Course 2022-2023 in Sustainable Finance

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¹The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Overview
The objective of this course is to understand the concepts of sustainable finance from the viewpoint of asset owners and managers.

Textbook
Handbook of Sustainable Finance
Thierry Roncalli
University Paris-Saclay
## General information

### Textbook

ResearchGate: [https://www.researchgate.net/publication/365355205](https://www.researchgate.net/publication/365355205)

### Slides

SSRN: [https://ssrn.com/abstract=4339823](https://ssrn.com/abstract=4339823)
ResearchGate: [https://www.researchgate.net/publication/367479551](https://www.researchgate.net/publication/367479551)

### Additional materials (\LaTeX + Figures + Matlab programs)

http://www.thierry-roncalli.com/SustainableFinanceCourse.html
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   - Many words, one concept
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   - Extensive use of acronyms

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   - Reporting frameworks
   - Regulatory framework

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   - The market share of ESG investing
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   - Corporate ESG data

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   - Other statistical methods
   - Performance evaluation criteria

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   - Rating migration matrix
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- ESG efficient frontier

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- Sovereign bonds
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   - Regulation of climate risk
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   - Natural capital and negative externalities

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   - Other IAMs

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   - Shared socioeconomic pathways
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- Carbon intensity

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- Carbon trend
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- Green revenue share
- Other greenness metrics
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- Other topics
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   - Equity portfolios
   - Bond portfolios

29 Climate portfolio allocation
   - Portfolio decarbonization
   - Net zero portfolio alignment
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30 Climate risk hedging
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32. Equity portfolios

33. Bond portfolios
   - $L_2$-norm risk measures
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- **Definition**
  - Chronic vs. acute risk

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  - General framework
  - Geolocation

- **Applications**
  - Cyclones and hurricanes
  - Floods
  - Other physical risks
Course 2022-2023 in Sustainable Finance
Lecture 1. Introduction

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“Sustainable finance refers to the process of taking environmental, social and governance (ESG) considerations into account when making investment decisions in the financial sector, leading to more long-term investments in sustainable economic activities and projects. **Environmental considerations** might include climate change mitigation and adaptation, as well as the environment more broadly, for instance the preservation of biodiversity, pollution prevention and the circular economy. **Social considerations** could refer to issues of inequality, inclusiveness, labour relations, investment in human capital and communities, as well as human rights issues. **The governance** of public and private institutions — including management structures, employee relations and executive remuneration — plays a fundamental role in ensuring the inclusion of social and environmental considerations in the decision-making process.” (European Commission).
Many words, one concept

Figure 1: Many words, one concept
Responsible investment (RI)

Responsible investment is an approach to investment that explicitly acknowledges the relevance to the investor of environmental, social and governance factors, and of the long-term health of the market as a whole.

Sustainable investing (SI)

Sustainable investing is an investment approach that considers environmental, social and governance factors in portfolio selection.

Socially responsible investing (SRI)

SRI is an investment strategy that is considered socially responsible, because it invests in companies that have ethical practices.

Environmental, Social and Governance (ESG)

Environmental, Social, and Corporate Governance (ESG) refers to the factors that measure the sustainability of an investment.
Sustainable Investing
≈
Socially Responsible Investing (SRI)
≈
Environmental, Social, and Governance (ESG)

Remark
Blue Finance ⊂ Green Finance, Climate Finance ⊂ Sustainable Finance
Historical perspective

- Responsible investment (RI): 2000's
- ESG investing (ESG): 2010's
- Sustainable finance (SF): 2020's

Why?
At the beginning, sustainable finance mainly concerns final investors and asset owners (ethics) ⇔ responsible investment

Then, it gains momentum in asset management ⇔ ESG investing

Finally, it spreads across all financial actors (e.g. issuers, banks, central banks, etc.) ⇔ Sustainable finance
ESG motivations

Figure 2: The raison d’être of ESG investing
A myriad of acronyms

How many acronyms do you know?
A myriad of acronyms

Many financial actors

ESG financial ecosystem

- Asset owners (pension funds, sovereign wealth funds (SWF), insurance and institutional investors, retail investors, etc.)
- Asset managers
- ESG rating agencies
- ESG index sponsors
- Banks
- ESG associations (GSIA, UNPRI, etc.)
- Regulators and international bodies (governments, financial and industry regulators, central banks, etc.)
- Issuers (equities, bonds, loans, etc.)
- Society and people

ESG Investing ⇔ ESG Financing (= Sustainable Finance)
The issuer point of view of ESG

Corporate financial performance (CFP)
- Friedman (1970)
- Shareholder theory
- Corporations have no social responsibility to the public or society
- Their only responsibility is to its shareholders (profit maximization)

Corporate social responsibility (CSR)
- Freeman (2010)
- Stakeholder theory
- Corporations create negative externalities
- They must have social and moral responsibilities
- Impact on the cost-of-capital and business risk
Sustainable investment forums

GSIA members

- The European Sustainable Investment Forum (Eurosif), http://www.eurosif.org
- Responsible Investment Association Australasia (RIAA), https://responsibleinvestment.org
- Responsible Investment Association Canada (RIA Canada), https://www.riacanada.ca
- UK Sustainable Investment & Finance Association (UKSIF), https://www.uksif.org
- The Forum for Sustainable & Responsible Investment (US SIF), https://www.ussif.org
- Dutch Association of Investors for Sustainable Development (VBDO), https://www.vbdo.nl/en/
- Japan Sustainable Investment Forum (JSIF), https://japansif.com/english
Sustainable investment forums

**Figure 3:** 2018 GSIA report

**Figure 4:** 2020 GSIA report
Initiatives

- Principles for responsible investment (PRI)
- Climate Action 100+
- Net zero alliances: (NZAOA, NZAM, PAII, NZBA, NZIA, etc) ⇒ GFANZ
PRI (or UNPRI)

Figure 5: Principles for Responsible Investment (PRI)

https://www.unpri.org
PRI (or UNPRI)

- Early 2005: UN Secretary-General Kofi Annan invited a group of the world’s largest institutional investors to join a process to develop the Principles for Responsible Investment
- April 2006: The Principles were launched at the New York Stock Exchange
- 6 ESG principles
- The 63 founding signatories are 32 asset owners\(^a\) and 31 asset managers\(^b\) and data providers\(^c\)

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\(^a\) AP2, CDC, CDPQ, CalPERS, ERAFP, FRR, IFC, NZSF, NGPF, PGGM, UNJSPF, USS, etc.

\(^b\) Amundi (CAAM), Sumitomo Trust, BNP PAM, Mitsubishi Trust, Threadneedle, Aviva, Candriam, etc.

\(^c\) Trucost, Vigeo, etc.
Signatories’ commitment

“As institutional investors, we have a duty to act in the best long-term interests of our beneficiaries. In this fiduciary role, we believe that environmental, social, and corporate governance (ESG) issues can affect the performance of investment portfolios (to varying degrees across companies, sectors, regions, asset classes and through time). We also recognise that applying these Principles may better align investors with broader objectives of society. Therefore, where consistent with our fiduciary responsibilities, we commit to the following:

- Principle 1: We will incorporate ESG issues into investment analysis and decision-making processes.
- Principle 2: We will be active owners and incorporate ESG issues into our ownership policies and practices.
- Principle 3: We will seek appropriate disclosure on ESG issues by the entities in which we invest.
- Principle 4: We will promote acceptance and implementation of the Principles within the investment industry.
- Principle 5: We will work together to enhance our effectiveness in implementing the Principles.
- Principle 6: We will each report on our activities and progress towards implementing the Principles.

The Principles for Responsible Investment were developed by an international group of institutional investors reflecting the increasing relevance of environmental, social and corporate governance issues to investment practices. The process was convened by the United Nations Secretary-General. In signing the Principles, we as investors publicly commit to adopt and implement them, where consistent with our fiduciary responsibilities. We also commit to evaluate the effectiveness and improve the content of the Principles over time. We believe this will improve our ability to meet commitments to beneficiaries as well as better align our investment activities with the broader interests of society. We encourage other investors to adopt the Principles.”

Source: https://www.unpri.org
Definition
ESG ecosystem
The Market of ESG Investing
Many financial actors
Reporting frameworks
Regulatory framework

PRI

Figure 6: PRI Signatory growth
Source: https://www.unpri.org
Rating agencies


Rating agencies

1. ESG scores and ratings
2. ESG data
3. ESG indices
Regulators: Who? Why?

**Table 1:** The supervision institutions in finance

<table>
<thead>
<tr>
<th></th>
<th>Banks</th>
<th>Insurers</th>
<th>Markets</th>
<th>All sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>BCBS</td>
<td>IAIS</td>
<td>IOSCO</td>
<td>FSB</td>
</tr>
<tr>
<td>EU</td>
<td>EBA/ECB</td>
<td>EIOPA</td>
<td>ESMA</td>
<td>ESFS</td>
</tr>
<tr>
<td>US</td>
<td>FDIC/FRB</td>
<td>FIO</td>
<td>SEC</td>
<td>FSOC</td>
</tr>
</tbody>
</table>

- Greenwashing
  - Explicit & deliberate greenwashing;
  - Unintentional greenwashing.

- Fiduciary duties
**ESG regulations**

*Figure 7: Who will regulate ESG? — The regulators viewpoint (MSCI, 2022)*

ESG regulations

Figure 8: Who will regulate ESG? — The regulated viewpoint (MSCI, 2022)

Visit the MSCI website

https://www.msci.com/who-will-regulate-esg

and obtain the detailed list of regulations by year, country, regulator, regulated investors, etc.
The example of central banks

Figure 9: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)

- Launched at the Paris One Planet Summit (OPS) on December 2017
- 8 founding members: Banco de Mexico, BoE, BdF, Dutch Central Bank, Buba, Swedish FSA, HKMA, MAS and PBOC
- As of March 19th 2021, the NGFS consists of 89 members (CBs, EBA, EIOPA, ESMA) and 13 observers (BCBS, IMF, IAIS, IOSCO)
The example of central banks

Go the NGFS website (https://www.ngfs.net) and download the NGFS climate scenarios: https://www.ngfs.net/en/publications/ngfs-climate-finance-research-portal

See also https://data.ene.iiasa.ac.at/ngfs (NGFS scenario explorer hosted by IIASA³)

³International Institute for Applied Systems Analysis
### Reporting frameworks

**Table 2**: List of the main reporting frameworks

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Acronym</th>
<th>Name</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>GC</td>
<td>UN Global Compact Initiative</td>
<td>2000/2000</td>
</tr>
<tr>
<td></td>
<td>GRI</td>
<td>Global Reporting Initiative</td>
<td>1997/2000</td>
</tr>
<tr>
<td></td>
<td>IIRC</td>
<td>International Integrated Reporting Council</td>
<td>2010/2013</td>
</tr>
<tr>
<td></td>
<td>ISSB</td>
<td>International Sustainability Standards Board</td>
<td>2021/2023</td>
</tr>
<tr>
<td></td>
<td>SASB</td>
<td>Sustainability Accounting Standards Board</td>
<td>2011/2016</td>
</tr>
<tr>
<td></td>
<td>SDGs</td>
<td>UN Sustainable Development Goals</td>
<td>2015/2016</td>
</tr>
<tr>
<td>Climate</td>
<td>CDP</td>
<td>Carbon Disclosure Project</td>
<td>2000/2000</td>
</tr>
<tr>
<td></td>
<td>CDSB</td>
<td>Climate Disclosure Standards Board</td>
<td>2007/2015</td>
</tr>
<tr>
<td></td>
<td>PCAF</td>
<td>Partnership for Carbon Accounting Financials</td>
<td>2019/2020</td>
</tr>
<tr>
<td></td>
<td>SBTi</td>
<td>Science Based Targets initiative</td>
<td>2015/2015</td>
</tr>
<tr>
<td></td>
<td>TCFD</td>
<td>Task Force on Climate-Related Financial Disclosures</td>
<td>2015/2017</td>
</tr>
</tbody>
</table>
Sustainable Development Goals

Figure 10: The SDGs icons

Source: https://sdgs.un.org/goals#icons.
### Sustainable Development Goals

#### Table 3: The 17 SDGs

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Description</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No poverty</td>
<td>End poverty in all its forms everywhere</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Zero hunger</td>
<td>End hunger, achieve food security and improved nutrition and promote sustainable agriculture</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Good health and well-being</td>
<td>Ensure healthy lives and promote well-being for all at all ages</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Quality education</td>
<td>Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gender equality</td>
<td>Achieve gender equality and empower all women and girls</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Clean water and sanitation</td>
<td>Ensure availability and sustainable management of water and sanitation for all</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Affordable and clean energy</td>
<td>Ensure access to affordable, reliable, sustainable and modern energy for all</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Decent work and economic growth</td>
<td>Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Industry, innovation and infrastructure</td>
<td>Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

## Sustainable Development Goals

**Table 4: The 17 SDGs**

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Description</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Reduced inequality</td>
<td>Reduce inequality within and among countries</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>11</td>
<td>Sustainable cities and communities</td>
<td>Make cities and human settlements inclusive, safe, resilient and sustainable</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>12</td>
<td>Responsible consumption and production</td>
<td>Ensure sustainable consumption and production patterns</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>13</td>
<td>Climate action</td>
<td>Take urgent action to combat climate change and its impacts</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Life below water</td>
<td>Conserve and sustainably use the oceans, seas and marine resources for sustainable development</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>15</td>
<td>Life on land</td>
<td>Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Peace, justice, and strong institutions</td>
<td>Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Partnerships for the goals</td>
<td>Strengthen the means of implementation and revitalize the Global Partnership for Sustainable Development</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

The GHG Protocol corporate standard classifies a company’s greenhouse gas emissions in three scopes⁴:

- **Scope 1**: Direct GHG emissions (○)
- **Scope 2**: Consumption of purchased energy (○○)
- **Scope 3**: Other indirect GHG emissions (●●)
  - **Scope 3 upstream**: emissions associated to the supply side
    1. First tier direct (●)
    2. Tier 2 and 3 suppliers (●●)
  - **Scope 3 downstream**: emissions associated with the product sold by the entity
    1. Use of the product (●●●)
    2. Waste disposal & recycling (●●●●)

---

⁴Measurement robustness: from ○○○○ (very high) to ●●●● (very low)
Each year, CDP sends a questionnaire to organizations and collects information on three environmental dimensions:

1. Climate change (based on the GHG Protocol)
2. Forest management
3. Water security
**Carbon Disclosure Project (CDP)**

**Table 5: Examples of 2019 carbon emissions and intensity**

<table>
<thead>
<tr>
<th>Company</th>
<th>Emission (in tCO$_2$e)</th>
<th>Revenue (in $mn)</th>
<th>Intensity (in tCO$_2$e/$mn)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_2$</td>
<td>$SC_3^{up}$</td>
</tr>
<tr>
<td>Amazon</td>
<td>5 760 000</td>
<td>5 500 000</td>
<td>20 054 722</td>
</tr>
<tr>
<td>Apple</td>
<td>50 549</td>
<td>862 127</td>
<td>27 624 282</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>64 829</td>
<td>280 789</td>
<td>1 923 307</td>
</tr>
<tr>
<td>BP</td>
<td>49 199 999</td>
<td>5 200 000</td>
<td>103 840 194</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>905 000</td>
<td>926 000</td>
<td>15 197 607</td>
</tr>
<tr>
<td>Danone</td>
<td>722 122</td>
<td>944 877</td>
<td>28 969 780</td>
</tr>
<tr>
<td>Exxon</td>
<td>111 000 000</td>
<td>9 000 000</td>
<td>107 282 831</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>81 655</td>
<td>692 299</td>
<td>3 101 582</td>
</tr>
<tr>
<td>LVMH</td>
<td>67 613</td>
<td>262 609</td>
<td>11 853 749</td>
</tr>
<tr>
<td>Microsoft</td>
<td>113 414</td>
<td>3 556 553</td>
<td>5 977 488</td>
</tr>
<tr>
<td>Nestle</td>
<td>3 291 303</td>
<td>3 206 495</td>
<td>61 262 078</td>
</tr>
<tr>
<td>Pfizer</td>
<td>734 638</td>
<td>762 840</td>
<td>4 667 225</td>
</tr>
<tr>
<td>Samsung Electronics</td>
<td>5 067 000</td>
<td>10 998 000</td>
<td>33 554 245</td>
</tr>
<tr>
<td>Volkswagen</td>
<td>4 494 066</td>
<td>5 973 894</td>
<td>65 335 372</td>
</tr>
<tr>
<td>Walmart</td>
<td>6 101 641</td>
<td>13 057 352</td>
<td>40 651 079</td>
</tr>
</tbody>
</table>

Source: Trucost (2022) & Authors’ calculations.
### Table 6: The 11 recommended disclosures (TCFD, 2017)

<table>
<thead>
<tr>
<th>Recommendation</th>
<th>#</th>
<th>Recommended Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governance</td>
<td>1</td>
<td>Board oversight</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Management’s role</td>
</tr>
<tr>
<td>Strategy</td>
<td>3</td>
<td>Risks and opportunities</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Impact on organization</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Resilience of strategy</td>
</tr>
<tr>
<td>Risk management</td>
<td>6</td>
<td>Risk ID and assessment processes</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Risk management processes</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Integration into overall risk management</td>
</tr>
<tr>
<td>Metrics and targets</td>
<td>9</td>
<td>Climate-related metrics</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Scope 1, 2, 3 GHG emissions</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Climate-related targets</td>
</tr>
</tbody>
</table>

Examples of recommended metrics

- GHG emissions (absolute scope 1, scope 2, and scope 3 GHG emissions; financed emissions by asset class; weighted average carbon intensity)
- Transition risks (volume of real estate collaterals highly exposed to transition risk; concentration of credit exposure to carbon-related assets; percent of revenue from coal mining)
- Physical risks (number and value of mortgage loans in 100-year flood zones; proportion of real assets exposed to 1:100 or 1:200 climate-related hazards)
- Climate-related opportunities (proportion of green buildings, green revenues)
- Capital deployment (green CAPEX)
- Internal carbon prices (internal carbon price, shadow carbon price)
- Remuneration
**Figure 11**: Total number of ESG regulations

Regulatory framework

Figure 12: Number of ESG regulations per region

European Union

- The action plan on sustainable finance (May 2018)
- The European Green Deal (December 2019)
- The Fit-for-55 package (July 2021)
- The REPowerEU plan or energy security package (May 2022)
European Union

- EU taxonomy regulation
- Climate benchmarks (PAB)
- Sustainable finance disclosure regulation (SFDR)
- MiFID II & IDD
- Corporate sustainability reporting directive (CSRD)
Figure 13: Sustainable finance — implementation timeline

European Union

Last updated: 26 September 2022
Figure 14: Sustainable finance — implementation timeline

- **First FMP PAI statement**: First reference period for the Financial Market Participant (FMP) first Principal Adverse Impact (PAI) statement on 30 June 2023 must be 1 Jan – 31 Dec 2022
- **First two environmental objectives**: Point (a) (climate change mitigation) and point (b) (climate change adaptation) of environmental objectives under Article 9 TR
- **All environmental objectives**: In addition to point (a) and (b) above, point (c) (the sustainable use and protection of water and marine resources), point (d) (the transition to a circular economy), point (e) (pollution prevention and control) and point (f) (the protection and restoration of biodiversity and ecosystems) of environmental objectives under Article 9 TR
- **Art 5 and Art 6 TR**: Transparency of environmentally sustainable investments (Article 5) and of financial products that promote environmental characteristics (Article 6) in pre-contractual disclosures and in periodic reports
- **Art 8 TR DA**: Transparency of undertakings in non-financial statements
- **COM adopted DA bundling SFDR and TR RTSs**: COM bundled all 13 RTS of the SFDR, including the new empowerments for RTS introduced by the TR in one single DA (Commission Delegated Regulation EU 2022/1288)
- **Companies currently subject to NFRD (Non-Financial Reporting Directive)**: requirements apply to financial years (FYs) starting on / after 1 January 2024, first reporting in 2025
- **Large companies not currently subject to NFRD: requirements apply to FYs starting on / after 1 January 2025, first reporting in 2026
- **Listed SMEs**: requirements apply to FYs starting on / after 1 January 2026, first reporting in 2027 (opt-out possible until 2029)
- **3rd country companies**: requirements apply to FYs starting on / after 1 January 2028, first reporting in 2029

Legend:
- Taxonomy Regulation (TR) L1
- Taxonomy Regulation Article 8 Delegated Act (DA)
- Sustainable Finance Disclosures Regulation (SFDR) L1
- SFDR RTS - Joint ESAs draft Regulatory Technical Standards (RTS)
- MiFID and IDD DAs
- UCITS and AIFMD DAs
- Corporate Sustainability Reporting Directive (CSRD) – final text
EU taxonomy regulation

1. Climate change mitigation
2. Climate change adaptation
3. Sustainable use and protection of water and marine resources
4. Transition to a circular economy
5. Pollution prevention and control
6. Protection and restoration of biodiversity and ecosystem
Climate benchmarks

The common principles are:

- A year-on-year self-decarbonization of 7% on average per annum, based on scope 1, 2 and 3 emissions
- A minimum carbon intensity reduction $\mathcal{R}^-$ compared to the investable universe
- A minimum exposure to sectors highly exposed to climate change

Two labels:

1. CTB: (climate transition benchmark) $\Rightarrow \mathcal{R}^- = 30$
2. PAB: (Paris aligned benchmark) $\Rightarrow \mathcal{R}^- = 50$
Article 6 (or non-ESG products)
It covers standard financial products that cannot be Article 8 or Article 9

Article 8 (or ESG products)
It corresponds to financial products which “promote, among other characteristics, environmental or social characteristics, or a combination of those characteristics, provided that the companies in which the investments are made follow good governance practices”

Article 9 (or sustainable products)
In addition to the points covered by Article 8, these financial products have a sustainable investment objective

+ SI, PAI, etc.
⇒ sustainable preferences
**Environmental factors:** (1) climate change mitigation; (2) climate change adaptation; (3) water and marine resources; (4) resource use and circular economy; (5) pollution; (6) biodiversity and ecosystems.

**Social factors:** (1) equal opportunities for all; (2) working conditions; (3) respect for human rights.

**Governance factors:** (1) role and composition of administrative, management and supervisory bodies; (2) business ethics and corporate culture, including anti-corruption and anti-bribery; (3) political engagements of the undertaking, including its lobbying activities; (4) management and quality of relationships with business partners.

*single materiality ≠ double materiality*
ESG strategies

1. Exclusion
   - Exclusion policy & negative (or worst-in-class) screening
2. Values
   - Norms-based screening
3. Selection
   - Positive (or best-in-class) screening
4. Thematic
   - Sustainability themed investing (e.g. green bonds)
5. Integration
   - ESG scoring is fully integrated in portfolio management
6. Engagement
   - Voting policy & shareholder activism
7. Impact
   - Impact investing

**Figure 15:** Categorisation of ESG strategies (Eurosif, 2019)
Exclusion/Negative Screening

The exclusion from a fund or portfolio of certain sectors, companies or practices based on specific ESG criteria (worst-in-class)

Examples:
- Systematic exclusion of issuers rated CCC
- Exclusion of issuers rated BB, B and CCC
- Sector exclusion (e.g., Energy)
- Sub-industry exclusion (e.g. Coal & Consumable Fuels)
- Exclusion list of individual issuers

Source: Global Sustainable Investment Alliance (2019)
Values/Norms-based Screening (and Red Flags)

Screening of investments against minimum standards of business practice based on international norms, such as those issued by the OECD, ILO, UN (Global Compact) and UNICEF\(^a\)

\(^a\)In Europe, the top exclusion criteria are (1) controversial weapons (Ottawa and Oslo treaties), (2) tobacco, (3) all weapons, (4) gambling, (5) pornography, (6) nuclear energy, (7) alcohol, (8) GMO and (9) animal testing (Eurosif, 2019)

Examples:

- Controversial sectors: controversial weapons, conventional weapons, civilian firearms, nuclear weapons, nuclear power, thermal coal, tobacco, alcohol, gambling, adult entertainment, genetically modified, fossil fuels production & reserves
- Many ETF funds
ESG strategies

Selection/Positive Screening

Investment in sectors, companies or projects selected for positive ESG performance relative to industry peers (best-in-class)

Examples:

- Selection of issuers rated **AAA**, **AA** and **A**
- Selection of issuers that have improved their rating (Momentum ESG strategy)

Source: Global Sustainable Investment Alliance (2019)
Thematic/Sustainability Themed Investing

Investment in themes or assets specifically related to sustainability (for example clean energy, green technology or sustainable agriculture)

Examples:

- Funds invested in Green Bonds
- Funds invested in Social Bonds
- Funds invested in Sustainable Infrastructure
- Funds invested in Natural Ressources

Source: Global Sustainable Investment Alliance (2019)
ESG Integration

The systematic and explicit inclusion by investment managers of environmental, social and governance factors into financial analysis

Examples:

- The stock picking score is a mix (50/50) of a fundamental score and an ESG score
- The fund must have an ESG score greater than the score of its benchmark

Source: Global Sustainable Investment Alliance (2019)
Corporate Engagement/Shareholder Action

The use of shareholder power to influence corporate behavior, including through direct corporate engagement (i.e., communicating with senior management and/or boards of companies), filing or co-filing shareholder proposals, and proxy voting that is guided by comprehensive ESG guidelines.

Source: Global Sustainable Investment Alliance (2019)

Examples:
- Voting policy
- Public divestment
- Biodiversity and deforestation financing
- Engagement with target companies on a specific subject (e.g., pay ratio or living wage)
- Escalated engagement: concerns public, proposing shareholder resolutions & litigation
ESG strategies

Impact Investing

Targeted investments aimed at solving social or environmental problems, and including community investing, where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose.

Examples:
- Funds with a Social Impact objective
- Funds invested in Green Bonds
- PAB and CTB ETFs

Source: Global Sustainable Investment Alliance (2019)
Impact Investing/Community Investing

• Impact Investing
  Investing to achieve positive, social and environmental impacts – requires measuring and reporting against these impacts, demonstrating the intentionality of investor and underlying asset/investee, and demonstrating the investor contribution

• Community Investing
  Where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose. Some community investing is impact investing, but community investing is broader and considers other forms of investing and targeted lending activities.

Source: Global Sustainable Investment Alliance (2021)
**Figure 16:** Sustainable investment assets at the start of 2016

- **Global Responsible Investment market in 2016:** $23 tn
- ~ 1/4 of global assets under management
- +25% Growth in 2 years

**Regions:**
- **USA:** $8.7 tn, 33% growth in 2 years
- **Canada:** $1.1 tn, 49% growth in 2 years
- **Europe:** $12.04 tn, 12% growth in 2 years
- **Japan:** $480 bn, (vs $7 bn in 2014)
- **Australia / NZ:** $500 bn, 247% growth in 2 years

Source: GSIA (2016).
Figure 17: Sustainable investment assets at the start of 2018

- **$ 30.7 tn**
  - **Global Responsible Investment major markets in 2018**
  - CANADA: $ 1.7 tn, 42% growth in 2 years
  - USA: $ 12.0 tn, 38% growth in 2 years
  - EUROPE: $ 14.1 tn, 11% growth in 2 years
  - JAPAN: $ 2.2 tn, (vs $ 474 bn in 2016)
  - AUSTRALIA / NZ: $ 0.7 tn, 46% growth in 2 years

- **~ 2/5**
  - ~ 2/5 of global assets under management

- **+34%**
  - +34% growth in 2 years

Figure 18: Sustainable investment assets at the start of 2020

- **USA**: $17tn, 48%* growth in 2 years
- **CANADA**: $2.4tn, 48%* growth in 2 years
- **EUROPE**: $12tn, 34%* growth in 2 years
- **JAPAN**: $2.8tn, 34% growth in 2 years
- **AUS / NZ**: $906bn, 25% growth in 2 years

Source: GSIA (2020).
**The market of ESG investing**

*Figure 19: Asset values of ESG strategies between 2014 and 2018*

The market of ESG investing

Table 7: ESG asset growth

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Exclusion</td>
<td>11.7%</td>
<td>14.6%</td>
<td>-24.0%</td>
<td>15 030</td>
</tr>
<tr>
<td>2</td>
<td>Values/Norms-based</td>
<td>19.0%</td>
<td>-13.1%</td>
<td>-11.5%</td>
<td>4 140</td>
</tr>
<tr>
<td>3</td>
<td>Selection</td>
<td>7.6%</td>
<td>50.1%</td>
<td>-24.9%</td>
<td>1 384</td>
</tr>
<tr>
<td>4</td>
<td>Thematic Investing</td>
<td>55.1%</td>
<td>92.0%</td>
<td>91.4%</td>
<td>1 948</td>
</tr>
<tr>
<td>5</td>
<td>Integration</td>
<td>17.4%</td>
<td>30.2%</td>
<td>43.6%</td>
<td>25 195</td>
</tr>
<tr>
<td>6</td>
<td>Engagement</td>
<td>18.9%</td>
<td>8.3%</td>
<td>6.8%</td>
<td>10 504</td>
</tr>
<tr>
<td>7</td>
<td>Impact Investing</td>
<td>56.8%</td>
<td>33.7%</td>
<td>-20.8%</td>
<td>352</td>
</tr>
</tbody>
</table>

The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Several issues:

- **E**: climate change mitigation, climate change adaptation, preservation of biodiversity, pollution prevention, circular economy
- **S**: inequality, inclusiveness, labor relations, investment in human capital and communities, human rights
- **G**: management structure, employee relations, executive remuneration

⇒ requires a lot of alternative data
Sovereign ESG data

Sovereign ESG framework

- World Bank
- Data may be downloaded at the following webpage:
- E: 27 variables
- S: 22 variables
- G: 18 variables
Table 8: The World Bank database of sovereign ESG indicators

**Environmental**
- Emissions & pollution (5)
- Natural capital endowment and management (6)
- Energy use & security (7)
- Environment/climate risk & resilience (6)
- Food security (3)

**Social**
- Education & skills (3)
- Employment (3)
- Demography (3)
- Poverty & inequality (4)
- Health & nutrition (5)
- Access to services (4)

**Governance**
- Human rights (2)
- Government effectiveness (2)
- Stability & rule of law (4)
- Economic environment (3)
- Gender (4)
- Innovation (3)
Table 9: Indicators of the environmental pillar (World Bank database)

- **Emissions & pollution**: (1) CO2 emissions (metric tons per capita); (2) GHG net emissions/removals by LUCF (Mt of CO2 equivalent); (3) Methane emissions (metric tons of CO2 equivalent per capita); (4) Nitrous oxide emissions (metric tons of CO2 equivalent per capita); (5) PM2.5 air pollution, mean annual exposure (micrograms per cubic meter);

- **Natural capital endowment & management**: (1) Adjusted savings: natural resources depletion (% of GNI); (2) Adjusted savings: net forest depletion (% of GNI); (3) Annual freshwater withdrawals, total (% of internal resources); (4) Forest area (% of land area); (5) Mammal species, threatened; (6) Terrestrial and marine protected areas (% of total territorial area);

- **Energy use & security**: (1) Electricity production from coal sources (% of total); (2) Energy imports, net (% of energy use); (3) Energy intensity level of primary energy (MJ/$2011 PPP GDP); (4) Energy use (kg of oil equivalent per capita); (5) Fossil fuel energy consumption (% of total); (6) Renewable electricity output (% of total electricity output); (7) Renewable energy consumption (% of total final energy consumption);

- **Environment/climate risk & resilience**: (1) Cooling degree days (projected change in number of degree Celsius); (2) Droughts, floods, extreme temperatures (% of population, average 1990-2009); (3) Heat Index 35 (projected change in days); (4) Maximum 5-day rainfall, 25-year return level (projected change in mm); (5) Mean drought index (projected change, unitless); (6) Population density (people per sq. km of land area);

- **Food security**: (1) Agricultural land (% of land area); (2) Agriculture, forestry, and fishing, value added (% of GDP); (3) Food production index (2004-2006 = 100);

Table 10: Indicators of the social pillar (World Bank database)

- **Education & skills**: (1) Government expenditure on education, total (% of government expenditure); (2) Literacy rate, adult total (% of people ages 15 and above); (3) School enrollment, primary (% gross);
- **Employment**: (1) Children in employment, total (% of children ages 7-14); (2) Labor force participation rate, total (% of total population ages 15-64) (modeled ILO estimate); (3) Unemployment, total (% of total labor force) (modeled ILO estimate);
- **Demography**: (1) Fertility rate, total (births per woman); (2) Life expectancy at birth, total (years); (3) Population ages 65 and above (% of total population);
- **Poverty & inequality**: (1) Annualized average growth rate in per capita real survey mean consumption or income, total population (%); (2) Gini index (World Bank estimate); (3) Income share held by lowest 20%; (4) Poverty headcount ratio at national poverty lines (% of population);
- **Health & nutrition**: (1) Cause of death, by communicable diseases and maternal, prenatal and nutrition conditions (% of total); (2) Hospital beds (per 1,000 people); (3) Mortality rate, under-5 (per 1,000 live births); (4) Prevalence of overweight (% of adults); (5) Prevalence of undernourishment (% of population);
- **Access to services**: (1) Access to clean fuels and technologies for cooking (% of population); (2) Access to electricity (% of population); (3) People using safely managed drinking water services (% of population); (4) People using safely managed sanitation services (% of population);

**Table 11:** Indicators of the governance pillar (World Bank database)

- **Human rights:** (1) Strength of legal rights index (0 = weak to 12 = strong); (2) Voice and accountability (estimate);
- **Government effectiveness:** (1) Government effectiveness (estimate); (2) Regulatory quality (estimate);
- **Stability & rule of law:** (1) Control of corruption (estimate); (2) Net migration; (3) Political stability and absence of violence/terrorism (estimate); (4) Rule of law (estimate);
- **Economic environment:** (1) Ease of doing business index (1 = most business-friendly regulations); (2) GDP growth (annual %); (3) Individuals using the internet (% of population);
- **Gender:** (1) Proportion of seats held by women in national parliaments (%); (2) Ratio of female to male labor force participation rate (%) (modeled ILO estimate); (3) School enrollment, primary and secondary (gross), gender parity index (GPI); (4) Unmet need for contraception (% of married women ages 15-49);
- **Innovation:** (1) Patent applications, residents; (2) Research and development expenditure (% of GDP); (3) Scientific and technical journal articles;

Where to find the data?

- National accounts statistics collected by OECD, United Nations Statistics Division (UNSD), etc.
- Internal departments and specialized databases of the World Bank: World Bank Open Data, Business Enabling Environment (BEE), Climate Change Knowledge Portal (CCKP), Global Electrification Database (GEP), etc.
- International organizations: Emission Database for Global Atmospheric Research (EDGAR), Food and Agriculture Organization FAO, International Energy Agency (IEA), International Labour Organization (ILO), World Health Organization (WHO), etc.
- NGOs: Climate Watch, etc.;
- Academic resources: International disasters database (EM-DAT) of the Centre for Research on the Epidemiology of Disasters (Université Catholique de Louvain), etc.
Other frameworks

The most known are FTSE (Beyond Ratings), Moody’s (Vigeo-Eiris), MSCI, Sustainalytics, RepRisk and Verisk Mapplecroft.

⇒ The average cross-correlation between data providers is equal to 85% for the ESG score, 42% for the environmental score, 85% for the social score and 71% for the governance score.
## Bias towards richest countries

### Table 12: Correlation of ESG scores with country's national income (GNI per capita)

<table>
<thead>
<tr>
<th>Factor</th>
<th>ESG</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISS</td>
<td>68%</td>
<td>7%</td>
<td>86%</td>
<td>77%</td>
</tr>
<tr>
<td>FTSE (Beyond Ratings)</td>
<td>91%</td>
<td>74%</td>
<td>88%</td>
<td>84%</td>
</tr>
<tr>
<td>MSCI</td>
<td>84%</td>
<td>10%</td>
<td>90%</td>
<td>77%</td>
</tr>
<tr>
<td>RepRisk</td>
<td>78%</td>
<td>79%</td>
<td>75%</td>
<td>37%</td>
</tr>
<tr>
<td>RobecoSAM</td>
<td>89%</td>
<td>82%</td>
<td>85%</td>
<td>85%</td>
</tr>
<tr>
<td>Sustainalytics</td>
<td>95%</td>
<td>83%</td>
<td>94%</td>
<td>93%</td>
</tr>
<tr>
<td>V.E</td>
<td>60%</td>
<td>23%</td>
<td>79%</td>
<td>39%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>81%</td>
<td>51%</td>
<td>85%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Source: Gratcheva et al. (2020).
The mushrooming growth of data

Figure 20: Palm oil production (2019)

The mushrooming growth of data

**Figure 21:** Palm oil imports (2019)


Source: UN Food and Agriculture Organization (FAO)
The mushrooming growth of data

**Figure 22:** Share of global annual deforestation (2015)

Source: Our World in Data, [https://ourworldindata.org/deforestation](https://ourworldindata.org/deforestation).
The mushrooming growth of data

Figure 23: Threatened mammal species (2018)

An example with the biodiversity risk

Figure 24: Global living planet index

Source: https://livingplanetindex.org/latest_results & Author’s calculation.
An example with the biodiversity risk

Some databases:
- the Red List Index (RLI)
- World Database on Protected Areas (WDPA)
- Integrated Biodiversity Assessment Tool (IBAT)
- Exploring Natural Capital Opportunities, Risks and Exposure (ENCORE)
- Etc.
Corporate ESG data

Data sources:

1. Corporate publications (self-reporting)
   - Annual reports
   - Corporate sustainability reports

2. Financial and regulatory filings (standardized reporting)
   - Mandatory reports (SFDR, CSRD, EUTR, etc.)
   - Non-mandatory frameworks (PRI, TCFD, CDP, etc.)

3. News and other media

4. NGO reports and websites

5. Company assessment and due diligence questionnaire (DDQ)

6. Internal models
Figure 25: From raw data to ESG pillars

- Raw data (or data points)
  - ESG Metrics
  - ESG Indicators
  - ESG Themes
  - ESG Pillars
### Table 13: An example of ESG criteria (corporate issuers)

<table>
<thead>
<tr>
<th>Environmental</th>
<th>Social</th>
<th>Governance</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Carbon emissions</td>
<td>- Employment conditions</td>
<td>- Board independence</td>
</tr>
<tr>
<td>- Energy use</td>
<td>- Community involvement</td>
<td>- Corporate behaviour</td>
</tr>
<tr>
<td>- Pollution</td>
<td>- Gender equality</td>
<td>- Audit and control</td>
</tr>
<tr>
<td>- Waste disposal</td>
<td>- Diversity</td>
<td>- Executive compensation</td>
</tr>
<tr>
<td>- Water use</td>
<td>- Stakeholder opposition</td>
<td>- Shareholder’ rights</td>
</tr>
<tr>
<td>- Renewable energy</td>
<td>- Access to medicine</td>
<td>- CSR strategy</td>
</tr>
<tr>
<td>- Green cars*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Green financing*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) means a specific criterion related to one or several sectors
(Green cars ⇒ Automobiles, Green financing ⇒ Financials)
Corporate ESG data

Some examples:

- Bloomberg rates 11,800 public companies. They use more than 120 ESG indicators and 2,000+ data points.
- ISS ESG rates about 10,000 issuers. They use more than 800 indicators and applies approximately 100 indicators per company.
- FTSE Russell rates about 7,200 securities. They use more than 300 indicators and 14 themes.
- MSCI rates 10,000 companies (14,000 issuers including subsidiaries) and 680,000 securities globally. They use 10 themes, 1000+ data points, 80 exposure metrics and 250+ management metrics.
- Refinitiv rates 12,000 public and private companies. They consider 10 themes. These themes are built using 186 metrics and 630+ data points.
- S&P Dow Jones Indices uses between 16 to 27 criteria scores, a questionnaire and 1,000 data points.
- Sustainalytics rates more than 16,300 companies. They consider 20 material ESG issues, based on 350+ indicators.
The race for alternative data

- Controversies ⇒ NLP (RepRisk, daily basis: 500,000+ documents, 100,000+ sources, 23 languages)
- Geospatial data ⇒ Physical risk
The divergence of corporate ESG ratings

Figure 26: ESG rating disagreement

This graph illustrates the ESG rating divergence. The horizontal axis indicates the value of the Sustainalytics rating as a benchmark for each firm (n = 924). Rating values by the other five raters are plotted on the vertical axis in different colors. For each rater, the distribution of values has been normalized to zero mean and unit variance. The Sustainalytics rating has discrete values that show up visually as vertical lines where several companies have the same rating value.

Source: Berg et al. (2022).
The divergence of corporate ESG ratings

Berg et al. (2022) identify three sources of divergence:

1. **Measurement** divergence refers to situation where rating agencies measure the same indicator using different ESG metrics (56%)

2. **Scope** divergence refers to situation where ratings are based on different set of ESG indicators (38%)

3. **Weight** divergence emerges when rating agencies take different views on the relative importance of ESG indicators” (6%)
## The divergence of corporate ESG ratings

Table 14: Rank correlation among ESG ratings

<table>
<thead>
<tr>
<th></th>
<th>MSCI</th>
<th>Refinitiv</th>
<th>S&amp;P Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refinitiv</td>
<td>43%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>S&amp;P Global</td>
<td>45%</td>
<td>69%</td>
<td>100%</td>
</tr>
<tr>
<td>Sustainalytics</td>
<td>53%</td>
<td>64%</td>
<td>69%</td>
</tr>
</tbody>
</table>

Source: Billio et al. (2021).
One-level tree structure

- $X_1, \ldots, X_m$ are $m$ features
- The score is linear:
  \[ S = \sum_{j=1}^{m} \omega_j X_j \]
- $\omega_j$ is the weight of the $j^{th}$ metric
One-level tree structure

- $X_1, \ldots, X_m$ are $m$ features
- The score is linear:
  \[ S = \sum_{j=1}^{m} \omega_j X_j \]
- $\omega_j$ is the weight of the $j^{\text{th}}$ metric
The Altman $Z$ score is equal to:

$$Z = 1.2 \cdot X_1 + 1.4 \cdot X_2 + 3.3 \cdot X_3 + 0.6 \cdot X_4 + 1.0 \cdot X_5$$

where the variables $X_j$ represent the following financial ratios:

<table>
<thead>
<tr>
<th>$X_j$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Working capital / Total assets</td>
</tr>
<tr>
<td>$X_2$</td>
<td>Retained earnings / Total assets</td>
</tr>
<tr>
<td>$X_3$</td>
<td>Earnings before interest and tax / Total assets</td>
</tr>
<tr>
<td>$X_4$</td>
<td>Market value of equity / Total liabilities</td>
</tr>
<tr>
<td>$X_5$</td>
<td>Sales / Total assets</td>
</tr>
</tbody>
</table>

$Z_i \Rightarrow Z_i^* = (Z_i - m_z) / \sigma_z \Rightarrow$ Decision rule
The intermediary scores are equal to:

\[ S_k^{(1)} = \sum_{j=1}^{m} \omega_{j,k} X_j \]

whereas the expression of the final score is:

\[ S := S_1^{(0)} = \sum_{k=1}^{m(1)} \omega_k^{(0)} S_k^{(1)} \]
Figure 27: A two-level non-overlapping tree

Level 1: $\omega_{1,1}^{(1)} = 50\%$; $\omega_{2,1}^{(1)} = 25\%$; $\omega_{3,1}^{(1)} = 25\%$; $\omega_{4,2}^{(1)} = 50\%$; $\omega_{5,2}^{(1)} = 50\%$; $\omega_{6,3}^{(1)} = 100\%$;

Level 0: $\omega_{1}^{(0)} = \omega_{2}^{(0)} = \omega_{3}^{(0)} = 33.33\%$;
Two-level tree structure

\[
\begin{align*}
S_1^{(1)} &= 0.5 \cdot X_1 + 0.25 \cdot X_2 + 0.25 \cdot X_3 \\
S_2^{(1)} &= 0.5 \cdot X_4 + 0.5 \cdot X_5 \\
S_3^{(1)} &= X_6
\end{align*}
\]

\[
S = \frac{S_1^{(1)} + S_2^{(1)} + S_3^{(1)}}{3}
\]
Two-level tree structure

**Figure 28:** A two-level overlapping tree graph

- **Level 1:** \(\omega_{1,1}^{(1)} = 50\%;\ \omega_{2,1}^{(1)} = 25\%;\ \omega_{3,1}^{(1)} = 25\%;\ \omega_{3,2}^{(1)} = 25\%;\ \omega_{4,2}^{(1)} = 25\%;\ \omega_{5,2}^{(1)} = 50\%;\ \omega_{6,3}^{(1)} = 100\%\; ;

- **Level 0:** \(\omega_{1}^{(0)} = \omega_{2}^{(0)} = \omega_{3}^{(0)} = 33.33\%\; ;

Thierry Roncalli
Course 2022-2023 in Sustainable Finance
Figure 29: Tree data structure
Tree and graph theory

- $L$ is the number of levels
- We have $S_j^{(L)} = X_j$
- The value of the $k^{th}$ node at level $\ell$ is given by:

$$S_k^{(\ell)} = \sum_{j=1}^{m(\ell+1)} \omega_{j,k} S_j^{(\ell+1)}$$
Figure 30: An example of ESG scoring tree (MSCI methodology)

Source: MSCI (2020)
Let $\omega(\ell)$ be the $m(\ell+1) \times m(\ell)$ matrix, whose elements are $\omega_{j,k}^{(\ell)}$ for $j = 1, \ldots, m(\ell+1)$ and $k = 1, \ldots, m(\ell)$.

The final score is equal to:

$$S = \omega^\top X$$

where:

$$\omega = \omega^{(L-1)} \cdots \omega^{(1)} \omega^{(0)}$$
Score normalization

If $X \sim F$, we obtain:

$$G(s) = \Pr \{S \leq s\}$$

$$= \Pr \{\omega^\top X \leq s\}$$

$$= \int \cdots \int 1 \{\omega^\top x \leq s\} \, dF(x)$$

$$= \int \cdots \int 1 \left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \, dF(x_1, \ldots, x_m)$$

$$= \int \cdots \int 1 \left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \, dC(F_1(x_1), \ldots, F_m(x_m))$$

Therefore, the distribution $G$ depends on the copula function $C$ and the marginals $(F_1, \ldots, F_m)$ of $F$

$$F_1 \equiv F_1 \equiv \ldots \equiv F_m \Rightarrow G \equiv F_1?$$
In the independent case, we obtain a convolution probability distribution:

\[
G(s) = \int \cdots \int 1\left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \prod_{j=1}^{m} dF_j(x_j)
\]

If \( X_j \sim N(\mu_j, \sigma_j^2) \), we have \( \omega_j X_j \sim N(\omega_j \mu_j, \omega_j^2 \sigma_j^2) \). We deduce that:

\[
S \sim N\left( \sum_{j=1}^{m} \omega_j \mu_j, \sum_{j=1}^{m} \omega_j^2 \sigma_j^2 \right) \equiv N(\omega^\top \mu, \omega^\top \Sigma \omega)
\]

where \( \mu = (\mu_1, \ldots, \mu_m) \) and \( \Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_m^2) \).
Score normalization

Figure 31: Probability distribution of the scores based on the previous tree
Exercise

We assume that \( X_1 \sim \mathcal{U}_{[0,1]} \) and \( X_2 \sim \mathcal{U}_{[0,1]} \) are two independent random variables. We consider the score \( S \) defined as:

\[
S = \frac{X_1 + X_2}{2}
\]
Score normalization

Figure 32: Geometric interpretation of the probability mass function

Case (a): \( 0 \leq s \leq 0.5 \)

Case (b): \( 0.5 \leq s \leq 1 \)
We deduce that:

\[ \Pr \{ S \leq s \} = \begin{cases} \frac{1}{2} (2s)^2 = 2s^2 & \text{if } 0 \leq s \leq \frac{1}{2} \\ 1 - \frac{1}{2} (2 - 2s)^2 = -1 + 4s - 2s^2 & \text{if } \frac{1}{2} \leq s \leq 1 \end{cases} \]

The density function is then:

\[ g(s) = \begin{cases} 4s & \text{if } 0 \leq s \leq \frac{1}{2} \\ 4 - 4s & \text{if } \frac{1}{2} \leq s \leq 1 \end{cases} \]

In the general case, we have:

\[ S = \frac{X_1 + X_2 + \cdots + X_m}{m} \sim \text{Bates} (m) \]
Figure 33: Probability density function of $S$ (uniform distribution)
Exercise

We assume that $X \sim \mathcal{N} (\mu, \Sigma)$ with $\mu_j = 0$, $\sigma_j = 1$ and $\rho_{j,k} = \rho$ for $j \neq k$. Show that:

$$\mathbb{E} [S] = 0$$

and

$$\text{var} (S) = \rho S^2 (w) + (1 - \rho) \mathcal{H} (\omega)$$

where $S (w) = \sum_{j=1}^{m} \omega_j$ is the sum index and $\mathcal{H} (\omega) = \sum_{j=1}^{m} \omega_j^2$ is the Herfindahl index. Deduce that:

$$\sigma_S = \sqrt{\rho + (1 - \rho) \mathcal{H} (\omega)}$$
How to normalize?

\[ S_k^{(\ell)} = \varphi \left( \sum_{j=1}^{m(\ell+1)} \omega_{j,k} S_j^{(\ell+1)} \right) \]
Score normalization

1. *m*-score normalization:

\[ m_i = \frac{x_i - x^-}{x^+ - x^-} \]

where \( x^- = \min x_i \) and \( x^+ = \max x_i \)

2. *q*-score normalization:

\[ q_i = H(x_i) \]

where \( H \) is the distribution function of \( X \)

3. *z*-score normalization:

\[ z_i = \frac{x_i - \mu}{\sigma} \]

where \( \mu \) and \( \sigma \) are the mathematical expectation and standard deviation of \( X \)

4. *b*-score normalization:

\[ b_i = B^{-1}(H(x_i); \alpha, \beta) \]

where \( B(\alpha, \beta) \) is the beta distribution
Probability integral transform (PIT)

If $X \sim H$ and is continuous, $Y = H(X)$ is a uniform random variable.

We have $Y \in [0, 1]$ and:

\[
\begin{align*}
\Pr \{ Y \leq y \} & = \Pr \{ H(X) \leq y \} \\
& = \Pr \{ X \leq H^{-1}(y) \} \\
& = H(H^{-1}(y)) \\
& = y
\end{align*}
\]
Score normalization

Computing the empirical distribution $\hat{H}$

- Let $\{x_1, x_2, \ldots, x_n\}$ be the sample
- We have:

$$q_i = \hat{H}(x_i) = \Pr\{X \leq x_i\} = \frac{\# \{x_j \leq x_i\}}{n_q}$$

- $n_q = n$ or $n_q = n + 1$?
Score normalization

Exercise

What is the normalization shape of this transformation?

\[ S = \frac{2}{1 + e^{-z}} - 1 \]

Hint: compute the density function.
Score normalization

Example

The data are normally distributed with mean $\mu = 5$ and standard deviation $\sigma = 2$. To map these data into a 0/1 score, we consider the following transform:

$$s := \varphi(x) = \Phi^{-1} \left( \Phi \left( \frac{x - 5}{2} \right); \alpha, \beta \right)$$
Score normalization

Figure 34: Transforming data into $b$-score

Transform function $s = \varphi(x)$

Probability density function of the score
Example

We consider the raw data of 9 companies that belong to the same industry. The first variable measures the carbon intensity of the scope 1 + 2 in 2020, while the second variable is the variation of carbon emissions between 2015 and 2020. We would like to create the score

\[ S \equiv 70\% \cdot X_1 + 30\% \cdot X_2. \]

<table>
<thead>
<tr>
<th>Firm</th>
<th>Carbon intensity in tCO₂e/$ mn</th>
<th>Carbon momentum in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.0</td>
<td>−3.0</td>
</tr>
<tr>
<td>2</td>
<td>38.6</td>
<td>−5.5</td>
</tr>
<tr>
<td>3</td>
<td>30.6</td>
<td>5.6</td>
</tr>
<tr>
<td>4</td>
<td>74.4</td>
<td>−1.3</td>
</tr>
<tr>
<td>5</td>
<td>97.1</td>
<td>−16.8</td>
</tr>
<tr>
<td>6</td>
<td>57.1</td>
<td>−3.5</td>
</tr>
<tr>
<td>7</td>
<td>132.4</td>
<td>8.5</td>
</tr>
<tr>
<td>8</td>
<td>92.5</td>
<td>−9.1</td>
</tr>
<tr>
<td>9</td>
<td>64.9</td>
<td>−4.6</td>
</tr>
</tbody>
</table>
Score normalization

- $q$-score 0/100
- $z$-score
- $qz = 100 \cdot \Phi(z)$
- $\frac{q}{100}$
- $\frac{q}{100}$
- $bz = \mathcal{B}^{-1}(\Phi(z); \alpha, \beta)$ where $\alpha = \beta = 2$
- $bz^* = \mathcal{B}^{-1}(\Phi(z); \alpha, \beta)$ where $\alpha = 2.5$ and $\beta = 1.5$. 
## Score normalization

**Table 15:** Computation of the score $S \equiv 70\% \cdot X_1 + 30\% \cdot X_2$ ($q$-score 0/100 normalization)

<table>
<thead>
<tr>
<th>#</th>
<th>$X_1$</th>
<th>$q_1$</th>
<th>$X_2$</th>
<th>$q_2$</th>
<th>$s$</th>
<th>$S$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.00</td>
<td>70.00</td>
<td>−3.00</td>
<td>60.00</td>
<td>67.00</td>
<td>80.00</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>38.60</td>
<td>20.00</td>
<td>−5.50</td>
<td>30.00</td>
<td>23.00</td>
<td>10.00</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>30.60</td>
<td>10.00</td>
<td>5.60</td>
<td>80.00</td>
<td>31.00</td>
<td>20.00</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>74.40</td>
<td>50.00</td>
<td>−1.30</td>
<td>70.00</td>
<td>56.00</td>
<td>60.00</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>97.10</td>
<td>80.00</td>
<td>−16.80</td>
<td>10.00</td>
<td>59.00</td>
<td>70.00</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>57.10</td>
<td>30.00</td>
<td>−3.50</td>
<td>50.00</td>
<td>36.00</td>
<td>30.00</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>132.40</td>
<td>90.00</td>
<td>8.50</td>
<td>90.00</td>
<td>90.00</td>
<td>90.00</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>92.50</td>
<td>60.00</td>
<td>−9.10</td>
<td>20.00</td>
<td>48.00</td>
<td>50.00</td>
<td>5</td>
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<tr>
<td>9</td>
<td>64.90</td>
<td>40.00</td>
<td>−4.60</td>
<td>40.00</td>
<td>40.00</td>
<td>40.00</td>
<td>4</td>
</tr>
<tr>
<td>Mean</td>
<td>75.73</td>
<td>50.00</td>
<td>−3.30</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td></td>
</tr>
<tr>
<td>Std-dev.</td>
<td>31.95</td>
<td>27.39</td>
<td>7.46</td>
<td>27.39</td>
<td>20.60</td>
<td>27.39</td>
<td></td>
</tr>
</tbody>
</table>
### Table 16: Computation of the score $S \equiv 70\% \cdot X_1 + 30\% \cdot X_2$ (z-score normalization)

<table>
<thead>
<tr>
<th>#</th>
<th>$X_1$</th>
<th>$z_1$</th>
<th>$X_2$</th>
<th>$z_2$</th>
<th>$s$</th>
<th>$S$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.00</td>
<td>0.572</td>
<td>-3.00</td>
<td>0.040</td>
<td>0.412</td>
<td>0.543</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>38.60</td>
<td>-1.162</td>
<td>-5.50</td>
<td>-0.295</td>
<td>-0.902</td>
<td>-1.188</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>30.60</td>
<td>-1.413</td>
<td>5.60</td>
<td>1.193</td>
<td>-0.631</td>
<td>-0.831</td>
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</tr>
<tr>
<td>4</td>
<td>74.40</td>
<td>-0.042</td>
<td>-1.30</td>
<td>0.268</td>
<td>0.051</td>
<td>0.067</td>
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<td>5</td>
<td>97.10</td>
<td>0.669</td>
<td>-16.80</td>
<td>-1.810</td>
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<td>-0.099</td>
<td>5</td>
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<tr>
<td>6</td>
<td>57.10</td>
<td>-0.583</td>
<td>-3.50</td>
<td>-0.027</td>
<td>-0.416</td>
<td>-0.548</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>132.40</td>
<td>1.774</td>
<td>8.50</td>
<td>1.582</td>
<td>1.716</td>
<td>2.261</td>
<td>9</td>
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<tr>
<td>8</td>
<td>92.50</td>
<td>0.525</td>
<td>-9.10</td>
<td>-0.778</td>
<td>0.134</td>
<td>0.177</td>
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<tr>
<td>9</td>
<td>64.90</td>
<td>-0.339</td>
<td>-4.60</td>
<td>-0.174</td>
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<td>-0.382</td>
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<tr>
<td>Mean</td>
<td>75.73</td>
<td>0.000</td>
<td>-3.30</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>Std-dev.</td>
<td>31.95</td>
<td>1.000</td>
<td>7.46</td>
<td>1.000</td>
<td>0.759</td>
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Table 17: Comparison of the different scoring methods

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<th>zq</th>
<th>bz</th>
<th>bz*</th>
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<td>S</td>
<td>R</td>
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<td>R</td>
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<td>9.19</td>
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<td>21.37</td>
<td>2</td>
</tr>
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<td>0.07</td>
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<td>54.13</td>
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<td>70.00</td>
<td>7</td>
<td>0.10</td>
<td>5</td>
<td>56.65</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>30.00</td>
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<td>0.55</td>
<td>3</td>
<td>24.42</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>90.00</td>
<td>9</td>
<td>2.26</td>
<td>9</td>
<td>98.04</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>50.00</td>
<td>5</td>
<td>0.18</td>
<td>7</td>
<td>60.39</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>40.00</td>
<td>4</td>
<td>0.38</td>
<td>4</td>
<td>30.96</td>
<td>4</td>
</tr>
<tr>
<td>Mean</td>
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<td>0.00</td>
<td>47.94</td>
<td>0.00</td>
<td>47.94</td>
<td>0.00</td>
</tr>
<tr>
<td>Std-dev.</td>
<td>27.39</td>
<td>1.00</td>
<td>28.79</td>
<td>0.82</td>
<td>28.79</td>
<td>0.82</td>
</tr>
</tbody>
</table>
An example with the CEO pay ratio

The CEO pay ratio is calculated by dividing the CEO’s compensation by the pay of the median employee. It is one of the key metrics for the G pillar. It has been imposed by the Dodd-Frank Act, which requires that publicly traded companies disclose:

1. the median total annual compensation of all employees other than the CEO;
2. the ratio of the CEO’s annual total compensation to that of the median employee;
3. the wage ratio of the CEO to the median employee.

⇒ the average S&P 500 company’s CEO-to-worker pay ratio was 324-to-1 in 2021 (AFL-CIO)
**Table 18: Examples of CEO pay ratio (June 2021)**

<table>
<thead>
<tr>
<th>Company name</th>
<th>P</th>
<th>R</th>
<th>Company name</th>
<th>P</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abercrombie &amp; Fitch</td>
<td>1954</td>
<td>4,293</td>
<td>Netflix</td>
<td>202931</td>
<td>190</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>9291</td>
<td>1,939</td>
<td>BlackRock</td>
<td>133644</td>
<td>182</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>11285</td>
<td>1,657</td>
<td>Pfizer</td>
<td>98972</td>
<td>181</td>
</tr>
<tr>
<td>Gap</td>
<td>6177</td>
<td>1,558</td>
<td>Goldman Sachs</td>
<td>138854</td>
<td>178</td>
</tr>
<tr>
<td>Alphabet</td>
<td>258708</td>
<td>1,085</td>
<td>MSCI</td>
<td>55857</td>
<td>165</td>
</tr>
<tr>
<td>Walmart</td>
<td>22484</td>
<td>983</td>
<td>Verisk Analytics</td>
<td>77055</td>
<td>117</td>
</tr>
<tr>
<td>Estee Lauder</td>
<td>30733</td>
<td>697</td>
<td>Facebook</td>
<td>247883</td>
<td>94</td>
</tr>
<tr>
<td>Ralph Lauren</td>
<td>21358</td>
<td>570</td>
<td>Invesco</td>
<td>125282</td>
<td>92</td>
</tr>
<tr>
<td>NIKE</td>
<td>25386</td>
<td>550</td>
<td>Boeing</td>
<td>158869</td>
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<tr>
<td>Citigroup</td>
<td>52988</td>
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<td>Citrix Systems</td>
<td>181769</td>
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<tr>
<td>PepsiCo</td>
<td>45896</td>
<td>368</td>
<td>Harley-Davidson</td>
<td>187157</td>
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<td>Microsoft</td>
<td>172512</td>
<td>249</td>
<td>Amazon.com</td>
<td>28848</td>
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<tr>
<td>Apple</td>
<td>57596</td>
<td>201</td>
<td>Berkshire Hathaway</td>
<td>65740</td>
<td>6</td>
</tr>
</tbody>
</table>

Source: [https://aflcio.org](https://aflcio.org) (June 2021)
An example with the CEO pay ratio

Figure 35: Histogram of the CEO pay ratio
Figure 36: Histogram of $z$-score applied to the CEO pay ratio
An example with the CEO pay ratio

What is the solution? Give the transform function \( y = \varphi(x) \).

Hint: use the beta distribution.
### Unsupervised learning

- Clustering ($K$-means, hierarchical clustering)
- Dimension reduction (PCA, NMF)
Other statistical methods

Supervised learning

- Discriminant analysis (LDA, QDA)
- Binary choice models (logistic regression, probit model)
- Regression models (OLS, lasso)

⇒ Advanced learning models (k-NN, neural networks and support vector machines) are not relevant in the case of ESG scoring

We need to define the response variable $Y$
Let $S_i(t)$ be the credit score of individual $i$ at time $t$

We have:

$$Y_i(t) = 1 \{ \tau_i \leq t + \delta \} = 1 \{ D_i(t + \delta) = 1 \}$$

where $\tau_i$ and $D_i$ are the default time and the default indicator function, and $\delta$ is the time horizon (e.g., one year)

The calibration problem of the credit scoring model is:

$$\Pr \{ Y_i(t) = 0 \} = f(S_i(t))$$

where $f$ is an increasing function
Application to ESG scoring models

- Let $S_i(t)$ be the ESG score of company $i$ at time $t$
- Endogenous response variable:
  - (a) Best-in-class oriented scoring system:
    \[ Y_i(t) = 1 \{ S_i(t + h) \geq s^* \} \]
    where $s^*$ is the best-in-class threshold
  - (b) Worst-in-class oriented scoring system:
    \[ Y_i(t) = 1 \{ S_i(t + h) \leq s^* \} \]
    where $s^*$ is the worst-in-class threshold
- Exogenous response variable
  - (c) Binary response:
    \[ Y_i(t) = 1 \{ C_i(t + h) \geq 0 \} \]
    where $C_i(t)$ is the controversy index
  - (d) Continuous response:
    \[ Y_i(t) = C_i(t + h) \]
- The calibration problem of the ESG scoring model is
  \[ \Pr \{ Y_i(t) = 0 \} = f(S_i(t)) \] or \[ Y_i(t) = f(S_i(t)) \]
  where the function $f$ is increasing for case (a) and decreasing for cases (b), (c) and (d)
Performance evaluation criteria

- ESG scoring and rating
  - Shannon entropy
  - Confusion matrix
  - Binary classification ratios (TPR, FNR, TNR, FPR, PPV, ACC, $F_1$)

- ESG scoring
  - Performance, selection and discriminant curves
  - ROC curve
  - Gini coefficient
Table 19: Credit rating system of S&P, Moody’s and Fitch

<table>
<thead>
<tr>
<th></th>
<th>Prime Maximum Safety</th>
<th>High Grade High Quality</th>
<th>Upper Medium Grade</th>
<th>Lower Medium Grade</th>
<th>Non Investment Grade Speculative</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P/Fitch</td>
<td>AAA</td>
<td>AA+</td>
<td>A+</td>
<td>BBB+</td>
<td>BB+</td>
</tr>
<tr>
<td>Moody’s</td>
<td>Aaa</td>
<td>AA</td>
<td>A</td>
<td>Baa1</td>
<td>Ba+</td>
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<td></td>
<td>AA−</td>
<td></td>
<td>Baa2</td>
<td>Ba2</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td></td>
<td>AA1</td>
<td></td>
<td>Baa3</td>
<td>Ba3</td>
</tr>
<tr>
<td>Moody’s</td>
<td></td>
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<tr>
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<th>Substantial Risk</th>
<th>In Poor Standing</th>
<th>Extremely Speculative</th>
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<tbody>
<tr>
<td>S&amp;P/Fitch</td>
<td>B+</td>
<td>B</td>
<td>CCC+</td>
<td>CC</td>
</tr>
<tr>
<td>Moody’s</td>
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<td>B1</td>
<td>Caa1</td>
<td>Ca</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>B</td>
<td>CCC</td>
<td></td>
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<tr>
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<td>B</td>
<td>B</td>
<td>CCC</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Caa2</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Caa3</td>
<td></td>
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<tr>
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<td></td>
</tr>
</tbody>
</table>
Amundi: A (high), B,... to G (low) — 7-grade scale
FTSE Russell: 0 (low), 1,... to 5 (high) — 6-grade scale
ISS ESG: 1 (high), 2,... to 10 (low) — 10-grade scale
MSCI: AAA (high), AA,... to CCC (low) — 7-grade scale
Refinitiv: A+ (high), A, A-, B+,... to D- (low) — 12-grade scale
RepRisk: AAA (high), AA,... to D (low) — 8-grade scale
Sustainalytics: 1 (low), 2,... to 5 (high) — 5-grade scale
ESG rating process

**Figure 37: From ESG score to ESG rating**

- **Risk Score** (numeric value)
- **Map**
- **Rating** (letter)

Two-step approach:

1. Specification of the map function:

   \[ \text{Map} : \Omega_S \rightarrow \Omega_R \]

   \[ S \mapsto R = \text{Map}(S) \]

   where \( \Omega_S \) is the support of ESG scores, \( \Omega_R \) is the ordered state space of ESG ratings and \( R \) is the ESG rating.

2. Validation (and the possible *forcing*) of the rating by the analyst.
ESG rating process

Example with the MSCI ESG rating system

- $\Omega_s = [0, 10]$
- $\Omega_R = \{\text{CCC, B, BB, BBB, A, AA, AAA}\}$
- The map function is defined as

$$Map(s) = \begin{cases} 
\text{CCC} & \text{if } s \in [0, \frac{10}{7}] \\
\text{B} & \text{if } s \in \left[\frac{10}{7}, \frac{20}{7}\right] \\
\text{BB} & \text{if } s \in \left[\frac{20}{7}, \frac{30}{7}\right] \\
\text{BBB} & \text{if } s \in \left[\frac{30}{7}, \frac{40}{7}\right] \\
\text{A} & \text{if } s \in \left[\frac{40}{7}, \frac{50}{7}\right] \\
\text{AA} & \text{if } s \in \left[\frac{50}{7}, \frac{60}{7}\right] \\
\text{AAA} & \text{if } s \in \left[\frac{60}{7}, 10\right] 
\end{cases}$$

$(0 - 1.429)$

$(1.429 - 2.857)$

$(2.857 - 4.286)$

$(4.286 - 5.714)$

$(5.714 - 7.143)$

$(7.143 - 8.571)$

$(8.571 - 10)$
The map function is an increasing piecewise function

\[ S \sim F \text{ and } S \in (s^-, s^+) \]

\( \{ s_0^* = s^-, s_1^*, \ldots, s_{K-1}^*, s_K^* = s^+ \} \) are the knots of the piecewise function

\( \Omega_R = \{ R_1, \ldots, R_K \} \) is the set of grades

\[ \Rightarrow \text{The frequency distribution of the ratings is given by:} \]

\[
p_k = \Pr \{ R = R_k \} = \Pr \{ s_{k-1}^* \leq S < s_k^* \} = F(s_k^*) - F(s_{k-1}^*)
\]
If we would like to build a rating system with pre-defined frequencies $(p_1, \ldots, p_K)$, we have to solve the following equation:

$$F(s_k^*) - F(s_{k-1}^*) = p_k$$

We deduce that:

$$F(s_k^*) = p_k + F(s_{k-1}^*)$$

$$= p_k + p_{k-1} + F(s_{k-2}^*)$$

$$= \left( \sum_{j=1}^{k} p_j \right) + F(s_0^*)$$

and:

$$s_k^* = F^{-1} \left( \sum_{j=1}^{k} p_j \right)$$
Exercise

- We assume that $S \sim U_{[a,b]}$
- Show that $p_k = K^{-1}$ if the rating system consists in $K$ equally-sized intervals
- Show that the knots of the map function are equal to:

$$s_k^* = a + (b - a) \left( \sum_{j=1}^{k} p_j \right)$$

when we impose pre-defined frequencies $(p_1, \ldots, p_K)$

- If we consider a 0/100 uniform score and $\Omega_R \times P = (CCC, 5\%), (B, 10\%), (BB, 20\%), (BBB, 30\%), (A, 20\%), (AA, 10\%), (AAA, 5\%)$, show that $s_{CCC}^* = 5$, $s_B^* = 15$, $s_{BB}^* = 35$, $s_{BBB}^* = 65$, $s_A^* = 85$ and $s_{AA}^* = 95$
For a $z$-score system ($S \sim \mathcal{N}(0, 1)$), we obtain:

$$p_k = \Phi(s_k^*) - \Phi(s_{k-1}^*)$$
Figure 38: Map function of a \( z \)-score (equal-space ratings)
ESG rating process

Figure 39: Map function of a $z$-score (equal-frequency ratings)
### Table 20: ESG migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
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<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
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<tr>
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<td>14.3%</td>
<td>14.3%</td>
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<tr>
<td>A</td>
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<td>14.3%</td>
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<td>14.3%</td>
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<td>14.3%</td>
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</table>

\[\Rightarrow I(R(t) \mid R(s)) = \ln 7\]
### Table 21: ESG migration matrix

<table>
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<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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<td>100%</td>
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</tbody>
</table>

\[ \Rightarrow \mathcal{I}(\mathcal{R}(t) \mid \mathcal{R}(s)) = 0 \]
Table 22: ESG migration matrix

<table>
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<th>B</th>
<th>CCC</th>
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<td>0%</td>
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<td>96%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>A</td>
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<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>BBB</td>
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<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
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</tr>
<tr>
<td>BB</td>
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<td>0%</td>
<td>2%</td>
<td>96%</td>
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<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
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<tr>
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<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4%</td>
<td>96%</td>
</tr>
</tbody>
</table>

\[ 0 < \mathbb{I}(\mathcal{R}(t) | \mathcal{R}(s)) \ll \ln 7 \]
A good reference on Markov chains is:

Rating migration matrix
Discrete time modeling

Definition

- $\mathcal{R}$ is a time-homogeneous Markov chain
- $\Omega_\mathcal{R} = \{R_1, \ldots, R_K\}$ is the state space of the chain
- $\mathcal{K} = \{1, \ldots, K\}$ is the corresponding index set
- The transition matrix is defined as $P = (p_{i,j})$
- $p_{i,j}$ is the probability that the entity migrates from rating $R_i$ to rating $R_j$
- The matrix $P$ satisfies the following properties:
  - $\forall i, j \in \mathcal{K}, p_{i,j} \geq 0$
  - $\forall i \in \mathcal{K}, \sum_{j=1}^{K} p_{i,j} = 1$
### Table 23: ESG migration matrix #1 (one-year transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
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<tbody>
<tr>
<td>AAA</td>
<td>92.76</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>AA</td>
<td>4.15</td>
<td>82.73</td>
<td>11.86</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
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<tr>
<td>A</td>
<td>0.18</td>
<td>15.47</td>
<td>72.98</td>
<td>10.46</td>
<td>0.82</td>
<td>0.09</td>
<td>0.00</td>
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<tr>
<td>BBB</td>
<td>0.07</td>
<td>1.32</td>
<td>19.60</td>
<td>69.49</td>
<td>9.03</td>
<td>0.42</td>
<td>0.07</td>
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<tr>
<td>BB</td>
<td>0.04</td>
<td>0.19</td>
<td>1.55</td>
<td>19.36</td>
<td>70.88</td>
<td>7.75</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.05</td>
<td>0.24</td>
<td>1.43</td>
<td>21.54</td>
<td>74.36</td>
<td>2.38</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
<td>0.44</td>
<td>2.21</td>
<td>13.24</td>
<td>83.89</td>
</tr>
</tbody>
</table>
The probability that the entity reaches the state $R_j$ at time $t$ given that it has reached the state $R_i$ at time $s$ is equal to:

$$p(s, i; t, j) = \Pr \{ \mathcal{R}(t) = R_j \mid \mathcal{R}(s) = R_i \} = p_{i,j}^{(t-s)}$$

We note $p_{i,j}^{(n)}$ the $n$-step transition probability:

$$p_{i,j}^{(n)} = \Pr \{ \mathcal{R}(t + n) = R_j \mid \mathcal{R}(t) = R_i \}$$

and the associated $n$-step transition matrix $P^{(n)} = \left( p_{i,j}^{(n)} \right)$
For $n = 2$, we obtain:

$$p_{i,j}^{(2)} = \Pr \{ \mathcal{R}(t + 2) = R_j \mid \mathcal{R}(t) = R_i \}$$

$$= \sum_{k=1}^{K} \Pr \{ \mathcal{R}(t + 2) = R_j, \mathcal{R}(t + 1) = R_k \mid \mathcal{R}(t) = R_i \}$$

$$= \sum_{k=1}^{K} \Pr \{ \mathcal{R}(t + 2) = R_j \mid \mathcal{R}(t + 1) = R_k \} \cdot \Pr \{ \mathcal{R}(t + 1) = R_k \mid \mathcal{R}(t) = R_i \}$$

$$= \sum_{k=1}^{K} p_{i,k} \cdot p_{k,j}$$
The forward Chapman-Kolmogorov equation is:

\[
p_{i,j}^{(n+m)} = \sum_{k=1}^{K} p_{i,k}^{(n)} \cdot p_{k,j}^{(m)} \quad \forall n, m > 0
\]

or \( P^{(n+m)} = P^{(n)} \cdot P^{(m)} \) with \( P^{(0)} = I \)

We have:

\[
P^{(n)} = P^{(n-1)} \cdot P^{(1)}
= P^{(n-2)} \cdot P^{(1)} \cdot P^{(1)}
= \prod_{t=1}^{n} P^{(1)}
= P^n
\]

We deduce that:

\[
p(t, i; t + n, j) = p_{i,j}^{(n)} = e_i^\top P^n e_j
\]
Table 24: Two-year transition probability in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>86.28</td>
<td>10.08</td>
<td>2.25</td>
<td>0.92</td>
<td>0.44</td>
<td>0.02</td>
<td>0.00</td>
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<tr>
<td>AA</td>
<td>7.30</td>
<td>70.52</td>
<td>18.68</td>
<td>2.67</td>
<td>0.66</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.95</td>
<td>24.24</td>
<td>57.16</td>
<td>15.20</td>
<td>2.19</td>
<td>0.25</td>
<td>0.01</td>
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<tr>
<td>BBB</td>
<td>0.21</td>
<td>5.06</td>
<td>28.22</td>
<td>52.11</td>
<td>12.93</td>
<td>1.33</td>
<td>0.14</td>
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<tr>
<td>BB</td>
<td>0.09</td>
<td>0.79</td>
<td>6.07</td>
<td>27.45</td>
<td>53.68</td>
<td>11.37</td>
<td>0.55</td>
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<tr>
<td>B</td>
<td>0.01</td>
<td>0.18</td>
<td>0.98</td>
<td>6.26</td>
<td>31.47</td>
<td>57.28</td>
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<td>1.32</td>
<td>6.31</td>
<td>21.13</td>
<td>70.70</td>
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</tbody>
</table>
Rating migration matrix

Discrete time modeling

We have:

\[ p_{\text{AAA,AAA}}^{(2)} = p_{\text{AAA,AAA}} \times p_{\text{AAA,AAA}} + p_{\text{AAA,AA}} \times p_{\text{AA,AAA}} + p_{\text{AAA,A}} \times p_{\text{A,AAA}} + p_{\text{AAA,BBB}} \times p_{\text{BBB,AAA}} + p_{\text{AAA,BB}} \times p_{\text{BB,AAA}} + p_{\text{AAA,B}} \times p_{\text{B,AAA}} + p_{\text{AAA,CCC}} \times p_{\text{CCC,AAA}} \]

\[ = 0.9276^2 + 0.0566 \times 0.0415 + 0.0090 \times 0.0018 + 0.0045 \times 0.0007 + 0.0023 \times 0.0004 \]

\[ = 86.28\% \]
### Table 25: Five-year transition probability in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
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<tbody>
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<td>AAA</td>
<td>70.45</td>
<td>18.69</td>
<td>6.97</td>
<td>2.61</td>
<td>1.08</td>
<td>0.18</td>
<td>0.01</td>
</tr>
<tr>
<td>AA</td>
<td>13.13</td>
<td>50.21</td>
<td>26.03</td>
<td>7.90</td>
<td>2.22</td>
<td>0.48</td>
<td>0.03</td>
</tr>
<tr>
<td>A</td>
<td>4.35</td>
<td>33.20</td>
<td>37.78</td>
<td>17.99</td>
<td>5.52</td>
<td>1.08</td>
<td>0.09</td>
</tr>
<tr>
<td>BBB</td>
<td>1.50</td>
<td>16.49</td>
<td>32.49</td>
<td>30.90</td>
<td>14.61</td>
<td>3.63</td>
<td>0.38</td>
</tr>
<tr>
<td>BB</td>
<td>0.50</td>
<td>5.98</td>
<td>17.83</td>
<td>30.10</td>
<td>31.35</td>
<td>12.85</td>
<td>1.39</td>
</tr>
<tr>
<td>B</td>
<td>0.15</td>
<td>1.90</td>
<td>7.40</td>
<td>18.95</td>
<td>35.11</td>
<td>31.26</td>
<td>5.23</td>
</tr>
<tr>
<td>CCC</td>
<td>0.05</td>
<td>0.64</td>
<td>2.55</td>
<td>6.93</td>
<td>17.96</td>
<td>38.54</td>
<td>43.33</td>
</tr>
</tbody>
</table>
Stationary distribution

- \( \pi_k^{(n)} = \Pr \{ \mathcal{R}(n) = R_k \} \) is the probability of the state \( R_k \) at time \( n \):

- \( \pi^{(n)} = (\pi_1^{(n)}, \ldots, \pi_K^{(n)}) \) satisfies \( \pi^{(n+1)} = P^\top \pi^{(n)} \)

- The Markov chain \( \mathcal{R} \) has a stationary distribution \( \pi^* \) if \( \pi^* = P^\top \pi^* \)

- \( T_k = \inf \{ n : \mathcal{R}(n) = R_k \mid \mathcal{R}(0) = R_k \} \) is the return period of state \( R_k \)

- The average return period is then equal to:

\[
\tau_k := \mathbb{E}[T_k] = \frac{1}{\pi_k^*}
\]
We obtain:

$$\pi^* = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%)$$

The average return periods are then equal to 5.6, 3.4, 4.0, 6.6, 12.0, 30.4 and 149.0 years

⇒ Best-in-class (or winning-) oriented system
Table 26: ESG migration matrix #2 (one-month transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>93.50</td>
<td>5.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>2.00</td>
<td>93.00</td>
<td>4.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.00</td>
<td>3.00</td>
<td>93.00</td>
<td>3.90</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.00</td>
<td>0.10</td>
<td>2.80</td>
<td>94.00</td>
<td>3.00</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>BB</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>3.50</td>
<td>94.50</td>
<td>1.80</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>3.70</td>
<td>96.00</td>
<td>0.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>98.50</td>
</tr>
</tbody>
</table>

⇒ The stationary distribution is
\( \pi^* = (3.11\%, 10.10\%, 17.46\%, 27.76\%, 25.50\%, 12.68\%, 3.39\%) \) and the average return periods are equal to 32.2, 9.9, 5.7, 3.6, 3.9, 7.9 and 29.5 years
⇒ balanced rating system
### Table 27: One-year probability transition in % (migration matrix #2)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>48.06</td>
<td>29.71</td>
<td>10.34</td>
<td>6.42</td>
<td>4.95</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>AA</td>
<td>11.65</td>
<td>49.25</td>
<td>24.10</td>
<td>9.60</td>
<td>4.87</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>A</td>
<td>2.02</td>
<td>17.51</td>
<td>49.67</td>
<td>24.72</td>
<td>5.52</td>
<td>0.54</td>
<td>0.03</td>
</tr>
<tr>
<td>BBB</td>
<td>0.27</td>
<td>3.53</td>
<td>17.46</td>
<td>55.50</td>
<td>20.21</td>
<td>2.88</td>
<td>0.16</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.60</td>
<td>4.21</td>
<td>23.43</td>
<td>57.45</td>
<td>13.27</td>
<td>1.01</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.08</td>
<td>0.74</td>
<td>5.94</td>
<td>27.10</td>
<td>64.18</td>
<td>1.96</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.07</td>
<td>0.57</td>
<td>4.22</td>
<td>5.77</td>
<td>5.85</td>
<td>83.51</td>
</tr>
</tbody>
</table>
### Rating migration matrix

**Discrete time modeling**

#### Table 28: One-month probability transition in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>99.36</td>
<td>0.53</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.39</td>
<td>98.31</td>
<td>1.26</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>-0.02</td>
<td>1.65</td>
<td>97.14</td>
<td>1.21</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.01</td>
<td>-0.07</td>
<td>2.28</td>
<td>96.72</td>
<td>1.06</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>BB</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.12</td>
<td>2.29</td>
<td>96.92</td>
<td>0.88</td>
<td>0.01</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>-0.15</td>
<td>2.45</td>
<td>97.42</td>
<td>0.25</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>1.37</td>
<td>98.53</td>
</tr>
</tbody>
</table>

⇒ Negative probabilities

**The ESG rating system is not Markovian!**
Mean hitting time

- Let $\mathcal{A} \subset \mathbb{K}$ be a given subset. The first hitting time of $\mathcal{A}$ is given by:

$$T(\mathcal{A}) = \inf \{n : R(n) \in \mathcal{A}\}$$

- The mean first hitting time to target $\mathcal{A}$ from state $k$ is defined as:

$$\tau_k(\mathcal{A}) = \mathbb{E}[T(\mathcal{A}) \mid R(0) = R_k]$$

- We can show that $\tau_k(\mathcal{A}) = 1 + \sum_{j=1}^{K} p_{k,j} \tau_j(\mathcal{A})$

- The solution is given by the LP problem:

$$\tau(\mathcal{A}) = \arg \min \sum_{k=1}^{K} x_k \quad \text{s.t.} \quad \begin{cases} 
  x_k = 0 & \text{if } k \in \mathcal{A} \\
  x_k = 1 + \sum_{j=1}^{K} p_{k,j} x_j & \text{if } k \notin \mathcal{A} \\
  x_k \geq 0 
\end{cases}$$
\[ B = \{\text{AAA, AA, A}\} \]
\[ \mathcal{W} = \{\text{BB, B, CCC}\} \]

<table>
<thead>
<tr>
<th>Rating system</th>
<th>$\mathcal{W}$-target</th>
<th>$B$-target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AAA</td>
<td>AA</td>
</tr>
<tr>
<td>#1</td>
<td>79.21</td>
<td>70.04</td>
</tr>
</tbody>
</table>
Rating migration matrix
Estimation of the transition matrix

Theoretical approach:
- Bayes theorem:
  \[ p_{i,j} = \frac{\Pr \{ \mathcal{R}(t+1) = R_j \mid \mathcal{R}(t) = R_i \} \cdot \Pr \{ \mathcal{R}(t) = R_i \}}{\Pr \{ \mathcal{R}(t) = R_i \}} \]

- We have seen that:
  \[ \Pr \{ \mathcal{R}(t) = R_k \} = F(s_k^*) - F(s_{k-1}^*) = p_k \]

- We deduce that:
  \[ p_{i,j} = \frac{C(F(s_i^*), F(s_j^*)) - C(F(s_{i-1}^*), F(s_j^*)) - C(F(s_i^*), F(s_{j-1}^*)) + C(F(s_{i-1}^*), F(s_{j-1}^*))}{F(s_i^*) - F(s_{i-1}^*)} \]

where \( C \) is the copula function of the random vector \((S(t), S(t+1))\)
Non-parametric approach:

$$\hat{p}_{i,j}(t) = \frac{\# \{ R(t+1) = R_j, R(t) = R_i \}}{\# \{ R(t) = R_i \}} = \frac{n_{i,j}(t)}{n_{i,.}(t)}$$

⇒ cohort method vs. pooling method
Table 29: Number of observations $n_{i,j}$ (migration matrix #1)

<table>
<thead>
<tr>
<th>$n_{i,j}$</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>$n_{i,·}(t)$</th>
<th>$\hat{p}_{i,·}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>2050</td>
<td>125</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2210</td>
<td>3.683%</td>
</tr>
<tr>
<td>AA</td>
<td>280</td>
<td>5580</td>
<td>800</td>
<td>60</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>6745</td>
<td>11.242%</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>1700</td>
<td>8020</td>
<td>1150</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>10990</td>
<td>18.317%</td>
</tr>
<tr>
<td>BBB</td>
<td>10</td>
<td>190</td>
<td>2820</td>
<td>10000</td>
<td>1300</td>
<td>60</td>
<td>10</td>
<td>14390</td>
<td>23.983%</td>
</tr>
<tr>
<td>BB</td>
<td>5</td>
<td>25</td>
<td>200</td>
<td>2500</td>
<td>9150</td>
<td>1000</td>
<td>30</td>
<td>12910</td>
<td>21.517%</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>5</td>
<td>25</td>
<td>150</td>
<td>2260</td>
<td>7800</td>
<td>250</td>
<td>10490</td>
<td>17.483%</td>
</tr>
<tr>
<td>CCC</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>300</td>
<td>1900</td>
<td>2265</td>
<td>3.775%</td>
</tr>
</tbody>
</table>

| $n_{·,j}(t)$ | 2365 | 7625 | 11890 | 13850 | 12875 | 9175 | 2190 | 60000      |
| $\hat{p}_{·,j}(t)$ | 3.942% | 12.708% | 19.817% | 23.133% | 21.458% | 15.292% | 3.650% | 100.00% |
For the migration matrix #1, we have:

$$\pi^* = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%)$$

The initial empirical distribution of ratings is:

$$\hat{\pi}^{(0)} = (3.683\%, 11.242\%, 18.317\%, 23.983\%, 21.517\%, 17.483\%, 3.775\%)$$

We have:

$$\hat{\pi}^{(1)} = \hat{P}^T \hat{\pi}^{(0)} = (3.942\%, 12.708\%, 19.817\%, 23.133\%, 21.458\%, 15.290\%, 3.650\%)$$
Figure 40: Dynamics of the probability distribution $\pi^{(n)}$ (migration matrix #1)
The transition matrix is defined as follows:

\[ P_{i,j}(s;t) = p(s,i;t,j) = \Pr \{ R(t) = R_j \mid R(s) = R_i \} \]

If \( R \) is a time-homogenous Markov, we have:

\[ P(t) = P(0;t) = \exp(t\Lambda) \]

\( \Lambda = (\lambda_{i,j}) \) is the Markov generator matrix \( \Lambda = (\lambda_{i,j}) \) where \( \lambda_{i,j} \geq 0 \) for all \( i \neq j \) and \( \lambda_{i,i} = -\sum_{j \neq i} K \lambda_{i,j} \)
Rating migration matrix
Continuous-time modeling

An example

- Rating system with three states: A (good rating), B (average rating) and C (bad rating)
- The Markov generator is equal to:

\[
\Lambda = \begin{pmatrix}
-0.30 & 0.20 & 0.10 \\
0.15 & -0.40 & 0.25 \\
0.10 & 0.15 & -0.25
\end{pmatrix}
\]
The one-year transition probability matrix is equal to:

\[ P(1) = e^{\Lambda} = \begin{pmatrix} 75.63\% & 14.84\% & 9.53\% \\ 11.63\% & 69.50\% & 18.87\% \\ 8.52\% & 11.73\% & 79.75\% \end{pmatrix} \]

For the two-year maturity, we get:

\[ P(2) = e^{2\Lambda} = \begin{pmatrix} 59.74\% & 22.65\% & 17.61\% \\ 18.49\% & 52.24\% & 29.27\% \\ 14.60\% & 18.76\% & 66.63\% \end{pmatrix} \]

We verify that \( P(2) = P(1) \cdot P(1) \) because:

\[ P(t) = e^{t\Lambda} = (e^{\Lambda})^t = P(1)^t \]

We have:

\[ P\left(\frac{1}{12}\right) = e^{\frac{1}{12}\Lambda} = \begin{pmatrix} 97.54\% & 1.62\% & 0.83 \\ 1.22\% & 96.74\% & 2.03 \\ 0.82\% & 1.22\% & 97.95 \end{pmatrix} \]
We consider the matrix function in the space $\mathbb{M}$ of square matrices:

$$f : \mathbb{M} \rightarrow \mathbb{M}$$

$$A \mapsto B = f(A)$$

For instance, if $f(x) = \sqrt{x}$ and $A$ is positive, we can define the matrix $B$ such that:

$$BB^* = B^*B = A$$

$B$ is called the square root of $A$ and we note $B = A^{1/2}$
We consider the following Taylor expansion:

\[
f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!}f''(x_0) + \ldots
\]

We can show that if the series converge for \( |x - x_0| < \alpha \), then the matrix \( f(A) \) defined by the following expression:

\[
f(A) = f(x_0) + (A - x_0I)f'(x_0) + \frac{(A - x_0I)^2}{2!}f''(x_0) + \ldots
\]

converges to the matrix \( B \) if \( |A - x_0I| < \alpha \) and we note \( B = f(A) \)
In the case of the exponential function, we have:

\[ f(x) = e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \]

We deduce that the exponential of the matrix \( A \) is equal to:

\[ B = e^A = \sum_{k=0}^{\infty} \frac{A^k}{k''!} \]

The logarithm of \( A \) is the matrix \( B \) such that \( e^B = A \) and we note \( B = \ln A \).
Let $A$ and $B$ be two $n \times n$ square matrices. We have the properties:

\[
\begin{align*}
    f(A^\top) &= f(A)^\top \\
    Af(A) &= f(A)A \\
    f(B^{-1}AB) &= B^{-1}f(A)B
\end{align*}
\]

It follows that:

\[
\begin{cases}
    e^{A^\top} = (e^A)^\top \\
    e^{B^{-1}AB} = B^{-1}e^AB \\
    Ae^B = e^BA & \text{if } AB = BA \\
    e^{A+B} = e^Ae^B = e^Be^A & \text{if } AB = BA
\end{cases}
\]
The Schur decomposition of the $n \times n$ matrix $A$ is equal to:

$$A = QTQ^*$$

where $Q$ is a unitary matrix and $T$ is an upper triangular matrix.

For transcendental functions, we have:

$$f(A) = Qf(T)Q^*$$

where $A = QTQ^*$ is the Schur decomposition of $A$. 
We have:

\[ \hat{\Lambda} = \frac{1}{t} \ln \left( \hat{P}(t) \right) \]

\[ \Rightarrow \hat{\Lambda} \text{ may not verify the Markov conditions: } \hat{\lambda}_{i,j} \geq 0 \text{ for all } i \neq j \text{ and } \sum_{j=1}^{K} \lambda_{i,j} = 0 \]
Table 30: Non-Markov generator $\Lambda' = \ln (P)$ of the migration matrix #1 (in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>-7.663</td>
<td>6.427</td>
<td>0.542</td>
<td>0.466</td>
<td>0.245</td>
<td>-0.016</td>
<td>-0.000</td>
</tr>
<tr>
<td>AA</td>
<td>4.770</td>
<td>-20.604</td>
<td>15.451</td>
<td>-0.001</td>
<td>0.318</td>
<td>0.066</td>
<td>-0.001</td>
</tr>
<tr>
<td>A</td>
<td>-0.267</td>
<td>20.259</td>
<td>-35.172</td>
<td>14.953</td>
<td>0.152</td>
<td>0.083</td>
<td>-0.008</td>
</tr>
<tr>
<td>BBB</td>
<td>0.102</td>
<td>-1.051</td>
<td>28.263</td>
<td>-40.366</td>
<td>13.100</td>
<td>-0.128</td>
<td>0.080</td>
</tr>
<tr>
<td>BB</td>
<td>0.032</td>
<td>0.307</td>
<td>-1.762</td>
<td>28.351</td>
<td>-37.889</td>
<td>10.832</td>
<td>0.129</td>
</tr>
<tr>
<td>B</td>
<td>-0.005</td>
<td>-0.008</td>
<td>0.503</td>
<td>-2.240</td>
<td>30.227</td>
<td>-31.482</td>
<td>3.006</td>
</tr>
<tr>
<td>CCC</td>
<td>0.000</td>
<td>-0.024</td>
<td>0.194</td>
<td>0.469</td>
<td>0.365</td>
<td>16.806</td>
<td>-17.810</td>
</tr>
</tbody>
</table>
The first approach consists in adding the negative values back into the diagonal values:

\[
\tilde{\lambda}_{i,j} = \max\left(\hat{\lambda}_{i,j}, 0\right) \quad i \neq j \\
\tilde{\lambda}_{i,i} = \hat{\lambda}_{i,i} + \sum_{j \neq i} \min\left(\hat{\lambda}_{i,j}, 0\right)
\]

The second estimator carries forward the negative values on the matrix entries which have the correct sign:

\[
G_i = |\hat{\lambda}_{i,i}| + \sum_{j \neq i} \max\left(\hat{\lambda}_{i,j}, 0\right), \quad B_i = \sum_{j \neq i} \max\left(-\hat{\lambda}_{i,j}, 0\right)
\]

\[
\tilde{\lambda}_{i,j} = \begin{cases} 
0 & \text{if } i \neq j \text{ and } \hat{\lambda}_{i,j} < 0 \\
\hat{\lambda}_{i,j} - B_i \frac{\hat{\lambda}_{i,j}}{|\hat{\lambda}_{i,j}|} / G_i & \text{if } G_i > 0 \\
\hat{\lambda}_{i,j} & \text{if } G_i = 0
\end{cases}
\]
### Table 31: Markov generator of the migration matrix #1 (in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>-7.679</td>
<td>6.427</td>
<td>0.542</td>
<td>0.466</td>
<td>0.245</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>AA</td>
<td>4.770</td>
<td>-20.606</td>
<td>15.451</td>
<td>0.000</td>
<td>0.318</td>
<td>0.066</td>
<td>0.000</td>
</tr>
<tr>
<td>A</td>
<td>0.000</td>
<td>20.259</td>
<td>-35.447</td>
<td>14.953</td>
<td>0.152</td>
<td>0.083</td>
<td>0.000</td>
</tr>
<tr>
<td>BBB</td>
<td>0.102</td>
<td>0.000</td>
<td>28.263</td>
<td>-41.545</td>
<td>13.100</td>
<td>0.000</td>
<td>0.080</td>
</tr>
<tr>
<td>BB</td>
<td>0.032</td>
<td>0.307</td>
<td>0.000</td>
<td>38.351</td>
<td>-39.651</td>
<td>10.832</td>
<td>0.129</td>
</tr>
<tr>
<td>B</td>
<td>0.000</td>
<td>0.000</td>
<td>0.503</td>
<td>0.000</td>
<td>30.227</td>
<td>-33.735</td>
<td>3.006</td>
</tr>
<tr>
<td>CCC</td>
<td>0.000</td>
<td>0.000</td>
<td>0.194</td>
<td>0.469</td>
<td>0.365</td>
<td>16.806</td>
<td>-17.834</td>
</tr>
</tbody>
</table>
Table 32: ESG migration Markov matrix #1 (one-year transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.75</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.17</td>
<td>82.73</td>
<td>11.85</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.40</td>
<td>15.51</td>
<td>72.79</td>
<td>10.39</td>
<td>0.81</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>BBB</td>
<td>0.12</td>
<td>2.11</td>
<td>19.60</td>
<td>68.69</td>
<td>8.91</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.43</td>
<td>2.79</td>
<td>19.25</td>
<td>69.65</td>
<td>7.61</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.09</td>
<td>0.65</td>
<td>2.98</td>
<td>21.21</td>
<td>72.71</td>
<td>2.35</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.02</td>
<td>0.25</td>
<td>0.58</td>
<td>2.19</td>
<td>13.09</td>
<td>83.87</td>
</tr>
</tbody>
</table>
### Table 33: Original migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.76</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.15</td>
<td>82.73</td>
<td>11.86</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.18</td>
<td>15.47</td>
<td>72.98</td>
<td>10.46</td>
<td>0.82</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.07</td>
<td>1.32</td>
<td>19.60</td>
<td>69.49</td>
<td>9.03</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.19</td>
<td>1.55</td>
<td>19.36</td>
<td>70.88</td>
<td>7.75</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.05</td>
<td>0.24</td>
<td>1.43</td>
<td>21.54</td>
<td>74.36</td>
<td>2.38</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
<td>0.44</td>
<td>2.21</td>
<td>13.24</td>
<td>83.89</td>
</tr>
</tbody>
</table>

### Table 34: New migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.75</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.17</td>
<td>82.73</td>
<td>11.85</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.40</td>
<td>15.51</td>
<td>72.79</td>
<td>10.39</td>
<td>0.81</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>BBB</td>
<td>0.12</td>
<td>2.11</td>
<td>19.60</td>
<td>68.69</td>
<td>8.91</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.43</td>
<td>2.79</td>
<td>19.25</td>
<td>69.65</td>
<td>7.61</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.09</td>
<td>0.65</td>
<td>2.98</td>
<td>21.21</td>
<td>72.71</td>
<td>2.35</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.02</td>
<td>0.25</td>
<td>0.58</td>
<td>2.19</td>
<td>13.09</td>
<td>83.87</td>
</tr>
</tbody>
</table>
Why it is important that ESG ratings satisfy the Markov property

- Lack of memory:

<table>
<thead>
<tr>
<th></th>
<th>$t-2$</th>
<th>$t-1$</th>
<th>$t$</th>
<th>$t+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>→ BBB</td>
<td>→ BBB</td>
<td>→ ?</td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td>→ BBB</td>
<td>→ BBB</td>
<td>→ ?</td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>→ BB</td>
<td>→ BBB</td>
<td>→ ?</td>
<td></td>
</tr>
</tbody>
</table>

- Non-Markov property:

$$\text{Pr} \{ \mathcal{R}_{c_1} (t+1) = R_j \mid \mathcal{R}_{c_1} (t) = R_i \} \neq \text{Pr} \{ \mathcal{R}_{c_2} (t+1) = R_j \mid \mathcal{R}_{c_2} (t) = R_i \}$$

for two different companies $c_1$ and $c_2$
How to perform a dynamic analysis?

- We deduce that:

\[ \pi_k(t, A) = \Pr \{ R(t) \in A \mid R(0) = k \} = \sum_{j \in A} e_k^T e^{t \Lambda} e_j \]

- Some properties

  - \( \partial_t \exp (\Lambda t) = \Lambda \exp (\Lambda t) \)
  - \( \partial_t^m \exp (\Lambda t) = \Lambda^m \exp (\Lambda t) \)
  - \( \int_0^t e^{\Lambda s} \, ds = (e^{\Lambda t} - I_K) \Lambda^{-1} \)

- For example, the "time density function" is given by:

\[ \pi_k^{(m)}(t, A) := \frac{\partial \pi_k(t, A)}{\partial t^m} = \sum_{j \in A} e_k^T \Lambda^m e^{t \Lambda} e_j \]
Figure 41: Probability $\pi_k(t, A)$ to reach $A$ at time $t$ (migration matrix #1)
Figure 42: Dynamic analysis (migration matrix #1)
### Table 35: Example of credit migration matrix (one-year probability transition in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.82</td>
<td>6.50</td>
<td>0.56</td>
<td>0.06</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.63</td>
<td>91.87</td>
<td>6.64</td>
<td>0.65</td>
<td>0.06</td>
<td>0.11</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.08</td>
<td>2.26</td>
<td>91.66</td>
<td>5.11</td>
<td>0.61</td>
<td>0.23</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>BBB</td>
<td>0.05</td>
<td>0.27</td>
<td>5.84</td>
<td>87.74</td>
<td>4.74</td>
<td>0.98</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.11</td>
<td>0.64</td>
<td>7.85</td>
<td>81.14</td>
<td>8.27</td>
<td>0.89</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.11</td>
<td>0.30</td>
<td>0.42</td>
<td>6.75</td>
<td>83.07</td>
<td>3.86</td>
<td>5.49</td>
</tr>
<tr>
<td>CCC</td>
<td>0.19</td>
<td>0.00</td>
<td>0.38</td>
<td>0.75</td>
<td>2.44</td>
<td>12.03</td>
<td>60.71</td>
<td>23.50</td>
</tr>
<tr>
<td>D</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

The trace statistics is equal to:

\[ \lambda(t) = \frac{\text{trace}(e^{t\Lambda})}{K} \]
Figure 43: Trace statistics of credit and ESG migration matrices
Course 2022-2023 in Sustainable Finance
Lecture 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2023

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6 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Mean-variance optimization problem

Model settings

- An investment universe of $n$ assets
- $w = (w_1, \ldots, w_n)$ is the vector of weights in the portfolio
- The portfolio is fully invested meaning that $\sum_{i=1}^{n} w_i = 1^T w = 1$
- $R = (R_1, \ldots, R_n)$ is the vector of asset returns
- We denote by $\mu = \mathbb{E}[R]$ and $\Sigma = \mathbb{E}\left[ (R - \mu)(R - \mu)^\top \right]$ the vector of expected returns and the covariance matrix of asset returns
We have:

\[ R(w) = \sum_{i=1}^{n} w_i R_i = w^\top R \]

The expected return \( \mu(w) := \mathbb{E}[R(w)] \) of the portfolio is equal to:

\[ \mu(w) = \mathbb{E}[w^\top R] = w^\top \mathbb{E}[R] = w^\top \mu \]

whereas its variance \( \sigma^2(w) := \text{var}(R(w)) \) is given by:

\[
\sigma^2(w) = \mathbb{E} \left[ (R(w) - \mu(w))(R(w) - \mu(w))^\top \right] \\
= \mathbb{E} \left[ w^\top (R - \mu) (R - \mu)^\top w \right] \\
= w^\top \Sigma w
\]
We can then formulate the investor’s financial problem as follows:

1. Maximizing the expected return of the portfolio under a volatility constraint ($\sigma$-problem):

$$\max \mu (w) \quad \text{s.t.} \quad \sigma (w) \leq \sigma^*$$

2. Or minimizing the volatility of the portfolio under a return constraint ($\mu$-problem):

$$\min \sigma (w) \quad \text{s.t.} \quad \mu (w) \geq \mu^*$$

$\Rightarrow$ The key idea of Markowitz was to transform the original non-linear optimization problems into a quadratic optimization problem.
Mean-variance optimization problem
Introducing the quadratic utility function

- The mean-variance (or quadratic) utility function is:

\[ U(w) := \mathbb{E}[R(w)] - \frac{\tilde{\gamma}}{2} \text{var}(R(w)) = w^\top \mu - \frac{\tilde{\gamma}}{2} w^\top \Sigma w \]

where \( \tilde{\gamma} \) is the absolute risk-aversion parameter

- We obtain the following problem:

\[ w^* (\tilde{\gamma}) = \arg \max \left\{ U(w) = w^\top \mu - \frac{\tilde{\gamma}}{2} w^\top \Sigma w \right\} \]

s.t. \( 1^\top w = 1 \)

- \( \tilde{\gamma} = 0 \Rightarrow \) maximum mean portfolio
- \( \tilde{\gamma} = \infty \Rightarrow \) minimum variance portfolio:

\[ w^* (\infty) = \arg \min \frac{1}{2} w^\top \Sigma w \quad \text{s.t.} \quad 1^\top w = 1 \]
In practice, professionals formulate the optimization problem as follows:

\[
\left( \nu^* (\gamma) \right) = \arg \min_{\nu} \frac{1}{2} \nu^T \Sigma \nu - \gamma \nu^T \mu \\
\text{s.t. } \mathbf{1}^T \nu = 1
\]

where \( \gamma = \bar{\gamma}^{-1} \) is called the risk-tolerance

This is a standard QP problem
Definition

The formulation of a standard QP problem is:

\[ w^* = \arg \min \frac{1}{2} w^\top Qw - w^\top R \]

u.c. \[ \begin{align*}
Aw &= B \\
Cw &\leq D \\
w^- &\leq w \leq w^+
\end{align*} \]

\[ \Rightarrow \text{We have } Q = \Sigma, \ R = \gamma \mu, \ A = 1^\top \text{ and } B = 1 \]
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

Mean-variance optimization problem
Illustration

Example #1
We consider an investment universe of five assets. Their expected returns
are equal to 5%, 7%, 6%, 10% and 8% while their volatilities are equal
to 18%, 20%, 22%, 25% and 30%. The correlation matrix of asset
returns is given by the following matrix:


100%
 70% 100%





30% 100%
C =  20%

 −30%

20%
10% 100%
0%
0%
0%
0% 100%

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Mean-variance optimization problem

**Figure 44: Efficient frontier (Example #1)**
The GMV portfolio is obtained with $\gamma = 0$

The solution is:

$$w_{\text{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

We have:

$$\sigma(w) \geq \sigma(w_{\text{gmv}}) = 10.40\% \quad \forall w$$
### Table 36: Solution of the Markowitz optimization problem (in %)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.00</th>
<th>0.10</th>
<th>0.20</th>
<th>0.50</th>
<th>1.00</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*(\gamma)$</td>
<td>66.35</td>
<td>58.25</td>
<td>50.14</td>
<td>25.84</td>
<td>-14.67</td>
<td>-338.72</td>
</tr>
<tr>
<td>$w_2^*(\gamma)$</td>
<td>-28.52</td>
<td>-22.67</td>
<td>-16.82</td>
<td>0.74</td>
<td>30.00</td>
<td>264.12</td>
</tr>
<tr>
<td>$w_3^*(\gamma)$</td>
<td>15.31</td>
<td>13.30</td>
<td>11.30</td>
<td>5.28</td>
<td>-4.74</td>
<td>-84.93</td>
</tr>
<tr>
<td>$w_4^*(\gamma)$</td>
<td>34.85</td>
<td>37.65</td>
<td>40.44</td>
<td>48.82</td>
<td>62.78</td>
<td>174.50</td>
</tr>
<tr>
<td>$w_5^*(\gamma)$</td>
<td>12.02</td>
<td>13.48</td>
<td>14.94</td>
<td>19.32</td>
<td>26.62</td>
<td>85.03</td>
</tr>
<tr>
<td>$\mu(w^*(\gamma))$</td>
<td>6.69</td>
<td>6.97</td>
<td>7.25</td>
<td>8.09</td>
<td>9.49</td>
<td>20.71</td>
</tr>
<tr>
<td>$\sigma(w^*(\gamma))$</td>
<td>10.40</td>
<td>10.53</td>
<td>10.93</td>
<td>13.35</td>
<td>19.71</td>
<td>84.38</td>
</tr>
</tbody>
</table>
Mean-variance optimization problem
How to solve the $\mu$-problem and the $\sigma$-problem?

- We have to find the optimal value of $\gamma$ such that $\mu(w^*(\gamma)) = \mu^*$ or $\sigma(w^*(\gamma)) = \sigma^*$
- We use the bisection algorithm
- If we target a portfolio with $\sigma^* = 15\%$, we know that $\gamma \in [0.5, 1]$. The optimal solution $w^*$ is $(14.06\%, 9.25\%, 2.37\%, 52.88\%, 21.44\%)$ and the bisection algorithm returns $\gamma = 0.6455$. In this case, we obtain $\mu(w^*(\gamma)) = 8.50\%$
- If we consider a $\mu$-problem with $\mu^* = 9\%$, we find $\gamma = 0.8252$, $w^* = (-0.50\%, 19.77\%, -1.23\%, 57.90\%, 24.07\%)$ and $\sigma(w^*(\gamma)) = 17.30\%$
Mean-variance optimization problem
Adding some constraints

- The Lagrange function of the optimization problem is equal to:

\[ L(w; \lambda_0) = \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu + \lambda_0 (1^\top w - 1) \]

where \( \lambda_0 \) is the Lagrange coefficients associated with the constraint \( 1^\top w = 1 \)
- The solution \( w^* \) verifies the following first-order conditions:

\[
\begin{cases}
\partial_w L(w; \lambda_0) = \Sigma w - \gamma \mu + \lambda_0 1 = 0 \\
\partial_{\lambda_0} L(w; \lambda_0) = 1^\top w - 1 = 0
\end{cases}
\]
- We obtain \( w = \Sigma^{-1} (\gamma \mu - \lambda_0 1) \). Because \( 1^\top w - 1 = 0 \), we have

\[
\gamma 1^\top \Sigma^{-1} \mu - \lambda_0 1^\top \Sigma^{-1} 1 = 1.
\]

It follows that:

\[
\lambda_0 = \frac{\gamma 1^\top \Sigma^{-1} \mu - 1}{1^\top \Sigma^{-1} 1}
\]
The solution is then:

\[
\mathbf{w}^*(\gamma) = w_{\text{gmv}} + \gamma w_{\text{lsp}}
\]

where:

- \( w_{\text{gmv}} = (\Sigma^{-1}\mathbf{1}) / (\mathbf{1}^T \Sigma^{-1} \mathbf{1}) \) is the global minimum variance portfolio
- \( w_{\text{lsp}} \) is a long/short cash-neutral portfolio such that \( \mathbf{1}^T w_{\text{lsp}} = 0 \)
Mean-variance optimization problem
Adding some constraints

- We could think that a QP solver is not required
- The analytical calculus gives:

\[ w_{\text{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%) \]

and:

\[ w_{\text{lsp}} = (-81.01\%, 58.53\%, -20.05\%, 27.93\%, 14.60\%) \]

- In practice, professionals consider other constraints:

\[
\begin{align*}
    w^*(\gamma) &= \arg\min \frac{1}{2} w^T \Sigma w - \gamma w^T \mu \\
    \text{s.t.} & \begin{cases} 
    1^T w = 1 \\
    w \in \Omega
    \end{cases}
\end{align*}
\]

where \( w \in \Omega \) corresponds to the set of restrictions

- No short-selling restriction (\( w_i \geq 0 \) and \( \Omega = [0, 1]^n \)) and asset bounds (\( w_i \leq w^+ \)) \( \Rightarrow \) No analytical solution (because of the KKT conditions) \( \Rightarrow \) QP solver
The tangency portfolio
Two-fund separation theorem

We consider a combination of the risk-free asset and a portfolio $w$:

$$R(\tilde{w}) = (1 - \alpha) r + \alpha R(w)$$

where:

- $r$ is the return of the risk-free asset
- $\tilde{w} = (\alpha w, 1 - \alpha)$ is a vector of dimension $(n + 1)$
- $\alpha \geq 0$ is the proportion of the wealth invested in the risky portfolio

⇒ It follows that

$$\mu(\tilde{w}) = (1 - \alpha) r + \alpha \mu(w) = r + \alpha (\mu(w) - r),$$

$$\sigma^2(\tilde{w}) = \alpha^2 \sigma^2(w)$$

and:

$$\mu(\tilde{w}) = r + \frac{(\mu(w) - r)}{\sigma(w)} \sigma(\tilde{w})$$
Figure 45: Capital market line (Example #1)
Let \( \text{SR}(w | r) \) be the Sharpe ratio of portfolio \( w \):

\[
\text{SR}(w | r) = \frac{\mu(w) - r}{\sigma(w)}
\]

We have:

\[
\frac{\mu(\tilde{w}) - r}{\sigma(\tilde{w})} = \frac{\mu(w) - r}{\sigma(w)} \iff \text{SR}(\tilde{w} | r) = \text{SR}(w | r)
\]

The tangency portfolio \( w^* \) satisfies:

\[
w^* = \arg \max \tan \theta(w)
\]
If we consider our example with $r = 3\%$, the composition of the tangency portfolio is:

$$w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)$$

and we have:

$$\begin{align*}
\mu (w^*) &= 7.51\% \\
\sigma (w^*) &= 11.50\% \\
SR (w^* | r) &= 0.39 \\
\theta (w^*) &= 21.40 \text{ degrees}
\end{align*}$$
The tangency portfolio
Augmented optimization problem

- When the risk-free asset belongs to the investment universe, the optimization problem becomes:

\[ \tilde{w}^* (\gamma) = \arg \min \frac{1}{2} \tilde{w}^T \tilde{\Sigma} \tilde{w} - \gamma \tilde{w}^T \tilde{\mu} \]

s.t. \[ \left\{ \begin{array}{l} \mathbf{1}^T \tilde{w} = 1 \\ \tilde{w} \in \Omega \end{array} \right. \]

where \( \tilde{w} = (w, w_r) \) is the augmented allocation vector of dimension \( n + 1 \).

- It follows that:

\[ \tilde{\Sigma} = \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \tilde{\mu} = \begin{pmatrix} \mu \\ r \end{pmatrix} \]
In the case where $\Omega = \mathbb{R}^{n+1}$, we can show that the optimal solution is equal to:

$$\tilde{w}^* (\gamma) = \alpha \cdot \begin{pmatrix} w^* \\ 0 \end{pmatrix} + (1 - \alpha) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $w^*$ is the tangency portfolio:

$$w^* = \frac{\Sigma^{-1}(\mu - r1)}{1^\top \Sigma^{-1}(\mu - r1)}$$

The proportion of risky assets is equal to

$$\alpha = \gamma \frac{1^\top \Sigma^{-1}(\mu - r1)}{1^\top \Sigma^{-1}(\mu - r1)}$$

The risk-tolerance coefficient associated to the tangency portfolio is given by:

$$\gamma(w^*) = \frac{1}{1^\top \Sigma^{-1}(\mu - r1)}$$
At the equilibrium, Sharpe (1964) showed that:

\[ \pi_i := \mu_i - r = \beta_i (\mu (w^*) - r) \]

where \( \pi_i \) is the risk premium of the asset \( i \) and:

\[ \beta_i = \frac{\text{cov} (R_i, R (w^*))}{\text{var} (R (w^*))} \]

We have:

\[ \beta (x \mid w) = \frac{\sigma (x, w)}{\sigma^2 (w)} = \frac{x^\top \Sigma w}{w^\top \Sigma w} \]

and:

\[ \beta_i = \beta (e_i \mid w) = \frac{e_i^\top \Sigma w}{w^\top \Sigma w} = \frac{(\Sigma w)_i}{w^\top \Sigma w} \]
Market equilibrium and CAPM
Risk premium and beta

In the case of Example #1, we have:

- \( w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%) \)
- \( (\mu(w^*) = 7.51\%, r = 3\%) \Rightarrow \mu(w^*) = 4.51\% \)

Table 37: Computation of the beta and risk premia (Example #1)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>( \mu(w) )</th>
<th>( \mu(w) - r )</th>
<th>( \beta(w \mid w^*) )</th>
<th>( \pi(w \mid w^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_1 )</td>
<td>5.00%</td>
<td>2.00%</td>
<td>0.444</td>
<td>2.00%</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>7.00%</td>
<td>4.00%</td>
<td>0.887</td>
<td>4.00%</td>
</tr>
<tr>
<td>( e_3 )</td>
<td>6.00%</td>
<td>3.00%</td>
<td>0.665</td>
<td>3.00%</td>
</tr>
<tr>
<td>( e_4 )</td>
<td>10.00%</td>
<td>7.00%</td>
<td>1.553</td>
<td>7.00%</td>
</tr>
<tr>
<td>( e_5 )</td>
<td>8.00%</td>
<td>5.00%</td>
<td>1.109</td>
<td>5.00%</td>
</tr>
<tr>
<td>( w_{ew} )</td>
<td>7.20%</td>
<td>4.20%</td>
<td>0.932</td>
<td>4.20%</td>
</tr>
<tr>
<td>( w_{gmv} )</td>
<td>6.69%</td>
<td>3.69%</td>
<td>0.817</td>
<td>3.69%</td>
</tr>
</tbody>
</table>
Jensen (1968) defined the alpha return as:

\[ R_{j,t} - r = \alpha_j + \beta_j (R_t (w_m) - r) + \varepsilon_{j,t} \]

where \( R_{j,t} \) is the return of the mutual fund \( j \) at time \( t \), \( R_t (w_m) \) is the return of the market portfolio and \( \varepsilon_{j,t} \) is an idiosyncratic risk.

More generally, the alpha is defined by the difference between the risk premium \( \pi (w) \) of portfolio \( w \) and the beta \( \beta (w) \) of the portfolio times the market risk premium \( \pi_m \):

\[
\alpha = (\mu(w) - r) - \beta(w | w_m) (\mu(w_m) - r)
= \pi(w) - \beta(w) \pi_m
\]
In the case of Example #1 & no short-selling constraint, we have:

- \( w^* = (33.62\%, 0\%, 8.79\%, 40.65\%, 16.95\%) \)
- \( \mu (w^*) = 7.63\%, r = 3\% \) \( \Rightarrow \mu (w^*) = 4.63\% \)

**Table 38: Computation of the alpha return (Example #1)**

| Portfolio | \( \mu (w) \) | \( \mu (w) - r \) | \( \beta (w | w^*) \) | \( \pi (w | w^*) \) | \( \alpha (w | w^*) \) |
|-----------|----------------|------------------|-----------------|-----------------|-----------------|
| e₁        | 5.00%          | 2.00%            | 0.432           | 2.00%           | 0.00%           |
| e₂        | 7.00%          | 4.00%            | 0.970           | 4.49%           | -0.49%          |
| e₃        | 6.00%          | 3.00%            | 0.648           | 3.00%           | 0.00%           |
| e₄        | 10.00%         | 7.00%            | 1.512           | 7.00%           | 0.00%           |
| e₅        | 8.00%          | 5.00%            | 1.080           | 5.00%           | 0.00%           |
| wₑₑ       | 7.20%          | 4.20%            | 0.929           | 4.30%           | -0.10%          |
| wₑₑ       | 6.69%          | 3.69%            | 0.766           | 3.55%           | 0.14%           |
Portfolio optimization in the presence of a benchmark
Utility function revisited

- $b$ is the benchmark
- The tracking error is:

$$
\epsilon = R(w) - R(b) = \sum_{i=1}^{n} w_i R_i - \sum_{i=1}^{n} b_i R_i = w^T R - b^T R = (w - b)^T R
$$

- The expected excess return is equal to:

$$
\mu (w \mid b) := \mathbb{E} [\epsilon] = (w - b)^T \mu
$$

- The volatility of the tracking error is defined as:

$$
\sigma (w \mid b) := \sigma (e) = \sqrt{(w - b)^T \Sigma (w - b)}
$$
The objective of the investor is then to maximize the expected tracking error with a constraint on the tracking error volatility:

$$w^* = \text{arg max } \mu (w \mid b) \quad \text{s.t.} \quad \begin{cases} 1^T x = 1 \\ \sigma (w \mid b) \leq \sigma^* \end{cases}$$

We have:

$$f (w \mid b) = \frac{1}{2} \sigma^2 (w \mid b) - \gamma \mu (w \mid b)$$

$$= \frac{1}{2} (w - b)^T \Sigma (w - b) - \gamma (w - b)^T \mu$$

$$= \frac{1}{2} w^T \Sigma w - w^T (\gamma \mu + \Sigma b) + \frac{1}{2} b^T \Sigma b + \gamma b^T \mu$$

(constant)
We have:

\[
Q = \Sigma \\
R = \gamma \mu + \Sigma b \\
A = 1^T \\
B = 1 \\
C = \\
D = \\
w^- = 0_n \text{ (if no short-selling)} \\
w^+ = 1_n \text{ (if no short-selling)}
\]
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

Portfolio optimization in the presence of a benchmark

Example #2
We consider an investment universe of four assets. Their expected
returns are equal to 5%, 6.5%, 8% and 6.5% while their volatilities are
equal to 15%, 20%, 25% and 30%. The correlation matrix of asset
returns is given by the following matrix:


100%
 10% 100%



C=

40%
70% 100%
50%
40%
80% 100%
The benchmark is b = (60%, 40%, 20%, −20%).

Thierry Roncalli

Course 2022-2023 in Sustainable Finance

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Figure 46: Efficient frontier with a benchmark (Example #2)
Figure 47: Tangency portfolio with respect to a benchmark (Example #2)

⇒ the tangency portfolio is equal to (46.56%, 33.49%, 39.95%, −20.00%)
Portfolio optimization in the presence of a benchmark

Information ratio

- We have:

$$\text{IR} (w \mid b) = \frac{\mu (w \mid b)}{\sigma (w \mid b)} = \frac{(w - b)^\top \mu}{\sqrt{(w - b)^\top \Sigma (w - b)}}$$

- If we consider a combination of the benchmark $b$ and the active portfolio $w$, the composition of the portfolio is:

$$x = (1 - \alpha) b + \alpha w$$

where $\alpha \geq 0$ is the proportion of wealth invested in the portfolio $w$

- It follows that:

$$\mu (x \mid b) = (x - b)^\top \mu = \alpha \mu (w \mid b)$$

and:

$$\sigma^2 (x \mid b) = (x - b)^\top \Sigma (x - b) = \alpha^2 \sigma^2 (w \mid b)$$

- We deduce that:

$$\mu (x \mid b) = \text{IR} (w \mid b) \cdot \sigma (x \mid b)$$
• Expected (or required) returns $\neq$ historical (or realised) returns:

$$\pi_i \neq R_i$$

• Difference between the unconstrained risk premium and the implied risk premium:

$$\pi_i \neq \tilde{\pi}_i$$
The Pastor-Stambaugh-Taylor model

Model settings

- The asset excess returns \( \tilde{R} = R - r = (\tilde{R}_1, \ldots, \tilde{R}_n) \) are normally distributed: \( \tilde{R} \sim \mathcal{N}(\pi, \Sigma) \)
- Each firm has an ESG characteristic \( G_i \), which is positive for *esg-friendly* (or *green*) firms and negative for *esg-unfriendly* (or *brown*) firms
- \( G_i > 0 \) induces positive social impact, while \( G_i < 0 \) induces negative externalities on the society
- Economy with a continuum of agents \( (j = 1, 2, \ldots, \infty) \)
- \( w_{i,j} \) is the fraction of the wealth invested by agent \( j \) in stock \( i \)
- \( w_j = (w_{1,j}, \ldots, w_{n,j}) \) is the allocation vector of agent \( j \)
The Pastor-Stambaugh-Taylor model

Model settings

- The relationship between the initial and terminal wealth $W_j$ and $\tilde{W}_j$ is given by:

$$\tilde{W}_j = \left(1 + r + w_j^\top \tilde{R}\right) W_j$$

- Exponential CARA utility function:

$$U \left(\tilde{W}_j, w_j\right) = -\exp \left(-\tilde{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)$$

where:

- $\tilde{\gamma}_j$ is the absolute risk-aversion
- $b_j = \varphi_j \mathcal{G}$ is the vector of nonpecuniary benefits ($\varphi_j \geq 0$)
The Pastor-Stambaugh-Taylor model

Optimal portfolio

- The expected utility is equal to:

\[
\mathbb{E} \left[ U \left( \tilde{W}_j, w_j \right) \right] = \mathbb{E} \left[ - \exp \left( -\tilde{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j \right) \right] \\
= \mathbb{E} \left[ - \exp \left( -\tilde{\gamma}_j \left( 1 + r + w_j^\top \tilde{R} \right) W_j - w_j^\top b_j W_j \right) \right] \\
= -e^{-\tilde{\gamma}_j(1+r)W_j} \mathbb{E} \left[ \exp \left( -\tilde{\gamma}_j w_j^\top \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \right) \right] \\
= e^{-\bar{\Gamma}_j(1+r)} \mathbb{E} \left[ \exp \left( -\bar{\Gamma}_j w_j^\top \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \right) \right]
\]

where \( \bar{\Gamma}_j = \tilde{\gamma}_j W_j \) is the nominal risk aversion

- We notice that \( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \sim \mathcal{N} \left( \pi + \tilde{\gamma}_j^{-1} b_j, \Sigma \right) \) and:

\[
-\bar{\Gamma}_j w_j^\top \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \sim \mathcal{N} \left( -\bar{\Gamma}_j w_j^\top \left( \pi + \tilde{\gamma}_j^{-1} b_j \right), \bar{\Gamma}_j^2 w_j^\top \Sigma w_j \right)
\]
We deduce that:

$$\mathbb{E} \left[ U \left( \tilde{W}_j, w_j \right) \right] = e^{-\bar{\Gamma}_j (1+r)} \exp \left( -\bar{\Gamma}_j w_j^\top \left( \pi + \bar{\gamma}_j^{-1} b_j \right) + \frac{1}{2} \bar{\Gamma}_j^2 w_j^\top \Sigma w_j \right)$$

The first-order condition is equal to:

$$-\bar{\Gamma}_j \left( \pi + \bar{\gamma}_j^{-1} b_j \right) + \bar{\Gamma}_j^2 \Sigma w_j = 0$$

Finally, Pastor et al. (2021) concluded that the optimal portfolio is:

$$w_j^* = \Gamma_j \Sigma^{-1} \left( \pi + \gamma_j b_j \right)$$

where $\Gamma_j = \bar{\Gamma}_j^{-1}$ and $\gamma_j = \bar{\gamma}_j^{-1}$ are the relative nominal and unitary risk-tolerance.
Maximizing the expected utility is equivalent to solve the classical Markowitz QP problem:

$$w_j^* (\gamma_j) = \arg \min \frac{1}{2} w_j^T \Sigma w_j - \gamma_j w_j^T \mu'$$

s.t. $1^T w_j = 1$

where

- $\gamma_j = \bar{\gamma}_j^{-1}$ is the relative risk tolerance
- $\mu' = \mu + \gamma_j b_j$ is the vector of modified expected returns
Example #3

We consider a universe of $n$ risky assets, where $n$ is an even number. The risk-free rate $r$ is set to 3%. We assume that the Sharpe ratio of these assets is the same and is equal to 20%. The volatility of asset $i$ is equal to $\sigma_i = 0.10 + 0.20 \cdot e^{-n^{-1}[0.5i]}$. The correlation between asset returns is constant: $C = C_n(\rho)$. The social impact of the firms is given by the vector $G$. When $G$ is not specified, it is equal to the cyclic vector $(+1\%, -1\%, +1\%, \ldots, +1\%, -1\%)$. This implies that half of the firms (green firms) have a positive social impact while the others (brown firms) have a negative impact.
Table 39: Mean-variance optimized portfolios with ESG preferences (Example #3, \( n = 6 \), \( \rho = 25\% \))

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>( \mathcal{G} = (1%, -1%, 1%, -1%, 1%, -1%) )</th>
<th>( \mathcal{G} = (10%, 5%, 2%, 3%, 25%, 30%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>44.97% 48.87% 58.65% 67.48%</td>
<td>44.97% 46.83% 28.69% 0.00%</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>44.97% 41.06% 19.60% 0.00%</td>
<td>44.97% 37.06% 9.17% 0.00%</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>5.03% 9.82% 21.75% 32.52%</td>
<td>5.03% 0.00% 0.00% 0.00%</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>5.03% 0.25% 0.00% 0.00%</td>
<td>5.03% 0.00% 0.00% 0.00%</td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>0.00% 0.00% 0.00% 0.00%</td>
<td>0.00% 0.83% 16.62% 21.09%</td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>0.00% 0.00% 0.00% 0.00%</td>
<td>0.00% 15.28% 45.53% 78.91%</td>
</tr>
<tr>
<td>( \mu (w^*) )</td>
<td>8.33% 8.33% 8.27% 8.22%</td>
<td>8.33% 8.23% 7.79% 7.43%</td>
</tr>
<tr>
<td>( \sigma (w^*) )</td>
<td>20.00% 20.09% 20.07% 21.56%</td>
<td>20.00% 19.33% 16.70% 19.17%</td>
</tr>
<tr>
<td>SR ((w^* \mid r))</td>
<td>0.27 0.27 0.26 0.24</td>
<td>0.27 0.27 0.29 0.23</td>
</tr>
</tbody>
</table>
The Pastor-Stambaugh-Taylor model

Optimal portfolio

Figure 48: Efficient frontier with ESG preferences (Example #3, \( n = 20 \), \( \rho = 25\% \))
The Pastor-Stambaugh-Taylor model

Risk premium

- $W = \int W_j \, dj$
- $\omega_j = W_j / W$ is the market share of the economic agent $j$
- $W_{i,j} = w_{i,j}^* W_j = w_{i,j}^* \omega_j W$
- We have:

$$W_i = \int_j W_{i,j} \, dj = \int_j w_{i,j}^* \omega_j W \, dj$$

- Let $w_m = (w_{1,m}, \ldots, w_{n,m})$ be the market portfolio. We have:

$$w_{i,m} = \frac{W_i}{W} = \int_j w_{i,j}^* \omega_j \, dj$$

and $\int_j \omega_j \, dj = 1$
The Pastor-Stambaugh-Taylor model

- The market clearing condition satisfies:

\[ w_m = \int_j \omega_j w_j^* \, dj \]
\[ = \int_j \omega_j \Gamma_j \Sigma^{-1} (\pi + \gamma_j b_j) \, dj \]
\[ = \int_j \omega_j \Gamma_j \Sigma^{-1} (\pi + \gamma_j \varphi_j \mathcal{G}) \, dj \]
\[ = \left( \int_j \Gamma_j \omega_j \, dj \right) \Sigma^{-1} \pi + \left( \int_j \omega_j \Gamma_j \psi_j \, dj \right) \Sigma^{-1} \mathcal{G} \]

where \( \psi_j = \gamma_j \varphi_j \)

- It follows that:

\[ w_m = \Gamma_m \Sigma^{-1} \pi + \Gamma_m \psi_m \Sigma^{-1} \mathcal{G} \]

where \( \Gamma_m = \int_j \Gamma_j \omega_j \, dj \) and \( \psi_m = \Gamma_m^{-1} \left( \int_j \omega_j \Gamma_j \psi_j \, dj \right) \) are the average risk tolerance and the weighted average of ESG preferences
The asset risk premia are equal to:

$$\pi = \frac{1}{\Gamma_m} \sum w_m - \psi_m G$$

while the market risk premium is defined as:

$$\pi_m = w_m^\top \pi$$

$$= \frac{1}{\Gamma_m} w_m^\top \sum w_m - \psi_m w_m^\top G$$

$$= \frac{1}{\Gamma_m} \sigma_m^2 - \psi_m G_m$$

where $\sigma_m = \sqrt{w_m^\top \sum w_m}$ and $G_m = w_m^\top G$ are the volatility and the green intensity (or greenness) of the market portfolio.
The Pastor-Stambaugh-Taylor model

The risk premium including the ESG sentiment is lower than the CAPM risk premium if the market ESG intensity is positive:

\[ G_m > 0 \implies \pi_m \leq \pi_m^{\text{capm}} \]

It is greater than the CAPM risk premium if the market ESG intensity is negative:

\[ G_m < 0 \implies \pi_m \geq \pi_m^{\text{capm}} \]

The gap \( \Delta \pi_m^{\text{esg}} := |\pi_m - \pi_m^{\text{capm}}| \) is an increasing function of the market ESG sentiment \( \psi_m \):

\[ \psi_m \uparrow \implies \Delta \pi_m^{\text{esg}} \uparrow \]
If we assume that $G_m \approx 0$, we have $\Gamma_m = \sigma_m^2 / \pi_m$.

$$\pi = \beta \pi_m - \psi_m G$$

and:

$$\alpha_i = \pi_i - \beta_i \pi_m = -\psi_m G_i$$

If $\psi_m > 0$, “green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas” (Pastor et al., 2021).
Example #4

We consider Example #3. The market is made up of two long-only investors ($j = 1, 2$): a non-ESG investor ($\varphi_1 = 0$) and an ESG investor ($\varphi_2 > 0$). We assume that they have the same risk tolerance $\gamma$. We note $W_1$ and $W_2$ their financial wealth, which is entirely invested in the risky assets. We assume that $W_1 = W_2 = 1$. 
The Pastor-Stambaugh-Taylor model

Risk premium

- The tangency portfolio is equal to:

\[
\begin{align*}
 w^* &= \frac{\Sigma^{-1}(\mu - r1)}{1^T \Sigma^{-1}(\mu - r1)} \\
 &= (15.04\%, 15.04\%, 16.65\%, 16.65\%, 18.31\%, 18.31\%)
\end{align*}
\]

- \( w_1^* = w^* \) and \( \gamma_1 = 1/(1^T \Sigma^{-1}(\mu - r1)) = 0.4558 \)

- \( \gamma_2 = \gamma_1 \) and:

\[
\begin{align*}
 w_2^* &= \arg\min \frac{1}{2} w^T \Sigma w - \gamma_2 w^T (\mu + \gamma_2 \varphi_2 \mathcal{G}) \\
\text{s.t.} & \left\{ \\
& 1^T w = 1 \\
& w \geq 0
\right\}
\end{align*}
\]

- We obtain

\[
 w_2^* = (18.86\%, 11.22\%, 21.33\%, 11.97\%, 23.96\%, 12.65\%)
\]
The Pastor-Stambaugh-Taylor model

The market portfolio is then equal to:

\[ w_m = \frac{W_1}{W} w_1^* + \frac{W_2}{W} w_2^* \]

\[ = (1 - \omega_{esg}) \cdot w_1^* + \omega_{esg} \cdot w_2^* \]

When \( W_1 = W_2 = 1 \), we obtain

\[ w_m = (16.95\%, 13.13\%, 18.99\%, 14.31\%, 21.13\%, 15.48\%) \]

\[ \mu_m = 7.86\% \]

\[ \sigma_m = 14.93\% \]

We deduce that:

\[ \beta = (1.15, 1.05, 1.04, 0.95, 0.95, 0.86) \]

\[ \pi = (5.58\%, 5.12\%, 5.06\%, 4.61\%, 4.62\%, 4.17\%) \]

\[ \alpha = (-19.09, 26.19, -19.43, 25.84, -19.72, 25.55) \] (in bps)
### Table 40: Computation of alpha returns (Example #4, $n = 6$, $\rho = 25\%$)

<table>
<thead>
<tr>
<th>$i$</th>
<th>Portfolio $w_1^*$</th>
<th>Portfolio $w_2^*$</th>
<th>Portfolio $w_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_i$ (in %)</td>
<td>$\beta_i$ (in %)</td>
<td>$\pi_i$ (in %)</td>
</tr>
<tr>
<td>1</td>
<td>15.04</td>
<td>1.11</td>
<td>5.39</td>
</tr>
<tr>
<td>2</td>
<td>15.04</td>
<td>1.11</td>
<td>5.39</td>
</tr>
<tr>
<td>3</td>
<td>16.65</td>
<td>1.00</td>
<td>4.87</td>
</tr>
<tr>
<td>4</td>
<td>16.65</td>
<td>1.00</td>
<td>4.87</td>
</tr>
<tr>
<td>5</td>
<td>18.31</td>
<td>0.91</td>
<td>4.43</td>
</tr>
<tr>
<td>6</td>
<td>18.31</td>
<td>0.91</td>
<td>4.43</td>
</tr>
</tbody>
</table>
**The Pastor-Stambaugh-Taylor model**

Risk premium

### Figure 49: Evolution of the alpha return with respect to the market share of ESG investors (Example #4, \(n = 6\), \(\rho = 25\%\))
“In equilibrium, green assets have low expected returns because investors enjoy holding them and because green assets hedge climate risk. Green assets nevertheless outperform when positive shocks hit the ESG factor, which captures shifts in customers’ tastes for green products and investors’ tastes for green holdings.” (Pastor et al., 2021).

- ESG risk premium?
- Green risk premium?
The Pastor-Stambaugh-Taylor model
What does equilibrium mean?

**Figure 50:** Impact of alpha returns on the underperformance probability

*Figure 50a:* Probability of underperformance $p_u(\Delta \alpha, t)$ in % (asset level)

*Figure 50b:* Probability of underperformance $p_u(\Delta \alpha, t)$ in % (asset level)

*Figure 50c:* Time $t(\Delta \alpha, p_u)$ in years (asset level)

*Figure 50d:* Time $t(\Delta \alpha, p_u)$ in years (portfolio level)
Extension of the PST model
The Avromov-Cheng-Liou-Tarelli model

- We have:
  \[
  \begin{pmatrix}
  \tilde{R} \\
  \tilde{S}
  \end{pmatrix}
  \sim \mathcal{N}
  \left( \begin{pmatrix}
  \pi \\
  \mu_s
  \end{pmatrix},
  \begin{pmatrix}
  \Sigma & \Sigma_{\pi,s} \\
  \Sigma_{s,\pi} & \Sigma_s
  \end{pmatrix}
  \right)
  \]

- The optimal solution is:
  \[
  w_j^* = \Gamma_j \Sigma^{-1} \left( \pi + \psi_j \mu_s \right) + \Gamma_j^{-1} \Omega_j \left( \pi + \psi_j \mu_s \right)
  \]
  PST solution + ESG uncertainty
Extension of the PST model
The Avromov-Cheng-Liou-Tarelli model

- If there is no ESG uncertainty \((S = \mu_s \text{ and } \Sigma_s = 0)\), the vector of risk premia is given by:

\[
\pi^{esg} = \beta \pi_m - \psi_m (\mu_s - \beta \bar{S}_m) \\
= \pi^{capm} - \psi_m (\mu_s - \beta \bar{S}_m)
\]

- If there is an uncertainty on ESG scores \((S \neq \mu_s \text{ and } \Sigma_s \neq 0)\), the vector of risk premia becomes:

\[
\tilde{\pi}^{esg} = \tilde{\beta} \tilde{\pi}_m - \psi_m (\tilde{\mu}_s - \tilde{\beta} \tilde{S}_m) \\
= \beta \pi_m + (\tilde{\beta} - \beta) \pi_m - \psi_m (\tilde{\mu}_s - \tilde{\beta} \tilde{S}_m)
\]
“In equilibrium, the market premium increases and demand for stocks declines under ESG uncertainty. In addition, the CAPM alpha and effective beta both rise with ESG uncertainty and the negative ESG-alpha relation weakens.” (Avramov et al., 2022).
### Extension of the PST model

#### Risk factor model

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Course 2022-2023 in Sustainable Finance
The Pedersen-Fitzgibbons-Pomorski model

Model settings

- $\tilde{R} = R - r \sim \mathcal{N}(\pi, \Sigma)$
- $S = (S_1, \ldots, S_n)$
- The terminal wealth is $\tilde{W} = \left(1 + r + w^T \tilde{R}\right) W$
- The model uses the mean-variance utility:

$$\mathcal{U} \left( \tilde{W}, w \right) = \mathbb{E} \left[ \tilde{W} \right] - \frac{\gamma}{2} \text{var} \left( \tilde{W} \right) + \zeta \left( S \left( w \right) \right) W$$

$$= \left(1 + r + w^T \pi - \frac{\gamma}{2} w^T \Sigma w + \zeta \left( w^T S \right) \right) W$$

where $\zeta$ is a function that depends on the investor.
- Optimizing the utility function is equivalent to find the mean-variance-esg optimized portfolio:

\[ w^* = \arg \max w^T \pi - \frac{\bar{\gamma}}{2} w^T \Sigma w + \zeta (w^T S) \]

s.t. \( 1^T w = 1 \)

- \( \sigma (w) = \sqrt{w^T \Sigma w} \)
- \( S (w) = w^T S \)
The optimization problem can be decomposed as follows:

\[
\begin{align*}
\mathbf{w}^* &= \arg \left\{ \max_{\mathbf{S}} \left\{ \max_{\bar{\sigma}} \left\{ \max_{\mathbf{w}} \left\{ f \left( \mathbf{w}; \pi, \Sigma, \mathbf{S} \right) \right\} \right\} \right\} \right\} \\
\text{where:} & \\
f \left( \mathbf{w}; \pi, \Sigma, \mathbf{S} \right) &= \mathbf{w}^\top \pi - \frac{\bar{\gamma}}{2} \sigma^2 \left( \mathbf{w} \right) + \zeta \left( \mathbf{S} \left( \mathbf{w} \right) \right) \\
\Omega &= \left\{ \mathbf{w} \in \mathbb{R}^n : 1^\top \mathbf{w} = 1, \sigma \left( \mathbf{w} \right) = \bar{\sigma}, \mathbf{S} \left( \mathbf{w} \right) = \bar{\mathbf{S}} \right\}
\end{align*}
\]
We consider the $\sigma - S$ problem:

$$
\begin{align*}
\mathbf{w}^* (\bar{\sigma}, \bar{S}) &= \arg \max \mathbf{w}^\top \pi \\
\text{s.t.} \quad \begin{cases}
1^\top \mathbf{w} = 1 \\
\mathbf{w}^\top \Sigma \mathbf{w} - \bar{\sigma}^2 = 0 \\
\mathbf{w}^\top (\mathbf{S} - \bar{S} \mathbf{1}) = 0
\end{cases}
\end{align*}
$$

The Lagrange function is:

$$
\mathcal{L} (\mathbf{w}; \lambda_1, \lambda_2) = \mathbf{w}^\top \pi + \lambda_1 (\mathbf{w}^\top \Sigma \mathbf{w} - \bar{\sigma}^2) + \lambda_2 (\mathbf{w}^\top (\mathbf{S} - \bar{S} \mathbf{1}))
$$

The first-order condition is:

$$
\frac{\partial \mathcal{L} (\mathbf{w}; \lambda_1, \lambda_2)}{\partial \mathbf{w}} = \pi + 2 \lambda_1 \Sigma \mathbf{w} + \lambda_2 (\mathbf{S} - \bar{S} \mathbf{1}) = 0
$$

We deduce that the optimal portfolio is given by:

$$
\mathbf{w} = -\frac{1}{2 \lambda_1} \Sigma^{-1} (\pi + \lambda_2 (\mathbf{S} - \bar{S} \mathbf{1}))
$$
The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

The second constraint \( w^T (S - \bar{S}1) = 0 \) implies that:

\[
(*) \quad \Leftrightarrow \quad (S - \bar{S}1)^T \frac{1}{2\lambda_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1)) = 0 \\
\Leftrightarrow \quad \lambda_2 = -\frac{(S - \bar{S}1)^T \Sigma^{-1} \pi}{(S - \bar{S}1)^T \Sigma^{-1} (S - \bar{S}1)} \\
\Leftrightarrow \quad \lambda_2 = \frac{\bar{S}(1^T \Sigma^{-1} \pi) - S^T \Sigma^{-1} \pi}{S^T \Sigma^{-1} S - 2\bar{S}(1^T \Sigma^{-1} S) + \bar{S}^2 (1^T \Sigma^{-1} 1)} \\
\Leftrightarrow \quad \lambda_2 = \frac{C_{1,\pi} \bar{S} - C_{s,\pi}}{C_{s,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2}
\]

where \( C_{x,y} \) is the compact notation for \( x^T \Sigma^{-1} y \) — \( C_{1,\pi} = 1^T \Sigma^{-1} \pi \), \( C_{s,\pi} = S^T \Sigma^{-1} \pi \), \( C_{s,s} = S^T \Sigma^{-1} S \), \( C_{1,s} = 1^T \Sigma^{-1} S \) and \( C_{1,1} = 1^T \Sigma^{-1} 1 \).
Using the first constraint $w^\top \Sigma w - \bar{\sigma}^2 = 0$, we deduce that:

$$\bar{\sigma}^2 = -\frac{1}{2\lambda_1} w^\top \Sigma \Sigma^{-1} (\pi + \lambda_2 (\mathcal{S} - \mathcal{S}1))$$

$$= -\frac{1}{2\lambda_1} (w^\top \pi + \lambda_2 w^\top (\mathcal{S} - \mathcal{S}1))$$

$$= -\frac{1}{2\lambda_1} w^\top \pi$$

$$= \frac{1}{4\lambda_1^2} \pi^\top \Sigma^{-1} (\pi + \lambda_2 (\mathcal{S} - \mathcal{S}1))$$

The first Lagrange coefficient is then equal to $(C_{\pi,\pi} = \pi^\top \Sigma^{-1} \pi)$:

$$\lambda_1 = -\frac{1}{2\bar{\sigma}} \sqrt{\pi^\top \Sigma^{-1} \pi + \lambda_2 (\pi^\top \Sigma^{-1} \mathcal{S} - \mathcal{S} (\pi^\top \Sigma^{-1} 1))}$$

$$= -\frac{1}{2\bar{\sigma}} \sqrt{C_{\pi,\pi} - \frac{(C_{1,\pi} \mathcal{S} - C_{s,\pi})^2}{C_{s,s} - 2C_{1,s} \mathcal{S} + C_{1,1} \mathcal{S}^2}}$$
The optimal portfolio is the product of the volatility $\bar{\sigma}$ and the vector $\varrho(\bar{S})$:

$$w^* (\bar{\sigma}, \bar{S}) = -\frac{1}{2\lambda_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))$$

$$= \bar{\sigma} \cdot \varrho(\bar{S})$$

where:

$$\varrho(\bar{S}) = \frac{1}{\lambda'_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))$$

and:

$$\lambda'_1 = \sqrt{C_{\pi,\pi} - \frac{(C_{1,\pi} \bar{S} - C_{s,\pi})^2}{C_{s,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2}}$$
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

Example #5
We consider an investment universe of four assets. Their expected
returns are equal to 6%, 7%, 8% and 10% while their volatilities are
equal to 15%, 20%, 25% and 30%. The correlation matrix of asset
returns is given by the following matrix:


100%
 20% 100%



C=

30%
50% 100%
40%
60%
70% 100%
The risk-free rate is set to 2%. The ESG score vector is
S = (3%, 2%, −2%, −3%).

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The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

• We obtain $C_{1,\pi} = 2.4864$, $C_{s,\pi} = 0.0425$, $C_{s,s} = 0.1274$, $C_{1,s} = 1.9801$, $C_{1,1} = 64.1106$ and $C_{\pi,\pi} = 0.1193$

• If we target $\bar{\sigma} = 20\%$ and $\bar{S} = 1\%$, we deduce that $\lambda_1 = -0.8514$ and $\lambda_2 = -0.1870$

• The optimal portfolio is then:

$$w^* (\bar{\sigma}, \bar{S}) = \begin{pmatrix} 59.31\% \\ 29.52\% \\ 21.76\% \\ 20.72\% \end{pmatrix}$$

• It follows that the portfolio is leveraged since we have $w_r = 1 - 1^T w = -31.31\%$
We verify that \( \sqrt{w^*(\bar{\sigma}, \bar{S})^\top \Sigma w^*(\bar{\sigma}, \bar{S})} = 20\% \) and 
\[
\left( w^*(\bar{\sigma}, \bar{S})^\top S \right) / \left( 1^\top w^*(\bar{\sigma}, \bar{S}) \right) = 1\% 
\]

We also notice that:

\[
\varrho(\bar{S}) = \begin{pmatrix} 2.9657 \\ 1.4759 \\ 1.0881 \\ 1.0358 \end{pmatrix}
\]

and verify that \( w^*(\bar{\sigma}, \bar{S}) = \bar{\sigma} \cdot \varrho(\bar{S}) \)

The portfolio is then leveraged when \( \bar{\sigma} \geq 1 / \left( 1^\top \varrho(\bar{S}) \right) = 17.75\% \).
The Pedersen-Fitzgibbons-Pomorski model
The Sharpe ratio of the optimal portfolio

- We rewrite the first-order condition as:

\[(\ast) \iff \pi + 2\lambda_1 \Sigma w + \lambda_2 (S - \bar{S}1) = 0 \]
\[
\iff w^T \pi + 2\lambda_1 w^T \Sigma w + \lambda_2 w^T (S - \bar{S}1) = 0 \\
\iff w^T \pi + 2\lambda_1 \bar{\sigma}^2 = 0 \\
\iff \lambda_1 = -\frac{1}{2} \frac{w^T \pi}{\bar{\sigma}^2} = -\frac{1}{2} \frac{\text{SR}(w | r)}{\bar{\sigma}}
\]

- We deduce that the Sharpe ratio of the optimal portfolio is:

\[
\text{SR}(w^* (\bar{\sigma}, \bar{S}) | r) = \sqrt{C_{\pi,\pi} - \left( \frac{C_{1,\pi} \bar{S} - C_{s,\pi}}{C_{s,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2} \right)^2} = \text{SR}(\bar{S} | \pi, \Sigma, S)
\]

- It depends on the asset parameters \(\pi, \Sigma, S\), the ESG objective \(\bar{S}\) of the investor, but not the volatility target \(\bar{\sigma}\).
The Pedersen-Fitzgibbons-Pomorski model
The Sharpe ratio of the optimal portfolio

Figure 51: Relationship between $\bar{S}$ and $\text{SR}(\bar{S} \mid \pi, \Sigma)$ (Example #5)
The Pedersen-Fitzgibbons-Pomorski model
The Sharpe ratio of the optimal portfolio

Using Example #5

- The Sharpe ratio of the optimal portfolio \( w^* (20\%, 1\%) \) is equal to 0.3406
- We have 
  \[
  \begin{align*}
  \text{SR} (w^* (\bar{\sigma}, -3\%) \mid r) &= 0.2724, \\
  \text{SR} (w^* (\bar{\sigma}, -2\%) \mid r) &= 0.2875, \\
  \text{SR} (w^* (\bar{\sigma}, -1\%) \mid r) &= 0.3052, \\
  \text{SR} (w^* (\bar{\sigma}, 0\%) \mid r) &= 0.3242, \\
  \text{SR} (w^* (\bar{\sigma}, 1\%) \mid r) &= 0.3406, \\
  \text{SR} (w^* (\bar{\sigma}, 2\%) \mid r) &= 0.3443, \quad \text{and} \quad \text{SR} (w^* (\bar{\sigma}, 3\%) \mid r) &= 0.3221
  \end{align*}
\]
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

- The objective function is equal to:

\[
f (w^* (\bar{\sigma}, \bar{S}) ; \pi, \Sigma, S) = \left( \frac{w^* (\bar{\sigma}, \bar{S})^\top}{\bar{\sigma}} \pi \right) \bar{\sigma} - \frac{\tilde{\gamma}}{2} \bar{\sigma}^2 + \zeta (\bar{S})
\]

- The \( \sigma \)-problem becomes:

\[(\star) = \max_{\bar{\sigma}} \left\{ \max_{w} \{ f (w; \pi, \Sigma, S) \text{ s.t. } w \in \Omega (\bar{\sigma}, \bar{S}) \} \right\}
\]

\[
= \max_{\bar{\sigma}} \left\{ \text{SR} (\bar{S} | \pi, \Sigma, S) \bar{\sigma} - \frac{\tilde{\gamma}}{2} \bar{\sigma}^2 + \zeta (\bar{S}) \right\}
\]

- The first-order condition is \( \text{SR} (\bar{S} | \pi, \Sigma, S) - \tilde{\gamma} \bar{\sigma} = 0 \) or

\[
\bar{\sigma} = \tilde{\gamma}^{-1} \text{SR} (\bar{S} | \pi, \Sigma, S)
\]
Theoretical models
Empirical results
Cost of capital
Modern portfolio theory
ESG risk premium
ESG efficient frontier

The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

- We have:

\[ f \left( w^* \left( \bar{\sigma}, \bar{S} \right); \pi, \Sigma, S \right) = \bar{\gamma}^{-1} \text{SR}^2 \left( \bar{S} \mid \pi, \Sigma, S \right) - \]
\[ \frac{1}{2} \bar{\gamma}^{-1} \text{SR}^2 \left( \bar{S} \mid \pi, \Sigma, S \right) + \zeta \left( \bar{S} \right) \]
\[ = \frac{1}{2} \bar{\gamma}^{-1} \left( \text{SR}^2 \left( \bar{S} \mid \pi, \Sigma, S \right) + 2\bar{\gamma}\zeta \left( \bar{S} \right) \right) \]

- We conclude that the \( S \)-problem becomes:

\[ S^* = \arg \max_{S} \left\{ \text{SR}^2 \left( \bar{S} \mid \pi, \Sigma, S \right) + 2\bar{\gamma}\zeta \left( \bar{S} \right) \right\} \]

- The optimal portfolio is \( w^* = w^* \left( \sigma^*, S^* \right) \) where \( S^* \) is the solution of the \( S \)-problem and \( \sigma^* = \bar{\gamma}^{-1} \text{SR} \left( S^* \mid \pi, \Sigma, S \right) \)
Pedersen et al. (2021) distinguished three groups of investors:

- Type-U or ESG-unaware investors have no ESG preference and do not use the information of ESG scores
- Type-A or ESG-aware investors have no ESG preference, but they use the ESG scores to update their views on the risk premia
- Type-M or ESG-motivated investors have ESG preferences, implying that they would like to have a high ESG score
Type-U investors hold the same portfolio:

$$w_U^* = \frac{\Sigma^{-1} \pi}{1^T \Sigma^{-1} \pi}$$

Type-A investors choose the optimal portfolio with the highest Sharpe ratio ($\zeta(s) = 0$) $\Rightarrow S_A^*$ is the optimal ESG score.

Type-M investors choose an optimal portfolio on the ESG-SR efficient frontier, with:

$$S_M^* \geq S_A^*$$

and:

$$\text{SR} (S_M^* | \pi, \Sigma, S) \leq \text{SR} (S_A^* | \pi, \Sigma, S)$$
The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Figure 52: Optimal portfolio for type-U investors (Example #5)
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

Figure 53: Optimal portfolio for type-A investors (Example #5)
For type-M investors, we first compute the function $\xi(\bar{S})$:

$$\xi(\bar{S}) = SR^2(\bar{S} | \pi, \Sigma, S) + 2\gamma \zeta(\bar{S})$$

The optimal portfolio corresponds to the optimal ESG score that maximizes $\xi(\bar{S})$.
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

Figure 54: Optimal portfolio for type-M investors when $\zeta(s) = s$ (Example #5)
Figure 55: Optimal portfolio for type-M investors when $\zeta(s) = 0.2\sqrt{\max(s, 0)}$
### Table 41: Optimal portfolios (Example #5)

<table>
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<tr>
<th>Statistics</th>
<th>Type-U</th>
<th>Type-A</th>
<th>Type-M ( \zeta (s) = s )</th>
<th>Type-M ( \zeta (s) = 0.2\sqrt{\max(s, 0)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\gamma} )</td>
<td>0.017</td>
<td>0.100</td>
<td>0.023 0.028 0.034</td>
<td>0.021 0.024 0.027</td>
</tr>
<tr>
<td>( S (w^*) )</td>
<td>0.139</td>
<td>0.345</td>
<td>0.682 0.329 0.203</td>
<td>0.687 0.339 0.221</td>
</tr>
<tr>
<td>( \sigma (w^*) )</td>
<td>0.345</td>
<td>0.345</td>
<td>0.341 0.329 0.305</td>
<td>0.343 0.339 0.332</td>
</tr>
<tr>
<td>SR ( (w^* \mid r) )</td>
<td>0.524</td>
<td>0.378</td>
<td>3.028 1.623 1.090</td>
<td>2.900 1.542 1.072</td>
</tr>
<tr>
<td>( w_1^* )</td>
<td>0.289</td>
<td>0.086</td>
<td>1.786 1.009 0.718</td>
<td>1.673 0.919 0.660</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>0.120</td>
<td>0.086</td>
<td>0.383 0.073 −0.056</td>
<td>0.464 0.169 0.065</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>0.067</td>
<td>0.048</td>
<td>−0.012 −0.144 −0.178</td>
<td>0.106 −0.035 −0.079</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>0.000</td>
<td>0.280</td>
<td>−4.184 −1.562 −0.574</td>
<td>−4.143 −1.596 −0.718</td>
</tr>
</tbody>
</table>
The Pedersen-Fitzgibbons-Pomorski model

Impact on asset returns

- If $\omega^U = 1$ and $\omega^A = \omega^M = 0$, then unconditional expected returns are given by the CAPM:
  \[ \mathbb{E}[R_i] - r = \beta_i (\mathbb{E}[R_m] - r) \]
  but conditional expected returns depend on the ESG scores:
  \[ \mathbb{E}[R_i | S] - r = \beta_i (\mathbb{E}[R_m] - r) + \theta \frac{S_i - S_m}{P_i} \]
  where $P_i$ is the asset price of asset $i$

- If $\omega^A = 1$ and $\omega^U = \omega^M = 0$, then the informational value of ESG scores is fully incorporated into asset prices, and we have:
  \[ \mathbb{E}[R_i | S] - r = \tilde{\beta}_i (\mathbb{E}[R_m | S] - r) \]
  where $\tilde{\beta}_i$ is the ESG-adjusted beta coefficient

- If $\omega^M = 1$ and $\omega^U = \omega^A = 0$, then the conditional expected return is given by:
  \[ \mathbb{E}[R_i | S] - r = \tilde{\beta}_i (\mathbb{E}[R_m | S] - r) + \lambda_2 (S_i - S_m) \]
“If all types of investors exist, then several things can happen. If a security has a higher ESG score, then, everything else equal, its expected return can be higher or lower. A higher ESG score increases the demand for the stock from type-M investors, leading to a higher price and, therefore, a lower required return […] Companies with poor ESG scores that are down-weighted by type-M investors will have lower prices and higher cost of capital. […] Furthermore, the force that can increase the expected return is that the higher ESG could be a favorable signal of firm fundamentals, and if many type-U investors ignore this, the fundamental signal perhaps would not be fully reflected in the price […] A future increase in ESG investing would lead to higher prices for high-ESG stocks […]. If these flows are unexpected (or not fully captured in the price for other reasons), then high-ESG stocks would experience a return boost during the period of this repricing of ESG. If these flows are expected, then expected returns should not be affected.” (Pedersen et al., 2021)
According to Coqueret (2022), we can classify the academic studies into four categories:

1. ESG improves performance
2. ESG does not impact performance
3. ESG is financially detrimental
4. The relationship between ESG and performance depends on many factors
What is the performance of ESG investing?

According to Friede et al. (2015), the first category dominates the other categories:

“[…] The results show that the business case for ESG investing is empirically very well founded. Roughly 90% of studies find a nonnegative ESG – CFP relation. More importantly, the large majority of studies reports positive findings. We highlight that the positive ESG impact on CFP appears stable over time. Promising results are obtained when differentiating for portfolio and non-portfolio studies, regions, and young asset classes for ESG investing such as emerging markets, corporate bonds, and green real estate.”

⇒ Many dimensions of CFP (cost of capital, G pillar, proxy variables, etc.)
We can also find many studies, whose conclusion is more neutral or negative: Barnett and Salomon (2006), Fabozzi et al. (2008), Hong and Kacperczyk (2009), Johnson et al. (2009), Capelle-Blancard and Monjon (2014), Matos (2020), etc.

⇒ Sin stocks

Mixed results
What is the performance of ESG investing?

- Generally, academic studies that analyze the relationship between ESG and performance are based on long-term historical data, typically the last 20 years or the last 30 years.

- Two issues:
  1. ESG investing was marginal 15+ years ago
  2. ESG data are not robust or relevant before 2010

- The relationship between ESG and performance is dynamic

- Sometimes, ESG may create performance, but sometimes not
Sorted-portolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date $t$, we rank the stocks according to their Amundi ESG $z$-score $s_{i,t}$
- We form the five quintile portfolios $Q_i$ for $i = 1, \ldots, 5$
- The portfolio $Q_i$ is invested during the period $]t, t+1[$:
  - $Q_1$ corresponds to the best-in-class portfolio (best scores)
  - $Q_5$ corresponds to the worst-in-class portfolio (worst scores)
- Quarterly rebalancing
- Universe: MSCI World Index
- Equally-weighted and sector-neutral portfolio (and region-neutral for the world universe)
Table 42: An illustrative example

<table>
<thead>
<tr>
<th>Asset</th>
<th>$S_i$</th>
<th>Rank</th>
<th>$Q_i$</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>-0.3</td>
<td>6</td>
<td>$Q_3$</td>
<td>+50%</td>
</tr>
<tr>
<td>#2</td>
<td>0.2</td>
<td>5</td>
<td>$Q_3$</td>
<td>+50%</td>
</tr>
<tr>
<td>#3</td>
<td>-1.0</td>
<td>7</td>
<td>$Q_4$</td>
<td>+50%</td>
</tr>
<tr>
<td>#4</td>
<td>1.5</td>
<td>3</td>
<td>$Q_2$</td>
<td>+50%</td>
</tr>
<tr>
<td>#5</td>
<td>-2.9</td>
<td>10</td>
<td>$Q_5$</td>
<td>+50%</td>
</tr>
<tr>
<td>#6</td>
<td>0.8</td>
<td>4</td>
<td>$Q_2$</td>
<td>+50%</td>
</tr>
<tr>
<td>#7</td>
<td>-1.4</td>
<td>8</td>
<td>$Q_4$</td>
<td>+50%</td>
</tr>
<tr>
<td>#8</td>
<td>2.3</td>
<td>2</td>
<td>$Q_1$</td>
<td>+50%</td>
</tr>
<tr>
<td>#9</td>
<td>2.8</td>
<td>1</td>
<td>$Q_1$</td>
<td>+50%</td>
</tr>
<tr>
<td>#10</td>
<td>-2.2</td>
<td>9</td>
<td>$Q_5$</td>
<td>+50%</td>
</tr>
</tbody>
</table>
Figure 56: Annualized return of ESG-sorted portfolios (MSCI North America)

Figure 57: Annualized return of ESG-sorted portfolios (MSCI North America)

**Figure 58: Annualized return of ESG-sorted portfolios (MSCI EMU)**

Figure 59: Annualized return of ESG-sorted portfolios (MSCI EMU)

**Table 43: Impact of ESG screening on sorted portfolio returns (2010 – 2017)**

<table>
<thead>
<tr>
<th>Period</th>
<th>Pillar</th>
<th>North America</th>
<th>EMU</th>
<th>Europe-ex-EMU</th>
<th>Japan</th>
<th>World</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010 – 2013</td>
<td>ESG</td>
<td>++</td>
<td>++</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>++</td>
<td>0</td>
<td>+</td>
<td>−</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>−</td>
<td>0</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>2014 – 2017</td>
<td>ESG</td>
<td>++</td>
<td>++</td>
<td>0</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>++</td>
<td>0</td>
<td>−</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+</td>
<td>++</td>
<td>0</td>
<td>+</td>
<td>++</td>
</tr>
</tbody>
</table>

Figure 60: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI North America)

Figure 61: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI EMU)

The impact of investment flows

- The 2014 break
  - Strong mobilization of the largest institutional European investors: NBIM, APG, PGGM, ERAFP, FRR, etc.
  - They are massively invested in European stocks and America stocks: NBIM $>$ CalPERS + CalSTRS + NYSCRF for U.S. stocks

- The 2018-2019 period
  - Implication of U.S. investors continues to be weak
  - Strong mobilization of medium (or tier two) institutional European investors, that have a low exposure on American stocks
  - Mobilization of European investors is not sufficient
**Figure 62:** The monotonous assumption of the ESG-performance relationship

- **(a) Return-based**
- **(b) Risk-based**
- **(c) Skewed-return**
- **(d) Skewed-risk**
Simulated results
Sorted portfolios

Figure 63: How to play ESG momentum?

- Above average
- Below average
- Exclusion zone

Q1 vs. Q2: less interesting
Q4 vs. Q5: more interesting
We note $b$ the benchmark, $\mathbf{S}$ the vector of ESG scores and $\Sigma$ the covariance matrix.

We consider the following optimization problem:

$$w^*(\gamma) = \arg \min_{w} \frac{1}{2} \sigma^2(w \mid b) - \gamma \mathbf{S}(w \mid b)$$

where $\sigma^2(w \mid b) = (w - b)^\top \Sigma (w - b)$ and $\mathbf{S}(w \mid b)$ are the ex-ante tracking error variance and the ESG excess score of portfolio $w$ with respect to the benchmark $b$.

Since we have:

$$\mathbf{S}(w \mid b) = (w - b)^\top \mathbf{S} = \mathbf{S}(w) - \mathbf{S}(b)$$

we obtain the following optimization function:

$$w^*(\gamma) = \arg \min_{w} \frac{1}{2} w^\top \Sigma w - w^\top (\gamma \mathbf{S} + \Sigma b)$$

The QP form is given by $Q = \Sigma$ and $R = \gamma \mathbf{S} + \Sigma b$. 
Figure 64: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

Figure 65: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, individual pillars)

Source: Bennani et al. (2018).
Figure 66: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

Figure 67: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2013, individual pillars)

Figure 68: Annualized excess return of ESG-optimized portfolios (MSCI World, 2014-2017, individual pillars)

Figure 69: Annualized excess return in bps of ESG-optimized portfolios (MSCI North America and EMU, 2010-2017)

The single-factor model is:

\[ R_{i,t} = \alpha_{i,j} + \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \]

where:
- \( R_{i,t} \) is the return of stock \( i \) at time \( t \)
- \( F_{j,t} \) is the value of the \( j^{\text{th}} \) common risk factor at time \( t \) (market, size, value, momentum, low-volatility, quality or ESG)
- \( \varepsilon_{i,t} \) is the idiosyncratic risk

The average proportion of the return variance explained by the common factor is given by:

\[ \bar{R}^2_j = \frac{1}{n} \sum_{i=1}^{n} R^2_{i,j} = \frac{1}{n} \sum_{i=1}^{n} \left( 1 - \frac{\text{var} (\varepsilon_{i,t})}{\text{var} (R_{i,t})} \right) \]
Table 44: Results of cross-section regression with long-only risk factors (single-factor linear regression model, average $R^2$)

<table>
<thead>
<tr>
<th>Factor</th>
<th>North America</th>
<th></th>
<th></th>
<th>Eurozone</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>40.8%</td>
<td>28.6%</td>
<td></td>
<td>42.8%</td>
<td>36.3%</td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>39.3%</td>
<td>26.1%</td>
<td></td>
<td>37.1%</td>
<td>23.3%</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>38.9%</td>
<td>26.7%</td>
<td></td>
<td>41.6%</td>
<td>33.6%</td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>39.6%</td>
<td>26.3%</td>
<td></td>
<td>40.8%</td>
<td>34.1%</td>
<td></td>
</tr>
<tr>
<td>Low-volatility</td>
<td>35.8%</td>
<td>25.1%</td>
<td></td>
<td>38.7%</td>
<td>33.4%</td>
<td></td>
</tr>
<tr>
<td>Quality</td>
<td>39.1%</td>
<td>26.6%</td>
<td></td>
<td>42.4%</td>
<td>34.6%</td>
<td></td>
</tr>
<tr>
<td>ESG</td>
<td>40.1%</td>
<td>27.4%</td>
<td></td>
<td>42.6%</td>
<td>35.3%</td>
<td></td>
</tr>
</tbody>
</table>

Source: Roncalli (2020).

- Specific risk has increased during the period 2014 – 2019
- Since 2014, we find that:
  - ESG $\succ$ Value $\succ$ Quality $\succ$ Momentum $\succ$ ... (North America)
  - ESG $\succ$ Quality $\succ$ Momentum $\succ$ Value $\succ$ ... (Eurozone)
Regression model

We have:

\[ R_{i,t} = \alpha_i + \sum_{j=1}^{m} \beta_{i,j}\mathcal{F}_{j,t} + \varepsilon_{i,t} \]

where \( m \) is the number of risk factors

- 1F = market
- 5F = size + value + momentum + low-volatility + quality
- 6F = 5F + ESG
### Multi-factor model

**Table 45:** Results of cross-section regression with long-only risk factors (multi-factor linear regression model, average $R^2$)

<table>
<thead>
<tr>
<th>Model</th>
<th>North America</th>
<th></th>
<th>Eurozone</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>40.8%</td>
<td>28.6%</td>
<td>42.8%</td>
<td>36.3%</td>
</tr>
<tr>
<td>5F model</td>
<td>46.1%</td>
<td>38.4%</td>
<td>49.5%</td>
<td>45.0%</td>
</tr>
<tr>
<td>6F model (5F + ESG)</td>
<td>46.7%</td>
<td>39.7%</td>
<td>50.1%</td>
<td>45.8%</td>
</tr>
</tbody>
</table>

Source: Roncalli (2020).

***p-value statistic for the MSCI Index (time-series, 2014 – 2019):***

- 6F = **Size**, Value, Momentum, Low-volatility, Quality, **ESG** (North America)
- 6F = **Size**, Value, Momentum, **Low-volatility**, Quality, ESG (Eurozone)
We use a lasso penalized regression in place of the traditional least squares regression:

\[
\{\hat{\alpha}_i, \hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,m}\} = \arg\min \left\{ \frac{1}{2} \text{var} (\varepsilon_{i,t}) + \lambda \| \beta_i \|_1 \right\}
\]

- Low-factor intensity ($\lambda \approx \infty$) $\Rightarrow$ we determine which risk factor is the most important
- When the factor intensity reaches 100% ($\lambda = 0$), we obtain the same results calculated previously with the linear regression
Factor selection

Figure 70: Factor picking (MSCI North America, 2014-2019, global score)

Source: Roncalli (2020).
Figure 71: Factor picking (MSCI EMU, 2014-2019, global score)

Source: Roncalli (2020).
What is the difference between alpha and beta?

α or β?

“[…] When an alpha strategy is massively invested, it has an enough impact on the structure of asset prices to become a risk factor.

[…] Indeed, an alpha strategy becomes a common market risk factor once it represents a significant part of investment portfolios and explains the cross-section dispersion of asset returns” (Roncalli, 2020)

- ESG remains an alpha strategy in North America
- ESG becomes a beta strategy (or a risk factor) in Europe
- Forward looking, ESG will be a beta strategy in North America
Table 46: Performance of ESG indexes (MSCI World, 2010 – 2022)

<table>
<thead>
<tr>
<th>Year</th>
<th>CW</th>
<th>ESG</th>
<th>SRI</th>
<th>Alpha (in bps)</th>
<th>CW</th>
<th>ESG</th>
<th>SRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>11.8</td>
<td>10.7</td>
<td>10.6</td>
<td>109</td>
<td>11</td>
<td>-114</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>-5.5</td>
<td>-5.4</td>
<td>-5.5</td>
<td>12</td>
<td>-19</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>15.8</td>
<td>14.5</td>
<td>13.2</td>
<td>-135</td>
<td>258</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>26.7</td>
<td>27.6</td>
<td>27.4</td>
<td>89</td>
<td>71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>4.9</td>
<td>4.9</td>
<td>3.9</td>
<td>-6</td>
<td>-102</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>-0.9</td>
<td>-1.1</td>
<td>-1.6</td>
<td>-23</td>
<td>-71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>7.5</td>
<td>7.3</td>
<td>7.7</td>
<td>-26</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td>22.4</td>
<td>21.0</td>
<td>23.6</td>
<td>-142</td>
<td>124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>-8.7</td>
<td>-7.8</td>
<td>-6.7</td>
<td>94</td>
<td>199</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2019</td>
<td>27.7</td>
<td>28.2</td>
<td>29.8</td>
<td>48</td>
<td>209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>15.9</td>
<td>15.3</td>
<td>19.9</td>
<td>-61</td>
<td>396</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2021</td>
<td>21.8</td>
<td>24.7</td>
<td>27.0</td>
<td>288</td>
<td>523</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>-18.1</td>
<td>-19.6</td>
<td>-22.5</td>
<td>-143</td>
<td>-436</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3Y: 4.9 5.0 5.7 2 73
5Y: 6.1 6.4 7.4 31 125
7Y: 8.5 8.5 9.6 1 110
10Y: 8.9 8.9 9.5 5 64

Source: MSCI, Factset & Author’s calculation.
Figure 72: Alpha return of several ESG equity indexes (in bps)

Source: MSCI, Factset & Author's calculation.
Bond markets ≠ stock markets

**Stocks**
- ESG scoring is incorporated in portfolio management
- ESG = long-term business risk ⇒ strongly impacts the equity
- Portfolio integration
- Managing the business risk

**Bonds**
- ESG integration is generally limited to exclusions
- ESG lowly impacts the debt
- Portfolio completion
- Fixed income = impact investing
- Development of pure play ESG securities (green and social bonds)

⇒ Stock holders are more ESG sensitive than bond holders because of the capital structure
ESG investment flows affect asset pricing differently

- Impact on carry (coupon effect)?
- Impact on price dynamics (credit spread/mark-to-market effect)?
- Buy-and-hold portfolios ≠ managed portfolios

The distinction between IG and HY bonds

- ESG and credit ratings are correlated
- There are more worst-in-class issuers in the HY universe, and best-in-class issuers in the IG universe
- Non-neutrality of the bond universe (bonds ≠ stocks)

Bond markets ≠ stock markets
Bond markets ≠ stock markets

Figure 73: Probability density function of ESG scores

- The average $z$-score for IG bonds is positive
- The average $z$-score for HY bonds is negative

Source: Ben Slimane et al. (2019).
Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date \( t \), we rank the bonds according to their Amundi ESG \( z \)-score
- We form the five quintile portfolios \( Q_i \) for \( i = 1, \ldots, 5 \)
- The portfolio \( Q_i \) is invested during the period \( [t, t + 1] \):
  - \( Q_1 \) corresponds to the best-in-class portfolio (best scores)
  - \( Q_5 \) corresponds to the worst-in-class portfolio (worst scores)
- Monthly rebalancing
- Universe: ICE (BofAML) Large Cap IG EUR Corporate Bond
- Sector-weighted and sector-neutral portfolio
- Within a sector, bonds are equally-weighted
Simulated results
Sorted portfolios

**Figure 74**: Annualized return in bps of the long short $Q_1 - Q_5$ strategy (IG, 2010 – 2019)

Source: Ben Slimane *et al.* (2019).
Table 47: Carry statistics (in bps)

<table>
<thead>
<tr>
<th>Period</th>
<th>Q1</th>
<th>Q5</th>
<th>Q1 − Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010 – 2013</td>
<td>175</td>
<td>192</td>
<td>−17</td>
</tr>
<tr>
<td>2014 – 2019</td>
<td>113</td>
<td>128</td>
<td>−15</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2019).
Figure 75: Annualized credit return in bps of ESG sorted portfolios (EUR IG, 2010 – 2019)

Source: Ben Slimane et al. (2019).
Simulated results

Optimized portfolios

- Portfolio \( w = (w_1, \ldots, w_n) \) and benchmark \( b = (b_1, \ldots, b_n) \)
- ESG score of the portfolio:

\[
S(w) = \sum_{i=1}^{n} w_i S_i
\]

- ESG excess score of portfolio \( w \) with respect to benchmark \( b \):

\[
S(w \mid b) = \sum_{i=1}^{n} (w_i - b_i) S_i = S(w) - S(b)
\]

- \( z \)-scores \( \Rightarrow S(w \mid b) > 0 \)
- Active or tracking risk \( R(w \mid b) \)
- The optimization problem becomes:

\[
w^*(\gamma) = \arg\min R(w \mid b) - \gamma S(w \mid b)
\]
Simulated results
Optimized portfolios

- The modified duration risk of portfolio \( w \) with respect to benchmark \( b \) is:

\[
\mathcal{R}_{MD} (x \mid b) = \sum_{j=1}^{n_S} \left( \left( \sum_{i \in \text{Sector}(j)} w_i \text{MD}_i \right) - \left( \sum_{i \in \text{Sector}(j)} b_i \text{MD}_i \right) \right)^2
\]

where \( n_S \) is the number of sectors and \( \text{MD}_i \) is the modified duration of bond \( i \)

- An alternative is to use the DTS risk measure:

\[
\mathcal{R}_{DTS} (x \mid b) = \sum_{j=1}^{n_S} \left( \left( \sum_{i \in \text{Sector}(j)} w_i \text{DTS}_i \right) - \left( \sum_{i \in \text{Sector}(j)} b_i \text{DTS}_i \right) \right)^2
\]

where \( \text{DTS}_i \) is the DTS of bond \( i \)

- Hybrid approach:

\[
\mathcal{R} (w \mid b) = \frac{1}{2} \mathcal{R}_{MD} (w \mid b) + \frac{1}{2} \mathcal{R}_{DTS} (w \mid b)
\]
Simulated results
Optimized portfolios

Figure 76: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2010 – 2013)

Source: Ben Slimane et al. (2019).
Figure 77: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2014 – 2016)

Source: Ben Slimane et al. (2019).
Figure 78: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2010 – 2013)

Source: Ben Slimane et al. (2019).
Figure 79: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2014 – 2016)

Source: Ben Slimane et al. (2019).
Table 48: Performance of ESG bond indexes (sovereign)

<table>
<thead>
<tr>
<th>Year</th>
<th>FTSE WGBI</th>
<th></th>
<th></th>
<th>FTSE EGBI</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Return</td>
<td>Alpha</td>
<td>Return</td>
<td>Alpha</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BM ESG</td>
<td>ESG</td>
<td>BM ESG</td>
<td>ESG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>4.61 4.31</td>
<td>−30</td>
<td>0.61 4.14</td>
<td>353</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>6.35 7.05</td>
<td>69</td>
<td>3.41 7.31</td>
<td>391</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>1.65 3.06</td>
<td>141</td>
<td>10.65 7.39</td>
<td>−326</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>−4.00 −2.95</td>
<td>105</td>
<td>2.21 −1.40</td>
<td>−362</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>−0.48 −0.22</td>
<td>26</td>
<td>13.19 11.44</td>
<td>−175</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>−3.57 −4.85</td>
<td>−128</td>
<td>1.65 0.39</td>
<td>−126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>1.60 1.02</td>
<td>−59</td>
<td>3.20 4.00</td>
<td>81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td>7.49 8.16</td>
<td>67</td>
<td>0.15 −0.47</td>
<td>−62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>−0.84 −1.41</td>
<td>−57</td>
<td>0.88 1.65</td>
<td>78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2019</td>
<td>5.90 5.56</td>
<td>−34</td>
<td>6.72 4.45</td>
<td>−227</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>10.11 10.90</td>
<td>79</td>
<td>5.03 4.11</td>
<td>−92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2021</td>
<td>−6.97 −7.15</td>
<td>−17</td>
<td>−3.54 −3.76</td>
<td>−21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>−18.26 −20.00</td>
<td>−173</td>
<td>−18.52 −19.06</td>
<td>−54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3Y</td>
<td>−5.75 −6.26</td>
<td>−51</td>
<td>−6.19 −6.74</td>
<td>−55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5Y</td>
<td>−2.54 −3.03</td>
<td>−49</td>
<td>−2.33 −2.95</td>
<td>−61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7Y</td>
<td>−0.58 −0.93</td>
<td>−35</td>
<td>−1.21 −1.63</td>
<td>−42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10Y</td>
<td>−1.22 −1.46</td>
<td>−24</td>
<td>0.77 −0.17</td>
<td>−94</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 49: Performance of ESG bond indexes (corporates)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bloomberg Euro Aggregate Corporate Return</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BM</td>
<td>SRI</td>
</tr>
<tr>
<td>2010</td>
<td>3.07</td>
<td>2.93</td>
</tr>
<tr>
<td>2011</td>
<td>1.49</td>
<td>1.17</td>
</tr>
<tr>
<td>2012</td>
<td>13.59</td>
<td>13.99</td>
</tr>
<tr>
<td>2013</td>
<td>2.37</td>
<td>2.49</td>
</tr>
<tr>
<td>2014</td>
<td>8.40</td>
<td>8.31</td>
</tr>
<tr>
<td>2015</td>
<td>−0.56</td>
<td>−0.59</td>
</tr>
<tr>
<td>2016</td>
<td>4.73</td>
<td>4.60</td>
</tr>
<tr>
<td>2017</td>
<td>2.41</td>
<td>2.47</td>
</tr>
<tr>
<td>2018</td>
<td>−1.25</td>
<td>−1.12</td>
</tr>
<tr>
<td>2019</td>
<td>6.24</td>
<td>6.01</td>
</tr>
<tr>
<td>2020</td>
<td>2.77</td>
<td>2.69</td>
</tr>
<tr>
<td>2021</td>
<td>−0.97</td>
<td>−0.96</td>
</tr>
<tr>
<td>2022</td>
<td>−13.65</td>
<td>−13.62</td>
</tr>
</tbody>
</table>

**3Y**: −4.21 | −4.22 | −4.18 | −4.23 | −1 | 3 | −2

**5Y**: −1.61 | −1.63 | −1.62 | −1.64 | −2 | −1 | −3

**7Y**: −0.16 | −0.19 | −0.20 | −0.19 | −3 | −4 | −3

**10Y**: 0.88 | 0.86 | 0.86 |  | | −2 | −1
Table 50: Performance of ESG bond indexes (corporates)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bloomberg US Corporate Return</th>
<th>Alpha</th>
<th>Bloomberg Global High Yield Return</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BM</td>
<td>SRI</td>
<td>S-SRI</td>
<td>ESG-S</td>
</tr>
<tr>
<td>2019</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2021</td>
<td>-1.04</td>
<td>-1.55</td>
<td>9.56</td>
<td>2.34</td>
</tr>
<tr>
<td>2022</td>
<td>-15.76</td>
<td>-15.12</td>
<td>-1.10</td>
<td>-13.86</td>
</tr>
</tbody>
</table>
Definition
<table>
<thead>
<tr>
<th>Theoretical models</th>
<th>Empirical results</th>
<th>Cost of capital</th>
<th>Equities</th>
<th>Corporate bonds</th>
<th>Sovereign bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equities
Correlation between Credit ratings and ESG ratings

Figure 80: Average ESG $z$-score with respect to the credit rating (2010 – 2019)
An integrated Credit-ESG model

We consider the following regression model:

\[
\ln \text{OAS}_{i,t} = \alpha_t + \beta_{\text{esg}} \cdot S_{i,t} + \beta_{\text{md}} \cdot \text{MD}_{i,t} + \sum_{j=1}^{N_{\text{Sector}}} \beta_{\text{Sector}}(j) \cdot \text{Sector}_{i,t}(j) + \\
\beta_{\text{sub}} \cdot \text{SUB}_{i,t} + \sum_{k=1}^{N_{\text{Rating}}} \beta_{\text{Rating}}(k) \cdot \text{Rating}_{i,t}(k) + \varepsilon_{i,t}
\]

where:

- \( S_{i,t} \) is the ESG z-score of Bond \( i \) at time \( t \)
- \( \text{SUB}_{i,t} \) is a dummy variable accounting for subordination of the bond
- \( \text{MD}_{i,t} \) is the modified duration
- \( \text{Sector}_{i,t}(j) \) is a dummy variable for the \( j^{\text{th}} \) sector
- \( \text{Rating}_{i,t}(k) \) is a dummy variable for the \( k^{\text{th}} \) rating
An integrated Credit-ESG model

Table 51: Results of the panel data regression model (EUR IG, 2010 – 2019)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ESG</td>
<td>E</td>
<td>S</td>
<td>G</td>
</tr>
<tr>
<td>$R^2$</td>
<td>60.0%</td>
<td>59.4%</td>
<td>59.5%</td>
<td>60.3%</td>
</tr>
<tr>
<td>Excess $R^2$</td>
<td>0.6%</td>
<td>0.0%</td>
<td>0.2%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\hat{\beta}_{esg}$</td>
<td>-0.05</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>t-statistic</td>
<td>-32</td>
<td>-7</td>
<td>-16</td>
<td>-39</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2020)

The assumption $H_0 : \beta_{esg} < 0$ is not rejected
**ESG cost of capital with min/max score bounds**

We calculate the difference between:

1. the funding cost of **the worst-in-class issuer** and
2. the funding cost of **the best-in-class issuer**

by assuming that:

- the two issuers have the same credit rating;
- the two issuers belong to the same sector;
- the two issuers have the same capital structure;
- the two issuers have the same debt maturity.

⇒ Two approaches:

1. **Theoretical approach**: ESG scores are set to $-3$ and $+3$ (not realistic)
2. **Empirical approach**: ESG scores are set to observed min/max score bounds (e.g. min/max = $-2.0/+1.9$ for Consumer Cyclical A-rated EUR, $-2.1/+3.2$ for Banking A-rated EUR, etc.)
### Table 52: ESG cost of capital (IG, 2014 – 2019)

<table>
<thead>
<tr>
<th>Sector</th>
<th>EUR</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AA</td>
<td>A</td>
</tr>
<tr>
<td>Banking</td>
<td>23</td>
<td>45</td>
</tr>
<tr>
<td>Basic</td>
<td>9</td>
<td>25</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>Communication</td>
<td>26</td>
<td>48</td>
</tr>
<tr>
<td>Consumer Cyclical</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>Consumer Non-Cyclical</td>
<td>15</td>
<td>29</td>
</tr>
<tr>
<td>Utility &amp; Energy</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>Average</td>
<td>12</td>
<td>31</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2020)
Motivation

- Financial analysis **versus/and** extra-financial analysis
- Sovereign risk ≠ Corporate risk
- Which ESG metrics are priced and not priced in by the market?
- What is the nexus between ESG analysis and credit analysis?
A Tale of Two Countries

- The example of Barbados and Jamaica
- Why the economic growth of two countries with the same economic development at time $t$ is different 10, 20 or 30 years later?
### Sovereign ESG themes

#### Environmental
- Biodiversity
- Climate change
- Commitment to environmental standards
- Energy mix
- Natural hazard
- Natural hazard outcome
- Non-renewable energy resources
- **Temperature**
- Water management

#### Social
- Civil unrest
- Demographics
- **Education**
- Gender
- Health
- Human rights
- **Income**
- Labour market standards
- Migration
- Water and electricity access

#### Governance
- Business environment and R&D
- **Governance effectiveness**
- Infrastructure and mobility
- International relations
- Justice
- **National security**
- Political stability

---
The economics of sovereign risk

Assessment of a country’s creditworthiness

- Confidence in the country? Only financial reasons?
- Country default risk cannot be summarized by only financial figures!
- Why some rich countries have to pay a credit risk premium?
- How to explain the large differences in Asia?
Single-factor analysis

Data

Endogenous variable
10Y sovereign bond yield

Explanatory variables
- 269 ESG variables grouped into 26 ESG thematics
- 183 indicators come from Verisk Maplecrof database, the 86 remaining metrics were retrieved from the World Bank, ILO, WHO, FAO, UN...
- 6 control variables: GDP Growth, Net Debt, Reserves, Account Balance, Inflation and Credit Rating

Panel dimensions
- 67 countries
- 2015–2020
Let $s_{i,t}$ be the bond yield spread of the country $i$ at time $t$. We consider the following regression model estimated by OLS:

$$s_{i,t} = \alpha + \beta x_{i,t} + \sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} + \epsilon_{i,t}$$

where $x_{i,t}$ is the ESG metric, $\gamma_k$ are coefficients, and $\epsilon_{i,t}$ is the error term.

and:

$$\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} = \gamma_1 g_{i,t} + \gamma_2 \pi_{i,t} + \gamma_3 d_{i,t} + \gamma_4 ca_{i,t} + \gamma_5 r_{i,t} + \gamma_6 R_{i,t}$$

where $g_{i,t}$ is the economic growth, $\pi_{i,t}$ is the inflation, $d_{i,t}$ is the debt ratio, $ca_{i,t}$ is the current account balance, $r_{i,t}$ is the reserve adequacy and $R_{i,t}$ is the credit rating.
### Table 53: 7 most relevant indicators of the single-factor analysis per pillar

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Thematic</th>
<th>Indicator</th>
<th>$\Delta R^2_c$</th>
<th>$F$-test</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate change</td>
<td></td>
<td>Climate change vulnerability (acute)</td>
<td>5.51%</td>
<td>57.19</td>
<td>1</td>
</tr>
<tr>
<td>Climate change</td>
<td></td>
<td>Climate change exposure (extreme)</td>
<td>4.80%</td>
<td>48.60</td>
<td>2</td>
</tr>
<tr>
<td>Water management</td>
<td></td>
<td>Agricultural water withdrawal</td>
<td>4.02%</td>
<td>47.10</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Climate change</td>
<td>Climate change sensitivity (acute)</td>
<td>3.95%</td>
<td>38.79</td>
<td>4</td>
</tr>
<tr>
<td>Biodiversity</td>
<td></td>
<td>Biodiversity threatening score</td>
<td>3.53%</td>
<td>35.32</td>
<td>5</td>
</tr>
<tr>
<td>Climate change</td>
<td></td>
<td>Climate change exposure (acute)</td>
<td>3.39%</td>
<td>32.95</td>
<td>6</td>
</tr>
<tr>
<td>Climate change</td>
<td></td>
<td>Climate change vulnerability (average)</td>
<td>3.11%</td>
<td>31.16</td>
<td>7</td>
</tr>
<tr>
<td>Human rights</td>
<td></td>
<td>Freedom of assembly</td>
<td>8.74%</td>
<td>89.58</td>
<td>1</td>
</tr>
<tr>
<td>Human rights</td>
<td></td>
<td>Extent of arbitrary unrest</td>
<td>8.04%</td>
<td>80.10</td>
<td>2</td>
</tr>
<tr>
<td>Human rights</td>
<td></td>
<td>Extent of torture and ill treatment</td>
<td>7.63%</td>
<td>75.48</td>
<td>3</td>
</tr>
<tr>
<td>S</td>
<td>Labour market standards</td>
<td>Severity of working time violations</td>
<td>7.21%</td>
<td>70.46</td>
<td>4</td>
</tr>
<tr>
<td>Labour market standards</td>
<td></td>
<td>Forced labour violations (extent)</td>
<td>6.10%</td>
<td>54.40</td>
<td>5</td>
</tr>
<tr>
<td>Labour market standards</td>
<td></td>
<td>Child labour (extent)</td>
<td>5.83%</td>
<td>54.68</td>
<td>6</td>
</tr>
<tr>
<td>Migration</td>
<td></td>
<td>Vulnerability of migrant workers</td>
<td>5.83%</td>
<td>53.76</td>
<td>7</td>
</tr>
<tr>
<td>National security</td>
<td></td>
<td>Severity of kidnappings</td>
<td>6.80%</td>
<td>64.49</td>
<td>1</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td></td>
<td>Ease of access to loans</td>
<td>6.77%</td>
<td>73.57</td>
<td>2</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td></td>
<td>Roads km</td>
<td>6.45%</td>
<td>63.66</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Business environment and R&amp;D</td>
<td>Capacity for innovation</td>
<td>5.65%</td>
<td>58.58</td>
<td>4</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td></td>
<td>Ethical behaviour of firms</td>
<td>5.37%</td>
<td>55.14</td>
<td>5</td>
</tr>
<tr>
<td>National security</td>
<td></td>
<td>Frequency of kidnappings</td>
<td>5.27%</td>
<td>48.49</td>
<td>6</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td></td>
<td>Physical connectivity</td>
<td>4.94%</td>
<td>50.76</td>
<td>7</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
### Table 54: Summary of the results

<table>
<thead>
<tr>
<th>Relevant</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature</strong></td>
<td>Climate change</td>
<td>Labour market standards</td>
<td>Infrastructure and mobility</td>
</tr>
<tr>
<td>Natural hazard outcome</td>
<td></td>
<td>Human rights</td>
<td>National security</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Migration</td>
<td>Justice</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Less relevant</th>
<th>Income</th>
<th>Education</th>
<th>Political stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water management</td>
<td>Water and electricity access</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy mix</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thierry Roncalli Course 2022-2023 in Sustainable Finance 343 / 1114
We consider the following multi-factor regression model:

$$s_{i,t} = \alpha + \sum_{j=1}^{m} \beta_{j} x_{i,t}^{(j)} + \sum_{k=1}^{6} \gamma_{k} z_{i,t}^{(k)} + \varepsilon_{i,t}$$

A 4-step process

1. We consider the significant variables of the single-factor analysis at the 1% level
2. We filter the variables selected at Step 1 in order to eliminate redundant variables in each ESG theme
3. We perform a lasso regression to retain the seven most relevant variables within each ESG pillar
4. We perform a multi-factor analysis ($m = 21 \Rightarrow m = 7$)
Multi-factor analysis
The collinearity issue

Table 55: Example of variables exhibiting high correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta R_c^2$</th>
<th>Correlation$_{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate change exposure (average)</td>
<td>2.12%</td>
<td>1.00 0.74 0.80 0.48 0.92 0.77</td>
</tr>
<tr>
<td>Climate change exposure (acute)</td>
<td>3.89%</td>
<td>0.74 1.00 0.65 0.51 0.73 0.89</td>
</tr>
<tr>
<td>Climate change exposure (extreme)</td>
<td>4.80%</td>
<td>0.80 0.65 1.00 0.54 0.79 0.71</td>
</tr>
<tr>
<td>Climate change sensitivity (average)</td>
<td>3.95%</td>
<td>0.48 0.51 0.54 1.00 0.76 0.81</td>
</tr>
<tr>
<td>Climate change vulnerability (average)</td>
<td>3.11%</td>
<td>0.92 0.73 0.79 0.76 1.00 0.89</td>
</tr>
<tr>
<td>Climate change vulnerability (acute)</td>
<td>5.51%</td>
<td>0.77 0.89 0.71 0.81 0.89 1.00</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)

Selecting the variables

1. For each variable, we identify the highest pairwise correlation
2. Among each couple, we retain the variable showing the highest $\Delta R_c^2$
3. Among these variables, we select the variable with the lowest correlation
Multi-factor analysis
The collinearity issue

Figure 81: Filtering process

Original dataset
Significant variables
Redundant variables
Lasso by pillar
Multi-factor estimation

269
184
74
21

Source: Semet et al. (2021)
### Table 56: Results after Step 3: Lasso regression pillar by pillar

<table>
<thead>
<tr>
<th>Rank</th>
<th>Pillar</th>
<th>Thematic</th>
<th>Variable</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Non-renewable energy resources</td>
<td>Total GHG emissions</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Biodiversity</td>
<td>Biodiversity threatening score</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Natural hazard</td>
<td>Severe storm hazard (absolute high extreme)</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Temperature</td>
<td>Temperature change</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Non-renewable energy resources</td>
<td>Fossil fuel intensity of the economy</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Natural hazard</td>
<td>Drought hazard (absolute high extreme)</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Commitment to environmental standards</td>
<td>Paris Agreement</td>
<td>E</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rank</th>
<th>Pillar</th>
<th>Thematic</th>
<th>Variable</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Migration</td>
<td>Vulnerability of migrant workers</td>
<td>S</td>
<td></td>
</tr>
</tbody>
</table>
| 2    | Demographics | Projected population change (5 years) | S | +
| 3    | Civil unrest | Frequency of civil unrest incidents | S | 
| 4    | Labor market standards | Index of labor standards | S | 
| 5    | Labor market standards | Right to join trade unions (protection) | S | 
| 6    | Human rights | Food import security | S | 
| 7    | Income | Average monthly wage | S | 

<table>
<thead>
<tr>
<th>Rank</th>
<th>Pillar</th>
<th>Thematic</th>
<th>Variable</th>
<th>Sign</th>
</tr>
</thead>
</table>
| 1    | International relationships | Exporting across borders (cost) | G | +
| 2    | Business environment and R&D | Ethical behaviour of firms | G | 
| 3    | National security | Severity of kidnappings | G | 
| 4    | Business environment and R&D | Capacity for innovation | G | 
| 5    | Infrastructure and mobility | Physical connectivity | G | 
| 6    | Infrastructure and mobility | Air transport departures | G | 
| 7    | Infrastructure and mobility | Rail lines km | G | 

*Source: Semet et al. (2021)*
### Multi-factor analysis

Global analysis - Lasso regression on the three pillars

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Indicator</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>Exporting across borders (cost)</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Severe storm hazard</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>Capacity for innovation</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Ethical behaviour of firms</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>Temperature change</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Severity of kidnappings</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>Drought hazard</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>Fossil fuel intensity of the economy</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>Biodiversity threatening score</td>
<td>9</td>
</tr>
<tr>
<td>S</td>
<td>Index of labor standards</td>
<td>10</td>
</tr>
</tbody>
</table>

**ESG pillar importance**

Source: Semet et al. (2021)
Table 57: Final multi-factor model

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\sigma} (\hat{\beta}) )</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ( \alpha )</td>
<td>2.834</td>
<td>0.180</td>
<td>15.72***</td>
<td>0.00</td>
</tr>
<tr>
<td>GDP growth ( g_{i,t} )</td>
<td>0.017</td>
<td>0.012</td>
<td>1.37</td>
<td>0.17</td>
</tr>
<tr>
<td>Inflation ( \pi_{i,t} )</td>
<td>0.048</td>
<td>0.007</td>
<td>6.64***</td>
<td>0.00</td>
</tr>
<tr>
<td>Debt ratio ( d_{i,t} )</td>
<td>-0.001</td>
<td>0.001</td>
<td>-1.71*</td>
<td>0.08</td>
</tr>
<tr>
<td>Current account balance ( ca_{i,t} )</td>
<td>-0.012</td>
<td>0.005</td>
<td>-2.45**</td>
<td>0.01</td>
</tr>
<tr>
<td>Reserve adequacy ( r_{i,t} )</td>
<td>0.005</td>
<td>0.007</td>
<td>0.74</td>
<td>0.45</td>
</tr>
<tr>
<td>Rating score ( \mathcal{R}_{i,t} )</td>
<td>-0.013</td>
<td>0.001</td>
<td>-9.08***</td>
<td>0.00</td>
</tr>
<tr>
<td>Exporting across borders (cost)</td>
<td>(4.05 \times 10^{-4})</td>
<td>(9.83 \times 10^{-5})</td>
<td>4.11***</td>
<td>0.00</td>
</tr>
<tr>
<td>Severe storm hazard (absolute high extreme)</td>
<td>-0.015</td>
<td>0.009</td>
<td>-1.66*</td>
<td>0.09</td>
</tr>
<tr>
<td>Capacity for innovation</td>
<td>-0.004</td>
<td>0.001</td>
<td>-4.99***</td>
<td>0.00</td>
</tr>
<tr>
<td>Ethical behavior of firms</td>
<td>-0.061</td>
<td>0.021</td>
<td>-2.79***</td>
<td>0.00</td>
</tr>
<tr>
<td>Temperature change</td>
<td>-0.149</td>
<td>0.042</td>
<td>-3.50***</td>
<td>0.00</td>
</tr>
<tr>
<td>Severity of kidnappings</td>
<td>-0.032</td>
<td>0.007</td>
<td>-4.25***</td>
<td>0.00</td>
</tr>
<tr>
<td>Drought hazard (absolute high extreme)</td>
<td>(3.33 \times 10^{-8})</td>
<td>(1.27 \times 10^{-8})</td>
<td>2.60***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\( \Delta R^2_c = 13.51\% \), \( F\)-test = 29.28***

Source: Semet et al. (2021)
Multi-factor analysis
High income vs middle income countries

High income
Middle income
### Multi-factor analysis

#### High income countries

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Indicator</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Fossil fuel intensity of the economy</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Temperature change</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>Cooling degree days annual average</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Capacity for innovation</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>Heat stress (future)</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Severity of kidnappings</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>Biodiversity threatening score</td>
<td>7</td>
</tr>
<tr>
<td>G</td>
<td>Efficacy of corporate boards</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>Total GHG emissions</td>
<td>9</td>
</tr>
<tr>
<td>S</td>
<td>Significant marginalized group</td>
<td>10</td>
</tr>
</tbody>
</table>

#### ESG pillar importance

![ESG pillar importance chart](chart.png)

- **Transition risk**
- **S** is lagging

Source: Semet *et al.* (2021)
## Multi-factor analysis

### Middle income countries

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Indicator</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Tsunami hazard</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Transport infrastructure exposed to natural hazards</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>Severity of kidnappings</td>
<td>3</td>
</tr>
<tr>
<td>S</td>
<td>Discrimination based on LGBT status</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>Air transport departures</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Exporting across borders (cost)</td>
<td>6</td>
</tr>
<tr>
<td>G</td>
<td>Index of labour standards</td>
<td>7</td>
</tr>
<tr>
<td>S</td>
<td>Vulnerability of migrant workers</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>Paris Agreement</td>
<td>9</td>
</tr>
<tr>
<td>G</td>
<td>Military expenditure (% of GDP)</td>
<td>10</td>
</tr>
</tbody>
</table>

### ESG pillar importance

- **Physical risk**
- **Social issues are priced**

Source: Semet et al. (2021)
Explaining credit ratings with ESG metrics

Statistical framework

We consider the logit model:

\[
\Pr \{ G_{i,t} = 1 \} = F \left( \theta_0 + \sum_{j=1}^{m} \beta_j x_{i,t}^{(j)} \right)
\]

where:

- \( G_{i,t} = 1 \) indicates if the country \( i \) is rated upper grade at time \( t \)
  - If the rating \( \geq \) A then \( G_{i,t} = 1 \)
  - If the rating \( \leq \) BBB then \( G_{i,t} = 0 \)

- \( F(z) \) is the logistic cumulative density function

- \( x_{i,t}^{(j)} \) is the \( j^{th} \) selected indicator

We note \( \theta_j = e^{\beta_j} \) is the odds-ratio coefficient

Lasso-penalized logit regression

Again, we perform a lasso regression to retain the seven most relevant variables for each ESG pillar and then we perform a multi-factor analysis
Theoretical models
Empirical results
Cost of capital
Equities
Corporate bonds
Sovereign bonds

Explaining credit ratings with ESG metrics
Lasso selection process

Table 58: List of selected ESG variables for the logistic regression

<table>
<thead>
<tr>
<th>Theme</th>
<th>Variable</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commitment to environmental standards</td>
<td>Domestic regulatory framework</td>
<td>1</td>
</tr>
<tr>
<td>Climate change</td>
<td>Climate change vulnerability (average)</td>
<td>2</td>
</tr>
<tr>
<td>Water management</td>
<td>Water import security (average)</td>
<td>3</td>
</tr>
<tr>
<td>Energy mix</td>
<td>Energy self sufficiency</td>
<td>4</td>
</tr>
<tr>
<td>Water management</td>
<td>Wastewater treatment index</td>
<td>5</td>
</tr>
<tr>
<td>Water management</td>
<td>Water intensity of the economy</td>
<td>6</td>
</tr>
<tr>
<td>Biodiversity</td>
<td>Biodiversity threatening score</td>
<td>7</td>
</tr>
<tr>
<td>Health</td>
<td>Health expenditure per capita</td>
<td>1</td>
</tr>
<tr>
<td>Water and electricity access</td>
<td>Public dissatisfaction with water quality</td>
<td>2</td>
</tr>
<tr>
<td>Education</td>
<td>Mean years of schooling of adults</td>
<td>3</td>
</tr>
<tr>
<td>Income</td>
<td>Base pay / value added per worker</td>
<td>4</td>
</tr>
<tr>
<td>Demographics</td>
<td>Urban population change (5 years)</td>
<td>5</td>
</tr>
<tr>
<td>Human rights</td>
<td>Basic food stuffs net imports per person</td>
<td>6</td>
</tr>
<tr>
<td>Human rights</td>
<td>Food import security</td>
<td>7</td>
</tr>
<tr>
<td>Government effectiveness</td>
<td>Government effectiveness index</td>
<td>1</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Venture capital availability</td>
<td>2</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>R&amp;D expenditure (% of GDP)</td>
<td>3</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td>Customs efficiency</td>
<td>4</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Enforcing a contract (time)</td>
<td>5</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Paying tax (process)</td>
<td>6</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Getting electricity (time)</td>
<td>7</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
Explaining credit ratings with ESG metrics


e-pillar

Table 59: Logit model with environmental variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma} (\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic regulatory framework</td>
<td>1.415</td>
<td>0.156</td>
<td>3.16***</td>
<td>0.00</td>
</tr>
<tr>
<td>Climate change vulnerability (average)</td>
<td>2.929</td>
<td>0.572</td>
<td>5.51***</td>
<td>0.00</td>
</tr>
<tr>
<td>Water import security (average)</td>
<td>1.385</td>
<td>0.147</td>
<td>3.07***</td>
<td>0.00</td>
</tr>
<tr>
<td>Energy self sufficiency</td>
<td>0.960</td>
<td>0.033</td>
<td>−1.16</td>
<td>0.24</td>
</tr>
<tr>
<td>Wastewater treatment index</td>
<td>1.011</td>
<td>0.008</td>
<td>1.36</td>
<td>0.17</td>
</tr>
<tr>
<td>Water intensity of the economy</td>
<td>1.000</td>
<td>0.000</td>
<td>−1.02</td>
<td>0.30</td>
</tr>
<tr>
<td>Biodiversity threatening score</td>
<td>0.887</td>
<td>0.026</td>
<td>−4.02***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell (\hat{\beta}) = -107.60$, AIC = 231.19, $R^2 = 49.1\%$, ACC = 83.6\%

Source: Semet et al. (2021)
### Table 60: Logit model with social variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma} (\hat{\theta}_j)$</th>
<th>$t$-student</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health expenditure per capita</td>
<td>1.001</td>
<td>0.000</td>
<td>3.47***</td>
<td>0.00</td>
</tr>
<tr>
<td>Public dissatisfaction with water quality</td>
<td>0.889</td>
<td>0.024</td>
<td>−4.27***</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean years of schooling of adults</td>
<td>2.710</td>
<td>0.583</td>
<td>4.64***</td>
<td>0.00</td>
</tr>
<tr>
<td>Base pay / value added per worker</td>
<td>0.000</td>
<td>0.000</td>
<td>−5.13***</td>
<td>0.00</td>
</tr>
<tr>
<td>Urban population change (5 years)</td>
<td>1.653</td>
<td>0.131</td>
<td>6.36***</td>
<td>0.00</td>
</tr>
<tr>
<td>Basic food stuffs net imports per person</td>
<td>0.996</td>
<td>0.001</td>
<td>−3.58***</td>
<td>0.00</td>
</tr>
<tr>
<td>Food import security</td>
<td>0.973</td>
<td>0.006</td>
<td>−4.33***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell (\hat{\beta}) = -72.41$, $\text{AIC} = 160.83$, $\mathcal{R}^2 = 65.6\%$, $\text{ACC} = 87.9\%$

Source: Semet et al. (2021)
Table 61: Logit model with governance variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma}(\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government effectiveness index</td>
<td>1.096</td>
<td>0.035</td>
<td>2.81***</td>
<td>0.00</td>
</tr>
<tr>
<td>Venture capital availability</td>
<td>1.020</td>
<td>0.005</td>
<td>4.16***</td>
<td>0.00</td>
</tr>
<tr>
<td>R&amp;D expenditure (% of GDP)</td>
<td>2.259</td>
<td>1.006</td>
<td>1.83*</td>
<td>0.06</td>
</tr>
<tr>
<td>Customs efficiency</td>
<td>2.193</td>
<td>1.657</td>
<td>1.04</td>
<td>0.29</td>
</tr>
<tr>
<td>Enforcing a contract (time)</td>
<td>0.997</td>
<td>0.001</td>
<td>-3.69***</td>
<td>0.00</td>
</tr>
<tr>
<td>Paying tax (process)</td>
<td>0.914</td>
<td>0.031</td>
<td>-2.63***</td>
<td>0.00</td>
</tr>
<tr>
<td>Getting electricity (time)</td>
<td>0.989</td>
<td>0.004</td>
<td>-2.73***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell\left(\hat{\beta}\right) = -67.78, \text{AIC} = 151.57, R^2 = 67.9\%, \text{ACC} = 90.1\%$

Source: Semet et al. (2021)
### Table 62: Logit model with the ESG selected variables

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma}(\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Domestic regulatory framework</td>
<td>2.881</td>
<td>2.108</td>
<td>1.44</td>
<td>0.14</td>
</tr>
<tr>
<td>E</td>
<td>Climate change vulnerability (average)</td>
<td>0.275</td>
<td>0.302</td>
<td>−1.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Water import security (average)</td>
<td>0.717</td>
<td>0.467</td>
<td>−0.50</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>Biodiversity threatening score</td>
<td>1.029</td>
<td>0.199</td>
<td>0.14</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>Health expenditure per capita</td>
<td>0.998</td>
<td>0.002</td>
<td>−1.10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Public dissatisfaction with water quality</td>
<td>1.332</td>
<td>0.269</td>
<td>1.41</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Mean years of schooling of adults</td>
<td>68.298</td>
<td>85.559</td>
<td>3.37**</td>
<td>0.00</td>
</tr>
<tr>
<td>S</td>
<td>Base pay / value added per worker</td>
<td>0.000</td>
<td>0.000</td>
<td>−1.07</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Urban population change (5 years)</td>
<td>3.976</td>
<td>1.857</td>
<td>2.95**</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Basic food stuffs net imports per person</td>
<td>0.990</td>
<td>0.004</td>
<td>−2.07**</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Food import security</td>
<td>0.803</td>
<td>0.067</td>
<td>−2.59***</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Government effectiveness index</td>
<td>1.751</td>
<td>0.412</td>
<td>2.37**</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Venture capital availability</td>
<td>1.099</td>
<td>0.035</td>
<td>2.93***</td>
<td>0.00</td>
</tr>
<tr>
<td>G</td>
<td>Enforcing a contract (time)</td>
<td>0.999</td>
<td>0.004</td>
<td>−0.31</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Paying tax (process)</td>
<td>0.846</td>
<td>0.096</td>
<td>−1.47</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Getting electricity (time)</td>
<td>0.882</td>
<td>0.037</td>
<td>−2.95***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$$\ell(\hat{\beta}) = -18.91, \ \text{AIC} = 71.83, \ \hat{R}^2 = 91.1\%, \ \text{ACC} = 96.7\%$$

Source: Semet et al. (2021)
Explaining credit ratings with ESG metrics
Prediction accuracy of credit ratings

Table 63: Summary of the results

<table>
<thead>
<tr>
<th></th>
<th>***</th>
<th>$R^2$</th>
<th>Accuracy</th>
<th>Sensitivity</th>
<th>Specificity</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>E*</td>
<td>4</td>
<td>48.02%</td>
<td>84.97%</td>
<td>86.90%</td>
<td>83.23%</td>
<td>230.04</td>
</tr>
<tr>
<td>S*</td>
<td>7</td>
<td>65.60%</td>
<td>87.90%</td>
<td>88.80%</td>
<td>86.90%</td>
<td>160.83</td>
</tr>
<tr>
<td>G*</td>
<td>4</td>
<td>67.70%</td>
<td>89.54%</td>
<td>91.72%</td>
<td>87.58%</td>
<td>150.65</td>
</tr>
<tr>
<td>ESG*</td>
<td>7</td>
<td>79.02%</td>
<td>92.50%</td>
<td>93.80%</td>
<td>91.30%</td>
<td>104.80</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)

⇒ Final model: Education, Demographics, Human rights, Government effectiveness, Business environment and R&D
Explaining credit ratings with ESG metrics
Prediction accuracy of credit ratings

Figure 82: Prediction accuracy (in %) of credit ratings

<table>
<thead>
<tr>
<th>Rating</th>
<th>Probability range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>83% – 100%</td>
</tr>
<tr>
<td>AA</td>
<td>67% – 82%</td>
</tr>
<tr>
<td>A</td>
<td>50% – 66%</td>
</tr>
<tr>
<td>BBB</td>
<td>39% – 49%</td>
</tr>
<tr>
<td>BB</td>
<td>29% – 38%</td>
</tr>
<tr>
<td>B</td>
<td>11% – 28%</td>
</tr>
<tr>
<td>C</td>
<td>0% – 10%</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
ESG and sovereign risk
Summary of the results

<table>
<thead>
<tr>
<th>What is directly priced by the bond market?</th>
<th>What is indirectly priced by credit rating agencies?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E} \succ \mathbb{G} \succ \mathbb{S}$</td>
<td>$\mathbb{G} \succ \mathbb{S} \succ \mathbb{E}$</td>
</tr>
<tr>
<td>Significant market-based ESG indicators $\neq$ Relevant CRA-based ESG indicators</td>
<td></td>
</tr>
</tbody>
</table>
| • High-income countries  
  Transition risk $\succ$ Physical risk | $\mathbb{E}$ metrics are second-order variables:  
  • Environmental standards  
  • Water management  
  • Biodiversity  
  • Climate change |
| • Middle-income countries  
  Physical risk $\succ$ Transition risk |  |

$\mathbb{S}$ matters for middle-income countries, especially for Gender inequality, Working conditions and Migration

National security, Infrastructure and mobility and International relationships are the relevant $\mathbb{G}$ metrics

Fundamental analysis: $\mathbb{R}_c^2 \approx 70\%$

Extra-financial analysis: $\Delta \mathbb{R}_c^2 \approx 13.5\%$

Education, Demographic and Human rights are prominent indicators for the $\mathbb{S}$ pillar

Government effectiveness, Business environment and R&D dominate the $\mathbb{G}$ pillar

Accuracy $> 95\%$

AAA, AA, B, CCC $\succ$ A $\succ$ BB $\succ$ BBB
Course 2022-2023 in Sustainable Finance
Lecture 4. Exercise
Equity Portfolio Optimization with ESG Scores

Thierry Roncalli

* Amundi Asset Management

* University of Paris-Saclay

March 2023

7 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
We consider the CAPM model:

\[ R_i - r = \beta_i (R_m - r) + \varepsilon_i \]

where \( R_i \) is the return of asset \( i \), \( R_m \) is the return of the market portfolio \( w_m \), \( r \) is the risk free asset, \( \beta_i \) is the beta of asset \( i \) with respect to the market portfolio and \( \varepsilon_i \) is the idiosyncratic risk of asset \( i \). We have \( R_m \perp \varepsilon_i \) and \( \varepsilon_i \perp \varepsilon_j \). We note \( \sigma_m \) the volatility of the market portfolio.

Let \( \tilde{\sigma}_i \), \( \mu_i \) and \( S_i \) be the idiosyncratic volatility, the expected return and the ESG score of asset \( i \). We use a universe of 6 assets with the following parameter values:

<table>
<thead>
<tr>
<th>Asset ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_i )</td>
<td>0.10</td>
<td>0.30</td>
<td>0.50</td>
<td>0.90</td>
<td>1.30</td>
<td>2.00</td>
</tr>
<tr>
<td>( \tilde{\sigma}_i ) (in %)</td>
<td>17.00</td>
<td>17.00</td>
<td>16.00</td>
<td>10.00</td>
<td>11.00</td>
<td>12.00</td>
</tr>
<tr>
<td>( \mu_i ) (in %)</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>5.50</td>
<td>7.50</td>
<td>11.00</td>
</tr>
<tr>
<td>( S_i )</td>
<td>1.10</td>
<td>1.50</td>
<td>2.50</td>
<td>–1.82</td>
<td>–2.35</td>
<td>–2.91</td>
</tr>
</tbody>
</table>

and \( \sigma_m = 20\% \). The risk-free return \( r \) is set to 1% and the expected return of the market portfolio \( w_m \) is equal to \( \mu_m = 6\% \).
Question 1

We assume that the CAPM is valid.
Question (a)

Calculate the vector $\mu$ of expected returns.
Using the CAPM, we have:

$$\mu_i = r + \beta_i (\mu_m - r)$$

For instance, we have:

$$\mu_1 = 1\% + 0.10 \times (6\% - 1\%) = 1.5\%$$

and:

$$\mu_2 = 1\% + 0.30 \times 5\% = 2.5\%$$

Finally, we obtain $$\mu = (1.5\%, 2.5\%, 3.5\%, 5.5\%, 7.5\%, 11\%)$$
Question (b)
Compute the covariance matrix $\Sigma$. Deduce the volatility $\sigma_i$ of the asset $i$ and find the correlation matrix $C = (\rho_{i,j})$ between asset returns.
• We have:

\[ \Sigma = \sigma_m^2 \beta \beta^T + D \]

where:

\[ D = \text{diag}(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_6^2) \]

• The numerical value of \( \Sigma \) is:

\[
\Sigma = \begin{pmatrix}
293 & 12 & 20 & 36 & 52 & 80 \\
12 & 325 & 60 & 108 & 156 & 240 \\
20 & 60 & 356 & 180 & 260 & 400 \\
36 & 108 & 180 & 424 & 468 & 720 \\
52 & 156 & 260 & 468 & 797 & 1040 \\
80 & 240 & 400 & 720 & 1040 & 1744
\end{pmatrix} \times 10^{-4}
\]
- We have:
  \[ \sigma_i = \sqrt{\Sigma_{i,i}} \]

- We obtain:
  \[ \sigma = (17.12\%, 18.03\%, 18.87\%, 20.59\%, 28.23\%, 41.76\%) \]

- We have:
  \[ \rho_{i,j} = \frac{\Sigma_{i,j}}{\sigma_i \sigma_j} \]

- We obtain the following correlation matrix expressed in %:

  \[
  \mathbf{C} = \begin{pmatrix}
  100.00 &  &  &  &  & \\
  3.89 & 100.00 &  &  &  & \\
  6.19 & 17.64 & 100.00 &  &  & \\
  10.21 & 29.09 & 46.33 & 100.00 &  & \\
  10.76 & 30.65 & 48.81 & 80.51 & 100.00 &  \\
  11.19 & 31.88 & 50.76 & 83.73 & 88.21 & 100.00
  \end{pmatrix}
  \]
Question (c)

Compute the tangency portfolio $w^*$. Calculate $\mu (w^*)$ and $\sigma (w^*)$. Deduce the Sharpe ratio and the ESG score of the tangency portfolio.
We have:

\[ w^* = \Sigma^{-1} (\mu - r1) \]

\[ = \frac{1}{1^\top \Sigma^{-1} (\mu - r1)} \]

\[ = \begin{pmatrix}
0.94% \\
2.81% \\
5.28% \\
24.34% \\
29.06% \\
37.57%
\end{pmatrix} \]

We deduce:

\[ \mu (w^*) = w^\top \mu = 7.9201\% \]

\[ \sigma (w^*) = \sqrt{w^\top \Sigma w^*} = 28.3487\% \]

\[ SR (w^* | r) = \frac{7.9201\% - 1\%}{28.3487\%} = 0.2441 \]

\[ S (w^*) = \sum_{i=1}^{6} w_i^* S_i = -2.0347 \]
Question (d)

Compute the beta coefficient \( \beta_i (w^*) \) of the six assets with respect to the tangency portfolio \( w^* \), and the implied expected return \( \tilde{\mu}_i \):

\[
\tilde{\mu}_i = r + \beta_i (w^*) (\mu (w^*) - r)
\]
• We have:

\[ \beta_i (w^*) = \frac{e_i^\top \Sigma w^*}{\sigma^2 (w^*)} \]

• We obtain:

\[ \beta (w^*) = \begin{pmatrix}
0.0723 \\
0.2168 \\
0.3613 \\
0.6503 \\
0.9393 \\
1.4451 \\
\end{pmatrix} \]

• The computation of \( \tilde{\mu}_i = r + \beta_i (w^*) (\mu (w^*) - r) \) gives:

\[ \tilde{\mu} = \begin{pmatrix}
1.50\% \\
2.50\% \\
3.50\% \\
5.50\% \\
7.50\% \\
11.00\% \\
\end{pmatrix} \]
Question (e)

Deduce the market portfolio $w_m$. Comment on these results.


- $\beta_i(w^*) \neq \beta_i(w_m)$ but risk premia are exact
- Let us assume that the allocation of $w_m$ is equal to $\alpha$ of the tangency portfolio $w^*$ and $1 - \alpha$ of the risk-free asset. We deduce that:

$$
\beta(w_m) = \frac{\sum w_m}{\sigma^2(w_m)} = \frac{\alpha \sum w^*}{\alpha^2 \sigma^2(w^*)} = \frac{1}{\alpha} \beta(w^*)
$$

- We have:

$$
\alpha = \frac{\beta_i(w^*)}{\beta_i(w_m)} = 72.25% 
$$

- The market portfolio $w_m$ is equal to $72.25\%$ of the tangency portfolio $w^*$ and $27.75\%$ of the risk-free asset
- We have:
  \[ \mu(w_m) = r + \alpha (\mu(w^*) - r) = 1\% + 72.25\% \times (7.9201\% - 1\%) = 6\% \]
  and:
  \[ \sigma(w_m) = \alpha \sigma(w^*) = 72.25\% \times 28.3487\% = 20.48\% \]

- We deduce that:
  \[ \text{SR}(w_m | r) = \frac{6\% - 1\%}{20.48\%} = 0.2441 \]

- We do not obtain the true value of the Sharpe ratio:
  \[ \text{SR}(w_m | r) = \frac{6\% - 1\%}{20\%} = 0.25 \]

- The tangency portfolio has an idiosyncratic risk:
  \[ \sqrt{w_m^T \left( \sigma_m^2 \beta \beta^T \right) w_m} = 20\% < \sigma(w_m) = 20.48\% \]
Question 2

We consider long-only portfolios and we also impose a minimum threshold $S^*$ for the portfolio ESG score:

$$ S(w) = w^T S \geq S^* $$
Question (a)

Let $\gamma$ be the risk tolerance. Write the mean-variance optimization problem.
We have:

$$w^* = \arg \min w^T \Sigma w - \gamma w^T \mu$$

subject to

$$1_6^T w = 1$$
$$w^T S \geq S^*$$
$$0_6 \leq w \leq 1_6$$
Question (b)
Find the QP form of the MVO problem.
The matrix form of the QP problem is:

\[ w^* = \arg \min \frac{1}{2} w^\top Q w - w^\top R \]

s.t. \[
\begin{align*}
Aw &= B \\
Cw &\leq D \\
w^- &\leq w \leq w^+
\end{align*}
\]

We deduce that \( Q = \Sigma, \ R = \gamma \mu, \ A = 1_6^\top, \ B = 1, \ C = -S^\top, \ D = -S^*, \ w^- = 0_6 \) and \( w^+ = 1_6 \)
Question (c)

Compare the efficient frontier when (1) there is no ESG constraint ($S^* = -\infty$), (2) we impose a positive ESG score ($S^* = 0$) and (3) the minimum threshold is set to 0.5 ($S^* = 0.5$). Comment on these results.
To compute the efficient frontier, we consider several values of \( \gamma \in [-1, 2] \).

For each value of \( \gamma \), we compute the optimal portfolio \( w^* \) and deduce its expected return \( \mu(w^*) \) and its volatility \( \sigma(w^*) \).
Figure 83: Impact of the minimum ESG score on the efficient frontier
Question (d)

For each previous cases, find the tangency portfolio \( w^* \) and the corresponding risk tolerance \( \gamma^* \). Compute then \( \mu (w^*) \), \( \sigma (w^*) \), \( \text{SR}(w^* | r) \) and \( S(w^*) \). Comment on these results.
Let $w^*(\gamma)$ be the MVO portfolio when the risk tolerance is equal to $\gamma$

By using a fine grid of $\gamma$ values, we can find the optimal value $\gamma^*$ by solving numerically the following optimization problem with the brute force algorithm:

$$\gamma^* = \arg \max_{\gamma} \frac{\mu(w^*(\gamma)) - r}{\sigma(w^*(\gamma))} \quad \text{for } \gamma \in [0, 2]$$

We deduce the tangency portfolio $w^* = w^*(\gamma^*)$
Table 64: Impact of the minimum ESG score on the efficient frontier

<table>
<thead>
<tr>
<th>$\gamma^*$</th>
<th>$-\infty$</th>
<th>0</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^*$ (in %)</td>
<td>1.1613</td>
<td>0.8500</td>
<td>0.8500</td>
</tr>
<tr>
<td></td>
<td>0.9360</td>
<td>9.7432</td>
<td>9.1481</td>
</tr>
<tr>
<td></td>
<td>2.8079</td>
<td>16.3317</td>
<td>19.0206</td>
</tr>
<tr>
<td>$\mu (w^*)$ (in %)</td>
<td>5.2830</td>
<td>31.0176</td>
<td>40.3500</td>
</tr>
<tr>
<td></td>
<td>24.3441</td>
<td>5.1414</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>29.0609</td>
<td>11.6028</td>
<td>3.8248</td>
</tr>
<tr>
<td></td>
<td>37.5681</td>
<td>26.1633</td>
<td>27.6565</td>
</tr>
<tr>
<td>$\sigma (w^*) (in%)$</td>
<td>7.9201</td>
<td>5.6710</td>
<td>5.3541</td>
</tr>
<tr>
<td>$\sigma (w^*) (in%)$</td>
<td>28.3487</td>
<td>19.8979</td>
<td>19.2112</td>
</tr>
<tr>
<td>$\overline{SR} (w^* \mid r)$</td>
<td>0.2441</td>
<td>0.2347</td>
<td>0.2266</td>
</tr>
<tr>
<td>$S (w^*)$</td>
<td>−2.0347</td>
<td>0.0000</td>
<td>0.5000</td>
</tr>
</tbody>
</table>
Question (e)

Draw the relationship between the minimum ESG score $S^*$ and the Sharpe ratio $\text{SR}(w^* | r)$ of the tangency portfolio.
We perform the same analysis as previously for several values $S^* \in [-2.5, 2.5]$

We verify that the Sharpe ratio is a decreasing function of $S^*$
Figure 84: Relationship between the minimum ESG score $S^*$ and the Sharpe ratio $\text{SR}(w^* | r)$ of the tangency portfolio.
Question (f)

We assume that the market portfolio $w_m$ corresponds to the tangency portfolio when $S^* = 0.5$. 
The market portfolio $w_m$ is then equal to:

$$w_m = \begin{pmatrix}
9.15 \\
19.02 \\
40.35 \\
0.00 \\
3.82 \\
27.66
\end{pmatrix}$$

We deduce that:

$$\mu(w_m) = 5.3541\%$$
$$\sigma(w_m) = 19.2112\%$$
$$SR(w_m | r) = 0.2266$$
$$S(w_m) = 0.5$$
Question (f).i

Compute the beta coefficient $\beta_i \left( w_m \right)$ and the implied expected return $\tilde{\mu}_i \left( w_m \right)$ for each asset. Deduce then the alpha return $\alpha_i$ of asset $i$. Comment on these results.
We have:

\[
\beta_i (w_m) = \frac{e_i^T \Sigma w_m}{\sigma^2 (w_m)}
\]

and:

\[
\tilde{\mu}_i (w_m) = r + \beta_i (w_m) (\mu (w_m) - r)
\]

We deduce that the alpha return is equal to:

\[
\alpha_i = \mu_i - \tilde{\mu}_i (w_m)
\]

\[
= (\mu_i - r) - \beta_i (w_m) (\mu (w_m) - r)
\]

We notice that \( \alpha_i < 0 \) for the first three assets and \( \alpha_i > 0 \) for the last three assets, implying that:

\[
\begin{cases} 
S_i > 0 \Rightarrow \alpha_i < 0 \\
S_i < 0 \Rightarrow \alpha_i > 0
\end{cases}
\]
Table 65: Computation of the alpha return due to the ESG constraint

<table>
<thead>
<tr>
<th>Asset</th>
<th>$\beta_i \ (w_m)$</th>
<th>$\tilde{\mu}_i \ (w_m)$ (in %)</th>
<th>$\tilde{\mu}_i \ (w_m) - r$ (in %)</th>
<th>$\alpha_i$ (in bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1660</td>
<td>1.7228</td>
<td>0.7228</td>
<td>−22.28</td>
</tr>
<tr>
<td>2</td>
<td>0.4321</td>
<td>2.8813</td>
<td>1.8813</td>
<td>−38.13</td>
</tr>
<tr>
<td>3</td>
<td>0.7518</td>
<td>4.2733</td>
<td>3.2733</td>
<td>−77.33</td>
</tr>
<tr>
<td>4</td>
<td>0.8494</td>
<td>4.6984</td>
<td>3.6984</td>
<td>80.16</td>
</tr>
<tr>
<td>5</td>
<td>1.2395</td>
<td>6.3967</td>
<td>5.3967</td>
<td>110.33</td>
</tr>
<tr>
<td>6</td>
<td>1.9955</td>
<td>9.6885</td>
<td>8.6885</td>
<td>131.15</td>
</tr>
</tbody>
</table>
Question (f).ii

We consider the equally-weighted portfolio $w_{ew}$. Compute its beta coefficient $\beta(w_{ew} \mid w_m)$, its implied expected return $\tilde{\mu}(w_{ew})$ and its alpha return $\alpha(w_{ew})$. Comment on these results.
We have:

\[ \beta (w_{ew} \mid w_m) = \frac{w_{ew}^\top \sum w_m}{\sigma^2 (w_m)} = 0.9057 \]

and:

\[ \tilde{\mu} (w_{ew}) = 1\% + 0.9057 \times (5.3541\% - 1\%) = 4.9435\% \]

We deduce that:

\[ \alpha (w_{ew}) = \mu (w_{ew}) - \tilde{\mu} (w_{ew}) = 5.25\% - 4.9435\% = 30.65 \text{ bps} \]

We verify that:

\[ \alpha (w_{ew}) = \sum_{i=1}^{6} w_{ew,i} \alpha_i = \frac{\sum_{i=1}^{6} \alpha_i}{6} = 30.65 \text{ bps} \]

The equally-weighted portfolio has a positive alpha because:

\[ S (w_{ew}) = -0.33 \ll S (w_m) = 0.50 \]
Question 3

The objective of the investor is twice. He would like to manage the tracking error risk of his portfolio with respect to the benchmark \( b = (15\%, 20\%, 19\%, 14\%, 15\%, 17\%) \) and have a better ESG score than the benchmark. Nevertheless, this investor faces a long-only constraint because he cannot leverage his portfolio and he cannot also be short on the assets.
Question (a)

What is the ESG score of the benchmark?
We have:

\[ S(b) = \sum_{i=1}^{6} b_i S_i = -0.1620 \]
We assume that the investor's portfolio is \( w = (10\%, 10\%, 30\%, 20\%, 20\%, 10\%) \). Compute the excess score \( S(w \mid b) \), the expected excess return \( \mu(w \mid b) \), the tracking error volatility \( \sigma(w \mid b) \) and the information ratio \( IR(w \mid b) \). Comment on these results.
We have:

\[
\begin{align*}
S (w \mid b) &= (w - b)^\top S = 0.0470 \\
\mu (w \mid b) &= (w - b)^\top \mu = -0.5 \text{ bps} \\
\sigma (w \mid b) &= \sqrt{(w - b)^\top \Sigma (w - b)} = 2.8423\% \\
IR (w \mid b) &= \frac{\mu (w \mid b)}{\sigma (w \mid b)} = -0.0018
\end{align*}
\]

The portfolio $w$ is not optimal since it improves the ESG score of the benchmark, but its information ratio is negative. Nevertheless, the expected excess return is close to zero (less than $-1$ bps).
Question (c)

Same question with the portfolio \( w = (10\%, 15\%, 30\%, 10\%, 15\%, 20\%) \).
We have: We have:

\[
\begin{align*}
\mathbf{S}(w | b) &= (w - b)^\top \mathbf{S} = 0.1305 \\
\mu(w | b) &= (w - b)^\top \mathbf{\mu} = 29.5 \text{ bps} \\
\sigma(w | b) &= \sqrt{(w - b)^\top \mathbf{\Sigma} (w - b)} = 2.4949\% \\
\text{IR}(w | b) &= \frac{\mu(w | b)}{\sigma(w | b)} = 0.1182
\end{align*}
\]
Question (d)

In the sequel, we assume that the investor has no return target. In fact, the objective of the investor is to improve the ESG score of the benchmark and control the tracking error volatility. We note $\gamma$ the risk tolerance. Give the corresponding esg-variance optimization problem.
The optimization problem is:

\[
  w^* \quad = \quad \arg \min \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b)
\]

\[
  \text{s.t.} \quad \begin{cases} 
    1^T w = 1 \\
    0 \leq w \leq 1
  \end{cases}
\]
Question (e)
Find the matrix form of the corresponding QP problem.
The objective function is equal to:

\[
(*) = \frac{1}{2} \sigma^2 (w | b) - \gamma S (w | b)
\]

\[
= \frac{1}{2} (w - b)^\top \Sigma (w - b) - \gamma (w - b)^\top S
\]

\[
= \frac{1}{2} w^\top \Sigma w - w^\top (\Sigma b + \gamma S) + \left( \gamma b^\top S + \frac{1}{2} b^\top \Sigma b \right)
\]

\[
\text{does not depend on } w
\]

We deduce that \( Q = \Sigma, R = \Sigma b + \gamma S, A = 1_6^\top, B = 1, w^- = 0_6 \)
and \( w^+ = 1_6 \)
Question (f)

Draw the esg-variance efficient frontier \((\sigma (w^* \mid b), \mathcal{S} (w^* \mid b))\) where \(w^*\) is an optimal portfolio.
We solve the QP problem for several values of $\gamma \in [0, 5\%]$ and obtain Figure 85.
Figure 85: Efficient frontier of tracking a benchmark with an ESG score objective
Question (g)

Find the optimal portfolio $w^*$ when we target a given tracking error volatility $\sigma^*$. The values of $\sigma^*$ are 0%, 1%, 2%, 3% and 4%.
• Using the QP numerical algorithm, we compute the optimal value $\sigma (w \mid b)$ for $\gamma = 0$ and $\gamma = 5\%$

• Then, we apply the bisection algorithm to find the optimal value $\gamma^*$ such that:

$$\sigma (w \mid b) = \sigma^*$$
Table 66: Solution of the $\sigma$-problem

<table>
<thead>
<tr>
<th>Target $\sigma^*$</th>
<th>0</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*$ (in bps)</td>
<td>0.000</td>
<td>4.338</td>
<td>8.677</td>
<td>13.015</td>
<td>18.524</td>
</tr>
<tr>
<td>$w^*$ (in %)</td>
<td>15.000</td>
<td>15.175</td>
<td>15.350</td>
<td>15.525</td>
<td>14.921</td>
</tr>
<tr>
<td></td>
<td>20.000</td>
<td>21.446</td>
<td>22.892</td>
<td>24.338</td>
<td>25.385</td>
</tr>
<tr>
<td></td>
<td>19.000</td>
<td>23.084</td>
<td>27.167</td>
<td>31.251</td>
<td>35.589</td>
</tr>
<tr>
<td></td>
<td>14.000</td>
<td>9.588</td>
<td>5.176</td>
<td>0.763</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>15.000</td>
<td>12.656</td>
<td>10.311</td>
<td>7.967</td>
<td>3.555</td>
</tr>
<tr>
<td></td>
<td>17.000</td>
<td>18.052</td>
<td>19.104</td>
<td>20.156</td>
<td>20.550</td>
</tr>
<tr>
<td>$S(\bar{w}^*</td>
<td>b)$</td>
<td>0.000</td>
<td>0.230</td>
<td>0.461</td>
<td>0.691</td>
</tr>
</tbody>
</table>
Question (h)

Find the optimal portfolio $w^*$ when we target a given excess score $S^*$.
The values of $S^*$ are 0, 0.1, 0.2, 0.3 and 0.4.
• Same method as previously with the following equation:

$$\mathcal{S}(w | b) = \mathcal{S}^*$$

• An alternative approach consists in solving the following optimization problem:

$$w^* = \arg \min \frac{1}{2} \sigma^2 (w | b)$$

subject to:

$$\begin{cases} 
1^T w = 1 \\
\mathcal{S}(w | b) = \mathcal{S}^* \\
0_6 \leq w \leq 1_6
\end{cases}$$

• We have: $Q = \Sigma$, $R = \Sigma b$, $A = \left( \begin{array}{c} 1^T_6 \\ \mathcal{S}^T \end{array} \right)$, $B = \left( \begin{array}{c} 1 \\ \mathcal{S}^* + \mathcal{S}^T b \end{array} \right)$,

$w^- = 0_6$ and $w^+ = 1_6$
### Table 67: Solution of the $S$-problem

<table>
<thead>
<tr>
<th>Target $S^*$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*$ (in bps)</td>
<td>0.000</td>
<td>1.882</td>
<td>3.764</td>
<td>5.646</td>
<td>7.528</td>
</tr>
<tr>
<td>$w^*$ (in %)</td>
<td>15.000</td>
<td>15.076</td>
<td>15.152</td>
<td>15.228</td>
<td>15.304</td>
</tr>
<tr>
<td></td>
<td>20.000</td>
<td>20.627</td>
<td>21.255</td>
<td>21.882</td>
<td>22.509</td>
</tr>
<tr>
<td></td>
<td>19.000</td>
<td>20.772</td>
<td>22.544</td>
<td>24.315</td>
<td>26.087</td>
</tr>
<tr>
<td></td>
<td>14.000</td>
<td>12.086</td>
<td>10.171</td>
<td>8.257</td>
<td>6.343</td>
</tr>
<tr>
<td></td>
<td>15.000</td>
<td>13.983</td>
<td>12.966</td>
<td>11.949</td>
<td>10.932</td>
</tr>
<tr>
<td></td>
<td>17.000</td>
<td>17.456</td>
<td>17.913</td>
<td>18.369</td>
<td>18.825</td>
</tr>
<tr>
<td>$\sigma(w^*</td>
<td>b)$ (in %)</td>
<td>0.000</td>
<td>0.434</td>
<td>0.868</td>
<td>1.301</td>
</tr>
</tbody>
</table>
Question (i)

We would like to compare the efficient frontier obtained in Question 3(f) with the efficient frontier when we implement a best-in-class selection or a worst-in-class exclusion. The selection strategy consists in investing only in the best three ESG assets, while the exclusion strategy implies no exposure on the worst ESG asset. Draw the three efficient frontiers. Comment on these results.
For the best-in-class strategy, the optimization problem becomes:

\[
\begin{align*}
    w^* &= \arg\min \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b) \\
    \text{s.t.} & \quad \begin{cases}
        \mathbf{1}_6^T w = 1 \\
        w_4 = w_5 = w_6 = 0 \\
        0 \leq w \leq \mathbf{1}_6
    \end{cases}
\end{align*}
\]

The QP form is defined by \( Q = \Sigma, \) \( R = \Sigma b + \gamma S, \) \( A = \mathbf{1}_6^T, \) \( B = 1, \) \( w^- = 0_6 \) and \( w^+ = \begin{pmatrix} 1_3 \\ 0_3 \end{pmatrix} \)
For the worst-in-class strategy, the optimization problem becomes:

\[ w^* = \arg \min w \in \mathbb{R}^n \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b) \]

s.t. \[
\begin{align*}
1^T w &= 1 \\
0 &= w_6 \\
0 \leq w &\leq 1 \\
\end{align*}
\]

The QP form is defined by \( Q = \Sigma, \ R = \Sigma b + \gamma S, \ A = 1^T_6, \ B = 1, \ w^- = 0_6 \) and \( w^+ = \begin{pmatrix} 1_5 \\ 0 \end{pmatrix} \)
• The efficient frontiers are reported in Figure 86
• The exclusion strategy has less impact than the selection strategy
• The selection strategy implies a high tracking error risk
**Figure 86:** Comparison of the efficient frontiers (ESG integration, best-in-class selection and worst-in-class exclusion)
Question (j)

Which minimum tracking error volatility must the investor accept to implement the best-in-class selection strategy? Give the corresponding optimal portfolio.
We solve the first problem of Question 3(i) with $\gamma = 0$

We obtain:

$$\sigma (w \mid b) \geq 11.17\%$$

The lower bound $\sigma (w^* \mid b) = 11.17\%$ corresponds to the following optimal portfolio:

$$w^* = \begin{pmatrix}
16.31\% \\
34.17\% \\
49.52\% \\
0\% \\
0\% \\
0\%
\end{pmatrix}$$
Remark

The impact of ESG scores on optimized portfolios depends on their relationship with expected returns, volatilities, correlations, beta coefficients, etc. In the previous exercise, the results are explained because the best-in-class assets are those with the lowest expected returns and beta coefficients while the worst-in-class assets are those with the highest expected returns and beta coefficients. For instance, we obtain a high tracking error risk for the best-in-class selection strategy, because the best-in-class assets have low volatilities and correlations with respect to worst-in-class assets, implying that it is difficult to replicate these last assets with the other assets.
Course 2022-2023 in Sustainable Finance
Lecture 5. Sustainable Financial Products, Impact Investing & Engagement

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2023

8 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
The big issue for an investor is:

How to avoid Greenwashing (& ESG washing)?

Greenwash (also greenwashing)

- Activities by a company or an organization that are intended to make people think that it is concerned about the environment, even if its real business actually harms the environment
- A common form of greenwash is to publicly claim a commitment to the environment while quietly lobbying to avoid regulation


In finance, greenwashing is understood as making misleading claims about environmental practices, performance or products
We must distinguish two types of risk:

- Explicit & deliberate greenwashing
  
  **Deliberate greenwashing** = mis-selling risk

- Unintentional greenwashing

  **Unintentional greenwashing** = misinterpretation risk
SRI Investment funds

Market

- Investment vehicles
  - Mutual funds
  - ETFs
  - Mandates & dedicated funds
- Investment strategies
  - Thematic strategies (e.g. water, social, wind energy, climate, plastic, etc.)
  - ESG-tilted strategies (e.g. exclusion, negative screening, best-in-class, enhanced ESG score, controlled tracking error, etc.)
  - Climate strategies (e.g. low carbon, 2°C alignment, activity exclusions\(^9\), etc.)
  - Sustainability-linked securities (e.g. green bonds, social bonds, etc.)

Both \(\alpha\) and \(\beta\) management

\(^9\)e.g. coal exploration, oil exploration, electricity generation with a high GHG intensity
## SRI Investment funds

### Market

<table>
<thead>
<tr>
<th>Mutual funds</th>
<th>ETFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amundi Climate Transition</td>
<td>Amundi Index MSCI Europe SRI UCITS ETF</td>
</tr>
<tr>
<td>Amundi ARI European Credit SRI</td>
<td>Amundi MSCI Emerging ESG Leaders UCITS ETF</td>
</tr>
<tr>
<td>AXA World Funds – Euro Bonds SRI</td>
<td>Amundi EURO ISTOXX Climate Paris Aligned PAB UCITS ETF</td>
</tr>
<tr>
<td>CPR Invest Social Impact</td>
<td>Lyxor New Energy UCITS ETF</td>
</tr>
<tr>
<td>Fidelity U.S. Sustainability Index</td>
<td>Lyxor World Water UCITS ETF</td>
</tr>
<tr>
<td>Fidelity Sustainable Water &amp; Waste</td>
<td>SPDR S&amp;P 500 ESG</td>
</tr>
<tr>
<td>Natixis ESG Dynamic Fund</td>
<td>First Trust Global Wind Energy ETF</td>
</tr>
<tr>
<td>Vanguard FTSE Social Index</td>
<td>Invesco S&amp;P 500 ESG UCITS ETF</td>
</tr>
<tr>
<td>Etc.</td>
<td>Etc.</td>
</tr>
</tbody>
</table>

---

**Thierry Roncalli**

Course 2022-2023 in Sustainable Finance
ESG represents **58% of the net new assets** (NNA) in the European ETF market

ESG fund assets reach $1.652 bn

- Europe: $1.343 bn (or 81.3%)
- US: $236.4 bn (or 14.3%)
- Asia: $43.1 bn (or 2.6%)

Net flows into sustainable mutual funds and ETFs in Q4 2020: $370 bn (or **+29% of assets**)

Net flows into sustainable mutual funds and ETFs in 2020

- Europe: $273 bn, almost double the total for 2019, almost 5 times more than in 2017
- US: $51.2 bn, more than double the total for 2019, almost 10 times more than in 2018

## European sustainable finance labels

- Novethic label (pioneer label in 2009, suspended in 2016)
- French SRI label — [https://www.lelabelisr.fr](https://www.lelabelisr.fr)
- FNG label (Germany) — [https://fng-siegel.org](https://fng-siegel.org)
- Towards Sustainability label (Belgium) — [https://www.towardssustainability.be](https://www.towardssustainability.be)
- LuxFLAG label (Luxembourg) — [https://www.luxflag.org](https://www.luxflag.org)
- Nordic Swan Ecolabel (Nordic countries) — [https://www.nordic-ecolabel.org](https://www.nordic-ecolabel.org)
- Umweltzeichen Ecolabel (Austria) — [https://www.umweltzeichen.at/en](https://www.umweltzeichen.at/en)
- French Greenfin label — [https://www.ecologie.gouv.fr/label-greenfin](https://www.ecologie.gouv.fr/label-greenfin)
Remark

According to Novethic (2020), 806 funds had a label at the end of December 2019. Nine months later, this number has increased by 392 and the AUM has be multiplied by 3.2!
“Today it is difficult for consumers, companies and other market actors to make sense of the many environmental labels and initiatives on the environmental performance of products and companies. There are more than 200 environmental labels active in the EU, and more than 450 active worldwide; there are more than 80 widely used reporting initiatives and methods for carbon emissions only. Some of these methods and initiatives are reliable, some not; they are variable in the issues they cover” (European Commission, 2020).

Source: https://ec.europa.eu/environment/eussd/index.htm
SRI Investment funds

Regulation

1. EU taxonomy regulation
2. Sustainable Finance disclosure regulation (SFDR)
3. Climate benchmarks
4. Sustainability preferences (MiFID II & IDD)
SRI Investment funds
Regulation

SFDR

- Article 6: Non-ESG funds (standard funds)
- Article 8: ESG funds (funds that promote \( E \) or \( S \) characteristics)
- Article 9: Sustainable funds (funds that have a sustainable investment objective: impact investing or reduction of carbon emissions)
New benchmark rules

- Climate transition benchmarks (CTB): high level of decarbonization (−30%), no controversial weapons and tobacco, high positive impact on climate change, etc.
- Paris-aligned benchmarks (PAB): high level of decarbonization (−50%), no controversial weapons and tobacco, no activities in coal, oil and natural gas, global warming below 2°, etc.

- MSCI Climate Paris Aligned Indexes — www.msci.com/esg/climate-paris-aligned-indexes
- FTSE TPI Climate Transition Index Series — www.ftserussell.com/products/indices/tpi-climate-transition
- STOXX Climate Transition Benchmark (CTB) and STOXX Paris-Aligned Benchmark (PAB) Indices — qontigo.com/solutions/climate-indices
### Table 68: Sustainable fixed-income market

<table>
<thead>
<tr>
<th>Theme</th>
<th>Label</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSS+</td>
<td>Green</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td></td>
<td>Social</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td></td>
<td>Sustainability</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td>Transition</td>
<td>Sustainability-Linked</td>
<td>Entity KPI-linked</td>
</tr>
<tr>
<td></td>
<td>Transition</td>
<td>Use of proceeds</td>
</tr>
</tbody>
</table>

Source: CBI (2022).
Figure 87: Issuance of GSS securities (in $ bn)

Source: https://www.climatebonds.net/market/data.
Green bonds

Definition

Green bonds are any type of bond instrument where the proceeds or an equivalent amount will be exclusively applied to finance or re-finance, in part or in full, new and/or existing eligible green projects and which are aligned with the four core components of the Green Bond Principles (GBP).

⇒ Green bonds are “regular” bonds\(^\text{10}\) aiming at funding projects with positive environmental and/or climate benefits

\(^\text{10}\) A regular bond pays regular interest to bondholders
Green Bonds Principles (GBP)

The 4 core components of the GBP are:

1. Use of proceeds
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

Green bonds
Green Bonds Principles

The use of proceeds includes:

- Renewable energy
- Energy efficiency
- Pollution prevention (e.g. GHG control, soil remediation, waste recycling)
- Sustainable management of living natural resources (e.g. sustainable agriculture, sustainable forestry, restoration of natural landscapes)
- Terrestrial and aquatic biodiversity conservation (e.g. protection of coastal, marine and watershed environments)
- Clean transportation
- Sustainable water management
- Climate change adaptation
- Eco-efficient products
- Green buildings
With respect to the **process for project evaluation and selection** (component 2), the issuer of a green bond should clearly communicate:

- the environmental sustainability objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the “balance sheet” and the allocation of funds\(^\text{11}\)
- An external review (not mandatory but highly recommended)

\(^{11}\)The proceeds should be credited to a sub-account
The **reporting** (component 4) must be based on the following pillars:

- **Transparency**
- **Description of the projects, allocated amounts and expected impacts**
- **Qualitative performance indicators**
- **Quantitative performance measures** (e.g. energy capacity, electricity generation, GHG emissions reduced/avoided, number of people provided with access to clean power, decrease in water use, reduction in the number of cars required)**
Standardization is strongly required by investors and regulators

- Green Bond Principles\(^{12}\) (ICMA, 2021)
- Climate Bonds Standard\(^{13}\) (CBI, 2019)
- EU Green Bond Standard (2021)
- China Green Bond Principles (PBOC, CBIRC, July 2022)
- Asean Green Bond Standards (ACMF, 2018)

\(^{12}\) The first version is published in January 2014

\(^{13}\) The first version is published in November 2011
Green bonds
Types of debt instruments

Asset-linked bond structures
- Regular bond
- Revenue bond
- Project bond
- Green loans

Asset-backed bond structures
- Securitized bond
- Project bond
- ABS/MBS/CLO/CDO
- Covered bond
Green bonds
Certification

- Second party opinion provided by ESG rating agencies (ISS, Sustainalytics, Vigeo-Eiris);
- Certification by specialized green bond entities (CBI, CICERO, DNV);
- Green bond assessment by statistical rating organizations (Moody’s, S&P).
Green bonds

Examples

- Solar bond by the City of San Francisco in 2001
- Equity-linked climate awareness bond by the European Investment Bank (EIB) in 2007
- First green bond issued by the World Bank (in collaboration with Skandinaviska Enskilda Banken) in November 2008
- First corporate green bonds: French utility company EDF ($1.8 bn) and Swedish real estate company Vasakronan ($120 bn)
- Toyota introduced the auto industry’s first-ever asset-backed green bond in 2014 ($1.75 bn)
- The Commonwealth of Massachusetts issued the first municipal green bond in 2013 ($100 mn)
- The first sovereign green are: Poland in December 2016 ($1 bn) and France\(^{14}\) in January 2017 ($10 bn)

\(^{14}\) Green OAT 1.75% 25 June 2039.
Green bonds
The green bond market

Green bond issuers
- Sovereigns (agencies, municipals, governments)
- Multilateral development banks (MDB)
- Energy and utility companies
- Banks
- Other corporates

Green bond investors
- Pension funds
- Sovereign wealth funds
- Insurance companies
- Asset managers
- Retail investors (e.g. employee savings plans)

Strong imbalance between supply and demand
Green bonds
The green bond market

Figure 88: Issuance and notional outstanding of green debt by market type

Source: https://www.climatebonds.net/market/data.
**Figure 89:** Issuance and notional outstanding of green debt by region

Source: https://www.climatebonds.net/market/data.
Figure 90: Issuance and notional outstanding of green debt by use of proceeds

Source: https://www.climatebonds.net/market/data.
Figure 91: Issuance and notional outstanding of green debt by issuer type

Source: https://www.climatebonds.net/market/data.
Green bonds
How to investing in green bonds

Example of green bond funds:
- Allianz IG green bond fund
- Amundi RI impact green bonds
- AXA WF ACT green bonds
- BNP Paribas green bond
- Calvert green bond fund
- Mirova global green bond fund
- TIAA-CREF green bond fund
- Etc.
Green bonds
How to investing in green bonds

List of green bond indices:
- Bloomberg Barclays MSCI Global Green Bond Index
- S&P Green Bond Index
- Solactive Green Bond Index
- ChinaBond China Climate-Aligned Bond Index:
- ICE BofA Green Index

⇒ ETF and index funds
“I show that investors respond positively to the issuance announcement, a response that is stronger for first-time issuers and bonds certified by third parties. The issuers improve their environmental performance post-issuance (i.e., higher environmental ratings and lower CO\textsubscript{2} emissions) and experience an increase in ownership by long-term and green investors. Overall, the findings are consistent with a signaling argument – by issuing green bonds, companies credibly signal their commitment toward the environment.” (Flammer, 2021, page 499).
Green bonds

The economics of green bonds

Green bonds = second-best instrument
Green bonds
The green bond premium

Definition

- The green bond premium (or greenium) is the difference in pricing between green bonds and regular bonds.
- The greenium is defined as:

\[ g = y(\text{GB}) - y(\text{CB}) \]

where \( y(\text{GB}) \) is the yield (or return) of the green bond and \( y(\text{CB}) \) is the yield (or return) of the conventional twin bond.
Green bonds
The green bond premium

- From the issuer’s point of view, a green bond issuance is more expensive than a conventional issuance due to the need for external review, regular reporting and impact assessments.
- From the investor’s point of view, there is no fundamental difference between a green bond and a conventional bond, meaning that one should consider a negative green bond premium as a market anomaly.
Green bonds
The green bond premium

Green twin bonds

- Introduced in 2020 by Germany
- Issuance of a green and conventional bond at the same time with the same characteristics
- Investors may swap the green bond with the conventional bond any time, but not vice-versa
- Liquidity of the green bond market ↑
Examples of twin bonds:

- On 3 September 2020, the 10-year German green bond with a coupon of 0.00% was priced 1 basis point below the 10-year conventional German bond.

- On 19 January 2022, Denmark issued a 10-year green bond with the same maturity, interest payment dates and coupon rate as the conventional 2031 Danish bond. The effective yield of the green bond was 5 basis points below the twin regular bond.
Green bonds
The green bond premium

Example #1

We consider a 10-year green bond $GB_1$ whose current price is equal to 91.35. The corresponding conventional twin bond is a 20-year regular bond, whose remaining maturity is exactly equal to ten years and its price is equal to 90.07%. We assume that the two bonds have the same coupon level, which is equal to 4%.
Computation of the greenium with the current yield:

- We have:
  \[ y(GB) = \frac{4}{91.35} = 4.379\% \]
  and:
  \[ y(CB) = \frac{4}{90.07} = 4.441\% \]
- We deduce that the greenium is equal to:
  \[ g = 4.441\% - 4.379\% = -6.2 \text{ bps} \]
Green bonds
The green bond premium

Computation of the greenium with the yield to maturity:

- We solve the equation:

\[
\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 91.35
\]

and find:

\[ y(GB) = 5\% \]

- We solve the equation:

\[
\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 90.07
\]

and find:

\[ y(CB) = 5.169\% \]

- We deduce that the greenium is equal to:

\[ g = 5\% - 5.169\% = -16.9 \text{ bps} \]
**Green bonds**

The green bond premium

**Figure 92:** Greenium in bps of the German green (twin) bond (DBR 0% 15/08/2030)

Source: ICE (2022).
Green bonds

The green bond premium

What about the greenium when the green bond is not a twin bond?

⇒ We must distinguish primary and secondary markets
Green bonds
The green bond premium

- In the primary market, the greenium is negative ($\approx 5 - 10$ bps on average)
- How to measure the persistence of the greenium in the secondary market?
Green bonds
The green bond premium

There are two approaches:

- **Bottom-up approach**
  - Compares the green bond of an issuer with a synthetic conventional bond of the same issuer
  - Same characteristics in terms of currency, seniority and duration

- **Top-down approach**
  - Compare a green bond index portfolio to a conventional bond index portfolio
  - Same characteristics in terms of currency, sector, credit quality and maturity
Green bonds
The green bond premium

**Bottom-up approach**

1. We filter all the conventional bonds, which has the same issuer, the same currency, and the same seniority of the green bond GB.

2. We select the two conventional bonds CB\(_1\) and CB\(_2\) which are the nearest in terms of modified duration:

   \[ | MD(GB) - MD(CB_j) |_{j\neq 1,2} \geq \sup_{j=1,2} | MD(GB) - MD(CB_j) | \]

3. We perform the linear interpolation/extrapolation of the two yields \( y(CB_1) \) and \( y(CB_2) \):

   \[ y(CB) = y(CB_1) + \frac{MD(GB) - MD(CB_1)}{MD(CB_2) - MD(CB_1)} (y(CB_2) - y(CB_1)) \]
Example #2

- We consider a green bond, whose modified duration is 8 years. Its yield return is equal to 132 bps.
- We can surround the green bond by two conventional bonds with modified duration 7 and 9.5 years. The yield is respectively equal to 125 and 148 bps.
- The interpolated yield is equal to:

\[ y(CB) = 125 + \frac{8 - 7}{9.5 - 7} (148 - 125) \]

\[ = 134.2 \text{ bps} \]

- It follows that the greenium is equal to \(-2.2 \text{ bps}\):

\[ g = 132 - 134.2 = -2.2 \text{ bps} \]
We consider a portfolio \( w = (w_1, \ldots, w_n) \) of green bonds. We perform a clustering analysis by considering the 4-uplets (Currency \( \times \) Sector \( \times \) Credit quality \( \times \) Maturity). Let \((C_h, S_j, R_k, M_l)\) be an observation for the 4-uplet (e.g. EUR, Financials, AAA, 1Y-3Y). We compute its weight:

\[
\omega_{h,j,k,l} = \sum_{i \in (C_h,S_j,R_k,M_l)} w_i
\]

The greenium is then defined as the weighted excess yield:

\[
g = \sum_{h,j,k,l} \omega_{h,j,k,l} \left( y_{h,j,k,l}(GB) - y_{h,j,k,l}(CB) \right)
\]
Main result (Ben Slimane et al., 2020)

The greenium is negative between $-5$ and $-2$ bps on average

Other results:

- Differences between sectors, currencies, maturities, regions and ratings
- Transatlantic divided between US and Europe
- The volatility of green bond portfolios are lower than the volatility of conventional bond portfolios $\Rightarrow$ identical Sharpe ratio since the last four years
- Time-varying property of the greenium
Green bonds
The green bond premium

Figure 93: Evolution of the greenium (in bps)

Source: Ben Slimane et al. (2020)
Green bonds

The green bond premium

Green financing $\iff$ green investing

1. Bond issuers have a competitive advantage to finance their environmental projects using green bonds instead of conventional bonds

2. Another premium? the “green bond issuer premium”
Social bonds

**Definition**

Social Bonds are any type of bond instrument where the proceeds, or an equivalent amount, will be exclusively applied to finance or re-finance in part or in full new and/or existing eligible social projects and which are aligned with the four core components of the Social Bond Principles (SBP).

Source: ICMA (2021), https://www.icmagroup.org/sustainable-finance
Social bonds
Social Bonds Principles

Social Bonds Principles (SBP)

The 4 core components of the SBP are:

1. Use of proceeds
   - Eligible social project categories
   - Target populations
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

The **eligible social projects categories** (component 1) are:

- Affordable basic infrastructure (e.g. clean drinking water, sanitation, clean energy)
- Access to essential services (e.g. health, education)
- Affordable housing (e.g. sustainable cities)
- Employment generation (e.g. pandemic crisis)
- Food security and sustainable food systems (e.g. nutritious and sufficient food, resilient agriculture)
- Socioeconomic advancement and empowerment (e.g. income inequality, gender inequality)
- Etc.
The **target populations** (component 1) are:

- Living below the poverty line
- Excluded and/or marginalised populations/communities
- People with disabilities
- Migrants and/or displaced persons
- Undereducated
- Unemployed
- Women and/or sexual and gender minorities
- Aging populations and vulnerable youth
- Etc.
Social bonds
Social Bonds Principles

With respect to the **process for project evaluation and selection** (component 2), the issuer of a social bond should clearly communicate:

- the social objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the “*balance sheet*” and the allocation of funds\(^{15}\)
- An external review (not mandatory but highly recommended)

\(^{15}\)The proceeds should be credited to a sub-account
The **reporting** (component 4) must be based on the following pillars:

- Transparency
- Description of the projects, allocated amounts and expected impacts
- Qualitative performance indicators
- Quantitative performance measures (e.g. number of beneficiaries)
Social bonds

Market

Figure 94: Issuance of social bonds

Source: https://www.climatebonds.net/market/data.
Instituto de Crédito Oficial (Spanish state-owned bank, March 2020)

“The Social Bond proceeds under ICO's Second – Floor facilities will be allocated to loans to finance small, medium and micro enterprises with an emphasis on employment creation or employment retention in: (1) specific economically underperforming regions of Spain; (2) specific municipalities of Spain facing depopulation; (3) regions affected by a natural disaster. [...] The target populations are SMEs in line with European Union’s standards.”
Pepper Money (non-bank lender in Australia and New Zealand, April 2022)

“The positive social impact of a Pepper Money eligible social project derives from its direct contribution to improving access to financial services and socio-economic empowerment, by using proprietary systems to make flexible loan solutions available to applicants who are not served by traditional banks. [...] Pepper Money is seeking to achieve positive social outcomes for a target population of Australians that lack access to essential financial services and experience inequitable access to and lack of control over assets. Pepper Money directly aims to address the positive social outcome of home ownership for borrowers who may have complexity in their income streams, gaps in their loan documentation or have adverse credit history. Traditionally, this cohort has been underserved by banks that rely on inflexible algorithmic loan application processing.”
Danone (French multinational food-products corporation, March 2018)

“The eligible project categories are: (1) research & innovation for advanced medical nutrition (target populations: infants, pregnant women, patients and elderly people with specific nutritional needs), (2) social inclusiveness (target populations: farmers, excluded and/or marginalised populations and/or communities, people living under the poverty line, rural communities in developing countries), (3) responsible farming and agriculture (target populations: milk producers, farmers), etc.”
Korian (European care group, October 2021)

“The proceeds of any instrument issued under the framework will be used […] to provide services, solutions, and technologies that will enable Korian to meet at least one of its social objectives: (1) to increase and improve long-term care nursing home capacity for dependent older adults; (2) to increase and improve medical capacity for people in need of medical support; (3) to increase and improve access to alternative, nonmedical services, technologies, and housing solutions that facilitate the retention of older adults’ autonomy; and (4) to improve the daily provision of care to and foster a safer living environment for its patients. […] Furthermore, Korian’s target populations are older adults, which Korian defines as being over 65 years of age, and those who are dependent on others for some degree of care, which is defined by the health authorities or insurance system of the respective country.”
JASSO (Japan Student Services Organization, July 2022)

“The social project categories concern the financing of the ‘Category 2 Scholarship Loans’ (interest-bearing scholarship loans that have to be repaid) while the target population is made up of students with financial difficulties.”
Other sustainability-related instruments
Sustainability bonds

Sustainability bond = GBP + SBP

Remark
According to CBI, the cumulative issuance of sustainability bonds reaches $620 bn at the end of June 2022
Sustainability-linked bond (SLB)

- Two principles:
  - = a sustainability bond (green/social)
  - + a step up coupon if the KPI is not satisfied
  ⇒ forward-looking performance-based instrument
- The financial characteristics of the bond depends on whether the issuer achieves predefined ESG objectives
- Those objectives are:
  1. measured through predefined Key Performance Indicators (KPI)
  2. assessed against predefined Sustainability Performance Targets (SPT)
ENEL General Purpose SDG Linked Bond

- SDG: 7 (affordable and clean energy), 13 (climate action), 9 (industry, innovation and infrastructure) and 11 (sustainable cities and communities)
- SDG 7 target: renewables installed capacity as of December 31, 2021 ≥ 55% (confirmed by external verifier)
- One time step up coupon of 25 bps if SDG 7 is not achieved
- On April 2022, the independent report produced by KPMG certifies that “the renewables installed capacity percentage as of December 31, 2021 is equal to 57.5%”.
Other sustainability-related instruments
Sustainability-linked bonds (SLB)

H&M sustainability-linked bond

- 18 February 2021
- €500 mn
- Maturity of 8.5 years
- The annual coupon rate is 25 bps
- The objectives to achieve by 2025 are:
  - KPI₁ Increase the share of recycled materials used to 30% (SPT₁)
  - KPI₂ Reduce emissions from the Group’s own operations (scopes 1+2) by 20% (SPT₂)
  - KPI₃ Reduce scope 3 emissions from fabric production, garment manufacturing, raw materials and upstream transport by 10% (SPT₃)
- The global KPI is equal to 40% × KPI₁ + 20% × KPI₂ + 40% × KPI₃
- The step-up of the coupons can consequently be 0%, 20%, 40%, 60%, 80% or 100% of the total step-up rate

Thierry Roncalli
According to Berrada et al. (2022), “the SLB market has grown strongly since its inception. [...] Bloomberg identifies a total of 434 outstanding bonds flagged as ‘sustainability-linked’ as of February 2022. In contrast, in 2018, there was only a single SLB. The amount raised through the single 2018 SLB issue was $0.22 bn, whereas the total amount raised through all SLBs issued in 2021 was approximately $160 bn”.

- The large majority of SLB issues address exclusively E issues (65%) or a combination of E, S and G issues (17%) or E and G issues (3%)
- The most frequent KPI concerns GHG emissions (40 %), followed by the issuer’s global ESG score (14 %)
Other sustainability-related instruments

Transition bonds

- Financial instruments to support the transition of an issuer, which has significant current carbon emissions
- Fund projects such as renewable energy developments, energy efficiency upgrades, etc.
- The final objective of the bond issuer is always to reduce their carbon emissions
- For example, transition bonds can be used to switch diesel powered ships to natural gas or to implement carbon capture and storage.
Sustainable real assets
What is the final motivation of the ESG investor?

Financial performance or/and extra-financial performance?
The key elements of impact investing are:

1. **Intentionality**
   The intention of an investor to generate a positive and measurable social and environmental impact

2. **Additionality**
   Fulfilling a positive impact beyond the provision of private capital

3. **Measurement**
   Being able to account for in a transparent way on the financial, social and environmental performance of investments

Source: Eurosif (2019)

The investor must be able to measure its impact from a quantitative point of view
Figure 95: Global Impact Investing Network (GIIN)

https://thegiin.org
The example of social impact bonds

Social impact bond (SIB) = pay-for-success bond ($\approx$ call option)

The Peterborough SIB

- On 18 March 2010, the UK Secretary of State for Justice announced a six-year SIB pilot scheme that will see around 3,000 short-term prisoners from Peterborough prison, serving less than 12 months, receiving intensive interventions both in prison and in the community.
- Funding from investors will be initially used to pay for the services.
- If reoffending is not reduced by at least 7.5%, the investors will receive no recompense.
The example of sustainability-linked bonds

Sustainability-linked\(^\text{16}\) (SLB) = **pay-for-failure bond** (\(\approx\) cap option)

**Risk taker**

SIB: investor viewpoint \(\neq\) SLB: issuer viewpoint

\(^{16}\text{See the examples of ENEL and H&M previously}\)
Measurement tools

Impact assessment and metrics

- Avoided CO2 emissions in tons per $M invested
- Amount of clean water produced by the project
- Number of children who are less obese
- Land management
- Affordable housing
- Job creation
- Construction of student housing
The sustainable development goals are a collection of 17 interlinked global goals designed to be a "blueprint to achieve a better and more sustainable future for all"

https://sdgs.un.org
Figure 96: The map of sustainable development goals
Figure 97: Mapping the SDGs across E, S, and G
Sustainable development goals (SDG)

Figure 98: Examples of sovereign SDG reports

The challenge of reporting

- Impact reporting and investment standards (IRIS) proposed by GIIN
- EU taxonomy on sustainable finance
- Non-financial reporting directive 2014/95/EU (NFRD)
- Carbon accounting
### The challenge of reporting

#### Table 69: Impact reporting of the CPR Invest — Social Impact fund

<table>
<thead>
<tr>
<th>Social indicator</th>
<th>Social indicator</th>
<th>Coverage ratio</th>
<th>Coverage ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Global Index</td>
<td>CPR Fund</td>
<td>Global Index</td>
</tr>
<tr>
<td>CEO pay ratio</td>
<td>333</td>
<td>114</td>
<td>82%</td>
</tr>
<tr>
<td>% of women in the board direction</td>
<td>18%</td>
<td>19%</td>
<td>79%</td>
</tr>
<tr>
<td>Hours of training</td>
<td>33 hours</td>
<td>39 hours</td>
<td>33%</td>
</tr>
<tr>
<td>Trade union rate</td>
<td>36%</td>
<td>45%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Source: CPR Asset Management (2021)
The challenge of reporting

- **Amundi ARI – Impact Green Bonds (Annual impact record 2020)**
  - GHG avoided emissions per €1 mn invested per year: 586.5 tCO$_2$e
  - GHG avoided emissions rebased per €1 mn invested per year: 882.7 tCO$_2$e

- **CPR Invest – Climate Action**
  - –69% of tCO$_2$e wrt MSCI ACWI

- **CPR Invest – Food For Generations**
  - **Water consumption**: 6,765 m$^3$/meur for the fund vs 13,258 for the benchmark and 18,869 for the universe
  - **Waste recycling ratio**: 71.14% for the fund vs 66.45% for the benchmark and 67.22% for the universe

Source: Amundi (2021) and CPR Asset Management (2021)
### Table 70: Impact investing reporting of the Amundi Finance & Solidarité fund

<table>
<thead>
<tr>
<th>Category</th>
<th>2020</th>
<th>Since inception (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>People housed</td>
<td>2,364</td>
<td>10,336</td>
</tr>
<tr>
<td>Job created/preserved</td>
<td>9,439</td>
<td>43,655</td>
</tr>
<tr>
<td>Care recipients</td>
<td>83,240</td>
<td>250,314</td>
</tr>
<tr>
<td>Trained people</td>
<td>18,702</td>
<td>59,686</td>
</tr>
<tr>
<td>Preserved agricultural farmland (hectare)</td>
<td>438</td>
<td>987</td>
</tr>
<tr>
<td>Waste recycling (ton)</td>
<td>82,590</td>
<td>219,287</td>
</tr>
<tr>
<td>Microcredit beneficiaries</td>
<td>60,171</td>
<td>276,514</td>
</tr>
</tbody>
</table>

Source: Amundi (2021)
The challenge of reporting

Figure 99: Companies’ portfolio contribution of the Finance & Solidarité fund

Source: Amundi (2021)
Voting $\subset$ Engagement $\subset$ Stewardship
**Figure 100**: Difference between stewardship and engagement reports

Amundi Stewardship Report (2021)

Amundi Engagement Report (2021)

“It guides investors on how to implement the PRI’s Principle 2, which sets out signatories’ commitment to stewardship, stating: we will be active owners and incorporate ESG issues into our ownership policies and practices. [...] The PRI defines stewardship as the use of influence by institutional investors to maximise overall long-term value including the value of common economic, social and environmental assets, on which returns and clients’ and beneficiaries’ interests depend.” (PRI, 2021).
Active ownership ≈ Engagement ≈ Shareholder activism

“investors who, dissatisfied with some aspect of a company’s management or operations, try to bring about change within the company without a change in control” Gillan and Starks (2000).
Definition

- Conflicting interests between shareholders and management (separation between ownership and control)
- Stakeholder theory (Freeman, 2004)

**Milton Friedman (1970)**

"the social responsibility of business is to increase its profits"

**Peter Drucker (1954)**

"leaders in every single institution and in every single sector . . . have two responsibilities. They are responsible and accountable for the performance of their institutions, and that requires them and their institutions to be concentrated, focused, limited. They are responsible also, however, for the community as a whole"
Shareholder activism can take various forms

1. Engage behind the scene with management and the board
2. Propose resolutions (shareholder proposals)
3. Vote (form coalition/express dissent/call back lent shares)
4. Voice displeasure publicly (in the media)
5. Initiate a takeover (acquire a sizable equity share)
6. Exit (sell shares, take an offsetting bet)

Source: Bekjarovski and Brière (2018)
“Behind the curtain engagement involves private communication between activist shareholders and the firm’s board or management, that tends to precede public measures such as vote, shareholder proposals and voice. In a sense, the existence of other forms of public activism can be taken as a signal that behind the scene engagements were unsuccessful. When it comes to environmental and social issues, writing to the board or management is a common method though which shareholders can express concern and attempt to influence corporate policy behind the curtain; alternatively, face to face meetings with management or non-executive directors are a more common behind the scene engagement method when it comes to governance.” Bekjurovski and Brière (2018).
Shareholder activism
Engage behind the scenes

Three families of engagement:

1. on-going engagement, where the goal for investors is to explain their ESG policy and collect information from the company. For instance, they can encourage companies to adopt best ESG practices, alert companies on ESG risks or better understand sectorial ESG challenges;

2. engagement for influence (or protest), where the goal is to express dissatisfaction with respect to some ESG issues, make recommendations to the firm and measure/control ESG progress of companies;

3. pre-AGM engagement, where the goal is to discuss with companies any resolution items that the investor may vote against.
Shareholder activism
Engage behind the scenes

The three steps of identification are:
1. List of engagement issues
2. Screening of companies
3. List of targeted companies

The different stages of engagement tracking are:
- Issues are raised to the company;
- Issues are acknowledged by the company;
- The company develops a strategy to address the issues;
- The company implements changes and the issues are resolved;
- The company did not solve the issues and the engagement failed.
According to the SEC (Securities Exchange Act Rule 14a-8, §240):

“a shareholder proposal or resolution is a recommendation or requirement that the company and/or its board of directors take action, which the shareholder intend to present at a meeting of the company’s shareholders. The proposal should state as clearly as possible the course of action that the shareholder believes the company should follow. If the proposal is placed on the company’s proxy card, the company must also provide in the form of proxy means for shareholders to specify by boxes a choice between approval or disapproval, or abstention.”
Threshold criteria:

- US: $2,000 + No-action letter
- France, Germany, and UK: 5% of the capital
- Italy: 2.5% of the capital
- Netherlands: 0.33%
- Spain: 3% of the capital

⇒ Collective shareholder proposals

Shareholder resolution = Escalation
Shareholder activism
Propose resolutions

Some figures (Russell 300 & 2022 proxy season)

- 98% of proposals are filed by the management, while less than 2% corresponds to shareholder resolutions;
- Only 60% of shareholder resolutions are voted; The other 40% are omitted, not presented, withdrawn or pending;
- The average number of proposals per company is around two;
- The proponents of shareholder resolutions are concentrated on a small number of investors or organisations (15 proponents were responsible of 75% of shareholder proposals);
- The repartition of shareholder proposals voted in 2022 was the following: 11% related to E issues, 41% related to S issues and 48% related to G issues.
Shareholder activism

Vote

- Historical perspectives
- Importance of voting associations and NGOs
- US ≻ Europe
- The concept of proxy voting
  - Institutional Shareholder Services (ISS)
  - Glass Lewis
- Say on Pay (2002)
  - Support rate for Russell 3000 companies: 87% in 2022 (from 15.4% to 99%)
  - Results for Germany, France and Spain
- Say on Climate (2020)
Figure 101: Average support rate of shareholder proposals (Russell 3000 companies)

Source: PwC’s Governance Insights Center (2022).
Shareholder activism

Vote

Some figures with Russell 3000 companies

- 555 shareholder resolutions have been voted
- Only 82 have received majority support
- This means that one shareholder resolution was adopted for 37 companies!

What is the efficiency of vote? ≠ What is the impact of vote?
Figure 102: Pass rate of shareholder proposals (Russell 3000 companies)

1970: Publication of the book *Exit, Voice, and Loyalty: Responses to Decline in Firms, Organizations, and States* by the economist Albert Hirschman

- Exist-voice model: exist **versus** voice or exit **and** voice
- Voice as a form of escalation
- Impact of collaborative engagement (e.g., Climate Action 100+)
- Increasing involvement of NGOs in the debate on engagement and greenwashing
Shareholder activism
Initiate a takeover

⇒ Hedge funds
Shareholder activism

**Exit** refers to the process of selling off investments in a particular company or industry

**Divestment** is a more general term that implies a significant exposure reduction

Divestment: Final step in an escalation strategy?
Figure 103: What kinds of institutions are divesting from fossil fuel?

Source: https://divestmentdatabase.org.
Case study: the Cambridge University endowment fund

“A dilemma faced by an increasing number of investors is whether to divest from environmentally damaging businesses or whether to enter into a dialogue with them. This predicament now has its epicentre in Cambridge, England, where the ancient University of Cambridge faces great pressure from students and staff to respond to the threat of climate breakdown. Having already received two reports on its approach to responsible investment, the university has appointed a new chief investment officer (CIO) who, alongside University Council and the wider university community, needs to consider the question of whether to divest from or to engage with fossil-fuel firms.” Chambers et al. (2020).
Case study: Church of England Pensions Board

In 2020, they engaged with 21 companies. At the end of the process, 12 companies were supposed to make sufficient progress, while 9 companies were added to the list of restricted investments. These divestments totalled £32.23 mn (wrt £3.7 bn of assets under management).
Case study: The Universities Superannuation Scheme (USS)

- USS manage about £90 bn
- In 2020, they excluded certain sectors: tobacco manufacturing; thermal coal mining (coal to be burned for electricity generation), specifically where they made up more than 25% of revenues, and certain controversial weapons
- The first exclusion was announced in May 2020
- Two years after, divestment from these sectors is completed
- Ethics for USS ⇒ USS should extend its divestment policy
Individual vs. collaborative engagement
The role of institutional investors
Impact of active ownership
Voting process

- "The company sets the agenda for the annual shareholder meeting;
- The custodian confirms the identity of the shareholders and the number of shares eligible for voting – often for a specific date ahead of the meeting (record date);
- Shareholders receive the meeting materials from the company (may be before or after the record date);
- Shareholders procuring proxy advisory services receive voting recommendations;
- Shareholders instruct the custodian on how to vote, often through a proxy voting service provider, within a deadline ahead of the shareholder meeting (cut-off date);
- Voting takes place at the shareholder meeting;
- Shareholders receive confirmation from the service provider that their voting instructions have been carried out."
Proxy voting
Figure 104: Voting Matters series of ShareAction

Source: https://shareaction.org.
## Statistics about ESG voting

### Table 71: Statistics of success rate shareholder resolutions

<table>
<thead>
<tr>
<th>Year</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of resolutions</td>
<td>64</td>
<td>102</td>
<td>144</td>
<td>249</td>
</tr>
<tr>
<td>Resolutions with majority support</td>
<td>3</td>
<td>15</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>Success rate (in %)</td>
<td>4.7</td>
<td>14.7</td>
<td>20.1</td>
<td>14.9</td>
</tr>
<tr>
<td>Average support rate (in %)</td>
<td>28.2</td>
<td>29.9</td>
<td>32.9</td>
<td>29.9</td>
</tr>
<tr>
<td>10%</td>
<td>6.5</td>
<td>9.2</td>
<td>7.2</td>
<td>9.4</td>
</tr>
<tr>
<td>25%</td>
<td>17.0</td>
<td>13.1</td>
<td>12.0</td>
<td>13.5</td>
</tr>
<tr>
<td>75%</td>
<td>37.7</td>
<td>42.6</td>
<td>42.8</td>
<td>40.3</td>
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<tr>
<td>90%</td>
<td>41.8</td>
<td>55.2</td>
<td>81.2</td>
<td>57.6</td>
</tr>
<tr>
<td>Average support rate (in %)</td>
<td>28.2</td>
<td>35.8</td>
<td>41.8</td>
<td>31.6</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>24.5</td>
<td>28.8</td>
<td>27.4</td>
<td></td>
</tr>
</tbody>
</table>

Figure 105: Histogram (in %) of support rates

### Table 72: Average support rate in % for ESG resolutions

<table>
<thead>
<tr>
<th>Topic</th>
<th>Method</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>Arithmetic</td>
<td>45.8</td>
<td>57.4</td>
<td>58.9</td>
<td>65.0</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>32.7</td>
<td>42.1</td>
<td>47.6</td>
<td>46.5</td>
</tr>
<tr>
<td>Environment</td>
<td>Arithmetic</td>
<td>45.8</td>
<td>61.0</td>
<td>66.0</td>
<td>64.8</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>32.7</td>
<td>44.7</td>
<td>55.8</td>
<td>48.8</td>
</tr>
<tr>
<td>Social</td>
<td>Arithmetic</td>
<td>53.3</td>
<td>55.2</td>
<td>62.7</td>
<td></td>
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<tr>
<td></td>
<td>Weighted</td>
<td>39.0</td>
<td>43.7</td>
<td>44.3</td>
<td></td>
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<tr>
<td>Pay &amp; politics</td>
<td>Arithmetic</td>
<td>39.0</td>
<td>43.7</td>
<td>71.5</td>
<td></td>
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<tr>
<td></td>
<td>Weighted</td>
<td></td>
<td></td>
<td>47.8</td>
<td></td>
</tr>
</tbody>
</table>

Statistics about ESG voting

Asset managers

Figure 106: Arithmetic average support rate in % per country and year

Overall

Environment

Social

Pay

Figure 107: Weighted average support rate in % per country and year

Table 73: Best performers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM</th>
<th>Overall</th>
<th>E</th>
<th>S</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Achmea IM</td>
<td>Netherlands</td>
<td>251</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>Impax AM</td>
<td>UK</td>
<td>56</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>BNP PAM</td>
<td>France</td>
<td>761</td>
<td>99</td>
<td>97</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>MN</td>
<td>Netherlands</td>
<td>193</td>
<td>99</td>
<td>97</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>Candriam</td>
<td>Luxembourg</td>
<td>180</td>
<td>98</td>
<td>97</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>PGGM</td>
<td>Netherlands</td>
<td>331</td>
<td>97</td>
<td>93</td>
<td>100</td>
<td>97</td>
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<tr>
<td>7</td>
<td>Man</td>
<td>UK</td>
<td>149</td>
<td>96</td>
<td>98</td>
<td>94</td>
<td>98</td>
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<tr>
<td>8</td>
<td>Robeco</td>
<td>Netherlands</td>
<td>228</td>
<td>95</td>
<td>94</td>
<td>94</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>Aviva Investors</td>
<td>UK</td>
<td>363</td>
<td>93</td>
<td>88</td>
<td>96</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>Amundi AM</td>
<td>France</td>
<td>2348</td>
<td>93</td>
<td>93</td>
<td>92</td>
<td>98</td>
</tr>
<tr>
<td>11</td>
<td>Nordea AM</td>
<td>Finland</td>
<td>333</td>
<td>91</td>
<td>93</td>
<td>89</td>
<td>90</td>
</tr>
<tr>
<td>12</td>
<td>Aegon AM</td>
<td>Netherlands</td>
<td>466</td>
<td>90</td>
<td>85</td>
<td>94</td>
<td>90</td>
</tr>
<tr>
<td>13</td>
<td>Federated Hermes</td>
<td>UK</td>
<td>672</td>
<td>89</td>
<td>88</td>
<td>87</td>
<td>90</td>
</tr>
<tr>
<td>14</td>
<td>Pictet AM</td>
<td>Switzerland</td>
<td>284</td>
<td>88</td>
<td>85</td>
<td>90</td>
<td>91</td>
</tr>
<tr>
<td>15</td>
<td>Legal &amp; General</td>
<td>Switzerland</td>
<td>1923</td>
<td>86</td>
<td>84</td>
<td>84</td>
<td>98</td>
</tr>
</tbody>
</table>

Source: ShareAction (2023) & Author’s calculations.
Table 74: Worst performers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM</th>
<th>Overall</th>
<th>E</th>
<th>S</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>Goldman Sachs AM</td>
<td>US</td>
<td>2 218</td>
<td></td>
<td>56</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>60</td>
<td>Baillie Gifford</td>
<td>UK</td>
<td>455</td>
<td></td>
<td>29</td>
<td>29</td>
<td>45</td>
</tr>
<tr>
<td>61</td>
<td>SSGA</td>
<td>US</td>
<td>4 140</td>
<td></td>
<td>30</td>
<td>31</td>
<td>22</td>
</tr>
<tr>
<td>62</td>
<td>BlackRock</td>
<td>US</td>
<td>10 014</td>
<td></td>
<td>28</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>63</td>
<td>T. Rowe Price</td>
<td>US</td>
<td>1 642</td>
<td></td>
<td>26</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>64</td>
<td>Fidelity Investments</td>
<td>US</td>
<td>4 520</td>
<td></td>
<td>23</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>65</td>
<td>Vanguard</td>
<td>US</td>
<td>8 274</td>
<td></td>
<td>12</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>66</td>
<td>Dimensional Fund Advisors</td>
<td>US</td>
<td>679</td>
<td></td>
<td>6</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>67</td>
<td>Santander AM</td>
<td>Spain</td>
<td>220</td>
<td></td>
<td>0</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>68</td>
<td>Walter Scott &amp; Partners</td>
<td>UK</td>
<td>95</td>
<td></td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: ShareAction (2023) & Author’s calculations.
Table 75: Ranking of the 25 largest asset managers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2019</td>
<td>2020</td>
</tr>
<tr>
<td>22</td>
<td>BlackRock</td>
<td>US</td>
<td>10014</td>
<td>7</td>
</tr>
<tr>
<td>25</td>
<td>Vanguard</td>
<td>US</td>
<td>8274</td>
<td>8</td>
</tr>
<tr>
<td>23</td>
<td>Fidelity Investments</td>
<td>US</td>
<td>4520</td>
<td>9</td>
</tr>
<tr>
<td>21</td>
<td>SSGA</td>
<td>US</td>
<td>4140</td>
<td>26</td>
</tr>
<tr>
<td>18</td>
<td>J.P. Morgan AM</td>
<td>US</td>
<td>2742</td>
<td>7</td>
</tr>
<tr>
<td>16</td>
<td>Capital Group</td>
<td>US</td>
<td>2716</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Amundi AM</td>
<td>France</td>
<td>2348</td>
<td>66</td>
</tr>
<tr>
<td>20</td>
<td>Goldman Sachs AM</td>
<td>US</td>
<td>2218</td>
<td>37</td>
</tr>
<tr>
<td>3</td>
<td>Legal &amp; General</td>
<td>UK</td>
<td>1923</td>
<td>82</td>
</tr>
<tr>
<td>24</td>
<td>T. Rowe Price</td>
<td>US</td>
<td>1642</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>Invesco</td>
<td>US</td>
<td>1611</td>
<td>34</td>
</tr>
<tr>
<td>12</td>
<td>Morgan Stanley IM</td>
<td>US</td>
<td>1566</td>
<td>79</td>
</tr>
<tr>
<td>14</td>
<td>Wellington Management</td>
<td>US</td>
<td>1426</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Northern Trust AM</td>
<td>US</td>
<td>1348</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>Nuveen AM</td>
<td>US</td>
<td>1271</td>
<td>62</td>
</tr>
<tr>
<td>8</td>
<td>UBS AM</td>
<td>Switzerland</td>
<td>1216</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>DWS</td>
<td>Germany</td>
<td>1055</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>AXA IM</td>
<td>France</td>
<td>1059</td>
<td>79</td>
</tr>
<tr>
<td>6</td>
<td>Schroders</td>
<td>UK</td>
<td>991</td>
<td>56</td>
</tr>
<tr>
<td>17</td>
<td>AllianceBernstein</td>
<td>US</td>
<td>779</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>AllianzGI</td>
<td>Germany</td>
<td>766</td>
<td>89</td>
</tr>
<tr>
<td>1</td>
<td>BNP PAM</td>
<td>France</td>
<td>761</td>
<td>48</td>
</tr>
<tr>
<td>19</td>
<td>Columbia Threadneedle</td>
<td>US</td>
<td>754</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Manulife IM</td>
<td>Canada</td>
<td>723</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>APG AM</td>
<td>Netherlands</td>
<td>721</td>
<td>72</td>
</tr>
</tbody>
</table>

Figure 108: Evolution of the support rate in % per asset manager

Main findings

1. "49 additional resolutions would have received majority support if the largest asset managers had voted in favour of them.

2. Voting performance has been stagnant in the US and the UK compared to 2021, while European asset managers have shown a large improvement.

3. Asset managers across the board are hesitant to back action-oriented resolutions, which would have the most transformative impact on environmental and social issues."
Figure 109: Ranking of the 36 say on climate resolutions with respect to the support rate in %

Source: ShareAction (2023) & Author’s calculations.
Statistics about ESG voting

Asset managers

3 case studies of Say on Climate resolutions

- Electricité de France or EDF (French energy company): 99.9%
- Barclays (British bank): 80.8%
- Woodside Energy Group Ltd. (Australian energy company): 51.03%
Statistics about ESG voting
Asset owners
Course 2022-2023 in Sustainable Finance
Lecture 6. Global Warning & Climate Change

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2023

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17 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Some definitions
Awareness of climate change impacts
The ecosystem of climate change

Climate financial risk

Climate risks transmission channels to financial stability

- The **physical risks** that arise from the increased frequency and severity of climate and weather related events that damage property and disrupt trade
- The **liability risks** stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible
- The **transition risks** that can arise through a sudden and disorderly adjustment to a low carbon economy

Speech by Mark Carney at the International Climate Risk Conference for Supervisors, Amsterdam, April 6, 2018

Physical and transition risks ⇔ E
Liability risks ⇔ S (and G ?)
Climate financial risk

Risks are transversal to financial risks

- **Carbon risk** (reputational and regulation risks) \(\Rightarrow\) economic, market and credit risks
- **Climate risk** (extreme weather events, natural disasters) \(\Rightarrow\) economic, operational, credit and market risks

Carbon/climate risks are part of risk management
Climate financial risk

Climate risks include transition risk and physical risks:

- Transition risk is defined as the financial risk associated with the transition to a low-carbon economy. It includes policy changes, reputational impacts, and shifts in market preferences, norms and technology.

- Physical risk is defined as the financial losses due to extreme weather events and climate disasters like flooding, sea level rise, wildfires, droughts and storms.
Global warming (≈ climate change)

Global warming is the long-term heating of Earth’s climate system observed since the pre-industrial period (between 1850 and 1900) due to human activities, primarily fossil fuel burning

NASA Global Climate Change — https://climate.nasa.gov
Global warming

Figure 110: Global temperature anomaly

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Global warming

Carbon risk
Carbon risks correspond to the potential financial losses due to greenhouse gas (or GHG) emissions, mainly CO₂ emissions (in a strengthening regulatory context)
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Global warming

GHG

Greenhouse gases absorb and emit radiation energy, causing the greenhouse effect$^a$:

1. Water vapour ($\text{H}_2\text{O}$)
2. Carbon dioxide ($\text{CO}_2$)
3. Methane ($\text{CH}_4$)
4. Nitrous oxide ($\text{N}_2\text{O}$)
5. Ozone ($\text{O}_3$)

$^a$Without greenhouse effect, the average temperature of Earth’s surface would be about $-18^\circ\text{C}$. With greenhouse effect, the current temperature of Earth’s surface is about $+15^\circ\text{C}$. 

Thierry Roncalli
Course 2022-2023 in Sustainable Finance
# Global warming

Table 76: Pros and cons of greenhouse gases

<table>
<thead>
<tr>
<th>GHG</th>
<th>Pros</th>
<th>Cons</th>
<th>Global warming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water vapour</td>
<td>Life</td>
<td>Pollution</td>
<td>✓</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>Photosynthesis</td>
<td>Explosive(^\text{18})</td>
<td>✓</td>
</tr>
<tr>
<td>Methane</td>
<td>Energy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>Dentist 😊</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ozone</td>
<td>UV rays</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{18}\text{And dangerous for human life}\)
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Global warming

Carbon equivalent

Carbon dioxide equivalent (or CO₂e) is a term for describing different GHG in a common unit

- A quantity of GHG can be expressed as CO₂e by multiplying the amount of the GHG by its global warming potential (GWP)
- 1 kg of carbone dioxide corresponds to 1 kg of CO₂
- 1 kg of methane corresponds to 28 kg of CO₂
- 1 kg of nitrous oxide corresponds to 273 kg of CO₂
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CO₂ emissions

Figure 111: Cumulative CO₂e emissions (in GtCO₂e)

Source: Data on CO₂ and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)
CO$_2$ emissions

Figure 112: Annual CO$_2$e emissions (in GtCO$_2$e)

Source: Data on CO$_2$ and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)
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**CO₂ emissions**

![CO₂ emissions per capita graph](https://github.com/owid/co2-data)

**Figure 113:** CO₂e emissions per capita (in tonnes per capita)

**Source:** Data on CO₂ and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)
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**CO₂ emissions**

![Figure 114: Share of CO₂e emissions (in %)](https://github.com/owid/co2-data)

*Source: Data on CO₂ and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)*
### Top options for reducing your carbon footprint

Average reduction per person per year in tonnes of CO2 equivalent

<table>
<thead>
<tr>
<th>Option</th>
<th>Reduction (tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live car-free</td>
<td>2.04</td>
</tr>
<tr>
<td>Battery electric car</td>
<td>1.95</td>
</tr>
<tr>
<td>One less long-haul flight per year</td>
<td>1.68</td>
</tr>
<tr>
<td>Renewable energy</td>
<td>1.6</td>
</tr>
<tr>
<td>Public transport</td>
<td>0.98</td>
</tr>
<tr>
<td>Refurbishment/renovation</td>
<td>0.895</td>
</tr>
<tr>
<td>Vegan diet</td>
<td>0.8</td>
</tr>
<tr>
<td>Heat pump</td>
<td>0.795</td>
</tr>
<tr>
<td>Improved cooking equipment</td>
<td>0.65</td>
</tr>
<tr>
<td>Renewable-based heating</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Source: Centre for Research into Energy Demand Solutions
Scientific evidence of global warming: a rocky road

- 1824: Joseph Fourier published the scientific article “Remarques générales sur les températures du globe terrestre et des espaces planétaires” ⇒ the greenhouse effect
- 1863: John Tyndall published the books “Heat Considered as a Mode of Motion” in 1863 and “Contributions to Molecular Physics in the Domain of Radiant Heat” in 1872
- 1896: Svante Arrhenius published the scientific article “On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground” ⇒ if the quantity of carbonic acid increases in geometric progression, the augmentation of the temperature will increase nearly in arithmetic progression
- 1958: Charles David Keeling started collecting carbon dioxide samples at the Mauna Loa Observatory (Hawai) ⇒ Keeling curve
- 2021: Klaus Hasselmann and Syukuro Manabe won the Nobel Prize in Physics for the physical modelling of Earth’s climate, quantifying variability and reliably predicting global warming
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Scientific evidence of global warming: a rocky road

Figure 115: Keeling curve

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Scientific evidence of global warming
From the Holocene to the Anthropocene
The physics of climate change
The Intergovernmental Panel on Climate Change (IPCC) is the United Nations body for assessing the science related to climate change. The IPCC was created to provide policymakers with regular scientific assessments on climate change, its implications and potential future risks, as well as to put forward adaptation and mitigation options. Website: https://www.ipcc.ch

Remark

IPCC is known as “Groupe d’experts intergouvernemental sur l’évolution du climat” (GIEC) in French.

⇒ Other international bodies: International Energy Agency (IEA), etc.
Past

- Global sea level rose by 19 cm over the period 1901-2010
- Global glacier volume loss is equivalent to 400 bn tons per year since 30 years

Future

- Global sea level could increase by 82 cm by 2100
- Global glacier volume could decrease by 85% by 2100

IPCC, Climate Change Synthesis Report (2014)
IPCC working groups

- The IPCC Working Group I (WGI) examines the physical science underpinning past, present, and future climate change.
- The IPCC Working Group II (WGII) assesses the impacts, adaptation and vulnerabilities related to climate change.
- The IPCC Working Group III (WGIII) focuses on climate change mitigation, assessing methods for reducing greenhouse gas emissions, and removing greenhouse gases from the atmosphere.
Some famous reports

- Global Warming of 1.5°C — www.ipcc.ch/sr15
IPCC scenarios

- Website: https://www.ipcc.ch/data
- AR5
- SR15
- AR6
**Carbon neutrality** (or net zero) means that any CO\textsubscript{2} released into the atmosphere from human activity is balanced by an equivalent amount being removed

Apple Commits to Become Carbon Neutral to by 2030  
Carbon dioxide removal

Carbon dioxide removal (CDR)

1. Nature-based solutions
   - Afforestation (creating new forests)
   - Reforestation (multiplying trees in old forests)
   - Restoration of peat bogs
   - Restoration of coastal and marine habitats

2. Enhanced natural processes
   - Land management and no-till agriculture, which avoids carbon release through soil disturbance
   - Better wildfire management
   - Ocean fertilisation to increase its capacity to absorb CO₂ (enhanced weathering)

3. Technology solutions
   - Bioenergy with carbon capture and storage (BECCS)
   - Direct air capture (DAC)
   - Carbon mineralization
Carbon dioxide removal

The example of peatlands

- Peatlands are the largest natural terrestrial carbon store
- The term “peatland” refers to peat soil and wetland habitats
- They cover only 3% of the Earth’s surface
- They store 600 GtCO$_2$e
  - $\approx 45\%$ of all soil carbon
  - $\approx 67\%$ of all atmosphere carbon
- A depth of one meter corresponds to 1000 years of carbon storage
- Natural peatlands store 0.37 GtCO$_2$e per year

Two issues:
1. Stopping the destruction
2. Restoring and rebuilding
Carbon offsetting ≠ carbon emissions reduction

**Definition**

“Carbon offsetting consists for an entity in compensating its own carbon emissions by providing for emissions reductions outside its business boundaries [...] It allows an entity to claim carbon reductions from projects financed either directly or indirectly through carbon credits” (Créhalet, 2021).
Carbon offsetting

Carbon offsetting mechanisms:

Suppliers of carbon offsets

Carbon credits

Purchasers of carbon offsets

⇒ Many issues: carbon credit issuance, double counting, leakage, certification, etc.

Examples with REDD+ projects:

- Reducing Emissions from Deforestation and Forest Degradation
- What will happen if the forest has burned down?
- Issues of land management (afforestation in one area can lead to a deforestation in another area)
Climate risk and missing factors

The example of permafrost

- The permafrost contains 1,700 billion tons of carbon, almost double the amount of carbon that is currently in the atmosphere.
- Arctic permafrost holds roughly 15 million gallons of mercury – at least twice the amount contained in the oceans, atmosphere and all other land combined.
- A global temperature rise of 1.5°C above current levels would be enough to start the thawing of permafrost in Siberia.
- The global warming will become out-of-control after this tipping point.
- The thawing of the permafrost also threatens to unlock disease-causing viruses long trapped in the ice.

⇒ The survival of Humanity becomes uncertain if the tipping point is reached
Regulation of climate risk

- UN, international bodies & coalitions
- Countries
- Cities
- Industry self-regulation
- Non-governmental organizations (NGO)
- Financial regulators

**Hard regulation ≠ soft regulation**
United Nations Climate Change Conference

- Conference of the Parties (COP)
- Dealing with climate change
- COP 26: Glasgow (November 2021)
The **Kyoto Protocol** is an international treaty that commits state parties to reduce GHG emissions, based on the scientific consensus that:

1. **Global warming is occurring**
2. It is likely that **human-made CO₂ emissions have caused it**
The **Paris Agreement** is an international treaty with the following goals:

1. Keep a global temperature rise this century well below 2°C above the pre-industrial levels
2. Pursue efforts to limit the temperature increase to 1.5°C
3. Increase the ability of countries to deal with the impacts of climate change
4. Make finance flows consistent with low GHG emissions and climate-resilient pathways

⇒ Nationally determined contributions (NDC)
### Table 77: CO₂ emissions by country

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>CO₂ emissions Total (in GT)</th>
<th>Share</th>
<th>CO₂ emissions Per capita (in MT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>China</td>
<td>10.06</td>
<td>28%</td>
<td>7.2</td>
</tr>
<tr>
<td>2</td>
<td>USA</td>
<td>5.41</td>
<td>15%</td>
<td>15.5</td>
</tr>
<tr>
<td>3</td>
<td>India</td>
<td>2.65</td>
<td>7%</td>
<td>1.8</td>
</tr>
<tr>
<td>4</td>
<td>Russia</td>
<td>1.71</td>
<td>5%</td>
<td>12.0</td>
</tr>
<tr>
<td>5</td>
<td>Japan</td>
<td>1.16</td>
<td>3%</td>
<td>8.9</td>
</tr>
<tr>
<td>6</td>
<td>Germany</td>
<td>0.75</td>
<td>2%</td>
<td>8.8</td>
</tr>
<tr>
<td>7</td>
<td>Iran</td>
<td>0.72</td>
<td>2%</td>
<td>8.3</td>
</tr>
<tr>
<td>8</td>
<td>South Korea</td>
<td>0.72</td>
<td>2%</td>
<td>12.1</td>
</tr>
<tr>
<td>9</td>
<td>Saudi Arabia</td>
<td>0.72</td>
<td>2%</td>
<td>17.4</td>
</tr>
<tr>
<td>10</td>
<td>Indonesia</td>
<td>0.72</td>
<td>2%</td>
<td>2.2</td>
</tr>
<tr>
<td>11</td>
<td>Canada</td>
<td>0.56</td>
<td>2%</td>
<td>15.1</td>
</tr>
<tr>
<td>15</td>
<td>Turkey</td>
<td>0.42</td>
<td>1%</td>
<td>4.7</td>
</tr>
<tr>
<td>17</td>
<td>United Kingdom</td>
<td>0.37</td>
<td>1%</td>
<td>5.8</td>
</tr>
<tr>
<td>19</td>
<td>France</td>
<td>0.33</td>
<td>1%</td>
<td>4.6</td>
</tr>
<tr>
<td>20</td>
<td>Italy</td>
<td>0.33</td>
<td>1%</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Source: Earth System Science Data, [https://earth-system-science-data.net](https://earth-system-science-data.net)

Paris Agreement: where we are?

- 194 states have signed the Agreement
- They represent about 80% of GHG emissions
- USA, Iran and Turkey have not signed the Agreement
### Figure 116: Paris Agreement assessments of aviation and shipping

Source: Climate Action Tracker (CAT), https://climateactiontracker.org
The Coalition of Finance Ministers for Climate Action

www.financeministersforclimate.org

- Commitment to implement fully the Paris Agreement
- Santiago Action Plan
- Helsinki principles (1. align, 2. share, 3. promote, 4. mainstream, 5. mobilize, 6. engage)
Regulation of climate risk

Coalitions

- **One Planet Summit**
  
  www.oneplanetsummit.fr

- **One Planet Sovereign Wealth Funds (OPSWF)**
  
  Funding members: Abu Dhabi Investment Authority (ADIA), Kuwait Investment Authority (KIA), NZ Superannuation Fund (NZSF), Public Investment Fund (PIF), Qatar Investment Authority (QIA), NBIM

  New members: BPifrance, CDP Equity, COFIDES, FONSIS, ISIF, KIC, Mubadala IC, NIIF, NIC NBK

- **One Planet Asset Managers**
  
  Funding members: Amundi AM, BlackRock, BNP PAM, GSAM, HSBC Global AM, Natixis IM, Northern Trust AM, SSGA

  New members: AXA IM, Invesco, Legal & General IM, Morgan Stanley IM, PIMCO UBS AM

- **One Planet Private Equity Funds**
  
  Members: Ardian, Carlyle Group, Global Infrastructure Partners, Macquarie Infrastructure and Real Assets (MIRA), SoftBank IA
The example of France

- August 2015: French Energy Transition for Green Growth Law (or Energy Transition Law)
- Roadmap to mitigate climate change and diversify the energy mix

Article 173 of the French Energy Transition Law

- The annual report of listed companies must include:
  - Financial risks related to the effects of climate change
  - The measures adopted by the company to reduce them
  - The consequences of climate change on the company’s activities

- New requirements for investors:
  - Disclosure of climate (and ESG) criteria into investment decision making process
  - Disclosure of the contribution to the energy transition and the global warming limitation international objective
  - Reporting on climate change-related risks (including both physical risks and transition risks), and GHG emissions of assets

- Banks and credit providers shall conduct climate stress testing
Regulation of climate risk

Carbon pricing

- Polluter pays principle
  - A carbon price is a cost applied to carbon pollution to encourage polluters to reduce the amount of GHG they emit into the atmosphere
  - Negative externality

- Two instruments of carbon pricing
  1. **Carbon tax**
  2. **Cap-and-trade** (CAT) or **emissions trading scheme** (ETS)

- Some examples
Regulation of climate risk
Carbon pricing

Figure 117: EU ETS carbon price* (in €/tCO₂)

(*) The carbon price reaches 34.43 euros a tonne on Monday 11, 2021
Regulation of climate risk

Carbon pricing

<table>
<thead>
<tr>
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</table>

Table 78: Carbon tax (in $/tCO₂)

Stranded Assets are assets that have suffered from unanticipated or premature write-downs, devaluations or conversion to liabilities

For example, a $2^\circC$ alignment implies to keep a large proportion of existing fossil fuel reserves in the ground (30% of oil reserves, 50% of gas reserves and 80% of coal)

Risk factors: Regulations, carbon prices, change in demand, social pressure, etc.

Example of the covid-19 crisis ⇒ air travel
Regulation of climate risk

Financial regulation

- Financial Stability Board (FSB)
- European Central Bank (ECB)
- The French Prudential Supervision and Resolution Authority (ACPR)
- The Prudential Regulation Authority (PRA)
- Network for Greening the Financial System (NGFS)
- Etc.
Task Force on Climate-related Financial Disclosures (TCFD)

- Established by the FSB in 2015 to develop a set of voluntary, consistent disclosure recommendations for use by companies in providing information to investors, lenders and insurance underwriters about their climate-related financial risks
- Website: www.fsb-tcfd.org
- Chairman: Michael R. Bloomberg (founder of Bloomberg L.P.)
- 31 members
- June 2017: Publication of the “Recommendations of the Task Force on Climate-related Financial Disclosures”
## Regulation of climate risk

**Financial regulation**

<table>
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<th>Recommendation</th>
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<td>Governance</td>
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<td>Board oversight</td>
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<td>Management’s role</td>
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<td>Strategy</td>
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<td>Risks and opportunities</td>
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<td>4</td>
<td>Impact on organization</td>
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<td>5</td>
<td>Resilience of strategy</td>
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<td>8</td>
<td>Integration into overall risk management</td>
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<tr>
<td>Metrics and targets</td>
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<td>Climate-related metrics</td>
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<td></td>
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<td>Scope 1, 2, 3 GHG emissions</td>
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<tr>
<td></td>
<td>11</td>
<td>Climate-related targets</td>
</tr>
</tbody>
</table>

*Table 79: The 11 recommended disclosures (TCFD, 2017)*
Some key findings of the 2020 Status Report (TCFD, 2020):

- Disclosure of climate-related financial information has increased since 2017, but continuing progress is needed
- Average level of disclosure across the Task Force’s 11 recommended disclosures was 40% for energy companies and 30% for materials and buildings companies
- Asset manager and asset owner reporting to their clients and beneficiaries, respectively, is likely insufficient
Climate stress testing

- ACPR (2020): Climate Risk Analysis and Supervision\(^{19}\)
- Bank of England (2021): Climate Biennial Exploratory Scenario (June 2021)

Top-down approach \(\neq\) bottom-up approach


Climate capital requirements

Green supporting factor

- Risk weights may depend on the green/brown nature of the credit
- Green loans
- Green supporting factor $\neq$ Brown penalising factor

Similar idea: Green Quantitative Easing (GQE)
Figure 118: In April 2021, Basel Committee publishes two reports on climate risk.
Climate capital requirements

In June 2022, Basel Committee publishes guidelines:

Principles for the effective management and supervision of climate-related financial risks
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Sustainable growth and climate change

“There is no Plan B, because there is no Planet B“

Ban Ki-moon, UN Secretary-General, September 2014

Is it a question of climate-related issues?
In fact, it is more an economic growth issue

“The Golden Rule of Accumulation: A Fable for Growthmen“

Nobel Prize in Economics, 2006
Sustainable growth and climate change

Environmental
Social
Governance

Economy
Sustainability
Growth

Adam Smith (1776)
An Inquiry into the Nature and Causes of The Wealth of Nations
The Solow growth model

The model

- Production function:
  \[ Y(t) = F(K(t), A(t) L(t)) \]
  where \( K(t) \) is the capital, \( L(t) \) is the labor and \( A(t) \) is the knowledge factor.

- Law of motion for the capital per unit of effective labor
  \[ k(t) = K(t) / (A(t) L(t)) \]
  
  \[ \frac{dk(t)}{dt} = s f(k(t)) - (g_L + g_A + \delta_K) k(t) \]

  where \( s \) is the saving rate, \( \delta_K \) is the depreciation rate of capital and \( g_A \) and \( g_L \) are the productivity and labor growth rates.
Golden rule with the Cobb-Douglas production and Hicks neutrality

The equilibrium to respect the ‘fairness’ between generations is:

\[
k^* = \left(\frac{s}{gL + gA + \delta K}\right)^{\frac{1}{1 - \alpha}}
\]

“Each generation in a boundless golden age of natural growth will prefer the same investment ratio, which is to say the same natural growth path” (Phelps, 1961, page 640).

“By a golden age I shall mean a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital-output ratio is stationary over time” (Phelps, 1961, page 639).
What is economic growth and what is the balanced growth path?

- There is a saving rate that maximizes consumption over time and between generations ("the fair rate to preserve future generations")
- Economic growth corresponds to the exponential growth of capital and output to answer the needs of the growing population
- Introducing human and natural capitals add constraints and therefore reduce growth!

Economic growth $\Rightarrow \begin{cases} \text{productivity} \uparrow \text{and labor} \uparrow \\ \text{maximization of consumption-based utility} \end{cases}$ function
What are the effects of environmental constraints on growth?

Introducing a decreasing natural capital (Romer, 2006)

The balanced growth path $g_Y^*$ is equal to:

$$g_Y^* = g_L + g_A - \frac{g_L + g_A + \delta_{N_c}}{1 - \alpha} \varphi$$

where $\delta_{N_c}$ is the depreciation rate of natural capital and $\varphi$ is the elasticity of output with respect to (normalized) natural capital $N_c(t)$

“The static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline” (Hotteling, 1931, page 138-139)

Accounting for environment... changes the definition of economic growth
Preferences modeling (Ramsey model)

- $\rho$ is the discount rate (time preference)
- $c(t)$ is the consumption per capita and $u$ is the CRRA utility function:

$$u(c(t)) = \begin{cases} 
\frac{1}{1-\theta}c(t)^{1-\theta} & \text{if } \theta > 0, \theta \neq 1 \\
\ln c(t) & \text{if } \theta = 1
\end{cases}$$

where $\theta$ is the risk aversion parameter

- Maximization of the welfare function:

$$\int_t^\infty e^{-\rho t} u(c(t)) \, dt$$
The discounting issue

Does the golden rule of saving rates hold in a Keynesian approach with discounted maximization of consumption?

- “There is still time to avoid the worst impacts of climate change, if we take strong action now” (Stern, 2007)
- “I got it wrong on climate change – it’s far, far worse” (Stern, 2013)

Figure 119: Discounted value of $100 loss

The value of a loss in 100 years almost disappears... while it is only the next generation!
Does consumption maximization make sense?

How many planets do we need?

To achieve the current levels of consumption for the world population, we need:

- US: 5 planets
- France: 3 planets
- India: 0.6 planet

Fairness between generations

Keynes

“In the long run, we are all dead“

John Maynard Keynes\(^a\), A Tract on Monetary Reform, 1923.

\(^a\)“Men will not always die quietly“, The Economic Consequences of the Peace, 1919.

Carney

“The Tragedy of the Horizon“

Mark Carney, Chairman of the Financial Stability Board, 2015

⇒ Back to the Golden Rule and the Fable for Growthmen...
Main categories

- **Optimization models**
  The inputs of these models are parameters and assumptions about the structure of the relationships between variables. The outputs provided by optimization process are scenarios depending on a set of constraints

- **Evaluation models**
  Based on exogenous scenarios, the outputs provide results from partial equilibriums between variables

Three main components of IAMs

1. Economic growth relationships
2. Dynamics of climate emissions
3. Objective function
Figure 120: Economic models of climate risk

- Production: Industry and business generate CO₂ emissions
- Climate change: Change in radiative warming; ocean currents; sea level rise; etc.
- Impact and damages: Losses on the entire economy
- Objective: Measures and tax policies to control CO₂ emissions
Modeling framework

1. Economic module
   1. Production function $\rightarrow$ GDP
   2. Impact of the climate risk on GDP (damage losses, mitigation and adaptation costs)
   3. The climate loss function depends on the temperature

2. Climate module
   1. Dynamics of GHG emissions
   2. Modeling of Atmospheric and lower ocean temperatures

3. Optimal control problem
   1. Maximization of the utility function
   2. We can test many variants
The most famous IAM is the **Dynamic Integrated model of Climate and the Economy** (or DICE) developed by William Nordhaus\(^\text{21}\).
The **gross production** $Y(t)$ is given by a Cobb-Douglas function:

$$Y(t) = A(t) K(t)^\gamma L(t)^{1-\gamma}$$

where:
- $A(t)$ is the total productivity factor
- $K(t)$ is the capital input
- $L(t)$ is the labor input
- $\gamma \in ]0, 1[$ measures the elasticity of the capital factor:

Climate change impacts the **net output**:

$$Q(t) = \Omega_{\text{climate}}(t) Y(t) \leq Y(t)$$

Classical identities $Q(t) = C(t) + I(t)$ and $I(t) = s(t) Q(t)$
The dynamics of the state variables are:

\[
\begin{align*}
A(t) &= (1 + g_A(t)) A(t - 1) \\
K(t) &= (1 - \delta_K) K(t - 1) + I(t) \\
L(t) &= (1 + g_L(t)) L(t - 1)
\end{align*}
\]

We have:

\[
\begin{align*}
g_A(t) &= \frac{1}{1 + \delta_A} g_A(t - 1) \\
g_L(t) &= \frac{1}{1 + \delta_L} g_L(t - 1)
\end{align*}
\]
Economic module

Labor input

**Example #1**

The world population was equal to 7.725 billion in 2019 and 7.805 billion in 2020. At the beginning of the 1970s, we estimate that the annual growth rate was equal to 2.045%. According to the United Nations, the global population could surpass 10 billion by 2100.
In 2020, the annual growth rate was equal to:

\[ g_L(2020) = \frac{L(2020)}{L(2019)} - 1 = \frac{7.805}{7.725} - 1 = 1.036\% \]

Since we have \( g_L(t) = \left( \frac{1}{1 + \delta_L} \right)^{t-t_0} \) \( g_L(t_0) \), we deduce that:

\[ \delta_L = \left( \frac{g_L(t_0)}{g_L(t)} \right)^{1/(t-t_0)} - 1 \]

An estimate of \( \delta_L \) is then:

\[ \delta_L = \left( \frac{g_L(1970)}{g_L(2020)} \right)^{1/30} - 1 = 2.292\% \]
Figure 121: Evolution of the labor input $L(t)$

- $\delta_L = 2.292\%$
- $\delta_L = 1.500\%$
- $\delta_L = 3.250\%$
Economic module

Labor input

Figure 122: Projection of the world population

Economic module
Labor input

- AR(1) model:

\[ g_L(t) = \phi g_L(t - 1) + \varepsilon(t) \]

We have

\[ \hat{\delta}_L = \frac{(1 - \hat{\phi})}{\hat{\phi}} \]

- Log-linear model:

\[ \ln g_L(t) = \beta_0 + \beta_1 (t - t_0) + \varepsilon(t) \]

We have:

\[ \hat{\delta}_L = e^{-\hat{\beta}_1} - 1 \]
Economic module

Figure 123: Population growth rate

### Table 80: Average productivity growth rate (in %)

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</table>

Source: Penn World Table 10.01 (Feenstra et al., 2015) & Author’s calculations.
Figure 124: Total factor productivity index (base 100 = 1960)

Source: Penn World Table 10.01 (Feenstra et al., 2015) & Author’s calculations.
**Figure 125: Dynamics of the TFP growth rate**

We use the following calibration rule: \( \delta_A = \sqrt{n}d - 1 \)
Penn World Table/IMF's ICSD

In 2019, we obtain $I(2019) = \$30.625$ tn, $K(2019) = \$318.773$ tn and $Y(2019) = \$124.418$ tn

We also have:

$$\delta_K(t) = \frac{K(t-1) - K(t) + I(t)}{K(t-1)}$$

and we obtain $\delta_K(2019) = 6.25\%$

To calibrate the initial value of $A(t)$, we inverse the Coob-Douglas function:

$$A(2019) = \frac{Y(t)}{K(t)^\gamma L(t)^{1-\gamma}} = \frac{124.418}{318.773^{0.30} \times 7.725^{0.70}} = 5.276$$

The saving rate $s(t)$ is exogenous
Figure 126: Historical estimates of \( I(t) \), \( K(t) \), \( Y(t) \) and \( \delta K(t) \)

Source: IMF Investment and Capital Stock Dataset (2021) & Author’s calculations.
Figure 127: Simulation of the DICE macroeconomic module

\[ I(t) \text{ (in $ tn)} \]

\[ K(t) \text{ (in $ tn)} \]

\[ L(t) \text{ (in bn)} \]

\[ Y(t) \text{ (in $ tn)} \]

- \( s(t) = 25\% \)
- \( \cdot s(t) = 15\% \)
The survival function is given by:

\[ \Omega_{\text{climate}} (t) = \Omega_D (t) \Omega_{\Lambda} (t) = \frac{1}{1 + D (t)} (1 - \Lambda (t)) \]

where:

- \( D (t) \geq 0 \) is the climate damage function (physical risk)
- \( \Lambda (t) \geq 0 \) is the mitigation or abatement cost (transition risk)
The cost $D(t)$ resulting from natural disasters depends on the atmospheric temperature $T_{AT}(t)$:

$$D(t) = \psi_1 T_{AT}(t) + \psi_2 T_{AT}(t)^2$$

The abatement cost function depends on the control variable $\mu(t)$:

$$\Lambda(t) = \theta_1(t) \mu(t)^\theta_2$$

The global impact of climate change is equal to:

$$\Omega_{climate}(t) = \frac{1 - \theta_1(t) \mu(t)^\theta_2}{1 + \psi_1 T_{AT}(t) + \psi_2 T_{AT}(t)^2}$$
Figure 128: Loss function due to climate damage costs
Figure 129: Abatement cost function

\[ \Lambda(t) \text{ (in %)} \]

\[ t \text{ (in years)} \]

- \( \mu(t) = 1 \)
- \( \mu(t) = 0.75 \)
- \( \mu(t) = 0.50 \)
- \( \mu(t) = 0 \)
The total GHG emissions depends on the production $Y(t)$ and the land use emissions $\mathcal{CE}_{\text{Land}}(t)$:

$$
\mathcal{CE}(t) = \mathcal{CE}_{\text{Industry}}(t) + \mathcal{CE}_{\text{Land}}(t)
$$

$$
= (1 - \mu(t))\sigma(t)Y(t) + \mathcal{CE}_{\text{Land}}(t)
$$

$\sigma(t)$ is the anthropogenic carbon intensity of the economy:

$$
\sigma(t) = (1 + g_{\sigma}(t))\sigma(t-1)
$$

where:

$$
g_{\sigma}(t) = \frac{1}{1 + \delta_{\sigma}}g_{\sigma}(t-1)
$$
Figure 130: Physical carbon pump

**Source:** ocean-climate.org.
We have:

\[
\begin{align*}
    CC_{\text{AT}}(t) &= \phi_{1,1}CC_{\text{AT}}(t-1) + \phi_{1,2}CC_{\text{UP}}(t-1) + \phi_1CE(t) \\
    CC_{\text{UP}}(t) &= \phi_{2,1}CC_{\text{AT}}(t-1) + \phi_{2,2}CC_{\text{UP}}(t-1) + \phi_{2,3}CC_{\text{LO}}(t-1) \\
    CC_{\text{LO}}(t) &= \phi_{3,2}CC_{\text{UP}}(t-1) + \phi_{3,3}CC_{\text{LO}}(t-1)
\end{align*}
\]

The dynamics of \( CC = (CC_{\text{AT}}, CC_{\text{UP}}, CC_{\text{LO}}) \) is a VAR(1) process:

\[
    CC(t) = \Phi_{CC}CC(t-1) + B_{CC}E(t)
\]

**Carbon cycle diffusion matrix**

We have:

\[
    \Phi_{CC} = \begin{pmatrix}
        91.20\% & 3.83\% & 0 \\
        8.80\% & 95.92\% & 0.03\% \\
        0 & 0.25\% & 99.97\%
    \end{pmatrix}
\]
Figure 131: Impulse response analysis ($\Delta CE = -1$ GtCO$_2$e)
We have:

\[ F_{\text{RAD}}(t) = \frac{\eta}{\ln 2} \ln \left( \frac{CC_{\text{AT}}(t)}{CC_{\text{AT}}(1750)} \right) + F_{\text{EX}}(t) \]

where:

- \( F_{\text{RAD}}(t) \) is the change in total radiative forcing of GHG emissions since 1750 (expressed in \( W/m^2 \))
- \( \eta \) is the temperature forcing parameter
- \( F_{\text{EX}}(t) \) is the exogenous forcing (other GHG emissions)
The climate system for temperatures is characterized by a two-layer system:

\[
\begin{align*}
\mathcal{T}_{\text{AT}} (t) &= \mathcal{T}_{\text{AT}} (t-1) + \xi_1 (\mathcal{F}_{\text{RAD}} (t) - \xi_2 \mathcal{T}_{\text{AT}} (t-1) - \\
&\quad \xi_3 (\mathcal{T}_{\text{AT}} (t-1) - \mathcal{T}_{\text{LO}} (t-1))) \\
\mathcal{T}_{\text{LO}} (t) &= \mathcal{T}_{\text{LO}} (t-1) + \xi_4 (\mathcal{T}_{\text{AT}} (t-1) - \mathcal{T}_{\text{LO}} (t-1))
\end{align*}
\]

Let \( \mathcal{T} = (\mathcal{T}_{\text{AT}}, \mathcal{T}_{\text{LO}}) \) be the temperature vector. We have:

\[
\mathcal{T} (t) = \Xi_T \mathcal{T} (t-1) + B_T \mathcal{F}_{\text{RAD}} (t)
\]
### Table 81: Output of the DICE climate module \((Y(t) = Y(t_0), \mu(t) = \mu(t_0))\)

<table>
<thead>
<tr>
<th>Year</th>
<th>CE(t)</th>
<th>σ(t)</th>
<th>CC_AT(t)</th>
<th>F_RAD(t)</th>
<th>T_AT(t)</th>
<th>T_LO(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>36.91</td>
<td>0.55</td>
<td>830.4</td>
<td>2.14</td>
<td>0.800</td>
<td>0.007</td>
</tr>
<tr>
<td>2015</td>
<td>36.25</td>
<td>0.55</td>
<td>825.7</td>
<td>2.14</td>
<td>0.900</td>
<td>0.027</td>
</tr>
<tr>
<td>2020</td>
<td>36.06</td>
<td>0.56</td>
<td>821.9</td>
<td>2.14</td>
<td>0.986</td>
<td>0.048</td>
</tr>
<tr>
<td>2025</td>
<td>35.97</td>
<td>0.57</td>
<td>818.9</td>
<td>2.14</td>
<td>1.061</td>
<td>0.072</td>
</tr>
<tr>
<td>2030</td>
<td>35.98</td>
<td>0.57</td>
<td>816.6</td>
<td>2.15</td>
<td>1.127</td>
<td>0.097</td>
</tr>
<tr>
<td>2035</td>
<td>36.05</td>
<td>0.58</td>
<td>814.9</td>
<td>2.16</td>
<td>1.186</td>
<td>0.122</td>
</tr>
<tr>
<td>2040</td>
<td>36.18</td>
<td>0.58</td>
<td>813.9</td>
<td>2.18</td>
<td>1.238</td>
<td>0.149</td>
</tr>
<tr>
<td>2045</td>
<td>36.36</td>
<td>0.59</td>
<td>813.3</td>
<td>2.20</td>
<td>1.286</td>
<td>0.176</td>
</tr>
<tr>
<td>2050</td>
<td>36.58</td>
<td>0.59</td>
<td>813.3</td>
<td>2.23</td>
<td>1.329</td>
<td>0.204</td>
</tr>
<tr>
<td>2055</td>
<td>36.82</td>
<td>0.60</td>
<td>813.6</td>
<td>2.26</td>
<td>1.370</td>
<td>0.232</td>
</tr>
<tr>
<td>2060</td>
<td>37.09</td>
<td>0.61</td>
<td>814.4</td>
<td>2.29</td>
<td>1.408</td>
<td>0.261</td>
</tr>
<tr>
<td>2065</td>
<td>37.39</td>
<td>0.61</td>
<td>815.4</td>
<td>2.32</td>
<td>1.445</td>
<td>0.289</td>
</tr>
<tr>
<td>2070</td>
<td>37.70</td>
<td>0.62</td>
<td>816.8</td>
<td>2.35</td>
<td>1.480</td>
<td>0.318</td>
</tr>
<tr>
<td>2075</td>
<td>38.02</td>
<td>0.62</td>
<td>818.4</td>
<td>2.39</td>
<td>1.514</td>
<td>0.347</td>
</tr>
<tr>
<td>2080</td>
<td>38.36</td>
<td>0.63</td>
<td>820.3</td>
<td>2.43</td>
<td>1.547</td>
<td>0.376</td>
</tr>
<tr>
<td>2085</td>
<td>38.71</td>
<td>0.64</td>
<td>822.4</td>
<td>2.46</td>
<td>1.580</td>
<td>0.406</td>
</tr>
<tr>
<td>2090</td>
<td>39.06</td>
<td>0.64</td>
<td>824.7</td>
<td>2.50</td>
<td>1.612</td>
<td>0.435</td>
</tr>
<tr>
<td>2095</td>
<td>39.43</td>
<td>0.65</td>
<td>827.1</td>
<td>2.55</td>
<td>1.645</td>
<td>0.464</td>
</tr>
<tr>
<td>2100</td>
<td>39.80</td>
<td>0.66</td>
<td>829.7</td>
<td>2.59</td>
<td>1.677</td>
<td>0.494</td>
</tr>
</tbody>
</table>
Climate module

Figure 132: Simulation of the DICE climate module

![Graphs showing simulation results](image-url)
Figure 133: The nightmare climate-economic scenario \((g_Y = 0\%, \mu(t) = 0)\)
The optimal control problem

Optimization problem

- The social welfare function $W$ is equal to:

$$W(s(t), \mu(t)) = \sum_{t=t_0+1}^{T} \frac{L(t)U(c(t))}{(1 + \rho)^{t-t_0}}$$

where $\rho$ is the (generational) discount rate and $c(t) = C(t)/L(t)$ is the consumption per capita

- $U(c) = (c^{1-\alpha} - 1) / (1 - \alpha)$ is the CRRA utility function

- The optimal control problem is then given by:

$$(s^*(t), \mu^*(t)) = \arg \max W(s(t), \mu(t))$$

s.t. \begin{cases} 
\text{DICE Equations} \\
\mu(t) \in [0, 1] \\
s(t) \in [0, 1]
\end{cases}$$
The optimal control problem

The important variables are:

- $T_{AT}(t)$ — Atmospheric temperature
- $\mu(t)$ — Control rate (mitigation policies)
- $CE(t)$ — Total emissions of GHG
- $SCC(t)$ — Social cost of carbon
“The most important single economic concept in the economics of climate change is the social cost of carbon (SCC). This term designates the economic cost caused by an additional tonne of carbon dioxide emissions or its equivalent. In a more precise definition, it is the change in the discounted value of economic welfare from an additional unit of COtwo-equivalent emissions. The SCC has become a central tool used in climate change policy, particularly in the determination of regulatory policies that involve greenhouse gas emissions.” (Nordhaus, 2017).
The social cost of carbon is then defined as:

\[ SCC(t) = \frac{\partial W(t)}{\partial CE(t)} = \frac{\partial C(t)}{\partial CE(t)} \]

It is expressed in $/tCO_2$
Figure 134: Optimal welfare scenario (DICE 2013R)

Limits of economic models
Integrated assessment models
Scenarios

Social cost of carbon (SCC)

Figure 135: $2^\circ \text{C}$ scenario (DICE 2013R)

Figure 136: Optimal welfare scenario (DICE 2016R)

Social cost of carbon (SCC)

Figure 137: $2^\circ$C scenario (DICE 2016R)

The tragedy of the horizon
Achieving the 2°C scenario

- In 2013, the DICE model suggested to reduce drastically CO₂ emissions...

- Since 2016, the 2°C trajectory is no longer feasible! (minimum ≈ 2.6°C)

- For many models, we now have:

\[ P(\Delta T > 2°C) > 95\% \]
### Social cost of carbon (SCC)

**Table 82:** Global SCC under different scenario assumptions (in $/tCO₂)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2050</th>
<th>CAGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>31.2</td>
<td>37.3</td>
<td>44.0</td>
<td>51.6</td>
<td>102.5</td>
<td>3.46%</td>
</tr>
<tr>
<td>Optimal</td>
<td>30.7</td>
<td>36.7</td>
<td>43.5</td>
<td>51.2</td>
<td>103.6</td>
<td>3.54%</td>
</tr>
<tr>
<td>2.5°C-max</td>
<td>184.4</td>
<td>229.1</td>
<td>284.1</td>
<td>351.0</td>
<td>1006.2</td>
<td>4.97%</td>
</tr>
<tr>
<td>2.5°C-mean</td>
<td>106.7</td>
<td>133.1</td>
<td>165.1</td>
<td>203.7</td>
<td>543.3</td>
<td>4.76%</td>
</tr>
</tbody>
</table>

In 2007, Nicholas Stern published a report called *The Economics of Climate Change: The Stern Review*

The Stern Review called for sharp and immediate action to stabilize greenhouse gases because:

"the benefits of strong, early action on climate change outweighs the costs"

The Stern Review proposes to use $\rho = 0.10\%$
Social cost of carbon (SCC)
The Stern-Nordhaus controversy

Figure 138: Discounted value of $10
The time (or generational) discount rate $\rho$ is also called the pure rate of time preference.

It is related to the Ramsey rule:

$$r = \rho + \alpha g$$

where:

- $r$ is the real interest rate
- $g = \frac{\partial c(t)}{c(t)}$ is the growth rate of per capita consumption
- $\alpha$ is the consumption elasticity of the utility function
We report the computations done by Dasgupta (2008):

<table>
<thead>
<tr>
<th>Model</th>
<th>$\rho$</th>
<th>$\alpha$</th>
<th>$g_c$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cline (1992)</td>
<td>0.0%</td>
<td>1.5</td>
<td>1.3%</td>
<td>2.05%</td>
</tr>
<tr>
<td>Nordhaus (2007)</td>
<td>3.0%</td>
<td>1.0</td>
<td>1.3%</td>
<td>4.30%</td>
</tr>
<tr>
<td>Stern (2007)</td>
<td>0.1%</td>
<td>1.0</td>
<td>1.3%</td>
<td>1.40%</td>
</tr>
</tbody>
</table>
### Table 83: Global SCC under different discount rate assumptions

<table>
<thead>
<tr>
<th>Discount rate</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2050</th>
<th>CAGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern</td>
<td>197.4</td>
<td>266.5</td>
<td>324.6</td>
<td>376.2</td>
<td>629.2</td>
<td>3.37%</td>
</tr>
<tr>
<td>Stern</td>
<td>30.7</td>
<td>36.7</td>
<td>43.5</td>
<td>51.2</td>
<td>103.6</td>
<td>3.54%</td>
</tr>
<tr>
<td>2.5%</td>
<td>128.5</td>
<td>140.0</td>
<td>152.0</td>
<td>164.6</td>
<td>235.7</td>
<td>1.75%</td>
</tr>
<tr>
<td>3%</td>
<td>79.1</td>
<td>87.3</td>
<td>95.9</td>
<td>104.9</td>
<td>156.6</td>
<td>1.97%</td>
</tr>
<tr>
<td>4%</td>
<td>36.3</td>
<td>40.9</td>
<td>45.8</td>
<td>51.1</td>
<td>81.7</td>
<td>2.34%</td>
</tr>
<tr>
<td>5%</td>
<td>19.7</td>
<td>22.6</td>
<td>25.7</td>
<td>29.1</td>
<td>49.2</td>
<td>2.65%</td>
</tr>
</tbody>
</table>

Some models

- AIM .......................... RCP 6.0
- DICE/RICE
- FUND
- GCAM
- IMACLIM (CIRED)
- IMAGE .......................... RCP 2.6
- MESSAGE .......................... RCP 8.5
- MiniCAM .......................... RCP 4.5
- PAGE
- REMIND
- RESPONSE (CIRED)
- WITCH
### Table 84: Main integrated assessment models

<table>
<thead>
<tr>
<th>Model</th>
<th>Reference</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stylized simple models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DICE</td>
<td>Nordhaus and Sztorc (2013)</td>
<td>Dynamic Integrated Climate-Economy</td>
</tr>
<tr>
<td>FUND</td>
<td>Anthoff and Tol (2014)</td>
<td>Climate Framework for Uncertainty, Negotiation and Distribution</td>
</tr>
<tr>
<td><strong>Complex models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GCAM</td>
<td>Calvin et al. (2019)</td>
<td>Global Change Assessment Model</td>
</tr>
<tr>
<td>GLOBIOM</td>
<td>Havlik et al. (2018)</td>
<td>Global Biosphere Management Model</td>
</tr>
<tr>
<td>IMACLIM-R</td>
<td>Sassi et al. (2010)</td>
<td>Integrated Model to Assess Climate Change</td>
</tr>
<tr>
<td>IMAGE</td>
<td>Stehfest et al. (2014)</td>
<td>Integrated Model to Assess the Greenhouse Effect</td>
</tr>
<tr>
<td>MAGICC</td>
<td>Meinshausen et al. (2011)</td>
<td>Model for the Assessment of Greenhouse Gas Induced Climate Change</td>
</tr>
<tr>
<td>MAgPIE</td>
<td>Dietrich et al. (2019)</td>
<td>Model of Agricultural Production and its Impact on the Environment</td>
</tr>
<tr>
<td>REMIND</td>
<td>Aboumahboub et al. (2020)</td>
<td>REgional Model of INvestments and Development</td>
</tr>
<tr>
<td>WITCH</td>
<td>Bosetti et al. (2006)</td>
<td>World Induced Technical Change Hybrid</td>
</tr>
</tbody>
</table>

Source: Grubb et al. (2021) & Author’s research.
The Leaders

- DICE
- FUND
- PAGE

⇒ SCC: PAGE ≻ DICE ≻ FUND
Stylized IAMs

Figure 139: Histogram of the 150,000 US Government SCC estimates for 2020 with a 3% discount rate

Source: Rose et al. (2017).
Stylized IAMs
The liability/fairness question

Liability

Growth

Environment

Social

Aristotle (384 BC – 322 BC)

$\Theta I K \Omega N \, N I K O M A X E I \Omega N$

Karl Marx and Friedrich Engels (1848)

The Communist Manifesto
Stylized IAMs
The liability/fairness question

Fairness

Du Contrat Social
Three types of inequalities

- Spatial (or regional) inequalities
- Social (or intra-generation) inequalities
- Time (or inter-generation) inequalities

⇒ These issues are highly related to liability risks:

“[…] liability risks stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible” (Mark Carney, 2018)

- Regional inequalities ⇒ lack of cooperation between countries (e.g., Glasgow COP 26)
- Social inequalities ⇒ climate action postponing (e.g., carbon tax in France)
The **Regional Integrated model of Climate and the Economy (RICE)** model is a sub-regional neoclassical climate economy model (Nordhaus and Yang, 1996)

⇒ **Sub-regional problem of welfare:**
- Each region of the world has a different utility functions
- The big issue is how the most developed regions can finance the transition to a low-carbon economy of the less developed regions

**Both spacial and time (inter-generation) inequalities**
The Nested Inequalities Climate-Economy (NICE) model integrates distributional differences of income (Dennig et al., 2015)

“[...] If the distribution of damage is less skewed to high income than the distribution of consumption, then weak or no climate policy will result in sufficiently large damages on the lower economic strata to eventually stop their welfare levels from improving, and instead cause them to decline” (Dennig et al., 2015)

Both social (intra-generation) and time (inter-generation) inequalities
Complex IAMs

Figure 140: Linkages between the major systems in GCAM

Source: Calvin et al. (2019).
Figure 141: The main land use sectors of GLOBIOM

Source: https://iiasa.github.io/GLOBIOM.
Figure 142: Overview of the IIASA IAM framework

Figure 143: The Remind-MAgPIE framework

Criticisms of integrated assessment models

“IAM-based analyses of climate policy create a perception of knowledge and precision that is illusory and can fool policymakers into thinking that the forecasts the models generate have some kind of scientific legitimacy” (Pindyck, 2017)

- Certain inputs, such as the discount rate, are arbitrary
- There is a lot of uncertainty about climate sensitivity and the temperature trajectory
- Modeling damage functions is arbitrary
- IAMs are unable to consider tail risk
Figure 144: Scenario evaluation

Climate scenario (input) \[\rightarrow\] Evaluation Process \[\rightarrow\] Economic scenario (output)
Climate scenarios

- The representative concentration pathways (RCPs) — IPCC AR5
- The IEA scenarios
- The 1.5°C scenarios — SR15
- The scenarios for the future published — IPCC AR6
Climate scenarios

The RCP scenarios

1. RCP 2.6: GHG emissions start declining by 2020 and go to zero by 2100 (IMAGE)
2. RCP 4.5: GHG emissions peak around 2040, and then decline (MiniCAM)
3. RCP 6.0: GHG emissions peak around 2080, and then decline (AIM)
4. RCP 8.5: GHG emissions continue to rise throughout the 21st century (MESSAGE)
**Figure 145:** Total radiative forcing (in $W/m^2$)

Source: [https://tntcat.iiasa.ac.at/RcpDb](https://tntcat.iiasa.ac.at/RcpDb).
Figure 146: Greenhouse gas concentration trajectory

Source: https://tntcat.iiasa.ac.at/RcpDb.
Climate scenarios

The RCP scenarios

Figure 147: Greenhouse gas emissions trajectory

Source: https://tntcat.iiasa.ac.at/RcpDb.
Figure 148: Total GHG emissions trajectory (in GtCO$_2$e)

Source: https://tntcat.iiasa.ac.at/RcpDb.
Climate scenarios

The IEA scenarios

Figure 149: Direct CO₂ emissions (in Gt)

Climate scenarios
The 1.5°C scenarios

Figure 150: IPCC 1.5°C scenarios of CO₂ emissions

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
**Figure 151**: Confidence interval of the average IPCC 1.5°C scenario

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Climate scenarios
The 1.5°C scenarios

Figure 152: IPCC 1.5°C scenarios of the global mean temperature

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Figure 153: Confidence interval of the exceedance probability $\Pr \{ T > 1.5^\circ C \}$

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Climate scenarios
The 1.5°C scenarios

Figure 154: Confidence interval of the exceedance probability $\Pr \{ T > 2^\circ C \}$

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Climate scenarios

The AR6 scenarios

The new dataset contains 188 models, 1,389 scenarios, 244 countries and regions, and 1,791 variables, which can be split into six main categories:

- **Agriculture**: agricultural demand, crop, food, livestock, production, etc.
- **Capital cost**: coal, electricity, gas, hydro, hydrogen, nuclear, etc.
- **Energy**: capacity, efficiency, final energy, lifetime, OM cost, primary/secondary energy, etc.
- **GHG impact**: carbon sequestration, concentration, emissions, forcing, temperature, etc.
- **Natural resources**: biodiversity, land cover, water consumption, etc.
- **Socio-economic variables**: capital formation, capital stock, consumption, discount rate, employment, expenditure, export, food demand, GDP, Gini coefficient, import, inequality, interest rate, investment, labour supply, policy cost, population, prices, production, public debt, government revenue, taxes, trade, unemployment, value added, welfare, etc.
Climate scenarios
The AR6 scenarios

Figure 155: Histogram of some AR6 output variables by 2100

Source: https://data.ene.iiasa.ac.at/ar6.
Figure 156: Histogram of some AR6 output variables by 2100

Source: https://data.ene.iiasa.ac.at/ar6.
“The SSP narratives [are] a set of five qualitative descriptions of future changes in demographics, human development, economy and lifestyle, policies and institutions, technology, and environment and natural resources. [...] Development of the narratives drew on expert opinion to (1) identify key determinants of the challenges [to mitigation and adaptation] that were essential to incorporate in the narratives and (2) combine these elements in the narratives in a manner consistent with scholarship on their inter-relationships. The narratives are intended as a description of plausible future conditions at the level of large world regions that can serve as a basis for integrated scenarios of emissions and land use, as well as climate impact, adaptation and vulnerability analyses.” (O’Neill et al., 2017)
Shared socioeconomic pathways

Figure 157: The shared socioeconomic pathways

Source: O’Neill et al. (2017).
Figure 158: The shared socioeconomic pathways

- **SSP1**: **Sustainability** (Taking the Green Road)
  - Low challenges for both mitigation and adaptation, rapid development

- **SSP2**: **Middle of the Road**
  - Moderate challenges for mitigation and adaptation

- **SSP3**: **Regional Rivalry** (A Rocky Road)
  - High challenges for both mitigation and adaptation — Concerns about competitiveness/security and regional conflicts pushing countries to focus on regional issues

- **SSP4**: **Inequality** (A Road Divided)
  - Low challenges for mitigation, high for adaptation — Unequal investment in human capital, concentration of power in a small business elite

- **SSP5**: **Fossil-fueled Development** (Taking the Highway)
  - High challenges for mitigation, low for adaptation

Source: O’Neill et al. (2017).
The mitigation/adaptation trade-off is obviously an environmental issue, but the SSPs encompass other environmental narratives, e.g. land use, energy efficiency and green economy.

The social dimension is the central theme of SSPs, and concerns demography, wealth, inequality & poverty, health, education, employment, and more generally the evolution of society. This explains that SSPs and SDGs are highly interconnected.

Finally, the governance dimension is present though two major themes: international fragmentation or cooperation, and the political/economic system, including corruption, stability, rule of law, etc.
Shared socioeconomic pathways

- SSP1: IMAGE (PBL)
- SSP2: MESSAGE-GLOBIOM (IIASA)
- SSP3: AIM/CGE (NIES)
- SSP4: GCAM (PNNL)
- SSP5: REMIND-MAGPIE (PIK) and WITCH-GLOBIOM (FEEM)
Figure 159: SSP demography projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 160: SSP economic projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 161: SSP environmental projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Shared socioeconomic pathways

**Figure 162:** SSP land use projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Shared socioeconomic pathways

**Figure 163:** Example of SSP regional differences

**Source:** https://tntcat.iiasa.ac.at/SspDb.
Figure 164: Gini coefficient projections by 2100

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 165: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)
The NGFS scenarios explore a set of six scenarios which are consistent with the NGFS framework (see figure) published in the First NGFS Comprehensive Report covering the following dimensions:

- **Orderly** scenarios assume climate policies are introduced early and become gradually more stringent. Both physical and transition risks are relatively subdued.
- **Disorderly** scenarios explore higher transition risk due to policies being delayed or divergent across countries and sectors. For example, carbon prices are typically higher for a given temperature outcome.
- **Hot house world** scenarios assume that some climate policies are implemented in some jurisdictions, but globally efforts are insufficient to halt significant global warming. The scenarios result in severe physical risk including irreversible impacts like sea-level rise.

Objectives and framework

The NGFS scenarios explore the impacts of climate change and climate policy with the aim of providing a common reference framework.
NGFS scenarios

- **Orderly scenarios**
  - #1 Net zero 2050 (NZ)
  - #2 Below 2°C (B2D)

- **Disorderly scenarios**
  - #3 Divergent net zero (DNZ)
  - #4 Delayed transition (DT)

- **Hot house world scenarios**
  - #5 Nationally determined contributions (NDC)
  - #6 Current policies (CP)
**Figure 167: Physical and transition risk level of NGFS scenarios**

<table>
<thead>
<tr>
<th>Category</th>
<th>Scenario</th>
<th>Physical risk</th>
<th>Transition risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Policy ambition</td>
<td>Policy reaction</td>
</tr>
<tr>
<td>Orderly</td>
<td>Net Zero 2050</td>
<td>1.4°C</td>
<td>Immediate and smooth</td>
</tr>
<tr>
<td></td>
<td>Below 2°C</td>
<td>1.6°C</td>
<td>Immediate and smooth</td>
</tr>
<tr>
<td>Disorderly</td>
<td>Divergent Net Zero</td>
<td>1.4°C</td>
<td>Immediate but divergent across sectors</td>
</tr>
<tr>
<td></td>
<td>Delayed Transition</td>
<td>1.6°C</td>
<td>Delayed</td>
</tr>
<tr>
<td>Hot house world</td>
<td>Nationally Determined Contributions (NDCs)</td>
<td>2.6°C</td>
<td>NDCs</td>
</tr>
<tr>
<td></td>
<td>Current Policies</td>
<td>3°C +</td>
<td>Non-currente policies</td>
</tr>
</tbody>
</table>
NGFS scenarios

Variables (economic)
- Central bank intervention rate
- Domestic demand
- Effective exchange rate
- Exchange rate
- Exports (goods and services)
- Gross Domestic Product (GDP)
- Gross domestic income
- Imports (goods and services)
- Inflation rate
- Long term & real interest rates
- Trend output for capacity utilisation
- Unemployment

Variables (energy)
- Coal price
- Gas price
- Oil price
- Quarterly consumption of coal
- Quarterly consumption of gas
- Quarterly consumption of oil
- Quarterly consumption of renewables
- Total energy consumption

Models (IPCC)
- Meta-model: NiGEM 1.21
- Sub-models:
  1. GCAM 5.3
  2. MESSAGE-GLOBIOM 1.1
  3. REMIND-MAgPIE 2.1-4.2

6 scenarios
- Net Zero 2050 (NZ)
- Below 2°C (B2D)
- Divergent Net Zero (DNZ)
- Delayed Transition (DT)
- Notionally Determined Contribution (NDC)
- Current Policies (CP)
Table 85: Impact of climate change on the GDP loss by 2050 (GCAM)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−3.09</td>
<td>−5.64</td>
<td>−2.35</td>
<td>−3.28</td>
<td>−5.15</td>
<td>−2.56</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−0.75</td>
<td>−3.66</td>
<td>−1.78</td>
<td>−0.89</td>
<td>−0.88</td>
<td></td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.84</td>
<td>−5.64</td>
<td>−6.00</td>
<td>−5.05</td>
<td>−6.03</td>
<td>−3.44</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−6.03</td>
<td>−5.09</td>
</tr>
</tbody>
</table>
**Table 86: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)**

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−2.05</td>
<td>−5.26</td>
<td>−1.55</td>
<td>−2.64</td>
<td>−4.78</td>
<td>−1.59</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−1.46</td>
<td>−10.00</td>
<td>−10.77</td>
<td>−1.39</td>
<td>−3.26</td>
<td></td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.51</td>
<td>−5.26</td>
<td>−11.53</td>
<td>−13.37</td>
<td>−6.16</td>
<td>−4.84</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td>−11.57</td>
<td>−13.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## NGFS scenarios

Table 87: Impact of climate change on the GDP loss by 2050 (REMIND-MAgPIE)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−2.24</td>
<td>−6.05</td>
<td>−1.67</td>
<td>−2.65</td>
<td>−5.41</td>
<td>−1.76</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−0.78</td>
<td></td>
<td>−3.01</td>
<td>−1.95</td>
<td>−0.33</td>
<td>−1.46</td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.02</td>
<td>−6.05</td>
<td>−4.68</td>
<td>−4.59</td>
<td>−5.73</td>
<td>−3.21</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td></td>
<td></td>
<td>−4.70</td>
<td>−4.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### NGFS scenarios

#### Table 88: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>−13.58</td>
<td>−7.50</td>
<td>−27.35</td>
<td>−29.37</td>
<td>−11.78</td>
<td>−18.36</td>
</tr>
<tr>
<td>Asia</td>
<td>−1.50</td>
<td>−7.29</td>
<td>−5.44</td>
<td>−8.76</td>
<td>−6.78</td>
<td>−1.38</td>
</tr>
<tr>
<td>Australia</td>
<td>−4.11</td>
<td>−3.90</td>
<td>−11.03</td>
<td>−11.74</td>
<td>−5.77</td>
<td>−5.19</td>
</tr>
<tr>
<td>Brazil</td>
<td>−4.43</td>
<td>−5.92</td>
<td>−13.15</td>
<td>−15.90</td>
<td>−6.67</td>
<td>−6.65</td>
</tr>
<tr>
<td>Canada</td>
<td>−1.02</td>
<td>−2.37</td>
<td>−15.07</td>
<td>−18.12</td>
<td>−4.33</td>
<td>−4.87</td>
</tr>
<tr>
<td>China</td>
<td>−2.33</td>
<td>−4.97</td>
<td>−5.13</td>
<td>−6.73</td>
<td>−4.67</td>
<td>−2.76</td>
</tr>
<tr>
<td>Developing Europe</td>
<td>−0.28</td>
<td>−3.11</td>
<td>−0.56</td>
<td>−7.38</td>
<td>−2.73</td>
<td>0.39</td>
</tr>
<tr>
<td>Europe</td>
<td>−1.02</td>
<td>−2.84</td>
<td>−9.64</td>
<td>−11.02</td>
<td>−4.01</td>
<td>−1.62</td>
</tr>
<tr>
<td>France</td>
<td>−1.15</td>
<td>−2.80</td>
<td>−8.35</td>
<td>−9.48</td>
<td>−3.68</td>
<td>−1.56</td>
</tr>
<tr>
<td>Germany</td>
<td>−0.77</td>
<td>−2.38</td>
<td>−8.58</td>
<td>−9.38</td>
<td>−3.63</td>
<td>−1.21</td>
</tr>
<tr>
<td>India</td>
<td>−3.45</td>
<td>−8.61</td>
<td>−16.43</td>
<td>−17.74</td>
<td>−8.71</td>
<td>−3.86</td>
</tr>
<tr>
<td>Italy</td>
<td>−0.15</td>
<td>−3.69</td>
<td>−9.23</td>
<td>−12.88</td>
<td>−4.85</td>
<td>−0.89</td>
</tr>
<tr>
<td>Japan</td>
<td>−1.26</td>
<td>−4.14</td>
<td>−7.16</td>
<td>−10.05</td>
<td>−4.61</td>
<td>−1.40</td>
</tr>
<tr>
<td>Latam</td>
<td>−4.35</td>
<td>−6.10</td>
<td>−12.70</td>
<td>−14.58</td>
<td>−6.97</td>
<td>−5.74</td>
</tr>
<tr>
<td>Middle East</td>
<td>−9.97</td>
<td>−7.98</td>
<td>−22.03</td>
<td>−21.96</td>
<td>−10.28</td>
<td>−15.24</td>
</tr>
<tr>
<td>Russia</td>
<td>−12.18</td>
<td>−2.26</td>
<td>−23.46</td>
<td>−23.80</td>
<td>−7.54</td>
<td>−17.11</td>
</tr>
<tr>
<td>South Africa</td>
<td>−2.02</td>
<td>−5.06</td>
<td>−7.24</td>
<td>−9.16</td>
<td>−5.38</td>
<td>−3.04</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.11</td>
<td>−3.49</td>
<td>−3.23</td>
<td>−7.57</td>
<td>−3.33</td>
<td>0.12</td>
</tr>
<tr>
<td>Spain</td>
<td>−2.41</td>
<td>−3.81</td>
<td>−12.49</td>
<td>−12.89</td>
<td>−5.41</td>
<td>−3.30</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.32</td>
<td>−2.25</td>
<td>−9.47</td>
<td>−10.35</td>
<td>−2.18</td>
<td>2.30</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>−0.86</td>
<td>−1.90</td>
<td>−6.50</td>
<td>−8.05</td>
<td>−2.56</td>
<td>−1.33</td>
</tr>
<tr>
<td>United States</td>
<td>−2.67</td>
<td>−4.38</td>
<td>−15.37</td>
<td>−17.66</td>
<td>−6.31</td>
<td>−4.36</td>
</tr>
<tr>
<td>World</td>
<td>−3.51</td>
<td>−5.26</td>
<td>−11.53</td>
<td>−13.37</td>
<td>−6.16</td>
<td>−4.84</td>
</tr>
</tbody>
</table>
Figure 168: GDP impact by 2050 (% change from baseline) — Delayed transition scenario
Figure 169: GDP impact by 2050 (% change from baseline) — Net zero 2050 scenario
NGFS scenarios

Figure 170: Impact of climate scenarios on economics (% change from baseline) — China
Figure 171: Impact of climate scenarios on economics (% change from baseline) — United States
Figure 172: Impact of climate scenarios on economics (% change from baseline) — France
Figure 173: Impact of climate scenarios on economics (% change from baseline) — United Kingdom
Course 2022-2023 in Sustainable Finance
Lecture 8. Climate Risk Measures

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*Amundi Asset Management22

*University of Paris-Saclay

March 2023

22The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
How to define the carbon footprint?

Wackernagel and Rees (1996) published the seminal book on the ecological footprint:

“the carbon footprint stands for a certain amount of gaseous emissions that are relevant to climate change and associated with human production or consumption activities”

Wiedmann and Minx (2008) proposed this definition:

“The carbon footprint is a measure of the exclusive total amount of carbon dioxide emissions that is directly and indirectly caused by an activity or is accumulated over the life stages of a product”
The carbon footprint is measured in carbon dioxide equivalent (CO₂e) ⇒ a common unit

We have:

\[
\text{equivalent mass of CO}_2 = \text{mass of the gas} \times \text{gwp of the gas}
\]

Examples (IPCC, AR5, 2013):
- 1 kg of methane corresponds to 28 kg of CO₂
- 1 kg of nitrous oxide corresponds to 265 kg of CO₂

The carbon footprint is equal to:

\[
m = \sum_{i=1}^{n} m_i \cdot \text{gwp}_i
\]

The units are: kgCO₂e, tCO₂e, ktCO₂e, MtCO₂e and GtCO₂e
Carbon footprint

Example #1

We consider a company $A$ that emits 3,017 tonnes of $CO_2$, 10 tonnes of $CH_4$ and 1.8 tonnes of $N_2O$. For the company $B$, the GHG emissions are respectively equal to 2,302 tonnes of $CO_2$, 32 tonnes of $CH_4$ and 3.0 tonnes of $N_2O$.

The mass of $CO_2$ equivalent for companies $A$ and $B$ is equal to:

$$m_A = 3017 \times 1 + 10 \times 28 + 1.8 \times 265 = 3,774 \text{ tCO}_2\text{e}$$

and:

$$m_B = 2302 \times 1 + 32 \times 28 + 3.0 \times 265 = 3,993 \text{ tCO}_2\text{e}$$
Estimation of the global warming potential

- According to IPCC (2007), GWP is defined as “the cumulative radiative forcing, both direct and indirect effects, over a specified time horizon resulting from the emission of a unit mass of gas related to some reference gas”.
- Each gas differs in their capacity to absorb the energy (radiative efficiency) and how long it stays in the atmosphere (lifetime).
- The impact of a gas on global warming depends on the combination of radiative efficiency and lifetime.
The mathematics of GWP

The mathematical definition of the global warming potential is:

\[ gwp_i(t) = \frac{Agwp_i(t)}{Agwp_0(t)} = \frac{\int_0^t RF_i(s) \, ds}{\int_0^t RF_0(s) \, ds} = \frac{\int_0^t A_i(s) S_i(s) \, ds}{\int_0^t A_0(s) S_0(s) \, ds} \]

where \( A_i(t) \) is the radiative efficiency value of gas \( i \), \( S_i(t) \) is the decay function and \( i = 0 \) is the reference gas (e.g., CO₂)

We assume that:

\[ S_i(t) = \sum_{j=1}^m a_{i,j} e^{-\lambda_{i,j} t} \]

where \( \sum_{j=1}^m a_{i,j} = 1 \)

We obtain:

\[ gwp_i(t) = \frac{A_i \sum_{j=1}^m a_{i,j} \lambda_{i,j}^{-1} (1 - e^{-\lambda_{i,j} t})}{A_0 \sum_{j=1}^m a_{0,j} \lambda_{0,j}^{-1} (1 - e^{-\lambda_{0,j} t})} \]
Estimation of the global warming potential

- Carbon dioxide
  - \( A_{\text{CO}_2} = 1.76 \times 10^{-18} \)
  - The impulse response function is:
    \[
    S_{\text{CO}_2}(t) = 0.2173 + 0.2240 \cdot \exp\left(-\frac{t}{394.4}\right) + 0.2824 \cdot \exp\left(-\frac{t}{36.54}\right) + 0.2763 \cdot \exp\left(-\frac{t}{4.304}\right)
    \]

- Methane
  - \( A_{\text{CH}_4} = 2.11 \times 10^{-16} \)
  - The impulse response function is:
    \[
    S_{\text{CH}_4}(t) = \exp\left(-\frac{t}{12.4}\right)
    \]
Estimation of the global warming potential

Figure 174: Fraction of gas remaining in the atmosphere

Source: Kleinberg(2020) & Author’s calculations.
Estimation of the global warming potential

Remark

- The decay function is a survival function
- The density function is equal to \( f_i(t) = -\partial_t S_i(t) \)
- Let \( \tau_i \) be random time that the gas remains in the atmosphere
- In the case of the exponential distribution \( \mathcal{E}(\lambda) \), we have

\[
S_i(t) = e^{-\lambda t} \\
f_i(t) = \lambda e^{-\lambda t} \\
\mathbb{E}[\tau_i] = \frac{1}{\lambda}
\]

\( \Rightarrow \) The survival function of the CH\(_4\) gas is exponential with a mean time equal to 12.4 years (\( \lambda = 1/12.4 \))
In the general case, the probability density function is equal to:

\[ f_i(t) = -\partial_t S_i(t) = \sum_{j=1}^{m} a_{i,j} \lambda_{i,j} e^{-\lambda_{i,j} t} \]

The mean time \( T_i \) is given by:

\[ T_i := \mathbb{E}[\tau_i] = \int_0^{\infty} s f_i(s) \, ds \]

\[ = \sum_{j=1}^{m} a_{i,j} \int_0^{\infty} \lambda_{i,j} s e^{-\lambda_{i,j} s} \, ds \]

\[ = \sum_{j=1}^{m} \frac{a_{i,j}}{\lambda_{i,j}} \]

Remark

We have \( T_{\text{CH}_4} = 12.4 \) years, but \( T_{\text{CO}_2} = \infty \)
Estimation of the global warming potential

Figure 175: Probability density function of the random time

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

**Remark**

- \( f_i(t) \) is an exponential mixture distribution where \( m \) is the number of mixture components
- \( \mathcal{E}(\lambda_{i,j}) \) is the probability distribution associated with the \( j^{th} \) component
- \( a_{i,j} \) is the mixture weight of the \( j^{th} \) component

We have:

\[
\tau_i = \mathbb{E}[\tau_i] = \sum_{j=1}^{m} a_{i,j} \mathbb{E}[\tau_{i,j}] = \sum_{j=1}^{m} a_{i,j} \tau_{i,j}
\]

For the CO\(_2\) gas, the exponential mixture distribution is defined by the following parameters:

<table>
<thead>
<tr>
<th>( j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{i,j} )</td>
<td>0.2173</td>
<td>0.2240</td>
<td>0.2824</td>
<td>0.2763</td>
</tr>
<tr>
<td>( \lambda_{i,j} \times 10^3 )</td>
<td>0.00</td>
<td>2.535</td>
<td>27.367</td>
<td>232.342</td>
</tr>
<tr>
<td>( \tau_{i,j} ) (in years)</td>
<td>( \infty )</td>
<td>394.4</td>
<td>36.54</td>
<td>4.304</td>
</tr>
</tbody>
</table>
We have $S_{\text{CO}_2}(\infty) = 21.73\%$!
Estimation of the global warming potential

Figure 177: Absolute global warming potential

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

Figure 178: Global warming potential for methane

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

We have:

- \( \text{Agwp}_{\text{CO}_2} (\infty) = \infty \)
- \( \text{Agwp}_{\text{CH}_4} (\infty) = A_{\text{CH}_4} \times T_{\text{CH}_4} \propto 2.11 \times 12.4 = 26.164 \)
- The instantaneous global warming potential of the methane is equal to:
  \[
  \text{gwp}_{\text{CH}_4} (0) = \frac{A_{\text{CH}_4}}{A_{\text{CO}_2}} = \frac{2.11 \times 10^{-16}}{1.76 \times 10^{-18}} \approx 119.9
  \]
- After 100 years, we obtain:
  \[
  \text{gwp}_{\text{CH}_4} (100) = 28.3853
  \]
  This is the IPCC value!
- Because of the persistent regime of the carbon dioxide, we have
  \( \text{gwp}_{\text{CH}_4} (\infty) = 0 \)
- We have:
  \[
  \text{gwp}_{\text{CH}_4} (t) \leq 1 \iff t \geq 6382 \text{ years}
  \]
Estimation of the global warming potential

Table 89: GWP values for 100-year time horizon

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>AR2</th>
<th>AR4</th>
<th>AR5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>21</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>N₂O</td>
<td>310</td>
<td>298</td>
<td>265</td>
</tr>
<tr>
<td>Sulphur hexafluoride</td>
<td>SF₆</td>
<td>23 900</td>
<td>22 800</td>
<td>23 500</td>
</tr>
<tr>
<td>Hydrofluorocarbons (HFC)</td>
<td>CHF₃</td>
<td>11 700</td>
<td>14 800</td>
<td>12 400</td>
</tr>
<tr>
<td></td>
<td>CH₂F₂</td>
<td>650</td>
<td>675</td>
<td>677</td>
</tr>
<tr>
<td></td>
<td>Etc.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perfluorocarbons (PFC)</td>
<td>CF₄</td>
<td>6 500</td>
<td>7 390</td>
<td>6 630</td>
</tr>
<tr>
<td></td>
<td>C₂F₆</td>
<td>9 200</td>
<td>12 200</td>
<td>11 100</td>
</tr>
<tr>
<td></td>
<td>Etc.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consolidation accounting at the company level

Two approaches:

1. Equity share approach
2. Control approach
   1. Financial control
   2. Operational control
### Table 90: Percent of reported GHG emissions under each consolidation method

<table>
<thead>
<tr>
<th>Accounting categories</th>
<th>GHG accounting based on equity share</th>
<th>financial control</th>
<th>operational control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholly owned asset</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Group companies/subsidiaries</td>
<td>OWNNR</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Associated/affiliated companies</td>
<td>OWNNR</td>
<td>0%</td>
<td>0%/100%</td>
</tr>
<tr>
<td>Joint ventures/partnerships</td>
<td>OWNNR</td>
<td>OWNNR</td>
<td>0%/100%</td>
</tr>
<tr>
<td>Fixed asset investments</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Franchises</td>
<td>OWNNR</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Source: GHG Protocol (2004, Table 1, page 19).

OWNR = Ownership ratio
For each company, the brown number corresponds to the carbon emissions in tCO$_2$e. The three figures at the right or left of the node corresponds respectively to the equity share, the financial control and the operational control.
Consolidation accounting at the company level

- **Equity share approach:**

\[
CE_A = 827 + 100\% \times 135 + 90\% \times 261 + 45\% \times 220 + 0\% \times 1385 + 90\% \times 75\% \times 63 + 90\% \times 50\% \times 179 + 45\% \times 33\% \times 37
\]

\[= 1424.4\text{tCO}_2\text{e}\]

- **Financial control approach:**

\[
CE_A = 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1385 + 100\% \times 100\% \times 63 + 100\% \times 50\% \times 179 + 100\% \times 0\% \times 37
\]

\[= 1595.50\text{tCO}_2\text{e}\]

- **Operational control approach:**

\[
CE_A = 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1385 + 100\% \times 100\% \times 63 + 100\% \times 0\% \times 179 + 100\% \times 0\% \times 37
\]

\[= 1506.00\text{tCO}_2\text{e}\]
Scope 1, 2 and 3 of carbon emissions

GHG Protocol (www.ghgprotocol.org/corporate-standard)

- **Scope 1** denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- **Scope 2** corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- **Scope 3** are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories\(^a\):
  - Upstream scope 3 emissions are defined as indirect carbon emissions related to purchased goods and services.
  - Downstream scope 3 emissions are defined as indirect carbon emissions related to sold goods and services.

\(^a\)The upstream value chain includes all activities related to the suppliers whereas the downstream value chain refers to post-manufacturing activities.
### Table 91: Examples of CDP reporting (\(CE\) in tCO\(_2\)e, year 2020)

<table>
<thead>
<tr>
<th>Scope</th>
<th>Category</th>
<th>Sub-category</th>
<th>Amazon</th>
<th>Danone</th>
<th>ENEL</th>
<th>Pfizer</th>
<th>Netflix</th>
<th>Walmart</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>9623 138</td>
<td>668 354</td>
<td>45 255 000</td>
<td>654 460</td>
<td>30 883</td>
<td>7 236 499</td>
</tr>
<tr>
<td>2</td>
<td>Location-based (2a)</td>
<td>Purchased goods and services</td>
<td>16 683 423</td>
<td>19 920 918</td>
<td>2 526 537</td>
<td>765 208</td>
<td>130 200 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Capital goods</td>
<td>13 202 065</td>
<td>191 894</td>
<td>116 366</td>
<td>645 328</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fuel and energy related activities</td>
<td>1 248 847</td>
<td>283 764</td>
<td>1 061 268</td>
<td>203 093</td>
<td>12 287</td>
<td>3 327 874</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream transportation and distribution</td>
<td>8 563 695</td>
<td>321 558</td>
<td>112 358</td>
<td>723 558</td>
<td>64 693</td>
<td>342 577</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waste generated in operations</td>
<td>16 628</td>
<td>152 789</td>
<td>3 161</td>
<td>14 940</td>
<td>869 927</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Business travel</td>
<td>313 043</td>
<td>35 128</td>
<td>41 439</td>
<td>37 439</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Employee commuting</td>
<td>306 033</td>
<td>48 414</td>
<td>19 116</td>
<td>3 500 000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream leased assets</td>
<td>1 223 903</td>
<td>30 522</td>
<td>131</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Downstream</td>
<td>Downstream transportation and distribution</td>
<td>2 785 676</td>
<td>1 627 090</td>
<td>7 295</td>
<td>5 099</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Processing of sold products</td>
<td>1 426 543</td>
<td>1 885 548</td>
<td>46 524 860</td>
<td>952</td>
<td>32 211 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use of sold products</td>
<td>0</td>
<td>782 649</td>
<td></td>
<td>349</td>
<td>130 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>End-of-life treatment of sold products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Downstream leased assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Franchises</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Investments</td>
<td>36 839</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scope 1 + 2a</td>
<td></td>
<td>18 642 924</td>
<td>1 533 064</td>
<td>50 245 685</td>
<td>1 206 037</td>
<td>59 468</td>
<td>18 268 299</td>
</tr>
<tr>
<td></td>
<td>Scope 1 + 2b</td>
<td></td>
<td>14 888 227</td>
<td>1 147 564</td>
<td>53 110 954</td>
<td>1 196 981</td>
<td>31 024</td>
<td>16 426 836</td>
</tr>
<tr>
<td></td>
<td>Scope 3 upstream</td>
<td></td>
<td>41 557 637</td>
<td>20 679 029</td>
<td>1 176 787</td>
<td>3 774 086</td>
<td>1 019 240</td>
<td>138 923 145</td>
</tr>
<tr>
<td></td>
<td>Scope 3 downstream</td>
<td></td>
<td>4 212 219</td>
<td>4 295 287</td>
<td>46 524 860</td>
<td>44 134</td>
<td>1 301</td>
<td>32 346 229</td>
</tr>
<tr>
<td></td>
<td>Scope 3</td>
<td></td>
<td>45 769 856</td>
<td>24 974 316</td>
<td>47 701 647</td>
<td>3 818 220</td>
<td>1 020 541</td>
<td>171 269 374</td>
</tr>
<tr>
<td></td>
<td>Scope 1 + 2a + 3</td>
<td></td>
<td>64 412 780</td>
<td>26 507 380</td>
<td>97 947 332</td>
<td>5 024 257</td>
<td>1 080 009</td>
<td>189 537 673</td>
</tr>
<tr>
<td></td>
<td>Scope 1 + 2b + 3</td>
<td></td>
<td>60 658 083</td>
<td>26 121 880</td>
<td>100 812 601</td>
<td>5 015 201</td>
<td>1 051 565</td>
<td>187 696 210</td>
</tr>
</tbody>
</table>

Source: CDP database as of 01/07/2022 & Author’s computation.
Scope 1, 2 and 3 of carbon emissions

CDP questionnaire for corporates

- www.cdp.net/en/guidance/guidance-for-companies
- HTML, Word and PDF formats
- 129 pages and 16 sections: $SC_1$ ($\S C6.1$), $SC_2$ ($\S C6.3$) and $SC_3$ emissions ($\S C6.5$) — emissions intensities ($\S C6.10$)
Computation of scope 1 emissions

- We allocate the activities to the three scopes
- Then, we apply an emission factor to each activity and each gas:

\[ E_{g,h} = A_h \cdot E \mathcal{F}_{g,h} \]

where \( A_h \) is the \( h^{th} \) activity rate (also called activity data) and \( E \mathcal{F}_{g,h} \) is the emission factor for the \( h^{th} \) activity and the \( g^{th} \) gas

- \( A_h \) can be measured in volume, weight, distance, duration, surface, etc.
- \( E_{g,h} \) is expressed in tonne
- \( E \mathcal{F}_{g,h} \) is measured in tonne per activity unit
- For each gas, we calculate the total emissions:

\[ E_g = \sum_{h=1}^{n_A} E_{g,h} = \sum_{h=1}^{n_A} A_h \cdot E \mathcal{F}_{g,h} \]

- Finally, we estimate the carbon emissions by applying the right GWP:

\[ CE = \sum_{g=1}^{n_G} gwp_g \cdot E_g \]
Tier methods

The choice of data inputs is codified by IPCC (2019):

- Tier 1 methods use global default emission factors;
- Tier 2 methods use country-level or region-specific emission factors;
- Tier 3 methods use directly monitored or site-specific emission factors.

⇒ IPCC Emission Factor Database, National Inventory Reports (NIRs), country emission factor databases, etc.

France

- The database of emission factors is managed by ADEME (Agence de l’Environnement et de la Maîtrise de l’Energie)
- It contains about 5300 validated emission factors
- [https://bilans-ges.ademe.fr](https://bilans-ges.ademe.fr)
Reporting of scope 1 emissions

GHG inventory document of Enel (2021)

- Scope 1 emissions expressed in ktCO₂e:

<table>
<thead>
<tr>
<th></th>
<th>CO₂</th>
<th>CH₄</th>
<th>N₂O</th>
<th>NF₃</th>
<th>SF₆</th>
<th>HFCs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity power</td>
<td>50,643.54</td>
<td>385.25</td>
<td>98.14</td>
<td>0.014</td>
<td>31.15</td>
<td>10.22</td>
<td>51,168.32</td>
</tr>
<tr>
<td>generation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity distribution</td>
<td>208.33</td>
<td>0.24</td>
<td>0.45</td>
<td></td>
<td>111.62</td>
<td></td>
<td>320.64</td>
</tr>
<tr>
<td>Real estate</td>
<td>79.87</td>
<td>0.22</td>
<td>1.24</td>
<td></td>
<td></td>
<td></td>
<td>81.30</td>
</tr>
<tr>
<td>Total</td>
<td>50,931.72</td>
<td>385.71</td>
<td>99.83</td>
<td>0.014</td>
<td>142.77</td>
<td>10.22</td>
<td>51,750.26</td>
</tr>
</tbody>
</table>

- The scope 1 emissions of Enel is equal to 51.75 MtCO₂e
## Scope 1 emissions

Table 92: Examples of emission factors (EFDB, IPCC)

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Gas</th>
<th>Region</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron and steel production</td>
<td>Integrated facility</td>
<td>CO₂</td>
<td>Canada</td>
<td>1.6</td>
<td>t/tonne</td>
</tr>
<tr>
<td></td>
<td>Electrode consumption from steel produced in electric arc furnaces</td>
<td>CO₂</td>
<td>Global</td>
<td>5.0</td>
<td>kg/tonne</td>
</tr>
<tr>
<td></td>
<td>Steel processing (rolling mills)</td>
<td>N₂O</td>
<td>Global</td>
<td>40</td>
<td>g/tonne</td>
</tr>
<tr>
<td>Manufacture of solid fuels</td>
<td>Metallurgical coke production</td>
<td>CO₂</td>
<td>Global</td>
<td>0.56</td>
<td>t/tonne</td>
</tr>
<tr>
<td>Fuel combustion activities</td>
<td>Crude oil</td>
<td>CH₄</td>
<td>Global</td>
<td>0.1</td>
<td>g/tonne</td>
</tr>
<tr>
<td></td>
<td>Natural gas</td>
<td>CO₂</td>
<td>Global</td>
<td>20</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td></td>
<td>Ethane</td>
<td>CO₂</td>
<td>Global</td>
<td>15.3</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td></td>
<td>Semiconductor manufacturing (silicon)</td>
<td>CF₄</td>
<td>Global</td>
<td>16.8</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td></td>
<td>Cement production</td>
<td>CO₂</td>
<td>Global</td>
<td>0.4985</td>
<td>t/tonne</td>
</tr>
<tr>
<td></td>
<td>Enteric fermentation</td>
<td>CH₄</td>
<td>Global</td>
<td>0.078</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Horses</td>
<td>Manure management (annual average temperature is less than 15°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>1.4</td>
<td>kg/head/year</td>
</tr>
<tr>
<td></td>
<td>Manure management (annual average temperature is between 15°C and 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>2.1</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Buffalo</td>
<td>Enteric fermentation</td>
<td>CH₄</td>
<td>Global</td>
<td>65</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Poultry</td>
<td>Manure management (annual average temperature is less than 15°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.117</td>
<td>kg/head/year</td>
</tr>
<tr>
<td></td>
<td>Manure management (annual average temperature is between 15°C and 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.157</td>
<td>kg/head/year</td>
</tr>
<tr>
<td></td>
<td>Manure management (annual average temperature is greater than 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.023</td>
<td>kg/head/year</td>
</tr>
</tbody>
</table>

Scope 2 emissions

Definition

Scope 2 is “an indirect emission category that includes GHG emissions from the purchased or acquired electricity, steam, heat, or cooling consumed” (GHG Protocol, 2015):

- **Electricity**
  People use electricity for operating machines, lighting, heating, cooling, electric vehicle charging, computers, electronics, public transportation systems, etc.

- **Steam**
  Industries use steam for mechanical work, heating, propulsion, driven turbines in electric power plants, etc.

- **Heat**
  Buildings use heat to control inside temperature and heat water, while the industrial sector uses heat for washing, cooking, sterilizing, drying, etc. Heat may be produced from electricity, solar heat processes or thermal combustion.

- **Cooling**
  It is produced from electricity or though the processes of forced air, conduction, convection, etc.
**Scope 2 emissions**

**Figure 180:** Energy production and consumption from owned/operated generation

**Figure 181:** Direct line energy transfer

Chapter 5: Identifying Scope 2 Emissions and Setting the Scope 2 Boundary

Electricity generators report any emissions from generation in scope 1, but most renewable or nuclear technology would report “zero” emissions from this generation. A grid operator or utility dispatches these generation units throughout the day on the basis of contracts, cost, and other factors. Because it is a shared network as opposed to a direct line, consumers may not be able to identify the specific power plant producing the energy they are using at any given time. Use of specified generation on the grid can only be determined contractually. Energy on the grid moves to the nearest point it can be used, and multiple regions can exchange power depending on the capacity and needs of these regions. Steam, heat, and cooling can also be delivered through a grid, often called a district energy system. Such systems provide energy to multiple consumers, though they often have only one generation facility and serve a more limited geographic area than electricity grids.

4. If some consumed electricity comes from owned/operated equipment, and some is purchased from the grid (Figure 5.4).

Some companies own, operate, or host energy generation sources such as solar panels or fuel cells on the premises of their building or in close proximity to where the energy is consumed. This arrangement is often termed “distributed generation” or “on-site” consumption, as it consists of generation units across decentralized locations (often Figure 5.3 electricity distribution on a grid).

Figure 182: Electricity production on a grid

Source: GHG Protocol (2015, Figure 5.4, page 38).
**Figure 183:** Facility consuming both energy generated on-site and purchased from the grid

---

Source: GHG Protocol (2015, Figure 5.3, page 37).

---

**Scope 2 emissions**

The company may consume some or all of the energy output from these generation facilities; sell excess energy output back to the grid; and purchase additional grid power to cover any remaining energy demand.

The owners/operator of a distributed generation facility may therefore have both scope 1 emissions from energy generation, as well as scope 2 emissions from any energy purchased from the grid, or consumed from on-site generation where attributes (e.g. certificates) are sold.

This arrangement impacts activity data as follows:

- **Determining the underlying activity data** (in MWh or kWh) in these systems may be challenging given the flux of electricity coming in or flowing out.
- Many markets utilize “net metering” for these systems, which allows grid purchases to be measured only as net of any energy exported to the grid. This net number may also be the basis for how costs are assessed.
- For accurate scope 2 GHG accounting, companies shall use the total—or gross—electricity purchases from the grid rather than grid purchases “net” of generation for the scope 2 calculation. A company’s total energy consumption would therefore include self-generated energy (any emissions reflected in scope 1) and total electricity purchased from the grid (electricity). It would exclude generation sold back to the grid.
- If a company cannot distinguish between its gross and net grid purchases, it should state and justify this in the inventory.

Table 5.1 illustrates the difference between total energy consumption and net energy consumption (if the reporter is a net grid consumer rather than producer). A negative
Computation of scope 2 emissions

Scope 2 emissions are calculated using activity data and emission factors expressed in MWh and tCO₂e/MWh:

\[ CE = \sum_s A_s \cdot \varepsilon F_s \]

where:
- \( A_s \) is the amount of purchased electricity for the energy generation source \( s \)
- \( \varepsilon F_s \) is the emission factor of the source \( s \)
Example #2

We consider a company, whose electricity consumption is equal to 2 000 MWh per year. The electricity comes from two sources: 60% from a direct line with an electricity supplier (source $S_1$) and 40% from the country grid (source $S_2$). The emission factors are respectively equal to 200 and 350 gCO$_2$e/kWh.
Computation of scope 2 emissions

- The electricity consumption from source $S_1$ is equal to $60\% \times 2000 = 1200$ MWh or $1200000$ kWh.
- We deduce that the carbon emissions from this source is:

$$CE(S_1) = (1.2 \times 10^6) \times 200 = 240 \times 10^6 \text{ gCO}_2\text{e} = 240 \text{ tCO}_2\text{e}$$

- For the second source, we obtain:

$$CE(S_2) = (0.8 \times 10^6) \times 350 = 280 \times 10^6 \text{ gCO}_2\text{e} = 280 \text{ tCO}_2\text{e}$$

- We deduce that the Scope 2 carbon emissions of this company is equal to $520$ tCO$_2$e.
Scope 2 emissions accounting

Two main methods:

- **Location-based method**
  In this approach, the company uses the average emission factor of the region or the country. For instance, if the electricity consumption is located in France, the company can use the emission intensity of the French energy mix;

- **Market-based method**
  This approach reflects the GHG emissions from the electricity that the company has chosen in the market. This means that the scope 2 carbon emissions will depend on the scope 1 carbon intensity of the electricity supplier
Figure 184: Emission factor in $g \text{CO}_2e/kWh$ of electricity generation (European Union, 1990 – 1992)

## Scope 2 emission factors

Table 93: Emission factor in g\(\text{CO}_2\text{e}/\text{kWh}\) of electricity generation in the world

<table>
<thead>
<tr>
<th>Region</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>484</td>
<td>Australia</td>
<td>531</td>
<td>Germany</td>
<td>354</td>
<td>Portugal</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>539</td>
<td>Canada</td>
<td>128</td>
<td>India</td>
<td>637</td>
<td>Russia</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>280</td>
<td>China</td>
<td>544</td>
<td>Iran</td>
<td>492</td>
<td>Spain</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>North America</td>
<td>352</td>
<td>Costa Rica</td>
<td>33</td>
<td>Italy</td>
<td>226</td>
<td>Switzerland</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>South America</td>
<td>204</td>
<td>Cuba</td>
<td>575</td>
<td>Japan</td>
<td>479</td>
<td>United Kingdom</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>World</td>
<td>442</td>
<td>France</td>
<td>58</td>
<td>Norway</td>
<td>26</td>
<td>United States</td>
<td>380</td>
<td></td>
</tr>
</tbody>
</table>

Source: https://ourworldindata.org/grapher/carbon-intensity-electricity
Example #3

We consider a French bank, whose activities are mainly located in France and the Western Europe. Below, we report the energy consumption (in MWh) by country:

<table>
<thead>
<tr>
<th>Country</th>
<th>Energy Consumption (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>125,807</td>
</tr>
<tr>
<td>Germany</td>
<td>71,890</td>
</tr>
<tr>
<td>Italy</td>
<td>197,696</td>
</tr>
<tr>
<td>Netherlands</td>
<td>18,152</td>
</tr>
<tr>
<td>Spain</td>
<td>61,106</td>
</tr>
<tr>
<td>UK</td>
<td>124,010</td>
</tr>
<tr>
<td>France</td>
<td>1,132,261</td>
</tr>
<tr>
<td>Ireland</td>
<td>125,807</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>33,069</td>
</tr>
<tr>
<td>Portugal</td>
<td>12,581</td>
</tr>
<tr>
<td>Switzerland</td>
<td>73,148</td>
</tr>
<tr>
<td>World</td>
<td>37,742</td>
</tr>
</tbody>
</table>
If we consider a Tier 1 approach, we can estimate the scope 2 emissions of the bank by computing the total activity data and multiplying by the global emission factor.

Since we have twelve sources, we obtain:

\[
A = \sum_{s=1}^{12} A_s = 125\,807 + 1\,132\,261 + \ldots + 37\,742 = 2\,013\,269 \text{ MWh}
\]

and:

\[
CE = A \cdot EF_{World} = (2\,013,269 \times 10^3) \times 442 = 889\,864\,898\,000 \text{ gCO}_2e = 889.86 \text{ ktCO}_2e
\]
Another Tier 1 approach is to consider the emission factor of the European Union, because the rest of the world represents less than 2% of the electricity consumption. Using $EF_{EU} = 275$, we obtain $CE = 553.65$ ktCO$_2$e
The third approach uses a Tier 2 method by considering the emission factor of each country.

We use the previous figures and the following emission factors: Belgium (143); Ireland (402); Luxembourg (68) and Netherlands (331).

We deduce that:

\[ CE = \sum_{s=1}^{12} A_s \cdot EF_s \]

\[ = (125,807 \times 143 + 1,132,261 \times 58 + \ldots + 124,010 \times 270 + 37,742 \times 442) \times \frac{10^3}{10^9} \]

\[ = 278.85 \text{ ktCO}_2\text{e} \]

⇒ The estimated scope 2 emissions of this bank are sensitive to the approach.
Example #4

We consider a Norwegian company, whose current electricity consumption is equal to 1351 Mwh. 60% of the electricity comes from the Norwegian hydroelectricity and the GO system guarantees that this green electricity emits 1 gCO₂e/kWh.

If we assume that the remaining 40% of the electricity consumption comes from the Norwegian grid²³, the market based scope 2 emissions of this company are equal to:

\[ \text{CE} = \frac{10^6 \times 60\% \times 1 + 10^6 \times 40\% \times 26}{10^6} \]

\[ = 11 \text{ ktCO}_2\text{e} \]

²³The emission factor for Norway is 26 gCO₂e/kWh.
## Computation of scope 2 emissions

**Table 94:** Emission factor in gCO$_2$e/KWh from electricity supply technologies (IPCC, 2014; UNECE, 2022)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Characteristic</th>
<th>IPCC</th>
<th>UNECE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Min–Max</td>
</tr>
<tr>
<td>Wind</td>
<td>Onshore</td>
<td>11</td>
<td>7–56</td>
</tr>
<tr>
<td></td>
<td>Offshore</td>
<td>12</td>
<td>8–35</td>
</tr>
<tr>
<td>Nuclear</td>
<td></td>
<td>12</td>
<td>3–110</td>
</tr>
<tr>
<td>Hydro power</td>
<td></td>
<td>24</td>
<td>1–2200</td>
</tr>
<tr>
<td>Solar power</td>
<td>CSP</td>
<td>27</td>
<td>9–63</td>
</tr>
<tr>
<td></td>
<td>Rooftop (PV)</td>
<td>41</td>
<td>26–60</td>
</tr>
<tr>
<td></td>
<td>Utility/ground (PV)</td>
<td>48</td>
<td>18–180</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38</td>
<td>6–79</td>
</tr>
<tr>
<td>Geothermal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biomass</td>
<td>Dedicated</td>
<td>230</td>
<td>130–420</td>
</tr>
<tr>
<td>Gas</td>
<td>CCS</td>
<td>169</td>
<td>90–370</td>
</tr>
<tr>
<td></td>
<td>Combined cycle</td>
<td>490</td>
<td>410–650</td>
</tr>
<tr>
<td></td>
<td></td>
<td>510–1170</td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>CCS</td>
<td>161</td>
<td>70–290</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td>820</td>
<td>740–650</td>
</tr>
</tbody>
</table>

CSP: concentrated solar power; PV: photovoltaic power; CCS: carbon capture and storage; PC: pulverized coal.
Reporting of scope 2 emissions

GHG inventory document of Enel (2021)

- The scope 2 emissions expressed in kt\(\text{CO}_2\text{e}\) are:

<table>
<thead>
<tr>
<th></th>
<th>Electricity purchased from the grid</th>
<th>Losses on the distribution grid</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location-based</td>
<td>1 336.67</td>
<td>2 966.52</td>
<td>4 303.18</td>
</tr>
<tr>
<td>Market-based</td>
<td>2 351.00</td>
<td>4 763.15</td>
<td>7 114.15</td>
</tr>
</tbody>
</table>
## Location-based versus market-based scope 2 emissions

Table 95: Statistics of CDP scope 2 emissions (2020)

<table>
<thead>
<tr>
<th></th>
<th>$CE_{loc} = 0$</th>
<th>$CE_{loc} = CE_{mkt} = 0$</th>
<th>$CE_{mkt} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0.89%</td>
<td>0.39%</td>
<td>8.78%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$CE_{loc} &gt; CE_{mkt}$</th>
<th>$CE_{loc} = CE_{mkt}$</th>
<th>$CE_{loc} &lt; CE_{mkt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>70.43%</td>
<td>9.48%</td>
<td>20.09%</td>
</tr>
<tr>
<td>Mean variation ratio</td>
<td>+43.89%</td>
<td>0.00%</td>
<td>−22.04%</td>
</tr>
</tbody>
</table>

Source: CDP database as of 01/07/2022 & Author’s computation.
Scope 3 categories

**Upstream**
1. Purchased goods and services
2. Capital goods
3. Fuel and energy related activities
4. Upstream transportation and distribution
5. Waste generated in operations
6. Business travel
7. Employee commuting
8. Upstream leased assets
9. Other upstream

**Downstream**
1. Downstream transportation and distribution
2. Processing of sold products
3. Use of sold products
4. End-of-life treatment of sold products
5. Downstream leased assets
6. Franchises
7. Investments
8. Other downstream
Scope 3 emissions are all the indirect emissions in the company’s value chain, apart from indirect emissions which are reported in scope 2:

1. **Purchased goods and services (not included in categories 2-8)**
   Extraction, production, and transportation of goods and services purchased or acquired by the company

2. **Capital goods**
   Extraction, production, and transportation of capital goods purchased or acquired by the company

3. **Fuel- and energy-related activities (not included in scopes 1 or 2)**
   Extraction, production, and transportation of fuels and energy purchased or acquired by the company

4. **Upstream transportation and distribution**
   Transportation and distribution of products purchased by the company between the company’s tier 1 suppliers and its own operations;
   Transportation and distribution services purchased by the company, including inbound logistics, outbound logistics (e.g., sold products), and transportation and distribution between the company’s own facilities
Scope 3 emissions

- **Waste generated in operations**
  Disposal and treatment of waste generated in the company’s operations

- **Business travel**
  Transportation of employees for business-related activities

- **Employee commuting**
  Transportation of employees between their homes and their work sites

- **Upstream leased assets**
  Operation of assets leased by the company (lessee)
Scope 3 emissions

9 Downstream transportation and distribution
Transportation and distribution of products sold by the company between the company’s operations and the end consumer (if not paid for by the company)

10 Processing of sold products
Processing of intermediate products sold by downstream companies (e.g., manufacturers)

11 Use of sold products
End use of goods and services sold by the company

12 End-of-life treatment of sold products
Waste disposal and treatment of products sold by the company at the end of their life
Scope 3 emissions

- **Downstream leased assets**
  Operation of assets owned by the company (lessor) and leased to other entities

- **Franchises**
  Operation of franchises reported by franchisor

- **Investments**
  Operation of investments (including equity and debt investments and project finance)
## Scope 3 emissions

### Table 96: Scope 3 emission factors for business travel and employee commuting (United States)

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>CO₂  (kg/unit)</th>
<th>CH₄  (g/unit)</th>
<th>N₂O  (g/unit)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger car</td>
<td>0.332</td>
<td>0.0070</td>
<td>0.0070</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Light-duty truck</td>
<td>0.454</td>
<td>0.0120</td>
<td>0.0090</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>0.183</td>
<td>0.0700</td>
<td>0.0070</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Intercity rail (northeast corridor)</td>
<td>0.058</td>
<td>0.0055</td>
<td>0.0007</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Intercity rail (other routes)</td>
<td>0.150</td>
<td>0.0117</td>
<td>0.0038</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Intercity rail (national average)</td>
<td>0.113</td>
<td>0.0092</td>
<td>0.0026</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Commuter rail</td>
<td>0.139</td>
<td>0.0112</td>
<td>0.0028</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Transit rail (subway, tram)</td>
<td>0.099</td>
<td>0.0084</td>
<td>0.0012</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Bus</td>
<td>0.056</td>
<td>0.0210</td>
<td>0.0009</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (short haul, &lt; 300 miles)</td>
<td>0.207</td>
<td>0.0064</td>
<td>0.0066</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (medium haul, 300-2300 miles)</td>
<td>0.129</td>
<td>0.0006</td>
<td>0.0041</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (long haul, &gt; 2300 miles)</td>
<td>0.163</td>
<td>0.0006</td>
<td>0.0052</td>
<td>passenger-mile</td>
</tr>
</tbody>
</table>


These factors are intended for use in the distance-based method defined in the Scope 3 Calculation Guidance. If fuel data are available, then the fuel-based method should be used.
## Scope 3 emissions

<table>
<thead>
<tr>
<th>Category</th>
<th>S3E</th>
<th>ADEME</th>
<th>Category</th>
<th>S3E</th>
<th>ADEME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2500</td>
<td>2300</td>
<td>Air transport</td>
<td>1970</td>
<td>1190</td>
</tr>
<tr>
<td>Construction</td>
<td>810</td>
<td>360</td>
<td>Education</td>
<td>310</td>
<td>120</td>
</tr>
<tr>
<td>Financial intermediation</td>
<td>140</td>
<td>110</td>
<td>Health and Social Work</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>560</td>
<td>320</td>
<td>Rubber and plastics</td>
<td>1270</td>
<td>800</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>300</td>
<td>170</td>
<td>Textiles</td>
<td>1100</td>
<td>600</td>
</tr>
</tbody>
</table>

Carbon emissions of investment portfolios

Two methods for measuring the carbon footprint of an investment portfolio:

1. Financed emissions approach
2. Ownership approach
The investor calculates the carbon emissions that are financed across both equity and debt.

EVIC is used to estimate the value of the enterprise. It is “the sum of the market capitalization of ordinary and preferred shares at fiscal year end and the book values of total debt and minorities interests” (TEG, 2019).

Let $W$ be the wealth invested in the company, the financed emissions are equal to:

$$CE(W) = \frac{W}{EVIC} \cdot CE$$

In the case of a portfolio $(W_1, \ldots, W_n)$ where $W_i$ is the wealth invested in company $i$, we have:

$$CE(W) = \sum_{i=1}^{n} CE_i(W_i) = \sum_{i=1}^{n} \frac{W_i}{EVIC_i} \cdot CE_i$$

$CE(W)$ is expressed in tCO$_2$e.
We break down the carbon emissions between the stockholders of the company.

We have:

$$\text{CE} (W) = \sum_{i=1}^{n} \frac{W_i}{MV_i} \cdot \text{CE}_i = \sum_{i=1}^{n} \varpi_i \cdot \text{CE}_i$$

where:

- $MV_i$ is the market value of company $i$
- $\varpi_i$ is the ownership ratio of the investor
Carbon emissions of investment portfolios
Ownership approach

- Let $W = \sum_{i=1}^{n} W_i$ be the portfolio value.
- The portfolio weight of asset $i$ is given by:
  \[ w_i = \frac{W_i}{W} \]

- We deduce that:
  \[ \varpi_i = \frac{W_i}{MV_i} = \frac{w_i \cdot W}{MV_i} \]

- It follows that:
  \[ CE(W) = \sum_{i=1}^{n} \frac{w_i \cdot W}{MV_i} CE_i = W \left( \sum_{i=1}^{n} \frac{w_i \cdot CE_i}{MV_i} \right) = W \left( \sum_{i=1}^{n} w_i \cdot CI_{i}^{MV} \right) \]
  where $CI_{i}^{MV}$ is the market value-based carbon intensity:
  \[ CI_{i}^{MV} = \frac{CE_i}{MV_i} \]

- $CE(W)$ is generally computed with $W = $1 mn and is expressed in tCO$_2$e (per $ mn$ invested)
The ownership approach is valid only for equity portfolios. To compute the market value (or the total market capitalization), we use the following approximation:

\[ MV = \frac{MC}{FP} \]

where \( MC \) and \( FP \) are the free float market capitalisation and percentage of the company.
**Example #5**

We consider a $100 mn investment portfolio with the following composition: $63.1 mn in company A, $16.9 mn in company B and $20.0 mn in company C. The data are the following:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Market capitalization (in $ bn)</th>
<th>31/12/2021</th>
<th>31/12/2022</th>
<th>31/01/2023</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>12.886</td>
<td>10.356</td>
<td>10.625</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>7.005</td>
<td>6.735</td>
<td>6.823</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>3.271</td>
<td>3.287</td>
<td>3.474</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Debt (in $ bn)</th>
<th>$FP (in %)</th>
<th>$SC_{1-2} (in ktCO₂e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.112</td>
<td>99.8</td>
<td>756.144</td>
</tr>
<tr>
<td>B</td>
<td>0.000</td>
<td>39.3</td>
<td>23.112</td>
</tr>
<tr>
<td>C</td>
<td>0.458</td>
<td>96.7</td>
<td>454.460</td>
</tr>
</tbody>
</table>
As of 31 January 2023, the EVIC value for company A is equal to:

$$\text{EVIC}_A = \frac{10356}{0.998} + 1112 = 11489 \text{ mn}$$

We deduce that the financed emissions are equal to:

$$CE_A (\$63.1 \text{ mn}) = \frac{63.1}{11489} \times 756.144 = 4.153 \text{ ktCO}_2\text{e}$$
If we assume that the investor has no bond in the portfolio, we can use the ownership approach:

$$w_A = \frac{63.1}{10,625/0.998} = 59.2695 \text{ bps}$$

The carbon emissions of the investment in company $A$ is then equal to:

$$CE_A ($63.1 \text{ mn}) = 59.2695 \times 10^{-4} \times 756.144 = 4.482 \text{ ktCO}_2e$$
Finally, we obtain the following results:

<table>
<thead>
<tr>
<th></th>
<th>Financed emissions</th>
<th>Carbon emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>4.153</td>
<td>4.482</td>
</tr>
<tr>
<td>Company B</td>
<td>0.023</td>
<td>0.022</td>
</tr>
<tr>
<td>Company C</td>
<td>2.356</td>
<td>2.530</td>
</tr>
<tr>
<td>Portfolio</td>
<td>6.532</td>
<td>7.034</td>
</tr>
</tbody>
</table>
Figure 185: 2019 carbon emissions per GICS sector in GtCO$_2$e (scopes 1 & 2)

Table 98: Breakdown (in %) of carbon emissions in 2019

<table>
<thead>
<tr>
<th>Sector</th>
<th>$SC_1$</th>
<th>$SC_2$</th>
<th>$SC_{1-2}$</th>
<th>$SC_{3}^{up}$</th>
<th>$SC_{3}^{down}$</th>
<th>$SC_3$</th>
<th>$SC_{1-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>0.1</td>
<td>5.1</td>
<td>0.8</td>
<td>1.5</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>1.7</td>
<td>9.7</td>
<td>2.9</td>
<td>14.1</td>
<td>10.2</td>
<td>10.8</td>
<td>9.1</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>2.3</td>
<td>6.7</td>
<td>2.9</td>
<td>18.6</td>
<td>1.6</td>
<td>4.4</td>
<td>4.1</td>
</tr>
<tr>
<td>Energy</td>
<td>15.0</td>
<td>8.5</td>
<td>14.0</td>
<td>14.1</td>
<td>40.1</td>
<td>36.0</td>
<td>31.2</td>
</tr>
<tr>
<td>Financials</td>
<td>0.7</td>
<td>1.8</td>
<td>0.9</td>
<td>2.6</td>
<td>1.8</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.3</td>
<td>1.7</td>
<td>0.5</td>
<td>2.6</td>
<td>0.2</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.2</td>
<td>8.9</td>
<td>10.0</td>
<td>15.6</td>
<td>24.2</td>
<td>22.8</td>
<td>20.0</td>
</tr>
<tr>
<td>Information Technology</td>
<td>0.6</td>
<td>6.8</td>
<td>1.5</td>
<td>4.9</td>
<td>2.3</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Materials</td>
<td>29.8</td>
<td>40.7</td>
<td>31.4</td>
<td>20.2</td>
<td>13.5</td>
<td>14.6</td>
<td>18.2</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.3</td>
<td>2.8</td>
<td>0.6</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Utilities</td>
<td>39.0</td>
<td>7.3</td>
<td>34.4</td>
<td>4.7</td>
<td>4.8</td>
<td>4.8</td>
<td>11.2</td>
</tr>
<tr>
<td><strong>Total (in GtCO$_2$e)</strong></td>
<td><strong>15.1</strong></td>
<td><strong>2.6</strong></td>
<td><strong>17.6</strong></td>
<td><strong>10.3</strong></td>
<td><strong>53.7</strong></td>
<td><strong>64.0</strong></td>
<td><strong>81.6</strong></td>
</tr>
</tbody>
</table>

Figure 186: 2019 carbon emissions per GICS sector in GtCO₂ₑ (scopes 1, 2 & 3 upstream)

Figure 187: 2019 carbon emissions per GICS sector in GtCO$_2$e (scopes 1, 2 & 3)

Source: Trucost (2022) & Barahhou et al. (2022)
Figure 188: Sector contribution in %

Figure 189: Histogram of 2019 carbon emissions (logarithmic scale, tCO$_2$e)

Negative and avoided emissions
Carbon intensity

- Carbon emissions = absolute carbon footprint in an absolute value
- Carbon intensity = relative carbon footprint

⇒ we normalize the carbon emissions by a size or activity unit
We can measure the carbon footprint of:

- countries by $t\text{CO}_2e$ per capita
- watching television by $\text{CO}_2e$ emissions per viewer-hour
- washing machines by $\text{kgCO}_2e$ per wash
- cars by $\text{kgCO}_2e$ per kilometer driven
- companies by $\text{ktCO}_2e$ per $1$ mn revenue
- etc.
Product carbon footprint (PCF)

- The product carbon footprint measures the relative carbon emissions of a product throughout its life cycle.
- Life cycle assessment (LCA), distinguishes two methods:
  1. **Cradle-to-gate** refers to the carbon footprint of a product from the moment it is produced (including the extraction of raw materials) to the moment it enters the store.
  2. **Cradle-to-grave** covers the entire life cycle of a product, including the use-phase and recycling.
### Physical intensity ratios

**Table 99:** Examples of product carbon footprint (in kgCO$_2$e per unit)

<table>
<thead>
<tr>
<th>Product</th>
<th>Category</th>
<th>Cradle-to-gate</th>
<th>Cradle-to-grave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screen</td>
<td>21.5 inches</td>
<td>222</td>
<td>236</td>
</tr>
<tr>
<td>Screen</td>
<td>23.8 inches</td>
<td>248</td>
<td>265</td>
</tr>
<tr>
<td>Computer</td>
<td>Laptop</td>
<td>156</td>
<td>169</td>
</tr>
<tr>
<td>Computer</td>
<td>Desktop</td>
<td>169</td>
<td>189</td>
</tr>
<tr>
<td>Computer</td>
<td>High performance</td>
<td>295</td>
<td>394</td>
</tr>
<tr>
<td>Smartphone</td>
<td>Classical</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Smartphone</td>
<td>5 inches</td>
<td>33</td>
<td>32</td>
</tr>
<tr>
<td>Oven</td>
<td>Built-in electric</td>
<td>187</td>
<td>319</td>
</tr>
<tr>
<td>Oven</td>
<td>Professional (combi steamer)</td>
<td>734</td>
<td>12 676</td>
</tr>
<tr>
<td>Washing machine</td>
<td>Capacity 5kg</td>
<td>248</td>
<td>468</td>
</tr>
<tr>
<td>Washing machine</td>
<td>Capacity 7kg</td>
<td>275</td>
<td>539</td>
</tr>
<tr>
<td>Shirt</td>
<td>Coton</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Shirt</td>
<td>Viscose</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Balloon</td>
<td>Football</td>
<td>3.4</td>
<td>5.1</td>
</tr>
<tr>
<td>Balloon</td>
<td>Basket-ball</td>
<td>3.6</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Corporate carbon footprint (CCF)

- Extension of the PCF to companies
- The CCF of a cement manufacturer is measured by the amount of GHG emissions per tonne of cement
- The CCF of airlines is measured by the amount of GHG emissions per RPK (revenue passenger kilometers, which is calculated by multiplying the number of paying passengers by the distance traveled)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport sector (aviation)</td>
<td>CO₂e/RPK</td>
<td>Revenue passenger kilometers</td>
</tr>
<tr>
<td>Transport sector (shipping)</td>
<td>CO₂e/RTK</td>
<td>Revenue tonne kilometers</td>
</tr>
<tr>
<td>Industry (cement)</td>
<td>CO₂e/t cement</td>
<td>Tonne of cement</td>
</tr>
<tr>
<td>Industry (steel)</td>
<td>CO₂e/t steel</td>
<td>Tonne of steel</td>
</tr>
<tr>
<td>Electricity</td>
<td>CO₂e/MWh</td>
<td>Megawatt hour</td>
</tr>
<tr>
<td>Buildings</td>
<td>CO₂e/SQM</td>
<td>Square meter</td>
</tr>
</tbody>
</table>
Monetary intensity ratios

Problem

- How to aggregate carbon footprint?
- Portfolio managers use monetary intensity ratios, which are defined as:

\[ CI = \frac{CE}{Y} \]

where \( CE \) is the company’s carbon emissions and \( Y \) is a monetary variable measuring its activity.
For instance, we can use revenues, sales, etc. to normalize carbon emissions:

\[
\begin{align*}
\text{CI}_{\text{Revenue}} & = \frac{CE}{\text{Revenue}} \\
\text{CI}_{\text{Sales}} & = \frac{CE}{\text{Sales}} \\
\text{CI}_{\text{EVIC}} & = \frac{CE}{\text{EVIC}} \\
\text{CI}_{\text{MV}} & = \frac{CE}{\text{MV}}
\end{align*}
\]

**Remark**

*The previous carbon emission metrics based on EVIC and market value can be viewed as carbon intensity metrics*
If we consider the EVIC-based approach, the carbon intensity of the portfolio is given by:

$$CI^{EVIC}(w) = CE^{EVIC}(W)$$

$$= \frac{1}{W} \sum_{i=1}^{n} \frac{W_i}{EVIC_i} \cdot CE_i$$

$$= \sum_{i=1}^{n} \frac{W_i}{W} \cdot \frac{CE_i}{EVIC_i}$$

$$= \sum_{i=1}^{n} w_i \cdot CI^{EVIC}_i$$

where $w = (w_1, \ldots, w_n)$ is the vector of portfolio weights.

In a similar way, we obtain:

$$CI^{MV}(w) = \sum_{i=1}^{n} w_i \cdot CI^{MV}_i$$
Non-additivity property of \( CI \)

- We consider the revenue-based carbon intensity (also called the economic carbon intensity)
- The carbon intensity of the portfolio is:

\[
CI_{\text{Revenue}}(w) = \frac{CE(w)}{Y(w)}
\]

where:

- \( CE(w) \) measures the carbon emissions of the portfolio:

\[
CE(w) = \sum_{i=1}^{n} W_i \cdot \frac{CE_i}{MV_i} = W \sum_{i=1}^{n} \frac{W_i}{MV_i} \cdot CE_i
\]

- \( Y(w) \) is the total revenue of the portfolio:

\[
Y(w) = \sum_{i=1}^{n} W_i \cdot \frac{Y_i}{MV_i} = W \sum_{i=1}^{n} \frac{W_i}{MV_i} \cdot Y_i
\]
Non-additivity property of $\mathbf{CI}$

- We deduce that:

$$\mathbf{CI}^{\text{Revenue}}(w) = \frac{\sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot \mathbf{CE}_i}{\sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot Y_i} = \sum_{i=1}^{n} w_i \cdot \omega_i \cdot \mathbf{CI}_i^{\text{Revenue}}$$

where $\omega_i$ is the ratio between the revenue per market value of company $i$ and the weighted average revenue per market value of the portfolio:

$$\omega_i = \frac{Y_i}{MV_i} \cdot \frac{1}{\sum_{k=1}^{n} w_k \cdot \frac{Y_k}{MV_k}}$$

- We conclude that:

$$\mathbf{CI}^{\text{Revenue}}(w) \neq \sum_{i=1}^{n} w_i \cdot \mathbf{CI}_i^{\text{Revenue}}$$
In order to avoid the previous problem, we generally use the weighted average carbon intensity (WACI) of the portfolio:

\[ CI_{\text{Revenue}}(w) = \sum_{i=1}^{n} w_i \cdot CI_i^{\text{Revenue}} \]

This method is the standard approach in portfolio management
Carbon intensity is always additive when we consider a given issuer:

\[
\text{CI}_i(\text{SC}_{1-3}) = \frac{CE_i(\text{SC}_1) + CE_i(\text{SC}_2)}{Y_i} + CE_i(\text{SC}_3) = CI_i(\text{SC}_1) + CI_i(\text{SC}_2) + CI_i(\text{SC}_3)
\]
Example #6

We assume that $C\mathbf{E}_1 = 5 \times 10^6 \text{ CO}_2 \text{e}$, $Y_1 = 0.2 \times 10^6$, $MV_1 = 10 \times 10^6$, $C\mathbf{E}_2 = 50 \times 10^6 \text{ CO}_2 \text{e}$, $Y_2 = 4 \times 10^6$ and $MV_2 = 10 \times 10^6$. We invest $W = 10 \text{ mn}$. 
Illustration

- We deduce that:

\[ CI_1 = \frac{5 \times 10^6}{0.2 \times 10^6} = 25.0 \text{ tCO}_2e/\$\text{ mn} \]

and

\[ CI_2 = 12.5 \text{ tCO}_2e/\$\text{ mn} \]

- We have:

\[
\begin{align*}
CE(w) &= W \left( w_1 \frac{CE_1}{MV_1} + w_2 \frac{CE_2}{MV_2} \right) \\
Y(w) &= W \left( w_1 \frac{Y_1}{MV_1} + w_2 \frac{Y_2}{MV_2} \right) \\
CI(w) &= w_1 CI_1 + w_2 CI_2
\end{align*}
\]
We obtain the following results:

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$CE(w)$ ($\times 10^6$ CO$_2$e)</th>
<th>$Y(w)$ ($\times $10$^6$)</th>
<th>$\frac{CE(w)}{Y(w)}$</th>
<th>$CI(w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100%</td>
<td>50.00</td>
<td>4.00</td>
<td>12.50</td>
<td>12.50</td>
</tr>
<tr>
<td>10%</td>
<td>90%</td>
<td>45.50</td>
<td>3.62</td>
<td>12.57</td>
<td>13.75</td>
</tr>
<tr>
<td>20%</td>
<td>80%</td>
<td>41.00</td>
<td>3.24</td>
<td>12.65</td>
<td>15.00</td>
</tr>
<tr>
<td>30%</td>
<td>70%</td>
<td>36.50</td>
<td>2.86</td>
<td>12.76</td>
<td>16.25</td>
</tr>
<tr>
<td>50%</td>
<td>50%</td>
<td>27.50</td>
<td>2.10</td>
<td>13.10</td>
<td>18.75</td>
</tr>
<tr>
<td>70%</td>
<td>30%</td>
<td>18.50</td>
<td>1.34</td>
<td>13.81</td>
<td>21.25</td>
</tr>
<tr>
<td>80%</td>
<td>20%</td>
<td>14.00</td>
<td>0.96</td>
<td>14.58</td>
<td>22.50</td>
</tr>
<tr>
<td>90%</td>
<td>10%</td>
<td>9.50</td>
<td>0.58</td>
<td>16.38</td>
<td>23.75</td>
</tr>
<tr>
<td>100%</td>
<td>0%</td>
<td>5.00</td>
<td>0.20</td>
<td>25.00</td>
<td>25.00</td>
</tr>
</tbody>
</table>

We notice that the weighted average carbon intensity can be very different than the economic carbon intensity.
The case of sovereign issuers

Remark

For sovereign issuers, the economic carbon intensity is measured in mega-tonnes of CO₂ₑ per million dollars of GDP while the physical carbon intensity unit is tCO₂ₑ per capita.
Figure 190: Histogram of 2019 carbon intensities (logarithmic scale, tCO$_2$e/$$ mn)

### Table 100: Examples of 2019 carbon emissions and intensities

<table>
<thead>
<tr>
<th>Company</th>
<th>Carbon emissions (in tCO₂e)</th>
<th>Revenue (in $ mn)</th>
<th>Intensity (in tCO₂e/$ mn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airbus</td>
<td>576,705</td>
<td>73,999</td>
<td>7.3</td>
</tr>
<tr>
<td>Allianz</td>
<td>46,745</td>
<td>135,279</td>
<td>0.3</td>
</tr>
<tr>
<td>Alphabet</td>
<td>111,283</td>
<td>161,857</td>
<td>0.7</td>
</tr>
<tr>
<td>Amazon</td>
<td>5,760,000</td>
<td>280,522</td>
<td>20.5</td>
</tr>
<tr>
<td>Apple</td>
<td>50,549</td>
<td>260,174</td>
<td>0.2</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>64,829</td>
<td>78,244</td>
<td>0.8</td>
</tr>
<tr>
<td>Boeing</td>
<td>611,001</td>
<td>76,559</td>
<td>8.0</td>
</tr>
<tr>
<td>BP</td>
<td>49,199,999</td>
<td>276,850</td>
<td>177.7</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>905,000</td>
<td>53,800</td>
<td>16.8</td>
</tr>
<tr>
<td>Danone</td>
<td>722,122</td>
<td>28,308</td>
<td>25.5</td>
</tr>
<tr>
<td>Enel</td>
<td>69,981,891</td>
<td>86,610</td>
<td>808.0</td>
</tr>
<tr>
<td>Exelon</td>
<td>111,000,000</td>
<td>255,583</td>
<td>434.3</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>81,655</td>
<td>115,627</td>
<td>0.7</td>
</tr>
<tr>
<td>Juventus</td>
<td>6,665</td>
<td>709</td>
<td>9.4</td>
</tr>
<tr>
<td>LVMH</td>
<td>67,613</td>
<td>60,083</td>
<td>1.1</td>
</tr>
<tr>
<td>Microsoft</td>
<td>113,414</td>
<td>125,843</td>
<td>0.9</td>
</tr>
<tr>
<td>Nestle</td>
<td>3,291,303</td>
<td>93,153</td>
<td>35.3</td>
</tr>
<tr>
<td>Netflix</td>
<td>38,481</td>
<td>20,156</td>
<td>1.9</td>
</tr>
<tr>
<td>NVIDIA</td>
<td>2,767</td>
<td>11,716</td>
<td>0.2</td>
</tr>
<tr>
<td>PepsiCo</td>
<td>3,552,415</td>
<td>67,161</td>
<td>52.9</td>
</tr>
<tr>
<td>Pfizer</td>
<td>734,638</td>
<td>51,750</td>
<td>14.2</td>
</tr>
<tr>
<td>Roche</td>
<td>288,157</td>
<td>64,154</td>
<td>4.5</td>
</tr>
<tr>
<td>Samsung Electronics</td>
<td>5,067,000</td>
<td>197,733</td>
<td>25.6</td>
</tr>
<tr>
<td>TotalEnergies</td>
<td>40,909,135</td>
<td>200,316</td>
<td>204.2</td>
</tr>
<tr>
<td>Toyota</td>
<td>2,522,987</td>
<td>272,608</td>
<td>9.3</td>
</tr>
<tr>
<td>Volkswagen</td>
<td>4,494,066</td>
<td>282,817</td>
<td>15.9</td>
</tr>
<tr>
<td>Walmart</td>
<td>6,101,641</td>
<td>514,405</td>
<td>11.9</td>
</tr>
</tbody>
</table>

### Table 101: Examples of 2019 carbon intensities

<table>
<thead>
<tr>
<th>Company</th>
<th>Intensity (in tCO₂e/$ mn)</th>
<th>$SC_1$</th>
<th>$SC_2$</th>
<th>$SC_{up}$</th>
<th>$SC_{down}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon</td>
<td></td>
<td>20.5</td>
<td>19.6</td>
<td>71.5</td>
<td>37.2</td>
</tr>
<tr>
<td><strong>Apple</strong></td>
<td></td>
<td>0.2</td>
<td>3.3</td>
<td>106.2</td>
<td>21.0</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td></td>
<td>0.8</td>
<td>3.6</td>
<td>24.6</td>
<td>0.0</td>
</tr>
<tr>
<td>BP</td>
<td></td>
<td>177.7</td>
<td>18.8</td>
<td>375.1</td>
<td>2104.5</td>
</tr>
<tr>
<td>Caterpillar</td>
<td></td>
<td>16.8</td>
<td>17.2</td>
<td>282.5</td>
<td>7472.0</td>
</tr>
<tr>
<td><strong>Danone</strong></td>
<td></td>
<td>25.5</td>
<td>33.4</td>
<td>1023.4</td>
<td>157.7</td>
</tr>
<tr>
<td>Exxon</td>
<td></td>
<td>434.3</td>
<td>35.2</td>
<td>419.8</td>
<td>2324.6</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td></td>
<td>0.7</td>
<td>6.0</td>
<td>26.8</td>
<td>133.6</td>
</tr>
<tr>
<td>LVMH</td>
<td></td>
<td>1.1</td>
<td>4.4</td>
<td>197.3</td>
<td>15.7</td>
</tr>
<tr>
<td>Microsoft</td>
<td></td>
<td>0.9</td>
<td>28.3</td>
<td>47.5</td>
<td>31.8</td>
</tr>
<tr>
<td><strong>Nestle</strong></td>
<td></td>
<td>35.3</td>
<td>34.4</td>
<td>657.6</td>
<td>363.9</td>
</tr>
<tr>
<td>Pfizer</td>
<td></td>
<td>14.2</td>
<td>14.7</td>
<td>90.2</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Samsung Electronics</strong></td>
<td></td>
<td>25.6</td>
<td>55.6</td>
<td>169.7</td>
<td>308.4</td>
</tr>
<tr>
<td>Volkswagen</td>
<td></td>
<td>15.9</td>
<td>21.1</td>
<td>231.0</td>
<td>1254.9</td>
</tr>
<tr>
<td>Walmart</td>
<td></td>
<td>11.9</td>
<td>25.4</td>
<td>79.0</td>
<td>62.9</td>
</tr>
</tbody>
</table>

Table 102: Carbon intensity in $tCO_2e/$ mn per GICS sector and sector contribution in % (MSCI World, June 2022)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$b_i$ (in %)</th>
<th>$SC_1$</th>
<th>$SC_{1-2}$</th>
<th>$SC_{1-3}^{up}$</th>
<th>$SC_{1-3}$</th>
<th>$SC_1$</th>
<th>$SC_{1-2}$</th>
<th>$SC_{1-3}^{up}$</th>
<th>$SC_{1-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>2</td>
<td>28</td>
<td>134</td>
<td>172</td>
<td>0.14</td>
<td>1.31</td>
<td>3.30</td>
<td>1.31</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>23</td>
<td>65</td>
<td>206</td>
<td>590</td>
<td>1.87</td>
<td>4.17</td>
<td>6.92</td>
<td>6.21</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>28</td>
<td>55</td>
<td>401</td>
<td>929</td>
<td>1.68</td>
<td>2.66</td>
<td>10.16</td>
<td>7.38</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>632</td>
<td>698</td>
<td>1006</td>
<td>6823</td>
<td>24.49</td>
<td>21.53</td>
<td>16.33</td>
<td>34.37</td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>13</td>
<td>19</td>
<td>52</td>
<td>244</td>
<td>1.33</td>
<td>1.58</td>
<td>2.28</td>
<td>3.34</td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>10</td>
<td>22</td>
<td>120</td>
<td>146</td>
<td>1.12</td>
<td>1.92</td>
<td>5.54</td>
<td>2.12</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>111</td>
<td>130</td>
<td>298</td>
<td>1662</td>
<td>8.38</td>
<td>7.83</td>
<td>9.43</td>
<td>16.38</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>7</td>
<td>23</td>
<td>112</td>
<td>239</td>
<td>1.13</td>
<td>3.03</td>
<td>7.57</td>
<td>5.06</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>478</td>
<td>702</td>
<td>1113</td>
<td>2,957</td>
<td>15.89</td>
<td>18.57</td>
<td>15.48</td>
<td>12.93</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>22</td>
<td>101</td>
<td>167</td>
<td>571</td>
<td>0.48</td>
<td>1.81</td>
<td>1.57</td>
<td>1.65</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>1,744</td>
<td>1,794</td>
<td>2,053</td>
<td>2,840</td>
<td>43.47</td>
<td>35.59</td>
<td>21.41</td>
<td>9.24</td>
</tr>
<tr>
<td>MSCI World</td>
<td></td>
<td>130</td>
<td>163</td>
<td>310</td>
<td>992</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI World EW</td>
<td></td>
<td>168</td>
<td>211</td>
<td>391</td>
<td>1,155</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let \( b = (b_1, \ldots, b_n) \) be the weights of the assets that belong to a benchmark.

Its weighted average carbon intensity is given by:

\[
CI(b) = \sum_{i=1}^{n} b_i \cdot CI_i
\]

where \( CI_i \) is the carbon intensity of asset \( i \).

If we focus on the carbon intensity for a given sector, we use the following formula:

\[
CI(\text{Sector}_j) = \frac{\sum_{i \in \text{Sector}_j} b_i \cdot CI_i}{\sum_{i \in \text{Sector}_j} b_i}
\]
The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period $[t_0, t]$. From a mathematical point of view, it corresponds to the signed area of the region bounded by the function $C\varepsilon(t)$:

$$CB(t_0, t) = \int_{t_0}^{t} C\varepsilon(s) \, ds$$
Below, we report the historical data of carbon emissions from 2010 to 2020. Moreover, the company has announced his carbon targets for the years until 2050

<table>
<thead>
<tr>
<th>$t$</th>
<th>CE $t$</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>4.800</td>
<td>4.950</td>
<td>5.100</td>
<td>5.175</td>
<td>5.175</td>
<td>5.175</td>
<td>5.175</td>
<td>5.175</td>
<td>5.100</td>
</tr>
<tr>
<td>2018</td>
<td>5.025</td>
<td>4.950</td>
<td>4.875</td>
<td>4.200</td>
<td>3.300</td>
<td>1.500</td>
<td>0.750</td>
<td>0.150</td>
<td></td>
</tr>
</tbody>
</table>

The asterisk * indicates that the company has announced a carbon target for this year.
Figure 191: Past, expected and net carbon budgets (Example #7)
Computation of the carbon budget

Numerical solution

- We consider the equally-spaced partition 
  \{[t_0, t_0 + \Delta t], \ldots, [t - \Delta t, t]\} of [t_0, t]
- Let \( m = \frac{t - t_0}{\Delta t} \) be the number of intervals
- We set \( CE_k = CE(t_0 + k\Delta t) \)
- The right Riemann approximation is:
  \[
  CB(t_0, t) = \int_{t_0}^{t} CE(s) \, ds \approx \sum_{k=1}^{m} CE(t_0 + k\Delta t) \Delta t = \Delta t \sum_{k=1}^{m} CE_k
  \]
- The left Riemann sum is:
  \[
  CB(t_0, t) \approx \Delta t \sum_{k=0}^{m-1} CE_k
  \]
- The midpoint rule is:
  \[
  CB(t_0, t) \approx \Delta t \sum_{k=1}^{m} CE \left( t_0 + \frac{k}{2} \Delta t \right)
  \]
If we use a constant linear reduction rate \( R(t_0, t) = R(t - t_0) \), we obtain the following analytical expression:

\[
CB(t_0, t) = \int_{t_0}^{t} (CE(t_0) - R(s - t_0)) \, ds = (t - t_0) CE(t_0) - \frac{(t - t_0)^2}{2} R
\]

In the case of a constant compound reduction rate:

\[
CE(t) = (1 - R)^{(t-t_0)} CE(t_0)
\]

we obtain:

\[
CB(t_0, t) = CE(t_0) \int_{t_0}^{t} (1 - R)^{(s-t_0)} \, ds = \frac{(1 - R)^{(t-t_0)} - 1}{\ln (1 - R)} CE(t_0)
\]
Computation of the carbon budget
Analytical solution: the case of a constant reduction rate

If we assume that $CE(t) = e^{-R(t-t_0)}CE(t_0)$, we have:

$$CB(t_0, t) = CE(t_0) \left[ -\frac{e^{-R(s-t_0)}}{R} \right]_t^{t_0} = CE(t_0) \frac{1 - e^{-R(t-t_0)}}{R}$$

Remark

If the carbon emissions increase at a positive growth rate $g$, we set $R = -g$. 
The remaining carbon budget $CB(2019, t)$ is:

- 580 GtCO$_2$e for a 50% probability of limiting warming to 1.5°C
- 420 GtCO$_2$e for a 66% probability
- 300 GtCO$_2$e for an 83% probability

IPCC (2018)
Computation of the carbon budget
Analytical solution: the case of a Linear function

- If we assume that \( CE(t) = \beta_0 + \beta_1 t \), we deduce that:

\[
CB(t_0, t) = \int_{t_0}^{t} (\beta_0 + \beta_1 s) \, ds \\
= \left[ \beta_0 s + \frac{1}{2} \beta_1 s^2 \right]_{t_0}^{t} \\
= \beta_0 (t - t_0) + \frac{1}{2} \beta_1 (t^2 - t_0^2)
\]

- We can extend this formula to a piecewise linear function:

\( CB(t_0, t) = \ldots \)
## Net zero emissions scenario (IEA)

### Table 104: IEA NZE scenario (in GtCO$_2$e)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>12.4</td>
<td>13.3</td>
<td>13.5</td>
<td>13.6</td>
<td>13.3</td>
<td>13.3</td>
<td>13.5</td>
<td>14.0</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>Buildings</td>
<td>2.89</td>
<td>2.78</td>
<td>2.9</td>
<td>2.84</td>
<td>2.87</td>
<td>2.91</td>
<td>2.95</td>
<td>2.98</td>
<td>3.01</td>
<td></td>
</tr>
<tr>
<td>Transport</td>
<td>7.01</td>
<td>7.13</td>
<td>7.18</td>
<td>7.37</td>
<td>7.5</td>
<td>7.72</td>
<td>7.88</td>
<td>8.08</td>
<td>8.25</td>
<td>8.29</td>
</tr>
<tr>
<td>Industry</td>
<td>8.06</td>
<td>8.47</td>
<td>8.57</td>
<td>8.71</td>
<td>8.78</td>
<td>8.71</td>
<td>8.56</td>
<td>8.52</td>
<td>8.72</td>
<td>8.9</td>
</tr>
<tr>
<td>Other</td>
<td>1.87</td>
<td>1.89</td>
<td>1.91</td>
<td>1.96</td>
<td>1.87</td>
<td>1.89</td>
<td>1.89</td>
<td>1.92</td>
<td>1.92</td>
<td>1.91</td>
</tr>
<tr>
<td>Gross emissions</td>
<td>32.2</td>
<td>33.3</td>
<td>33.7</td>
<td>34.4</td>
<td>34.5</td>
<td>34.5</td>
<td>34.5</td>
<td>35.9</td>
<td>35.9</td>
<td></td>
</tr>
<tr>
<td>BECCS/DACCS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Net emissions</td>
<td>32.2</td>
<td>33.3</td>
<td>33.7</td>
<td>34.4</td>
<td>34.5</td>
<td>34.5</td>
<td>34.5</td>
<td>35.9</td>
<td>35.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2035</th>
<th>2040</th>
<th>2045</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>13.5</td>
<td>10.8</td>
<td>5.82</td>
<td>2.12</td>
<td>−0.08</td>
<td>−0.31</td>
<td>−0.37</td>
</tr>
<tr>
<td>Buildings</td>
<td>2.86</td>
<td>2.43</td>
<td>1.81</td>
<td>1.21</td>
<td>0.69</td>
<td>0.32</td>
<td>0.12</td>
</tr>
<tr>
<td>Transport</td>
<td>7.15</td>
<td>7.23</td>
<td>5.72</td>
<td>4.11</td>
<td>2.69</td>
<td>1.5</td>
<td>0.69</td>
</tr>
<tr>
<td>Industry</td>
<td>8.48</td>
<td>8.14</td>
<td>6.89</td>
<td>5.25</td>
<td>3.48</td>
<td>1.8</td>
<td>0.52</td>
</tr>
<tr>
<td>Other</td>
<td>1.91</td>
<td>1.66</td>
<td>0.91</td>
<td>0.09</td>
<td>−0.46</td>
<td>−0.82</td>
<td>−0.96</td>
</tr>
<tr>
<td>Gross emissions</td>
<td>33.9</td>
<td>30.3</td>
<td>21.5</td>
<td>13.7</td>
<td>7.77</td>
<td>4.3</td>
<td>1.94</td>
</tr>
<tr>
<td>BECCS/DACCS</td>
<td>0</td>
<td>−0.06</td>
<td>−0.32</td>
<td>−0.96</td>
<td>−1.46</td>
<td>−1.8</td>
<td>−1.94</td>
</tr>
<tr>
<td>Net emissions</td>
<td>33.9</td>
<td>30.2</td>
<td>21.1</td>
<td>12.8</td>
<td>6.32</td>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: IEA (2021, Figure 2.3, page 55)
Figure 193: CO$_2$ emissions by sector in the IEA NZE scenario (in GtCO$_2$e)

Source: IEA (2021) & Author’s calculations
<table>
<thead>
<tr>
<th>$t$</th>
<th>Electricity</th>
<th>Buildings</th>
<th>Transport</th>
<th>Industry</th>
<th>Other</th>
<th>Gross emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>74.4</td>
<td>50.2</td>
<td>43.7</td>
<td>16.2</td>
<td>10.8</td>
<td>195.4</td>
</tr>
<tr>
<td>2030</td>
<td>115.9</td>
<td>87.8</td>
<td>76.0</td>
<td>26.8</td>
<td>17.3</td>
<td>324.9</td>
</tr>
<tr>
<td>2040</td>
<td>140.9</td>
<td>140.0</td>
<td>117.6</td>
<td>39.1</td>
<td>18.8</td>
<td>466.6</td>
</tr>
<tr>
<td>2045</td>
<td>139.9</td>
<td>153.2</td>
<td>128.1</td>
<td>41.6</td>
<td>15.6</td>
<td>496.8</td>
</tr>
<tr>
<td>2050</td>
<td>138.2</td>
<td>159.0</td>
<td>133.6</td>
<td>42.7</td>
<td>11.2</td>
<td>512.4</td>
</tr>
</tbody>
</table>

Source: IEA (2021) & Author’s calculations
The linear trend model is defined by:

\[ CE(t) = \beta_0 + \beta_1 t + u(t) \]

where \( u(t) \sim \mathcal{N}(0, \sigma_u^2) \)

- OLS estimation
- The projected carbon trajectory is given by:

\[ CE^{Trend}(t) = \hat{CE}(t) = \hat{\beta}_0 + \hat{\beta}_1 t \]
We have:

\[ \widehat{CE}(0) = \hat{\beta}_0 \]

Base year: \( t_0 \)

The linear trend model becomes:

\[ CE(t) = \beta'_0 + \beta'_1 (t - t_0) + u(t) \]

We have the following relationships:

\[
\begin{cases}
\beta'_0 = \beta_0 + \beta_1 t_0 \\
\beta'_1 = \beta_1
\end{cases}
\]
Carbon trend
Linear trend model

Example #8

Below, we report the evolution of scope 1 + 2 carbon emissions for company A:

<table>
<thead>
<tr>
<th>Year</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE(t)$</td>
<td>57.8</td>
<td>58.4</td>
<td>57.9</td>
<td>55.1</td>
<td>51.6</td>
<td>48.3</td>
<td>47.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE(t)$</td>
<td>46.1</td>
<td>44.4</td>
<td>42.7</td>
<td>41.4</td>
<td>40.2</td>
<td>41.9</td>
<td>45.0</td>
</tr>
</tbody>
</table>
Carbon trend
Linear trend model

We obtain the following estimates:

- $\hat{\beta}_0 = 2.970.43$, $\hat{\beta}_1 = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2007$, $\hat{\beta}_0' = 57.85$, $\hat{\beta}_1' = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2020$, $\hat{\beta}_0' = 38.99$, $\hat{\beta}_1' = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- The two estimated models are coherent:

$$CE_{Trend}^{trend}(t) = 38.99 - 1.4512 \times (t - 2020)$$
$$= 2.970.43 - 1.4512 \times t$$

- We have:

$$CE_{Trend}^{trend}(2025) = 38.99 - 1.4512 \times 5 = 31.73 \text{ MtCO}_2\text{e}$$

- We have $CE(2020) = 45.0 \gg CE(2020) = 38.99$
- The rescaled model has the following expression:

$$CE_{Trend}^{trend}(t) = 45 - 1.4512 \times (t - 2020)$$
Figure 194: Linear carbon trend (Example #8)
The log-linear trend model is:

$$\ln CE(t) = \gamma_0 + \gamma_1 (t - t_0) + \nu(t)$$

Let $Y(t) = \ln CE(t)$ be the logarithmic transform of the carbon emissions.

OLS estimation using $Y(t)$
Carbon trend

Log-linear trend model

- We have:

\[ \hat{CE}(t) = \exp \left( \hat{Y}(t) \right) = \exp (\hat{\gamma}_0 + \hat{\gamma}_1 (t - t_0)) = \hat{CE}(t_0) \exp (\hat{\gamma}_1 (t - t_0)) \]

where \( \hat{CE}(t_0) = \exp (\hat{\gamma}_0) \)

- The mathematical expectation of \( CE(t) \) is equal to:

\[
\mathbb{E}[CE(t)] = \mathbb{E} \left[ e^{\hat{Y}(t)} \right] \\
= \mathbb{E} \left[ \mathcal{LN} (\gamma_0 + \gamma_1 (t - t_0), \sigma^2) \right] \\
= \exp \left( \gamma_0 + \gamma_1 (t - t_0) + \frac{1}{2} \sigma^2 \right) \\
= \hat{CE}(t_0) \exp (\hat{\gamma}_1 (t - t_0)) \\
\]

where \( \hat{CE}(t_0) = \exp (\hat{\gamma}_0 + \frac{1}{2} \hat{\sigma}^2) \)

- The rescaled log-linear trend model is:

\[ CE^{\text{Trend}}(t) = CE(t_0) \exp (\hat{\gamma}_1 (t - t_0)) \]
Interpretation of the slope

- \( \beta_1 \) is the absolute variation of carbon emissions:
  \[
  \frac{\partial C\mathcal{E}(t)}{\partial t} = \beta_1
  \]
  implying that the relative variation of carbon emissions is:
  \[
  \frac{\partial C\mathcal{E}(t)}{C\mathcal{E}(t)} = \frac{\beta_1}{C\mathcal{E}(t)}
  \]

- \( \gamma_1 \) is the relative variation of carbon emissions:
  \[
  \frac{\partial C\mathcal{E}(t)}{C\mathcal{E}(t)} = \frac{\partial \ln C\mathcal{E}(t)}{\partial t} = \gamma_1
  \]
Example #8:

- We obtain the following results: $\hat{\gamma}_0 = 3.6800$, $\hat{\gamma}_1 = -2.95\%$ and $\hat{\sigma}_v = 0.0520$

- $\hat{\mathcal{CE}}(2020) = 39.65$ MtCO$_2$e without the correction of the variance bias

- $\hat{\mathcal{CE}}(2020) = 39.70$ MtCO$_2$e with the correction of the variance bias
Carbon trend
Log-linear trend model

Figure 195: Log-linear carbon trend (Example #8)
Example #9

We consider several historical trajectories of scope 1 carbon emissions:

<table>
<thead>
<tr>
<th>Year</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>2011</td>
<td>11.1</td>
<td>10.2</td>
<td>9.9</td>
<td>9.5</td>
</tr>
<tr>
<td>2012</td>
<td>10.5</td>
<td>10.5</td>
<td>9.5</td>
<td>9.0</td>
</tr>
<tr>
<td>2013</td>
<td>12.5</td>
<td>11.0</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>2014</td>
<td>13.0</td>
<td>10.8</td>
<td>9.3</td>
<td>8.3</td>
</tr>
<tr>
<td>2015</td>
<td>14.8</td>
<td>10.8</td>
<td>8.8</td>
<td>8.1</td>
</tr>
<tr>
<td>2016</td>
<td>16.0</td>
<td>13.0</td>
<td>8.7</td>
<td>7.7</td>
</tr>
<tr>
<td>2017</td>
<td>16.5</td>
<td>12.5</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>2018</td>
<td>17.0</td>
<td>13.5</td>
<td>9.0</td>
<td>7.0</td>
</tr>
<tr>
<td>2019</td>
<td>17.5</td>
<td>13.6</td>
<td>8.0</td>
<td>6.1</td>
</tr>
<tr>
<td>2020</td>
<td>19.8</td>
<td>13.6</td>
<td>8.2</td>
<td>6.2</td>
</tr>
</tbody>
</table>
Linear vs. log-linear trend model

Figure 196: Log-linear vs. linear carbon trend (Example #9)
The linear trend model can be written as:

\[
\begin{align*}
    y(t) &= \mu(t) + u(t) \\
    \mu(t) &= \mu(t-1) + \beta_1
\end{align*}
\]

where \( u(t) \sim \mathcal{N}(0, \sigma_u^2) \)

We have \( y(t) = \beta_0 + \beta_1 t + u(t) \) where \( \beta_0 = \mu(t_0) - \beta_1 t_0 \)

The local linear trend model is defined as:

\[
\begin{align*}
    y(t) &= \mu(t) + u(t) \\
    \mu(t) &= \mu(t-1) + \beta_1 (t - 1) + \eta(t) \\
    \beta_1(t) &= \beta_1(t-1) + \zeta(t)
\end{align*}
\]

where \( \eta(t) \sim \mathcal{N}(0, \sigma_\eta^2) \) and \( \zeta(t) \sim \mathcal{N}(0, \sigma_\zeta^2) \)

The stochastic trend \( \mu(t) \) and slope \( \beta_1(t) \) are estimated with KF
Example #8

- We estimate the parameters \((\sigma_u, \sigma_\eta, \sigma_\zeta)\) by maximizing the Whittle log-likelihood function.
- We obtain \(\hat{\sigma}_u = 0.7022\), \(\hat{\sigma}_\eta = 0.7019\) and \(\hat{\sigma}_\zeta = 0.8350\).
- The standard deviation of the stochastic slope variation \(\beta_1(t) - \beta_1(t - 1)\) is then equal to 0.8350 MtCO\(_2\)e.
Table 107: Kalman filter estimation of the stochastic trend (Example #8)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\mathbf{CE}(t)$</th>
<th>$\hat{\beta}_1(t)$ (RLS)</th>
<th>$\hat{\beta}_1(t)$ (KF)</th>
<th>$\mu(t)$ KF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>57.80</td>
<td>0.0000</td>
<td>57.80</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>58.40</td>
<td>0.2168</td>
<td>58.25</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>57.90</td>
<td>0.0500</td>
<td>−0.0441</td>
<td>58.00</td>
</tr>
<tr>
<td>2010</td>
<td>55.10</td>
<td>−0.8600</td>
<td>−1.3941</td>
<td>55.56</td>
</tr>
<tr>
<td>2011</td>
<td>51.60</td>
<td>−1.5700</td>
<td>−2.6080</td>
<td>52.01</td>
</tr>
<tr>
<td>2012</td>
<td>48.30</td>
<td>−2.0200</td>
<td>−3.1288</td>
<td>48.47</td>
</tr>
<tr>
<td>2013</td>
<td>47.10</td>
<td>−2.0929</td>
<td>−2.2977</td>
<td>46.82</td>
</tr>
<tr>
<td>2014</td>
<td>46.10</td>
<td>−2.0321</td>
<td>−1.5508</td>
<td>45.85</td>
</tr>
<tr>
<td>2015</td>
<td>44.40</td>
<td>−1.9817</td>
<td>−1.5029</td>
<td>44.38</td>
</tr>
<tr>
<td>2016</td>
<td>42.70</td>
<td>−1.9406</td>
<td>−1.5887</td>
<td>42.73</td>
</tr>
<tr>
<td>2017</td>
<td>41.40</td>
<td>−1.8891</td>
<td>−1.4655</td>
<td>41.36</td>
</tr>
<tr>
<td>2018</td>
<td>40.20</td>
<td>−1.8329</td>
<td>−1.3202</td>
<td>40.15</td>
</tr>
<tr>
<td>2019</td>
<td>41.90</td>
<td>−1.6824</td>
<td>0.1339</td>
<td>41.41</td>
</tr>
<tr>
<td>2020</td>
<td>45.00</td>
<td>−1.4512</td>
<td>1.7701</td>
<td>44.45</td>
</tr>
</tbody>
</table>
Carbon momentum

- We have:

\[ CM_{\text{Long}} (t) = \frac{\hat{\beta}_1 (t)}{CE (t)} \]

or:

\[ CM_{\text{Long}} (t) = \hat{\gamma}_1 (t) \]
## Statistics

**Table 108:** Statistics (in %) of carbon momentum $\mathcal{C}_M^{Long}(t)$ (MSCI World index, 1995 – 2021, linear trend)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Carbon emissions $\mathcal{S}_C$</th>
<th>Carbon intensity $\mathcal{S}_C^{up}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.0 1.6 2.3</td>
<td>−4.8 −2.4 −1.3</td>
</tr>
<tr>
<td>Negative</td>
<td>49.9 41.1 29.4</td>
<td>76.0 69.6 75.6</td>
</tr>
<tr>
<td>Positive</td>
<td>50.1 58.9 70.6</td>
<td>24.0 30.4 24.4</td>
</tr>
<tr>
<td>&lt; −10%</td>
<td>23.4 15.8 5.8</td>
<td>36.0 25.0 5.7</td>
</tr>
<tr>
<td>&lt; −5%</td>
<td>32.1 22.2 10.6</td>
<td>48.6 36.7 13.4</td>
</tr>
<tr>
<td>&gt; +5%</td>
<td>22.9 27.5 23.6</td>
<td>6.2 7.3 2.7</td>
</tr>
<tr>
<td>&gt; +10%</td>
<td>9.2 9.5 8.0</td>
<td>2.3 2.6 1.0</td>
</tr>
</tbody>
</table>

Source: Trucost database (2022) & Authors’ calculations.
Table 109: Statistics (in %) of carbon momentum $CM^\text{Long}(t)$ (MSCI World index, 1995 – 2021, log-linear trend)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Carbon emissions</th>
<th>Carbon intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_{1-2}$</td>
</tr>
<tr>
<td>Median</td>
<td>$-0.1$</td>
<td>$1.7$</td>
</tr>
<tr>
<td>Negative</td>
<td>$50.6$</td>
<td>$40.3$</td>
</tr>
<tr>
<td>Positive</td>
<td>$49.4$</td>
<td>$59.7$</td>
</tr>
<tr>
<td>$&lt; -10%$</td>
<td>$13.6$</td>
<td>$8.0$</td>
</tr>
<tr>
<td>$&lt; -5%$</td>
<td>$26.6$</td>
<td>$16.9$</td>
</tr>
<tr>
<td>$&gt; +5%$</td>
<td>$29.8$</td>
<td>$35.9$</td>
</tr>
<tr>
<td>$&gt; +10%$</td>
<td>$16.9$</td>
<td>$19.4$</td>
</tr>
</tbody>
</table>

Source: Trucost database (2022) & Authors’ calculations.
The PAC framework

- Participation
- Ambition
- Credibility
The $\mathcal{PAC}$ framework requires three time series:

- The historical pathway of carbon emission
- The reduction targets announced by the company
- The market-based sector scenario associated to the company that defines the decarbonization pathway

\[
\mathcal{CT} = \left\{ \mathcal{R}^{Target} (t_0, t_k), k = 1, \ldots, n_T \right\}
\]

\[
\mathcal{CS} = \left\{ \mathcal{R}^{Scenario} (t_0, t_k), k = 1, \ldots, n_S \right\}
\]
The PAC framework

**Table 110:** Reduction rates of the IEA NZE scenario (base year = 2020)

<table>
<thead>
<tr>
<th>Year</th>
<th>Electricity</th>
<th>Industry</th>
<th>Transport</th>
<th>Buildings</th>
<th>Other</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>20.0</td>
<td>4.0</td>
<td>-1.1</td>
<td>15.0</td>
<td>13.1</td>
<td>10.6</td>
</tr>
<tr>
<td>2030</td>
<td>56.9</td>
<td>18.8</td>
<td>20.0</td>
<td>36.7</td>
<td>52.4</td>
<td>36.6</td>
</tr>
<tr>
<td>2035</td>
<td>84.3</td>
<td>38.1</td>
<td>42.5</td>
<td>57.7</td>
<td>95.3</td>
<td>59.6</td>
</tr>
<tr>
<td>2040</td>
<td>100.0</td>
<td>59.0</td>
<td>62.4</td>
<td>75.9</td>
<td>100.0</td>
<td>77.1</td>
</tr>
<tr>
<td>2045</td>
<td>100.0</td>
<td>78.8</td>
<td>79.0</td>
<td>88.8</td>
<td>100.0</td>
<td>87.3</td>
</tr>
<tr>
<td>2050</td>
<td>100.0</td>
<td>93.9</td>
<td>90.3</td>
<td>95.8</td>
<td>100.0</td>
<td>94.3</td>
</tr>
</tbody>
</table>

Source: IEA (2021) & Author’s calculations.
The PAC framework

The 3 questions of the PAC framework

1. Is the trend of the issuer in line with the scenario?
2. Is the commitment of the issuer to fight climate change ambitious?
3. Is the target setting of the company relevant and robust, or is it a form of greenwashing?
Example #10

- We consider Example #8
- Company A has announced the following targets:
  1. $R^T_{\text{Target}} (2020, 2025) = 40\%$
  2. $R^T_{\text{Target}} (2020, 2030) = 50\%$
  3. $R^T_{\text{Target}} (2020, 2035) = 75\%$
  4. $R^T_{\text{Target}} (2020, 2040) = 80\%$
  5. $R^T_{\text{Target}} (2020, 2050) = 90\%$
- Company A is an utility corporation $\Rightarrow$ we use the IEA NZE scenario for the sector Electricity
### Table 111: Comparison of carbon budgets (Example #10, base year = 2020)

<table>
<thead>
<tr>
<th>Year</th>
<th>Trend (linear)</th>
<th>Trend (log-linear)</th>
<th>Target</th>
<th>Scenario (global)</th>
<th>Scenario (electricity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>207</td>
<td>209</td>
<td>180</td>
<td>213</td>
<td>203</td>
</tr>
<tr>
<td>2030</td>
<td>377</td>
<td>390</td>
<td>304</td>
<td>385</td>
<td>341</td>
</tr>
<tr>
<td>2035</td>
<td>512</td>
<td>546</td>
<td>388</td>
<td>502</td>
<td>407</td>
</tr>
<tr>
<td>2040</td>
<td>610</td>
<td>680</td>
<td>439</td>
<td>573</td>
<td>425</td>
</tr>
<tr>
<td>2045</td>
<td>671</td>
<td>796</td>
<td>478</td>
<td>613</td>
<td>425</td>
</tr>
<tr>
<td>2050</td>
<td>697</td>
<td>896</td>
<td>506</td>
<td>634</td>
<td>425</td>
</tr>
</tbody>
</table>
The PAC framework

Figure 197: Carbon trend, targets and NZE scenario of company A

Source: IEA (2021) & Author’s calculations.
Assessment of the PAC pillars

Figure 198: Illustration of the participation, ambition and credibility pillars
Temperature scoring system

Figure 199: The $\mathcal{PAC}$ scoring system

(a) Model student?

(b) Black sheep?

(c) Shy child?

(d) Greenwashing?
Illustration

Figure 200: Carbon emissions, trend, targets and NZE scenario (Company B)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 201: Carbon emissions, trend, targets and NZE scenario (Company C)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 202: Carbon emissions, trend, targets and NZE scenario (Company D)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 203: Carbon emissions, trend, targets and NZE scenario (median analysis, global universe)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 204: Carbon emissions, trend, targets and NZE scenario (median analysis, sector universe)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Greenness measures

- Brown intensity: $\mathcal{BI}$
- Green intensity: $\mathcal{GI}$
- We have $\mathcal{BI} \in [0, 1]$, $\mathcal{GI} \in [0, 1]$ and $0 \leq \mathcal{BI} + \mathcal{GI} \leq 1$
- Most of the time, we have $\mathcal{BI} + \mathcal{GI} \neq 1$

Very brown  Brown  Neutral  Green  Very green
Greenness measures

**Figure 205: Several taxonomies**

(a) Green activities

(b) Brown activities

(c) All activities
Definition

The EU taxonomy for sustainable activities is “a classification system, establishing a list of environmentally sustainable economic activities.”
These economic activities must have a substantive contribution to at least one of the following six environmental objectives:

1. climate change mitigation
2. climate change adaptation
3. sustainable use and protection of water and marine resources
4. transition to a circular economy
5. pollution prevention and control
6. protection and restoration of biodiversity and ecosystem
A business activity must also meet two other criteria to qualify as sustainable:

- The activity must do no significant harm to the other environmental objectives (DNSH constraint)
- It must comply with minimum social safeguards (MS constraint)
Figure 206: EU taxonomy for sustainable activities

1a. SC
- Substantially contribute to at least one of the six objectives

1b. TSC
- Comply with Technical Screening Criteria

2. DNSH
- Do No Significant Harm to any other five objectives

3. MS
- Comply with Minimum (Social) Safeguards
We have:

\[ GI = \frac{GR}{TR} \cdot (1 - P) \cdot 1 \{ S \geq S^* \} \]

where:
- \( GR \) is the green revenue deduced from the six environmentally sustainable objectives
- \( TR \) is the total revenue
- \( P \) is the penalty coefficient reflecting the DNSH constraint
- \( S \) is the minimum safeguard score
- \( S^* \) is the threshold
The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

\[
\text{GRS} = \frac{\text{GR}}{\text{TR}}
\]

By construction, we have \( 0 \leq \text{GRS} \leq 1 \)

This measure is then impacted by the DNSH coefficient

The two extreme cases are:

\[
\left\{ \begin{array}{l}
P = 1 \Rightarrow \text{GI} = \text{GRS} \\
P = 0 \Rightarrow \text{GI} = 0
\end{array} \right.
\]

We have \( 0 \leq \text{GI} = \text{GRS} \cdot (1 - P) \leq \text{GRS} \)

The indicator function \( 1 \{ s \geq s^* \} \) is a binary all-or-nothing variable:

\[
S < S^* \Rightarrow \text{GI} = 0
\]
Example #11

We consider a company in the hydropower sector which has five production sites. Below, we indicate the power density efficiency, the GHG emissions, the DNSH compliance with respect to the biodiversity and the corresponding revenue:

<table>
<thead>
<tr>
<th>Site</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency (in Watt per m²)</td>
<td>3.2</td>
<td>3.5</td>
<td>3.3</td>
<td>5.6</td>
<td>4.2</td>
</tr>
<tr>
<td>GHG emissions (in gCO₂e per kWh)</td>
<td>35</td>
<td>103</td>
<td>45</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>Biodiversity DNSH compliance</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Revenue (in $ mn)</td>
<td>103</td>
<td>256</td>
<td>89</td>
<td>174</td>
<td>218</td>
</tr>
</tbody>
</table>
Green revenue share

- The total revenue is equal to:
  \[ TR = 103 + 256 + 89 + 174 + 218 = \$840 \text{ mn} \]

- The fourth site does not pass the technical screening, because the power density is above 5 Watt per \( m^2 \).

- The second site does not also comply because it has a GHG emissions greater than 100 gCO\(_2\)e per kWh.

- We deduce that the green revenue is equal to:
  \[ GR = 103 + 89 + 218 = \$410 \text{ mn} \]

- We conclude that the green revenue share is equal to 48.8%.

- According to the EU green taxonomy, the green intensity is lower because the last site is close to a biodiversity area and has a negative impact:
  \[ GI = \frac{103 + 89}{840} = 22.9\% \]
## Statistics

**Table 112:** Statistics in % of green revenue share (MSCI ACWI IMI, June 2022)

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency</th>
<th>0</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
<th>Max</th>
<th>Mean</th>
<th>Avg</th>
<th>Wgt</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>9.82</td>
<td>1.47</td>
<td>0.96</td>
<td>0.75</td>
<td>0.00</td>
<td>0.00</td>
<td>2.85</td>
<td>100.00</td>
<td>1.36</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>14.10</td>
<td>1.45</td>
<td>0.65</td>
<td>0.31</td>
<td>0.00</td>
<td>1.25</td>
<td>6.12</td>
<td>100.00</td>
<td>1.39</td>
<td>3.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>4.84</td>
<td>1.68</td>
<td>1.02</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>1.16</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>4.79</td>
<td>0.30</td>
<td>0.10</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>99.69</td>
<td>0.32</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>1.00</td>
<td>0.39</td>
<td>0.20</td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>98.47</td>
<td>0.26</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>4.75</td>
<td>0.28</td>
<td>0.11</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>99.98</td>
<td>0.29</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>27.85</td>
<td>5.82</td>
<td>3.17</td>
<td>1.68</td>
<td>0.42</td>
<td>11.82</td>
<td>30.36</td>
<td>100.00</td>
<td>4.78</td>
<td>5.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: MSCI (2022) & Barahhou (2022)

\[
F(x) = \text{Pr}\{GRS > x\}, \quad Q(\alpha) = \inf\{x : \text{Pr}\{GRS \leq x\} \geq \alpha\}, \quad \text{arithmetic average} \\
\frac{1}{n-1} \sum_{i=1}^{n} GRS_i \quad \text{and weighted mean} \quad GRS(b) = \sum_{i=1}^{n} b_i GRS_i
\]
• The green revenue share of the MSCI World index is equal to 5.24%.
• The green revenue share of the Bloomberg Global Investment Grade Corporate Bond index is equal to 3.49%.
• Alessi and Battiston (2022) estimated “a greenness of about 2.8% for EU financial markets.”
<table>
<thead>
<tr>
<th>Carbon footprint</th>
<th>Green taxonomy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic risk measures</td>
<td>Green revenue share</td>
</tr>
<tr>
<td><strong>Greenness measures</strong></td>
<td><strong>Other greenness metrics</strong></td>
</tr>
</tbody>
</table>

**Green capex**
Green-to-brown ratio
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Climate transition risk

Definition

- Transition risks arise from the sudden shift towards a low-carbon economy
- Such transitions could mean that some sectors of the economy face big shifts in asset values or higher costs of doing business

“It’s not that policies stemming from deals like the Paris Climate Agreement are bad for our economy — in fact, the risk of delaying action altogether would be far worse. Rather, it’s about the speed of transition to a greener economy — and how this affects certain sectors and financial stability” (Bank of England, 2021)
The carbon footprint approach assumes that the **climate-related market risk** of a company is measured by its **current carbon intensity**.

...But the **market perception** of the climate change may be different.
Climate transition risk

**Fundamental-based analysis**
- Carbon footprint and pathway are measured by CO$_2$ emissions
- They are fundamental data

**Market-based analysis**
- Financial market’s perception of the potentially reduced impact of climate policies’ on securities issued by corporations
- These carbon risk metrics use market data
- How an increase in carbon prices and taxes influences the credit risk of the issuer?
- How sensitive the asset price is to a carbon market factor?
Two main pricing systems:
- Carbon tax
- Emissions trading system (ETS)

Underlying idea
- A high carbon tax impacts the creditworthiness of corporates
- This impact is different from one issuer to another one
- Identifying for each company the carbon price that would lead the default probability in the Merton model to exceed a certain threshold
Based on the assumptions that the enterprise value $V$ is proportional to the earnings before interest, taxes, depreciation, and amortization (EBITDA) and that the debt $D$ remains constant, we can define the carbon price margin as\footnote{The parameter $\theta$ is the threshold of default probability}:

$$\text{CPM}_i = \left( 1 - \exp \left( \sigma_i \sqrt{\tau} \Phi (-\theta) - \left( r + \frac{1}{2} \sigma^2_i \right) \tau \right) \frac{D_i}{V_i} \right) \frac{\text{EBITDA}_i}{\text{CE}_i}$$

where $\sigma_i$ is the volatility of the enterprise value, $\tau$ is the maturity and $r$ is the risk-free rate.
Carbon tax
Energy mix

- How to measure the environmental performance of an utility company?
- How to measure the environmental performance of a country?
- How to assess a company located in a country with a bad energy mix?
This figure presents the energy generation breakdown for some countries. We can distinguish countries that rely on hydroelectric power (Brazil, Norway), nuclear (France, Switzerland) and mixed solutions (Canada, Germany, Spain, USA).

(*) Each grid circle represents 20% of energy generation. The scale of the radar chart is then 40% for Canada, Germany, Spain and USA, 60% for China, France and Switzerland, 80% for Brazil and 100% for Norway.
Implied temperature rating
Introduced by Harris (2015) and Görgen et al. (2019)

The underlying idea of the carbon beta is to estimate the sensitivity of the stock return with respect to a carbon/climate risk factor

Climate risk is not only an idiosyncratic risk for the issuer, but also a systematic risk factor like the Fama-French-Carhart market factors
Carbon beta

**Cross-section factor**
- Long/short portfolio
- Long on stocks highly exposed to carbon risk
- Short on stocks lowly exposed to carbon risk
- The value of the factor is the return of the L/S portfolio
- High carbon beta = highly exposed to carbon risk

**Time-series factor**
- Synthetic index that represents the financial perception of climate risk
- Textual analysis of climate change-related news published by newspapers and media
- High carbon beta = highly exposed to carbon risk

*Risk measure = carbon beta*
Let $R_i(t)$ be the return of stock $i$ at time $t$. We assume that:

$$R_i(t) = \alpha_i(t) + \beta_{i,mkt}(t) R_{mkt}(t) + \sum_{j=1}^{m} \beta_{i,j}(t) R_j(t) + \beta_{i,Carbon}(t) R_{Carbon}(t) + \varepsilon_i(t)$$

where $R_{mkt}(t)$ is the return of the market risk factor, $R_j(t)$ is the return of the $j^{th}$ alternative risk factor, $R_{Carbon}(t)$ is the return of the carbon risk factor and $\varepsilon_i(t)$ is a white noise process.

Remark

The carbon risk factor corresponds to a long/short portfolio between “green” and “brown” stocks.
Engle et al. (2020) proposed a related approach where the carbon risk factor is replaced by a climate risk news index $\mathcal{I}_{\text{Climate}}$:

$$R_i(t) = \alpha_i(t) + \beta_{i,mkt}(t) R_{\text{mkt}}(t) + \sum_{j=1}^{m} \beta_{i,F_j}(t) R_{F_j}(t) + \beta_{i,\text{Climate}}(t) \mathcal{I}_{\text{Climate}}(t) + \epsilon_i(t)$$

Remark

*The climate index $\mathcal{I}_{\text{Climate}}$ corresponds to a time series that measures the sentiment about the climate change. It is built using text mining and natural language processing (NLP)*
Carbon beta
The carbon risk factor approach

**Goal**
The main objective is to define a *market* measure of carbon risk

**Three-step approach**
- Defining a brown green score (BGS) for each stock (scoring model)
- Building a brown minus green factor (Fama-French approach)
- Estimating the carbon beta of a stock with respect to the BMG factor (Multi-factor regression analysis)

\[
\text{Carbon beta} = \text{market measure of carbon risk} \\
\neq \\
\text{Carbon intensity} = \text{fundamental measure of carbon risk}
\]
The market perception of a carbon risk measure depends on several dimensions: sector, country, etc.
Carbon beta
The carbon risk factor approach

Systematic carbon risk
- Common risk
- Carbon beta

Idiosyncratic carbon risk
- Specific risk
- Carbon intensity

Market measure (≈ general carbon risk exposure, e.g. market repricing risk)

Fundamental measure (≈ specific carbon risk exposure, e.g. reputational risk)
**Carbon beta**

The carbon risk factor approach

<table>
<thead>
<tr>
<th></th>
<th>Green</th>
<th>Neutral</th>
<th>Brown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>SG</td>
<td>SN</td>
<td>SB</td>
</tr>
<tr>
<td>Big</td>
<td>BG</td>
<td>BN</td>
<td>BB</td>
</tr>
</tbody>
</table>

The BMG factor return $R_{\text{bmg}}(t)$ is derived from the Fama-French method:

$$R_{\text{bmg}}(t) = \frac{1}{2} (R_{\text{SB}}(t) + R_{\text{BB}}(t)) - \frac{1}{2} (R_{\text{SG}}(t) + R_{\text{BG}}(t))$$

where the returns of each portfolio $R_j(t)$ (small green SG, big green BG, small brown SB, big brown BB) is value-weighted by the market capitalisation

⇒ The BMG factor is a Fama-French risk factor based on a scoring system (brown green score or BGS)
Figure 208: Cumulative performance of the BMG factor

Source: Görgen et al. (2019).
Carbon beta
The carbon risk factor approach

Figure 209: Box plots of the carbon sensitivities\textsuperscript{26}

\textsuperscript{26}The box plots provide the median, the quartiles and the 5\% and 95\% quantiles

Source: Roncalli et al. (2020).
Carbon beta
The carbon risk factor approach

Relative carbon risk
- The right measure is $\beta_{bmg}$
- Sign matters
- **Negative exposure** is preferred

Absolute carbon risk
- The right measure is $|\beta_{bmg}|$
- Sign doesn’t matter
- **Zero exposure** is preferred

Two examples
1. We consider three portfolios with a carbon beta of $-0.30$, $-0.05$ and $+0.30$ respectively
2. We consider two portfolios with the following characteristics:
   - The value of the carbon beta is $+0.10$ and the stock dispersion of carbon beta is $0.20$
   - The value of the carbon beta is $-0.30$ and the stock dispersion of carbon beta is $1.50$

⇒ Impact of portfolio management and theory
Climate beta
The climate index approach

- Two main references: Engle et al. (2020) & Ardia et al. (2021)
- We recall that brown assets must exhibit a positive risk premium
- Nevertheless, “[…] If ESG concerns strengthen unexpectedly and sufficiently, green assets outperform brown ones despite having lower expected returns” (Pástor et al., 2021)
- Academics proxy concerns about climate change using climate indices based on news
Figure 210: Media Climate Change Concerns (MCCC) index

This figure displays the daily MCCC index (gray points) together with its 30-day moving average (bold line) for January 2003 to June 2018. We also report several major events related to climate change (in boxes). The observations before January 1, 2010 (i.e., at the left of the black dotted line) are considered to be forward-looking, since the data from that period is used to compute the source-specific standard deviation estimate necessary to normalize the source-specific indices before aggregation into the MCCC index. The observations from January 1, 2010 to the end of the time series (i.e., at the right of the black dotted line) are not forward-looking and correspond to the period for our main analysis.

Source: Ardia et al. (2021).
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Quadratic programming

Definition

We have:

\[ x^* = \arg \min \frac{1}{2} x^\top Q x - x^\top R \]

s.t. \[
\begin{cases}
Ax = B \\
Cx \leq D \\
x^- \leq x \leq x^+
\end{cases}
\]

where \( x \) is a \( n \times 1 \) vector, \( Q \) is a \( n \times n \) matrix, \( R \) is a \( n \times 1 \) vector, \( A \) is a \( n_A \times n \) matrix, \( B \) is a \( n_A \times 1 \) vector, \( C \) is a \( n_C \times n \) matrix, \( D \) is a \( n_C \times 1 \) vector, and \( x^- \) and \( x^+ \) are two \( n \times 1 \) vectors.
A quadratic form is a polynomial with terms all of degree two

\[ QF(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j} x_i x_j = x^\top A x \]

Canonical form

\[ QF(x_1, \ldots, x_n) = \frac{1}{2} (x^\top A x + x^\top A^\top x) = \frac{1}{2} x^\top (A + A^\top) x = \frac{1}{2} x^\top Q x \]

Generalized quadratic form

\[ QF(x; Q, R, c) = \frac{1}{2} x^\top Q x - x^\top R + c \]
Quadratic form
Main properties

1. \( \varphi \cdot QF(w; Q, R, c) = QF(w; \varphi Q, \varphi R, \varphi c) \)
2. \( QF(x; Q_1, R_1, c_1) + QF(x; Q_2, R_2, c_2) = QF(x; Q_1 + Q_2, R_1 + R_2, c_1 + c_2) \)
3. \( QF(x - y; Q, R, c) = QF(x; Q, R + Qy, \frac{1}{2}y^TQy + y^TR + c) \)
4. \( QF(x - y; Q, R, c) = QF(y; Q, Qx - R, \frac{1}{2}x^TQx - x^TR + c) \)
5. \( \frac{1}{2} \sum_{i=1}^{n} q_i x_i^2 = QF(x; D(q), 0_n, 0) \) where \( q = (q_1, \ldots, q_n) \) is a \( n \times 1 \) vector and \( D(q) = \text{diag}(q) \)
6. \( \frac{1}{2} \sum_{i=1}^{n} q_i (x_i - y_i)^2 = QF(x; D(q), D(q) y, \frac{1}{2}y^TD(q)y) \)
7. \( \frac{1}{2} (\sum_{i=1}^{n} q_i x_i)^2 = QF(x; T(q), 0_n, 0) \) where \( T(q) = qq^T \)
8. \( \frac{1}{2} (\sum_{i=1}^{n} q_i (x_i - y_i))^2 = QF(x; T(q), T(q) y, \frac{1}{2}y^TT(q)y) \)
Quadratic form
Main properties

We note $\omega = (\omega_1, \ldots, \omega_n)$ where $\omega_i = 1 \{ i \in \Omega \}$

1. $\frac{1}{2} \sum_{i \in \Omega} q_i x_i^2 = QF \left( x; D (\omega \circ q), 0_n, 0 \right)$

2. $\frac{1}{2} \sum_{i \in \Omega} q_i (x_i - y_i)^2 =
   QF \left( x; D (\omega \circ q), D (\omega \circ q) y, \frac{1}{2} y^T D (\omega \circ q) y \right)$

3. $\frac{1}{2} \left( \sum_{i \in \Omega} q_i x_i \right)^2 = QF \left( x; T (\omega \circ q), 0_n, 0 \right)$

4. $\frac{1}{2} \left( \sum_{i \in \Omega} q_i (x_i - y_i) \right)^2 =
   QF \left( x; T (\omega \circ q), T (\omega \circ q) y, \frac{1}{2} y^T T (\omega \circ q) y \right)$

5. $D (\omega \circ q) = \text{diag} (\omega \circ q) = D (\omega) D (q)$

6. $T (\omega \circ q) = (\omega \circ q) (\omega \circ q)^T = (\omega \omega^T) \circ qq^T = T (\omega) \circ T (q)$
Mean-variance optimization

The long-only mean-variance optimization problem is given by:

\[ w^* = \arg \min \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu \]

subject to

\[ \begin{cases} 1_n^\top w = 1 \\ 0_n \leq w \leq 1_n \end{cases} \]

where:

- \( \gamma \) is the risk-tolerance coefficient
- the equality constraint is the budget constraint (\( \sum_{i=1}^n w_i = 1 \))
- the bounds correspond to the no short-selling restriction (\( w_i \geq 0 \))

**QP form**

\[ Q = \Sigma, \ R = \gamma \mu, \ A = 1_n^\top, \ B = 1, \ w^- = 0_n \text{ and } w^+ = 1 \]
Equity portfolio
Basic optimization problems

Tracking error optimization

The tracking error optimization problem is defined as:

\[
w^* = \arg \min w^\top \Sigma w - w^\top (\gamma \mu + \Sigma b)
\]

\[
\text{s.t. } \begin{cases}
1^\top_n w = 1 \\
0_n \leq w \leq 1_n
\end{cases}
\]

QP form

\[
Q = \Sigma, \quad R = \gamma \mu + \Sigma b, \quad A = 1_n^\top, \quad B = 1, \quad w^- = 0_n \text{ and } w^+ = 1
\]

⇒ Portfolio replication: \[ R = \Sigma b \]
Specification of the constraints

Sector weight constraint

- We have
  \[ s_j^- \leq \sum_{i \in \text{Sector}_j} w_i \leq s_j^+ \]

- \( s_j \) is the \( n \times 1 \) sector-mapping vector: \( s_{i,j} = 1 \{ i \in \text{Sector}_j \} \)

- We notice that:
  \[ \sum_{i \in \text{Sector}_j} w_i = s_j^\top w \]

- We deduce that:
  \[ s_j^- \leq \sum_{i \in \text{Sector}_j} w_i \leq s_j^+ \iff \begin{cases} s_j^- \leq s_j^\top w \\ s_j^\top w \leq s_j^+ \end{cases} \iff \begin{cases} -s_j^\top w \leq -s_j^- \\ s_j^\top w \leq s_j^+ \end{cases} \]

**QP form**

\[
\begin{pmatrix} -s_j^- \\ s_j^\top \end{pmatrix} w \leq \begin{pmatrix} -s_j^- \\ s_j^+ \end{pmatrix}
\begin{pmatrix} C \\ D \end{pmatrix}
\]
Specifying the score constraint:

- General constraint:
  \[
  \sum_{i=1}^{n} w_i S_i \geq S^* \iff -S^T w \leq -S^*
  \]

**QP form**

- \( C = -S^T \)
- \( D = -S^* \)
Specification of the constraints

Score constraint

- Sector-specific constraint:

\[
\sum_{i \in \text{Sector}_j} w_i S_i \geq S_j^* \iff \sum_{i=1}^{n} 1 \{ i \in \text{Sector}_j \} \cdot w_i S_i \geq S_j^*
\]

\[
\iff \sum_{i=1}^{n} s_{i,j} w_i S_i \geq S_j^*
\]

\[
\iff \sum_{i=1}^{n} w_i \cdot (s_{i,j} S_i) \geq S_j^*
\]

\[
\iff (s_j \circ S)^\top w \geq S_j^*
\]

**QP form**

- \( C = -(s_j \circ S)^\top \)
- \( D = -S_j^* \)
## Example #1

- The capitalization-weighted equity index is composed of 8 stocks.
- The weights are equal to 23%, 19%, 17%, 13%, 9%, 8%, 6% and 5%.
- The ESG score, carbon intensity and sector of the eight stocks are the following:

<table>
<thead>
<tr>
<th>Stock</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>−1.20</td>
<td>0.80</td>
<td>2.75</td>
<td>1.60</td>
<td>−2.75</td>
<td>−1.30</td>
<td>0.90</td>
<td>−1.70</td>
</tr>
<tr>
<td>$CT$</td>
<td>125</td>
<td>75</td>
<td>254</td>
<td>822</td>
<td>109</td>
<td>17</td>
<td>341</td>
<td>741</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Example #1 (Cont’d)

- The stock volatilities are equal to 22%, 20%, 25%, 18%, 35%, 23%, 13% and 29%
- The correlation matrix is given by:

\[
\begin{bmatrix}
100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 \\
80 & 100 & 70 & 75 & 100 & 70 & 70 & 80 \\
70 & 75 & 80 & 100 & 70 & 70 & 80 & 80 \\
60 & 65 & 80 & 100 & 85 & 70 & 70 & 80 \\
70 & 50 & 70 & 80 & 60 & 60 & 60 & 80 \\
50 & 60 & 70 & 80 & 80 & 80 & 70 & 70 \\
70 & 50 & 70 & 70 & 70 & 70 & 70 & 80 \\
60 & 65 & 70 & 70 & 70 & 70 & 70 & 70 \\
\end{bmatrix}
\]
We have:

$$w^* = \arg \min \frac{1}{2} w^T Q w - w^T R$$

s.t. \[
\begin{align*}
   Aw &= B \\
   Cw &\leq D \\
   w^- &\leq w \leq w^+
\end{align*}
\]
Using $\Sigma_{i,j} = C_{i,j}\sigma_i\sigma_j$, we obtain:

$$Q = \Sigma = 10^{-4} \times$$

\[
\begin{pmatrix}
484.00 & 352.00 & 385.00 & 237.60 & 539.00 & 253.00 & 200.20 & 382.80 \\
352.00 & 400.00 & 375.00 & 234.00 & 350.00 & 276.00 & 130.00 & 377.00 \\
385.00 & 375.00 & 625.00 & 360.00 & 612.50 & 402.50 & 227.50 & 507.50 \\
237.60 & 234.00 & 360.00 & 324.00 & 535.50 & 331.20 & 175.50 & 391.50 \\
539.00 & 350.00 & 612.50 & 535.50 & 1225.00 & 483.00 & 364.00 & 659.75 \\
253.00 & 276.00 & 402.50 & 331.20 & 483.00 & 529.00 & 149.50 & 466.90 \\
200.20 & 130.00 & 227.50 & 175.50 & 364.00 & 149.50 & 169.00 & 301.60 \\
382.80 & 377.00 & 507.50 & 391.50 & 659.75 & 466.90 & 301.60 & 841.00 \\
\end{pmatrix}
\]
Equity portfolios

Objective function

We have:

\[ R = \sum b = \begin{pmatrix} 3.74 \\ 3.31 \\ 4.39 \\ 3.07 \\ 5.68 \\ 3.40 \\ 2.02 \\ 4.54 \end{pmatrix} \times 10^{-2} \]
Equity portfolios
Constraint specification (bounds)

- The portfolio is long-only

**QP form**

- \( w^- = 0_8 \)
- \( w^+ = 1_8 \)
Equity portfolios
Constraint specification (equality)

- The budget constraint $\sum_{i=1}^{8} w_i = 1 \Rightarrow$ a first linear equation
  $A_0 w = B_0$

  **QP form**
  - $A_0 = 1_8^T$
  - $B_0 = 1$
We can impose the sector neutrality of the portfolio meaning that:

\[
\sum_{i \in \text{Sector}_j} w_i = \sum_{i \in \text{Sector}_j} b_i
\]

The sector neutrality constraint can be written as:

\[
\begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} w = \begin{pmatrix}
B_1 \\
B_2
\end{pmatrix}
\]

**QP form**

- \( A_1 = s_1^T = (1 1 0 0 1 0 1 0) \)
- \( A_2 = s_2^T = (0 0 1 1 0 1 0 1) \)
- \( B_1 = s_1^T b = \sum_{i \in \text{Sector}_1} b_i \)
- \( B_2 = s_2^T b = \sum_{i \in \text{Sector}_2} b_i \)
Equity portfolios
Constraint specification (inequality)

- We can impose a relative reduction of the benchmark carbon intensity:
  \[ CI(w) \leq (1 - R) CI(b) \Leftrightarrow C_1 w \leq D_1 \]

  **QP form**
  - \( C_1 = CI^T \) (because \( CI(w) = CI^T w \))
  - \( D_1 = (1 - R) CI(b) \)

- We can impose an absolute increase of the benchmark ESG score:
  \[ S(w) \geq S(b) + \Delta S^* \]

  Since \( S(w) = S^T w \), we deduce that \( C_2 w \leq D_2 \)

  **QP form**
  - \( C_2 = -S^T \)
  - \( D_2 = -(S(b) + \Delta S^*) \)
### Portfolio Optimization in Practice

**Climate Portfolio Allocation**  
**Climate Risk Hedging**  
**Quadratic Programming (QP) Problem**  
**Equity Portfolios**  
**Bond Portfolios**

---

#### Equity Portfolios

**Combination of Constraints**

<table>
<thead>
<tr>
<th>Set of constraints</th>
<th>Carbon intensity</th>
<th>ESG score</th>
<th>Sector neutrality</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>✓</td>
<td></td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_1$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td>✓</td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_2$</td>
<td>$D_2$</td>
</tr>
<tr>
<td>#3</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_1$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>#4</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_1$</td>
<td>$D_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_1$</td>
<td>$B_1$</td>
<td>$C_2$</td>
<td>$D_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_2$</td>
<td>$B_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Thierry Roncalli**

Course 2022-2023 in Sustainable Finance
### Equity portfolios

#### Results

**Table 113:** \( \mathcal{R} = 30\% \) and \( \Delta \mathcal{S}^* = 0.50 \) (Example #1)

<table>
<thead>
<tr>
<th>Weights (in %)</th>
<th>Benchmark</th>
<th>Set #1</th>
<th>Set #2</th>
<th>Set #3</th>
<th>Set #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>23.00</td>
<td>18.17</td>
<td>25.03</td>
<td>8.64</td>
<td>12.04</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>19.00</td>
<td>24.25</td>
<td>14.25</td>
<td>29.27</td>
<td>23.76</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>17.00</td>
<td>16.92</td>
<td>21.95</td>
<td>26.80</td>
<td>30.55</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>13.00</td>
<td>2.70</td>
<td>27.30</td>
<td>1.48</td>
<td>2.25</td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>9.00</td>
<td>12.31</td>
<td>3.72</td>
<td>10.63</td>
<td>8.51</td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>8.00</td>
<td>11.23</td>
<td>1.34</td>
<td>6.30</td>
<td>10.20</td>
</tr>
<tr>
<td>( w_7^* )</td>
<td>6.00</td>
<td>11.28</td>
<td>1.68</td>
<td>16.87</td>
<td>12.69</td>
</tr>
<tr>
<td>( w_8^* )</td>
<td>5.00</td>
<td>3.15</td>
<td>4.74</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Benchmark</th>
<th>Set #1</th>
<th>Set #2</th>
<th>Set #3</th>
<th>Set #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma (w^* \mid b) ) (in %)</td>
<td>0.00</td>
<td>0.50</td>
<td>1.18</td>
<td>1.90</td>
<td>2.12</td>
</tr>
<tr>
<td>( \mathcal{CI} (w^*) )</td>
<td>261.72</td>
<td>183.20</td>
<td>367.25</td>
<td>183.20</td>
<td>183.20</td>
</tr>
<tr>
<td>( \mathcal{R} (w^* \mid b) ) (in %)</td>
<td>30.00</td>
<td>-40.32</td>
<td>30.00</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>( \mathcal{S} (w^*) )</td>
<td>0.17</td>
<td>0.05</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>( \mathcal{S} (w^*) - \mathcal{S} (b) )</td>
<td>-0.12</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( w^* (\text{Sector}_1) ) (in %)</td>
<td>57.00</td>
<td>66.00</td>
<td>44.67</td>
<td>65.41</td>
<td>57.00</td>
</tr>
<tr>
<td>( w^* (\text{Sector}_2) ) (in %)</td>
<td>43.00</td>
<td>34.00</td>
<td>55.33</td>
<td>34.59</td>
<td>43.00</td>
</tr>
</tbody>
</table>
The carbon intensity of the $j^{th}$ sector within the portfolio $w$ is:

$$CI(w; \text{Sector}_j) = \sum_{i \in \text{Sector}_j} \tilde{w}_i CI_i$$

where $\tilde{w}_i$ is the normalized weight in the sector bucket:

$$\tilde{w}_i = \frac{w_i}{\sum_{k \in \text{Sector}_j} w_k}$$

Another expression of $CI(w; \text{Sector}_j)$ is:

$$CI(w; \text{Sector}_j) = \frac{\sum_{i \in \text{Sector}_j} w_i CI_i}{\sum_{i \in \text{Sector}_j} w_i} = (s_j \circ CI)^\top w$$
If we consider the constraint $\mathbf{CI}(w; \text{Sector}_j) \leq \mathbf{CI}_j^*$, we obtain:

\[(*) \iff \mathbf{CI}(w; \text{Sector}_j) \leq \mathbf{CI}_j^* \]
\[\iff (\mathbf{s}_j \odot \mathbf{CI})^\top w \leq \mathbf{CI}_j^* (\mathbf{s}_j^\top w) \]
\[\iff ((\mathbf{s}_j \odot \mathbf{CI}) - \mathbf{CI}_j^* \mathbf{s}_j)^\top w \leq 0 \]
\[\iff (\mathbf{s}_j \odot (\mathbf{CI} - \mathbf{CI}_j^* ))^\top w \leq 0 \]

**QP form**

- $C = (\mathbf{s}_j \odot (\mathbf{CI} - \mathbf{CI}_j^* ))^\top$
- $D = 0$
Equity portfolios
Dealing with constraints on relative weights

Example #2
- Example #1
- We would like to reduce the carbon footprint of the benchmark by 30%
- We impose the sector neutrality
Portfolio optimization in practice
Climate portfolio allocation
Climate risk hedging

Quadratic programming (QP) problem
Equity portfolios
Bond portfolios

Equity portfolios
Dealing with constraints on relative weights

QP form


1 1
A= 1 1
0 0

100%
B =  57%
43%
C=

125

1 1
0 0
1 1


1
1
0

1
0
1

1
1
0



1
0 
1



75

254

822

109

17

341

741



D = 183.2040

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Course 2022-2023 in Sustainable Finance

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Equity portfolios
Dealing with constraints on relative weights

- The optimal solution is:
  \[ w^* = (21.54\%, 18.50\%, 21.15\%, 3.31\%, 10.02\%, 15.26\%, 6.94\%, 3.27\%) \]

- \( \sigma(w^* | b) = 112 \text{ bps} \)

- \( CI(w^*) = 183.20 \text{ vs. } CI(b) = 261.72 \)

**BUT**

\[
\begin{align*}
CI(w^*; S_{sector_1}) &= 132.25 \\
CI(w^*; S_{sector_2}) &= 250.74
\end{align*}
\]

versus

\[
\begin{align*}
CI(b; S_{sector_1}) &= 128.54 \\
CI(b; S_{sector_2}) &= 438.26
\end{align*}
\]

The global reduction of 30% is explained by:

- an increase of 2.89% of the carbon footprint for the first sector
- a decrease of 42.79% of the carbon footprint for the second sector
Equity portfolios
Dealing with constraints on relative weights

- We impose $\mathcal{R}_1 = 20\%$

**QP form**

\[
C = \begin{pmatrix}
\mathbf{s}_1 \circ (CI - (1 - \mathcal{R}_1)CI(b; Sector_1))^T
\end{pmatrix} = \\
\begin{pmatrix}
125 & 75 & 254 & 822 & 109 & 17 & 341 & 741 \\
22.1649 & -27.8351 & 0 & 0 & 6.1649 & 0 & 238.1649 & 0
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
183.2040 \\
0
\end{pmatrix}
\]
Solving the new QP problem gives the following optimal portfolio:

\[ w^* = (22.70\%, 22.67\%, 19.23\%, 5.67\%, 11.39\%, 14.50\%, 0.24\%, 3.61\%) \]

- \( \sigma (w^* \mid b) = 144 \text{ bps} \)
- \( CI (w^*) = 183.20 \)
  - \( CI (w^*; Sector_1) = 102.84 \text{ (reduction of 20\%)} \)
  - \( CI (w^*; Sector_2) = 289.74 \text{ (reduction of 33.89\%)} \)
Risk measure of a bond portfolio

- We consider a zero-coupon bond, whose price and maturity date are $B(t, T)$ and $T$:
  
  $$B_t(t, T) = e^{-(r(t)+s(t))(T-t)+L(t)}$$

  where $r(t)$, $s(t)$ and $L(t)$ are the interest rate, the credit spread and the liquidity premium

- We deduce that:

  $$d \ln B(t, T) = -(T-t) \, dr(t) - (T-t) \, ds(t) + dL(t)$$

  $$= -D \, dr(t) - (D \, s(t)) \, \frac{ds(t)}{s(t)} + dL(t)$$

  $$= -D \, dr(t) - DTS(t) \, \frac{ds(t)}{s(t)} + dL(t)$$

  where:

  - $D = T - t$ is the remaining maturity (or duration)
  - $DTS(t)$ is the duration-times-spread factor
If we assume that \( r(t) \), \( s(t) \) and \( L(t) \) are independent, the risk of the defaultable bond is equal to:

\[
\sigma^2 (\ln B(t, T)) = D^2 \sigma_r^2 + (DTS(t))^2 \sigma_s^2 + \sigma_L^2
\]

Three risk components

\[
\sigma^2 (\ln B(t, T)) = D^2 \sigma_r^2 + (DTS(t))^2 \sigma_s^2 + \sigma_L^2
\]

\( \Rightarrow \) The historical volatility of a bond price is not a relevant risk measure
Bond portfolio optimization
Without a benchmark

- Duration risk:

\[ \text{MD}(w) = \sum_{i=1}^{n} w_i \text{MD}_i \]

- DTS risk:

\[ \text{DTS}(w) = \sum_{i=1}^{n} w_i \text{DTS}_i \]

- Clustering approach = generalization of the sector approach, e.g. (EUR, Financials, AAA to A−, 1Y-3Y)

- We have:

\[ \text{MD}_j(w) = \sum_{i \in \text{Sector}_j} w_i \text{MD}_i \]

and:

\[ \text{DTS}_j(w) = \sum_{i \in \text{Sector}_j} w_i \text{DTS}_i \]
Bond portfolio optimization
Without a benchmark

Objective function without a benchmark

We have:

\[
\mathbf{w^*} = \arg \min_{\mathbf{w}} \frac{\phi_{MD}}{2} \sum_{j=1}^{n_{sector}} (\text{MD}_j(\mathbf{w}) - \text{MD}_{j}^*)^2 + \frac{\phi_{DTS}}{2} \sum_{j=1}^{n_{sector}} (\text{DTS}_j(\mathbf{w}) - \text{DTS}_{j}^*)^2 - \gamma \sum_{i=1}^{n} w_i C_i
\]

where:

- \( \phi_{MD} \geq 0 \) and \( \phi_{DTS} \geq 0 \) indicate the relative weight of each risk component
- \( C_i \) is the expected carry of bond \( i \) and \( \gamma \) is the risk-tolerance coefficient
Bond portfolio optimization

Without a benchmark

**QP form**

\[
\begin{align*}
w^* &= \arg \min_{w} \mathcal{QF}(w; Q, R, c) \\
\text{s.t.} \quad &
\left\{ \begin{array}{l}
1_n^\top w = 1 \\
0_n \leq w \leq 1_n
\end{array} \right.
\end{align*}
\]

where \( \mathcal{QF}(w; Q, R, c) \) is the quadratic form of the objective function.
Bond portfolio optimization
Without a benchmark

We have:

\[
\frac{1}{2} (\text{MD}_j (w) - \text{MD}_j^*)^2 = \frac{1}{2} \left( \sum_{i \in \text{Sector}_j} w_i \text{MD}_i - \text{MD}_j^* \right)^2
\]

\[
= \frac{1}{2} \left( \sum_{i=1}^{n} s_{i,j} w_i \text{MD}_i - \text{MD}_j^* \right)^2
\]

\[
= \frac{1}{2} \left( \sum_{i=1}^{n} s_{i,j} \text{MD}_i w_i \right)^2 - w^\top (s_j \circ \text{MD}) \text{MD}_j^* + \frac{1}{2} \text{MD}_j^*^2
\]

\[
= QF \left( w; \sum_{i=1}^{n} s_{i,j} \text{MD}_i w_i, \sum_{i=1}^{n} s_{i,j} \text{MD}_i w_i, \frac{1}{2} \text{MD}_j^*^2 \right)
\]

where \( \text{MD} = (\text{MD}_1, \ldots, \text{MD}_n) \) is the vector of modified durations and \( \sum_{i=1}^{n} s_{i,j} \text{MD}_i w_i \) is the vector of modified durations and \( \mathcal{T} (u) = uu^\top \)
Bond portfolio optimization
Without a benchmark

We deduce that:

\[
\frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} (\text{MD}_j (w) - \text{MD}_j^*)^2 = QF (w; Q_{\text{MD}}, R_{\text{MD}}, c_{\text{MD}})
\]

where:

\[
\begin{align*}
Q_{\text{MD}} &= \sum_{j=1}^{n_{\text{sector}}} T (s_j \circ \text{MD}) \\
R_{\text{MD}} &= \sum_{j=1}^{n_{\text{sector}}} (s_j \circ \text{MD}) \text{MD}_j^* \\
c_{\text{MD}} &= \frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} \text{MD}_j^*^2
\end{align*}
\]
Bond portfolio optimization
Without a benchmark

In a similar way, we have:

\[
\frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} (\text{DTS}_j(w) - \text{DTS}_j^*)^2 = Q\mathcal{F}(w; Q_{\text{DTS}}, R_{\text{DTS}}, c_{\text{DTS}})
\]

where:

\[
\begin{align*}
Q_{\text{DTS}} &= \sum_{j=1}^{n_{\text{sector}}} \mathcal{T}(s_j \circ \text{DTS}) \\
R_{\text{MD}} &= \sum_{j=1}^{n_{\text{sector}}} (s_j \circ \text{DTS}) \text{DTS}_j^* \\
c_{\text{DTS}} &= \frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} \text{DTS}_j^{*2}
\end{align*}
\]
We have:

$$-\gamma \sum_{i=1}^{n} w_i C_i = \gamma QF(w; 0_{n,n}, C, 0) = QF(w; 0_{n,n}, \gamma C, 0)$$

where $C = (C_1, \ldots, C_n)$ is the vector of expected carry values.
Bond portfolio optimization
Without a benchmark

Quadratic form of the objective function

The function to optimize is:

$$QF (w; Q, R, c) = \phi_{MD} QF (w; Q_{MD}, R_{MD}, c_{MD}) + \phi_{DTS} QF (w; Q_{DTS}, R_{DTS}, c_{DTS}) + QF (w; 0_{n,n}, \gamma C, 0)$$

where:

$$\left\{ \begin{array}{l} Q = \phi_{MD} Q_{MD} + \phi_{DTS} Q_{DTS} \\ R = \gamma C + \phi_{MD} R_{MD} + \phi_{DTS} R_{DTS} \\ c = \phi_{MD} c_{MD} + \phi_{DTS} c_{DTS} \end{array} \right.$$. 
The MD- and DTS-based tracking error variances are equal to:

$$ R_{MD} (w \mid b) = \sigma_{MD}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in Sector_j} (w_i - b_i) MD_i \right)^2 $$

and:

$$ R_{DTS} (w \mid b) = \sigma_{DTS}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in Sector_j} (w_i - b_i) DTS_i \right)^2 $$

This means that $MD_j^* = \sum_{i \in Sector_j} b_i MD_i$ and $DTS_j^* = \sum_{i \in Sector_j} b_i DTS_i$.

The active share risk is defined as:

$$ R_{AS} (w \mid b) = \sigma_{AS}^2 (w \mid b) = \sum_{i=1}^{n} (w_i - b_i)^2 $$
Bond portfolio optimization
With a benchmark

Objective function with a benchmark

The optimization problem becomes:

\[ w^* = \arg \min \frac{1}{2} \mathcal{R}(w | b) - \gamma \sum_{i=1}^{n} (w_i - b_i) c_i \]

s.t. \[ \begin{cases} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{cases} \]

where the synthetic risk measure is equal to:

\[ \mathcal{R}(w | b) = \varphi_{AS} \mathcal{R}_{AS}(w | b) + \varphi_{MD} \mathcal{R}_{MD}(w | b) + \varphi_{DTS} \mathcal{R}_{DTS}(w | b) \]
Bond portfolio optimization
With a benchmark

We can show that

$$w^* = \arg \min_{w} QF(w; Q(b), R(b), c(b))$$

subject to

$$\begin{cases} 
1^T_n w = 1 \\
0_n \leq w \leq 1_n 
\end{cases}$$

where:

$$\begin{cases} 
Q(b) = \phi_{AS} Q_{AS}(b) + \phi_{MD} Q_{MD}(b) + \phi_{DTS} Q_{DTS}(b) \\
R(b) = \gamma C + \phi_{AS} R_{AS}(b) + \phi_{MD} R_{MD}(b) + \phi_{DTS} R_{DTS}(b) \\
c(b) = \gamma b^T C + \phi_{AS} c_{AS}(b) + \phi_{MD} c_{MD}(b) + \phi_{DTS} c_{DTS}(b) 
\end{cases}$$

$$Q_{AS}(b) = I_n, \quad R_{AS}(b) = b, \quad c_{AS}(b) = \frac{1}{2} b^T b, \quad Q_{MD}(b) = Q_{MD},$$

$$R_{MD}(b) = Q_{MD} b = R_{MD}, \quad c_{MD}(b) = \frac{1}{2} b^T Q_{MD} b = c_{MD},$$

$$Q_{DTS}(b) = Q_{DTS}, \quad R_{DTS}(b) = Q_{DTS} b = R_{DTS}, \text{ and}$$

$$c_{DTS}(b) = \frac{1}{2} b^T Q_{DTS} b = c_{DTS}$$
Example #3

We consider an investment universe of 9 corporate bonds with the following characteristics\(^a\):

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_i)</td>
<td>21</td>
<td>19</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>(CI_i)</td>
<td>111</td>
<td>52</td>
<td>369</td>
<td>157</td>
<td>18</td>
<td>415</td>
<td>17</td>
<td>253</td>
<td>900</td>
</tr>
<tr>
<td>(MD_i)</td>
<td>3.16</td>
<td>6.48</td>
<td>3.54</td>
<td>9.23</td>
<td>6.40</td>
<td>2.30</td>
<td>8.12</td>
<td>7.96</td>
<td>5.48</td>
</tr>
<tr>
<td>(DTS_i)</td>
<td>107</td>
<td>255</td>
<td>75</td>
<td>996</td>
<td>289</td>
<td>45</td>
<td>620</td>
<td>285</td>
<td>125</td>
</tr>
<tr>
<td>(Sector)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

We impose that \(0.25 \times b_i \leq w_i \leq 4 \times b_i\). We have \(\varphi_{AS} = 100\), \(\varphi_{MD} = 25\) and \(\varphi_{DTS} = 0.001\).

\(^a\)The units are: \(b_i\) in %, \(CI_i\) in tCO\(_2\)e/$ mn, \(MD_i\) in years and \(DTS_i\) in bps
The optimization problem is defined as:

$$w^* (\mathcal{R}) = \arg \min \frac{1}{2} w^T Q(b) w - w^T R(b)$$

subject to:

$$1_9^T w = 1$$

$$C \mathcal{I}^T w \leq (1 - \mathcal{R}) C \mathcal{I} (b)$$

$$\frac{b}{4} \leq w \leq 4b$$

where $\mathcal{R}$ is the reduction rate.
Since the bonds are ordering by sectors, $Q(b)$ is a block diagonal matrix:

$$Q(b) = \begin{pmatrix} Q_1 & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & Q_2 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & Q_3 \end{pmatrix} \times 10^3$$

where:

$$Q_1 = \begin{pmatrix} 0.3611 & 0.5392 & 0.2877 \\ 0.5392 & 1.2148 & 0.5926 \\ 0.2877 & 0.5926 & 0.4189 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 3.2218 & 1.7646 & 0.5755 \\ 1.7646 & 1.2075 & 0.3810 \\ 0.5755 & 0.3810 & 0.2343 \end{pmatrix}$$

and:

$$Q_3 = \begin{pmatrix} 2.1328 & 1.7926 & 1.1899 \\ 1.7926 & 1.7653 & 1.1261 \\ 1.1899 & 1.1261 & 0.8664 \end{pmatrix}$$

$$R(b) = (2.243, 4.389, 2.400, 6.268, 3.751, 1.297, 2.354, 2.120, 1.424) \times 10^2$$
Bond portfolio optimization
With a benchmark

Table 114: Weights in % of optimized bond portfolios (Example #3)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>21.00</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>6.00</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>(w^* (10%))</td>
<td>21.92</td>
<td>19.01</td>
<td>15.53</td>
<td>11.72</td>
<td>11.68</td>
<td>7.82</td>
<td>6.68</td>
<td>4.71</td>
<td>0.94</td>
</tr>
<tr>
<td>(w^* (30%))</td>
<td>26.29</td>
<td>20.24</td>
<td>10.90</td>
<td>10.24</td>
<td>16.13</td>
<td>3.74</td>
<td>9.21</td>
<td>2.50</td>
<td>0.75</td>
</tr>
<tr>
<td>(w^* (50%))</td>
<td>27.48</td>
<td>23.97</td>
<td>4.00</td>
<td>6.94</td>
<td>22.70</td>
<td>2.00</td>
<td>11.15</td>
<td>1.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 115: Risk statistics of optimized bond portfolios (Example #3)

| Portfolio | \(\text{AS}_{\text{Sector}}\) (in %) | MD \((w)\) (in years) | DTS \((w)\) (in bps) | \(\sigma_{\text{AS}} (w | b)\) (in %) | \(\sigma_{\text{MD}} (w | b)\) (in years) | \(\sigma_{\text{DTS}} (w | b)\) (in bps) | \(\text{CI} (w)\) gCO\(_2\)e/$|
|-----------|---------------------|------------------|------------------|-------------------|------------------|------------------|----------------|
| \(b\)    | 0.00                | 5.43             | 290.18           | 0.00              | 0.00             | 0.00             | 184.39         |
| \(w^* (10\%)\) | 3.00               | 5.45             | 293.53           | 2.62              | 0.02             | 3.80             | 165.95         |
| \(w^* (30\%)\) | 14.87              | 5.58             | 303.36           | 10.98             | 0.10             | 14.49            | 129.07         |
| \(w^* (50\%)\) | 28.31              | 5.73             | 302.14           | 21.21             | 0.19             | 30.11            | 92.19          |
Figure 211: Relationship between the reduction rate and the tracking risk (Example #3)
Advanced optimization problems

Large bond universe

- QP: $n \leq 5000$ (the dimension of $Q$ is $n \times n$)
- LP: $n \gg 10^6$

- Some figures as of 31/01/2023
  - MSCI World Index (DM): $n = 1508$ stocks
  - MSCI World IMI (DM): $n = 5942$ stocks
  - MSCI World AC (DM + EM): $n = 2882$ stocks
  - MSCI World AC IMI (DM + EM): $n = 7928$ stocks
  - Bloomberg Global Aggregate Total Return Index: $n = 28799$ securities
  - ICE BOFA Global Broad Market Index: $n = 33575$ securities

- Trick: $\mathcal{L}_2$-norm risk measures $\Rightarrow \mathcal{L}_1$-norm risk measures
We replace the synthetic risk measure by:

\[ \mathcal{D}(w \mid b) = \varphi'_{\text{AS}} \mathcal{D}_{\text{AS}}(w \mid b) + \varphi'_{\text{MD}} \mathcal{D}_{\text{MD}}(w \mid b) + \varphi'_{\text{DTS}} \mathcal{D}_{\text{DTS}}(w \mid b) \]

where:

\[ \mathcal{D}_{\text{AS}}(w \mid b) = \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| \]

\[ \mathcal{D}_{\text{MD}}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \right| \]

\[ \mathcal{D}_{\text{DTS}}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \right| \]
The optimization problem becomes:

\[ w^* = \arg \min_{w} D(w \mid b) - \gamma \sum_{i=1}^{n} (w_i - b_i) C_i \]

s.t. \[
\begin{aligned}
    1_n^T w &= 1 \\
    0_n &\leq w &\leq 1_n
\end{aligned}
\]
Absolute value trick

If \( c_i \geq 0 \), then:

\[
\min \sum_{i=1}^{n} c_i |f_i(x)| + g(x) \iff \begin{cases} 
\min \sum_{i=1}^{n} c_i \tau_i + g(x) \\
\text{s.t.} \begin{cases} 
|f_i(x)| \leq \tau_i \\
\tau_i \geq 0
\end{cases}
\end{cases}
\]

The problem becomes linear:

\[
|f_i(x)| \leq \tau_i \iff -\tau_i \leq f_i(x) \wedge f_i(x) \leq \tau_i
\]
Advanced optimization problems
Large bond universe

Linear programming

The standard formulation of a linear programming problem is:

\[ x^* = \arg \min c^T x \]

s.t. \[ \begin{cases} Ax = b \\ Cx \leq D \\ x^- \leq x \leq x^+ \end{cases} \]

where \( x \) is a \( n \times 1 \) vector, \( c \) is a \( n \times 1 \) vector, \( A \) is a \( n_A \times n \) matrix, \( B \) is a \( n_A \times 1 \) vector, \( C \) is a \( n_C \times n \) matrix, \( D \) is a \( n_C \times 1 \) vector, and \( x^- \) and \( x^+ \) are two \( n \times 1 \) vectors.
Advanced optimization problems
Large bond universe

We have:

\[ w^* = \arg \min \left( \frac{1}{2} \varphi'_\text{AS} \sum_{i=1}^{n} \tau_{i,w} + \varphi'_\text{MD} \sum_{j=1}^{n} \tau_{j,\text{MD}} + \varphi'_\text{DTS} \sum_{j=1}^{n} \tau_{j,\text{DTS}} - \gamma \sum_{i=1}^{n} (w_i - b_i) C_i \right) \]

1. \[ \mathbf{1}_n^\top w = 1 \]
2. \[ 0_n \leq w \leq 1_n \]
3. \[ |w_i - b_i| \leq \tau_{i,w} \]
4. \[ \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \leq \tau_{j,\text{MD}} \]
5. \[ \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \leq \tau_{j,\text{DTS}} \]

s.t. \[ \tau_{i,w} \geq 0, \tau_{j,\text{MD}} \geq 0, \tau_{j,\text{DTS}} \geq 0 \]
Advanced optimization problems
Large bond universe

\[ |w_i - b_i| \leq \tau_{i,w} \iff \begin{cases} w_i - \tau_{i,w} \leq b_i \\ -w_i - \tau_{i,w} \leq -b_i \end{cases} \]
Advanced optimization problems
Large bond universe

\[(*) \iff \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \right| \leq \tau_{j, \text{MD}}\]

\[\iff -\tau_{j, \text{MD}} \leq \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \leq \tau_{j, \text{MD}}\]

\[\iff -\tau_{j, \text{MD}} + \sum_{i \in \text{Sector}_j} b_i \text{MD}_i \leq \sum_{i \in \text{Sector}_j} w_i \text{MD}_i \leq \tau_{j, \text{MD}} + \sum_{i \in \text{Sector}_j} b_i \text{MD}_i\]

\[\iff -\tau_{j, \text{MD}} + \text{MD}_j^* \leq (s_j \circ \text{MD})^\top w \leq \tau_{j, \text{MD}} + \text{MD}_j^*\]

\[\iff \begin{cases} (s_j \circ \text{MD})^\top w - \tau_{j, \text{MD}} \leq \text{MD}_j^* \\ - (s_j \circ \text{MD})^\top w - \tau_{j, \text{MD}} \leq -\text{MD}_j^* \end{cases}\]
### Advanced optimization problems

**Large bond universe**

\[
\sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \leq \tau_{j,\text{DTS}} \iff \begin{cases}
(s_j \circ \text{DTS})^\top w - \tau_{j,\text{DTS}} \leq \text{DTS}_j^* \\
-(s_j \circ \text{DTS})^\top w - \tau_{j,\text{DTS}} \leq -\text{DTS}_j^*
\end{cases}
\]
Advanced optimization problems

LP formulation

- $x$ is a vector of dimension $n_x = 2 \times (n + n_{sector})$:

$$
x = \begin{pmatrix}
w \\
\tau_w \\
\tau_{MD} \\
\tau_{DTS}
\end{pmatrix}$$
The vector $c$ is equal to:

$$
c = \begin{pmatrix}
-\gamma C \\
\frac{1}{2} \varphi'_{AS} 1_n \\
\varphi'_{MD} 1_{nSector} \\
\varphi'_{DTS} 1_{nSector}
\end{pmatrix}
$$
The linear equality constraint $Ax = B$ is defined by:

$$A = \begin{pmatrix} 1_n^T & 0_n^T & 0_{nS}^T & 0_{nS}^T \end{pmatrix}$$

and:

$$B = 1$$
The linear inequality constraint $C x \leq D$ is defined by:

$$C = \begin{pmatrix}
I_n & -I_n & 0_{n, n_{sector}} & 0_{n, n_{sector}} \\
-I_n & -I_n & 0_{n, n_{sector}} & 0_{n, n_{sector}} \\
C_{MD} & 0_{n_{sector}, n} & -I_{n_{sector}} & 0_{n_{sector}, n_{sector}} \\
-C_{MD} & 0_{n_{sector}, n} & -I_{n_{sector}} & 0_{n_{sector}, n_{sector}} \\
C_{DTS} & 0_{n_{sector}, n} & 0_{n_{sector}, n_{sector}} & -I_{n_{sector}} \\
-C_{DTS} & 0_{n_{sector}, n} & 0_{n_{sector}, n_{sector}} & -I_{n_{sector}}
\end{pmatrix}$$

End:

$$D = \begin{pmatrix}
b \\
-b \\
\text{MD}^* \\
-\text{MD}^* \\
\text{DTS}^* \\
-\text{DTS}^*
\end{pmatrix}$$
Advanced optimization problems

LP formulation

- \( C_{MD} \) and \( C_{DTS} \) are two \( n_{sector} \times n \) matrices, whose elements are:

\[
(C_{MD})_{j,i} = s_{i,j} \text{MD}_i
\]

and:

\[
(C_{DTS})_{j,i} = s_{i,j} \text{DTS}_i
\]

- We have:

\[
\text{MD}^* = (\text{MD}_1^*, \ldots, \text{MD}_{n_{sector}}^*)
\]

and

\[
\text{DTS}^* = (\text{DTS}_1^*, \ldots, \text{DTS}_{n_{sector}}^*)
\]
The bounds are:

\[ x^- = 0_{n_x} \]

and:

\[ x^+ = \infty \cdot 1_{n_x} \]
Advanced optimization problems
LP formulation

- Additional constraints:

\[
\begin{align*}
A'w &= B' \\
C'w &\leq D' \iff
\begin{pmatrix}
A' & 0_{n_A, n_x-n} \\
C' & 0_{n_A, n_x-n}
\end{pmatrix} x &= B' \\
\begin{pmatrix}
A' \\
C'
\end{pmatrix} x &\leq D'
\end{align*}
\]
Toy example

We consider a toy example with four corporate bonds:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$ (in %)</td>
<td>35</td>
<td>15</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$CI_i$ (in tCO$_2$e/$\text{mn}$)</td>
<td>117</td>
<td>284</td>
<td>162.5</td>
<td>359</td>
</tr>
<tr>
<td>MD$_i$ (in years)</td>
<td>3.0</td>
<td>5.0</td>
<td>2.0</td>
<td>6.0</td>
</tr>
<tr>
<td>DTS$_i$ (in bps)</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

We would like to reduce the carbon footprint by 20%, and we set $\varphi'_{AS} = 100$, $\varphi'_{MD} = 25$ and $\varphi'_{DTS} = 1$.
We have $n = 4$, $n_{sector} = 2$ and:

$$x = \left( w_1, w_2, w_3, w_4, \tau_{w_1}, \tau_{w_2}, \tau_{w_3}, \tau_{w_4}, \tau_{MD_1}, \tau_{MD_2}, \tau_{DTS_1}, \tau_{DTS_2} \right)$$

Since the vector $C$ is equal to $0_4$, we obtain:

$$c = (0, 0, 0, 0, 50, 50, 50, 50, 25, 25, 1, 1)$$
The equality system $Ax = B$ is defined by:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{pmatrix}$$

and:

$$B = 1$$
The inequality system $Cx \leq D$ is given by:

$$
C = \begin{pmatrix}
3 & 5 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 2 & 6 & 0 & -1 & 0 & 0 \\
-3 & -5 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -2 & -6 & 0 & -1 & 0 & 0 \\
100 & 150 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 200 & 250 & 0 & 0 & 0 & -1 \\
-100 & -150 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -200 & -250 & 0 & 0 & 0 & -1 \\
117 & 284 & 162.5 & 359 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

and:

$$
D = (0.35, 0.15, 0.2, 0.3, -0.35, -0.15, -0.2, -0.3, \ldots, 1.8, 2.2, -1.8, -2.2, 57.5, 115, -57.5, -115, 179)
$$
The last row of $Cx \leq D$ corresponds to the carbon footprint constraint.

We have:

$$CI(b) = 223.75 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

and:

$$(1 - R) CI(b) = 0.80 \times 223.75 = 179.00 \text{ tCO}_2\text{e}/\$ \text{ mn}$$
We solve the LP program, and we obtain the following solution:

\[
\begin{aligned}
    w^* &= (47.34\%, 0\%, 33.3\%, 19.36\%) \\
    \tau_w^* &= (12.34\%, 15\%, 13.3\%, 10.64\%) \\
    \tau_{MD}^* &= (0.3798, 0.3725) \\
    \tau_{DTS}^* &= (10.1604, 0)
\end{aligned}
\]
Advanced optimization problems
Large bond universe

- Interpretation of $\tau^*_w$:

$$w^* \pm \tau^*_w = \begin{pmatrix} 47.34\% \\ 0.00\% \\ 33.30\% \\ 19.36\% \end{pmatrix} \pm \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix} \begin{pmatrix} 12.34\% \\ 15.00\% \\ 13.30\% \\ 10.64\% \end{pmatrix} = \begin{pmatrix} 35\% \\ 15\% \\ 20\% \\ 30\% \end{pmatrix} = b$$

- Interpretation of $\tau^*_\text{MD}$:

$$\left( \frac{\sum_{i \in \text{Sector}_1} w^*_i \times \text{MD}_i}{\sum_{i \in \text{Sector}_2} w^*_i \times \text{MD}_i} \right) \pm \tau^*_\text{MD} = \begin{pmatrix} 1.42 \\ 1.83 \end{pmatrix} \pm \begin{pmatrix} 0.38 \\ 0.37 \end{pmatrix} = \begin{pmatrix} 1.80 \\ 2.20 \end{pmatrix} = \begin{pmatrix} \text{MD}^*_1 \\ \text{MD}^*_2 \end{pmatrix}$$

- Interpretation of $\tau^*_\text{DTS}$:

$$\left( \frac{\sum_{i \in \text{Sector}_1} w^*_i \times \text{DTS}_i}{\sum_{i \in \text{Sector}_2} w^*_i \times \text{DTS}_i} \right) \pm \tau^*_\text{DTS} = \begin{pmatrix} 47.34 \\ 115.00 \end{pmatrix} \pm \begin{pmatrix} 10.16 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 57.50 \\ 115.00 \end{pmatrix} = \begin{pmatrix} \text{DTS}^*_1 \\ \text{DTS}^*_2 \end{pmatrix}$$
Advanced optimization problems
Large bond universe

Example #4 (Example #3 again)

We consider an investment universe of 9 corporate bonds with the following characteristics:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$</td>
<td>21</td>
<td>19</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$CI_i$</td>
<td>111</td>
<td>52</td>
<td>369</td>
<td>157</td>
<td>18</td>
<td>415</td>
<td>17</td>
<td>253</td>
<td>900</td>
</tr>
<tr>
<td>$MD_i$</td>
<td>3.16</td>
<td>6.48</td>
<td>3.54</td>
<td>9.23</td>
<td>6.40</td>
<td>2.30</td>
<td>8.12</td>
<td>7.96</td>
<td>5.48</td>
</tr>
<tr>
<td>$DTS_i$</td>
<td>107</td>
<td>255</td>
<td>75</td>
<td>996</td>
<td>289</td>
<td>45</td>
<td>620</td>
<td>285</td>
<td>125</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

We impose that $0.25 \times b_i \leq w_i \leq 4 \times b_i$ and assume that

$\varphi'_AS = \varphi AS = 100$,  $\varphi'_MD = \varphi MD = 25$ and  $\varphi'_DTS = \varphi DTS = 0.001$

---

The units are: $b_i$ in %, $CI_i$ in tCO$_2$/$ mn, $MD_i$ in years and $DTS_i$ in bps
Table 116: Weights in % of optimized bond portfolios (Example #4)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>21.00</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>6.00</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>$w^* (10%)$</td>
<td>21.70</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>7.46</td>
<td>4.00</td>
<td>0.84</td>
</tr>
<tr>
<td>$w^* (30%)$</td>
<td>34.44</td>
<td>19.00</td>
<td>4.00</td>
<td>11.65</td>
<td>11.98</td>
<td>6.65</td>
<td>7.52</td>
<td>4.00</td>
<td>0.75</td>
</tr>
<tr>
<td>$w^* (50%)$</td>
<td>33.69</td>
<td>19.37</td>
<td>4.00</td>
<td>3.91</td>
<td>24.82</td>
<td>2.00</td>
<td>10.46</td>
<td>1.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 117: Risk statistics of optimized bond portfolios (Example #4)

| Portfolio | AS$_{sector}$ (in %) | MD $(w)$ (in years) | DTS $(w)$ (in bps) | $\sigma_{AS} (w | b)$ (in %) | $\sigma_{MD} (w | b)$ (in years) | $\sigma_{DTS} (w | b)$ (in bps) | CI$(w)$ gCO$_2$/\$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.00</td>
<td>5.43</td>
<td>290.18</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>184.39</td>
</tr>
<tr>
<td>$w^* (10%)$</td>
<td>2.16</td>
<td>5.45</td>
<td>297.28</td>
<td>2.16</td>
<td>0.02</td>
<td>7.10</td>
<td>165.95</td>
</tr>
<tr>
<td>$w^* (30%)$</td>
<td>15.95</td>
<td>5.43</td>
<td>300.96</td>
<td>15.95</td>
<td>0.00</td>
<td>13.20</td>
<td>129.07</td>
</tr>
<tr>
<td>$w^* (50%)$</td>
<td>31.34</td>
<td>5.43</td>
<td>268.66</td>
<td>31.34</td>
<td>0.00</td>
<td>65.12</td>
<td>92.19</td>
</tr>
</tbody>
</table>
Equity portfolios

Threshold approach

The optimization problem is:

\[ w^* = \arg \min \frac{1}{2} (w - b)^\top \Sigma (w - b) \]

s.t. \[
\begin{align*}
1_n^\top w &= 1 \\
w &\in \Omega \\
0_n &\leq w \leq 1_n \\
CI (w) &\leq (1 - R) CI (b)
\end{align*}
\]
Order-statistic approach

- \(C\mathcal{I}_{i:n}\) is the order statistics of \((C\mathcal{I}_1, \ldots, C\mathcal{I}_n)\):

\[
\min C\mathcal{I}_i = C\mathcal{I}_{1:n} \leq C\mathcal{I}_{2:n} \leq \cdots \leq C\mathcal{I}_{i:n} \leq \cdots \leq C\mathcal{I}_{n:n} = \max C\mathcal{I}_i
\]

- The carbon intensity bound \(C\mathcal{I}^{(m,n)}\) is defined as:

\[
C\mathcal{I}^{(m,n)} = C\mathcal{I}_{n-m+1:n}
\]

where \(C\mathcal{I}_{n-m+1:n}\) is the \((n - m + 1)\)-th order statistic of \((C\mathcal{I}_1, \ldots, C\mathcal{I}_n)\)

- Exclusion process:

\[
C\mathcal{I}_i \geq C\mathcal{I}^{(m,n)} \Rightarrow w_i = 0
\]
Order-statistic approach (Cont’d)

The optimization problem is:

\[
\begin{align*}
    w^* &= \arg \min \frac{1}{2} (w - b)^\top \Sigma (w - b) \\
    \text{s.t.} & \quad \begin{cases}
        \mathbf{1}_n^\top w = 1 \\
        w \in \Omega \\
        0_n \leq w \leq \mathbf{1} \\
        \mathbf{C}_I < \mathbf{C}_I^{(m,n)}
    \end{cases}
\end{align*}
\]
Equity portfolios

Naive approach

We re-weight the remaining assets:

\[
w_i^* = \frac{1 \left\{ CI_i < CI^{(m,n)} \right\} \cdot b_i}{\sum_{k=1}^n 1 \left\{ CI_k < CI^{(m,n)} \right\} \cdot b_k}
\]
Example #5

We consider a capitalization-weighted equity index, which is composed of eight stocks. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in tCO$_2$e/$\text{mn}$) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the market one-factor model: the beta $\beta_i$ of each stock is equal to 0.30, 1.80, 0.85, 0.83, 1.47, 0.94, 1.67 and 1.08, the idiosyncratic volatilities $\tilde{\sigma}_i$ are respectively equal to 10%, 5%, 6%, 12%, 15%, 4%, 8% and 7%, and the estimated market volatility $\sigma_m$ is 18%.
The covariance matrix is:

\[ \Sigma = \beta \beta^\top \sigma^2_m + D \]

where:

1. \( \beta \) is the vector of beta coefficients
2. \( \sigma^2_m \) is the variance of the market portfolio
3. \( D = diag (\tilde{\sigma}^2_1, \ldots, \tilde{\sigma}^2_n) \) is the diagonal matrix of idiosyncratic variances
Table 118: Optimal decarbonization portfolios (Example #5, threshold approach)

|     | 0   | 10  | 20  | 30  | 40  | 50  | CI
|-----|-----|-----|-----|-----|-----|-----|---
| $R_1^*$ | 20.00 | 20.54 | 21.14 | 21.86 | 22.58 | 22.96 | 100.5
| $w_2^*$ | 19.00 | 19.33 | 19.29 | 18.70 | 18.11 | 17.23 | 97.2
| $w_3^*$ | 17.00 | 15.67 | 12.91 | 8.06  | 3.22  | 0.00  | 250.4
| $w_4^*$ | 13.00 | 12.28 | 10.95 | 8.74  | 6.53  | 3.36  | 352.3
| $w_5^*$ | 12.00 | 12.26 | 12.60 | 13.07 | 13.53 | 14.08 | 27.1
| $w_6^*$ | 8.00  | 11.71 | 16.42 | 22.57 | 28.73 | 34.77 | 54.2
| $w_7^*$ | 6.00  | 6.36  | 6.69  | 7.00  | 7.30  | 7.59  | 78.6
| $w_8^*$ | 5.00  | 1.86  | 0.00  | 0.00  | 0.00  | 0.00  | 426.7
| $\sigma (w^* | b)$ | 0.00 | 30.01 | 61.90 | 104.10 | 149.65 | 196.87
| $CI (w)$ | 160.57 | 144.52 | 128.46 | 112.40 | 96.34 | 80.29
| $R (w | b)$ | 0.00 | 10.00 | 20.00 | 30.00 | 40.00 | 50.00

The reduction rate and the weights are expressed in % whereas the tracking error volatility is measured in bps.
## Table 119: Optimal decarbonization portfolios (Example #5, order-statistic approach)

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>( CI_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>20.00</td>
<td>20.40</td>
<td>22.35</td>
<td>26.46</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>19.00</td>
<td>19.90</td>
<td>20.07</td>
<td>20.83</td>
<td>7.57</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>17.00</td>
<td>17.94</td>
<td>21.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>13.00</td>
<td>13.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>12.00</td>
<td>12.12</td>
<td>12.32</td>
<td>12.79</td>
<td>13.04</td>
<td>14.26</td>
<td>18.78</td>
<td>100.00</td>
<td>27.1</td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>8.00</td>
<td>10.04</td>
<td>17.14</td>
<td>32.38</td>
<td>74.66</td>
<td>75.12</td>
<td>81.22</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>( w_7^* )</td>
<td>6.00</td>
<td>6.37</td>
<td>6.70</td>
<td>7.53</td>
<td>4.73</td>
<td>10.62</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>( w_8^* )</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
</tbody>
</table>

The reduction rate, the weights and the tracking error volatility are expressed in %
Equity portfolios

Table 120: Optimal decarbonization portfolios (Example #5, naive approach)

<table>
<thead>
<tr>
<th>m</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$CI_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>21.05</td>
<td>24.39</td>
<td>30.77</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>20.00</td>
<td>23.17</td>
<td>29.23</td>
<td>42.22</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>17.89</td>
<td>20.73</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>13.68</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>12.63</td>
<td>14.63</td>
<td>18.46</td>
<td>26.67</td>
<td>46.15</td>
<td>60.00</td>
<td>100.00</td>
<td>27.1</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>8.42</td>
<td>9.76</td>
<td>12.31</td>
<td>17.78</td>
<td>30.77</td>
<td>40.00</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>6.32</td>
<td>7.32</td>
<td>9.23</td>
<td>13.33</td>
<td>23.08</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
</tbody>
</table>

$\sigma(\begin{bmatrix} w^* \end{bmatrix} | b)$

| 0.00 | 0.39 | 1.85 | 3.04 | 9.46 | 8.08 | 8.65 | 15.41 |

$CI(\begin{bmatrix} w \end{bmatrix})$

| 160.57 | 146.57 | 113.95 | 78.26 | 68.38 | 47.32 | 37.94 | 27.10 |

$R(\begin{bmatrix} w \end{bmatrix} | b)$

| 0.00 | 8.72 | 29.04 | 51.26 | 57.41 | 70.53 | 76.37 | 83.12 |

The reduction rate, the weights and the tracking error volatility are expressed in %.
**Figure 212:** Efficient decarbonization frontier (Example #5)
Figure 213: Efficient decarbonization frontier of the interpolated naive approach (Example #5)
Example #6

We consider a debt-weighted bond index, which is composed of eight bonds. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in $tCO_2e/\$ mn) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the modified duration which is respectively equal to 3.1, 6.6, 7.2, 5, 4.7, 2.1, 8.1 and 2.6 years, and the duration-times-spread factor, which is respectively equal to 100, 155, 575, 436, 159, 145, 804 and 365 bps. There are two sectors. Bonds #1, #3, #4 and #8 belong to Sector$_1$ while Bonds #2, #5, #6 and #7 belong to Sector$_2$. 
Table 121: Optimal decarbonization portfolios (Example #6, threshold approach)

<table>
<thead>
<tr>
<th>( R )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>( CI )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>20.00</td>
<td>21.62</td>
<td>23.93</td>
<td>26.72</td>
<td>30.08</td>
<td>33.44</td>
<td>100.5</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>19.00</td>
<td>18.18</td>
<td>16.98</td>
<td>14.18</td>
<td>7.88</td>
<td>1.58</td>
<td>97.2</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>17.00</td>
<td>18.92</td>
<td>21.94</td>
<td>22.65</td>
<td>16.82</td>
<td>11.00</td>
<td>250.4</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>13.00</td>
<td>11.34</td>
<td>5.35</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>12.00</td>
<td>13.72</td>
<td>16.14</td>
<td>21.63</td>
<td>33.89</td>
<td>46.14</td>
<td>27.1</td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>8.00</td>
<td>9.60</td>
<td>10.47</td>
<td>10.06</td>
<td>7.21</td>
<td>4.36</td>
<td>54.2</td>
</tr>
<tr>
<td>( w_7^* )</td>
<td>6.00</td>
<td>5.56</td>
<td>5.19</td>
<td>4.75</td>
<td>4.11</td>
<td>3.48</td>
<td>78.6</td>
</tr>
<tr>
<td>( w_8^* )</td>
<td>5.00</td>
<td>1.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
</tbody>
</table>

| \( AS_{sector} \) | 0.00 | 6.87 | 15.49 | 24.07 | 31.97 | 47.58 |        |
| \( MD (w) \)      | 5.48 | 5.49 | 5.45  | 5.29  | 4.90  | 4.51  |        |
| \( DTS (w) \)     | 301.05 | 292.34 | 282.28 | 266.12 | 236.45 | 206.78 |        |
| \( \sigma_{AS} (w\mid b) \) | 0.00 | 5.57 | 12.31 | 19.82 | 30.04 | 43.58 |        |
| \( \sigma_{MD} (w\mid b) \) | 0.00 | 0.01 | 0.04  | 0.17  | 0.49  | 0.81  |        |
| \( \sigma_{DTS} (w\mid b) \) | 0.00 | 8.99 | 19.29 | 35.74 | 65.88 | 96.01 |        |
| \( CI (w) \)       | 160.57 | 144.52 | 128.46 | 112.40 | 96.34 | 80.29 |        |
| \( R (w\mid b) \)    | 0.00 | 10.00 | 20.00 | 30.00 | 40.00 | 50.00 |        |
### Table 122: Optimal decarbonization portfolios (Example #6, order-statistic approach)

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$CI_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>20.83</td>
<td>24.62</td>
<td>64.64</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>18.60</td>
<td>18.13</td>
<td>21.32</td>
<td>3.32</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>17.79</td>
<td>26.30</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>14.53</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>12.89</td>
<td>13.96</td>
<td>6.00</td>
<td>36.57</td>
<td>41.27</td>
<td>41.27</td>
<td>100.00</td>
<td>27.1</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>9.74</td>
<td>11.85</td>
<td>0.00</td>
<td>60.11</td>
<td>58.73</td>
<td>58.73</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>5.62</td>
<td>5.15</td>
<td>8.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
<tr>
<td>$\text{AS}_{\text{sector}}$</td>
<td>0.00</td>
<td>5.78</td>
<td>19.72</td>
<td>49.00</td>
<td>76.68</td>
<td>80.00</td>
<td>80.00</td>
<td>88.00</td>
<td></td>
</tr>
<tr>
<td>$\text{MD (w)}$</td>
<td>5.48</td>
<td>5.52</td>
<td>5.54</td>
<td>4.77</td>
<td>3.27</td>
<td>3.17</td>
<td>3.17</td>
<td>4.70</td>
<td></td>
</tr>
<tr>
<td>$\text{DTS (w)}$</td>
<td>301.05</td>
<td>295.08</td>
<td>284.71</td>
<td>171.82</td>
<td>150.45</td>
<td>150.78</td>
<td>150.78</td>
<td>159.00</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{AS}} (w \mid b)$</td>
<td>0.00</td>
<td>5.73</td>
<td>17.94</td>
<td>50.85</td>
<td>66.96</td>
<td>68.63</td>
<td>68.63</td>
<td>95.33</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{MD}} (w \mid b)$</td>
<td>0.00</td>
<td>0.03</td>
<td>0.04</td>
<td>0.63</td>
<td>2.66</td>
<td>2.64</td>
<td>2.64</td>
<td>3.21</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{DTS}} (w \mid b)$</td>
<td>0.00</td>
<td>6.21</td>
<td>16.87</td>
<td>128.04</td>
<td>197.22</td>
<td>197.29</td>
<td>197.29</td>
<td>199.22</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{CI} (w)$</td>
<td>160.57</td>
<td>147.94</td>
<td>122.46</td>
<td>93.63</td>
<td>45.72</td>
<td>43.02</td>
<td>43.02</td>
<td>27.10</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{R} (w \mid b)$</td>
<td>0.00</td>
<td>7.87</td>
<td>23.74</td>
<td>41.69</td>
<td>71.53</td>
<td>73.21</td>
<td>73.21</td>
<td>83.12</td>
<td></td>
</tr>
</tbody>
</table>
Decarbonization scenario per sector:

\[ \mathcal{CI}(w; \text{Sector}_j) \leq (1 - \mathcal{R}_j) \mathcal{CI}(b; \text{Sector}_j) \]

We have:

\[ (s_j \circ (\mathcal{CI} - \mathcal{CI}^*_j))^\top w \leq 0 \]

where \( \mathcal{CI}^*_j = (1 - \mathcal{R}_j) \mathcal{CI}(b; \text{Sector}_j) \)
Sector-specific constraints
Sector scenario

QP form

\[
C = \begin{pmatrix}
(s_1 \circ (CI - CI^*))^T \\
\vdots \\
(s_j \circ (CI - CI^*))^T \\
\vdots \\
(s_{n_{\text{sector}}} \circ (CI - CI^*_{n_{\text{sector}}}))^T
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
(1 - R_1) CI (b; Sector_1) \\
\vdots \\
(1 - R_j) CI (b; Sector_j) \\
\vdots \\
(1 - R_{n_{\text{sector}}}) CI (b; Sector_{n_{\text{sector}}})
\end{pmatrix}
\]
## Sector-specific constraints

### Sector scenario

**Table 123:** Carbon intensity and threshold in $t\text{CO}_2e/\$ \text{mn}$ per GICS sector (MSCI World, 2030)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$C_{\text{T}(b; \text{Sector}_j)}$</th>
<th>$R_j$ (in %)</th>
<th>$C_{\text{T}}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_{1-2}$</td>
<td>$SC_{1-3}^{\text{up}}$</td>
</tr>
<tr>
<td>Communication Services</td>
<td>2</td>
<td>28</td>
<td>134</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>23</td>
<td>65</td>
<td>206</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>28</td>
<td>55</td>
<td>401</td>
</tr>
<tr>
<td>Energy</td>
<td>632</td>
<td>698</td>
<td>1,006</td>
</tr>
<tr>
<td>Financials</td>
<td>13</td>
<td>19</td>
<td>52</td>
</tr>
<tr>
<td>Health Care</td>
<td>10</td>
<td>22</td>
<td>120</td>
</tr>
<tr>
<td>Industrials</td>
<td>111</td>
<td>130</td>
<td>298</td>
</tr>
<tr>
<td>Information Technology</td>
<td>7</td>
<td>23</td>
<td>112</td>
</tr>
<tr>
<td>Materials</td>
<td>478</td>
<td>702</td>
<td>1,113</td>
</tr>
<tr>
<td>Real Estate</td>
<td>22</td>
<td>101</td>
<td>167</td>
</tr>
<tr>
<td>Utilities</td>
<td>1,744</td>
<td>1,794</td>
<td>2,053</td>
</tr>
<tr>
<td>MSCI World</td>
<td>130</td>
<td>163</td>
<td>310</td>
</tr>
</tbody>
</table>
Asset weight deviation constraint:

\[ \Omega := C_1 (m_w^-, m_w^+) = \{ w : m_w^- b \leq w \leq m_w^+ b \} \]

Sector weight deviation constraint:

\[ \Omega := C_2 (m_s^-, m_s^+) = \left\{ \forall j : m_s^- \sum_{i \in \text{Sector}_j} b_i \leq \sum_{i \in \text{Sector}_j} w_i \leq m_s^+ \sum_{i \in \text{Sector}_j} b_i \right\} \]

4. \( C_2 (m_s) = C_2 (1/m_s, m_s) \)

5. \( C_3 (m_w^-, m_w^+, m_s) = C_1 (m_w^-, m_w^+) \cap C_2 (m_s) \)
Sector-specific constraints
Sector and weight deviation constraints (bond portfolio)

1. Modified duration constraint:

\[ \Omega := \mathcal{C}_1 = \{ w : \text{MD} (w) = \text{MD} (b) \} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i) \text{MD}_i = 0 \right\} \]

2. DTS constraint

\[ \Omega := \mathcal{C}_2 = \{ w : \text{DTS} (w) = \text{DTS} (b) \} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i) \text{DTS}_i = 0 \right\} \]

3. Maturity/rating buckets:

\[ \Omega := \left\{ w : \sum_{i \in \text{Bucket}_j} (x_i - b_i) = 0 \right\} \]

- \( \mathcal{C}_3' \): \( \text{Bucket}_j \) is the \( j^{th} \) maturity bucket, e.g., 0–1, 1–3, 3–5, 5–7, 7–10 and 10+
- \( \mathcal{C}_4' \): \( \text{Bucket}_j \) is the \( j^{th} \) rating category, e.g., AAA–AA (AAA, AA+, AA and AA–), A (A+, A and A–) and BBB (BBB+, BBB, BBB–)
Two types of sectors:

1. High climate impact sectors (HCIS): 
   “sectors that are key to the low-carbon transition” (TEG, 2019)

2. Low climate impact sectors (LCIS)

Let $\mathcal{HCIS}(w) = \sum_{i \in \text{HCIS}} w_i$ be the HCIS weight of portfolio $w$:

$$\mathcal{HCIS}(w) \geq \mathcal{HCIS}(b)$$
## Sector-specific constraints

**HCIS constraint**

### Table 124: Weight and carbon intensity when applying the HCIS filter (MSCI World, June 2022)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index ( b_j )</th>
<th>HCIS ( b'_j )</th>
<th>( SC_1 ) Index</th>
<th>( SC_1 )</th>
<th>( SC_{1-2} )</th>
<th>( SC_{1-3}^{up} )</th>
<th>( SC_{1-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>0.00</td>
<td>2</td>
<td>28</td>
<td>134</td>
<td>172</td>
<td></td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>8.01</td>
<td>23</td>
<td>65</td>
<td>206</td>
<td>590</td>
<td>189</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>7.80</td>
<td>28</td>
<td>55</td>
<td>401</td>
<td>929</td>
<td>401</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>4.99</td>
<td>632</td>
<td>698</td>
<td>1006</td>
<td>6823</td>
<td>6823</td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>0.00</td>
<td>13</td>
<td>19</td>
<td>52</td>
<td>244</td>
<td></td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>9.98</td>
<td>10</td>
<td>22</td>
<td>120</td>
<td>146</td>
<td>177</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>7.96</td>
<td>111</td>
<td>130</td>
<td>298</td>
<td>1662</td>
<td>1921</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>10.67</td>
<td>7</td>
<td>23</td>
<td>112</td>
<td>239</td>
<td>390</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>4.28</td>
<td>478</td>
<td>702</td>
<td>1113</td>
<td>2957</td>
<td>2957</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>2.90</td>
<td>22</td>
<td>101</td>
<td>167</td>
<td>571</td>
<td>571</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>3.21</td>
<td>1744</td>
<td>1794</td>
<td>2053</td>
<td>2840</td>
<td>2840</td>
</tr>
<tr>
<td><strong>MSCI World</strong></td>
<td><strong>100.00</strong></td>
<td><strong>59.79</strong></td>
<td><strong>130</strong></td>
<td><strong>163</strong></td>
<td><strong>310</strong></td>
<td><strong>992</strong></td>
<td><strong>1498</strong></td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Empirical results (equity portfolios)

Figure 214: Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $SC_{1-2}$)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Empirical results (equity portfolios)

**Figure 215:** Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $SC_{1-3}$)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Barahhou et al. (2022) consider the basic optimization problem:

$$w^* = \arg\min \frac{1}{2} (w - b)^\top \Sigma (w - b)$$

s.t. \begin{align*}
CI(w) &\leq (1 - \mathcal{R}) CI(b) \\
 w &\in \Omega_0 \cap \Omega
\end{align*}

What is the impact of constraints $\Omega_0 \cap \Omega$?
Equity portfolios

Figure 216: Impact of the carbon scope on the tracking error volatility (MSCI World, June 2022, $C_0$ constraint)

Source: MSCI (2022), Trucost (2022) & Barahou et al. (2022)
## Equity Portfolios

**Table 125:** Sector allocation in % (MSCI World, June 2022, scope $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>7.95</td>
<td>8.15</td>
<td>8.42</td>
<td>8.78</td>
<td>9.34</td>
<td>10.13</td>
<td>12.27</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>10.69</td>
<td>10.69</td>
<td>10.65</td>
<td>10.52</td>
<td>10.23</td>
<td>9.62</td>
<td>6.74</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>7.80</td>
<td>7.69</td>
<td>7.48</td>
<td>7.11</td>
<td>6.35</td>
<td>5.03</td>
<td>1.77</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>4.14</td>
<td>3.65</td>
<td>3.10</td>
<td>2.45</td>
<td><strong>1.50</strong></td>
<td><strong>0.49</strong></td>
<td><strong>0.00</strong></td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>14.53</td>
<td>15.17</td>
<td>15.94</td>
<td>16.90</td>
<td><strong>18.39</strong></td>
<td><strong>20.55</strong></td>
<td><strong>28.62</strong></td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>14.74</td>
<td>15.09</td>
<td>15.50</td>
<td>16.00</td>
<td>16.78</td>
<td>17.77</td>
<td>17.69</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>9.28</td>
<td>9.01</td>
<td>8.71</td>
<td>8.36</td>
<td>7.79</td>
<td>7.21</td>
<td>6.03</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>21.68</td>
<td>22.03</td>
<td>22.39</td>
<td>22.88</td>
<td>23.51</td>
<td>24.12</td>
<td>24.02</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>3.78</td>
<td>3.46</td>
<td>3.06</td>
<td>2.56</td>
<td><strong>1.85</strong></td>
<td><strong>1.14</strong></td>
<td><strong>0.24</strong></td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>3.12</td>
<td>3.27</td>
<td>3.41</td>
<td>3.57</td>
<td>3.72</td>
<td>3.71</td>
<td>2.51</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>2.28</td>
<td>1.79</td>
<td>1.36</td>
<td>0.90</td>
<td><strong>0.54</strong></td>
<td><strong>0.24</strong></td>
<td><strong>0.12</strong></td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)

Portfolio decarbonization = strategy **long on Financials** and **short on Energy, Materials and Utilities**
Figure 217: Impact of $C_1$ constraint on the tracking error volatility (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Figure 218: Impact of $C_2$ and $C_3$ constraints (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Figure 219: Tracking error volatility with $C_3 (0, 10, 2)$ constraint (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
First approach

The carbon footprint contribution of the $m$ worst performing assets is:

$$
\text{CFC}^{(m,n)} = \sum_{i=1}^{n} 1 \left\{ C_{i} \geq C_{n-m+1} \right\} \cdot b_{i} C_{i} \text{Cl} (b)
$$

where $C_{n-m+1:n}$ is the $(n - m + 1)$-th order statistic.
Equity portfolios

Figure 220: Carbon footprint contribution $CFC^{(m,n)}$ in % (MSCI World, June 2022, first approach)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Second approach

- Another definition:

\[
CFC^{(m,n)} = \frac{\sum_{i=1}^{n} \mathbb{1}\left\{ CIC_i \geq CIC^{(m,n)} \right\} \cdot b_i C\mathcal{I}_i}{CI(b)}
\]

where \( CIC_i = b_i C\mathcal{I}_i \) and \( CIC^{(m,n)} = CIC_{n-m+1:n} \)

- Weight contribution:

\[
WC^{(m,n)} = \sum_{i=1}^{n} \mathbb{1}\left\{ CIC_i \geq CIC^{(m,n)} \right\} \cdot b_i
\]
Figure 221: Carbon footprint contribution $CFC^{(m,n)}$ in % (MSCI World, June 2022, second approach)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
### Table 126: Carbon footprint contribution $CFC^{(m,n)}_1$ in % (MSCI World, June 2022, second approach, $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$m$</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td></td>
<td>0.44</td>
<td>0.44</td>
<td>0.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Staples</td>
<td></td>
<td>0.78</td>
<td>1.37</td>
<td>2.44</td>
<td>2.93</td>
<td>4.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td></td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>3.75</td>
<td>4.44</td>
<td>4.92</td>
<td>5.62</td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td></td>
<td>9.61</td>
<td>17.35</td>
<td>23.78</td>
<td>29.56</td>
<td>31.78</td>
<td>33.02</td>
<td>33.89</td>
<td></td>
</tr>
<tr>
<td>Health Care</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.72</td>
</tr>
<tr>
<td>Industrials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td>Information Technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.98</td>
</tr>
<tr>
<td>Materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.08</td>
</tr>
<tr>
<td>Real Estate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.77</td>
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<tr>
<td>Utilities</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.81</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.08</td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
### Table 127: Weight contribution $\mathcal{W}(m,n)$ in % (MSCI World, June 2022, second approach, $\mathcal{SC}_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$b_j$ (in %)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>0.58</td>
<td>1.79</td>
<td>2.44</td>
<td>4.51</td>
<td>5.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>0.70</td>
<td>0.70</td>
<td>1.90</td>
<td>2.50</td>
<td>2.84</td>
<td>3.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>1.71</td>
<td>2.25</td>
<td>2.96</td>
<td>3.62</td>
<td>3.99</td>
<td>4.33</td>
<td>4.65</td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.34</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td></td>
<td>0.06</td>
<td>0.32</td>
<td>0.70</td>
<td>0.96</td>
<td>1.20</td>
<td>4.12</td>
<td></td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>0.16</td>
<td>4.70</td>
<td>8.42</td>
<td>8.78</td>
<td>11.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>0.29</td>
<td>0.29</td>
<td>0.47</td>
<td>0.88</td>
<td>1.10</td>
<td>1.40</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.29</strong></td>
<td><strong>2.71</strong></td>
<td><strong>3.30</strong></td>
<td><strong>5.49</strong></td>
<td><strong>14.50</strong></td>
<td><strong>21.32</strong></td>
<td><strong>26.63</strong></td>
<td><strong>41.24</strong></td>
<td></td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
The order-statistic optimization problem is:

\[
w^* = \arg \min \frac{1}{2} (w - b)^\top \Sigma (w - b)
\]

s.t. \[
\begin{align*}
1_n^\top w &= 1 \\
0_n &\leq w \leq w^{(m,n)}
\end{align*}
\]

where the upper bound \(w^{(m,n)}\) is equal to \(1 \{CI < CI^{(m,n)}\}\) for the first ordering approach and \(1 \{CIC < CIC^{(m,n)}\}\) for the second ordering approach.
Equity portfolios

- The naive method is:

\[
    w_i^* = \frac{e_i b_i}{\sum_{k=1}^{n} e_k b_k}
\]

where \( e_i \) is defined as \( 1 \left\{ CI_i < CI^{(m,n)} \right\} \) for the first ordering approach and \( 1 \left\{ CIC_i < CIC^{(m,n)} \right\} \) for the second ordering approach.
Figure 222: Tracking error volatility (MSCI World, June 2022, $SC_{1-3}$, first ordering method)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Figure 223: Tracking error volatility (MSCI World, June 2022, $SC_{1−3}$, second ordering method)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
The optimization problem is:

\[ w^* = \arg \min \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| + 50 \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \right| \text{DTS}_i \]

s.t. \[ \begin{align*}
CI(w) &\leq (1 - R) CI(b) \\
w &\in C_0 \cap C_1' \cap C_3' \cap C_4'
\end{align*} \]
Figure 224: Impact of the carbon scope on the active share in % (ICE Global Corp., June 2022)

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
Figure 225: Impact of the carbon scope on the DTS risk in bps (ICE Global Corp., June 2022)

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
Table 128: Sector allocation in % (ICE Global Corp., June 2022, scope $\mathcal{SC}_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.34</td>
<td>7.35</td>
<td>7.34</td>
<td>7.37</td>
<td>7.43</td>
<td>7.43</td>
<td>7.31</td>
<td>7.30</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>5.97</td>
<td>5.97</td>
<td>5.96</td>
<td>5.94</td>
<td>5.93</td>
<td>5.46</td>
<td>4.48</td>
<td>3.55</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.02</td>
<td>5.39</td>
<td>4.06</td>
</tr>
<tr>
<td>Energy</td>
<td>6.49</td>
<td>5.49</td>
<td>4.42</td>
<td>3.84</td>
<td>3.69</td>
<td>3.23</td>
<td>2.58</td>
<td>2.52</td>
</tr>
<tr>
<td>Financials</td>
<td>33.91</td>
<td>34.64</td>
<td>35.66</td>
<td>35.96</td>
<td>36.09</td>
<td>37.36</td>
<td>38.86</td>
<td>39.00</td>
</tr>
<tr>
<td>Health Care</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.52</td>
<td>7.48</td>
</tr>
<tr>
<td>Information Technology</td>
<td>5.57</td>
<td>5.57</td>
<td>5.59</td>
<td>5.59</td>
<td>5.60</td>
<td>5.60</td>
<td>5.52</td>
<td>5.27</td>
</tr>
<tr>
<td>Materials</td>
<td>3.44</td>
<td>3.43</td>
<td>3.31</td>
<td>3.18</td>
<td>3.12</td>
<td>2.64</td>
<td>2.25</td>
<td>1.86</td>
</tr>
<tr>
<td>Real Estate</td>
<td>4.76</td>
<td>4.74</td>
<td>4.74</td>
<td>4.74</td>
<td>4.74</td>
<td>4.66</td>
<td>4.61</td>
<td>3.93</td>
</tr>
<tr>
<td>Utilities</td>
<td>10.06</td>
<td>9.89</td>
<td>9.82</td>
<td>9.64</td>
<td>8.52</td>
<td>8.04</td>
<td>7.92</td>
<td>6.88</td>
</tr>
</tbody>
</table>

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
Course 2022-2023 in Sustainable Finance
Lecture 11. Exercise — Equity and Bond Portfolio Optimization with Green Preferences

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2023

28The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
We consider an investment universe of 8 issuers. In the table below, we report the carbon emissions $C\mathcal{E}_{i,j}$ (in ktCO$_2$e) of these companies and their revenues $Y_i$ (in $\text{bn}$), and we indicate in the last row whether the company belongs to sector $\text{Sector}_1$ or $\text{Sector}_2$:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C\mathcal{E}_{i,1}$</td>
<td>75</td>
<td>5000</td>
<td>720</td>
<td>50</td>
<td>2500</td>
<td>25</td>
<td>30000</td>
<td>5</td>
</tr>
<tr>
<td>$C\mathcal{E}_{i,2}$</td>
<td>75</td>
<td>5000</td>
<td>1030</td>
<td>350</td>
<td>4500</td>
<td>5</td>
<td>2000</td>
<td>64</td>
</tr>
<tr>
<td>$C\mathcal{E}_{i,3}$</td>
<td>24000</td>
<td>15000</td>
<td>1210</td>
<td>550</td>
<td>500</td>
<td>187</td>
<td>30000</td>
<td>199</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>300</td>
<td>328</td>
<td>125</td>
<td>100</td>
<td>200</td>
<td>102</td>
<td>107</td>
<td>25</td>
</tr>
<tr>
<td>$\text{Sector}$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The benchmark $b$ of this investment universe is defined as:

$$b = (22\%, 19\%, 17\%, 13\%, 11\%, 8\%, 6\%, 4\%)$$

In what follows, we consider long-only portfolios.
Question 1

We want to compute the carbon intensity of the benchmark.
Question (a)

Compute the carbon intensities $CI_{i,j}$ of each company $i$ for the scopes 1, 2 and 3.
We have:

\[ CI_{i,j} = \frac{CE_{i,j}}{Y_i} \]

For instance, if we consider the 8\textsuperscript{th} issuer, we have\textsuperscript{29}:

\[ CI_{8,1} = \frac{CE_{8,1}}{Y_8} = \frac{5}{25} = 0.20 \text{ tCO}_2\text{e}/\$ \text{ mn} \]
\[ CI_{8,2} = \frac{CE_{8,2}}{Y_8} = \frac{64}{25} = 2.56 \text{ tCO}_2\text{e}/\$ \text{ mn} \]
\[ CI_{8,3} = \frac{CE_{8,3}}{Y_8} = \frac{199}{25} = 7.96 \text{ tCO}_2\text{e}/\$ \text{ mn} \]

\textsuperscript{29}Because 1 ktCO\textsubscript{2}e/$ bn = 1 tCO\textsubscript{2}e/$ mn.
Since we have:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CE_{i,1} )</td>
<td>75</td>
<td>5000</td>
<td>720</td>
<td>50</td>
<td>2500</td>
<td>25</td>
<td>30000</td>
<td>5</td>
</tr>
<tr>
<td>( CE_{i,2} )</td>
<td>75</td>
<td>5000</td>
<td>1030</td>
<td>350</td>
<td>4500</td>
<td>5</td>
<td>2000</td>
<td>64</td>
</tr>
<tr>
<td>( CE_{i,3} )</td>
<td>24000</td>
<td>15000</td>
<td>1210</td>
<td>550</td>
<td>500</td>
<td>187</td>
<td>30000</td>
<td>199</td>
</tr>
<tr>
<td>( Y_i )</td>
<td>300</td>
<td>328</td>
<td>125</td>
<td>100</td>
<td>200</td>
<td>102</td>
<td>107</td>
<td>25</td>
</tr>
</tbody>
</table>

we obtain:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CI_{i,1} )</td>
<td>0.25</td>
<td>15.24</td>
<td>5.76</td>
<td>0.50</td>
<td>12.50</td>
<td>0.25</td>
<td>280.37</td>
<td>0.20</td>
</tr>
<tr>
<td>( CI_{i,2} )</td>
<td>0.25</td>
<td>15.24</td>
<td>8.24</td>
<td>3.50</td>
<td>22.50</td>
<td>0.05</td>
<td>18.69</td>
<td>2.56</td>
</tr>
<tr>
<td>( CI_{i,3} )</td>
<td>80.00</td>
<td>45.73</td>
<td>9.68</td>
<td>5.50</td>
<td>2.50</td>
<td>1.83</td>
<td>280.37</td>
<td>7.96</td>
</tr>
</tbody>
</table>
Question (b)

Deduce the carbon intensities $CI_{i,j}$ of each company $i$ for the scopes $1 + 2$ and $1 + 2 + 3$. 
We have:

\[ CI_{i,1-2} = \frac{CE_{i,1} + CE_{i,2}}{Y_i} = CI_{i,1} + CI_{i,2} \]

and:

\[ CI_{i,1-3} = CI_{i,1} + CI_{i,2} + CI_{i,3} \]

We deduce that:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI_{i,1}</td>
<td>0.25</td>
<td>15.24</td>
<td>5.76</td>
<td>0.50</td>
<td>12.50</td>
<td>0.25</td>
<td>280.37</td>
<td>0.20</td>
</tr>
<tr>
<td>CI_{i,1-2}</td>
<td>0.50</td>
<td>30.49</td>
<td>14.00</td>
<td>4.00</td>
<td>35.00</td>
<td>0.29</td>
<td>299.07</td>
<td>2.76</td>
</tr>
<tr>
<td>CI_{i,1-3}</td>
<td>80.50</td>
<td>76.22</td>
<td>23.68</td>
<td>9.50</td>
<td>37.50</td>
<td>2.12</td>
<td>579.44</td>
<td>10.72</td>
</tr>
</tbody>
</table>
Question (c)

Deduce the weighted average carbon intensity (WACI) of the benchmark if we consider the scope 1 + 2 + 3.
We have:

\[ CI(b) = \sum_{i=1}^{8} b_i CI_i \]

\[ = 0.22 \times 80.50 + 0.19 \times 76.2195 + 0.17 \times 23.68 + 0.13 \times 9.50 + \]

\[ 0.11 \times 37.50 + 0.08 \times 2.1275 + 0.06 \times 579.4393 + 0.04 \times 10.72 \]

\[ = 76.9427 \text{ tCO}_2e/\$ \text{ mn} \]
Question (d)

We assume that the market capitalization of the benchmark portfolio is equal to $10$ tn and we invest $1$ bn.
Question (d).i

Deduce the market capitalization of each company (expressed in $ bn).
We have:

\[ b_i = \frac{MC_i}{\sum_{k=1}^{8} MC_k} \]

and \( \sum_{k=1}^{8} MC_k = $10 \text{ tn.} \) We deduce that:

\[ MC_i = 10 \times b_i \]

We obtain the following values of market capitalization expressed in $ bn:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC_i</td>
<td>2200</td>
<td>1900</td>
<td>1700</td>
<td>1300</td>
<td>1100</td>
<td>800</td>
<td>600</td>
<td>400</td>
</tr>
</tbody>
</table>
Question (d).ii

Compute the ownership ratio for each asset (expressed in bps).
Let $W$ be the wealth invested in the benchmark portfolio $b$. The wealth invested in asset $i$ is equal to $b_iW$. We deduce that the ownership ratio is equal to:

$$\omega_i = \frac{b_iW}{MC_i} = \frac{b_iW}{b_i \sum_{k=1}^{n} MC_k} = \frac{W}{\sum_{k=1}^{n} MC_k}$$

When we invest in a capitalization-weighted portfolio, the ownership ratio is the same for all the assets. In our case, we have:

$$\omega_i = \frac{1}{10 \times 1000} = 0.01\%$$

The ownership ratio is equal to 1 basis point.
Question (d).iii

Compute the carbon emissions of the benchmark portfolio\(^a\) if we invest $1$ bn and we consider the scope $1 + 2 + 3$.

\(^a\)We assume that the float percentage is equal to 100% for all the 8 companies.
Using the financed emissions approach, the carbon emissions of our investment is equal to:

\[ CE \text{ ($1 \text{ bn})} = 0.01\% \times (75 + 75 + 24000) + 0.01\% \times (5000 + 5000 + 15000) + \ldots + 0.01\% \times (5 + 64 + 199) = 12.3045 \text{ ktCO}_2\text{e} \]
Question (d).iv

Compare the (exact) carbon intensity of the benchmark portfolio with the WACI value obtained in Question 1.(c).
We compute the revenues of our investment:

\[ Y \text{ ($1 bn)} = 0.01\% \sum_{i=1}^{8} Y_i = $0.1287 \text{ bn} \]

We deduce that the exact carbon intensity is equal to:

\[ CI \text{ ($1 bn)} = \frac{CE \text{ ($1 bn)}}{Y \text{ ($1 bn)}} = \frac{12.3045}{0.1287} = 95.6061 \text{ tCO}_2e/\$ \text{ mn} \]

We notice that the WACI of the benchmark underestimates the exact carbon intensity of our investment by 19.5%:

\[ 76.9427 < 95.6061 \]
Question 2

We want to manage an equity portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $R$. We assume that the volatility of the stocks is respectively equal to 22%, 20%, 25%, 18%, 40%, 23%, 13% and 29%. The correlation matrix between these stocks is given by:

$$
\rho = \begin{pmatrix}
100 & 100 & 100 \\
80 & 100 & 100 \\
70 & 75 & 100 \\
60 & 65 & 80 & 100 \\
70 & 50 & 70 & 85 & 100 \\
50 & 60 & 70 & 80 & 60 & 100 \\
70 & 50 & 70 & 75 & 80 & 50 & 100 \\
60 & 65 & 70 & 75 & 65 & 70 & 60 & 100
\end{pmatrix}
$$
Question (a)

Compute the covariance matrix $\Sigma$. 
The covariance matrix $\Sigma = (\Sigma_{i,j})$ is defined by:

$$\Sigma_{i,j} = \rho_{i,j} \sigma_i \sigma_j$$

We obtain the following numerical values (expressed in bps):

$$\Sigma = \begin{pmatrix}
484.0 & 352.0 & 385.0 & 237.6 & 616.0 & 253.0 & 200.2 & 382.8 \\
352.0 & 400.0 & 375.0 & 234.0 & 400.0 & 276.0 & 130.0 & 377.0 \\
385.0 & 375.0 & 625.0 & 360.0 & 700.0 & 402.5 & 227.5 & 507.5 \\
237.6 & 234.0 & 360.0 & 324.0 & 612.0 & 331.2 & 175.5 & 391.5 \\
616.0 & 400.0 & 700.0 & 612.0 & 1600.0 & 552.0 & 416.0 & 754.0 \\
253.0 & 276.0 & 402.5 & 331.2 & 552.0 & 529.0 & 149.5 & 466.9 \\
200.2 & 130.0 & 227.5 & 175.5 & 416.0 & 149.5 & 169.0 & 226.2 \\
382.8 & 377.0 & 507.5 & 391.5 & 754.0 & 466.9 & 226.2 & 841.0
\end{pmatrix}$$
Question (b)
Write the optimization problem if the objective function is to minimize the tracking error risk under the constraint of carbon intensity reduction.
The tracking error variance of portfolio $w$ with respect to benchmark $b$ is equal to:

$$\sigma^2(w \mid b) = (w - b)^\top \Sigma (w - b)$$

The carbon intensity constraint has the following expression:

$$\sum_{i=1}^8 w_i CI_i \leq (1 - R) CI(b)$$

where $R$ is the reduction rate and $CI(b)$ is the carbon intensity of the benchmark. Let $CI^* = (1 - R) CI(b)$ be the target value of the carbon footprint. The optimization problem is then:

$$w^* = \arg \min \frac{1}{2} \sigma^2(w \mid b)$$

s.t. \begin{align*}
\sum_{i=1}^8 w_i CI_i & \leq CI^* \\
\sum_{i=1}^8 w_i & = 1 \\
0 & \leq w_i \leq 1
\end{align*}

We add the second and third constraints in order to obtain a long-only portfolio.
Question (c)
Give the QP formulation of the optimization problem.
The objective function is equal to:

\[ f(w) = \frac{1}{2} \sigma^2 (w | b) = \frac{1}{2} (w - b)^\top \Sigma (w - b) = \frac{1}{2} w^\top \Sigma w - w^\top \Sigma b + \frac{1}{2} b^\top \Sigma b \]

while the matrix form of the carbon intensity constraint is:

\[ CI^\top w \leq CI^* \]

where \( CI = (CI_1, \ldots, CI_8) \) is the column vector of carbon intensities. Since \( b^\top \Sigma b \) is a constant and does not depend on \( w \), we can cast the previous optimization problem into a QP problem:

\[
\begin{align*}
    w^* &= \arg \min \frac{1}{2} w^\top Qw - w^\top R \\
    \text{s.t.} \quad & \begin{cases}
        Aw = B \\
        Cw \leq D \\
        w^- \leq w \leq w^+
    \end{cases}
\end{align*}
\]

We have \( Q = \Sigma, \ R = \Sigma b, \ A = \mathbf{1}_8^\top, \ B = 1, \ C = CI^\top, \ D = CI^*, \ w^- = \mathbf{0}_8 \) and \( w^+ = \mathbf{1}_8 \).
Question (d)

\( R \) is equal to 20%. Find the optimal portfolio if we target scope 1 + 2. What is the value of the tracking error volatility?
We have:

\[ CI(b) = 0.22 \times 0.50 + 0.19 \times 30.4878 + \ldots + 0.04 \times 2.76 \]
\[ = 30.7305 \text{ tCO}_2e/\$ \text{ mn} \]

We deduce that:

\[ CI^* = (1 - R) CI(b) = 0.80 \times 30.7305 = 24.5844 \text{ tCO}_2e/\$ \text{ mn} \]

Therefore, the inequality constraint of the QP problem is:

\[
\begin{pmatrix}
0.50 & 30.49 & 14.00 & 4.00 & 35.00 & 0.29 & 299.07 & 2.76
\end{pmatrix}
\begin{pmatrix}
w_1 \\
w_2 \\
\vdots \\
w_7 \\
w_8
\end{pmatrix} \leq 24.5844
We obtain the following optimal solution:

\[
\begin{pmatrix}
23.4961 \\
17.8129 \\
17.1278 \\
15.4643 \\
10.4037 \\
7.5903 \\
4.0946 \\
4.0104
\end{pmatrix}
\]

The minimum tracking error volatility \( \sigma (w^* \mid b) \) is equal to 15.37 bps.
Question (e)

Same question if $R$ is equal to 30%, 50%, and 70%.
Table 129: Solution of the equity optimization problem (scope $SC_{1-2}$)

<table>
<thead>
<tr>
<th>$R$</th>
<th>0%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>23.4961</td>
<td>24.2441</td>
<td>25.7402</td>
<td>30.4117</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.8129</td>
<td>17.2194</td>
<td>16.0323</td>
<td>9.8310</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>17.1278</td>
<td>17.1917</td>
<td>17.3194</td>
<td>17.8348</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>15.4643</td>
<td>16.6964</td>
<td>19.1606</td>
<td>23.3934</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.4037</td>
<td>10.1055</td>
<td>9.5091</td>
<td>7.1088</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>7.5903</td>
<td>7.3854</td>
<td>6.9757</td>
<td>6.7329</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>4.0946</td>
<td>3.1418</td>
<td>1.2364</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>4.0104</td>
<td>4.0157</td>
<td>4.0261</td>
<td>4.6874</td>
</tr>
</tbody>
</table>

| $CI(w)$ | 30.7305 | 24.5844 | 21.5114 | 15.3653 | 9.2192 |
| $\sigma(w|b)$ | 0.00 | 15.37 | 23.05 | 38.42 | 72.45 |
In Table 129, we report the optimal solution $w^*$ (expressed in %) of the optimization problem for different values of $R$. We also indicate the carbon intensity of the portfolio (in tCO$_2$e/$mn$) and the tracking error volatility (in bps). For instance, if $R$ is set to 50%, the weights of assets #1, #3, #4 and #8 increase whereas the weights of assets #2, #5, #6 and #7 decrease. The carbon intensity of this portfolio is equal to 15.3653 tCO$_2$e/$mn$. The tracking error volatility is below 40 bps, which is relatively low.
Question (f)

We target scope 1 + 2 + 3. Find the optimal portfolio if \( R \) is equal to 20%, 30%, 50% and 70%. Give the value of the tracking error volatility for each optimized portfolio.
In this case, the inequality constraint $C_w \leq D$ is defined by:

$$C = c \mathbf{1}_{1-3}^T = \begin{pmatrix} 80.5000 \\ 76.2195 \\ 23.6800 \\ 9.5000 \\ 37.5000 \\ 2.1275 \\ 579.4393 \\ 10.7200 \end{pmatrix}^T$$

and:

$$D = (1 - \mathcal{R}) \times 76.9427$$
We obtain the results given in Table 130.

**Table 130: Solution of the equity optimization problem (scope $SC_{1−3}$)**

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>0%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>23.9666</td>
<td>24.9499</td>
<td>26.4870</td>
<td>13.6749</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.4410</td>
<td>16.6615</td>
<td>8.8001</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>17.1988</td>
<td>17.2981</td>
<td>19.4253</td>
<td>24.1464</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>16.5034</td>
<td>18.2552</td>
<td>25.8926</td>
<td>41.0535</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.2049</td>
<td>9.8073</td>
<td>7.1330</td>
<td>3.5676</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>7.4169</td>
<td>7.1254</td>
<td>7.0659</td>
<td>8.8851</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>3.2641</td>
<td>1.8961</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>4.0043</td>
<td>4.0065</td>
<td>5.1961</td>
<td>8.6725</td>
</tr>
<tr>
<td>$CI(w)$</td>
<td>76.9427</td>
<td>61.5541</td>
<td>53.8599</td>
<td>38.4713</td>
<td>23.0828</td>
</tr>
<tr>
<td>$\sigma(w \mid b)$</td>
<td>0.00</td>
<td>21.99</td>
<td>32.99</td>
<td>104.81</td>
<td>414.48</td>
</tr>
</tbody>
</table>
Question (g)

Compare the optimal solutions obtained in Questions 2.(e) and 2.(f).
Figure 226: Impact of the scope on the tracking error volatility
Figure 227: Impact of the scope on the portfolio allocation (in %)
In Figure 226, we report the relationship between the reduction rate $\mathcal{R}$ and the tracking error volatility $\sigma (w \mid b)$. The choice of the scope has little impact when $\mathcal{R} \leq 45\%$. Then, we notice a high increase when we consider the scope $1 + 2 + 3$. The portfolio’s weights are given in Figure 227. For assets #1 and #3, the behavior is divergent when we compare scopes $1 + 2$ and $1 + 2 + 3$. 
Question 3

We want to manage a bond portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $R$. We use the scope $1 + 2 + 3$. In the table below, we report the modified duration $MD_i$ and the duration-times-spread factor $DTS_i$ of each corporate bond $i$:

<table>
<thead>
<tr>
<th>Asset</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MD_i$ (in years)</td>
<td>3.56</td>
<td>7.48</td>
<td>6.54</td>
<td>10.23</td>
<td>2.40</td>
<td>2.30</td>
<td>9.12</td>
<td>7.96</td>
</tr>
<tr>
<td>$DTS_i$ (in bps)</td>
<td>103</td>
<td>155</td>
<td>75</td>
<td>796</td>
<td>89</td>
<td>45</td>
<td>320</td>
<td>245</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Question 3 (Cont’d)

We remind that the active risk can be calculated using three functions. For the active share, we have:

\[ R_{AS} (w \mid b) = \sigma_{AS}^2 (w \mid b) = \sum_{i=1}^{n} (w_i - b_i)^2 \]

We also consider the MD-based tracking error risk:

\[ R_{MD} (w \mid b) = \sigma_{MD}^2 (w \mid b) = \sum_{j=1}^{n_{Sector}} \left( \sum_{i \in Sector_j} (w_i - b_i) MD_i \right)^2 \]

and the DTS-based tracking error risk:

\[ R_{DTS} (w \mid b) = \sigma_{DTS}^2 (w \mid b) = \sum_{j=1}^{n_{Sector}} \left( \sum_{i \in Sector_j} (w_i - b_i) DTS_i \right)^2 \]
Question 3 (Cont’d)

Finally, we define the synthetic risk measure as a combination of AS, MD and DTS active risks:

\[
\mathcal{R}(w \mid b) = \varphi_{AS} \mathcal{R}_{AS}(w \mid b) + \varphi_{MD} \mathcal{R}_{MD}(w \mid b) + \varphi_{DTS} \mathcal{R}_{DTS}(w \mid b)
\]

where \( \varphi_{AS} \geq 0, \varphi_{MD} \geq 0 \) and \( \varphi_{DTS} \geq 0 \) indicate the weight of each risk. In what follows, we use the following numerical values: \( \varphi_{AS} = 100, \varphi_{MD} = 25 \) and \( \varphi_{DTS} = 1 \). The reduction rate \( \mathcal{R} \) of the weighted average carbon intensity is set to 50% for the scope 1 + 2 + 3.
Question (a)

Compute the modified duration $\text{MD}(b)$ and the duration-times-spread factor $\text{DTS}(b)$ of the benchmark.
We have:

$$\text{MD} (b) = \sum_{i=1}^{n} b_i \text{MD}_i$$

$$= 0.22 \times 3.56 + 0.19 \times 7.48 + \ldots + 0.04 \times 7.96$$

$$= 5.96 \text{ years}$$

and:

$$\text{DTS} (b) = \sum_{i=1}^{n} b_i \text{DTS}_i$$

$$= 0.22 \times 103 + 0.19 \times 155 + \ldots + 0.04 \times 155$$

$$= 210.73 \text{ bps}$$
Question (b)

Let $w_{\text{ew}}$ be the equally-weighted portfolio. Compute $MD(w_{\text{ew}})$, $DTS(w_{\text{ew}})$, $\sigma_{\text{AS}}(w_{\text{ew}} | b)$, $\sigma_{\text{MD}}(w_{\text{ew}} | b)$ and $\sigma_{\text{DTS}}(w_{\text{ew}} | b)$.

\[ a \text{Precise the corresponding unit (years, bps or %) for each metric.} \]
We have:

\[
\begin{align*}
\text{MD}(w_{\text{ew}}) &= 6.20 \text{ years} \\
\text{DTS}(w_{\text{ew}}) &= 228.50 \text{ bps} \\
\sigma_{\text{AS}}(w_{\text{ew}} | b) &= 17.03\% \\
\sigma_{\text{MD}}(w_{\text{ew}} | b) &= 1.00 \text{ years} \\
\sigma_{\text{DTS}}(w_{\text{ew}} | b) &= 36.19 \text{ bps}
\end{align*}
\]
Question (c)

We consider the following optimization problem:

\[ w^* = \arg\min w \ \frac{1}{2} R_{AS} (w \mid b) \]

subject to:

\[
\begin{align*}
\sum_{i=1}^{n} w_i &= 1 \\
MD (w) &= MD (b) \\
DTS (w) &= DTS (b) \\
CI (w) &\leq (1 - R) CI (b) \\
0 &\leq w_i \leq 1
\end{align*}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( w^* \). Compute \( MD (w^*) \), \( DTS (w^*) \), \( \sigma_{AS} (w^* \mid b) \), \( \sigma_{MD} (w^* \mid b) \) and \( \sigma_{DTS} (w^* \mid b) \).
We have:

$$R_{AS} (w \mid b) = (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 + (w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2$$

The objective function is then:

$$f (w) = \frac{1}{2} R_{AS} (w \mid b)$$

The optimal solution is equal to:

$$w^* = (17.30\%, 17.41\%, 20.95\%, 14.41\%, 10.02\%, 11.09\%, 0\%, 8.81\%)$$

The risk metrics are:

$$\begin{align*}
\text{MD} (w^*) &= 5.96 \text{ years} \\
\text{DTS} (w^*) &= 210.73 \text{ bps} \\
\sigma_{AS} (w^* \mid b) &= 10.57\% \\
\sigma_{MD} (w^* \mid b) &= 0.43 \text{ years} \\
\sigma_{DTS} (w^* \mid b) &= 15.21 \text{ bps}
\end{align*}$$
Question (d)

We consider the following optimization problem:

\[ w^* = \arg \min \frac{\varphi_{AS}}{2} R_{AS} (w | b) + \frac{\varphi_{MD}}{2} R_{MD} (w | b) \]

\[
\begin{cases}
\sum_{i=1}^{n} w_i = 1 \\
DTS(w) = DTS(b) \\
CI(w) \leq (1 - R) CI(b) \\
0 \leq w_i \leq 1
\end{cases}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( w^* \). Compute \( MD(w^*) \), \( DTS(w^*) \), \( \sigma_{AS}(w^* | b) \), \( \sigma_{MD}(w^* | b) \) and \( \sigma_{DTS}(w^* | b) \).
We have\(^{30}\):

\[
\mathcal{R}_{\text{MD}} (w \mid b) = \left( \sum_{i=1,3,4,6} (w_i - b_i) \text{MD}_i \right)^2 + \left( \sum_{i=2,5,7,8} (w_i - b_i) \text{MD}_i \right)^2
\]

\[
= \left( \sum_{i=1,3,4,6} w_i \text{MD}_i - \sum_{i=1,3,4,6} b_i \text{MD}_i \right)^2 + \left( \sum_{i=2,5,7,8} w_i \text{MD}_i - \sum_{i=2,5,7,8} b_i \text{MD}_i \right)^2
\]

\[
= (3.56w_1 + 6.54w_3 + 10.23w_4 + 2.30w_6 - 3.4089)^2 + (7.48w_2 + 2.40w_5 + 9.12w_7 + 7.96w_8 - 2.5508)^2
\]

The objective function is then:

\[
f (w) = \frac{\varphi_{\text{AS}}}{2} \mathcal{R}_{\text{AS}} (w \mid b) + \frac{\varphi_{\text{MD}}}{2} \mathcal{R}_{\text{MD}} (w \mid b)
\]

\(^{30}\)We verify that 3.4089 + 2.5508 = 5.9597 years.
The optimal solution is equal to:

\[ w^* = (16.31\%, 18.44\%, 17.70\%, 13.82\%, 11.67\%, 11.18\%, 0\%, 10.88\%) \]

The risk metrics are:

\[
\begin{align*}
\mathbb{E}_2 (w^*) &= 5.93 \text{ years} \\
\text{DTS} (w^*) &= 210.73 \text{ bps} \\
\sigma_{\text{AS}} (w^* | b) &= 11.30\% \\
\sigma_{\text{MD}} (w^* | b) &= 0.03 \text{ years} \\
\sigma_{\text{DTS}} (w^* | b) &= 3.70 \text{ bps}
\end{align*}
\]
Question (e)

We consider the following optimization problem:

\[ w^* = \arg \min \frac{1}{2} \mathcal{R} (w \mid b) \]
\[
\text{s.t. } \begin{cases} 
\sum_{i=1}^{n} w_i = 1 \\
\mathcal{C} \mathcal{I} (w) \leq (1 - \mathcal{R}) \mathcal{C} \mathcal{I} (b) \\
0 \leq w_i \leq 1 
\end{cases}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( w^* \). Compute \( \text{MD} (w^*) \), \( \text{DTS} (w^*) \), \( \sigma_{\text{AS}} (w^* \mid b) \), \( \sigma_{\text{MD}} (w^* \mid b) \) and \( \sigma_{\text{DTS}} (w^* \mid b) \).
We have\textsuperscript{31}:

\[
\mathcal{R}_{\text{DTS}} (w \mid b) = \left( \sum_{i=1,3,4,6} (w_i - b_i) \text{DTS}_i \right)^2 + \left( \sum_{i=2,5,7,8} (w_i - b_i) \text{DTS}_i \right)^2
\]

\[
= (103w_1 + 75w_3 + 796w_4 + 45w_6 - 142.49)^2 + (155w_2 + 89w_5 + 320w_7 + 245w_8 - 68.24)^2
\]

The objective function is then:

\[
f (w) = \frac{\varphi_{\text{AS}}}{2} \mathcal{R}_{\text{AS}} (w \mid b) + \frac{\varphi_{\text{MD}}}{2} \mathcal{R}_{\text{MD}} (w \mid b) + \frac{\varphi_{\text{DTS}}}{2} \mathcal{R}_{\text{DTS}} (w \mid b)
\]

\textsuperscript{31}We verify that 142.49 + 68.24 = 210.73 bps.
The optimal solution is equal to:

\[ w^* = (16.98\%, 17.21\%, 18.26\%, 13.45\%, 12.10\%, 9.46\%, 0\%, 12.55\%) \]

The risk metrics are:

\[
\begin{cases}
MD (w^*) = 5.97 \text{ years} \\
DTS (w^*) = 210.68 \text{ bps} \\
\sigma_{AS} (w^* \mid b) = 11.94\% \\
\sigma_{MD} (w^* \mid b) = 0.03 \text{ years} \\
\sigma_{DTS} (w^* \mid b) = 0.06 \text{ bps}
\end{cases}
\]
Question (f)

Comment on the results obtained in Questions 3.(c), 3.(d) and 3.(e).
### Table 131: Solution of the bond optimization problem (scope $\mathbf{SC}_{1-3}$)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Benchmark</th>
<th>3.(c)</th>
<th>3.(d)</th>
<th>3.(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>17.3049</td>
<td>16.3102</td>
<td>16.9797</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.4119</td>
<td>18.4420</td>
<td>17.2101</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>20.9523</td>
<td>17.6993</td>
<td>18.2582</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>14.4113</td>
<td>13.8195</td>
<td>13.4494</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.0239</td>
<td>11.6729</td>
<td>12.1008</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>11.0881</td>
<td>11.1792</td>
<td>9.4553</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>8.8075</td>
<td>10.8769</td>
<td>12.5464</td>
</tr>
<tr>
<td>$\text{MD}(w)$</td>
<td>5.9597</td>
<td>5.9597</td>
<td>5.9344</td>
<td>5.9683</td>
</tr>
<tr>
<td>$\text{DTS}(w)$</td>
<td>210.7300</td>
<td>210.7300</td>
<td>210.7300</td>
<td>210.6791</td>
</tr>
<tr>
<td>$\sigma_{\text{AS}}(w</td>
<td>b)$</td>
<td>0.0000</td>
<td>10.5726</td>
<td>11.3004</td>
</tr>
<tr>
<td>$\sigma_{\text{MD}}(w</td>
<td>b)$</td>
<td>0.0000</td>
<td>0.4338</td>
<td>0.0254</td>
</tr>
<tr>
<td>$\sigma_{\text{DTS}}(w</td>
<td>b)$</td>
<td>0.0000</td>
<td>15.2056</td>
<td>3.7018</td>
</tr>
<tr>
<td>$\mathbf{CI}(w)$</td>
<td>76.9427</td>
<td>38.4713</td>
<td>38.4713</td>
<td>38.4713</td>
</tr>
</tbody>
</table>
Question (g)
How to find the previous solution of Question 3.(e) using a QP solver?
The goal is to write the objective function into a quadratic function:

\[
f(w) = \frac{\varphi_{AS}}{2} R_{AS}(w \mid b) + \frac{\varphi_{MD}}{2} R_{MD}(w \mid b) + \frac{\varphi_{DTS}}{2} R_{DTS}(w \mid b)
\]

\[
= \frac{1}{2} w^\top Q(b) w - w^\top R(b) + c(b)
\]

where:

\[
R_{AS}(w \mid b) = (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 + (w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2
\]

\[
R_{MD}(w \mid b) = (3.56w_1 + 6.54w_3 + 10.23w_4 + 2.30w_6 - 3.4089)^2 + (7.48w_2 + 2.40w_5 + 9.12w_7 + 7.96w_8 - 2.5508)^2
\]

\[
R_{DTS}(w \mid b) = (103w_1 + 75w_3 + 796w_4 + 45w_6 - 142.49)^2 + (155w_2 + 89w_5 + 320w_7 + 245w_8 - 68.24)^2
\]
We use the analytical approach which is described in Section 11.1.2 on pages 332-339. Moreover, we rearrange the universe such that the first fourth assets belong to the first sector and the last fourth assets belong to the second sector. In this case, we have:

\[
W = \begin{pmatrix}
W_1, W_3, W_4, W_6, \overbrace{W_2, W_5, W_7, W_8}^\text{Sector}_2, \overbrace{W_1, W_3, W_4, W_6}^\text{Sector}_1
\end{pmatrix}
\]
The matrix $Q(b)$ is block-diagonal:

$$Q(b) = \begin{pmatrix} Q_1 & 0_{4,4} \\ 0_{4,4} & Q_2 \end{pmatrix}$$

where the matrices $Q_1$ and $Q_2$ are equal to:

$$Q_1 = \begin{pmatrix} 11025.8400 & 8307.0600 & 82898.4700 & 4839.7000 \\ 8307.0600 & 6794.2900 & 61372.6050 & 3751.0500 \\ 82898.4700 & 61372.6050 & 636332.3225 & 36408.2250 \\ 4839.7000 & 3751.0500 & 36408.2250 & 2257.2500 \end{pmatrix}$$

and:

$$Q_2 = \begin{pmatrix} 25523.7600 & 14243.8000 & 51305.4400 & 39463.5200 \\ 14243.8000 & 8165.0000 & 29027.2000 & 22282.6000 \\ 51305.4400 & 29027.2000 & 104579.3600 & 80214.8800 \\ 39463.5200 & 22282.6000 & 80214.8800 & 61709.0400 \end{pmatrix}$$
The vector $R(b)$ is defined as follows:

$$R(b) = \begin{pmatrix} 15001.8621 \\ 11261.1051 \\ 114306.8662 \\ 6616.0617 \\ 11073.1996 \\ 6237.4080 \\ 22424.3824 \\ 17230.4092 \end{pmatrix}$$

Finally, the value of $c(b)$ is equal to:

$$c(b) = 12714.3386$$
Using a QP solver, we obtain the following numerical solution:

\[
\begin{pmatrix}
  w_1 \\
  w_3 \\
  w_4 \\
  w_6 \\
  w_2 \\
  w_5 \\
  w_7 \\
  w_8 \\
\end{pmatrix}
= \begin{pmatrix}
  16.9796 \\
  18.2582 \\
  13.4494 \\
  9.4553  \\
  17.2102 \\
  12.1009 \\
  0.0000  \\
  12.5464 \\
\end{pmatrix} \times 10^{-2}
\]

We observe some small differences (after the fifth digit) because the QP solver is more efficient than a traditional nonlinear solver.
Question 4

We consider a variant of Question 3 and assume that the synthetic risk measure is:

\[ \mathcal{D}(w \mid b) = \varphi_{AS}\mathcal{D}_{AS}(w \mid b) + \varphi_{MD}\mathcal{D}_{MD}(w \mid b) + \varphi_{DTS}\mathcal{D}_{DTS}(w \mid b) \]

where:

\[ \mathcal{D}_{AS}(w \mid b) = \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| \]

\[ \mathcal{D}_{MD}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \right| \]

\[ \mathcal{D}_{DTS}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \right| \]
Question (a)
Define the corresponding optimization problem when the objective is to minimize the active risk and reduce the carbon intensity of the benchmark by $\mathcal{R}$.
The optimization problem is:

\[ w^* = \arg \min_D (w \mid b) \]

s.t. \[
\begin{align*}
\mathbf{1}_8^T w &= 1 \\
\mathbf{C} \mathbf{I}^T w &\leq (1 - \mathbf{R}) \mathbf{C} \mathbf{I} (b) \\
\mathbf{0}_8 &\leq w \leq \mathbf{1}_8
\end{align*}
\]
Question (b)

Give the LP formulation of the optimization problem.
We use the absolute value trick and obtain the following optimization problem:

$$
w^* = \arg \min \frac{1}{2} \varphi_{AS} \sum_{i=1}^{8} \tau_{i,w} + \varphi_{MD} \sum_{j=1}^{2} \tau_{j,MD} + \varphi_{DTS} \sum_{j=1}^{2} \tau_{j,DTS}
$$

subject to

$$
\begin{align*}
1_8^\top w &= 1 \\
0_8 &\leq w \leq 1_8 \\
CI^\top w &\leq (1 - R) CI (b) \\
|w_i - b_i| &\leq \tau_{i,w} \\
|\sum_{i \in \text{sector}_j} (w_i - b_i) \text{MD}_i| &\leq \tau_{j,MD} \\
|\sum_{i \in \text{sector}_j} (w_i - b_i) \text{DTS}_i| &\leq \tau_{j,DTS} \\
\tau_{i,w} &\geq 0, \tau_{j,MD} \geq 0, \tau_{j,DTS} \geq 0
\end{align*}
$$
We can now formulate this problem as a standard LP problem:

\[
\begin{align*}
x^* &= \arg\min \quad c^\top x \\
\text{s.t.} \quad &\begin{cases} 
Ax = B \\
Cx \leq D \\
x^- \leq x \leq x^+
\end{cases}
\end{align*}
\]

where \( x \) is the \( 20 \times 1 \) vector defined as follows:

\[
x = \begin{pmatrix}
w \\
\tau_w \\
\tau_{MD} \\
\tau_{DTS}
\end{pmatrix}
\]
The $20 \times 1$ vector $c$ is equal to:

$$c = \begin{pmatrix} 0_8 \\ \frac{1}{2} \varphi_{AS} 1_8 \\ \varphi_{MD} 1_2 \\ \varphi_{DTS} 1_2 \end{pmatrix}$$

The equality constraint is defined by $A = \left( 1_8^T \ 0_8^T \ 0_2^T \ 0_2^T \right)$ and $B = 1$. The bounds are $x^- = 0_{20}$ and $x^+ = \infty \cdot 1_{20}$. 
For the inequality constraint, we have\(^{32}\):

\[
Cx \leq D \iff \begin{pmatrix}
I_8 & -I_8 & 0_{8,2} & 0_{8,2} \\
-I_8 & -I_8 & 0_{8,2} & 0_{8,2} \\
C_{MD} & 0_{2,8} & -I_2 & 0_{2,2} \\
-C_{MD} & 0_{2,8} & -I_2 & 0_{2,2} \\
C_{DTS} & 0_{2,8} & 0_{2,2} & -I_2 \\
-C_{DTS} & 0_{2,8} & 0_{2,2} & -I_2 \\
CI^\top & 0_{1,8} & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
b \\
-b \\
MD^* \\
-MD^* \\
DTS^* \\
-DTS^* \\
(1 - R)CI(b) \\
\end{pmatrix} \leq \begin{pmatrix}
b \\
-b \\
MD^* \\
-MD^* \\
DTS^* \\
-DTS^* \\
(1 - R) CI(b) \\
\end{pmatrix}
\]

where:

\[
C_{MD} = \begin{pmatrix}
3.56 & 0.00 & 6.54 & 10.23 & 0.00 & 2.30 & 0.00 & 0.00 \\
0.00 & 7.48 & 0.00 & 0.00 & 2.40 & 0.00 & 9.12 & 7.96 \\
\end{pmatrix}
\]

and:

\[
C_{DTS} = \begin{pmatrix}
103 & 0 & 75 & 796 & 0 & 45 & 0 & 0 \\
0 & 155 & 0 & 0 & 89 & 0 & 320 & 245 \\
\end{pmatrix}
\]

The \(2 \times 1\) vectors \(MD^*\) and \(DTS^*\) are respectively equal to \((3.4089, 2.5508)\) and \((142.49, 68.24)\).\(^ {32}\)
Question (c)

Find the optimal portfolio when $R$ is set to 50%. Compare the solution with this obtained in Question 3.(e).
We obtain the following solution:

\[ w^* = (18.7360, 15.8657, 17.8575, 13.2589, 11, 9.4622, 0, 13.8196) \times 10^{-2} \]

\[ \tau^*_w = (3.2640, 3.1343, 0.8575, 0.2589, 0, 1.4622, 6, 9.8196) \times 10^{-2} \]

\[ \tau_{MD} = (0, 0) \]

\[ \tau_{DTS} = (0, 0) \]
Table 132: Solution of the bond optimization problem (scope $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Benchmark</th>
<th>3.(e)</th>
<th>4.(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>16.9796</td>
<td>18.7360</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.2102</td>
<td>15.8657</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>18.2582</td>
<td>17.8575</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>13.4494</td>
<td>13.2589</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>12.1009</td>
<td>11.0000</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>9.4553</td>
<td>9.4622</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>12.5464</td>
<td>13.8196</td>
</tr>
<tr>
<td>MD($w$)</td>
<td>5.9597</td>
<td>5.9683</td>
<td>5.9597</td>
</tr>
<tr>
<td>DTS($w$)</td>
<td>210.7300</td>
<td>210.6791</td>
<td>210.7300</td>
</tr>
<tr>
<td>$\sigma_{AS}(w \mid b)$</td>
<td>0.0000</td>
<td>11.9400</td>
<td>12.4837</td>
</tr>
<tr>
<td>$\sigma_{MD}(w \mid b)$</td>
<td>0.0000</td>
<td>0.0308</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_{DTS}(w \mid b)$</td>
<td>0.0000</td>
<td>0.0561</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{D}_{AS}(w \mid b)$</td>
<td>0.0000</td>
<td>25.6203</td>
<td>24.7964</td>
</tr>
<tr>
<td>$\mathcal{D}_{MD}(w \mid b)$</td>
<td>0.0000</td>
<td>0.0426</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{D}_{DTS}(w \mid b)$</td>
<td>0.0000</td>
<td>0.0608</td>
<td>0.0000</td>
</tr>
<tr>
<td>CI($w$)</td>
<td>76.9427</td>
<td>38.4713</td>
<td>38.4713</td>
</tr>
</tbody>
</table>
In Table 132, we compare the two solutions\textsuperscript{33}. They are very close. In fact, we notice that the LP solution matches perfectly the MD and DTS constraints, but has a higher AS risk $\sigma_{AS} (w | b)$. If we note the two solutions $w^* (\mathcal{L}_1)$ and $w^* (\mathcal{L}_2)$, we have:

\[
\begin{align*}
R (w^* (\mathcal{L}_2) | b) &= 1.4524 < R (w^* (\mathcal{L}_1) | b) = 1.5584 \\
D (w^* (\mathcal{L}_2) | b) &= 13.9366 > D (w^* (\mathcal{L}_1) | b) = 12.3982
\end{align*}
\]

There is a trade-off between the $\mathcal{L}_1$- and $\mathcal{L}_2$-norm risk measures. This is why we cannot say that one solution dominates the other.

\textsuperscript{33}The units are the following: % for the weights $w_i$, and the active share metrics $\sigma_{AS} (w | b)$ and $D_{AS} (w | b)$; years for the modified duration metrics $MD (w)$, $\sigma_{MD} (w | b)$ and $D_{MD} (w | b)$; bps for the duration-times-spread metrics $DTS (w)$, $\sigma_{DTS} (w | b)$ and $D_{DTS} (w | b)$; $tCO_2e/\$ mn for the carbon intensity $DTS (w)$. 
Course 2022-2023 in Sustainable Finance
Lecture 12. Physical Risk Modeling

Thierry Roncalli

* Amundi Asset Management
* University of Paris-Saclay

March 2023

34 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
“Responsible investors have paid more attention to the transition risk than to the physical risk. However, recent events show that physical risk is also a big concern. It corresponds to the financial losses that really come from climate change, and not from the adaptation of the economy to prevent them. It includes droughts, floods, storms, etc.” (Le Guenedal and Roncalli, 2022).
<table>
<thead>
<tr>
<th>Definition</th>
<th>Chronic vs. acute risk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Statistical modeling of physical risk</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Chronic risk**
<table>
<thead>
<tr>
<th>Acute risk</th>
</tr>
</thead>
</table>

**Definition**
Statistical modeling of physical risk

**Applications**

**Chronic vs. acute risk**
Figure 228: Physical risk modeling
Statistical modeling of physical risk

Climate variable and data source

- The climate data source is the set \( \Theta_s = \{\theta (\lambda, \varphi, z, t)\} \)
- \( \theta = (\theta_1, \ldots, \theta_k) \) is a vector of \( k \) climate variables such as temperature, pressure or wind speed
- Each variable \( \theta_k \) has four coordinates:
  1. Latitude \( \lambda \)
  2. Longitude \( \varphi \)
  3. Height (or altitude) \( z \)
  4. Time \( t \)
- Three types of sources:
  1. Meteorological records
  2. Reanalysis
  3. Historical simulations by a climate model
Figure 229: Slice* of wind speed (07/11/2013, tropical cyclone Haiyan)

Source: Modern-Era Retrospective analysis for Research and Applications, Version 2, Global Modeling and Assimilation Office, NASA.

* This is a slice of the MERRA-2 reanalysis at a height of 10 meters on 7th November 2013. The red dot is the location of the eye of the tropical cyclone Haiyan, which affected more than 10 million people in the Philippines.
We first have define the sensitivity of the intensity of extreme events to climate change.

Let $E[I(\Theta_s(C))]$ be the expected intensity of the event in the scenario associated with the GHG concentration $C$.

The sensitivity of the event is equal to:

$$\Delta I(C) = E[I(\Theta_s(C))] - I(\Theta_s(C_0))$$

where $I(\Theta_s(C_0))$ is the current intensity or the reference intensity in a scenario where climate objectives are met.

For instance, we know that the maximum wind of tropical cyclones increases by more than 10% in scenarios with a high GHG concentration.
The asset value of the portfolio can then be written as:

$$\Psi(t) = \sum_{j=1}^{n} x_j \Psi_j(\lambda, \varphi, t)$$

where $\Psi_j(\lambda, \varphi, t)$ is the geolocated asset value estimated at time $t$ and $x_j$ is the weight of asset $j$ in the portfolio.

This requires the geolocation of the portfolio.
Figure 230: Geolocation of world power plants by energy source

Source: Global Power Database version 1.3 (June 2021).
Vulnerability

- The damage function $\Omega_j(I) \in [0, 1]$ is the fraction of property loss with respect to the intensity
- It is generally calibrated on past damages (insurance claims, economic loss, etc.) and disasters
The physical risk implied by the concentration scenario \( C \) is equal to:

\[
\Delta \text{Loss} (t, C) = \beta \cdot \mathcal{D} \mathcal{D} (t, C) = \beta \sum_{j=1}^{n} x_j \Psi_j (\lambda, \varphi, t) \Omega_j (\Delta I (t, C))
\]

- \( \Delta \text{Loss} (t, C) \) is the relative loss due to the events on the portfolio.
- \( \beta \) is the transmission factor of the direct damage \( \mathcal{D} \mathcal{D} (t, C) \) on the underlying to the loss of financial value in the investment portfolio.
- For example, if the facilities of an energy producer are damaged at 50%, the securities issued by this company will be impacted at 50% \( \times \beta \).
Climate hazard location
<table>
<thead>
<tr>
<th>Definition</th>
<th>General framework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical modeling of physical risk</td>
<td>Geolocation</td>
</tr>
</tbody>
</table>

**Asset location**
Applications
Tropical cyclone damage modeling


Two main modules:
- Simulation and generation of tropical cyclones under a given climate change scenario
- Geolocation of assets, damage modeling and loss estimation
**Applications**

Tropical cyclone damage modeling

**Figure 231:** What is a cyclone?

Figure 232: Modeling framework (Module 1)

Source: Le Guenedal et al. (2021).
Figure 233: Sample of storms (ERA-5 climate data)

Source: Le Guenedal et al. (2021).
Physics of cyclones

- Wind pressure relationship (Bloemendaal et al., 2020):
  \[ V = a(P_{env} - P_c)^b \]

- Maximum potential intensity (Holland, 1997; Emanuel, 1999):
  \[ MPI = f(y, SST, T_{tropo}, MSLP, RH, P_c) \]

- Maximum pressure drop (Bloemendaal et al., 2020):
  \[ MPD \sim P_{env} - P_c = A + Be^{C(SST-T_0)} \quad T_0 = 30^\circ C \]

- Pressure incremental variation (James and Mason, 2005):
  \[ \Delta_t P_c(t) = c_0 + c_1 \Delta_t P_c(t - 1) + c_2 e^{-c_3(P_c(t) - MPI(x,y,t))} + \epsilon(P_c, t) \]
  \[ \epsilon(P_c, t) \sim \mathcal{N}(0, \sigma_{P_c}^2) \]

- Decay function (Kaplan and DeMaria, 1995):
  \[ V(t_L) = V_b + (R \cdot V_0 - V_b)e^{-\alpha t} - C \]
  where \( C = m \left( \ln \frac{D}{D_0} \right) + b, \quad m = \tilde{c}_1 t_L (t_{0,L} - t_L) \) and \( b = d_1 t_L (t_{0,L} - t_L) \)
The cyclone simulation database must be sensitive to the climate change scenario.
Figure 235: GDP decomposition of North America (or physical asset values) (Litpop database)

Source: Le Guenedal et al. (2021).
Applications
Tropical cyclone damage modeling

**Figure 236: The case of Katrina (2005)**

![Map showing the path of Katrina (2005) with physical asset values and wind speed.](map.png)

*Physical asset values (mUSD)*

- 148.41316
- 0.13534
- 0.00012
- 0.00000

*Katrina (2005) wind speed*

- 150
- 125
- 100
- 75
- 50
- 25

Figure 237: The grid approach

Source: Le Guenedal et al. (2021).
Applications

Tropical cyclone damage modeling

Figure 238: Average global losses

Source: Le Guenedal et al. (2021).
### Applications

#### Tropical cyclone damage modeling

<table>
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<th>SSP2</th>
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<th>RCP 4.5</th>
<th>RCP 8.5</th>
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<td>+153%</td>
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<td>+360%</td>
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</tr>
</tbody>
</table>

**Remark**

- There are simulations that lead to annual losses that easily exceed 2 or 3 trillion dollars per year
- 1 Katrina = $180 billion in 2005

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**Table 133:** Average increase of financial losses per year

Source: Le Guenedal et al. (2021).
Floods
Drought
Water stress
Extreme heat
Wildfire
Course 2022-2023 in Sustainable Finance
Lecture 13. Climate Stress Testing and Risk Management

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