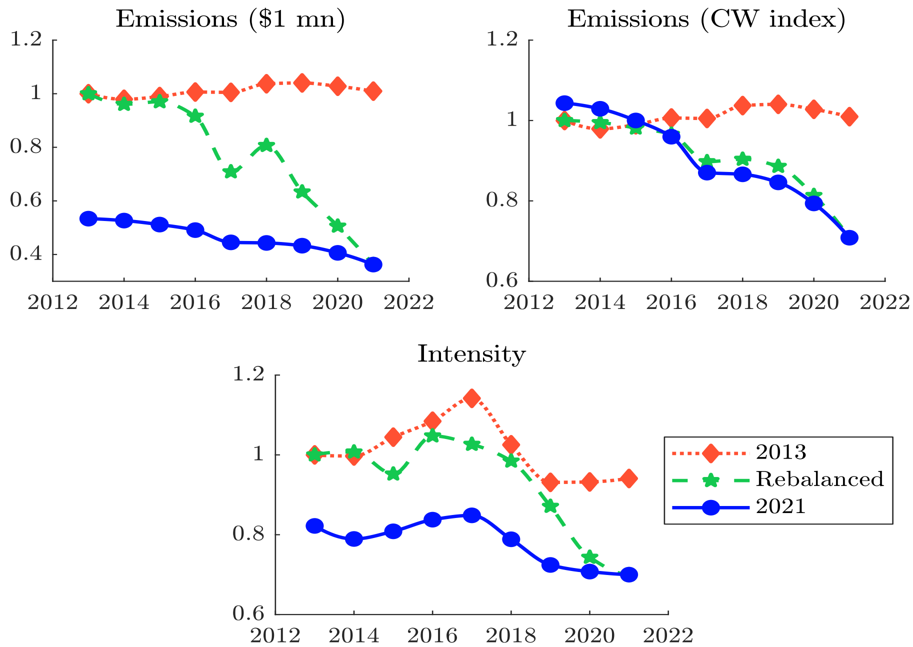
Figure 39: Rank correlation matrix of the $\mathcal{PAC}$ scoring system



The scoring system is based on the q-scores with $s_i = -\hat{\beta}_{i,1} / \boldsymbol{\mathcal{CE}}_i\,(2020)$ for the participation score $\mathcal{PS}_i$, $s_i = \left(\boldsymbol{\mathcal{CB}}_i^{\mathcal{T}rend}\,(2020, 2030) - \boldsymbol{\mathcal{CB}}_i^{\mathcal{T}arget}\,(2020, 2030)\right) / \boldsymbol{\mathcal{CE}}_i\,(2020)$ for the ambition score $\mathcal{AS}_i$, and $s_i = 1 - m_i^{Slope}$ for the credibility score $\mathcal{CS}_i$.

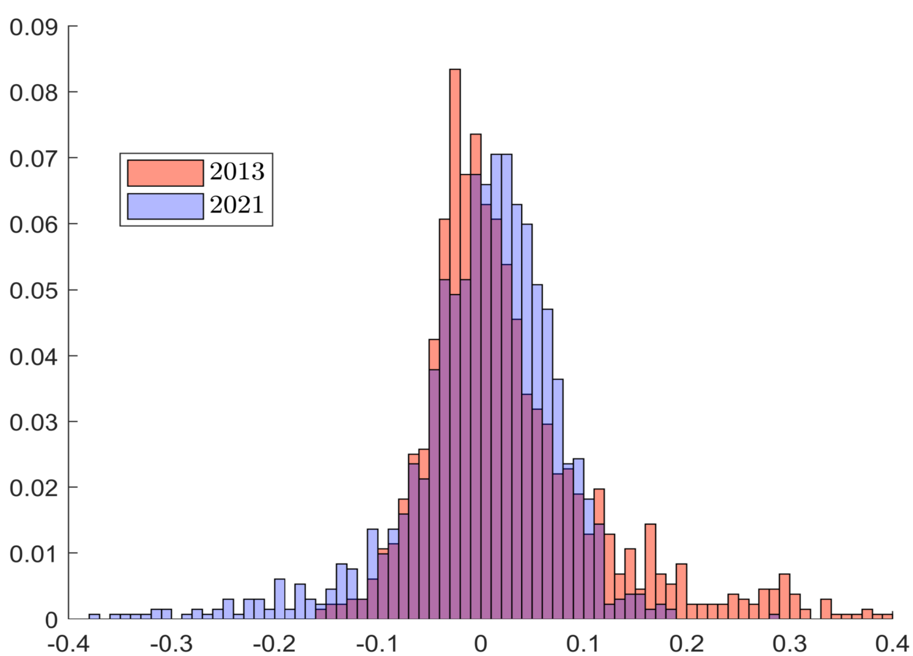
Source: CDP database (2021) & Authors' calculations.

Figure 40: Scope 1 + 2 carbon emissions and intensity (MSCI World index, $h = 1$ year)



Source: MSCI (2021), Trucost reporting year (2021) & Authors' calculations.

Figure 41: Histogram of the rescaled slope $\hat{\beta}_{i,1}$ (MSCI World index, 2013-2021)



Source: MSCI (2021), Trucost reporting year (2021) & Authors' calculations.