Course 2023-2024 in Sustainable Finance & Climate Change

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2024

1The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Overview
The objective of this course is to understand the concepts of sustainable finance from the viewpoint of asset owners and managers.

Textbook

[Image of the Handbook of Sustainable Finance by Thierry Roncalli]
General information

Textbook

SSRN: https://ssrn.com/abstract=4277875
ResearchGate: https://www.researchgate.net/publication/365355205

Slides

SSRN: https://ssrn.com/abstract=4730853
ResearchGate: https://www.researchgate.net/publication/???
http://www.thierry-roncalli.com/SustainableFinanceCourse.html

Additional materials (\LaTeX + Figures + Matlab programs)

http://www.thierry-roncalli.com/SustainableFinanceCourse.html
Part 1. Introduction

1. Definition
   - Many words, one concept
   - Historical perspective
   - Extensive use of acronyms

2. ESG ecosystem
   - Many financial actors
   - Reporting frameworks
   - Regulatory framework

3. The Market of ESG Investing
   - ESG strategies
   - The market share of ESG investing
Part 2. ESG Scoring

4 Data and variables
   - Sovereign ESG data
   - Corporate ESG data

5 Scoring system
   - Tree-based scoring method
   - Other statistical methods
   - Performance evaluation criteria

6 Rating system
   - Definition
   - ESG rating process
   - Rating migration matrix
Part 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

7 Theoretical models
   - Modern portfolio theory
   - ESG risk premium
   - ESG efficient frontier

8 Empirical results
   - Equity markets
   - ESG and factor investing
   - Fixed-income markets

9 Cost of capital
   - Equities
   - Corporate bonds
   - Sovereign bonds
Part 3. Exercise — Equity Portfolio Optimization with ESG Scores

10 CAPM and implied expected returns

11 Mean-variance optimization with ESG scores

12 Benchmark with ESG scores
Part 4. Sustainable Financial Products

13 SRI investment funds
- Market
- Labels
- Regulation

14 Green and social bonds
- Green bonds
- Social bonds
- Other sustainability-related instruments

15 Sustainable real assets
Part 5. Impact Investing

- **Definition**
  - Motivations
  - Sustainable development goals (SDGs)
  - The challenge of reporting

- **Thematic funds**

- **Biodiversity risk**
Part 6. Engagement & Voting Policy

19 Active ownership
   - The various forms of active ownership
   - Impact of active ownership

20 ESG voting
   - Voting process
   - Proxy voting
   - Defining a voting policy

21 ESG voting statistics
   - Asset managers
   - Asset owners
Part 8. Awareness of Climate Change Impacts

22 Scientific evidence of global warming
  - Greenhouse effect
  - Global warming

23 From the Holocene to the Anthropocene?
  - Anthropocene
  - Geological history of the climate
  - Anthropogenic factors of climate change

24 The physics of climate change
  - Energy balance models
  - Climate sensitivity and feedback
  - Tipping points
Part 9. The Ecosystem of Climate Change

25 Scientists
- Intergovernmental Panel on Climate Change
- Climate research institutions
- Scientific journals

26 Conferences of the Parties
- Earth Summit & UNFCCC
- Kyoto Protocol
- Paris Climate Agreement

27 Regulation policies
- European Union
- United States
- Other countries
Part 10. Economic Models & Climate Change

Integrated assessment models
- The DICE model
- Social cost of carbon
- Other IAMs

Scenarios
- Climate scenarios
- Shared socioeconomic pathways
- NGFS scenarios

Environmentally-extended input-output model
- Input-output analysis
- Estimation of indirect emissions
- Taxation, pass-through and price dynamics
Part 11. Climate Risk Measures

31 Carbon footprint
   - Global warming potential
   - Carbon emissions
   - Carbon intensity

32 Dynamic risk measures
   - Carbon budget
   - Carbon trend
   - The PAC framework

33 Greenness measures
   - Green taxonomy
   - Green revenue share
   - Other greenness metrics
Part 12. Transition Risk Modeling

34 Regulation policy
- Economic theory of negative externalities
- Carbon pricing
- Emissions trading system

35 Equity valuation
- Carbon pricing
- Stranded assets

36 Financing and technological risks
- Technology
- Financing
Part 13. Climate Portfolio Construction

37 Portfolio optimization in practice
- Quadratic programming (QP) problem
- Equity portfolios
- Bond portfolios

38 Portfolio decarbonization
- Equity and bond portfolios
- Sector-specific constraints
- Empirical results

39 Net-zero investing
- Integrated approach
- Core satellite approach
Part 13. Exercise — Equity and Bond Portfolio Optimization with Green Preferences

40 Carbon intensity of the benchmark

41 Equity portfolios

42 Bond portfolios
   - $L_2$-norm risk measures
   - $L_1$-norm risk measures
Part 14. Physical Risk Modeling

43 Modeling of physical risk
- General circulation models
- Statistical models
- Geolocation

44 Extreme weather modeling
- Cyclones and hurricanes
- Floods
- Other physical risks

45 Impact of climate-related physical risks
- Agriculture and food security
- Insurance and economic costs
- Other risks
Part 15. Climate Stress Testing and Risk Management

46 Market risk
- Commodity market
- Stock market
- Climate value-at-risk

47 Credit risk
- Mortgage and loan portfolios
- Bond portfolios
- Capital requirements

48 Stress testing
- Corporates
- Banks
- Insurance companies
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- **Lecture 1: Introduction**
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
“Sustainable finance refers to the process of taking environmental, social and governance (ESG) considerations into account when making investment decisions in the financial sector, leading to more long-term investments in sustainable economic activities and projects. Environmental considerations might include climate change mitigation and adaptation, as well as the environment more broadly, for instance the preservation of biodiversity, pollution prevention and the circular economy. Social considerations could refer to issues of inequality, inclusiveness, labour relations, investment in human capital and communities, as well as human rights issues. The governance of public and private institutions — including management structures, employee relations and executive remuneration — plays a fundamental role in ensuring the inclusion of social and environmental considerations in the decision-making process.” (European Commission).
Many words, one concept
Responsible investment (RI)

Responsible investment is an approach to investing that explicitly recognizes the relevance to the investor of environmental, social and governance factors, as well as the long-term health of the market as a whole.

Sustainable investing (SI)

Sustainable investing is an investment approach that considers environmental, social and governance factors in portfolio selection.

Socially responsible investing (SRI)

SRI is an investment strategy that is considered socially responsible, because it invests in companies that have ethical practices.

Environmental, Social and Governance (ESG)

Environmental, Social, and Corporate Governance (ESG) refers to the factors that measure the sustainability of an investment.
Sustainable Investing
≈
Socially Responsible Investing (SRI)
≈
Environmental, Social, and Governance (ESG)

Remark
Blue Finance ⊂ Green Finance, Climate Finance ⊂ Sustainable Finance
Historical perspective

- Responsible investment (RI): 2000’s
- ESG investing (ESG): 2010’s
- Sustainable finance (SF): 2020’s

Why?
Historical perspective

- At the beginning, sustainable finance mainly concerned final investors and asset owners (ethics) ⇒ **responsible investment**
- Then, it gains momentum in asset management ⇒ **ESG investing**
- Finally, it spreads across all financial actors (e.g. issuers, banks, central banks, etc.) ⇒ **Sustainable finance**
Historical perspective

- In 1758, the Quaker Philadelphia yearly meeting prohibited its members from participating in the slave trade (buying or selling humans).
- The first SRI mutual fund (Pioneer Fund) was created in 1928 by Philip Carret for evangelical Protestants.
- During the Vietnam War, shareholders organized resolutions against the production of napalm and Agent Orange (e.g., Dow Chemical).
- Creation of the Pax World Fund in 1971 (mixing both financial and social criteria for the stock selection).
- Emergence of the concept of “sin stocks”

Faith-based investing = Do no harm
The Friedman-Freeman debate

- 1953: Development of corporate social responsibility (CSR)
- Friedman doctrine & the **shareholder theory**
  
  "There is one and only one social responsibility of business — to use its resources and engage in activities designed to increase its profits so long as it stays within the rules of the game, which is to say, engages in open and free competition without deception or fraud." (Friedman, 1962, 1970).

- Freeman doctrine & the **stakeholder theory** (shareholders vs. customers, suppliers, employees, local communities, government agencies, financiers, etc.)

  ⇒ Corporate social performance (CSP) & corporate financial performance (CFP).
UN Global Compact (GC, July 2000)

The 10 principles are:

- **Human rights**
  1. Businesses should support and respect the protection of internationally proclaimed human rights;
  2. Ensure that they are not complicit in human rights abuses.

- **Labor**
  3. Businesses should uphold the freedom of association and the effective recognition of the right to collective bargaining;
  4. The elimination of all forms of forced and compulsory labor;
  5. The effective abolition of child labor;
  6. The elimination of discrimination in respect of employment and occupation.

- **Environment**
  7. Businesses should support a precautionary approach to environmental challenges;
  8. Take initiatives to promote greater environmental responsibility;

- **Anti-corruption**
  10. Businesses should work against corruption in all its forms, including extortion and bribery.

From 2004 to 2008, the UN Global Compact, the IFC & the Swiss government sponsored a series of conferences “Who Cares Wins” for investment professionals ⇒ the term ESG was first coined in the 2004 WCW report.
In 1986, the US Congress passed the "Comprehensive Anti-Apartheid Act", which banned new investments in South Africa.

The Sudan Divestment Task Force (SDTF) was formed in 2005 to coordinate and provide resources for the Sudan divestment campaign in response to the genocide occurring in the Darfur region.

The US "Sudan Accountability and Divestment Act" came into force in December 2007.
The Intergovernmental Panel on Climate Change (IPCC) was established in 1988 by the World Meteorological Organization (WMO) and the United Nations Environment Programme (UNEP).

- Publication of AR1-AR6
- June 1992: Earth Summit in Rio de Janeiro (Brazil)
  - Convention on Biological Diversity (CBD)
  - UN Framework Convention on Climate Change (UNFCCC)
    - COP3: Kyoto Protocol in 1997
    - COP21: Paris Agreement in 2015
Figure 1: The raison d’être of ESG investing
A myriad of acronyms

How many acronyms do you know?
A myriad of acronyms

Many financial actors

ESG financial ecosystem

- Asset owners (pension funds, sovereign wealth funds (SWF), insurance and institutional investors, retail investors, etc.)
- Asset managers
- ESG rating agencies
- ESG index sponsors
- Banks
- ESG associations (GSIA, UNPRI, etc.)
- Regulators and international bodies (governments, financial and industry regulators, central banks, etc.)
- Issuers (equities, bonds, loans, etc.)
- Society and people

ESG Investing ⇔ ESG Financing (= Sustainable Finance)
The issuer point of view of ESG

Corporate financial performance (CFP)
- Friedman (1970)
- Shareholder theory
- Corporations have no social responsibility to the public or society
- Their only responsibility is to its shareholders (profit maximization)

Corporate social responsibility (CSR)
- Freeman (2010)
- Stakeholder theory
- Corporations create negative externalities
- They must have social and moral responsibilities
- Impact on the cost-of-capital and business risk
Sustainable investment forums

GSIA members

- The European Sustainable Investment Forum (Eurosif), http://www.eurosif.org
- Responsible Investment Association Australasia (RIAA), https://responsibleinvestment.org
- Responsible Investment Association Canada (RIA Canada), https://www.riacanada.ca
- UK Sustainable Investment & Finance Association (UKSIF), https://www.uksif.org
- The Forum for Sustainable & Responsible Investment (US SIF), https://www.ussif.org
- Dutch Association of Investors for Sustainable Development (VBDO), https://www.vbdo.nl/en/
- Japan Sustainable Investment Forum (JSIF), https://japansif.com/english
Sustainable investment forums

Figure 2: 2020 & 2022 GSIA reports
Initiatives

- Principles for responsible investment (PRI)
- Climate Action 100+
- Net zero alliances: (NZAOA, NZAM, PAII, NZBA, NZIA, etc) ⇒ GFANZ
Figure 3: Principles for Responsible Investment (PRI)

https://www.unpri.org
PRI (or UNPRI)

- Early 2005: UN Secretary-General Kofi Annan invited a group of the world’s largest institutional investors to join a process to develop the Principles for Responsible Investment
- April 2006: The Principles were launched at the New York Stock Exchange
- 6 ESG principles
- The 63 founding signatories are 32 asset owners\(^a\) and 31 asset managers\(^b\) and data providers\(^c\)

\(^a\) AP2, CDC, CDPQ, CalPERS, ERAFP, FRR, IFC, NZSF, NGPF, PGGM, UNJSPF, USS, etc.
\(^b\) Amundi (CAAM), Sumitomo Trust, BNP PAM, Mitsubishi Trust, Threadneedle, Aviva, Candriam, etc.
\(^c\) Trucost, Vigeo, etc.
Signatories’ commitment

“As institutional investors, we have a duty to act in the best long-term interests of our beneficiaries. In this fiduciary role, we believe that environmental, social, and corporate governance (ESG) issues can affect the performance of investment portfolios (to varying degrees across companies, sectors, regions, asset classes and through time). We also recognise that applying these Principles may better align investors with broader objectives of society. Therefore, where consistent with our fiduciary responsibilities, we commit to the following:

- Principle 1: We will incorporate ESG issues into investment analysis and decision-making processes.
- Principle 2: We will be active owners and incorporate ESG issues into our ownership policies and practices.
- Principle 3: We will seek appropriate disclosure on ESG issues by the entities in which we invest.
- Principle 4: We will promote acceptance and implementation of the Principles within the investment industry.
- Principle 5: We will work together to enhance our effectiveness in implementing the Principles.
- Principle 6: We will each report on our activities and progress towards implementing the Principles.

The Principles for Responsible Investment were developed by an international group of institutional investors reflecting the increasing relevance of environmental, social and corporate governance issues to investment practices. The process was convened by the United Nations Secretary-General. In signing the Principles, we as investors publicly commit to adopt and implement them, where consistent with our fiduciary responsibilities. We also commit to evaluate the effectiveness and improve the content of the Principles over time. We believe this will improve our ability to meet commitments to beneficiaries as well as better align our investment activities with the broader interests of society. We encourage other investors to adopt the Principles.”

Source: https://www.unpri.org
PRI

Figure 4: PRI Signatory growth

Source: https://www.unpri.org
Regulators: Who? Why?

Table 1: The supervision institutions in finance

<table>
<thead>
<tr>
<th></th>
<th>Banks</th>
<th>Insurers</th>
<th>Markets</th>
<th>All sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>BCBS</td>
<td>IAIS</td>
<td>IOSCO</td>
<td>FSB</td>
</tr>
<tr>
<td>EU</td>
<td>EBA/ECB</td>
<td>EIOPA</td>
<td>ESMA</td>
<td>ESFS</td>
</tr>
<tr>
<td>US</td>
<td>FDIC/FRB</td>
<td>FIO</td>
<td>SEC</td>
<td>FSOC</td>
</tr>
</tbody>
</table>

- Greenwashing
  - Explicit & deliberate greenwashing;
  - Unintentional greenwashing.

- Fiduciary duties
ESG regulations

**Figure 5**: Who will regulate ESG? — The regulators viewpoint (MSCI, 2022)

Figure 6: Who will regulate ESG? — The regulated viewpoint (MSCI, 2022)

Visit the MSCI website

https://www.msci.com/who-will-regulate-esg

and obtain the detailed list of regulations
by year, country, regulator, regulated investors, etc.
The example of central banks

Figure 7: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)

- Launched at the Paris One Planet Summit (OPS) on December 2017
- 8 founding members: Banco de Mexico, BoE, BdF, Dutch Central Bank, Buba, Swedish FSA, HKMA, MAS and PBOC
- As of March 19th 2021, the NGFS consists of 89 members (CBs, EBA, EIOPA, ESMA) and 13 observers (BCBS, IMF, IAIS, IOSCO)
The example of central banks

Go the NGFS website (https://www.ngfs.net) and download the NGFS climate scenarios: https://www.ngfs.net/en/publications/ngfs-climate-finance-research-portal

See also https://data.ene.iiasa.ac.at/ngfs (NGFS scenario explorer hosted by IIASA³)

³International Institute for Applied Systems Analysis
Rating agencies


- **Consolidation of the industry (2010-2020):** ISS ESG, Moody’s, MSCI, Refinitiv, Reprisk, S&P Global, Sustainalytics.
Rating agencies

Consolidation in the 2010s:

- Vigeo and Eiris merged in October 2015 to form Vigeo-Eiris (V.E), which is acquired by Moody’s in April 2019
- In September 2015 and March 2018, ISS acquired Ethix SRI Advisors and Oekom to form ISS ESG solutions (ISS-ethix, ISS-climate and ISS-oekom). In November 2020, ISS is majority owned by Deutsche Börse Group
- In February and November 2009, RiskMetrics acquired Innovest and KLD. RiskMetrics is bought by MSCI in 2010, which creates MSCI ESG Research LLC
- In September 2009, DSR and Jantzi Research merged to form Sustainalytics. In the 2010s, Sustainalytics acquired Responsible Research (Singapore), ESG Analytics (Switzerland), Solaron (India) and GES (Sweden). In April 2020, Sustainalytics becomes a wholly-owned subsidiary of Morningstar.
- S&P Global acquired Trucost in October 2016 and the ESG ratings business of RobecoSAM in November 2019
Rating agencies

- ESG data
- ESG scores and ratings
- ESG indices
Table 2: List of the main reporting frameworks

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Acronym</th>
<th>Name</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>GC</td>
<td>UN Global Compact Initiative</td>
<td>2000/2000</td>
</tr>
<tr>
<td></td>
<td>GRI</td>
<td>Global Reporting Initiative</td>
<td>1997/2000</td>
</tr>
<tr>
<td></td>
<td>IIRC</td>
<td>International Integrated Reporting Council</td>
<td>2010/2013</td>
</tr>
<tr>
<td></td>
<td>ISSB</td>
<td>International Sustainability Standards Board</td>
<td>2021/2023</td>
</tr>
<tr>
<td></td>
<td>SASB</td>
<td>Sustainability Accounting Standards Board</td>
<td>2011/2016</td>
</tr>
<tr>
<td></td>
<td>SDGs</td>
<td>UN Sustainable Development Goals</td>
<td>2015/2016</td>
</tr>
<tr>
<td>Climate</td>
<td>CDP</td>
<td>Carbon Disclosure Project</td>
<td>2000/2000</td>
</tr>
<tr>
<td></td>
<td>CDSB</td>
<td>Climate Disclosure Standards Board</td>
<td>2007/2015</td>
</tr>
<tr>
<td></td>
<td>PCAF</td>
<td>Partnership for Carbon Accounting Financials</td>
<td>2019/2020</td>
</tr>
<tr>
<td></td>
<td>SBTi</td>
<td>Science Based Targets initiative</td>
<td>2015/2015</td>
</tr>
<tr>
<td></td>
<td>TCFD</td>
<td>Task Force on Climate-Related Financial Disclosures</td>
<td>2015/2017</td>
</tr>
</tbody>
</table>
On 26 June 2023, ISSB published the two IFRS standards that will take effect in January 2024:

- IFRS S1 general requirements for disclosure of sustainability-related financial information
- IFRS S2 climate-related disclosures
Sustainable Development Goals

Figure 8: The SDGs icons

Source: https://sdgs.un.org/goals#icons.
## Sustainable Development Goals

### Table 3: The 17 SDGs

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Description</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No poverty</td>
<td>End poverty in all its forms everywhere</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Zero hunger</td>
<td>End hunger, achieve food security and improved nutrition and promote sustainable agriculture</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Good health and well-being</td>
<td>Ensure healthy lives and promote well-being for all at all ages</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Quality education</td>
<td>Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gender equality</td>
<td>Achieve gender equality and empower all women and girls</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Clean water and sanitation</td>
<td>Ensure availability and sustainable management of water and sanitation for all</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Affordable and clean energy</td>
<td>Ensure access to affordable, reliable, sustainable and modern energy for all</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Decent work and economic growth</td>
<td>Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Industry, innovation and infrastructure</td>
<td>Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

## Sustainable Development Goals

### Table 4: The 17 SDGs

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Description</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Reduced inequality</td>
<td>Reduce inequality within and among countries</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>11</td>
<td>Sustainable cities and communities</td>
<td>Make cities and human settlements inclusive, safe, resilient and sustainable</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Responsible consumption and production</td>
<td>Ensure sustainable consumption and production patterns</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>13</td>
<td>Climate action</td>
<td>Take urgent action to combat climate change and its impacts</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Life below water</td>
<td>Conserve and sustainably use the oceans, seas and marine resources for sustainable development</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Life on land</td>
<td>Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Peace, justice, and strong institutions</td>
<td>Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Partnerships for the goals</td>
<td>Strengthen the means of implementation and revitalize the Global Partnership for Sustainable Development</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

The GHG Protocol corporate standard classifies a company’s greenhouse gas emissions in three scopes⁴:

- **Scope 1**: Direct GHG emissions (○)
- **Scope 2**: Consumption of purchased energy (○○)
- **Scope 3**: Other indirect GHG emissions (●●)
  - **Scope 3 upstream**: emissions associated to the supply side
    1. First tier direct (●)
    2. Tier 2 and 3 suppliers (●●)
  - **Scope 3 downstream**: emissions associated with the product sold by the entity
    1. Use of the product (●●●)
    2. Waste disposal & recycling (●●●●)

---

⁴Measurement robustness: from ○○○○ (very high) to ●●●● (very low)
Carbon Disclosure Project (CDP)

Each year, CDP sends a questionnaire to organizations and collects information on three environmental dimensions:

1. Climate change (based on the GHG Protocol)
2. Forest management
3. Water security
## Table 5: Examples of 2019 carbon emissions and intensity

<table>
<thead>
<tr>
<th>Company</th>
<th>SC1</th>
<th>SC2</th>
<th>SC3up</th>
<th>SC3down</th>
<th>Revenue (in $ mn)</th>
<th>SC1</th>
<th>SC2</th>
<th>SC3up</th>
<th>SC3down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon</td>
<td>5760000</td>
<td>5500000</td>
<td>20054722</td>
<td>10438551</td>
<td>280522</td>
<td>20.5</td>
<td>19.6</td>
<td>71.5</td>
<td>37.2</td>
</tr>
<tr>
<td>Apple</td>
<td>50549</td>
<td>862127</td>
<td>27624282</td>
<td>5470771</td>
<td>260174</td>
<td>0.2</td>
<td>3.3</td>
<td>106.2</td>
<td>21.0</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>64829</td>
<td>280789</td>
<td>1923307</td>
<td>1884</td>
<td>78244</td>
<td>0.8</td>
<td>3.6</td>
<td>24.6</td>
<td>0.0</td>
</tr>
<tr>
<td>BP</td>
<td>49199999</td>
<td>5200000</td>
<td>103840194</td>
<td>582639687</td>
<td>276850</td>
<td>177.7</td>
<td>18.8</td>
<td>375.1</td>
<td>2104.5</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>9050000</td>
<td>9260000</td>
<td>15197607</td>
<td>401993744</td>
<td>53800</td>
<td>16.8</td>
<td>17.2</td>
<td>282.5</td>
<td>7472.0</td>
</tr>
<tr>
<td>Danone</td>
<td>722122</td>
<td>944877</td>
<td>28969780</td>
<td>4464773</td>
<td>28308</td>
<td>25.5</td>
<td>33.4</td>
<td>1023.4</td>
<td>157.7</td>
</tr>
<tr>
<td>Exxon</td>
<td>111000000</td>
<td>9000000</td>
<td>107282831</td>
<td>594131943</td>
<td>255583</td>
<td>434.3</td>
<td>35.2</td>
<td>419.8</td>
<td>2324.6</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>81655</td>
<td>692299</td>
<td>3101582</td>
<td>15448469</td>
<td>115627</td>
<td>0.7</td>
<td>6.0</td>
<td>26.8</td>
<td>133.6</td>
</tr>
<tr>
<td>LVMH</td>
<td>67613</td>
<td>262609</td>
<td>11853749</td>
<td>942520</td>
<td>60083</td>
<td>1.1</td>
<td>4.4</td>
<td>197.3</td>
<td>15.7</td>
</tr>
<tr>
<td>Microsoft</td>
<td>113414</td>
<td>3556553</td>
<td>5977488</td>
<td>4003770</td>
<td>125843</td>
<td>0.9</td>
<td>28.3</td>
<td>47.5</td>
<td>31.8</td>
</tr>
<tr>
<td>Nestle</td>
<td>3291303</td>
<td>3206495</td>
<td>61262078</td>
<td>33900606</td>
<td>93153</td>
<td>35.3</td>
<td>34.4</td>
<td>657.6</td>
<td>363.9</td>
</tr>
<tr>
<td>Pfizer</td>
<td>734638</td>
<td>762840</td>
<td>4667225</td>
<td>133468</td>
<td>51750</td>
<td>14.2</td>
<td>14.7</td>
<td>90.2</td>
<td>2.6</td>
</tr>
<tr>
<td>Samsung Electronics</td>
<td>5067000</td>
<td>10998000</td>
<td>33554245</td>
<td>60978947</td>
<td>197733</td>
<td>25.6</td>
<td>55.6</td>
<td>169.7</td>
<td>308.4</td>
</tr>
<tr>
<td>Volkswagen</td>
<td>4494066</td>
<td>5973894</td>
<td>65335372</td>
<td>354913446</td>
<td>282817</td>
<td>15.9</td>
<td>21.1</td>
<td>231.0</td>
<td>1254.9</td>
</tr>
<tr>
<td>Walmart</td>
<td>6101641</td>
<td>13057352</td>
<td>40651079</td>
<td>32346229</td>
<td>514405</td>
<td>11.9</td>
<td>25.4</td>
<td>79.0</td>
<td>62.9</td>
</tr>
</tbody>
</table>

Source: Trucost (2022) & Authors' calculations.
- TCFD = Task Force on Climate Related Financial Disclosures
- TCFD is established by the FSB in 2015 and chaired by Michael Bloomberg
- TCFD ⇒ ISSB
Table 6: The 11 recommended disclosures (TCFD, 2017)

<table>
<thead>
<tr>
<th>Recommendation</th>
<th>#</th>
<th>Recommended Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governance</td>
<td>1</td>
<td>Board oversight</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Management’s role</td>
</tr>
<tr>
<td>Strategy</td>
<td>3</td>
<td>Risks and opportunities</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Impact on organization</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Resilience of strategy</td>
</tr>
<tr>
<td>Risk management</td>
<td>6</td>
<td>Risk ID and assessment processes</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Risk management processes</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Integration into overall risk management</td>
</tr>
<tr>
<td>Metrics and targets</td>
<td>9</td>
<td>Climate-related metrics</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Scope 1, 2, 3 GHG emissions</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Climate-related targets</td>
</tr>
</tbody>
</table>

Examples of recommended metrics

- GHG emissions (absolute scope 1, scope 2, and scope 3 GHG emissions; financed emissions by asset class; weighted average carbon intensity)
- Transition risks (volume of real estate collaterals highly exposed to transition risk; concentration of credit exposure to carbon-related assets; percent of revenue from coal mining)
- Physical risks (number and value of mortgage loans in 100-year flood zones; proportion of real assets exposed to 1:100 or 1:200 climate-related hazards)
- Climate-related opportunities (proportion of green buildings, green revenues)
- Capital deployment (green CAPEX)
- Internal carbon prices (internal carbon price, shadow carbon price)
- Remuneration
Figure 9: Examples of TCFD reports

Amundi (2021)

BlackRock (2021)

Engie (2022)

Sanofi (2021)
Regulatory framework

**Figure 10:** Total number of ESG regulations

Regulatory framework

**Figure 11: Number of ESG regulations per region**

The European Union

- The action plan on sustainable finance (May 2018)
- The European Green Deal (December 2019)
- The Fit-for-55 package (July 2021)
- The REPowerEU plan or energy security package (May 2022)
European Union

- EU taxonomy regulation
- Climate benchmarks (PAB)
- Sustainable finance disclosure regulation (SFDR)
- MiFID II & IDD
- Corporate sustainability reporting directive (CSRD)
**Figure 12: Sustainable finance — implementation timeline**

- **2021**
  - 1 Jan '22 - 31 Dec '22: Financial undertakings disclose proportion of assets exposed to taxonomy non-eligible and eligible economic activities under Art 8 TR DA.
  - 30 Jun '21: Large FMPs must comply with Art 4 SFDR - transparency of sustainability impacts at entity level (can no longer ‘explain’).
  - 1 Aug '22: Sustainability related provisions under UCITS and AIFMD DAs apply.
  - 2 Aug '22: Sustainability related provisions under MiFID and IDD DAs apply.
  - 10 Sep '22: ESAs to take stock of voluntary disclosures under SFDR (to be submitted every year).

- **2022**
  - 1 Jan '22 - 31 Dec '23: Financial undertakings disclose the full KPIs on taxonomy-alignment under Art 8 TR DA.
  - 6 April '22: COM adopts draft DA building on SFDR and TR RTSs.
  - 1 Jan '22: Periodic product disclosures in Art 11(1)-(3) SFDR start to apply.
  - 2 Jan '23: Product disclosures under Art 5 and 6 TR start to apply for the ‘first two environmental objectives’.
  - 13 Jul '22: First report of COM on application of TR (to be submitted every three years).

- **2023**
  - 1 Jan '23: Non-financial undertakings start disclosing the full KPIs on taxonomy-alignment under Art 8 TR DA.
  - 1 Jan '23: Application of SFDR RTS, including Art 5 and 6 TR product disclosures for ‘all environmental objectives’.
  - 1 Jan '24: Financial undertakings start disclosing the full KPIs on taxonomy-alignment under Art 8 TR DA.
  - 1 Jan '24: First report of COM on application of TR (to be submitted every three years).
  - 30 Jun '23: Non-financial undertakings disclose taxonomy-eligible and non-taxonomy-eligible activities under Art 8 TR DA.

- **2024**
  - 1 Jan '25: CSRD applies to companies currently subject to NFRD.
  - 1 Jan '25: CSRD applies to listed SMEs.
  - 1 Jan '25: CSRD applies to third country companies.

- **2025**
  - 1 Jan '26: CSRD applies to large companies not currently subject to NFRD.

- **2026**
  - 1 Jan '27: CSRD applies to large companies not currently subject to NFRD.

- **2027**
  - 1 Jan '28: CSRD applies to large companies not currently subject to NFRD.

- **2028**
  - 1 Jan '29: CSRD applies to large companies not currently subject to NFRD.

**Last updated: 26 September 2022**

*Enterprise size: Large (≥ 500 employees)*

*Thresholds: CSRD applies to large companies not currently subject to NFRD.*
Figure 13: Sustainable finance — implementation timeline

- **First FMP PAI statement**: First reference period for the Financial Market Participant (FMP) first Principal Adverse Impact (PAI) statement on 30 June 2023 must be 1 Jan – 31 Dec 2022
- **First two environmental objectives**: Point (a) (climate change mitigation) and point (b) (climate change adaptation) of environmental objectives under Art 9 TR
- **All environmental objectives**: In addition to point (a) and (b) above, point (c) (the sustainable use and protection of water and marine resources), point (d) (the transition to a circular economy), point (e) (pollution prevention and control) and point (f) (the protection and restoration of biodiversity and ecosystems) of environmental objectives under Article 9 TR
- **Art 5 and Art 6 TR**: Transparency of environmentally sustainable investments (Article 5) and of financial products that promote environmental characteristics (Article 6) in pre-contractual disclosures and in periodic reports
- **Art 8 TR DA**: Transparency of undertakings in non-financial statements
- **COM adopted DA bundling SFDR and TR RTSs**: COM bundled all 13 RTS of the SFDR, including the new empowerments for RTS introduced by the TR in one single DA (Commission Delegated Regulation EU 2022/1288)
- **Companies currently subject to NFRD (Non-Financial Reporting Directive)**: requirements apply to financial years (FYs) starting on / after 1 January 2024, first reporting in 2025
- **Large companies not currently subject to NFRD**: requirements apply to FYs starting on / after 1 January 2025, first reporting in 2026
- **Listed SMEs**: requirements apply to FYs starting on / after 1 January 2026, first reporting in 2027 (opt-out possible until 2029)
- **3rd country companies**: requirements apply to FYs starting on / after 1 January 2028, first reporting in 2029

Legend:
- Taxonomy Regulation (TR) L1
- Taxonomy Regulation Article 8 Delegated Act (DA)
- Sustainable Finance Disclosures Regulation (SFDR) L1
- SFDR RTS - Joint ESAs draft Regulatory Technical Standards (RTS)
- MiFID and IDD DAs
- UCITS and AIFMD DAs
- Corporate Sustainability Reporting Directive (CSRD) – final text

European Commission evaluation Reports
ESAs Report on voluntary disclosures under SFDR
EU taxonomy regulation

- Climate change mitigation
- Climate change adaptation
- Sustainable use and protection of water and marine resources
- Transition to a circular economy
- Pollution prevention and control
- Protection and restoration of biodiversity and ecosystem
EU taxonomy regulation

Figure 14: EU taxonomy for sustainable activities

1a. SC  Substantially contribute to at least one of the six objectives

1b. TSC  Comply with Technical Screening Criteria

2. DNSH  Do No Significant Harm to any other five objectives

3. MS  Comply with Minimum (Social) Safeguards
EU taxonomy regulation

EU taxonomy KPIs

- Green turnover (sales/revenues)
- Green CapEx (capital expenditure)
- Green OpEx (operating expenses)
EU taxonomy regulation

- Large public-interest companies already subject to the NFRD
- Large companies that are not presently subject to the NFRD, meeting two out of three CSRD criteria (250+ employees, balance sheet of €25+ mn, net turnover of €50+ mn)
- Listed SMEs and other undertakings
- Financial market participants
Climate benchmarks

The common principles are:

- A year-on-year self-decarbonization of 7% on average per annum, based on scope 1, 2 and 3 emissions
- A minimum carbon intensity reduction $\mathcal{R}^-$ compared to the investable universe
- A minimum exposure to sectors highly exposed to climate change

Two labels:

- CTB: (climate transition benchmark) $\Rightarrow \mathcal{R}^- = 30\%$
- PAB: (Paris aligned benchmark) $\Rightarrow \mathcal{R}^- = 50\%$
- Article 6 (or non-ESG products)
  It covers standard financial products that cannot be Article 8 or Article 9

- Article 8 (or ESG products)
  It corresponds to financial products which "promote, among other characteristics, environmental or social characteristics, or a combination of those characteristics, provided that the companies in which the investments are made follow good governance practices"

- Article 9 (or sustainable products)
  In addition to the points covered by Article 8, these financial products have a sustainable investment objective
  + SI (sustainable investment): S or/and E objectives
  + PAI (principal adverse impact): 64 PAIs (18 mandatory, 22 E 24 S)
### SFDR

<table>
<thead>
<tr>
<th>Corporates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate and other environment-related indicators</td>
<td></td>
</tr>
<tr>
<td>1. GHG emissions</td>
<td></td>
</tr>
<tr>
<td>2. Carbon footprint</td>
<td></td>
</tr>
<tr>
<td>3. GHG intensity of investee companies</td>
<td></td>
</tr>
<tr>
<td>4. Exposure to companies active in the fossil fuel sector</td>
<td></td>
</tr>
<tr>
<td>5. Share of non renewable energy consumption and production</td>
<td></td>
</tr>
<tr>
<td>6. Energy consumption intensity per high impact climate sector</td>
<td></td>
</tr>
<tr>
<td>7. Activities negatively affecting biodiversity sensitive areas</td>
<td></td>
</tr>
<tr>
<td>8. Emissions to water</td>
<td></td>
</tr>
<tr>
<td>9. Hazardous waste ratio</td>
<td></td>
</tr>
</tbody>
</table>

| Social and employee, respect for human rights, anti-corruption and anti-bribery matters |                                                                 |
| 10. Violations of UN Global Compact principles and OECD Guidelines for Multinational Enterprises |                                                                 |
| 11. Lack of processes and compliance mechanisms to monitor compliance with UN Global Compact principles and OECD Guidelines for MNEs |                                                                 |
| 12. Unadjusted gender pay gap                   |                                                                 |
| 13. Board gender diversity                      |                                                                 |
| 14. Exposure to controversial weapons (anti personnel mines, cluster munitions, chemical and biological weapons) |                                                                 |

| Sovereigns and supranationals                   |                                                                 |
| 15. GHG intensity (Climate indicator)           |                                                                 |
| 16. Investee countries subject to social violations (social indicator) |                                                                 |

| Real estate assets                              |                                                                 |
| Climate and other environment-related indicators |                                                                 |
| 17. Exposure to fossil fuels through real estate assets |                               |
| 18. Exposure to energy-inefficient real estate assets |                                             |

**Table 7: The 18 mandatory principal adverse indicators**
MiFID II & IDD

- MiFID II Suitability Test
- Integration of sustainability preferences to define the suitable product
- Integration of ESG criteria in the product governance

If the client has any sustainability preferences (yes/no), it has to choose one or a combination of the criteria below:

- Minimum percentage in environmentally sustainable investments aligned to the EU Taxonomy
- Minimum percentage invested in sustainable investments as defined in the SFDR (Articles 8 and 9)
- Quantitative/qualitative elements of principal adverse impacts defined by the client
Environmental factors: (1) climate change mitigation; (2) climate change adaptation; (3) water and marine resources; (4) resource use and circular economy; (5) pollution; (6) biodiversity and ecosystems.

Social factors: (1) equal opportunities for all; (2) working conditions; (3) respect for human rights.

Governance factors: (1) role and composition of administrative, management and supervisory bodies; (2) business ethics and corporate culture, including anti-corruption and anti-bribery; (3) political engagements of the undertaking, including its lobbying activities; (4) management and quality of relationships with business partners.

single materiality ≠ double materiality
- **CSRD** will replace progressively NFRD (Non-Financial Reporting Directive)

- Under the NFRD, large companies have to publish information related to environmental matters, social matters and treatment of employees, respect for human rights, anti-corruption and bribery, and diversity on company boards (in terms of age, gender, educational and professional background)

- The NFRD reporting rules apply to large public-interest companies with more than 500 employees (approximately 11,700 companies across the EU)
Who need to comply?

- EU companies (two of the following three conditions)
  - 250+ employees
  - Balance sheet of €25+ mn
  - Net turnover of €50+ mn

- Non-EU based companies with a net turnover of €150+ mn in the European Union
ESG strategies

1. Exclusion
   - Exclusion policy & negative (or worst-in-class) screening

2. Values
   - Norms-based screening

3. Selection
   - Positive (or best-in-class) screening

4. Thematic
   - Sustainability themed investing (e.g. green bonds)

5. Integration
   - ESG scoring is fully integrated in portfolio management

6. Engagement
   - Voting policy & shareholder activism

7. Impact
   - Impact investing

Figure 15: Categorisation of ESG strategies (Eurosif, 2019)
Exclusion/Negative Screening

The exclusion from a fund or portfolio of certain sectors, companies or practices based on specific ESG criteria (worst-in-class)

Examples:

- Systematic exclusion of issuers rated CCC
- Exclusion of issuers rated BB, B and CCC
- Sector exclusion (e.g., Energy)
- Sub-industry exclusion (e.g. Coal & Consumable Fuels)
- Exclusion list of individual issuers

Source: Global Sustainable Investment Alliance (2019)
Values/Norms-based Screening (and Red Flags)

Screening of investments against minimum standards of business practice based on international norms, such as those issued by the OECD, ILO, UN (Global Compact) and UNICEF\(^a\)

\(^a\)In Europe, the top exclusion criteria are (1) controversial weapons (Ottawa and Oslo treaties), (2) tobacco, (3) all weapons, (4) gambling, (5) pornography, (6) nuclear energy, (7) alcohol, (8) GMO and (9) animal testing (Eurosif, 2019)

Examples:

- Controversial sectors: controversial weapons, conventional weapons, civilian firearms, nuclear weapons, nuclear power, thermal coal, tobacco, alcohol, gambling, adult entertainment, genetically modified, fossil fuels production & reserves
- Many ETF funds

Source: Global Sustainable Investment Alliance (2019)
Definition

ESG ecosystem

The Market of ESG Investing

ESG strategies

The market share of ESG investing

ESG strategies

Selection/Positive Screening

Investment in sectors, companies or projects selected for positive ESG performance relative to industry peers (best-in-class)

Examples:

- Selection of issuers rated AAA, AA and A
- Selection of issuers that have improved their rating (Momentum ESG strategy)

Source: Global Sustainable Investment Alliance (2019)
The Market of ESG Investing

Thematic/Sustainability Themed Investing

Investment in themes or assets specifically related to sustainability (for example clean energy, green technology or sustainable agriculture)

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Funds invested in Green Bonds
- Funds invested in Social Bonds
- Funds invested in Sustainable Infrastructure
- Funds invested in Natural Resources
ESG Integration

The systematic and explicit inclusion by investment managers of environmental, social and governance factors into financial analysis

Examples:

- The stock picking score is a mix (50/50) of a fundamental score and an ESG score
- The fund must have an ESG score greater than the score of its benchmark

Source: Global Sustainable Investment Alliance (2019)
Corporate Engagement/Shareholder Action

The use of shareholder power to influence corporate behavior, including through direct corporate engagement (i.e., communicating with senior management and/or boards of companies), filing or co-filing shareholder proposals, and proxy voting that is guided by comprehensive ESG guidelines.

Examples:
- Voting policy
- Public divestment
- Biodiversity and deforestation financing
- Engagement with target companies on a specific subject (e.g., pay ratio or living wage)
- Escalated engagement: concerns public, proposing shareholder resolutions & litigation

Source: Global Sustainable Investment Alliance (2019)
**Impact Investing**

Targeted investments aimed at solving social or environmental problems, and including community investing, where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose.

*Source: Global Sustainable Investment Alliance (2019)*

Examples:

- Funds with a Social Impact objective
- Funds invested in Green Bonds
- PAB and CTB ETFs
Impact Investing/Community Investing

- **Impact Investing**
  Investing to achieve positive, social and environmental impacts – requires measuring and reporting against these impacts, demonstrating the intentionality of investor and underlying asset/investee, and demonstrating the investor contribution.

- **Community Investing**
  Where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose. Some community investing is impact investing, but community investing is broader and considers other forms of investing and targeted lending activities.

Source: Global Sustainable Investment Alliance (2021)
The market of ESG investing

Figure 16: Sustainable investment assets at the start of 2016

- **$23 tn**
  - Global Responsible Investment market in 2016

- **53%**
  - Europe
  - $12.04 tn
  - 12% growth in 2 years
  - Source: GSIA (2016)

- **38%**
  - USA
  - $8.7 tn
  - 33% growth in 2 years

- **5%**
  - Canada
  - $1.1 tn
  - 49% growth in 2 years

- **2%**
  - Japan
  - $480 bn
  - (vs $7 bn in 2014)

- **247%**
  - Australia / NZ
  - $500 bn
  - 247% growth in 2 years

- **+25%**
  - Growth in 2 years

Source: GSIA (2016).
The market of ESG investing

Figure 17: Sustainable investment assets at the start of 2018

- **USA**: $12.0 tn, 38% growth in 2 years
- **Canada**: $1.7 tn, 42% growth in 2 years
- **Australia / NZ**: $0.7 tn, 46% growth in 2 years
- **Europe**: $14.1 tn, 11% growth in 2 years
- **Japan**: $2.2 tn (vs $474 bn in 2016), 7% growth

~ 2/5 of global assets under management

+34% growth in 2 years

Figure 18: Sustainable investment assets at the start of 2020

- **$35.3tn**
  - Global RI market in 2020

- **35.9%**
  - of total global AUM

- **+15%**
  - Growth in 2 years

**Source:** GSIA (2020)
The market of ESG investing

**Figure 19**: Asset values of ESG strategies between 2014 and 2018

### Table 8: ESG asset growth

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Exclusion</td>
<td>11.7%</td>
<td>14.6%</td>
<td>-24.0%</td>
<td>15 030</td>
</tr>
<tr>
<td>2</td>
<td>Values/Norms-based</td>
<td>19.0%</td>
<td>-13.1%</td>
<td>-11.5%</td>
<td>4 140</td>
</tr>
<tr>
<td>3</td>
<td>Selection</td>
<td>7.6%</td>
<td>50.1%</td>
<td>-24.9%</td>
<td>1 384</td>
</tr>
<tr>
<td>4</td>
<td>Thematic Investing</td>
<td>55.1%</td>
<td>92.0%</td>
<td>91.4%</td>
<td>1 948</td>
</tr>
<tr>
<td>5</td>
<td>Integration</td>
<td>17.4%</td>
<td>30.2%</td>
<td>43.6%</td>
<td>25 195</td>
</tr>
<tr>
<td>6</td>
<td>Engagement</td>
<td>18.9%</td>
<td>8.3%</td>
<td>6.8%</td>
<td>10 504</td>
</tr>
<tr>
<td>7</td>
<td>Impact Investing</td>
<td>56.8%</td>
<td>33.7%</td>
<td>-20.8%</td>
<td>352</td>
</tr>
</tbody>
</table>

Course 2023-2024 in Sustainable Finance
Lecture 2. ESG Scoring

Thierry Roncalli*

* Amundi Asset Management
* University of Paris-Saclay

March 2024

5 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- **Lecture 2: ESG Scoring**
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Several issues:

- **E**: climate change mitigation, climate change adaptation, preservation of biodiversity, pollution prevention, circular economy
- **S**: inequality, inclusiveness, labor relations, investment in human capital and communities, human rights
- **G**: management structure, employee relations, executive remuneration

⇒ requires a lot of alternative data
Sovereign ESG data

Sovereign ESG framework

- World Bank
- Data may be downloaded at the following webpage:
- **E**: 27 variables
- **S**: 22 variables
- **G**: 18 variables
Table 9: The World Bank database of sovereign ESG indicators

<table>
<thead>
<tr>
<th>Environmental</th>
<th>Social</th>
<th>Governance</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Emissions &amp; pollution (5)</td>
<td>• Education &amp; skills (3)</td>
<td>• Human rights (2)</td>
</tr>
<tr>
<td>• Natural capital endowment and management (6)</td>
<td>• Employment (3)</td>
<td>• Government effectiveness (2)</td>
</tr>
<tr>
<td>• Energy use &amp; security (7)</td>
<td>• Demography (3)</td>
<td>• Stability &amp; rule of law (4)</td>
</tr>
<tr>
<td>• Environment/ climate risk &amp; resilience (6)</td>
<td>• Poverty &amp; inequality (4)</td>
<td>• Economic environment (3)</td>
</tr>
<tr>
<td>• Food security (3)</td>
<td>• Health &amp; nutrition (5)</td>
<td>• Gender (4)</td>
</tr>
<tr>
<td></td>
<td>• Access to services (4)</td>
<td>• Innovation (3)</td>
</tr>
</tbody>
</table>
Table 10: Indicators of the environmental pillar (World Bank database)

- **Emissions & pollution**: (1) CO2 emissions (metric tons per capita); (2) GHG net emissions/removals by LUCF (Mt of CO2 equivalent); (3) Methane emissions (metric tons of CO2 equivalent per capita); (4) Nitrous oxide emissions (metric tons of CO2 equivalent per capita); (5) PM2.5 air pollution, mean annual exposure (micrograms per cubic meter);

- **Natural capital endowment & management**: (1) Adjusted savings: natural resources depletion (% of GNI); (2) Adjusted savings: net forest depletion (% of GNI); (3) Annual freshwater withdrawals, total (% of internal resources); (4) Forest area (% of land area); (5) Mammal species, threatened; (6) Terrestrial and marine protected areas (% of total territorial area);

- **Energy use & security**: (1) Electricity production from coal sources (% of total); (2) Energy imports, net (% of energy use); (3) Energy intensity level of primary energy (MJ/$2011 PPP GDP); (4) Energy use (kg of oil equivalent per capita); (5) Fossil fuel energy consumption (% of total); (6) Renewable electricity output (% of total electricity output); (7) Renewable energy consumption (% of total final energy consumption);

- **Environment/climate risk & resilience**: (1) Cooling degree days (projected change in number of degree Celsius); (2) Droughts, floods, extreme temperatures (% of population, average 1990-2009); (3) Heat Index 35 (projected change in days); (4) Maximum 5-day rainfall, 25-year return level (projected change in mm); (5) Mean drought index (projected change, unitless); (6) Population density (people per sq. km of land area);

- **Food security**: (1) Agricultural land (% of land area); (2) Agriculture, forestry, and fishing, value added (% of GDP); (3) Food production index (2004-2006 = 100);

Table 11: Indicators of the social pillar (World Bank database)

- **Education & skills**: (1) Government expenditure on education, total (% of government expenditure); (2) Literacy rate, adult total (% of people ages 15 and above); (3) School enrollment, primary (% gross);
- **Employment**: (1) Children in employment, total (% of children ages 7-14); (2) Labor force participation rate, total (% of total population ages 15-64) (modeled ILO estimate); (3) Unemployment, total (% of total labor force) (modeled ILO estimate);
- **Demography**: (1) Fertility rate, total (births per woman); (2) Life expectancy at birth, total (years); (3) Population ages 65 and above (% of total population);
- **Poverty & inequality**: (1) Annualized average growth rate in per capita real survey mean consumption or income, total population (%); (2) Gini index (World Bank estimate); (3) Income share held by lowest 20%; (4) Poverty headcount ratio at national poverty lines (% of population);
- **Health & nutrition**: (1) Cause of death, by communicable diseases and maternal, prenatal and nutrition conditions (% of total); (2) Hospital beds (per 1,000 people); (3) Mortality rate, under-5 (per 1,000 live births); (4) Prevalence of overweight (% of adults); (5) Prevalence of undernourishment (% of population);
- **Access to services**: (1) Access to clean fuels and technologies for cooking (% of population); (2) Access to electricity (% of population); (3) People using safely managed drinking water services (% of population); (4) People using safely managed sanitation services (% of population);

Table 12: Indicators of the governance pillar (World Bank database)

- **Human rights**: (1) Strength of legal rights index (0 = weak to 12 = strong); (2) Voice and accountability (estimate);

- **Government effectiveness**: (1) Government effectiveness (estimate); (2) Regulatory quality (estimate);

- **Stability & rule of law**: (1) Control of corruption (estimate); (2) Net migration; (3) Political stability and absence of violence/terrorism (estimate); (4) Rule of law (estimate)

- **Economic environment**: (1) Ease of doing business index (1 = most business-friendly regulations); (2) GDP growth (annual %); (3) Individuals using the internet (% of population);

- **Gender**: (1) Proportion of seats held by women in national parliaments (%); (2) Ratio of female to male labor force participation rate (%) (modeled ILO estimate); (3) School enrollment, primary and secondary (gross), gender parity index (GPI); (4) Unmet need for contraception (% of married women ages 15-49);

- **Innovation**: (1) Patent applications, residents; (2) Research and development expenditure (% of GDP); (3) Scientific and technical journal articles;

Where to find the data?

- National accounts statistics collected by OECD, United Nations Statistics Division (UNSD), etc.
- Internal departments and specialized databases of the World Bank: World Bank Open Data, Business Enabling Environment (BEE), Climate Change Knowledge Portal (CCKP), Global Electrification Database (GEP), etc.
- International organizations: Emission Database for Global Atmospheric Research (EDGAR), Food and Agriculture Organization (FAO), International Energy Agency (IEA), International Labour Organization (ILO), World Health Organization (WHO), etc.
- NGOs: Climate Watch, etc.;
- Academic resources: International disasters database (EM-DAT) of the Centre for Research on the Epidemiology of Disasters (Université Catholique de Louvain), etc.
The most known are FTSE (Beyond Ratings), Moody’s (Vigeo-Eiris), MSCI, Sustainalytics, RepRisk and Verisk Mapplecroft.

⇒ The average cross-correlation between data providers is equal to 85% for the ESG score, 42% for the environmental score, 85% for the social score and 71% for the governance score.
**Table 13:** Correlation of ESG scores with country’s national income (GNI per capita)

<table>
<thead>
<tr>
<th>Factor</th>
<th>ESG</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISS</td>
<td>68%</td>
<td>7%</td>
<td>86%</td>
<td>77%</td>
</tr>
<tr>
<td>FTSE (Beyond Ratings)</td>
<td>91%</td>
<td>74%</td>
<td>88%</td>
<td>84%</td>
</tr>
<tr>
<td>MSCI</td>
<td>84%</td>
<td>10%</td>
<td>90%</td>
<td>77%</td>
</tr>
<tr>
<td>RepRisk</td>
<td>78%</td>
<td>79%</td>
<td>75%</td>
<td>37%</td>
</tr>
<tr>
<td>RobecoSAM</td>
<td>89%</td>
<td>82%</td>
<td>85%</td>
<td>85%</td>
</tr>
<tr>
<td>Sustainalytics</td>
<td>95%</td>
<td>83%</td>
<td>94%</td>
<td>93%</td>
</tr>
<tr>
<td>V.E</td>
<td>60%</td>
<td>23%</td>
<td>79%</td>
<td>39%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>81%</td>
<td>51%</td>
<td>85%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Source: Gratcheva et al. (2020).
The mushrooming growth of data

**Figure 20:** Palm oil production (2019)

The mushrooming growth of data

**Figure 21:** Palm oil imports (2019)

The mushrooming growth of data

**Figure 22:** Share of global annual deforestation (2015)

The mushrooming growth of data

**Figure 23**: Threatened mammal species (2018)

![Map of threatened mammal species, 2018.](image)

Threatened mammal species, 2018
Mammal species are mammals excluding whales and porpoises. Threatened species are those classified on the Red List as Critically Endangered, Endangered or Vulnerable. They are at high or greater risk of extinction in the wild.

An example with the biodiversity risk

**Figure 24:** Global living planet index

Source: https://livingplanetindex.org/latest_results & Author’s calculation.
An example with the biodiversity risk

Some databases:

- the Red List Index (RLI)
- World Database on Protected Areas (WDPA)
- Integrated Biodiversity Assessment Tool (IBAT)
- Exploring Natural Capital Opportunities, Risks and Exposure (ENCORE)
- Etc.
Corporate ESG data

Data sources:

- Corporate publications (self-reporting)
  - Annual reports
  - Corporate sustainability reports
- Financial and regulatory filings (standardized reporting)
  - Mandatory reports (SFDR, CSRD, EUTR, etc.)
  - Non-mandatory frameworks (PRI, TCFD, CDP, etc.)
- News and other media
- NGO reports and websites
- Company assessment and due diligence questionnaire (DDQ)
- Internal models
Figure 25: From raw data to ESG pillars
Table 14: An example of ESG criteria (corporate issuers)

<table>
<thead>
<tr>
<th>Environmental</th>
<th>Social</th>
<th>Governance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon emissions</td>
<td>Employment conditions</td>
<td>Board independence</td>
</tr>
<tr>
<td>Energy use</td>
<td>Community involvement</td>
<td>Corporate behaviour</td>
</tr>
<tr>
<td>Pollution</td>
<td>Gender equality</td>
<td>Audit and control</td>
</tr>
<tr>
<td>Waste disposal</td>
<td>Diversity</td>
<td>Executive compensation</td>
</tr>
<tr>
<td>Water use</td>
<td>Stakeholder opposition</td>
<td>Shareholder’s rights</td>
</tr>
<tr>
<td>Renewable energy</td>
<td>Access to medicine</td>
<td>CSR strategy</td>
</tr>
<tr>
<td>Green cars*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green financing*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) means a specific criterion related to one or several sectors
(Green cars ⇒ Automobiles, Green financing ⇒ Financials)
Corporate ESG data

Some examples:

- Bloomberg rates 11,800 public companies. They use more than 120 ESG indicators and 2,000+ data points.
- ISS ESG rates about 10,000 issuers. They use more than 800 indicators and applies approximately 100 indicators per company.
- FTSE Russell rates about 7,200 securities. They use more than 300 indicators and 14 themes.
- MSCI rates 10,000 companies (14,000 issuers including subsidiaries) and 680,000 securities globally. They use 10 themes, 1,000+ data points, 80 exposure metrics and 250+ management metrics.
- Refinitiv rates 12,000 public and private companies. They consider 10 themes. These themes are built using 186 metrics and 630+ data points.
- S&P Dow Jones Indices uses between 16 to 27 criteria scores, a questionnaire and 1,000 data points.
- Sustainalytics rates more than 16,300 companies. They consider 20 material ESG issues, based on 350+ indicators.
The race for alternative data

- Controversies ⇒ NLP (RepRisk, daily basis: 500,000+ documents, 100,000+ sources, 23 languages)
- Geospatial data ⇒ Physical risk
The divergence of corporate ESG ratings

Figure 26: ESG rating disagreement

This graph illustrates the ESG rating divergence. The horizontal axis indicates the value of the Sustainalytics rating as a benchmark for each firm (n = 924). Rating values by the other five raters are plotted on the vertical axis in different colors. For each rater, the distribution of values has been normalized to zero mean and unit variance. The Sustainalytics rating has discrete values that show up visually as vertical lines where several companies have the same rating value.

Source: Berg et al. (2022).
Berg et al. (2022) identify three sources of divergence:

- **Measurement** divergence refers to situation where rating agencies measure the same indicator using different ESG metrics (56%)
- **Scope** divergence refers to situation where ratings are based on different set of ESG indicators (38%)
- **Weight** divergence emerges when rating agencies take different views on the relative importance of ESG indicators” (6%)
The divergence of corporate ESG ratings

Table 15: Rank correlation among ESG ratings

<table>
<thead>
<tr>
<th></th>
<th>MSCI</th>
<th>Refinitiv</th>
<th>S&amp;P Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Refinitiv</td>
<td>43%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>S&amp;P Global</td>
<td>45%</td>
<td>69%</td>
<td>100%</td>
</tr>
<tr>
<td>Sustainalytics</td>
<td>53%</td>
<td>64%</td>
<td>69%</td>
</tr>
</tbody>
</table>

Source: Billio et al. (2021).
One-level tree structure

- $X_1, \ldots, X_m$ are $m$ features
- The score is linear:
  \[ S = \sum_{j=1}^{m} \omega_j X_j \]
- $\omega_j$ is the weight of the $j^{th}$ metric
One-level tree structure

- $X_1, \ldots, X_m$ are $m$ features
- The score is linear:

$$S = \sum_{j=1}^{m} \omega_j X_j$$

- $\omega_j$ is the weight of the $j^{th}$ metric
The Altman $Z$ score is equal to:

$$Z = 1.2 \cdot X_1 + 1.4 \cdot X_2 + 3.3 \cdot X_3 + 0.6 \cdot X_4 + 1.0 \cdot X_5$$

where the variables $X_j$ represent the following financial ratios:

<table>
<thead>
<tr>
<th>$X_j$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Working capital / Total assets</td>
</tr>
<tr>
<td>$X_2$</td>
<td>Retained earnings / Total assets</td>
</tr>
<tr>
<td>$X_3$</td>
<td>Earnings before interest and tax / Total assets</td>
</tr>
<tr>
<td>$X_4$</td>
<td>Market value of equity / Total liabilities</td>
</tr>
<tr>
<td>$X_5$</td>
<td>Sales / Total assets</td>
</tr>
</tbody>
</table>

$$Z_i \Rightarrow Z_i^* = (Z_i - m_z) / \sigma_z \Rightarrow \text{Decision rule}$$
The intermediary scores are equal to:

\[ S_k^{(1)} = \sum_{j=1}^{m} \omega_{j,k}^{(1)} X_j \]

whereas the expression of the final score is:

\[ S := S_1^{(0)} = \sum_{k=1}^{m(1)} \omega_k^{(0)} S_k^{(1)} \]
Figure 27: A two-level non-overlapping tree

- Level 1: $\omega_{1,1}^{(1)} = 50\%$; $\omega_{2,1}^{(1)} = 25\%$; $\omega_{3,1}^{(1)} = 25\%$; $\omega_{4,2}^{(1)} = 50\%$; $\omega_{5,2}^{(1)} = 50\%$; $\omega_{6,3}^{(1)} = 100\%$
- Level 0: $\omega_1^{(0)} = \omega_2^{(0)} = \omega_3^{(0)} = 33.33\%$
Two-level tree structure

\[
\begin{align*}
S_1^{(1)} &= 0.5 \cdot X_1 + 0.25 \cdot X_2 + 0.25 \cdot X_3 \\
S_2^{(1)} &= 0.5 \cdot X_4 + 0.5 \cdot X_5 \\
S_3^{(1)} &= X_6 \\
S &= \frac{S_1^{(1)} + S_2^{(1)} + S_3^{(1)}}{3}
\end{align*}
\]
Two-level tree structure

**Figure 28: A two-level overlapping tree graph**

- Level 1:
  - $\omega_{1,1}^{(1)} = 50\%$; $\omega_{2,1}^{(1)} = 25\%$; $\omega_{3,1}^{(1)} = 25\%$; $\omega_{3,2}^{(1)} = 25\%$; $\omega_{4,2}^{(1)} = 25\%$; $\omega_{5,2}^{(1)} = 50\%$; $\omega_{6,3}^{(1)} = 100\%$;
  - Level 0: $\omega_{1}^{(0)} = \omega_{2}^{(0)} = \omega_{3}^{(0)} = 33.33\%$;
Figure 29: Tree data structure
- $L$ is the number of levels
- We have $S_j^{(L)} = X_j$
- The value of the $k^{th}$ node at level $\ell$ is given by:

\[
S_k^{(\ell)} = \sum_{j=1}^{m_{(\ell+1)}} \omega_{j,k} S_j^{(\ell+1)}
\]
Figure 30: An example of ESG scoring tree (MSCI methodology)

Source: MSCI (2020)
Let $\omega^{(\ell)}$ be the $m(\ell+1) \times m(\ell)$ matrix, whose elements are $\omega^{(\ell)}_{j,k}$ for $j = 1, \ldots, m(\ell+1)$ and $k = 1, \ldots, m(\ell)$.

The final score is equal to:

$$S = \omega^\top X$$

where:

$$\omega = \omega^{(L-1)} \cdots \omega^{(1)} \omega^{(0)}$$
Score normalization

If $X \sim F$, we obtain:

$$G(s) = \Pr \{S \leq s\} = \Pr \{\omega^T X \leq s\} = \int \cdots \int 1 \{\omega^T x \leq s\} \, dF(x)$$

$$= \int \cdots \int 1 \left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \, dF(x_1, \ldots, x_m)$$

$$= \int \cdots \int 1 \left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \, dC(F_1(x_1), \ldots, F_m(x_m))$$

Therefore, the distribution $G$ depends on the copula function $C$ and the marginals $(F_1, \ldots, F_m)$ of $F$

$$F_1 \equiv F_1 \equiv \ldots \equiv F_m \Rightarrow G \equiv F_1?$$
Score normalization

In the independent case, we obtain a convolution probability distribution:

\[
G(s) = \int \cdots \int 1 \left\{ \sum_{j=1}^{m} \omega_j x_j \leq s \right\} \prod_{j=1}^{m} dF_j(x_j)
\]

If \( X_j \sim \mathcal{N} (\mu_j, \sigma^2_j) \), we have \( \omega_j X_j \sim \mathcal{N} (\omega_j \mu_j, \omega^2_j \sigma^2_j) \). We deduce that:

\[
S \sim \mathcal{N} \left( \sum_{j=1}^{m} \omega_j \mu_j, \sum_{j=1}^{m} \omega^2_j \sigma^2_j \right) \equiv \mathcal{N} (\omega^\top \mu, \omega^\top \Sigma \omega)
\]

where \( \mu = (\mu_1, \ldots, \mu_m) \) and \( \Sigma = \text{diag} (\sigma^2_1, \ldots, \sigma^2_m) \).
Score normalization

Figure 31: Probability distribution of the scores based on the previous tree
We assume that $X_1 \sim U_{[0,1]}$ and $X_2 \sim U_{[0,1]}$ are two independent random variables. We consider the score $S$ defined as:

$$S = \frac{X_1 + X_2}{2}$$
Score normalization

**Figure 32**: Geometric interpretation of the probability mass function

Case (a): $0 \leq s \leq 0.5$

Case (b): $0.5 \leq s \leq 1$
Score normalization

We deduce that:

\[
\Pr \{ S \leq s \} = \begin{cases} 
\frac{1}{2} (2s)^2 = 2s^2 & \text{if } 0 \leq s \leq \frac{1}{2} \\
1 - \frac{1}{2} (2 - 2s)^2 = -1 + 4s - 2s^2 & \text{if } \frac{1}{2} \leq s \leq 1 
\end{cases}
\]

The density function is then:

\[
g(s) = \begin{cases} 
4s & \text{if } 0 \leq s \leq \frac{1}{2} \\
4 - 4s & \text{if } \frac{1}{2} \leq s \leq 1 
\end{cases}
\]

In the general case, we have:

\[
S = \frac{X_1 + X_2 + \cdots + X_m}{m} \sim \text{Bates} (m)
\]
Score normalization

Figure 33: Probability density function of $S$ (uniform distribution)
Exercise

We assume that $X \sim \mathcal{N} (\mu, \Sigma)$ with $\mu_j = 0$, $\sigma_j = 1$ and $\rho_{j,k} = \rho$ for $j \neq k$. Show that:

$$\mathbb{E} [S] = 0$$

and

$$\text{var} (S) = \rho S^2 (w) + (1 - \rho) H (\omega)$$

where $S (w) = \sum_{j=1}^{m} \omega_j$ is the sum index and $H (\omega) = \sum_{j=1}^{m} \omega_j^2$ is the Herfindahl index. Deduce that:

$$\sigma_S = \sqrt{\rho + (1 - \rho) H (\omega)}$$
Score normalization

How to normalize?

$$S_k^{(\ell)} = \varphi \left( \sum_{j=1}^{m(\ell+1)} \omega_{j,k} S_j^{(\ell+1)} \right)$$
Score normalization

- **m-score normalization:**
  \[ m_i = \frac{x_i - x^-}{x^+ - x^-} \]
  where \( x^- = \min x_i \) and \( x^+ = \max x_i \)

- **q-score normalization:**
  \[ q_i = H(x_i) \]
  where \( H \) is the distribution function of \( X \)

- **z-score normalization:**
  \[ z_i = \frac{x_i - \mu}{\sigma} \]
  where \( \mu \) and \( \sigma \) are the mathematical expectation and standard deviation of \( X \)

- **b-score normalization:**
  \[ b_i = B^{-1}(H(x_i); \alpha, \beta) \]
  where \( B(\alpha, \beta) \) is the beta distribution
Probability integral transform (PIT)

If $X \sim H$ and is continuous, $Y = H(X)$ is a uniform random variable.

We have $Y \in [0, 1]$ and:

\[
Pr\{Y \leq y\} = Pr\{H(X) \leq y\} \\
= Pr\{X \leq H^{-1}(y)\} \\
= H(H^{-1}(y)) \\
= y
\]
Score normalization

Computing the empirical distribution $\hat{A}$

- Let $\{x_1, x_2, \ldots, x_n\}$ be the sample
- We have:

$$q_i = \hat{A}(x_i) = \Pr\{X \leq x_i\} = \frac{\# \{x_j \leq x_i\}}{n_q}$$

- $n_q = n$ or $n_q = n + 1$?
Score normalization

Exercise

What is the normalization shape of this transformation?

\[ S = \frac{2}{1 + e^{-z}} - 1 \]

Hint: compute the density function.
Example

The data are normally distributed with mean $\mu = 5$ and standard deviation $\sigma = 2$. To map these data into a 0/1 score, we consider the following transform:

$$ s := \varphi(x) = B^{-1} \left( \Phi \left( \frac{x - 5}{2} \right); \alpha, \beta \right) $$
Score normalization

Figure 34: Transforming data into $b$-score

Transform function $s = \varphi(x)$

Probability density function of the score
Example

We consider the raw data of 9 companies that belong to the same industry. The first variable measures the carbon intensity of the scope 1 + 2 in 2020, while the second variable is the variation of carbon emissions between 2015 and 2020. We would like to create the score 

\[ S \equiv 70\% \cdot X_1 + 30\% \cdot X_2. \]

<table>
<thead>
<tr>
<th>Firm</th>
<th>Carbon intensity in tCO$_2$/mn</th>
<th>Carbon momentum in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.0</td>
<td>-3.0</td>
</tr>
<tr>
<td>2</td>
<td>38.6</td>
<td>-5.5</td>
</tr>
<tr>
<td>3</td>
<td>30.6</td>
<td>5.6</td>
</tr>
<tr>
<td>4</td>
<td>74.4</td>
<td>-1.3</td>
</tr>
<tr>
<td>5</td>
<td>97.1</td>
<td>-16.8</td>
</tr>
<tr>
<td>6</td>
<td>57.1</td>
<td>-3.5</td>
</tr>
<tr>
<td>7</td>
<td>132.4</td>
<td>8.5</td>
</tr>
<tr>
<td>8</td>
<td>92.5</td>
<td>-9.1</td>
</tr>
<tr>
<td>9</td>
<td>64.9</td>
<td>-4.6</td>
</tr>
</tbody>
</table>
Score normalization

- **q-score** 0/100
- **z-score**
- \( qz = 100 \cdot \Phi(z) \)
- \( zq = \Phi^{-1} \left( \frac{q}{100} \right) \)
- \( bz = B^{-1} \left( \Phi(z) ; \alpha, \beta \right) \) where \( \alpha = \beta = 2 \)
- \( bz^* = B^{-1} \left( \Phi(z) ; \alpha, \beta \right) \) where \( \alpha = 2.5 \) and \( \beta = 1.5 \).
Table 16: Computation of the score $S = 70\% \cdot X_1 + 30\% \cdot X_2$ ($q$-score 0/100 normalization)

<table>
<thead>
<tr>
<th>#</th>
<th>$X_1$</th>
<th>$q_1$</th>
<th>$X_2$</th>
<th>$q_2$</th>
<th>$s$</th>
<th>$S$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.00</td>
<td>70.00</td>
<td>-3.00</td>
<td>60.00</td>
<td>67.00</td>
<td>80.00</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>38.60</td>
<td>20.00</td>
<td>-5.50</td>
<td>30.00</td>
<td>23.00</td>
<td>10.00</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>30.60</td>
<td>10.00</td>
<td>5.60</td>
<td>80.00</td>
<td>31.00</td>
<td>20.00</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>74.40</td>
<td>50.00</td>
<td>-1.30</td>
<td>70.00</td>
<td>56.00</td>
<td>60.00</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>97.10</td>
<td>80.00</td>
<td>-16.80</td>
<td>10.00</td>
<td>59.00</td>
<td>70.00</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>57.10</td>
<td>30.00</td>
<td>-3.50</td>
<td>50.00</td>
<td>36.00</td>
<td>30.00</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>132.40</td>
<td>90.00</td>
<td>8.50</td>
<td>90.00</td>
<td>90.00</td>
<td>90.00</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>92.50</td>
<td>60.00</td>
<td>-9.10</td>
<td>20.00</td>
<td>48.00</td>
<td>50.00</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>64.90</td>
<td>40.00</td>
<td>-4.60</td>
<td>40.00</td>
<td>40.00</td>
<td>40.00</td>
<td>4</td>
</tr>
<tr>
<td>Mean</td>
<td>75.73</td>
<td>50.00</td>
<td>-3.30</td>
<td>50.00</td>
<td>50.00</td>
<td>50.00</td>
<td></td>
</tr>
<tr>
<td>Std-dev.</td>
<td>31.95</td>
<td>27.39</td>
<td>7.46</td>
<td>27.39</td>
<td>20.60</td>
<td>27.39</td>
<td></td>
</tr>
</tbody>
</table>
### Table 17: Computation of the score \( S \equiv 70\% \cdot X_1 + 30\% \cdot X_2 \) (\( z \)-score normalization)

<table>
<thead>
<tr>
<th>#</th>
<th>( X_1 )</th>
<th>( z_1 )</th>
<th>( X_2 )</th>
<th>( z_2 )</th>
<th>( s )</th>
<th>( S )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.00</td>
<td>0.572</td>
<td>-3.00</td>
<td>0.040</td>
<td>0.412</td>
<td>0.543</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>38.60</td>
<td>-1.162</td>
<td>-5.50</td>
<td>-0.295</td>
<td>-0.902</td>
<td>-1.188</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>30.60</td>
<td>-1.413</td>
<td>5.60</td>
<td>1.193</td>
<td>-0.631</td>
<td>-0.831</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>74.40</td>
<td>-0.042</td>
<td>-1.30</td>
<td>0.268</td>
<td>0.051</td>
<td>0.067</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>97.10</td>
<td>0.669</td>
<td>-16.80</td>
<td>-1.810</td>
<td>-0.075</td>
<td>-0.099</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>57.10</td>
<td>-0.583</td>
<td>-3.50</td>
<td>-0.027</td>
<td>-0.416</td>
<td>-0.548</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>132.40</td>
<td>1.774</td>
<td>8.50</td>
<td>1.582</td>
<td>1.716</td>
<td>2.261</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>92.50</td>
<td>0.525</td>
<td>-9.10</td>
<td>-0.778</td>
<td>0.134</td>
<td>0.177</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>64.90</td>
<td>-0.339</td>
<td>-4.60</td>
<td>-0.174</td>
<td>-0.290</td>
<td>-0.382</td>
<td>4</td>
</tr>
<tr>
<td>Mean</td>
<td>75.73</td>
<td>0.000</td>
<td>-3.30</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Std-dev.</td>
<td>31.95</td>
<td>1.000</td>
<td>7.46</td>
<td>1.000</td>
<td>0.759</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>
### Table 18: Comparison of the different scoring methods

<table>
<thead>
<tr>
<th>#</th>
<th>( S )</th>
<th>( R )</th>
<th>( S )</th>
<th>( R )</th>
<th>( S )</th>
<th>( R )</th>
<th>( S )</th>
<th>( R )</th>
<th>( S )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80.00</td>
<td>8</td>
<td>0.54</td>
<td>8</td>
<td>76.27</td>
<td>8</td>
<td>0.84</td>
<td>8</td>
<td>0.66</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>10.00</td>
<td>1</td>
<td>-1.19</td>
<td>1</td>
<td>9.19</td>
<td>1</td>
<td>-1.28</td>
<td>1</td>
<td>0.20</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>20.00</td>
<td>2</td>
<td>-0.83</td>
<td>2</td>
<td>21.37</td>
<td>2</td>
<td>-0.84</td>
<td>2</td>
<td>0.29</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>60.00</td>
<td>6</td>
<td>0.07</td>
<td>6</td>
<td>54.13</td>
<td>5</td>
<td>0.25</td>
<td>6</td>
<td>0.52</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>70.00</td>
<td>7</td>
<td>-0.10</td>
<td>5</td>
<td>56.65</td>
<td>6</td>
<td>0.52</td>
<td>7</td>
<td>0.51</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>30.00</td>
<td>3</td>
<td>-0.55</td>
<td>3</td>
<td>24.42</td>
<td>3</td>
<td>-0.52</td>
<td>3</td>
<td>0.34</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>90.00</td>
<td>9</td>
<td>2.26</td>
<td>9</td>
<td>98.04</td>
<td>9</td>
<td>1.28</td>
<td>9</td>
<td>0.93</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>50.00</td>
<td>5</td>
<td>0.18</td>
<td>7</td>
<td>60.39</td>
<td>7</td>
<td>0.00</td>
<td>5</td>
<td>0.56</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>40.00</td>
<td>4</td>
<td>-0.38</td>
<td>4</td>
<td>30.96</td>
<td>4</td>
<td>-0.25</td>
<td>4</td>
<td>0.39</td>
<td>4</td>
</tr>
<tr>
<td>Mean</td>
<td>50.00</td>
<td>0.00</td>
<td>47.94</td>
<td>0.00</td>
<td>0.49</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std-dev.</td>
<td>27.39</td>
<td>1.00</td>
<td>28.79</td>
<td>0.82</td>
<td>0.22</td>
<td>0.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The CEO pay ratio is calculated by dividing the CEO’s compensation by the pay of the median employee. It is one of the key metrics for the G pillar. It has been imposed by the Dodd-Frank Act, which requires that publicly traded companies disclose:

1. the median total annual compensation of all employees other than the CEO;
2. the ratio of the CEO’s annual total compensation to that of the median employee;
3. the wage ratio of the CEO to the median employee.

⇒ the average S&P 500 company’s CEO-to-worker pay ratio was 324-to-1 in 2021 (AFL-CIO)
## An example with the CEO pay ratio

Table 19: Examples of CEO pay ratio (June 2021)

<table>
<thead>
<tr>
<th>Company name</th>
<th>$P$</th>
<th>$R$</th>
<th>Company name</th>
<th>$P$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abercrombie &amp; Fitch</td>
<td>1954</td>
<td>4,293</td>
<td>Netflix</td>
<td>202 931</td>
<td>190</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>9,291</td>
<td>1,939</td>
<td>BlackRock</td>
<td>133 644</td>
<td>182</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>11,285</td>
<td>1,657</td>
<td>Pfizer</td>
<td>98,972</td>
<td>181</td>
</tr>
<tr>
<td>Gap</td>
<td>6,177</td>
<td>1,558</td>
<td>Goldman Sachs</td>
<td>138 854</td>
<td>178</td>
</tr>
<tr>
<td>Alphabet</td>
<td>258 708</td>
<td>1,085</td>
<td>MSCI</td>
<td>55,857</td>
<td>165</td>
</tr>
<tr>
<td>Walmart</td>
<td>22 484</td>
<td>983</td>
<td>Verisk Analytics</td>
<td>77,055</td>
<td>117</td>
</tr>
<tr>
<td>Estee Lauder</td>
<td>30,733</td>
<td>697</td>
<td>Facebook</td>
<td>247 883</td>
<td>94</td>
</tr>
<tr>
<td>Ralph Lauren</td>
<td>21,358</td>
<td>570</td>
<td>Invesco</td>
<td>125 282</td>
<td>92</td>
</tr>
<tr>
<td>NIKE</td>
<td>25,386</td>
<td>550</td>
<td>Boeing</td>
<td>158 869</td>
<td>90</td>
</tr>
<tr>
<td>Citigroup</td>
<td>52,988</td>
<td>482</td>
<td>Citrix Systems</td>
<td>181 769</td>
<td>80</td>
</tr>
<tr>
<td>PepsiCo</td>
<td>45,896</td>
<td>368</td>
<td>Harley-Davidson</td>
<td>187 157</td>
<td>59</td>
</tr>
<tr>
<td>Microsoft</td>
<td>172,512</td>
<td>249</td>
<td>Amazon.com</td>
<td>28,848</td>
<td>58</td>
</tr>
<tr>
<td>Apple</td>
<td>57,596</td>
<td>201</td>
<td>Berkshire Hathaway</td>
<td>65,740</td>
<td>6</td>
</tr>
</tbody>
</table>

Source: https://aflcio.org (June 2021)
An example with the CEO pay ratio

Figure 35: Histogram of the CEO pay ratio
An example with the CEO pay ratio

Figure 36: Histogram of $z$-score applied to the CEO pay ratio
An example with the CEO pay ratio

What is the solution? Give the transform function $y = \varphi (x)$.

Hint: use the beta distribution.
Other statistical methods

Unsupervised learning

- Clustering (*K*-means, hierarchical clustering)
- Dimension reduction (PCA)
### Supervised learning

- Discriminant analysis (LDA, QDA)
- Binary choice models (logistic regression, probit model)
- Regression models (OLS, lasso)

⇒ Advanced learning models (k-NN, neural networks and support vector machines) are not relevant in the case of ESG scoring

We need to define the response variable $Y$
Example with credit scoring models

- Let $S_i(t)$ be the credit score of individual $i$ at time $t$
- We have:

$$Y_i(t) = 1 \{ \tau_i \leq t + \delta \} = 1 \{ D_i(t + \delta) = 1 \}$$

where $\tau_i$ and $D_i$ are the default time and the default indicator function, and $\delta$ is the time horizon (e.g., one year)

- The calibration problem of the credit scoring model is:

$$\Pr \{ Y_i(t) = 0 \} = f(S_i(t))$$

where $f$ is an increasing function
Application to ESG scoring models

- Let \( S_i(t) \) be the ESG score of company \( i \) at time \( t \)
- Endogenous response variable:
  - (a) Best-in-class oriented scoring system:
    \[
    Y_i(t) = 1 \{ S_i(t + h) \geq s^* \}
    \]
    where \( s^* \) is the best-in-class threshold
  - (b) Worst-in-class oriented scoring system:
    \[
    Y_i(t) = 1 \{ S_i(t + h) \leq s^* \}
    \]
    where \( s^* \) is the worst-in-class threshold
- Exogenous response variable
  - (c) Binary response:
    \[
    Y_i(t) = 1 \{ C_i(t + h) \geq 0 \}
    \]
    where \( C_i(t) \) is the controversy index
  - (d) Continuous response:
    \[
    Y_i(t) = C_i(t + h)
    \]
- The calibration problem of the ESG scoring model is
  \[
  \Pr \{ Y_i(t) = 0 \} = f (S_i(t)) \text{ or } Y_i(t) = f (S_i(t))
  \]
  where the function \( f \) is increasing for case (a) and decreasing for cases (b), (c) and (d)
Performance evaluation criteria

- ESG scoring and rating
  - Shannon entropy
  - Confusion matrix
  - Binary classification ratios (TPR, FNR, TNR, FPR, PPV, ACC, F₁)
- ESG scoring
  - Performance, selection and discriminant curves
  - ROC curve
  - Gini coefficient
Let \((X, Y)\) be a random vector where
\[
p_{i,j} = \Pr \{X = x_i, Y = y_j\},
\]
\[
p_i = \Pr \{X = x_i\} \quad \text{and} \quad p_j = \Pr \{Y = y_j\}.
\]
The Shannon entropy of the discrete random variable \(X\) is given by:
\[
\mathcal{I}(X) = - \sum_{i=1}^{n} p_i \ln p_i
\]
We have the property \(0 \leq \mathcal{I}(X) \leq \ln n\)
The entropy is equal to zero if there is a state \(i\) such that \(p_i = 1\) and is equal to \(\ln n\) in the case of the uniform distribution \(p_i = 1/n\)
The Shannon entropy is a measure of the average information of the system
The lower the Shannon entropy, the more informative the system
Shannon entropy

- For a random vector \((X, Y)\), we have:

\[
\mathcal{I} (X, Y) = - \sum_{i=1}^{n} \sum_{j=1}^{n} p_{i,j} \ln p_{i,j}
\]

- We deduce that the conditional information of \(Y\) given \(X\) is equal to:

\[
\mathcal{I} (Y | X) = \mathbb{E} [\mathcal{I} (Y | X = x)] = - \sum_{i=1}^{n} \sum_{j=1}^{n} p_{i,j} \ln \frac{p_{i,j}}{p_i} = \mathcal{I} (X, Y) - \mathcal{I} (X)
\]

- if \(X\) and \(Y\) are independent, we have \(\mathcal{I} (Y | X) = \mathcal{I} (Y)\) and \(\mathcal{I} (X, Y) = \mathcal{I} (Y) + \mathcal{I} (X)\)

- if \(X\) and \(Y\) are perfectly dependent, we have \(\mathcal{I} (Y | X) = 0\) and \(\mathcal{I} (X, Y) = \mathcal{I} (X)\)

- The amount of information obtained about one random variable, by the other random variable, is measured by the mutual information:

\[
\mathcal{I} (X \cap Y) = \mathcal{I} (Y) + \mathcal{I} (X) - \mathcal{I} (X, Y) = \sum_{i=1}^{n} \sum_{j=1}^{n} p_{i,j} \ln \frac{p_{i,j}}{p_i p_j}
\]
Shannon entropy

Figure 37: Examples of Shannon entropy calculation

\[ I(X) = I(Y) = 1.792 \]
\[ I(X,Y) = 3.584 \]
\[ I(X \cap Y) = 0 \]

\[ I(X) = I(Y) = 1.792 \]
\[ I(X,Y) = 1.792 \]
\[ I(X \cap Y) = 1.792 \]
Shannon entropy

Figure 38: Examples of Shannon entropy calculation

\[ I(X) = I(Y) = 1.683 \]
\[ I(X, Y) = 2.774 \]
\[ I(X \cap Y) = 0.593 \]

\[ I(X) = 1.658, \quad I(Y) = 1.328 \]
\[ I(X, Y) = 2.236 \]
\[ I(X \cap Y) = 0.750 \]
Let \( S \) and \( Y \) be the score and the control variable.

For instance, \( Y \) is a binary random variable that may indicate a bad ESG risk (\( Y = 0 \)) or a good ESG risk (\( Y = 1 \)).

\( Y \) may also correspond to classes defined by some quantiles.

We can measure the information of the system \((S, Y)\).

We can compare two scores \( S_1 \) and \( S_2 \) by using the statistical measures \( I(S_1 \cap Y) \) and \( I(S_2 \cap Y) \).
Shannon entropy
Application to (credit/ESG) scoring

$S_1$: $\mathcal{I}(S_1) = 1.767$, $\mathcal{I}(Y) = 1.609$, $\mathcal{I}(S_1, Y) = 2.614$, $\mathcal{I}(S_1 \cap Y) = 0.763$

$S_2$: $\mathcal{I}(S_2) = 1.771$, $\mathcal{I}(Y) = 1.609$, $\mathcal{I}(S_2, Y) = 2.745$, $\mathcal{I}(S_2 \cap Y) = 0.636$

$S_1 \succ S_2$
The control variable $Y$ can take two values:

1. $Y = 0$ corresponds to a bad risk (or bad signal)
2. $Y = 1$ corresponds to a good risk (or good signal)

We assume that the probability $\Pr \{ Y = 1 \mid \mathbf{S} \geq s \}$ is increasing with respect to the level $s \in [0, 1]$, which corresponds to the rate of acceptance.

The decision rule is the following:

- If the score of the observation is above the threshold $s$, the observation is selected.
- If the score of the observation is below the threshold $s$, the observation is not selected.

If $s$ is equal to one, we select no observation. If $s$ is equal to zero, we select all the observations.
The performance curve is the parametric function $y = P(x)$ defined by:

\[
\begin{align*}
  x(s) &= \Pr\{S \geq s\} \\
  y(s) &= \frac{\Pr\{Y = 0 \mid S \geq s\}}{\Pr\{Y = 0\}}
\end{align*}
\]

where $x(s)$ corresponds to the proportion of selected observations and $y(s)$ corresponds to the ratio between the proportion of selected bad risks and the proportion of bad risks in the population.

- The score is efficient if the ratio is below one.
- If $y(s) > 1$, the score selects more bad risks than those we can find in the population.
- If $y(s) = 1$, the score is random and the performance is equal to zero. In this case, the selected population is representative of the total population.
The selection curve is the parametric curve \( y = S(x) \) defined by:

\[
\begin{align*}
  x(s) &= \Pr\{S \geq s\} \\
  y(s) &= \Pr\{S \geq s \mid Y = 0\}
\end{align*}
\]

where \( y(s) \) corresponds to the ratio of observations that are wrongly selected.

- We would like that the curve \( y = S(x) \) is located below the bisecting line \( y = x \) in order to verify that \( \Pr\{S \geq s \mid Y = 0\} < \Pr\{S \geq s\} \).
- The performance and selection curves are related as follows:

\[
S(x) = xP(x)
\]
The discriminant curve is the parametric curve \( y = D(x) \) defined by:

\[
D(x) = g_1(g_0^{-1}(x))
\]

where:

\[
g_y(s) = \Pr\{S \geq s \mid Y = y\}
\]

- It represents the proportion of good risks in the selected population with respect to the proportion of bad risks in the selected population.
- The score is said to be discriminant if the curve \( y = D(x) \) is located above the bisecting line \( y = x \).
The previous parametric curves do not depend on the probability distribution of the score $S$, but only on the ranking of the observations.

They are then invariant if we apply an increasing function to the score.

We have the following properties:

1. The performance curve (respectively, the selection curve) is located below the line $y = 1$ (respectively, the bisecting line $y = x$) if and only if $\text{cov}(f(Y), g(S)) \geq 0$ for any increasing functions $f$ and $g$.

2. The performance curve is increasing if and only if:

   $$\text{cov}(f(Y), g(S) \mid S \geq s) \geq 0$$

   for any increasing functions $f$ and $g$, and any threshold level $s$.

3. The selection curve is convex if and only if $\mathbb{E}[f(Y) \mid S = s]$ is increasing with respect to the threshold level $s$ for any increasing function $f$. 

Thierry Roncalli
Course 2023-2024 in Sustainable Finance 174 / 1665
A score is perfect or optimal if there is a threshold level \( s^* \) such that 

\[
\Pr\{Y = 1 \mid S \geq s^*\} = 1 \quad \text{and} \quad \Pr\{Y = 0 \mid S < s^*\} = 1.
\]

It separates the population between good and bad risks. Graphically, the selection curve of a perfect score is equal to:

\[
y = \mathbb{1}\{x > \Pr\{Y = 1\}\} \cdot \left(1 + \frac{x - 1}{\Pr\{Y = 0\}}\right)
\]

Using the relationship \( S(x) = xP(x) \), we deduce that the performance curve of a perfect score is given by:

\[
y = \mathbb{1}\{x > \Pr\{Y = 1\}\} \cdot \left(\frac{x - \Pr\{Y = 1\}}{x \cdot \Pr\{Y = 0\}}\right)
\]

For the discriminant curve, a perfect score satisfies \( D(x) = 1 \).

When the score is random, we have \( S(x) = D(x) = x \) and \( P(x) = 1 \).
Properties

Figure 39: Performance, selection and discriminant curves
Comparing two scores $S_1$ and $S_2$

- The score $S_1$ is more performing on the population $P_1$ than the score $S_2$ on the population $P_2$ if and only if the performance (or selection) curve of $(S_1, P_1)$ is below the performance (or selection) curve of $(S_2, P_2)$.

- The score $S_1$ is more discriminatory on the population $P_1$ than the score $S_2$ on the population $P_2$ if and only if the discriminant curve of $(S_1, P_1)$ is above the discriminant curve of $(S_2, P_2)$. 
Comparing two scores $S_1$ and $S_2$

Figure 40: The score $S_1$ is better than the score $S_2$
Comparing two scores $S_1$ and $S_2$

**Figure 41:** Illustration of the partial ordering between two scores

$$y = S(x)$$

$S_{\text{score}} S_1$

$S_{\text{score}} S_2$
Kolmogorov-Smirnov test
Gini coefficient
Kolmogorov-Smirnov test

- We consider the cumulative distribution functions:

  \[ F_0(s) = \Pr\{S \leq s \mid Y = 0\} \]

  and:

  \[ F_1(s) = \Pr\{S \leq s \mid Y = 1\} \]

- The score \( S \) is relevant if we have the stochastic dominance order \( F_0 \succeq F_1 \).

- In this case, the score quality is measured by the Kolmogorov-Smirnov statistic:

  \[ KS = \max_s |F_0(s) - F_1(s)| \]

- It takes the value 1 if the score is perfect.
Kolmogorov-Smirnov test

Figure 42: Comparison of the distributions $F_0(s)$ and $F_1(s)$
The Lorenz curve $y = \mathcal{L}(x)$ is the parametric curve defined by:

$$\begin{cases} 
    x = \Pr\{X \leq x\} \\
    y = \Pr\{Y \leq y \mid X \leq x\}
\end{cases}$$

The Lorenz curve has two limit cases

- If the wealth is perfectly concentrated, one individual holds 100% of the total wealth
- If the wealth is perfectly allocated between all the individuals, the corresponding Lorenz curve is the bisecting line
Figure 43: An example of Lorenz curve
We define the Gini coefficient by:

\[
Gini(\mathcal{L}) = \frac{A}{A+B} = 1 - 2 \int_0^1 \mathcal{L}(x) \, dx
\]

where \(A\) is the area between the Lorenz curve and the curve of perfect equality, and \(B\) is the area between the curve of perfect concentration and the Lorenz curve.

- We have \(0 \leq Gini(\mathcal{L}) \leq 1\).
- The Gini coefficient is equal to zero in the case of perfect equality and one in the case of perfect concentration.
Application to scoring

- The selection curve is a Lorenz curve. We recall that
  \( F(s) = \Pr\{S \leq s\} \), \( F_0(s) = \Pr\{S \leq s \mid Y = 0\} \) and
  \( F_1(s) = \Pr\{S \leq s \mid Y = 1\} \). The selection curve is defined by the
  following parametric coordinates:

  \[
  \begin{align*}
  x(s) &= 1 - F(s) \\
  y(s) &= 1 - F_0(s)
  \end{align*}
  \]

  The selection curve measures the capacity of the score for not
  selecting bad risks

- The precision curve is the Lorenz curve that measures the capacity
  of the score for selecting good risks

  \[
  \begin{align*}
  x(s) &= \Pr\{S \geq s\} = 1 - F(s) \\
  y(s) &= \Pr\{S \geq s \mid Y = 1\} = 1 - F_1(s)
  \end{align*}
  \]
The receiver operating characteristic (ROC) curve is defined by:

\[
\begin{align*}
    x(s) &= \Pr \{ S \geq s \mid Y = 0 \} = 1 - F_0(s) \\
    y(s) &= \Pr \{ S \geq s \mid Y = 1 \} = 1 - F_1(s)
\end{align*}
\]

The AUC measure, which corresponds to the area under the ROC curve, gives the same information than the Gini coefficient since they are related by the equation:

\[
Gini \ (ROC) = 2 \times AUC \ (ROC) - 1
\]
Application to scoring

Figure 44: Selection, precision and ROC curves
Choice of the optimal cut-off

The choice of the optimal cut-off $s^*$ depends on the objective function. For instance, we can calibrate $s^*$ in order to achieve a minimum universe size of ESG assets. We can also fix a given selection rate. From a statistical point of view, we must distinguish the construction of the scoring model and the decision rule. In statistical learning, we generally consider three datasets: the training set, the validation set and the test set. The training set is used for calibrating the model and its parameters whereas the validation set helps to avoid overfitting. But the decision rule is based on the test set.
A confusion matrix is a special case of contingency matrix.

Each row of the matrix represents the frequency in a predicted class while each column represents the frequency in an actual class.

Using the test set, it takes the following form:

<table>
<thead>
<tr>
<th></th>
<th>$Y = 0$</th>
<th>$Y = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S &lt; s$</td>
<td>$n_{0,0}$</td>
<td>$n_{0,1}$</td>
</tr>
<tr>
<td>$S \geq s$</td>
<td>$n_{1,0}$</td>
<td>$n_{1,1}$</td>
</tr>
</tbody>
</table>

$n_0 = n_{0,0} + n_{1,0}$  
$n_1 = n_{0,1} + n_{1,1}$

where $n_{i,j}$ represents the number of observations of the cell $(i, j)$.
### Confusion matrix

<table>
<thead>
<tr>
<th></th>
<th>$Y = 0$</th>
<th>$Y = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S &lt; s$</td>
<td>It is rejected and it is a bad risk (true negative)</td>
<td>It is rejected, but it is a good risk (false negative)</td>
</tr>
<tr>
<td>$S \geq s$</td>
<td>It is accepted, but it is a bad risk (false positive)</td>
<td>It is accepted and it is a good risk (true positive)</td>
</tr>
</tbody>
</table>

The cells $(S < s, Y = 0)$ and $(S \geq s, Y = 1)$ correspond to observations that are well-classified: true negative (TN) and true positive (TP). The cells $(S \geq s, Y = 0)$ and $(S < s, Y = 1)$ correspond to two types of errors:

- **A false positive (FP) can induce a future loss, because the risk can materialize:** this is a type I error
- **A false negative (FN) potentially corresponds to a an opportunity cost:** this is a type II error
Classification ratios

True Positive Rate \[ TPR = \frac{TP}{TP + FN} \]
False Negative Rate \[ FNR = \frac{FN}{FN + TP} = 1 - TPR \]
True Negative Rate \[ TNR = \frac{TN}{TN + FP} \]
False Positive Rate \[ FPR = \frac{FP}{FP + TN} = 1 - TNR \]
Positive Predictive Value \[ PPV = \frac{TP}{TP + FP} \]

- The true positive rate (TPR) is also known as the sensitivity or the recall. It measures the proportion of real good risks that are correctly predicted good risk.
- The precision or the positive predictive value (PPV) measures the proportion of predicted good risks that are correctly real good risk.
The accuracy considers the classification of both negatives and positives:

$$\text{ACC} = \frac{TP + TN}{P + N} = \frac{TP + TN}{TP + FN + TN + FP}$$

The $F_1$ score is the harmonic mean of precision and sensitivity:

$$F_1 = \frac{2}{1/\text{precision} + 1/\text{sensitivity}} = \frac{2 \cdot \text{PPV} \cdot \text{TPR}}{\text{PPV} + \text{TPR}}$$

The $\phi$ coefficient or the Matthews correlation coefficient (MCC) is a measure of association between $S$ and $Y$:

$$\phi = \text{MCC} = \frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP + FP) \cdot (TP + FN) \cdot (TN + FP) \cdot (TN + FN)}}$$

$S$ and $Y$ are positively associated if most of the observations fall along the diagonal cells.
Let $S_i(t)$ be the ESG score of company $i$ at time $t$. The endogenous response variable can be defined as follows:

<table>
<thead>
<tr>
<th>Scoring system</th>
<th>Risk class</th>
<th>$Y_i(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best-in-class oriented</td>
<td>Good risk</td>
<td>$1 { S_i(t + \delta) \geq s^* }$</td>
</tr>
<tr>
<td>Worst-in-class oriented</td>
<td>Bad risk</td>
<td>$1 { S_i(t + \delta) \leq s^* }$</td>
</tr>
</tbody>
</table>

where $s^*$ is the best-in-class/worst-in-class threshold to determine. $Y_i(t)$ is endogenous because it depends on the future value of the score. Here, the backtesting procedure can be seen as a stability test of the ESG scoring system. An alternative is to use an exogenous response variable based on controversies. For example, to predict bad risks, we can use the binary response $Y_i(t) = 1 \{ C_i(t + \delta) \geq 0 \}$ where $C_i(t)$ is the controversy index.
Backtesting of unsupervised scoring systems

Dynamic analysis

- We consider the past momentum \( M_i(t, h) = S_i(t) - S_i(t - h) \) where \( h \) is typically the year.
- The response variable is based on the future momentum \( S_i(t + \delta) - S_i(t) \).
Illustration using an ESG scoring system

We consider four risk classes:

<table>
<thead>
<tr>
<th>Risk class</th>
<th>Definition</th>
<th>$Y_i(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst-in-class</td>
<td>$S_i(t) \leq \hat{F}^{-1}(20%)$</td>
<td>$\mathbb{1}{S_i(t + \delta) \leq s^*}$</td>
</tr>
<tr>
<td>Bad risk</td>
<td>$S_i(t) \leq \bar{S}$</td>
<td>$\mathbb{1}{S_i(t + \delta) \leq s^*}$</td>
</tr>
<tr>
<td>Good risk</td>
<td>$S_i(t) \geq \bar{S}$</td>
<td>$\mathbb{1}{S_i(t + \delta) \geq s^*}$</td>
</tr>
<tr>
<td>Best-in-class</td>
<td>$S_i(t) \geq \hat{F}^{-1}(80%)$</td>
<td>$\mathbb{1}{S_i(t + \delta) \geq s^*}$</td>
</tr>
</tbody>
</table>

where $\hat{F}$ is the empirical distribution of the score and $\bar{S}$ is the average of scores.
Illustration using an ESG scoring system

Table 20: Optimal cut-off $s^*$ (MSCI World)

<table>
<thead>
<tr>
<th>Risk class</th>
<th>$\delta = 3$ months</th>
<th>$\delta = 12$ months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACC</td>
<td>$F_1$</td>
</tr>
<tr>
<td>Worst-in-class</td>
<td>$-0.91$</td>
<td>$-0.61$</td>
</tr>
<tr>
<td>Bad risk</td>
<td>$-0.01$</td>
<td>$0.18$</td>
</tr>
<tr>
<td>Good risk</td>
<td>$-0.02$</td>
<td>$-0.18$</td>
</tr>
<tr>
<td>Best-in-class</td>
<td>$1.05$</td>
<td>$0.79$</td>
</tr>
</tbody>
</table>
**Table 21:** Optimal cut-off $s^*$ (MSCI EM)

<table>
<thead>
<tr>
<th>Risk class</th>
<th>$\delta = 3$ months</th>
<th>$\delta = 12$ months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACC</td>
<td>$F_1$</td>
</tr>
<tr>
<td>Worst-in-class</td>
<td>$-1.87$</td>
<td>$-1.19$</td>
</tr>
<tr>
<td>Bad risk</td>
<td>$0.13$</td>
<td>$0.23$</td>
</tr>
<tr>
<td>Good risk</td>
<td>$0.13$</td>
<td>$-0.15$</td>
</tr>
<tr>
<td>Best-in-class</td>
<td>$0.48$</td>
<td>$0.14$</td>
</tr>
</tbody>
</table>
Illustration using an ESG scoring system

Remark

Theoretically, the optimal cut-off is \( s^* = \Phi^{-1}(20\%) = -0.8416 \) for the worst-in-class category, \( s^* = \mathbb{E}[N(0, 1)] = 0 \) for the bad-risk and good-risk categories and \( s^* = \Phi^{-1}(80\%) = 0.8416 \) for the best-in-class category, because the backtesting procedure concerns \( z \)-scores.
Illustration using an ESG scoring system

Figure 45: Backtesting of ESG scores (worst-in-class & bad risk, MSCI World)
Illustration using an ESG scoring system

Figure 46: Backtesting of ESG scores (best-in-class & good risk, MSCI World)
Illustration using an ESG scoring system

Figure 47: Backtesting of ESG scores (worst-in-class & bad risk, MSCI EM)
Illustration using an ESG scoring system

Figure 48: Backtesting of ESG scores (best-in-class & good risk, MSCI EM)
### Table 22: Credit rating system of S&P, Moody's and Fitch

<table>
<thead>
<tr>
<th>Prime</th>
<th>High Grade</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Safety</td>
<td>High Quality</td>
<td>Medium Grade</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td>Moody’s</td>
<td></td>
</tr>
<tr>
<td>AAA</td>
<td>AA+</td>
<td>A+</td>
</tr>
<tr>
<td>Aaa</td>
<td>AA</td>
<td>A</td>
</tr>
<tr>
<td>A-</td>
<td>AA-</td>
<td>A-</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td>Moody’s</td>
<td></td>
</tr>
<tr>
<td>BBB+</td>
<td>BBB</td>
<td>BB+</td>
</tr>
<tr>
<td>Baa1</td>
<td>Baa2</td>
<td>Ba1</td>
</tr>
<tr>
<td>Baa3</td>
<td>Baa3</td>
<td>Ba2</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td>Moody’s</td>
<td></td>
</tr>
<tr>
<td>B+</td>
<td>B</td>
<td>BB</td>
</tr>
<tr>
<td>B1</td>
<td>B2</td>
<td>Ba</td>
</tr>
<tr>
<td>B3</td>
<td>Baa1</td>
<td>Ba2</td>
</tr>
<tr>
<td>CCC+</td>
<td>CCC</td>
<td>CCC</td>
</tr>
<tr>
<td>Caa1</td>
<td>Caa2</td>
<td>Caa3</td>
</tr>
<tr>
<td>Ca</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower</th>
<th>Non Investment Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium Grade</td>
<td>Speculative</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td>Moody’s</td>
</tr>
<tr>
<td>BBB+</td>
<td>BBB</td>
</tr>
<tr>
<td>Baa1</td>
<td>Baa2</td>
</tr>
<tr>
<td>Baa3</td>
<td>Baa3</td>
</tr>
<tr>
<td>S&amp;P/Fitch</td>
<td>Moody’s</td>
</tr>
<tr>
<td>B+</td>
<td>B</td>
</tr>
<tr>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>B3</td>
<td>Baa1</td>
</tr>
<tr>
<td>CCC+</td>
<td>CCC</td>
</tr>
<tr>
<td>Caa1</td>
<td>Caa2</td>
</tr>
<tr>
<td>Ca</td>
<td></td>
</tr>
</tbody>
</table>
Amundi: A (high), B,... to G (low) — 7-grade scale
FTSE Russell: 0 (low), 1,... to 5 (high) — 6-grade scale
ISS ESG: 1 (high), 2,... to 10 (low) — 10-grade scale
MSCI: AAA (high), AA,... to CCC (low) — 7-grade scale
Refinitiv: A+ (high), A, A-, B+,... to D- (low) — 12-grade scale
RepRisk: AAA (high), AA,... to D (low) — 8-grade scale
Sustainalytics: 1 (low), 2,... to 5 (high) — 5-grade scale
Figure 49: From ESG score to ESG rating

Two-step approach:

1. Specification of the map function:

\[
\text{Map} : \Omega_S \rightarrow \Omega_R \\
\mathcal{S} \mapsto \mathcal{R} = \text{Map}(\mathcal{S})
\]

where \(\Omega_S\) is the support of ESG scores, \(\Omega_R\) is the ordered state space of ESG ratings and \(\mathcal{R}\) is the ESG rating.

2. Validation (and the possible forcing) of the rating by the analyst.
ESG rating process

Example with the MSCI ESG rating system

- $\Omega_S = [0, 10]$
- $\Omega_R = \{\text{CCC, B, BB, BBB, A, AA, AAA}\}$
- The map function is defined as

$$\text{Map}(s) = \begin{cases} 
\text{CCC} & \text{if } s \in [0, \frac{10}{7}] \\
\text{B} & \text{if } s \in [\frac{10}{7}, \frac{20}{7}] \\
\text{BB} & \text{if } s \in [\frac{20}{7}, \frac{30}{7}] \\
\text{BBB} & \text{if } s \in [\frac{30}{7}, \frac{40}{7}] \\
\text{A} & \text{if } s \in [\frac{40}{7}, \frac{50}{7}] \\
\text{AA} & \text{if } s \in [\frac{50}{7}, \frac{60}{7}] \\
\text{AAA} & \text{if } s \in [\frac{60}{7}, 10] 
\end{cases}$$

Thierry Roncalli
Course 2023-2024 in Sustainable Finance 207 / 1665
The map function is an increasing piecewise function

\( S \sim F \) and \( S \in (s^-, s^+) \)

\( \{s_0^* = s^-, s_1^*, \ldots, s_{K-1}^*, s_K^* = s^+\} \) are the knots of the piecewise function

\( \Omega_R = \{R_1, \ldots, R_K\} \) is the set of grades

\( \Rightarrow \) The frequency distribution of the ratings is given by:

\[
p_k = \Pr \{R = R_k\} = \Pr \{s_{k-1}^* \leq S < s_k^*\} = F(s_k^*) - F(s_{k-1}^*)
\]
If we would like to build a rating system with pre-defined frequencies \((p_1, \ldots, p_K)\), we have to solve the following equation:

\[ F(s_k^*) - F(s_{k-1}^*) = p_k \]

We deduce that:

\[
F(s_k^*) = p_k + F(s_{k-1}^*) \\
= p_k + p_{k-1} + F(s_{k-2}^*) \\
= \left( \sum_{j=1}^{k} p_j \right) + F(s_0^*)
\]

and:

\[
s_k^* = F^{-1}\left( \sum_{j=1}^{k} p_j \right)
\]
Exercise

- We assume that $S \sim U_{[a,b]}$
- Show that $p_k = K^{-1}$ If the rating system consists in $K$ equally-sized intervals
- Show that the knots of the map function are equal to:

$$s_k^* = a + (b - a) \left( \sum_{j=1}^{k} p_j \right)$$

when we impose pre-defined frequencies $(p_1, \ldots, p_K)$
- If we consider a 0/100 uniform score and $\Omega_{\mathcal{R}} \times \mathcal{P} = (\text{CCC}, 5\%)$, (B, 10\%), (BB, 20\%), (BBB, 30\%), (A, 20\%), (AA, 10\%), (AAA, 5\%), show that $s^*_\text{CCC} = 5$, $s^*_B = 15$, $s^*_BB = 35$, $s^*_BBB = 65$, $s^*_A = 85$ and $s^*_AA = 95$
For a $z$-score system ($S \sim N(0, 1)$), we obtain:

$$p_k = \Phi (s_k^*) - \Phi (s_{k-1}^*)$$
Figure 50: Map function of a $z$-score (equal-space ratings)
Figure 51: Map function of a $z$-score (equal-frequency ratings)
Table 23: ESG migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>AA</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>A</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>BBB</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>BB</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>B</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
<tr>
<td>CCC</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
<td>14.3%</td>
</tr>
</tbody>
</table>

⇒ I(\mathcal{R}(t) | \mathcal{R}(s)) = \ln 7
Table 24: ESG migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>AA</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>A</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>BBB</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>BB</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>CCC</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

⇒ Ι(\(R(t) | R(s)\)) = 0
Table 25: ESG migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>96%</td>
<td>4%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>AA</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>A</td>
<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>BBB</td>
<td>0%</td>
<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>BB</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
<td>0%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>2%</td>
<td>96%</td>
<td>2%</td>
</tr>
<tr>
<td>CCC</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4%</td>
<td>96%</td>
</tr>
</tbody>
</table>

\[ \Rightarrow 0 < \mathcal{I}(\mathcal{R}(t) \mid \mathcal{R}(s)) \ll \ln 7 \]
A good reference on Markov chains is:

Rating migration matrix

Discrete time modeling

Definition

- $\mathcal{R}$ is a time-homogeneous Markov chain
- $\Omega_{\mathcal{R}} = \{R_1, \ldots, R_K\}$ is the state space of the chain
- $\mathbb{K} = \{1, \ldots, K\}$ is the corresponding index set
- The transition matrix is defined as $P = (p_{i,j})$
- $p_{i,j}$ is the probability that the entity migrates from rating $R_i$ to rating $R_j$
- The matrix $P$ satisfies the following properties:
  - $\forall i, j \in \mathbb{K}, p_{i,j} \geq 0$
  - $\forall i \in \mathbb{K}, \sum_{j=1}^{K} p_{i,j} = 1$
Table 26: ESG migration matrix #1 (one-year transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.76</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.15</td>
<td>82.73</td>
<td>11.86</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.18</td>
<td>15.47</td>
<td>72.98</td>
<td>10.46</td>
<td>0.82</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.07</td>
<td>1.32</td>
<td>19.60</td>
<td>69.49</td>
<td>9.03</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.19</td>
<td>1.55</td>
<td>19.36</td>
<td>70.88</td>
<td>7.75</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.05</td>
<td>0.24</td>
<td>1.43</td>
<td>21.54</td>
<td>74.36</td>
<td>2.38</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
<td>0.44</td>
<td>2.21</td>
<td>13.24</td>
<td>83.89</td>
</tr>
</tbody>
</table>
The probability that the entity reaches the state $R_j$ at time $t$ given that it has reached the state $R_i$ at time $s$ is equal to:

$$p(s, i; t, j) = \Pr \{ \mathcal{R}(t) = R_j \mid \mathcal{R}(s) = R_i \} = p_{i,j}^{(t-s)}$$

We note $p_{i,j}^{(n)}$ the $n$-step transition probability:

$$p_{i,j}^{(n)} = \Pr \{ \mathcal{R}(t + n) = R_j \mid \mathcal{R}(t) = R_i \}$$

and the associated $n$-step transition matrix $P^{(n)} = \begin{pmatrix} p_{i,j}^{(n)} \end{pmatrix}$
For $n = 2$, we obtain:

$$p_{i,j}^{(2)} = \Pr \{ R(t + 2) = R_j \mid R(t) = R_i \}$$

$$= \sum_{k=1}^{K} \Pr \{ R(t + 2) = R_j, R(t + 1) = R_k \mid R(t) = R_i \}$$

$$= \sum_{k=1}^{K} \Pr \{ R(t + 2) = R_j \mid R(t + 1) = R_k \} \cdot \Pr \{ R(t + 1) = R_k \mid R(t) = R_i \}$$

$$= \sum_{k=1}^{K} p_{i,k} \cdot p_{k,j}$$
The forward Chapman-Kolmogorov equation is:

$$p_{i,j}^{(n+m)} = \sum_{k=1}^{K} p_{i,k}^{(n)} \cdot p_{k,j}^{(m)} \quad \forall n, m > 0$$

or $P^{(n+m)} = P^{(n)} \cdot P^{(m)}$ with $P^{(0)} = I$

We have:

$$P^{(n)} = P^{(n-1)} \cdot P^{(1)}$$
$$= P^{(n-2)} \cdot P^{(1)} \cdot P^{(1)}$$
$$= \prod_{t=1}^{n} P^{(1)}$$
$$= P^{n}$$

We deduce that:

$$p(t, i; t + n, j) = p_{i,j}^{(n)} = e_i^\top P^n e_j$$
### Table 27: Two-year transition probability in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>86.28</td>
<td>10.08</td>
<td>2.25</td>
<td>0.92</td>
<td>0.44</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>7.30</td>
<td>70.52</td>
<td>18.68</td>
<td>2.67</td>
<td>0.66</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.95</td>
<td>24.24</td>
<td>57.16</td>
<td>15.20</td>
<td>2.19</td>
<td>0.25</td>
<td>0.01</td>
</tr>
<tr>
<td>BBB</td>
<td>0.21</td>
<td>5.06</td>
<td>28.22</td>
<td>52.11</td>
<td>12.93</td>
<td>1.33</td>
<td>0.14</td>
</tr>
<tr>
<td>BB</td>
<td>0.09</td>
<td>0.79</td>
<td>6.07</td>
<td>27.45</td>
<td>53.68</td>
<td>11.37</td>
<td>0.55</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.18</td>
<td>0.98</td>
<td>6.26</td>
<td>31.47</td>
<td>57.28</td>
<td>3.82</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.05</td>
<td>0.50</td>
<td>1.32</td>
<td>6.31</td>
<td>21.13</td>
<td>70.70</td>
</tr>
</tbody>
</table>
We have:

\[ p_{AAA,AAA}^{(2)} = p_{AAA,AAA} \times p_{AAA,AAA} + p_{AAA,AA} \times p_{AA,AAA} + p_{AAA,A} \times p_{A,AAA} + \]
\[ p_{AAA,BBB} \times p_{BBB,AAA} + p_{AAA,BB} \times p_{BB,AAA} + \]
\[ p_{AAA,B} \times p_{B,AAA} + p_{AAA,CCC} \times p_{CCC,AAA} \]

\[ = 0.9276^2 + 0.0566 \times 0.0415 + 0.0090 \times 0.0018 + \]
\[ 0.0045 \times 0.0007 + 0.0023 \times 0.0004 \]

\[ = 86.28\% \]
### Table 28: Five-year transition probability in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>70.45</td>
<td>18.69</td>
<td>6.97</td>
<td>2.61</td>
<td>1.08</td>
<td>0.18</td>
<td>0.01</td>
</tr>
<tr>
<td>AA</td>
<td>13.13</td>
<td>50.21</td>
<td>26.03</td>
<td>7.90</td>
<td>2.22</td>
<td>0.48</td>
<td>0.03</td>
</tr>
<tr>
<td>A</td>
<td>4.35</td>
<td>33.20</td>
<td>37.78</td>
<td>17.99</td>
<td>5.52</td>
<td>1.08</td>
<td>0.09</td>
</tr>
<tr>
<td>BBB</td>
<td>1.50</td>
<td>16.49</td>
<td>32.49</td>
<td>30.90</td>
<td>14.61</td>
<td>3.63</td>
<td>0.38</td>
</tr>
<tr>
<td>BB</td>
<td>0.50</td>
<td>5.98</td>
<td>17.83</td>
<td>30.10</td>
<td>31.35</td>
<td>12.85</td>
<td>1.39</td>
</tr>
<tr>
<td>B</td>
<td>0.15</td>
<td>1.90</td>
<td>7.40</td>
<td>18.95</td>
<td>35.11</td>
<td>31.26</td>
<td>5.23</td>
</tr>
<tr>
<td>CCC</td>
<td>0.05</td>
<td>0.64</td>
<td>2.55</td>
<td>6.93</td>
<td>17.96</td>
<td>38.54</td>
<td>43.33</td>
</tr>
</tbody>
</table>
Rating migration matrix
Discrete time modeling

Stationary distribution

- \( \pi^{(n)}_k = \Pr \{ R(n) = R_k \} \) is the probability of the state \( R_k \) at time \( n \):
- \( \pi^{(n)} = (\pi_1^{(n)}, \ldots, \pi_K^{(n)}) \) satisfies \( \pi^{(n+1)} = P^\top \pi^{(n)} \)
- The Markov chain \( R \) has a stationary distribution \( \pi^* \) if \( \pi^* = P^\top \pi^* \)
- \( T_k = \inf \{ n : R(n) = R_k \mid R(0) = R_k \} \) is the return period of state \( R_k \)
- The average return period is then equal to:

\[
\tau_k := \mathbb{E}[T_k] = \frac{1}{\pi^*_k}
\]
We obtain:

\[ \pi^* = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%) \]

- The average return periods are then equal to 5.6, 3.4, 4.0, 6.6, 12.0, 30.4 and 149.0 years

⇒ Best-in-class (or winning-) oriented system
Table 29: ESG migration matrix #2 (one-month transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>93.50</td>
<td>5.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>2.00</td>
<td>93.00</td>
<td>4.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.00</td>
<td>3.00</td>
<td>93.00</td>
<td>3.90</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.00</td>
<td>0.10</td>
<td>2.80</td>
<td>94.00</td>
<td>3.00</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>BB</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>3.50</td>
<td>94.50</td>
<td>1.80</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>3.70</td>
<td>96.00</td>
<td>0.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>98.50</td>
</tr>
</tbody>
</table>

⇒ The stationary distribution is

\[ \pi^* = (3.11\%, 10.10\%, 17.46\%, 27.76\%, 25.50\%, 12.68\%, 3.39\%) \]

and the average return periods are equal to 32.2, 9.9, 5.7, 3.6, 3.9, 7.9 and 29.5 years

⇒ balanced rating system
Table 30: One-year probability transition in % (migration matrix #2)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>48.06</td>
<td>29.71</td>
<td>10.34</td>
<td>6.42</td>
<td>4.95</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>AA</td>
<td>11.65</td>
<td>49.25</td>
<td>24.10</td>
<td>9.60</td>
<td>4.87</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>A</td>
<td>2.02</td>
<td>17.51</td>
<td>49.67</td>
<td>24.72</td>
<td>5.52</td>
<td>0.54</td>
<td>0.03</td>
</tr>
<tr>
<td>BBB</td>
<td>0.27</td>
<td>3.53</td>
<td>17.46</td>
<td>55.50</td>
<td>20.21</td>
<td>2.88</td>
<td>0.16</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.60</td>
<td>4.21</td>
<td>23.43</td>
<td>57.45</td>
<td>13.27</td>
<td>1.01</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.08</td>
<td>0.74</td>
<td>5.94</td>
<td>27.10</td>
<td>64.18</td>
<td>1.96</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.07</td>
<td>0.57</td>
<td>4.22</td>
<td>5.77</td>
<td>5.85</td>
<td>83.51</td>
</tr>
</tbody>
</table>
### Table 31: One-month probability transition in % (migration matrix #1)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>99.36</td>
<td>0.53</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.39</td>
<td>98.31</td>
<td>1.26</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>-0.02</td>
<td>1.65</td>
<td>97.14</td>
<td>1.21</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.01</td>
<td>-0.07</td>
<td>2.28</td>
<td>96.72</td>
<td>1.06</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>BB</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.12</td>
<td>2.29</td>
<td>96.92</td>
<td>0.88</td>
<td>0.01</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>-0.15</td>
<td>2.45</td>
<td>97.42</td>
<td>0.25</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>1.37</td>
<td>98.53</td>
</tr>
</tbody>
</table>

⇒ Negative probabilities

The ESG rating system is not Markovian!
Mean hitting time

- Let \( \mathcal{A} \subset K \) be a given subset. The first hitting time of \( \mathcal{A} \) is given by:

\[
T(\mathcal{A}) = \inf \{ n : R(n) \in \mathcal{A} \}
\]

- The mean first hitting time to target \( \mathcal{A} \) from state \( k \) is defined as:

\[
\tau_k(\mathcal{A}) = \mathbb{E}[T(\mathcal{A}) | R(0) = R_k]
\]

- We can show that \( \tau_k(\mathcal{A}) = 1 + \sum_{j=1}^{K} p_{k,j} \tau_j(\mathcal{A}) \)

- The solution is given by the LP problem:

\[
\tau(\mathcal{A}) = \arg \min \sum_{k=1}^{K} x_k \quad \text{s.t.} \quad \begin{cases}
  x_k = 0 & \text{if } k \in \mathcal{A} \\
  x_k = 1 + \sum_{j=1}^{K} p_{k,j} x_j & \text{if } k \notin \mathcal{A} \\
  x_k \geq 0
\end{cases}
\]
- $B = \{\text{AAA, AA, A}\}$
- $\mathcal{W} = \{\text{BB, B, CCC}\}$

<table>
<thead>
<tr>
<th>Rating system</th>
<th>$\mathcal{W}$-target</th>
<th>$B$-target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AAA</td>
<td>AA</td>
</tr>
<tr>
<td>#1</td>
<td>79.21</td>
<td>70.04</td>
</tr>
</tbody>
</table>
Rating migration matrix

Estimation of the transition matrix

Theoretical approach:

- Bayes theorem:

\[
p_{i,j} = \frac{\Pr \{ R(t+1) = R_j \mid R(t) = R_i \}}{\Pr \{ R(t) = R_i \}}
\]

We have seen that:

\[
\Pr \{ R(t) = R_k \} = F(s^*_k) - F(s^*_{k-1}) = p_k
\]

We deduce that:

\[
p_{i,j} = \frac{\mathcal{C}(F(s^*_i), F(s^*_j)) - \mathcal{C}(F(s^*_{i-1}), F(s^*_j)) - \mathcal{C}(F(s^*_i), F(s^*_{j-1})) + \mathcal{C}(F(s^*_{i-1}), F(s^*_{j-1}))}{F(s^*_i) - F(s^*_{i-1})}
\]

where \( \mathcal{C} \) is the copula function of the random vector \((S(t), S(t+1))\)
Non-parametric approach:

$$\hat{p}_{i,j}(t) = \frac{\# \{ R(t+1) = R_j, R(t) = R_i \}}{\# \{ R(t) = R_i \}} = \frac{n_{i,j}(t)}{n_{i,.}(t)}$$

⇒ cohort method vs. pooling method
### Table 32: Number of observations $n_{i,j}$ (migration matrix #1)

<table>
<thead>
<tr>
<th>$n_{i,j}$</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>$n_{i,.}(t)$</th>
<th>$\hat{p}_{i,.}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>2050</td>
<td>125</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2210</td>
<td>3.683%</td>
</tr>
<tr>
<td>AA</td>
<td>280</td>
<td>5580</td>
<td>800</td>
<td>60</td>
<td>20</td>
<td>5</td>
<td>0</td>
<td>6745</td>
<td>11.242%</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>1700</td>
<td>8020</td>
<td>1150</td>
<td>90</td>
<td>10</td>
<td>0</td>
<td>10990</td>
<td>18.317%</td>
</tr>
<tr>
<td>BBB</td>
<td>10</td>
<td>190</td>
<td>2820</td>
<td>10000</td>
<td>1300</td>
<td>60</td>
<td>10</td>
<td>14390</td>
<td>23.983%</td>
</tr>
<tr>
<td>BB</td>
<td>5</td>
<td>25</td>
<td>200</td>
<td>2500</td>
<td>9150</td>
<td>1000</td>
<td>30</td>
<td>12910</td>
<td>21.517%</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>5</td>
<td>25</td>
<td>150</td>
<td>2260</td>
<td>7800</td>
<td>250</td>
<td>10490</td>
<td>17.483%</td>
</tr>
<tr>
<td>CCC</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>300</td>
<td>1900</td>
<td>2265</td>
<td>3.775%</td>
</tr>
<tr>
<td>$n_{.j}(t)$</td>
<td>2365</td>
<td>7625</td>
<td>11890</td>
<td>13850</td>
<td>12875</td>
<td>9175</td>
<td>2190</td>
<td>60000</td>
<td>100.00%</td>
</tr>
<tr>
<td>$\hat{p}_{.j}(t)$</td>
<td>3.942%</td>
<td>12.708%</td>
<td>19.817%</td>
<td>23.133%</td>
<td>21.458%</td>
<td>15.292%</td>
<td>3.650%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For the migration matrix #1, we have:

$$\pi^* = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%)$$

The initial empirical distribution of ratings is:

$$\hat{\pi}^{(0)} = (3.683\%, 11.242\%, 18.317\%, 23.983\%, 21.517\%, 17.483\%, 3.775\%)$$

We have:

$$\hat{\pi}^{(1)} = \hat{P}^T \hat{\pi}^{(0)}$$

Figure 52: Dynamics of the probability distribution $\pi(n)$ (migration matrix #1)
The transition matrix is defined as follows:

\[ P_{i,j}(s; t) = p(s, i; t, j) = \Pr \{ R(t) = R_j \mid R(s) = R_i \} \]

If \( R \) is a time-homogenous Markov, we have:

\[ P(t) = P(0; t) = \exp(t\Lambda) \]

\( \Lambda = (\lambda_{i,j}) \) is the Markov generator matrix \( \Lambda = (\lambda_{i,j}) \) where \( \lambda_{i,j} \geq 0 \) for all \( i \neq j \) and \( \lambda_{i,i} = -\sum_{j \neq i}^{K} \lambda_{i,j} \).
An example

- Rating system with three states: A (good rating), B (average rating) and C (bad rating)
- The Markov generator is equal to:

\[ \Lambda = \begin{pmatrix} -0.30 & 0.20 & 0.10 \\ 0.15 & -0.40 & 0.25 \\ 0.10 & 0.15 & -0.25 \end{pmatrix} \]
The one-year transition probability matrix is equal to:

$$ P(1) = e^{\Lambda} = \begin{pmatrix} 75.63\% & 14.84\% & 9.53\% \\ 11.63\% & 69.50\% & 18.87\% \\ 8.52\% & 11.73\% & 79.75\% \end{pmatrix} $$

For the two-year maturity, we get:

$$ P(2) = e^{2\Lambda} = \begin{pmatrix} 59.74\% & 22.65\% & 17.61\% \\ 18.49\% & 52.24\% & 29.27\% \\ 14.60\% & 18.76\% & 66.63\% \end{pmatrix} $$

We verify that $P(2) = P(1) \cdot P(1)$ because:

$$ P(t) = e^{t\Lambda} = (e^{\Lambda})^t = P(1)^t $$

We have:

$$ P\left(\frac{1}{12}\right) = e^{\frac{1}{12}\Lambda} = \begin{pmatrix} 97.54\% & 1.62\% & 0.83 \\ 1.22\% & 96.74\% & 2.03 \\ 0.82\% & 1.22\% & 97.95 \end{pmatrix} $$
We consider the matrix function in the space $\mathbb{M}$ of square matrices:

$$f : \mathbb{M} \rightarrow \mathbb{M}$$

$$A \mapsto B = f(A)$$

For instance, if $f(x) = \sqrt{x}$ and $A$ is positive, we can define the matrix $B$ such that:

$$BB^* = B^*B = A$$

$B$ is called the square root of $A$ and we note $B = A^{1/2}$
We consider the following Taylor expansion:

\[ f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \ldots \]

We can show that if the series converge for \( |x - x_0| < \alpha \), then the matrix \( f(A) \) defined by the following expression:

\[ f(A) = f(x_0) + (A - x_0 I) f'(x_0) + \frac{(A - x_0 I)^2}{2!} f''(x_0) + \ldots \]

converges to the matrix \( B \) if \( |A - x_0 I| < \alpha \) and we note \( B = f(A) \)
In the case of the exponential function, we have:

\[ f(x) = e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \]

We deduce that the exponential of the matrix \( A \) is equal to:

\[ B = e^A = \sum_{k=0}^{\infty} \frac{A^k}{k''!} \]

The logarithm of \( A \) is the matrix \( B \) such that \( e^B = A \) and we note \( B = \ln A \).
Let $A$ and $B$ be two $n \times n$ square matrices. We have the properties:

\[
\begin{align*}
    f(A^\top) &= f(A)^\top \\
    Af(A) &= f(A)A \\
    f(B^{-1}AB) &= B^{-1}f(A)B
\end{align*}
\]

It follows that:

\[
\begin{cases}
    e^{A^\top} = (e^A)^\top \\
    e^{B^{-1}AB} = B^{-1}e^AB \\
    Ae^B = e^BA & \text{if } AB = BA \\
    e^{A+B} = e^Ae^B = e^Be^A & \text{if } AB = BA
\end{cases}
\]
The Schur decomposition of the $n \times n$ matrix $A$ is equal to:

$$A = QTQ^*$$

where $Q$ is a unitary matrix and $T$ is an upper triangular matrix.

For transcendental functions, we have:

$$f(A) = Qf(T)Q^*$$

where $A = QTQ^*$ is the Schur decomposition of $A$. 
Estimation of the Markov generator

We have:

\[ \hat{\Lambda} = \frac{1}{t} \ln \left( \hat{P}(t) \right) \]

⇒ \( \hat{\Lambda} \) may not verify the Markov conditions: \( \hat{\lambda}_{i,j} \geq 0 \) for all \( i \neq j \) and
\[ \sum_{j=1}^{K} \lambda_{i,j} = 0 \]
Table 33: Non-Markov generator $\Lambda' = \ln(P)$ of the migration matrix #1 (in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>-7.663</td>
<td>6.427</td>
<td>0.542</td>
<td>0.466</td>
<td>0.245</td>
<td>-0.016</td>
<td>-0.000</td>
</tr>
<tr>
<td>AA</td>
<td>4.770</td>
<td>-20.604</td>
<td>15.451</td>
<td>-0.001</td>
<td>0.318</td>
<td>0.066</td>
<td>-0.001</td>
</tr>
<tr>
<td>A</td>
<td>-0.267</td>
<td>20.259</td>
<td>-35.172</td>
<td>14.953</td>
<td>0.152</td>
<td>0.083</td>
<td>-0.008</td>
</tr>
<tr>
<td>BBB</td>
<td>0.102</td>
<td>-1.051</td>
<td>28.263</td>
<td>-40.366</td>
<td>13.100</td>
<td>-0.128</td>
<td>0.080</td>
</tr>
<tr>
<td>BB</td>
<td>0.032</td>
<td>0.307</td>
<td>-1.762</td>
<td>28.351</td>
<td>-37.889</td>
<td>10.832</td>
<td>0.129</td>
</tr>
<tr>
<td>B</td>
<td>-0.005</td>
<td>-0.008</td>
<td>0.503</td>
<td>-2.240</td>
<td>30.227</td>
<td>-31.482</td>
<td>3.006</td>
</tr>
<tr>
<td>CCC</td>
<td>0.000</td>
<td>-0.024</td>
<td>0.194</td>
<td>0.469</td>
<td>0.365</td>
<td>16.806</td>
<td>-17.810</td>
</tr>
</tbody>
</table>
The first approach consists in adding the negative values back into the diagonal values:

\[
\begin{align*}
\bar{\lambda}_{i,j} &= \max(\hat{\lambda}_{i,j}, 0) \quad i \neq j \\
\bar{\lambda}_{i,i} &= \hat{\lambda}_{i,i} + \sum_{j \neq i} \min(\hat{\lambda}_{i,j}, 0)
\end{align*}
\]

The second estimator carries forward the negative values on the matrix entries which have the correct sign:

\[
\begin{align*}
G_i &= |\hat{\lambda}_{i,i}| + \sum_{j \neq i} \max(\hat{\lambda}_{i,j}, 0), \\
B_i &= \sum_{j \neq i} \max(-\hat{\lambda}_{i,j}, 0), \\
\tilde{\lambda}_{i,j} &= \begin{cases} \\
0 & \text{if } i \neq j \text{ and } \hat{\lambda}_{i,j} < 0 \\
\hat{\lambda}_{i,j} - B_i & G_i > 0 \\
\hat{\lambda}_{i,j} & G_i = 0
\end{cases}
\end{align*}
\]
### Table 34: Markov generator of the migration matrix #1 (in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>-7.679</td>
<td>6.427</td>
<td>0.542</td>
<td>0.466</td>
<td>0.245</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>AA</td>
<td>4.770</td>
<td>-20.606</td>
<td>15.451</td>
<td>0.000</td>
<td>0.318</td>
<td>0.066</td>
<td>0.000</td>
</tr>
<tr>
<td>A</td>
<td>0.000</td>
<td>20.259</td>
<td>-35.447</td>
<td>14.953</td>
<td>0.152</td>
<td>0.083</td>
<td>0.000</td>
</tr>
<tr>
<td>BBB</td>
<td>0.102</td>
<td>0.000</td>
<td>28.263</td>
<td>-41.545</td>
<td>13.100</td>
<td>0.000</td>
<td>0.080</td>
</tr>
<tr>
<td>BB</td>
<td>0.032</td>
<td>0.307</td>
<td>0.000</td>
<td>38.351</td>
<td>-39.651</td>
<td>10.832</td>
<td>0.129</td>
</tr>
<tr>
<td>B</td>
<td>0.000</td>
<td>0.000</td>
<td>0.503</td>
<td>0.000</td>
<td>30.227</td>
<td>-33.735</td>
<td>3.006</td>
</tr>
<tr>
<td>CCC</td>
<td>0.000</td>
<td>0.000</td>
<td>0.194</td>
<td>0.469</td>
<td>0.365</td>
<td>16.806</td>
<td>-17.834</td>
</tr>
</tbody>
</table>
### Table 35: ESG migration Markov matrix #1 (one-year transition probability in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.75</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.17</td>
<td>82.73</td>
<td>11.85</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.40</td>
<td>15.51</td>
<td>72.79</td>
<td>10.39</td>
<td>0.81</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>BBB</td>
<td>0.12</td>
<td>2.11</td>
<td>19.60</td>
<td>68.69</td>
<td>8.91</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.43</td>
<td>2.79</td>
<td>19.25</td>
<td>69.65</td>
<td>7.61</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.09</td>
<td>0.65</td>
<td>2.98</td>
<td>21.21</td>
<td>72.71</td>
<td>2.35</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.02</td>
<td>0.25</td>
<td>0.58</td>
<td>2.19</td>
<td>13.09</td>
<td>83.87</td>
</tr>
</tbody>
</table>
### Table 36: Original migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.76</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.15</td>
<td>82.73</td>
<td>11.86</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.18</td>
<td>15.47</td>
<td>72.98</td>
<td>10.46</td>
<td>0.82</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>BBB</td>
<td>0.07</td>
<td>1.32</td>
<td>19.60</td>
<td>69.49</td>
<td>9.03</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.19</td>
<td>1.55</td>
<td>19.36</td>
<td>70.88</td>
<td>7.75</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.05</td>
<td>0.24</td>
<td>1.43</td>
<td>21.54</td>
<td>74.36</td>
<td>2.38</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.00</td>
<td>0.22</td>
<td>0.44</td>
<td>2.21</td>
<td>13.24</td>
<td>83.89</td>
</tr>
</tbody>
</table>

### Table 37: New migration matrix

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.75</td>
<td>5.66</td>
<td>0.90</td>
<td>0.45</td>
<td>0.23</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>4.17</td>
<td>82.73</td>
<td>11.85</td>
<td>0.89</td>
<td>0.30</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.40</td>
<td>15.51</td>
<td>72.79</td>
<td>10.39</td>
<td>0.81</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>BBB</td>
<td>0.12</td>
<td>2.11</td>
<td>19.60</td>
<td>68.69</td>
<td>8.91</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.43</td>
<td>2.79</td>
<td>19.25</td>
<td>69.65</td>
<td>7.61</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.09</td>
<td>0.65</td>
<td>2.98</td>
<td>21.21</td>
<td>72.71</td>
<td>2.35</td>
</tr>
<tr>
<td>CCC</td>
<td>0.00</td>
<td>0.02</td>
<td>0.25</td>
<td>0.58</td>
<td>2.19</td>
<td>13.09</td>
<td>83.87</td>
</tr>
</tbody>
</table>
Why it is important that ESG ratings satisfy the Markov property

- Lack of memory:

<table>
<thead>
<tr>
<th></th>
<th>( t-2 )</th>
<th>( t-1 )</th>
<th>( t )</th>
<th>( t+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>BBB</td>
<td>BBB</td>
<td>BBB</td>
<td>?</td>
</tr>
<tr>
<td>BBB</td>
<td>BBB</td>
<td>BBB</td>
<td>BBB</td>
<td>?</td>
</tr>
<tr>
<td>BB</td>
<td>BB</td>
<td>BBB</td>
<td>BBB</td>
<td>?</td>
</tr>
</tbody>
</table>

- Non-Markov property:

\[
\Pr \{ R_{c_1}(t+1) = R_j \mid R_{c_1}(t) = R_i \} \neq \Pr \{ R_{c_2}(t+1) = R_j \mid R_{c_2}(t) = R_i \}
\]

for two different companies \( c_1 \) and \( c_2 \)
How to perform a dynamic analysis?

- We deduce that:

\[ \pi_k (t, A) = \Pr \{ R (t) \in A \mid R (0) = k \} = \sum_{j \in A} e_k^\top e^t \Lambda e_j \]

- Some properties

  - \[ \partial_t \exp (\Lambda t) = \Lambda \exp (\Lambda t) \]
  - \[ \partial_t^m \exp (\Lambda t) = \Lambda^m \exp (\Lambda t) \]
  - \[ \int_0^t e^{\Lambda s} \, ds = (e^{\Lambda t} - I_K) \Lambda^{-1} \]

- For example, the "time density function" is given by:

\[ \pi_k^{(m)} (t, A) := \frac{\partial \pi_k (t, A)}{\partial t^m} = \sum_{j \in A} e_k^\top \Lambda^m e^t \Lambda e_j \]
Figure 53: Probability $\pi_k(t, A)$ to reach $A$ at time $t$ (migration matrix #1)
Figure 54: Dynamic analysis (migration matrix #1)

πₖ(t, AAA) in %

πₖ(t, CCC) in %

∂ₜπₖ(t, AAA) in bps

∂ₜπₖ(t, CCC) in bps
Table 38: Example of credit migration matrix (one-year probability transition in %)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>92.82</td>
<td>6.50</td>
<td>0.56</td>
<td>0.06</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>AA</td>
<td>0.63</td>
<td>91.87</td>
<td>6.64</td>
<td>0.65</td>
<td>0.06</td>
<td>0.11</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>A</td>
<td>0.08</td>
<td>2.26</td>
<td>91.66</td>
<td>5.11</td>
<td>0.61</td>
<td>0.23</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>BBB</td>
<td>0.05</td>
<td>0.27</td>
<td>5.84</td>
<td>87.74</td>
<td>4.74</td>
<td>0.98</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>BB</td>
<td>0.04</td>
<td>0.11</td>
<td>0.64</td>
<td>7.85</td>
<td>81.14</td>
<td>8.27</td>
<td>0.89</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.11</td>
<td>0.30</td>
<td>0.42</td>
<td>6.75</td>
<td>83.07</td>
<td>3.86</td>
<td>5.49</td>
</tr>
<tr>
<td>CCC</td>
<td>0.19</td>
<td>0.00</td>
<td>0.38</td>
<td>0.75</td>
<td>2.44</td>
<td>12.03</td>
<td>60.71</td>
<td>23.50</td>
</tr>
<tr>
<td>D</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

The trace statistics is equal to:

\[ \lambda(t) = \frac{\text{trace} \left( e^{t\Lambda} \right)}{K} \]
Figure 55: Trace statistics of credit and ESG migration matrices
Course 2023-2024 in Sustainable Finance
Lecture 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2024

---

6 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
An investment universe of $n$ assets

- $w = (w_1, \ldots, w_n)$ is the vector of weights in the portfolio
- The portfolio is fully invested meaning that $\sum_{i=1}^{n} w_i = \mathbf{1}^\top w = 1$
- $R = (R_1, \ldots, R_n)$ is the vector of asset returns
- We denote by $\mu = \mathbb{E} [R]$ and $\Sigma = \mathbb{E} \left[ (R - \mu) (R - \mu)^\top \right]$ the vector of expected returns and the covariance matrix of asset returns
We have:

\[ R(w) = \sum_{i=1}^{n} w_i R_i = w^\top R \]

The expected return \( \mu(w) := \mathbb{E}[R(w)] \) of the portfolio is equal to:

\[ \mu(w) = \mathbb{E}[w^\top R] = w^\top \mathbb{E}[R] = w^\top \mu \]

whereas its variance \( \sigma^2(w) := \text{var}(R(w)) \) is given by:

\[ \sigma^2(w) = \mathbb{E} \left[ (R(w) - \mu(w))(R(w) - \mu(w))^\top \right] \]

\[ = \mathbb{E} \left[ w^\top (R - \mu)(R - \mu)^\top w \right] \]

\[ = w^\top \Sigma w \]
We can then formulate the investor’s financial problem as follows:

1. Maximizing the expected return of the portfolio under a volatility constraint (\(\sigma\)-problem):

\[
\max \mu(w) \quad \text{s.t.} \quad \sigma(w) \leq \sigma^* 
\]

2. Or minimizing the volatility of the portfolio under a return constraint (\(\mu\)-problem):

\[
\min \sigma(w) \quad \text{s.t.} \quad \mu(w) \geq \mu^* 
\]

\(\Rightarrow\) The key idea of Markowitz was to transform the original non-linear optimization problems into a quadratic optimization problem
The mean-variance (or quadratic) utility function is:

\[ U(w) := \mathbb{E} [R(w)] - \frac{\bar{\gamma}}{2} \text{var}(R(w)) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w \]

where \( \bar{\gamma} \) is the absolute risk-aversion parameter.

We obtain the following problem:

\[
\begin{align*}
\mathbf{w}^* (\bar{\gamma}) &= \arg \max \left\{ U(w) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w \right\} \\
\text{s.t. } 1^\top w &= 1 
\end{align*}
\]

\( \bar{\gamma} = 0 \Rightarrow \) maximum mean portfolio

\( \bar{\gamma} = \infty \Rightarrow \) minimum variance portfolio:

\[
\begin{align*}
\mathbf{w}^* (\infty) &= \arg \min \frac{1}{2} w^\top \Sigma w \\
\text{s.t. } 1^\top w &= 1
\end{align*}
\]
In practice, professionals formulate the optimization problem as follows:

$$w^* (\gamma) = \arg\min \frac{1}{2} w^T \Sigma w - \gamma w^T \mu$$

s.t. $1^T w = 1$

where $\gamma = \gamma^{-1}$ is called the risk-tolerance

This is a standard QP problem
Definition

The formulation of a standard QP problem is:

\[ w^* = \arg \min \frac{1}{2} w^\top Q w - w^\top R \]

\[ \text{u.c.} \quad \begin{cases} Aw = B \\ Cw \leq D \\ w^- \leq w \leq w^+ \end{cases} \]

\[ \Rightarrow \text{We have } Q = \Sigma, \ R = \gamma \mu, \ A = 1^\top \text{ and } B = 1 \]
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

Mean-variance optimization problem
Illustration

Example #1
We consider an investment universe of five assets. Their expected returns
are equal to 5%, 7%, 6%, 10% and 8% while their volatilities are equal to
18%, 20%, 22%, 25% and 30%. The correlation matrix of asset returns
is given by the following matrix:


100%
 70% 100%





C =  20% 30% 100%

 −30% 20% 10% 100%

0%
0%
0%
0% 100%

Thierry Roncalli

Course 2023-2024 in Sustainable Finance

267 / 1665


Mean-variance optimization problem

Illustration

Figure 56: Efficient frontier (Example #1)
The GMV portfolio is obtained with $\gamma = 0$

The solution is:

$$w_{\text{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

We have:

$$\sigma(w) \geq \sigma(w_{\text{gmv}}) = 10.40\% \quad \forall w$$
Table 39: Solution of the Markowitz optimization problem (in %)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.00</th>
<th>0.10</th>
<th>0.20</th>
<th>0.50</th>
<th>1.00</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^* (\gamma)$</td>
<td>66.35</td>
<td>58.25</td>
<td>50.14</td>
<td>25.84</td>
<td>-14.67</td>
<td>-338.72</td>
</tr>
<tr>
<td>$w_2^* (\gamma)$</td>
<td>-28.52</td>
<td>-22.67</td>
<td>-16.82</td>
<td>0.74</td>
<td>30.00</td>
<td>264.12</td>
</tr>
<tr>
<td>$w_3^* (\gamma)$</td>
<td>15.31</td>
<td>13.30</td>
<td>11.30</td>
<td>5.28</td>
<td>-4.74</td>
<td>-84.93</td>
</tr>
<tr>
<td>$w_4^* (\gamma)$</td>
<td>34.85</td>
<td>37.65</td>
<td>40.44</td>
<td>48.82</td>
<td>62.78</td>
<td>174.50</td>
</tr>
<tr>
<td>$w_5^* (\gamma)$</td>
<td>12.02</td>
<td>13.48</td>
<td>14.94</td>
<td>19.32</td>
<td>26.62</td>
<td>85.03</td>
</tr>
<tr>
<td>$\mu (w^* (\gamma))$</td>
<td>6.69</td>
<td>6.97</td>
<td>7.25</td>
<td>8.09</td>
<td>9.49</td>
<td>20.71</td>
</tr>
<tr>
<td>$\sigma (w^* (\gamma))$</td>
<td>10.40</td>
<td>10.53</td>
<td>10.93</td>
<td>13.35</td>
<td>19.71</td>
<td>84.38</td>
</tr>
</tbody>
</table>
Mean-variance optimization problem
How to solve the $\mu$-problem and the $\sigma$-problem?

- We have to find the optimal value of $\gamma$ such that $\mu(w^*(\gamma)) = \mu^*$ or $\sigma(w^*(\gamma)) = \sigma^*$
- We use the bisection algorithm
- If we target a portfolio with $\sigma^* = 15\%$, we know that $\gamma \in [0.5, 1]$. The optimal solution $w^*$ is $(14.06\%, 9.25\%, 2.37\%, 52.88\%, 21.44\%)$ and the bisection algorithm returns $\gamma = 0.6455$. In this case, we obtain $\mu(w^*(\gamma)) = 8.50\%$
- If we consider a $\mu$-problem with $\mu^* = 9\%$, we find $\gamma = 0.8252$, $w^* = (-0.50\%, 19.77\%, -1.23\%, 57.90\%, 24.07\%)$ and $\sigma(w^*(\gamma)) = 17.30\%$
Theoretical models
Empirical results
Cost of capital
Modern portfolio theory
ESG risk premium
ESG efficient frontier

Mean-variance optimization problem
Adding some constraints

- The Lagrange function of the optimization problem is equal to:

\[ \mathcal{L}(w; \lambda_0) = \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu + \lambda_0 (1^\top w - 1) \]

where \( \lambda_0 \) is the Lagrange coefficients associated with the constraint \( 1^\top w = 1 \)

- The solution \( w^* \) verifies the following first-order conditions:

\[
\begin{align*}
\partial_w \mathcal{L}(w; \lambda_0) &= \Sigma w - \gamma \mu + \lambda_0 1 = 0 \\
\partial_{\lambda_0} \mathcal{L}(w; \lambda_0) &= 1^\top w - 1 = 0
\end{align*}
\]

- We obtain \( w = \Sigma^{-1} (\gamma \mu - \lambda_0 1) \). Because \( 1^\top w - 1 = 0 \), we have \( \gamma 1^\top \Sigma^{-1} \mu - \lambda_0 1^\top \Sigma^{-1} 1 = 1 \). It follows that:

\[
\lambda_0 = \frac{\gamma 1^\top \Sigma^{-1} \mu - 1}{1^\top \Sigma^{-1} 1}
\]
The solution is then:

\[
  w^*(\gamma) = \frac{\Sigma^{-1}1}{1^T \Sigma^{-1}1} + \gamma \frac{1^T \Sigma^{-1}1 \Sigma^{-1} \mu - (1^T \Sigma^{-1} \mu) \Sigma^{-1}1}{1^T \Sigma^{-1}1} \\
  = w_{gmv} + \gamma w_{lsp}
\]

where:
- \( w_{gmv} = (\Sigma^{-1}1) / (1^T \Sigma^{-1}1) \) is the global minimum variance portfolio
- \( w_{lsp} \) is a long/short cash-neutral portfolio such that \( 1^T w_{lsp} = 0 \)
Mean-variance optimization problem
Adding some constraints

- We could think that a QP solver is not required
- The analytical calculus gives:

\[ w_{\text{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%) \]
and:

\[ w_{\text{lsp}} = (-81.01\%, 58.53\%, -20.05\%, 27.93\%, 14.60\%) \]

- In practice, professionals consider other constraints:

\[ w^*(\gamma) = \arg \min \frac{1}{2} w^T \Sigma w - \gamma w^T \mu \]
\[ \text{s.t.} \begin{cases} 1^T w = 1 \\ w \in \Omega \end{cases} \]

where \( w \in \Omega \) corresponds to the set of restrictions

- No short-selling restriction (\( w_i \geq 0 \) and \( \Omega = [0, 1]^n \)) and asset bounds (\( w_i \leq w^+ \)) \( \Rightarrow \) No analytical solution (because of the KKT conditions) \( \Rightarrow \) QP solver
We consider a combination of the risk-free asset and a portfolio $w$:

$$R(\tilde{w}) = (1 - \alpha) r + \alpha R(w)$$

where:

- $r$ is the return of the risk-free asset
- $\tilde{w} = (\alpha w, 1 - \alpha)$ is a vector of dimension $(n + 1)$
- $\alpha \geq 0$ is the proportion of the wealth invested in the risky portfolio

$\Rightarrow$ It follows that $\mu(\tilde{w}) = (1 - \alpha) r + \alpha \mu(w) = r + \alpha (\mu(w) - r)$,

$$\sigma^2(\tilde{w}) = \alpha^2 \sigma^2(w)$$ and:

$$\mu(\tilde{w}) = r + \frac{\mu(w) - r}{\sigma(w)} \sigma(\tilde{w})$$
The tangency portfolio
Two-fund separation theorem

Figure 57: Capital market line (Example #1)
The tangency portfolio
Two-fund separation theorem

- Let $SR(w | r)$ be the Sharpe ratio of portfolio $w$:

$$SR(w | r) = \frac{\mu(w) - r}{\sigma(w)}$$

- We have:

$$\frac{\mu(\tilde{w}) - r}{\sigma(\tilde{w})} = \frac{\mu(w) - r}{\sigma(w)} \iff SR(\tilde{w} | r) = SR(w | r)$$

- The tangency portfolio $w^*$ satisfies:

$$w^* = \arg \max \tan \theta(w)$$
If we consider our example with \( r = 3\% \), the composition of the tangency portfolio is:

\[
w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)
\]

and we have:

\[
\begin{align*}
\mu (w^*) &= 7.51\% \\
\sigma (w^*) &= 11.50\% \\
\text{SR} (w^* \mid r) &= 0.39 \\
\theta (w^*) &= 21.40 \text{ degrees}
\end{align*}
\]
When the risk-free asset belongs to the investment universe, the optimization problem becomes:

$$\tilde{w}^* (\gamma) = \arg \min \frac{1}{2} \tilde{w}^T \tilde{\Sigma} \tilde{w} - \gamma \tilde{w}^T \tilde{\mu}$$

s.t. $$\begin{cases} 1^T \tilde{w} = 1 \\ \tilde{w} \in \Omega \end{cases}$$

where $$\tilde{w} = (w, w_r)$$ is the augmented allocation vector of dimension $$n + 1$$

It follows that:

$$\tilde{\Sigma} = \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix}$$ and $$\tilde{\mu} = \begin{pmatrix} \mu \\ r \end{pmatrix}$$
In the case where $\Omega = \mathbb{R}^{n+1}$, we can show that the optimal solution is equal to:

$$\tilde{w}^*(\gamma) = \alpha \cdot \begin{pmatrix} w^* \\ 0 \end{pmatrix} + (1 - \alpha) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $w^*$ is the tangency portfolio:

$$w^* = \frac{\Sigma^{-1}(\mu - r1)}{1^\top \Sigma^{-1}(\mu - r1)}$$

The proportion of risky assets is equal to

$$\alpha = \gamma \frac{1}{1^\top \Sigma^{-1}(\mu - r1)}$$

The risk-tolerance coefficient associated to the tangency portfolio is given by:

$$\gamma(w^*) = \frac{1}{1^\top \Sigma^{-1}(\mu - r1)}$$
At the equilibrium, Sharpe (1964) showed that:

$$\pi_i := \mu_i - r = \beta_i (\mu (w^*) - r)$$

where $\pi_i$ is the risk premium of the asset $i$ and:

$$\beta_i = \frac{\text{cov} (R_i, R (w^*))}{\text{var} (R (w^*))}$$

We have:

$$\beta (x \mid w) = \frac{\sigma (x, w)}{\sigma^2 (w)} = \frac{x^\top \Sigma w}{w^\top \Sigma w}$$

and:

$$\beta_i = \beta (e_i \mid w) = \frac{e_i^\top \Sigma w}{w^\top \Sigma w} = \frac{(\Sigma w)_i}{w^\top \Sigma w}$$
In the case of Example #1, we have:

- \( w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%) \)
- \( (\mu(w^*) = 7.51\%, r = 3\%) \Rightarrow \mu(w^*) = 4.51\% \)

**Table 40:** Computation of the beta and risk premia (Example #1)

| Portfolio | \( \mu(w) \) | \( \mu(w) - r \) | \( \beta(w | w^*) \) | \( \pi(w | w^*) \) |
|-----------|----------------|-------------------|----------------------|-------------------|
| \( e_1 \) | 5.00\%         | 2.00\%            | 0.444                | 2.00\%            |
| \( e_2 \) | 7.00\%         | 4.00\%            | 0.887                | 4.00\%            |
| \( e_3 \) | 6.00\%         | 3.00\%            | 0.665                | 3.00\%            |
| \( e_4 \) | 10.00\%        | 7.00\%            | 1.553                | 7.00\%            |
| \( e_5 \) | 8.00\%         | 5.00\%            | 1.109                | 5.00\%            |
| \( w_{ew} \) | 7.20\%      | 4.20\%            | 0.932                | 4.20\%            |
| \( w_{gmv} \) | 6.69\%       | 3.69\%            | 0.817                | 3.69\%            |
Jensen (1968) defined the alpha return as:

\[ R_{j,t} - r = \alpha_j + \beta_j (R_t (w_m) - r) + \varepsilon_{j,t} \]

where \( R_{j,t} \) is the return of the mutual fund \( j \) at time \( t \), \( R_t (w_m) \) is the return of the market portfolio and \( \varepsilon_{j,t} \) is an idiosyncratic risk.

More generally, the alpha is defined by the difference between the risk premium \( \pi (w) \) of portfolio \( w \) and the beta \( \beta (w) \) of the portfolio times the market risk premium \( \pi_m \):

\[
\alpha = (\mu (w) - r) - \beta (w | w_m) (\mu (w_m) - r) \\
= \pi (w) - \beta (w) \pi_m
\]
In the case of Example #1 & no short-selling constraint, we have:

- \( w^* = (33.62\%, 0\%, 8.79\%, 40.65\%, 16.95\%) \)
- \( (\mu(w^*) = 7.63\%, r = 3\%) \Rightarrow \mu(w^*) = 4.63\% \)

| Portfolio | \( \mu(w) \) | \( \mu(w) - r \) | \( \beta(w | w^*) \) | \( \pi(w | w^*) \) | \( \alpha(w | w^*) \) |
|-----------|-------------|-----------------|-----------------|-----------------|-----------------|
| \( e_1 \) | 5.00%       | 2.00%           | 0.432           | 2.00%           | 0.00%           |
| \( e_2 \) | 7.00%       | 4.00%           | 0.970           | 4.49%           | -0.49%          |
| \( e_3 \) | 6.00%       | 3.00%           | 0.648           | 3.00%           | 0.00%           |
| \( e_4 \) | 10.00%      | 7.00%           | 1.512           | 7.00%           | 0.00%           |
| \( e_5 \) | 8.00%       | 5.00%           | 1.080           | 5.00%           | 0.00%           |
| \( w_{ew} \) | 7.20%       | 4.20%           | 0.929           | 4.30%           | -0.10%          |
| \( w_{gmv} \) | 6.69%       | 3.69%           | 0.766           | 3.55%           | 0.14%           |
Portfolio optimization in the presence of a benchmark

Utility function revisited

- $b$ is the benchmark
- The tracking error is:

$$\epsilon = R(w) - R(b) = \sum_{i=1}^{n} w_i R_i - \sum_{i=1}^{n} b_i R_i = w^T R - b^T R = (w - b)^T R$$

- The expected excess return is equal to:

$$\mu(w \mid b) := \mathbb{E}[\epsilon] = (w - b)^T \mu$$

- The volatility of the tracking error is defined as:

$$\sigma(w \mid b) := \sigma(e) = \sqrt{(w - b)^T \Sigma (w - b)}$$
The objective of the investor is then to maximize the expected tracking error with a constraint on the tracking error volatility:

\[ w^* = \arg \max \mu(w \mid b) \quad \text{s.t.} \quad \begin{cases} 1^T x = 1 \\ \sigma(w \mid b) \leq \sigma^* \end{cases} \]

We have:

\[
f(w \mid b) = \frac{1}{2} \sigma^2(w \mid b) - \gamma \mu(w \mid b)
\]

\[
= \frac{1}{2} (w - b)^T \Sigma (w - b) - \gamma (w - b)^T \mu
\]

\[
= \frac{1}{2} w^T \Sigma w - w^T (\gamma \mu + \Sigma b) + \frac{1}{2} b^T \Sigma b + \gamma b^T \mu
\]

constant
We have:

\[
\begin{align*}
Q &= \Sigma \\
R &= \gamma \mu + \Sigma b \\
A &= 1^T \\
B &= 1 \\
C &= \\
D &= \\
w^- &= 0_n \text{ (if no short-selling)} \\
w^+ &= 1_n \text{ (if no short-selling)}
\end{align*}
\]
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

Portfolio optimization in the presence of a benchmark

Example #2
We consider an investment universe of four assets. Their expected
returns are equal to 5%, 6.5%, 8% and 6.5% while their volatilities are
equal to 15%, 20%, 25% and 30%. The correlation matrix of asset
returns is given by the following matrix:


100%
 10% 100%



C=

40% 70% 100%
50% 40% 80% 100%
The benchmark is b = (60%, 40%, 20%, −20%).

Thierry Roncalli

Course 2023-2024 in Sustainable Finance

288 / 1665


Figure 58: Efficient frontier with a benchmark (Example #2)
Figure 59: Tangency portfolio with respect to a benchmark (Example #2)

⇒ the tangency portfolio is equal to (46.56%, 33.49%, 39.95%, −20.00%)
We have:

\[
\text{IR} (w \mid b) = \frac{\mu(w \mid b)}{\sigma(w \mid b)} = \frac{(w - b)^\top \mu}{\sqrt{(w - b)^\top \Sigma (w - b)}}
\]

If we consider a combination of the benchmark \(b\) and the active portfolio \(w\), the composition of the portfolio is:

\[
x = (1 - \alpha) b + \alpha w
\]

where \(\alpha \geq 0\) is the proportion of wealth invested in the portfolio \(w\).

It follows that:

\[
\mu(x \mid b) = (x - b)^\top \mu = \alpha \mu(w \mid b)
\]

and:

\[
\sigma^2(x \mid b) = (x - b)^\top \Sigma (x - b) = \alpha^2 \sigma^2(w \mid b)
\]

We deduce that:

\[
\mu(x \mid b) = \text{IR} (w \mid b) \cdot \sigma(x \mid b)
\]
ESG risk premium

- Expected (or required) returns ≠ historical (or realised) returns:
  \[ \pi_i \neq R_i \]

- Difference between the unconstrained risk premium and the implied risk premium:
  \[ \pi_i \neq \tilde{\pi}_i \]
The Pastor-Stambaugh-Taylor model

Model settings

- The asset excess returns $\tilde{R} = R - r = (\tilde{R}_1, \ldots, \tilde{R}_n)$ are normally distributed: $\tilde{R} \sim N(\pi, \Sigma)$
- Each firm has an ESG characteristic $G_i$, which is positive for *esg-friendly* (or *green*) firms and negative for *esg-unfriendly* (or *brown*) firms
- $G_i > 0$ induces positive social impact, while $G_i < 0$ induces negative externalities on the society
- Economy with a continuum of agents ($j = 1, 2, \ldots, \infty$)
- $w_{i,j}$ is the fraction of the wealth invested by agent $j$ in stock $i$
- $w_j = (w_{1,j}, \ldots, w_{n,j})$ is the allocation vector of agent $j$
The relationship between the initial and terminal wealth $W_j$ and $\tilde{W}_j$ is given by:

$$\tilde{W}_j = \left(1 + r + w_j^\top \tilde{R}\right) W_j$$

Exponential CARA utility function:

$$\mathcal{U} \left(\tilde{W}_j, w_j\right) = -\exp \left(-\tilde{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)$$

where:

- $\tilde{\gamma}_j$ is the absolute risk-aversion
- $b_j = \varphi_j \mathcal{G}$ is the vector of nonpecuniary benefits ($\varphi_j \geq 0$)
The expected utility is equal to:

\[
E \left[ u \left( \tilde{W}_j, w_j \right) \right] = E \left[ -\exp \left( -\tilde{\gamma}_j \tilde{W}_j - w_j^\top b_j \tilde{W}_j \right) \right] \\
= E \left[ -\exp \left( -\tilde{\gamma}_j \left( 1 + r + w_j^\top \hat{R} \right) w_j - w_j^\top b_j w_j \right) \right] \\
= -e^{-\tilde{\gamma}_j(1+r)W_j}E \left[ \exp \left( -\tilde{\gamma}_j w_j^\top W_j \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \right) \right] \\
= e^{-\bar{\Gamma}_j(1+r)}E \left[ \exp \left( -\bar{\Gamma}_j w_j^\top \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \right) \right]
\]

where $\bar{\Gamma}_j = \tilde{\gamma}_j W_j$ is the nominal risk aversion

We notice that $\tilde{R} + \tilde{\gamma}_j^{-1} b_j \sim \mathcal{N} \left( \pi + \tilde{\gamma}_j^{-1} b_j, \Sigma \right)$ and:

\[
-\bar{\Gamma}_j w_j^\top \left( \tilde{R} + \tilde{\gamma}_j^{-1} b_j \right) \sim \mathcal{N} \left( -\bar{\Gamma}_j w_j^\top \left( \pi + \tilde{\gamma}_j^{-1} b_j \right), \bar{\Gamma}_j^2 w_j^\top \Sigma w_j \right)
\]
The Pastor-Stambaugh-Taylor model

Optimal portfolio

- We deduce that:
  \[
  \mathbb{E} \left[ U \left( \tilde{w}_j, w_j \right) \right] = e^{-\tilde{\Gamma}_j(1+r)} \exp \left( -\tilde{\Gamma}_j w_j^\top \left( \pi + \tilde{\gamma}_j^{-1} b_j \right) + \frac{1}{2} \tilde{\Gamma}_j^2 w_j^\top \Sigma w_j \right)
  \]

- The first-order condition is equal to:
  \[
  -\tilde{\Gamma}_j \left( \pi + \tilde{\gamma}_j^{-1} b_j \right) + \tilde{\Gamma}_j^2 \Sigma w_j = 0
  \]

  Finally, Pastor et al. (2021) concluded that the optimal portfolio is:
  \[
  w_j^\star = \Gamma_j \Sigma^{-1} \left( \pi + \gamma_j b_j \right)
  \]

  where \( \Gamma_j = \tilde{\Gamma}_j^{-1} \) and \( \gamma_j = \tilde{\gamma}_j^{-1} \) are the relative nominal and unitary risk-tolerance.
Maximizing the expected utility is equivalent to solve the classical Markowitz QP problem:

\[ w_j^* (\gamma_j) = \arg\min \frac{1}{2} w_j^T \Sigma w_j - \gamma_j w_j^T \mu' \]
\[ \text{s.t. } 1^T w_j = 1 \]

where

- \( \gamma_j = \gamma_j^{-1} \) is the relative risk tolerance
- \( \mu' = \mu + \gamma_j b_j \) is the vector of modified expected returns
Example #3

We consider a universe of $n$ risky assets, where $n$ is an even number. The risk-free rate $r$ is set to 3%. We assume that the Sharpe ratio of these assets is the same and is equal to 20%. The volatility of asset $i$ is equal to $\sigma_i = 0.10 + 0.20 \cdot e^{-n^{-1}[0.5i]}$. The correlation between asset returns is constant: $\mathbb{C} = \mathbb{C}_n(\rho)$. The social impact of the firms is given by the vector $\mathbf{G}$. When $\mathbf{G}$ is not specified, it is equal to the cyclic vector $(+1\%, -1\%, +1\%, \ldots, +1\%, -1\%)$. This implies that half of the firms (green firms) have a positive social impact while the others (brown firms) have a negative impact.
### The Pastor-Stambaugh-Taylor model

#### Optimal portfolio

**Table 42**: Mean-variance optimized portfolios with ESG preferences (Example #3, $n = 6$, $\rho = 25\%$)

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\mathcal{G} = (1%, -1%, 1%, -1%, 1%, -1%)$</th>
<th>$\mathcal{G} = (10%, 5%, 2%, 3%, 25%, 30%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00% 1.00% 5.00% 50.00%</td>
<td>0.00% 0.50% 1.00% 2.00%</td>
</tr>
<tr>
<td>$w_1^*$</td>
<td>44.97% 48.87% 58.65% 67.48%</td>
<td>44.97% 46.83% 28.69% 0.00%</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>44.97% 41.06% 19.60% 0.00%</td>
<td>44.97% 37.06% 9.17% 0.00%</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>5.03% 9.82% 21.75% 32.52%</td>
<td>5.03% 0.00% 0.00% 0.00%</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>5.03% 0.25% 0.00% 0.00%</td>
<td>5.03% 0.00% 0.00% 0.00%</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>0.00% 0.00% 0.00% 0.00%</td>
<td>0.00% 0.83% 16.62% 21.09%</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>0.00% 0.00% 0.00% 0.00%</td>
<td>0.00% 15.28% 45.53% 78.91%</td>
</tr>
<tr>
<td>$\mu (w^*)$</td>
<td>8.33% 8.33% 8.27% 8.22%</td>
<td>8.33% 8.23% 7.79% 7.43%</td>
</tr>
<tr>
<td>$\sigma (w^*)$</td>
<td>20.00% 20.09% 20.07% 21.56%</td>
<td>20.00% 19.33% 16.70% 19.17%</td>
</tr>
<tr>
<td>$\text{SR} (w^*</td>
<td>r)$</td>
<td>0.27 0.27 0.26 0.24</td>
</tr>
</tbody>
</table>
Figure 60: Efficient frontier with ESG preferences (Example #3, n = 20, $\rho = 25\%$)
The Pastor-Stambaugh-Taylor model

- \( W = \int W_j \, dj \)
- \( \omega_j = \frac{W_j}{\mathcal{W}} \) is the market share of the economic agent \( j \)
- \( \mathcal{W}_{i,j} = w_{i,j}^* \mathcal{W} = w_{i,j}^* \omega_j \mathcal{W} \)
- We have:
  \[
  \mathcal{W}_i = \int \mathcal{W}_{i,j} \, dj = \int w_{i,j}^* \omega_j \mathcal{W} \, dj
  \]
- Let \( \mathbf{w}_m = (w_{1,m}, \ldots, w_{n,m}) \) be the market portfolio. We have:
  \[
  w_{i,m} = \frac{\mathcal{W}_i}{\mathcal{W}} = \int w_{i,j}^* \omega_j \, dj
  \]

and \( \int \omega_j \, dj = 1 \)
The Pastor-Stambaugh-Taylor model

- The market clearing condition satisfies:

\[ w_m = \int \omega_j w_j^* \, dj \]

\[ = \int \omega_j \Gamma_j \Sigma^{-1} (\pi + \gamma_j b_j) \, dj \]

\[ = \int \omega_j \Gamma_j \Sigma^{-1} (\pi + \gamma_j \varphi_j \gamma^0) \, dj \]

\[ = \left( \int \Gamma_j \omega_j \, dj \right) \Sigma^{-1} \pi + \left( \int \omega_j \Gamma_j \psi_j \, dj \right) \Sigma^{-1} \gamma^0 \]

where \( \psi_j = \gamma_j \varphi_j \)

- It follows that:

\[ w_m = \Gamma_m \Sigma^{-1} \pi + \Gamma_m \psi_m \Sigma^{-1} \gamma^0 \]

where \( \Gamma_m = \int \Gamma_j \omega_j \, dj \) and \( \psi_m = \Gamma_m^{-1} \left( \int \omega_j \Gamma_j \psi_j \, dj \right) \) are the average risk tolerance and the weighted average of ESG preferences.
The asset risk premia are equal to:

\[ \pi = \frac{1}{\Gamma_m} \Sigma w_m - \psi_m G \]

while the market risk premium is defined as:

\[
\pi_m = w_m^\top \pi \\
= \frac{1}{\Gamma_m} w_m^\top \Sigma w_m - \psi_m w_m^\top G \\
= \frac{1}{\Gamma_m} \sigma_m^2 - \psi_m G_m
\]

where \( \sigma_m = \sqrt{w_m^\top \Sigma w_m} \) and \( G_m = w_m^\top G \) are the volatility and the green intensity (or greenness) of the market portfolio.
The risk premium including the ESG sentiment is lower than the CAPM risk premium if the market ESG intensity is positive:

\[ G_m > 0 \implies \pi_m \leq \pi_{m}^{\text{capm}} \]

It is greater than the CAPM risk premium if the market ESG intensity is negative:

\[ G_m < 0 \implies \pi_m \geq \pi_{m}^{\text{capm}} \]

The gap \( \Delta \pi_m^{\text{esg}} := |\pi_m - \pi_{m}^{\text{capm}}| \) is an increasing function of the market ESG sentiment \( \psi_m \):

\[ \psi_m \uparrow \implies \Delta \pi_m^{\text{esg}} \uparrow \]
If we assume that $G_m \approx 0$, we have $\Gamma_m = \sigma^2_m / \pi_m$.

$$\pi = \beta \pi_m - \psi_m G$$

and:

$$\alpha_i = \pi_i - \beta_i \pi_m = -\psi_m G_i$$

If $\psi_m > 0$, “green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas” (Pastor et al., 2021).
Example #4

We consider Example #3. The market is made up of two long-only investors \((j = 1, 2)\): a non-ESG investor \((\varphi_1 = 0)\) and an ESG investor \((\varphi_2 > 0)\). We assume that they have the same risk tolerance \(\gamma\). We note \(W_1\) and \(W_2\) their financial wealth, which is entirely invested in the risky assets. We assume that \(W_1 = W_2 = 1\).
The tangency portfolio is equal to:

\[ w^* = \frac{\Sigma^{-1} (\mu - r1)}{1^\top \Sigma^{-1} (\mu - r1)} \]

\[ = (15.04\%, 15.04\%, 16.65\%, 16.65\%, 18.31\%, 18.31\%) \]

- \( w_1^* = w^* \) and \( \gamma_1 = 1/ (1^\top \Sigma^{-1} (\mu - r1)) = 0.4558 \)
- \( \gamma_2 = \gamma_1 \) and:

\[ w_2^* = \arg\min \frac{1}{2} w^\top \Sigma w - \gamma_2 w^\top (\mu + \gamma_2 \varphi_2 \mathcal{G}) \]

s.t. \[ \begin{align*}
1^\top w &= 1 \\
\mathbf{0} &
\end{align*} \]

- We obtain

\[ w_2^* = (18.86\%, 11.22\%, 21.33\%, 11.97\%, 23.96\%, 12.65\%) \]
The Pastor-Stambaugh-Taylor model
Risk premium

- The market portfolio is then equal to:

\[ w_m = \frac{W_1}{W} w_1^* + \frac{W_2}{W} w_2^* = (1 - \omega^{\text{esg}}) \cdot w_1^* + \omega^{\text{esg}} \cdot w_2^* \]

- When \( W_1 = W_2 = 1 \), we obtain

\[ w_m = (16.95\%, 13.13\%, 18.99\%, 14.31\%, 21.13\%, 15.48\%) \]

\[ \mu_m = 7.86\% \]

\[ \sigma_m = 14.93\% \]

- We deduce that:

\[ \beta = (1.15, 1.05, 1.04, 0.95, 0.95, 0.86) \]

\[ \pi = (5.58\%, 5.12\%, 5.06\%, 4.61\%, 4.62\%, 4.17\%) \]

\[ \alpha = (-19.09, 26.19, -19.43, 25.84, -19.72, 25.55) \text{ (in bps)} \]
### Table 43: Computation of alpha returns (Example #4, $n = 6$, $\rho = 25\%$)

<table>
<thead>
<tr>
<th>$i$</th>
<th>Portfolio $w_1^*$</th>
<th></th>
<th>Portfolio $w_2^*$</th>
<th></th>
<th>Portfolio $w_m$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_i$ (in %)</td>
<td>$\beta_i$ (in %)</td>
<td>$\pi_i$ (in %)</td>
<td>$w_i$ (in %)</td>
<td>$\beta_i$ (in %)</td>
<td>$\pi_i$ (in %)</td>
</tr>
<tr>
<td>1</td>
<td>15.04</td>
<td>1.11</td>
<td>5.39</td>
<td>18.86</td>
<td>1.17</td>
<td>5.69</td>
</tr>
<tr>
<td>2</td>
<td>15.04</td>
<td>1.11</td>
<td>5.39</td>
<td>11.22</td>
<td>0.99</td>
<td>4.80</td>
</tr>
<tr>
<td>3</td>
<td>16.65</td>
<td>1.00</td>
<td>4.87</td>
<td>21.33</td>
<td>1.07</td>
<td>5.18</td>
</tr>
<tr>
<td>4</td>
<td>16.65</td>
<td>1.00</td>
<td>4.87</td>
<td>11.97</td>
<td>0.88</td>
<td>4.30</td>
</tr>
<tr>
<td>5</td>
<td>18.31</td>
<td>0.91</td>
<td>4.43</td>
<td>23.96</td>
<td>0.98</td>
<td>4.76</td>
</tr>
<tr>
<td>6</td>
<td>18.31</td>
<td>0.91</td>
<td>4.43</td>
<td>12.65</td>
<td>0.80</td>
<td>3.87</td>
</tr>
</tbody>
</table>
Figure 61: Evolution of the alpha return with respect to the market share of ESG investors (Example #4, $n = 6$, $\rho = 25\%$)
“In equilibrium, green assets have low expected returns because investors enjoy holding them and because green assets hedge climate risk. Green assets nevertheless outperform when positive shocks hit the ESG factor, which captures shifts in customers’ tastes for green products and investors’ tastes for green holdings.” (Pastor et al., 2021).

- ESG risk premium?
- Green risk premium?
Figure 62: Impact of alpha returns on the underperformance probability

The Pastor-Stambaugh-Taylor model
What does equilibrium mean?
We have:

\[
\begin{pmatrix}
\tilde{R} \\
S
\end{pmatrix}
\sim \mathcal{N}
\left(
\begin{pmatrix}
\pi \\
\mu_s
\end{pmatrix},
\begin{pmatrix}
\Sigma & \Sigma_{\pi,s} \\
\Sigma_{s,\pi} & \Sigma_s
\end{pmatrix}
\right)
\]

The optimal solution is:

\[
w_j^* = \Gamma_j \Sigma^{-1} (\pi + \psi_j \mu_s) + \Gamma_j^{-1} \Omega_j (\pi + \psi_j \mu_s)
\]

- PST solution
- ESG uncertainty
If there is no ESG uncertainty \( (\mathbf{S} = \mu_s \text{ and } \Sigma_s = 0) \), the vector of risk premia is given by:

\[
\pi^{\text{esg}} = \beta \pi_m - \psi_m (\mu_s - \beta \bar{\mathbf{S}}_m)
\]

\[
= \pi^{\text{capm}} - \psi_m (\mu_s - \beta \bar{\mathbf{S}}_m)
\]

If there is an uncertainty on ESG scores \( (\mathbf{S} \neq \mu_s \text{ and } \Sigma_s \neq 0) \), the vector of risk premia becomes:

\[
\tilde{\pi}^{\text{esg}} = \beta \tilde{\pi}_m - \psi_m (\tilde{\mu}_s - \beta \tilde{\mathbf{S}}_m)
\]

\[
= \beta \pi_m + \left( \beta - \bar{\beta} \right) \pi_m - \psi_m (\tilde{\mu}_s - \beta \tilde{\mathbf{S}}_m)
\]
“In equilibrium, the market premium increases and demand for stocks declines under ESG uncertainty. In addition, the CAPM alpha and effective beta both rise with ESG uncertainty and the negative ESG-alpha relation weakens.” (Avramov et al., 2022).
Extension of the PST model

Risk factor model
The Pedersen-Fitzgibbons-Pomorski model

Model settings

- \( \tilde{R} = R - r \sim \mathcal{N}(\pi, \Sigma) \)
- \( S = (S_1, \ldots, S_n) \)

The terminal wealth is

\[
\tilde{W} = \left( 1 + r + w^\top \tilde{R} \right) W
\]

The model uses the mean-variance utility:

\[
U(\tilde{W}, w) = \mathbb{E}[\tilde{W}] - \frac{\tilde{\gamma}}{2} \text{var}(\tilde{W}) + \zeta(S(w)) W
\]

\[
= \left( 1 + r + w^\top \pi - \frac{\tilde{\gamma}}{2} w^\top \Sigma w + \zeta(w^\top S) \right) W
\]

where \( \zeta \) is a function that depends on the investor.
Optimizing the utility function is equivalent to find the mean-variance-esg optimized portfolio:

\[
    w^* = \arg \max_w w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta (w^\top S)
\]

s.t. \( \mathbf{1}^\top w = 1 \)

- \( \sigma (w) = \sqrt{w^\top \Sigma w} \)
- \( S (w) = w^\top S \)
The optimization problem can be decomposed as follows:

\[ w^* = \arg \left\{ \max \{ \max \{ \max \{ f(w; \pi, \Sigma, \mathcal{S}) \text{ s.t. } w \in \Omega(\bar{\sigma}, \bar{\mathcal{S}}) \} \} \} \right\} \]

where:

\[ f(w; \pi, \Sigma, \mathcal{S}) = w^T \pi - \frac{\gamma}{2} \sigma^2(w) + \zeta(\mathcal{S}(w)) \]

and:

\[ \Omega = \{ w \in \mathbb{R}^n : 1^T w = 1, \sigma(w) = \bar{\sigma}, \mathcal{S}(w) = \bar{\mathcal{S}} \} \]
The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- We consider the $\sigma - S$ problem:

$$w^* (\bar{\sigma}, \bar{S}) = \arg \max w^\top \pi$$

s.t. \[
\begin{align*}
1^\top w &= 1 \\
ww^\top \Sigma w - \bar{\sigma}^2 &= 0 \\
w^\top (S - \bar{S}1) &= 0
\end{align*}
\]

- The Lagrange function is:

$$L (w; \lambda_1, \lambda_2) = w^\top \pi + \lambda_1 (w^\top \Sigma w - \bar{\sigma}^2) + \lambda_2 (w^\top (S - \bar{S}1))$$

- The first-order condition is:

$$\frac{\partial L (w; \lambda_1, \lambda_2)}{\partial w} = \pi + 2\lambda_1 \Sigma w + \lambda_2 (S - \bar{S}1) = 0$$

- We deduce that the optimal portfolio is given by:

$$w = -\frac{1}{2\lambda_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))$$
The second constraint $w^T (S - \bar{S}1) = 0$ implies that:

$$(*) \iff (S - \bar{S}1)^T \frac{1}{2\lambda_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1)) = 0$$

$$\iff \lambda_2 = - \frac{(S - \bar{S}1)^T \Sigma^{-1} \pi}{(S - \bar{S}1)^T \Sigma^{-1} (S - \bar{S}1)}$$

$$\iff \lambda_2 = \frac{\bar{S} (1^T \Sigma^{-1} \pi) - S^T \Sigma^{-1} \pi}{S^T \Sigma^{-1} S - 2\bar{S} (1^T \Sigma^{-1} S) + \bar{S}^2 (1^T \Sigma^{-1} 1)}$$

$$\iff \lambda_2 = \frac{C_{1,\pi} \bar{S} - C_{S,\pi}}{C_{S,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2}$$

where $C_{x,y}$ is the compact notation for $x^T \Sigma^{-1} y$ — $C_{1,\pi} = 1^T \Sigma^{-1} \pi$, $C_{S,\pi} = S^T \Sigma^{-1} \pi$, $C_{S,s} = S^T \Sigma^{-1} S$, $C_{1,s} = 1^T \Sigma^{-1} S$ and $C_{1,1} = 1^T \Sigma^{-1} 1$.
The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

Using the first constraint \( w^\top \Sigma w - \bar{\sigma}^2 = 0 \), we deduce that:

\[
\begin{align*}
\bar{\sigma}^2 &= - \frac{1}{2\lambda_1} w^\top \Sigma \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1)) \\
&= - \frac{1}{2\lambda_1} (w^\top \pi + \lambda_2 w^\top (S - \bar{S}1)) \\
&= - \frac{1}{2\lambda_1} w^\top \pi \\
&= \frac{1}{4\lambda_1^2} \pi^\top \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))
\end{align*}
\]

The first Lagrange coefficient is then equal to \( (C_{\pi,\pi} = \pi^\top \Sigma^{-1} \pi) \):

\[
\begin{align*}
\lambda_1 &= -\frac{1}{2\bar{\sigma}} \sqrt{\pi^\top \Sigma^{-1} \pi + \lambda_2 (\pi^\top \Sigma^{-1} S - \bar{S}) (\pi^\top \Sigma^{-1} 1)} \\
&= -\frac{1}{2\bar{\sigma}} \sqrt{C_{\pi,\pi} - \frac{(C_{1,\pi} \bar{S} - C_{s,\pi})^2}{C_{s,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2}}
\end{align*}
\]
The optimal portfolio is the product of the volatility $\bar{\sigma}$ and the vector $\varrho (\bar{S})$:

$$w^* (\bar{\sigma}, \bar{S}) = -\frac{1}{2\lambda_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))$$

$$= \bar{\sigma} \cdot \varrho (\bar{S})$$

where:

$$\varrho (\bar{S}) = \frac{1}{\lambda'_1} \Sigma^{-1} (\pi + \lambda_2 (S - \bar{S}1))$$

and:

$$\lambda'_1 = \sqrt{C_{\pi,\pi} - \frac{(C_{1,\pi} \bar{S} - C_{s,\pi})^2}{C_{s,s} - 2C_{1,s} \bar{S} + C_{1,1} \bar{S}^2}}$$
Theoretical models
Empirical results
Cost of capital

Modern portfolio theory
ESG risk premium
ESG efficient frontier

The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

Example #5
We consider an investment universe of four assets. Their expected
returns are equal to 6%, 7%, 8% and 10% while their volatilities are
equal to 15%, 20%, 25% and 30%. The correlation matrix of asset
returns is given by the following matrix:


100%
 20% 100%


C=
 30% 50% 100%

40% 60% 70% 100%
The risk-free rate is set to 2%. The ESG score vector is
S = (3%, 2%, −2%, −3%).

Thierry Roncalli

Course 2023-2024 in Sustainable Finance

324 / 1665


The Pedersen-Fitzgibbons-Pomorski model
Optimal portfolio

We obtain $C_{1,\pi} = 2.4864$, $C_{s,\pi} = 0.0425$, $C_{s,s} = 0.1274$, $C_{1,s} = 1.9801$, $C_{1,1} = 64.1106$ and $C_{\pi,\pi} = 0.1193$

If we target $\bar{\sigma} = 20\%$ and $\bar{\mathcal{S}} = 1\%$, we deduce that $\lambda_1 = -0.8514$ and $\lambda_2 = -0.1870$

The optimal portfolio is then:

$$w^* (\bar{\sigma}, \bar{\mathcal{S}}) = \begin{pmatrix} 59.31\% \\ 29.52\% \\ 21.76\% \\ 20.72\% \end{pmatrix}$$

It follows that the portfolio is leveraged since we have $w_r = 1 - \mathbf{1}^T w = -31.31\%$
We verify that \( \sqrt{w^* (\bar{\sigma}, \bar{S})^\top \Sigma w^* (\bar{\sigma}, \bar{S})} = 20\% \) and 
\[
\left( w^* (\bar{\sigma}, \bar{S})^\top S \right) / \left( 1^\top w^* (\bar{\sigma}, \bar{S}) \right) = 1\%
\]

We also notice that:

\[
\varrho (\bar{S}) = \begin{pmatrix}
2.9657 \\
1.4759 \\
1.0881 \\
1.0358
\end{pmatrix}
\]

and verify that \( w^* (\bar{\sigma}, \bar{S}) = \bar{\sigma} \cdot \varrho (\bar{S}) \)

The portfolio is then leveraged when \( \bar{\sigma} \geq 1 / (1^\top \varrho (\bar{S})) = 17.75\% \).
We rewrite the first-order condition as:

\[
(*) \iff \pi + 2\lambda_1 \Sigma w + \lambda_2 (\mathcal{S} - \tilde{\mathcal{S}} \mathbf{1}) = 0
\]
\[
\iff w^T \pi + 2\lambda_1 w^T \Sigma w + \lambda_2 w^T (\mathcal{S} - \tilde{\mathcal{S}} \mathbf{1}) = 0
\]
\[
\iff w^T \pi + 2\lambda_1 \bar{\sigma}^2 = 0
\]
\[
\iff \lambda_1 = -\frac{1}{2} \frac{w^T \pi}{\bar{\sigma}^2} = -\frac{1}{2} \frac{\text{SR}(w | r)}{\bar{\sigma}}
\]

We deduce that the Sharpe ratio of the optimal portfolio is:

\[
\text{SR}(w^* (\bar{\sigma}, \tilde{\mathcal{S}}) | r) = \sqrt{C_{\pi,\pi} - \frac{(C_{1,\pi} \tilde{\mathcal{S}} - C_{s,\pi})^2}{C_{s,s} - 2C_{1,s} \tilde{\mathcal{S}} + C_{1,1} \tilde{\mathcal{S}}^2}} = \text{SR}(\tilde{\mathcal{S}} | \pi, \Sigma, \mathcal{S})
\]

It depends on the asset parameters $\pi$, $\Sigma$, $\mathcal{S}$, the ESG objective $\tilde{\mathcal{S}}$ of the investor, but not the volatility target $\bar{\sigma}$.
Figure 63: Relationship between $\bar{S}$ and $\text{SR} (\bar{S} \mid \pi, \Sigma)$ (Example #5)
Using Example #5

- The Sharpe ratio of the optimal portfolio \( w^* (20\%, 1\%) \) is equal to 0.3406

- We have 
  
  \[
  \text{SR} (w^* (\bar{\sigma}, -3\%) | r) = 0.2724, \\
  \text{SR} (w^* (\bar{\sigma}, -2\%) | r) = 0.2875, \\
  \text{SR} (w^* (\bar{\sigma}, -1\%) | r) = 0.3052, \\
  \text{SR} (w^* (\bar{\sigma}, 0\%) | r) = 0.3242, \\
  \text{SR} (w^* (\bar{\sigma}, 1\%) | r) = 0.3406, \\
  \text{SR} (w^* (\bar{\sigma}, 2\%) | r) = 0.3443, \text{ and } \text{SR} (w^* (\bar{\sigma}, 3\%) | r) = 0.3221
  \]
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

The objective function is equal to:

\[ f \left( w^* \left( \tilde{\sigma}, \tilde{S} \right); \pi, \Sigma, S \right) = \left( \frac{w^* \left( \tilde{\sigma}, \tilde{S} \right)^T \pi}{\tilde{\sigma}} \right) \tilde{\sigma} - \frac{\tilde{\gamma}}{2} \tilde{\sigma}^2 + \zeta \left( \tilde{S} \right) \]

\[ = \text{SR} \left( \tilde{S} \mid \pi, \Sigma, S \right) \tilde{\sigma} - \frac{\tilde{\gamma}}{2} \tilde{\sigma}^2 + \zeta \left( \tilde{S} \right) \]

The \( \sigma \)-problem becomes:

\[ (*) = \max_{\tilde{\sigma}} \left\{ \max_w \left\{ f \left( w; \pi, \Sigma, S \right) \text{ s.t. } w \in \Omega \left( \tilde{\sigma}, \tilde{S} \right) \right\} \right\} \]

\[ = \max_{\tilde{\sigma}} \left\{ \text{SR} \left( \tilde{S} \mid \pi, \Sigma, S \right) \tilde{\sigma} - \frac{\tilde{\gamma}}{2} \tilde{\sigma}^2 + \zeta \left( \tilde{S} \right) \right\} \]

The first-order condition is

\[ \text{SR} \left( \tilde{S} \mid \pi, \Sigma, S \right) - \tilde{\gamma} \tilde{\sigma} = 0 \text{ or } \tilde{\sigma} = \frac{1}{\tilde{\gamma}} \text{SR} \left( \tilde{S} \mid \pi, \Sigma, S \right) \]
We have:

\[ f \left( w^* (\bar{\sigma}, \bar{S}) ; \pi, \Sigma, S \right) = \frac{\gamma^{-1}}{2} \text{SR}^2 (\bar{S} \mid \pi, \Sigma, S) - \]

\[ \frac{1}{2} \gamma^{-1} \text{SR}^2 (\bar{S} \mid \pi, \Sigma, S) + \zeta (\bar{S}) \]

\[ = \frac{1}{2} \gamma^{-1} \left( \text{SR}^2 (\bar{S} \mid \pi, \Sigma, S) + 2\gamma \zeta (\bar{S}) \right) \]

We conclude that the \( S \)-problem becomes:

\[ S^* = \arg \max_{\bar{S}} \left\{ \text{SR}^2 (\bar{S} \mid \pi, \Sigma, S) + 2\gamma \zeta (\bar{S}) \right\} \]

The optimal portfolio is \( w^* = w^* (\sigma^*, S^*) \) where \( S^* \) is the solution of the \( S \)-problem and \( \sigma^* = \gamma^{-1} \text{SR} (S^* \mid \pi, \Sigma, S) \)
Pedersen et al. (2021) distinguished three groups of investors:

- **Type-U** or ESG-unaware investors have no ESG preference and do not use the information of ESG scores.
- **Type-A** or ESG-aware investors have no ESG preference, but they use the ESG scores to update their views on the risk premia.
- **Type-M** or ESG-motivated investors have ESG preferences, implying that they would like to have a high ESG score.
Type-U investors hold the same portfolio:

\[ w_U^* = \frac{\Sigma^{-1}_\pi}{\mathbf{1}^\top \Sigma^{-1}_\pi} \]

Type-A investors choose the optimal portfolio with the highest Sharpe ratio \((\zeta(s) = 0) \Rightarrow S_A^*\) is the optimal ESG score.

Type-M investors choose an optimal portfolio on the ESG-SR efficient frontier, with:

\[ S_M^* \geq S_A^* \]

and:

\[ \text{SR} \left( S_M^* \mid \pi, \Sigma, \mathcal{S} \right) \leq \text{SR} \left( S_A^* \mid \pi, \Sigma, \mathcal{S} \right) \]
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

Figure 64: Optimal portfolio for type-U investors (Example #5)
The Pedersen-Fitzgibbons-Pomorski model
The ESG-SR frontier

Figure 65: Optimal portfolio for type-A investors (Example #5)
For type-M investors, we first compute the function $\xi(\tilde{S})$:

$$\xi(\tilde{S}) = SR^2(\tilde{S} \mid \pi, \Sigma, S) + 2\gamma \zeta(\tilde{S})$$

The optimal portfolio corresponds to the optimal ESG score that maximizes $\xi(\tilde{S})$.
Figure 66: Optimal portfolio for type-M investors when $\zeta(s) = s$ (Example #5)
Figure 67: Optimal portfolio for type-M investors when $\zeta(s) = 0.2\sqrt{\max(s, 0)}$
### Table 44: Optimal portfolios (Example #5)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Type-U</th>
<th>Type-A</th>
<th>Type-M</th>
<th>Type-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$s^*$</td>
<td>$s^*$</td>
<td>$s^*$</td>
<td>$s^*$</td>
</tr>
<tr>
<td>$S(s^*)$</td>
<td>0.017</td>
<td>0.017</td>
<td>0.023</td>
<td>0.023</td>
</tr>
<tr>
<td>$\sigma(s^*)$</td>
<td>0.139</td>
<td>0.100</td>
<td>0.682</td>
<td>0.329</td>
</tr>
<tr>
<td>SR($w^*</td>
<td>r$)</td>
<td>0.345</td>
<td>0.345</td>
<td>0.341</td>
</tr>
<tr>
<td>$w_1^*$</td>
<td>0.524</td>
<td>0.378</td>
<td>3.028</td>
<td>1.623</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>0.289</td>
<td>0.208</td>
<td>1.786</td>
<td>1.099</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>0.120</td>
<td>0.086</td>
<td>0.383</td>
<td>0.073</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>0.067</td>
<td>0.048</td>
<td>-0.012</td>
<td>-0.144</td>
</tr>
<tr>
<td>$w_r^*$</td>
<td>0.000</td>
<td>0.280</td>
<td>-4.184</td>
<td>-1.562</td>
</tr>
</tbody>
</table>
The Pedersen-Fitzgibbons-Pomorski model
Impact on asset returns

- If $\omega^U = 1$ and $\omega^A = \omega^M = 0$, then unconditional expected returns are given by the CAPM:
  \[ \mathbb{E} [R_i] - r = \beta_i (\mathbb{E} [R_m] - r) \]
  but conditional expected returns depend on the ESG scores:
  \[ \mathbb{E} [R_i | \mathcal{S}] - r = \beta_i (\mathbb{E} [R_m] - r) + \theta \frac{S_i - S_m}{P_i} \]
  where $P_i$ is the asset price of asset $i$

- If $\omega^A = 1$ and $\omega^U = \omega^M = 0$, then the informational value of ESG scores is fully incorporated into asset prices, and we have:
  \[ \mathbb{E} [R_i | \mathcal{S}] - r = \tilde{\beta}_i (\mathbb{E} [R_m | \mathcal{S}] - r) \]
  where $\tilde{\beta}_i$ is the ESG-adjusted beta coefficient

- If $\omega^M = 1$ and $\omega^U = \omega^A = 0$, then the conditional expected return is given by:
  \[ \mathbb{E} [R_i | \mathcal{S}] - r = \tilde{\beta}_i (\mathbb{E} [R_m | \mathcal{S}] - r) + \lambda_2 (S_i - S_m) \]
“If all types of investors exist, then several things can happen. If a security has a higher ESG score, then, everything else equal, its expected return can be higher or lower. A higher ESG score increases the demand for the stock from type-M investors, leading to a higher price and, therefore, a lower required return [...] Companies with poor ESG scores that are down-weighted by type-M investors will have lower prices and higher cost of capital. [...] Furthermore, the force that can increase the expected return is that the higher ESG could be a favorable signal of firm fundamentals, and if many type-U investors ignore this, the fundamental signal perhaps would not be fully reflected in the price [...] A future increase in ESG investing would lead to higher prices for high-ESG stocks [...] If these flows are unexpected (or not fully captured in the price for other reasons), then high-ESG stocks would experience a return boost during the period of this repricing of ESG. If these flows are expected, then expected returns should not be affected.”

(Pedersen et al., 2021)
What is the performance of ESG investing?

According to Coqueret (2022), we can classify the academic studies into four categories:

1. ESG improves performance
2. ESG does not impact performance
3. ESG is financially detrimental
4. The relationship between ESG and performance depends on many factors
According to Friede et al. (2015), the first category dominates the other categories:

“ [...] The results show that the business case for ESG investing is empirically very well founded. Roughly 90% of studies find a nonnegative ESG – CFP relation. More importantly, the large majority of studies reports positive findings. We highlight that the positive ESG impact on CFP appears stable over time. Promising results are obtained when differentiating for portfolio and non-portfolio studies, regions, and young asset classes for ESG investing such as emerging markets, corporate bonds, and green real estate.”

⇒ Many dimensions of CFP (cost of capital, G pillar, proxy variables, etc.)
We can also find many studies, whose conclusion is more neutral or negative: Barnett and Salomon (2006), Fabozzi et al. (2008), Hong and Kacperczyk (2009), Johnson et al. (2009), Capelle-Blancard and Monjon (2014), Matos (2020), etc.

⇒ Sin stocks

Mixed results
What is the performance of ESG investing?

- Generally, academic studies that analyze the relationship between ESG and performance are based on long-term historical data, typically the last 20 years or the last 30 years.
- Two issues:
  1. ESG investing was marginal 15+ years ago
  2. ESG data are not robust or relevant before 2010
- The relationship between ESG and performance is dynamic
- Sometimes, ESG may create performance, but sometimes not
Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date \( t \), we rank the stocks according to their Amundi ESG z-score \( s_{i,t} \)
- We form the five quintile portfolios \( Q_i \) for \( i = 1, \ldots, 5 \)
- The portfolio \( Q_i \) is invested during the period \( [t, t+1] \):
  - \( Q_1 \) corresponds to the best-in-class portfolio (best scores)
  - \( Q_5 \) corresponds to the worst-in-class portfolio (worst scores)
- Quarterly rebalancing
- Universe: MSCI World Index
- Equally-weighted and sector-neutral portfolio (and region-neutral for the world universe)
Table 45: An illustrative example

<table>
<thead>
<tr>
<th>Asset</th>
<th>$S_i$</th>
<th>Rank</th>
<th>$Q_i$</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>−0.3</td>
<td>6</td>
<td>$Q_3$</td>
<td>+50%</td>
</tr>
<tr>
<td>#2</td>
<td>0.2</td>
<td>5</td>
<td>$Q_3$</td>
<td>+50%</td>
</tr>
<tr>
<td>#3</td>
<td>−1.0</td>
<td>7</td>
<td>$Q_4$</td>
<td>+50%</td>
</tr>
<tr>
<td>#4</td>
<td>1.5</td>
<td>3</td>
<td>$Q_2$</td>
<td>+50%</td>
</tr>
<tr>
<td>#5</td>
<td>−2.9</td>
<td>10</td>
<td>$Q_5$</td>
<td>+50%</td>
</tr>
<tr>
<td>#6</td>
<td>0.8</td>
<td>4</td>
<td>$Q_2$</td>
<td>+50%</td>
</tr>
<tr>
<td>#7</td>
<td>−1.4</td>
<td>8</td>
<td>$Q_4$</td>
<td>+50%</td>
</tr>
<tr>
<td>#8</td>
<td>2.3</td>
<td>2</td>
<td>$Q_1$</td>
<td>+50%</td>
</tr>
<tr>
<td>#9</td>
<td>2.8</td>
<td>1</td>
<td>$Q_1$</td>
<td>+50%</td>
</tr>
<tr>
<td>#10</td>
<td>−2.2</td>
<td>9</td>
<td>$Q_5$</td>
<td>+50%</td>
</tr>
</tbody>
</table>
Figure 68: Annualized return of ESG-sorted portfolios (MSCI North America)

Figure 69: Annualized return of ESG-sorted portfolios (MSCI North America)

Figure 70: Annualized return of ESG-sorted portfolios (MSCI EMU)

Figure 71: Annualized return of ESG-sorted portfolios (MSCI EMU)

Environmental

Social

Governance

### Table 46: Impact of ESG screening on sorted portfolio returns (2010 – 2017)

<table>
<thead>
<tr>
<th>Period</th>
<th>Pillar</th>
<th>North America</th>
<th>EMU</th>
<th>Europe-ex-EMU</th>
<th>Japan</th>
<th>World</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010 – 2013</td>
<td>ESG</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>2014 – 2017</td>
<td>ESG</td>
<td>++</td>
<td>++</td>
<td>0</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>++</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>+</td>
<td>++</td>
<td>0</td>
<td>+</td>
<td>++</td>
</tr>
</tbody>
</table>

Simulated results
Sorted portfolios

Figure 72: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI North America)

Figure 73: Annualized return of long/short $Q_1 – Q_5$ sorted portfolios (MSCI EMU)

The impact of investment flows

- The 2014 break
  - Strong mobilization of the largest institutional European investors: NBIM, APG, PGGM, ERAFP, FRR, etc.
  - They are massively invested in European stocks and America stocks: NBIM ≻ CalPERS + CalSTRS + NYSCRF for U.S. stocks

- The 2018-2019 period
  - Implication of U.S. investors continues to be weak
  - Strong mobilization of medium (or tier two) institutional European investors, that have a low exposure on American stocks
  - Mobilization of European investors is not sufficient
Figure 74: The monotonous assumption of the ESG-performance relationship
Figure 75: How to play ESG momentum?

- Q2 vs. Q1: less interesting
- Q4 vs. Q5: more interesting
We note \( b \) the benchmark, \( \mathbf{S} \) the vector of ESG scores and \( \Sigma \) the covariance matrix

We consider the following optimization problem:

\[
\mathbf{w}^* (\gamma) = \arg\min \frac{1}{2} \sigma^2 (\mathbf{w} | b) - \gamma \mathbf{S} (\mathbf{w} | b)
\]

where \( \sigma^2 (\mathbf{w} | b) = (\mathbf{w} - \mathbf{b})^\top \Sigma (\mathbf{w} - \mathbf{b}) \) and \( \mathbf{S} (\mathbf{w} | b) \) are the ex-ante tracking error variance and the ESG excess score of portfolio \( \mathbf{w} \) with respect to the benchmark \( \mathbf{b} \)

Since we have:

\[
\mathbf{S} (\mathbf{w} | \mathbf{b}) = (\mathbf{w} - \mathbf{b})^\top \mathbf{S} = \mathbf{S} (\mathbf{w}) - \mathbf{S} (\mathbf{b})
\]

we obtain the following optimization function:

\[
\mathbf{w}^* (\gamma) = \arg\min \frac{1}{2} \mathbf{w}^\top \Sigma \mathbf{w} - \mathbf{w}^\top (\gamma \mathbf{S} + \Sigma \mathbf{b})
\]

The QP form is given by \( Q = \Sigma \) and \( R = \gamma \mathbf{S} + \Sigma \mathbf{b} \)
Figure 76: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

Figure 77: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, individual pillars)

Figure 78: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

Figure 79: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2013, individual pillars)

Figure 80: Annualized excess return of ESG-optimized portfolios (MSCI World, 2014-2017, individual pillars)

**Figure 81:** Annualized excess return in bps of ESG-optimized portfolios (MSCI North America and EMU, 2010-2017)

The single-factor model is:

\[ R_{i,t} = \alpha_{i,j} + \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \]

where:
- \( R_{i,t} \) is the return of stock \( i \) at time \( t \)
- \( F_{j,t} \) is the value of the \( j^{th} \) common risk factor at time \( t \) (market, size, value, momentum, low-volatility, quality or ESG)
- \( \varepsilon_{i,t} \) is the idiosyncratic risk

The average proportion of the return variance explained by the common factor is given by:

\[ \tilde{R}_j^2 = \frac{1}{n} \sum_{i=1}^{n} R_{i,j}^2 = \frac{1}{n} \sum_{i=1}^{n} \left( 1 - \frac{\text{var} (\varepsilon_{i,t})}{\text{var} (R_{i,t})} \right) \]
### Table 47: Results of cross-section regression with long-only risk factors (single-factor linear regression model, average $R^2$)

<table>
<thead>
<tr>
<th>Factor</th>
<th>North America</th>
<th></th>
<th>Eurozone</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>40.8%</td>
<td>28.6%</td>
<td>42.8%</td>
<td>36.3%</td>
</tr>
<tr>
<td>Size</td>
<td>39.3%</td>
<td>26.1%</td>
<td>37.1%</td>
<td>23.3%</td>
</tr>
<tr>
<td>Value</td>
<td>38.9%</td>
<td>26.7%</td>
<td>41.6%</td>
<td>33.6%</td>
</tr>
<tr>
<td>Momentum</td>
<td>39.6%</td>
<td>26.3%</td>
<td>40.8%</td>
<td>34.1%</td>
</tr>
<tr>
<td>Low-volatility</td>
<td>35.8%</td>
<td>25.1%</td>
<td>38.7%</td>
<td>33.4%</td>
</tr>
<tr>
<td>Quality</td>
<td>39.1%</td>
<td>26.6%</td>
<td>42.4%</td>
<td>34.6%</td>
</tr>
<tr>
<td>ESG</td>
<td>40.1%</td>
<td>27.4%</td>
<td>42.6%</td>
<td>35.3%</td>
</tr>
</tbody>
</table>

Source: Roncalli (2020).

- Specific risk has increased during the period 2014 – 2019
- Since 2014, we find that:
  - ESG $\succ$ Value $\succ$ Quality $\succ$ Momentum $\succ$ ... (North America)
  - ESG $\succ$ Quality $\succ$ Momentum $\succ$ Value $\succ$ ... (Eurozone)
Multi-factor model

Regression model

We have:

\[ R_{i,t} = \alpha_i + \sum_{j=1}^{m} \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \]

where \( m \) is the number of risk factors

- 1F = market
- 5F = size + value + momentum + low-volatility + quality
- 6F = 5F + ESG
### Table 48: Results of cross-section regression with long-only risk factors
(multi-factor linear regression model, average $R^2$)

<table>
<thead>
<tr>
<th>Model</th>
<th>North America</th>
<th></th>
<th>Eurozone</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>40.8%</td>
<td>28.6%</td>
<td>42.8%</td>
<td>36.3%</td>
</tr>
<tr>
<td>5F model</td>
<td>46.1%</td>
<td>38.4%</td>
<td>49.5%</td>
<td>45.0%</td>
</tr>
<tr>
<td>6F model (5F + ESG)</td>
<td>46.7%</td>
<td>39.7%</td>
<td>50.1%</td>
<td>45.8%</td>
</tr>
</tbody>
</table>

Source: Roncalli (2020).

***p-value statistic for the MSCI Index (time-series, 2014 – 2019):
- $6F = \text{Size}, \text{Value}, \text{Momentum}, \text{Low-volatility}, \text{Quality}, \text{ESG}$ (North America)
- $6F = \text{Size}, \text{Value}, \text{Momentum}, \text{Low-volatility}, \text{Quality}, \text{ESG}$ (Eurozone)
**Factor selection**

- We use a lasso penalized regression is used in place of the traditional least squares regression:

\[
\{\hat{\alpha}_i, \hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,m}\} = \arg\min \left\{ \frac{1}{2} \text{var}(\varepsilon_{i,t}) + \lambda \|\beta_i\|_1 \right\}
\]

- Low-factor intensity \((\lambda \approx \infty)\) \(\Rightarrow\) we determine which risk factor is the most important

- When the factor intensity reaches 100% \((\lambda = 0)\), we obtain the same results calculated previously with the linear regression
Factor selection

**Figure 82:** Factor picking (MSCI North America, 2014-2019, global score)

Source: Roncalli (2020).
**Factor selection**

**Figure 83**: Factor picking (MSCI EMU, 2014-2019, global score)

Source: Roncalli (2020).
What is the difference between alpha and beta?

α or β?

"[...] When an alpha strategy is massively invested, it has an enough impact on the structure of asset prices to become a risk factor. [...] Indeed, an alpha strategy becomes a common market risk factor once it represents a significant part of investment portfolios and explains the cross-section dispersion of asset returns" (Roncalli, 2020)

- ESG remains an alpha strategy in North America
- ESG becomes a beta strategy (or a risk factor) in Europe
- Forward looking, ESG will be a beta strategy in North America
### Table 49: Performance of ESG indexes (MSCI World, 2010 – 2022)

<table>
<thead>
<tr>
<th>Year</th>
<th>Return (in %)</th>
<th>Alpha (in bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CW</td>
<td>ESG</td>
</tr>
<tr>
<td>2010</td>
<td>11.8</td>
<td>10.7</td>
</tr>
<tr>
<td>2011</td>
<td>−5.5</td>
<td>−5.4</td>
</tr>
<tr>
<td>2012</td>
<td>15.8</td>
<td>14.5</td>
</tr>
<tr>
<td>2013</td>
<td>26.7</td>
<td>27.6</td>
</tr>
<tr>
<td>2014</td>
<td>4.9</td>
<td>4.9</td>
</tr>
<tr>
<td>2015</td>
<td>−0.9</td>
<td>−1.1</td>
</tr>
<tr>
<td>2016</td>
<td>7.5</td>
<td>7.3</td>
</tr>
<tr>
<td>2017</td>
<td>22.4</td>
<td>21.0</td>
</tr>
<tr>
<td>2018</td>
<td>−8.7</td>
<td>−7.8</td>
</tr>
<tr>
<td>2019</td>
<td>27.7</td>
<td>28.2</td>
</tr>
<tr>
<td>2020</td>
<td>15.9</td>
<td>15.3</td>
</tr>
<tr>
<td>2021</td>
<td>21.8</td>
<td>24.7</td>
</tr>
<tr>
<td>2022</td>
<td>−18.1</td>
<td>−19.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>CW</th>
<th>ESG</th>
<th>SRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>3Y</td>
<td>4.9</td>
<td>5.0</td>
<td>5.7</td>
</tr>
<tr>
<td>5Y</td>
<td>6.1</td>
<td>6.4</td>
<td>7.4</td>
</tr>
<tr>
<td>7Y</td>
<td>8.5</td>
<td>8.5</td>
<td>9.6</td>
</tr>
<tr>
<td>10Y</td>
<td>8.9</td>
<td>8.9</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Source: MSCI, Factset & Author’s calculation.
Figure 84: Alpha return of several ESG equity indexes (in bps)

Source: MSCI, Factset & Author’s calculation.
Bond markets ≠ stock markets

**Stocks**
- ESG scoring is incorporated in portfolio management
- ESG = long-term business risk ⇒ strongly impacts the equity
- Portfolio integration
- Managing the business risk

**Bonds**
- ESG integration is generally limited to exclusions
- ESG lowly impacts the debt
- Portfolio completion
- Fixed income = impact investing
- Development of pure play ESG securities (green and social bonds)

⇒ Stockholders are more ESG sensitive than bond holders because of the capital structure
ESG investment flows affect asset pricing differently

- Impact on carry (coupon effect)?
- Impact on price dynamics (credit spread/mark-to-market effect)?
- Buy-and-hold portfolios ≠ managed portfolios

The distinction between IG and HY bonds

- ESG and credit ratings are correlated
- There are more worst-in-class issuers in the HY universe, and best-in-class issuers in the IG universe
- Non-neutrality of the bond universe (bonds ≠ stocks)
Bond markets $\neq$ stock markets

Figure 85: Probability density function of ESG scores

- The average $z$-score for IG bonds is positive
- The average $z$-score for HY bonds is negative

Source: Ben Slimane et al. (2019).
Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date $t$, we rank the bonds according to their Amundi ESG z-score
- We form the five quintile portfolios $Q_i$ for $i = 1, \ldots, 5$
- The portfolio $Q_i$ is invested during the period $[t, t+1]$:
  - $Q_1$ corresponds to the best-in-class portfolio (best scores)
  - $Q_5$ corresponds to the worst-in-class portfolio (worst scores)
- Monthly rebalancing
- Universe: ICE (BofAML) Large Cap IG EUR Corporate Bond
- Sector-weighted and sector-neutral portfolio
- Within a sector, bonds are equally-weighted
**Figure 86:** Annualized return in bps of the long short $Q_1 - Q_5$ strategy (IG, 2010 – 2019)

Source: Ben Slimane *et al.* (2019).
Table 50: Carry statistics (in bps)

<table>
<thead>
<tr>
<th>Period</th>
<th>$Q_1$</th>
<th>$Q_5$</th>
<th>$Q_1 - Q_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010 – 2013</td>
<td>175</td>
<td>192</td>
<td>-17</td>
</tr>
<tr>
<td>2014 – 2019</td>
<td>113</td>
<td>128</td>
<td>-15</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2019).
Figure 87: Annualized credit return in bps of ESG sorted portfolios (EUR IG, 2010 – 2019)

Source: Ben Slimane et al. (2019).
Simulated results
Optimized portfolios

- Portfolio $w = (w_1, \ldots, w_n)$ and benchmark $b = (b_1, \ldots, b_n)$
- ESG score of the portfolio:
  \[
  S(w) = \sum_{i=1}^{n} w_i S_i
  \]
- ESG excess score of portfolio $w$ with respect to benchmark $b$:
  \[
  S(w \mid b) = \sum_{i=1}^{n} (w_i - b_i) S_i = S(w) - S(b)
  \]
- z-scores $\Rightarrow S(w \mid b) > 0$
- Active or tracking risk $R(w \mid b)$
- The optimization problem becomes:
  \[
  w^*(\gamma) = \arg\min R(w \mid b) - \gamma S(w \mid b)
  \]
Simulated results
Optimized portfolios

- The modified duration risk of portfolio $w$ with respect to benchmark $b$ is:

$$R_{MD}(x \mid b) = \sum_{j=1}^{n_S} \left( \left( \sum_{i \in \text{Sector}(j)} w_i \cdot MD_i \right) - \left( \sum_{i \in \text{Sector}(j)} b_i \cdot MD_i \right) \right)^2$$

where $n_S$ is the number of sectors and $MD_i$ is the modified duration of bond $i$

- An alternative is to use the DTS risk measure:

$$R_{DTS}(x \mid b) = \sum_{j=1}^{n_S} \left( \left( \sum_{i \in \text{Sector}(j)} w_i \cdot DTS_i \right) - \left( \sum_{i \in \text{Sector}(j)} b_i \cdot DTS_i \right) \right)^2$$

where $DTS_i$ is the DTS of bond $i$

- Hybrid approach:

$$R(w \mid b) = \frac{1}{2} R_{MD}(w \mid b) + \frac{1}{2} R_{DTS}(w \mid b)$$
Figure 88: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2010 – 2013)

Source: Ben Slimane et al. (2019).
Figure 89: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2014 – 2016)

Source: Ben Slimane et al. (2019).
Figure 90: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2010 – 2013)

Source: Ben Slimane et al. (2019).
**Figure 91**: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2014 – 2016)

Source: Ben Slimane *et al.* (2019).
Table 51: Performance of ESG bond indexes (sovereign)

<table>
<thead>
<tr>
<th>Year</th>
<th>FTSE WGBI</th>
<th>FTSE EGBI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Return</td>
<td>Alpha</td>
</tr>
<tr>
<td></td>
<td>BM ESG ESG</td>
<td>BM ESG ESG</td>
</tr>
<tr>
<td>2010</td>
<td>4.61 4.31  −30</td>
<td>0.61 4.14  353</td>
</tr>
<tr>
<td>2011</td>
<td>6.35 7.05  69</td>
<td>3.41 7.31  391</td>
</tr>
<tr>
<td>2012</td>
<td>1.65 3.06  141</td>
<td>10.65 7.39 −326</td>
</tr>
<tr>
<td>2013</td>
<td>−4.00 −2.95 105</td>
<td>2.21 −1.40 −362</td>
</tr>
<tr>
<td>2014</td>
<td>−0.48 −0.22  26</td>
<td>13.19 11.44 −175</td>
</tr>
<tr>
<td>2015</td>
<td>−3.57 −4.85 −128</td>
<td>1.65 0.39 −126</td>
</tr>
<tr>
<td>2016</td>
<td>1.60 1.02  −59</td>
<td>3.20 4.00  81</td>
</tr>
<tr>
<td>2017</td>
<td>7.49 8.16  67</td>
<td>0.15 −0.47 −62</td>
</tr>
<tr>
<td>2018</td>
<td>−0.84 −1.41 −57</td>
<td>0.88 1.65  78</td>
</tr>
<tr>
<td>2019</td>
<td>5.90 5.56  −34</td>
<td>6.72 4.45 −227</td>
</tr>
<tr>
<td>2020</td>
<td>10.11 10.90 79</td>
<td>5.03 4.11 −92</td>
</tr>
<tr>
<td>2021</td>
<td>−6.97 −7.15 −17</td>
<td>−3.54 −3.76 −21</td>
</tr>
<tr>
<td>2022</td>
<td>−18.26 −20.00 −173</td>
<td>−18.52 −19.06 −54</td>
</tr>
<tr>
<td>3Y</td>
<td>−5.75 −6.26 −51</td>
<td>−6.19 −6.74 −55</td>
</tr>
<tr>
<td>5Y</td>
<td>−2.54 −3.03 −49</td>
<td>−2.33 −2.95 −61</td>
</tr>
<tr>
<td>7Y</td>
<td>−0.58 −0.93 −35</td>
<td>−1.21 −1.63 −42</td>
</tr>
<tr>
<td>10Y</td>
<td>−1.22 −1.46 −24</td>
<td>0.77 −0.17 −94</td>
</tr>
</tbody>
</table>
### Table 52: Performance of ESG bond indexes (corporates)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bloomberg Euro Aggregate Corporate Return</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BM</td>
<td>SRI</td>
</tr>
<tr>
<td>2010</td>
<td>3.07</td>
<td>2.93</td>
</tr>
<tr>
<td>2011</td>
<td>1.49</td>
<td>1.17</td>
</tr>
<tr>
<td>2013</td>
<td>2.37</td>
<td>2.49</td>
</tr>
<tr>
<td>2014</td>
<td>8.40</td>
<td>8.31</td>
</tr>
<tr>
<td>2015</td>
<td>-0.56</td>
<td>-0.59</td>
</tr>
<tr>
<td>2016</td>
<td>4.73</td>
<td>4.60</td>
</tr>
<tr>
<td>2017</td>
<td>2.41</td>
<td>2.47</td>
</tr>
<tr>
<td>2018</td>
<td>-1.25</td>
<td>-1.12</td>
</tr>
<tr>
<td>2019</td>
<td>6.24</td>
<td>6.01</td>
</tr>
<tr>
<td>2020</td>
<td>2.77</td>
<td>2.69</td>
</tr>
<tr>
<td>2021</td>
<td>-0.97</td>
<td>-0.96</td>
</tr>
<tr>
<td>3Y</td>
<td>-4.21</td>
<td>-4.22</td>
</tr>
<tr>
<td>5Y</td>
<td>-1.61</td>
<td>-1.63</td>
</tr>
<tr>
<td>7Y</td>
<td>-0.16</td>
<td>-0.19</td>
</tr>
<tr>
<td>10Y</td>
<td>0.88</td>
<td>0.86</td>
</tr>
</tbody>
</table>
## Table 53: Performance of ESG bond indexes (corporates)

<table>
<thead>
<tr>
<th>Year</th>
<th>Bloomberg US Corporate Return</th>
<th>Alpha</th>
<th>Bloomberg Global High Yield Return</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BM</td>
<td>SRI</td>
<td>S-SRI</td>
<td>ESG-S</td>
</tr>
<tr>
<td>2019</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2021</td>
<td>−1.04</td>
<td>−1.55</td>
<td>9.56</td>
<td>2.34</td>
</tr>
<tr>
<td>2022</td>
<td>−15.76</td>
<td>−15.12</td>
<td>−1.10</td>
<td>−13.86</td>
</tr>
</tbody>
</table>
Definition
<table>
<thead>
<tr>
<th>Theoretical models</th>
<th>Equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical results</td>
<td>Corporate bonds</td>
</tr>
<tr>
<td>Cost of capital</td>
<td>Sovereign bonds</td>
</tr>
</tbody>
</table>
Correlation between Credit ratings and ESG ratings

Figure 92: Average ESG $\varepsilon$-score with respect to the credit rating (2010 – 2019)
An integrated Credit-ESG model

We consider the following regression model:

\[
\ln \text{OAS}_{i,t} = \alpha_t + \beta_{\text{esg}} \cdot S_{i,t} + \beta_{\text{md}} \cdot \text{MD}_{i,t} + \sum_{j=1}^{N_{\text{Sector}}} \beta_{\text{Sector}} (j) \cdot \text{Sector}_{i,t} (j) + \\
\beta_{\text{sub}} \cdot \text{SUB}_{i,t} + \sum_{k=1}^{N_{\text{Rating}}} \beta_{\text{Rating}} (k) \cdot \text{Rating}_{i,t} (k) + \varepsilon_{i,t}
\]

where:
- \( S_{i,t} \) is the ESG z-score of Bond \( i \) at time \( t \)
- \( \text{SUB}_{i,t} \) is a dummy variable accounting for subordination of the bond
- \( \text{MD}_{i,t} \) is the modified duration
- \( \text{Sector}_{i,t} (j) \) is a dummy variable for the \( j^{\text{th}} \) sector
- \( \text{Rating}_{i,t} (k) \) is a dummy variable for the \( k^{\text{th}} \) rating
An integrated Credit-ESG model

Table 54: Results of the panel data regression model (EUR IG, 2010 – 2019)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ESG</td>
<td>E</td>
<td>S</td>
<td>G</td>
<td>ESG</td>
<td>E</td>
<td>S</td>
<td>G</td>
</tr>
<tr>
<td>$R^2$</td>
<td>60.0%</td>
<td>59.4%</td>
<td>59.5%</td>
<td>60.3%</td>
<td>66.3%</td>
<td>65.0%</td>
<td>65.2%</td>
<td>64.6%</td>
</tr>
<tr>
<td>Excess $R^2$</td>
<td>0.6%</td>
<td>0.0%</td>
<td>0.2%</td>
<td>1.0%</td>
<td>4.0%</td>
<td>2.6%</td>
<td>2.9%</td>
<td>2.3%</td>
</tr>
<tr>
<td>$\hat{\beta}_{esg}$</td>
<td>-0.05</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.07</td>
<td>-0.09</td>
<td>-0.08</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
<tr>
<td>t-statistic</td>
<td>-32</td>
<td>-7</td>
<td>-16</td>
<td>-39</td>
<td>-124</td>
<td>-98</td>
<td>-104</td>
<td>-92</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2020)

The assumption $\mathcal{H}_0 : \beta_{esg} < 0$ is not rejected
ESG cost of capital with min/max score bounds

We calculate the difference between:

1. the funding cost of the **worst-in-class issuer** and
2. the funding cost of the **best-in-class issuer**

by assuming that:

- the two issuers have the same credit rating;
- the two issuers belong to the same sector;
- the two issuers have the same capital structure;
- the two issuers have the same debt maturity.

⇒ Two approaches:

- **Theoretical approach:** ESG scores are set to $-3$ and $+3$ (not realistic)
- **Empirical approach:** ESG scores are set to observed min/max score bounds (e.g. min/max = $-2.0/+1.9$ for Consumer Cyclical A-rated EUR, $-2.1/+3.2$ for Banking A-rated EUR, etc.)
## ESG cost of capital with min/max score bounds

### Table 55: ESG cost of capital (IG, 2014 – 2019)

<table>
<thead>
<tr>
<th>Industry</th>
<th>EUR AA</th>
<th>EUR A</th>
<th>EUR BBB</th>
<th>EUR Average</th>
<th>USD AA</th>
<th>USD A</th>
<th>USD BBB</th>
<th>USD Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banking</td>
<td>23</td>
<td>45</td>
<td>67</td>
<td>45</td>
<td>11</td>
<td>19</td>
<td>33</td>
<td>21</td>
</tr>
<tr>
<td>Basic</td>
<td>9</td>
<td>25</td>
<td>44</td>
<td>26</td>
<td>5</td>
<td>15</td>
<td>34</td>
<td>18</td>
</tr>
<tr>
<td>Capital Goods</td>
<td>8</td>
<td>32</td>
<td>42</td>
<td>27</td>
<td>6</td>
<td>15</td>
<td>26</td>
<td>16</td>
</tr>
<tr>
<td>Communication</td>
<td>26</td>
<td>48</td>
<td>37</td>
<td>37</td>
<td>5</td>
<td>11</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>Consumer Cyclical</td>
<td>3</td>
<td>26</td>
<td>43</td>
<td>28</td>
<td>2</td>
<td>8</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>Consumer Non-Cyclical</td>
<td>15</td>
<td>29</td>
<td>31</td>
<td>25</td>
<td>6</td>
<td>12</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>Utility &amp; Energy</td>
<td>12</td>
<td>32</td>
<td>56</td>
<td>33</td>
<td>9</td>
<td>14</td>
<td>31</td>
<td>18</td>
</tr>
<tr>
<td>Average</td>
<td>12</td>
<td>31</td>
<td>48</td>
<td><strong>31</strong></td>
<td>7</td>
<td>13</td>
<td>26</td>
<td><strong>15</strong></td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2020)
ESG and sovereign risk

Motivation

- Financial analysis **versus**/and extra-financial analysis
- Sovereign risk ≠ Corporate risk
- Which ESG metrics are priced and not priced in by the market?
- What is the nexus between ESG analysis and credit analysis?
The economics of sovereign risk

A Tale of Two Countries

- The example of Barbados and Jamaica
- Why the economic growth of two countries with the same economic development at time \( t \) is different 10, 20 or 30 years later?
Sovereign ESG themes

Environmental
- Biodiversity
- Climate change
- Commitment to environmental standards
- Energy mix
- Natural hazard
- Natural hazard outcome
- Non-renewable energy resources
- Temperature
- Water management

Social
- Civil unrest
- Demographics
- Education
- Gender
- Health
- Human rights
- Income
- Labour market standards
- Migration
- Water and electricity access

Governance
- Business environment and R&D
- Governance effectiveness
- Infrastructure and mobility
- International relations
- Justice
- National security
- Political stability
Assessment of a country’s creditworthiness

- Confidence in the country? Only financial reasons?
- Country default risk cannot be summarized by only financial figures!
- Why some rich countries have to pay a credit risk premium?
- How to explain the large differences in Asia?
## Endogenous variable

10Y sovereign bond yield

## Explanatory variables

- 269 ESG variables grouped into 26 ESG thematics
- 183 indicators come from Verisk Maplecroft database, the 86 remaining metrics were retrieved from the World Bank, ILO, WHO, FAO, UN...
- 6 control variables: GDP Growth, Net Debt, Reserves, Account Balance, Inflation and **Credit Rating**

## Panel dimensions

- 67 countries
- 2015–2020
Single-factor analysis

Regression model

Let $s_{i,t}$ be the bond yield spread of the country $i$ at time $t$. We consider the following regression model estimated by OLS:

$$s_{i,t} = \alpha + \beta x_{i,t} + \sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} + \epsilon_{i,t}$$

where $z_{i,t}^{(k)}$ is the ESG metric.

Control variables/
Fundamental model

and:

$$\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} = \gamma_1 g_{i,t} + \gamma_2 \pi_{i,t} + \gamma_3 d_{i,t} + \gamma_4 ca_{i,t} + \gamma_5 r_{i,t} + \gamma_6 R_{i,t}$$

where $g_{i,t}$ is the economic growth, $\pi_{i,t}$ is the inflation, $d_{i,t}$ is the debt ratio, $ca_{i,t}$ is the current account balance, $r_{i,t}$ is the reserve adequacy and $R_{i,t}$ is the credit rating.
### Table 56: 7 most relevant indicators of the single-factor analysis per pillar

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Thematic</th>
<th>Indicator</th>
<th>$\Delta R^2_c$</th>
<th>F-test</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate change</td>
<td>Climate change</td>
<td>Climate change vulnerability (acute)</td>
<td>5.51%</td>
<td>57.19</td>
<td>1</td>
</tr>
<tr>
<td>Climate change</td>
<td>Climate change</td>
<td>Climate change exposure (extreme)</td>
<td>4.80%</td>
<td>48.60</td>
<td>2</td>
</tr>
<tr>
<td>Water management</td>
<td>Agricultural water withdrawal</td>
<td></td>
<td>4.02%</td>
<td>47.10</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Climate change</td>
<td>Climate change sensitivity (acute)</td>
<td>3.95%</td>
<td>38.79</td>
<td>4</td>
</tr>
<tr>
<td>Biodiversity</td>
<td>Biodiversity threatening score</td>
<td></td>
<td>3.53%</td>
<td>35.32</td>
<td>5</td>
</tr>
<tr>
<td>Climate change</td>
<td>Climate change exposure (acute)</td>
<td></td>
<td>3.39%</td>
<td>32.95</td>
<td>6</td>
</tr>
<tr>
<td>Climate change</td>
<td>Climate change vulnerability (average)</td>
<td></td>
<td>3.11%</td>
<td>31.16</td>
<td>7</td>
</tr>
<tr>
<td>Human rights</td>
<td>Freedom of assembly</td>
<td></td>
<td>8.74%</td>
<td>89.58</td>
<td>1</td>
</tr>
<tr>
<td>Human rights</td>
<td>Extent of arbitrary unrest</td>
<td></td>
<td>8.04%</td>
<td>80.10</td>
<td>2</td>
</tr>
<tr>
<td>Human rights</td>
<td>Extent of torture and ill treatment</td>
<td></td>
<td>7.63%</td>
<td>75.48</td>
<td>3</td>
</tr>
<tr>
<td>S</td>
<td>Labour market standards</td>
<td>Severity of working time violations</td>
<td>7.21%</td>
<td>70.46</td>
<td>4</td>
</tr>
<tr>
<td>Labour market standards</td>
<td>Forced labour violations (extent)</td>
<td></td>
<td>6.10%</td>
<td>54.40</td>
<td>5</td>
</tr>
<tr>
<td>Labour market standards</td>
<td>Child labour (extent)</td>
<td></td>
<td>5.83%</td>
<td>54.68</td>
<td>6</td>
</tr>
<tr>
<td>Migration</td>
<td>Vulnerability of migrant workers</td>
<td></td>
<td>5.83%</td>
<td>53.76</td>
<td>7</td>
</tr>
<tr>
<td>National security</td>
<td>Severity of kidnappings</td>
<td></td>
<td>6.80%</td>
<td>54.49</td>
<td>1</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Ease of access to loans</td>
<td></td>
<td>6.77%</td>
<td>73.57</td>
<td>2</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td>Roads km</td>
<td></td>
<td>6.45%</td>
<td>63.66</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Business environment and R&amp;D</td>
<td>Capacity for innovation</td>
<td>5.65%</td>
<td>58.58</td>
<td>4</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Ethical behaviour of firms</td>
<td></td>
<td>5.37%</td>
<td>55.14</td>
<td>5</td>
</tr>
<tr>
<td>National security</td>
<td>Frequency of kidnappings</td>
<td></td>
<td>5.27%</td>
<td>48.49</td>
<td>6</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td>Physical connectivity</td>
<td></td>
<td>4.94%</td>
<td>50.76</td>
<td>7</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
### Table 57: Summary of the results

<table>
<thead>
<tr>
<th>Relevant</th>
<th>E</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Temperature</strong></td>
<td>Labour market standards</td>
<td>Infrastructure and mobility</td>
</tr>
<tr>
<td></td>
<td>Climate change</td>
<td>Human rights</td>
<td>National security</td>
</tr>
<tr>
<td></td>
<td>Natural hazard outcome</td>
<td>Migration</td>
<td>Justice</td>
</tr>
<tr>
<td>Less relevant</td>
<td>Water management</td>
<td><strong>Income</strong></td>
<td>Political stability</td>
</tr>
<tr>
<td></td>
<td>Energy mix</td>
<td><strong>Education</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Water and electricity access</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We consider the following multi-factor regression model:

\[ s_{i,t} = \alpha + \sum_{j=1}^{m} \beta_j x_{i,t}^{(j)} + \sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} + \varepsilon_{i,t} \]

**ESG variables / Extra-financial model**

**Control variables / Fundamental model**

**A 4-step process**

1. We consider the significant variables of the single-factor analysis at the 1% level
2. We filter the variables selected at Step 1 in order to eliminate redundant variables in each ESG theme
3. We perform a lasso regression to retain the seven most relevant variables within each ESG pillar
4. We perform a multi-factor analysis \((m = 21 \Rightarrow m = 7)\)
### Table 58: Example of variables exhibiting high correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta R^2_c$</th>
<th>Correlation$_{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate change exposure (average)</td>
<td>2.12%</td>
<td>1.00 0.74 0.80 0.48 0.92 0.77</td>
</tr>
<tr>
<td>Climate change exposure (acute)</td>
<td>3.89%</td>
<td>0.74 1.00 0.65 0.51 0.73 0.89</td>
</tr>
<tr>
<td>Climate change exposure (extreme)</td>
<td>4.80%</td>
<td>0.80 0.65 1.00 0.54 0.79 0.71</td>
</tr>
<tr>
<td>Climate change sensitivity (average)</td>
<td>3.95%</td>
<td>0.48 0.51 0.54 1.00 0.76 0.81</td>
</tr>
<tr>
<td>Climate change vulnerability (average)</td>
<td>3.11%</td>
<td>0.92 0.73 0.79 0.76 1.00 0.89</td>
</tr>
<tr>
<td>Climate change vulnerability (acute)</td>
<td>5.51%</td>
<td>0.77 0.89 0.71 0.81 0.89 1.00</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)

### Selecting the variables

1. For each variable, we identify the highest pairwise correlation
2. Among each couple, we retain the variable showing the highest $\Delta R^2_c$
3. Among these variables, we select the variable with the lowest correlation
Figure 93: Filtering process

Original dataset

Significant variables

Redundant variables

Lasso by pillar

Multi-factor estimation

269

184

74

21

Source: Semet et al. (2021)
### Table 59: Results after Step 3: Lasso regression pillar by pillar

<table>
<thead>
<tr>
<th>Rank</th>
<th>Pillar</th>
<th>Thematic</th>
<th>Variable</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Non-renewable energy resources</td>
<td>Total GHG emissions</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Biodiversity</td>
<td>Biodiversity threatening score</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Natural hazard</td>
<td>Severe storm hazard (absolute high extreme)</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Temperature</td>
<td>Temperature change</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Non-renewable energy resources</td>
<td>Fossil fuel intensity of the economy</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Natural hazard</td>
<td>Drought hazard (absolute high extreme)</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Commitment to environmental standards</td>
<td>Paris Agreement</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Migration</td>
<td>Vulnerability of migrant workers</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Demographics</td>
<td>Projected population change (5 years)</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Civil unrest</td>
<td>Frequency of civil unrest incidents</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Labor market standards</td>
<td>Index of labor standards</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Labor market standards</td>
<td>Right to join trade unions (protection)</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Human rights</td>
<td>Food import security</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Income</td>
<td>Average monthly wage</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>International relationships</td>
<td>Exporting across borders (cost)</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Business environment and R&amp;D</td>
<td>Ethical behaviour of firms</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>National security</td>
<td>Severity of kidnappings</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Business environment and R&amp;D</td>
<td>Capacity for innovation</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Infrastructure and mobility</td>
<td>Physical connectivity</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Infrastructure and mobility</td>
<td>Air transport departures</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Infrastructure and mobility</td>
<td>Rail lines km</td>
<td>−</td>
<td></td>
</tr>
</tbody>
</table>

*Source: Semet et al. (2021)*
### Multi-factor analysis

Global analysis - Lasso regression on the three pillars

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Indicator</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>Exporting across borders (cost)</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Severe storm hazard</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>Capacity for innovation</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Ethical behaviour of firms</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>Temperature change</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Severity of kidnappings</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>Drought hazard</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>Fossil fuel intensity of the economy</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>Biodiversity threatening score</td>
<td>9</td>
</tr>
<tr>
<td>S</td>
<td>Index of labor standards</td>
<td>10</td>
</tr>
</tbody>
</table>

### ESG pillar importance

Source: Semet et al. (2021)
### Table 60: Final multi-factor model

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\sigma}(\hat{\beta})$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $\alpha$</td>
<td>2.834</td>
<td>0.180</td>
<td>15.72***</td>
<td>0.00</td>
</tr>
<tr>
<td>GDP growth $g_{i,t}$</td>
<td>0.017</td>
<td>0.012</td>
<td>1.37</td>
<td>0.17</td>
</tr>
<tr>
<td>Inflation $\pi_{i,t}$</td>
<td>0.048</td>
<td>0.007</td>
<td>6.64***</td>
<td>0.00</td>
</tr>
<tr>
<td>Debt ratio $d_{i,t}$</td>
<td>-0.001</td>
<td>0.001</td>
<td>-1.71*</td>
<td>0.08</td>
</tr>
<tr>
<td>Current account balance $ca_{i,t}$</td>
<td>-0.012</td>
<td>0.005</td>
<td>-2.45**</td>
<td>0.01</td>
</tr>
<tr>
<td>Reserve adequacy $r_{i,t}$</td>
<td>0.005</td>
<td>0.007</td>
<td>0.74</td>
<td>0.45</td>
</tr>
<tr>
<td>Rating score $R_{i,t}$</td>
<td>-0.013</td>
<td>0.001</td>
<td>-9.08***</td>
<td>0.00</td>
</tr>
<tr>
<td>Exporting across borders (cost)</td>
<td>$4.05e^{-04}$</td>
<td>$9.83e^{-05}$</td>
<td>4.11***</td>
<td>0.00</td>
</tr>
<tr>
<td>Severe storm hazard (absolute high extreme)</td>
<td>-0.015</td>
<td>0.009</td>
<td>-1.66*</td>
<td>0.09</td>
</tr>
<tr>
<td>Capacity for innovation</td>
<td>-0.004</td>
<td>0.001</td>
<td>-4.99***</td>
<td>0.00</td>
</tr>
<tr>
<td>Ethical behavior of firms</td>
<td>-0.061</td>
<td>0.021</td>
<td>-2.79***</td>
<td>0.00</td>
</tr>
<tr>
<td>Temperature change</td>
<td>-0.149</td>
<td>0.042</td>
<td>-3.50***</td>
<td>0.00</td>
</tr>
<tr>
<td>Severity of kidnappings</td>
<td>-0.032</td>
<td>0.007</td>
<td>-4.25***</td>
<td>0.00</td>
</tr>
<tr>
<td>Drought hazard (absolute high extreme)</td>
<td>$3.33e^{-08}$</td>
<td>$1.27e^{-08}$</td>
<td>2.60***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\Delta R^2_c = 13.51\%$, $F$-test = 29.28***

Source: Semet et al. (2021)
Multi-factor analysis
High income vs middle income countries

Thierry Roncalli
Course 2023-2024 in Sustainable Finance
### Multi-factor analysis

#### High income countries

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Indicator</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Fossil fuel intensity of the economy</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Temperature change</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>Cooling degree days annual average</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>Capacity for innovation</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>Heat stress (future)</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Severity of kidnappings</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>Biodiversity threatening score</td>
<td>7</td>
</tr>
<tr>
<td>G</td>
<td>Efficacy of corporate boards</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>Total GHG emissions</td>
<td>9</td>
</tr>
<tr>
<td>S</td>
<td>Significant marginalized group</td>
<td>10</td>
</tr>
</tbody>
</table>

#### ESG pillar importance

![ESG pillar importance chart](chart.png)

- Transition risk
- **S** is lagging

Source: Semet et al. (2021)
Physical risk

Social issues are priced
Explaining credit ratings with ESG metrics

Statistical framework

We consider the logit model:

\[
\Pr \{ G_{i,t} = 1 \} = F \left( \beta_0 + \sum_{j=1}^{m} \beta_j x_{i,t}^{(j)} \right)
\]

where:

- \( G_{i,t} = 1 \) indicates if the country \( i \) is rated upper grade at time \( t \)
  - If the rating \( \geq A \) then \( G_{i,t} = 1 \)
  - if the rating \( \leq BBB \) then \( G_{i,t} = 0 \)
- \( F(z) \) is the logistic cumulative density function
- \( x_{i,t}^{(j)} \) is the \( j^{th} \) selected indicator

We note \( \theta_j = e^{\beta_j} \) is the odds-ratio coefficient

Lasso-penalized logit regression

Again, we perform a lasso regression to retain the seven most relevant variables for each ESG pillar and then we perform a multi-factor analysis
**Table 61**: List of selected ESG variables for the logistic regression

<table>
<thead>
<tr>
<th>Theme</th>
<th>Variable</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commitment to environmental standards</td>
<td>Domestic regulatory framework</td>
<td>1</td>
</tr>
<tr>
<td>Climate change</td>
<td>Climate change vulnerability (average)</td>
<td>2</td>
</tr>
<tr>
<td>Water management</td>
<td>Water import security (average)</td>
<td>3</td>
</tr>
<tr>
<td>Energy mix</td>
<td>Energy self sufficiency</td>
<td>4</td>
</tr>
<tr>
<td>Water management</td>
<td>Wastewater treatment index</td>
<td>5</td>
</tr>
<tr>
<td>Water management</td>
<td>Water intensity of the economy</td>
<td>6</td>
</tr>
<tr>
<td>Biodiversity</td>
<td>Biodiversity threatening score</td>
<td>7</td>
</tr>
<tr>
<td>Health</td>
<td>Health expenditure per capita</td>
<td>1</td>
</tr>
<tr>
<td>Water and electricity access</td>
<td>Public dissatisfaction with water quality</td>
<td>2</td>
</tr>
<tr>
<td>Education</td>
<td>Mean years of schooling of adults</td>
<td>3</td>
</tr>
<tr>
<td>Income</td>
<td>Base pay / value added per worker</td>
<td>4</td>
</tr>
<tr>
<td>Demographics</td>
<td>Urban population change (5 years)</td>
<td>5</td>
</tr>
<tr>
<td>Human rights</td>
<td>Basic food stuffs net imports per person</td>
<td>6</td>
</tr>
<tr>
<td>Human rights</td>
<td>Food import security</td>
<td>7</td>
</tr>
<tr>
<td>Government effectiveness</td>
<td>Government effectiveness index</td>
<td>1</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Venture capital availability</td>
<td>2</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>R&amp;D expenditure (% of GDP)</td>
<td>3</td>
</tr>
<tr>
<td>Infrastructure and mobility</td>
<td>Customs efficiency</td>
<td>4</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Enforcing a contract (time)</td>
<td>5</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Paying tax (process)</td>
<td>6</td>
</tr>
<tr>
<td>Business environment and R&amp;D</td>
<td>Getting electricity (time)</td>
<td>7</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
### Explaining credit ratings with ESG metrics

#### E pillar

**Table 62: Logit model with environmental variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma}(\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic regulatory framework</td>
<td>1.415</td>
<td>0.156</td>
<td>3.16***</td>
<td>0.00</td>
</tr>
<tr>
<td>Climate change vulnerability (average)</td>
<td>2.929</td>
<td>0.572</td>
<td>5.51***</td>
<td>0.00</td>
</tr>
<tr>
<td>Water import security (average)</td>
<td>1.385</td>
<td>0.147</td>
<td>3.07***</td>
<td>0.00</td>
</tr>
<tr>
<td>Energy self sufficiency</td>
<td>0.960</td>
<td>0.033</td>
<td>−1.16</td>
<td>0.24</td>
</tr>
<tr>
<td>Wastewater treatment index</td>
<td>1.011</td>
<td>0.008</td>
<td>1.36</td>
<td>0.17</td>
</tr>
<tr>
<td>Water intensity of the economy</td>
<td>1.000</td>
<td>0.000</td>
<td>−1.02</td>
<td>0.30</td>
</tr>
<tr>
<td>Biodiversity threatening score</td>
<td>0.887</td>
<td>0.026</td>
<td>−4.02***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell(\hat{\beta}) = −107.60$, AIC = 231.19, $R^2 = 49.1\%$, ACC = 83.6%

*Source: Semet et al. (2021)*
### Table 63: Logit model with social variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma}(\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health expenditure per capita</td>
<td>1.001</td>
<td>0.000</td>
<td>3.47***</td>
<td>0.00</td>
</tr>
<tr>
<td>Public dissatisfaction with water quality</td>
<td>0.889</td>
<td>0.024</td>
<td>−4.27***</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean years of schooling of adults</td>
<td>2.710</td>
<td>0.583</td>
<td>4.64***</td>
<td>0.00</td>
</tr>
<tr>
<td>Base pay / value added per worker</td>
<td>0.000</td>
<td>0.000</td>
<td>−5.13***</td>
<td>0.00</td>
</tr>
<tr>
<td>Urban population change (5 years)</td>
<td>1.653</td>
<td>0.131</td>
<td>6.36***</td>
<td>0.00</td>
</tr>
<tr>
<td>Basic food stuffs net imports per person</td>
<td>0.996</td>
<td>0.001</td>
<td>−3.58***</td>
<td>0.00</td>
</tr>
<tr>
<td>Food import security</td>
<td>0.973</td>
<td>0.006</td>
<td>−4.33***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell(\hat{\beta}) = -72.41, \text{ AIC} = 160.83, \mathcal{R}^2 = 65.6\%, \text{ ACC} = 87.9\%$

*Source: Semet et al. (2021)*
Explaining credit ratings with ESG metrics

G pillar

Table 64: Logit model with governance variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma} (\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government effectiveness index</td>
<td>1.096</td>
<td>0.035</td>
<td>2.81***</td>
<td>0.00</td>
</tr>
<tr>
<td>Venture capital availability</td>
<td>1.020</td>
<td>0.005</td>
<td>4.16***</td>
<td>0.00</td>
</tr>
<tr>
<td>R&amp;D expenditure (% of GDP)</td>
<td>2.259</td>
<td>1.006</td>
<td>1.83*</td>
<td>0.06</td>
</tr>
<tr>
<td>Customs efficiency</td>
<td>2.193</td>
<td>1.657</td>
<td>1.04</td>
<td>0.29</td>
</tr>
<tr>
<td>Enforcing a contract (time)</td>
<td>0.997</td>
<td>0.001</td>
<td>−3.69***</td>
<td>0.00</td>
</tr>
<tr>
<td>Paying tax (process)</td>
<td>0.914</td>
<td>0.031</td>
<td>−2.63***</td>
<td>0.00</td>
</tr>
<tr>
<td>Getting electricity (time)</td>
<td>0.989</td>
<td>0.004</td>
<td>−2.73***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell \left( \hat{\beta} \right) = −67.78$, AIC = 151.57, $R^2 = 67.9\%$, ACC = 90.1%

Source: Semet et al. (2021)
Explaining credit ratings with ESG metrics
E, S and G pillars

Table 65: Logit model with the ESG selected variables

<table>
<thead>
<tr>
<th>Pillar</th>
<th>Variable</th>
<th>$\hat{\theta}_j$</th>
<th>$\hat{\sigma} (\hat{\theta}_j)$</th>
<th>t-student</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Domestic regulatory framework</td>
<td>2.881</td>
<td>2.108</td>
<td>1.44</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Climate change vulnerability (average)</td>
<td>0.275</td>
<td>0.302</td>
<td>−1.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Water import security (average)</td>
<td>0.717</td>
<td>0.467</td>
<td>−0.50</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>Biodiversity threatening score</td>
<td>1.029</td>
<td>0.199</td>
<td>0.14</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>Health expenditure per capita</td>
<td>0.998</td>
<td>0.002</td>
<td>−1.10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Public dissatisfaction with water quality</td>
<td>1.332</td>
<td>0.269</td>
<td>1.41</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Mean years of schooling of adults</td>
<td>68.298</td>
<td>85.559</td>
<td>3.37***</td>
<td>0.00</td>
</tr>
<tr>
<td>S</td>
<td>Base pay / value added per worker</td>
<td>0.000</td>
<td>0.000</td>
<td>−1.07</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Urban population change (5 years)</td>
<td>3.976</td>
<td>1.857</td>
<td>2.95***</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Basic food stuffs net imports per person</td>
<td>0.990</td>
<td>0.004</td>
<td>−2.07**</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Food import security</td>
<td>0.803</td>
<td>0.067</td>
<td>−2.59***</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Government effectiveness index</td>
<td>1.751</td>
<td>0.412</td>
<td>2.37**</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Venture capital availability</td>
<td>1.099</td>
<td>0.035</td>
<td>2.93***</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Enforcing a contract (time)</td>
<td>0.999</td>
<td>0.004</td>
<td>−0.31</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Paying tax (process)</td>
<td>0.846</td>
<td>0.096</td>
<td>−1.47</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Getting electricity (time)</td>
<td>0.882</td>
<td>0.037</td>
<td>−2.95***</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$\ell (\hat{\beta}) = −18.91$, $AIC = 71.83$, $R^2 = 91.1\%$, $ACC = 96.7\%$

Source: Semet et al. (2021)
Explaining credit ratings with ESG metrics
Prediction accuracy of credit ratings

Table 66: Summary of the results

<table>
<thead>
<tr>
<th></th>
<th>***</th>
<th>$R^2$</th>
<th>Accuracy</th>
<th>Sensitivity</th>
<th>Specificity</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>E*</td>
<td>4</td>
<td>48.02%</td>
<td>84.97%</td>
<td>86.90%</td>
<td>83.23%</td>
<td>230.04</td>
</tr>
<tr>
<td>S*</td>
<td>7</td>
<td>65.60%</td>
<td>87.90%</td>
<td>88.80%</td>
<td>86.90%</td>
<td>160.83</td>
</tr>
<tr>
<td>G*</td>
<td>4</td>
<td>67.70%</td>
<td>89.54%</td>
<td>91.72%</td>
<td>87.58%</td>
<td>150.65</td>
</tr>
<tr>
<td>ESG*</td>
<td>7</td>
<td>79.02%</td>
<td>92.50%</td>
<td>93.80%</td>
<td>91.30%</td>
<td>104.80</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)

⇒ Final model: Education, Demographics, Human rights, Government effectiveness, Business environment and R&D
Explaining credit ratings with ESG metrics
Prediction accuracy of credit ratings

Figure 94: Prediction accuracy (in %) of credit ratings

<table>
<thead>
<tr>
<th>Rating</th>
<th>Probability range</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>83% – 100%</td>
</tr>
<tr>
<td>AA</td>
<td>67% – 82%</td>
</tr>
<tr>
<td>A</td>
<td>50% – 66%</td>
</tr>
<tr>
<td>BBB</td>
<td>39% – 49%</td>
</tr>
<tr>
<td>BB</td>
<td>29% – 38%</td>
</tr>
<tr>
<td>B</td>
<td>11% – 28%</td>
</tr>
<tr>
<td>C</td>
<td>0% – 10%</td>
</tr>
</tbody>
</table>

Source: Semet et al. (2021)
### ESG and sovereign risk

#### Summary of the results

<table>
<thead>
<tr>
<th>What is directly priced by the bond market?</th>
<th>What is indirectly priced by credit rating agencies?</th>
</tr>
</thead>
<tbody>
<tr>
<td>[E] &gt; [G] &gt; [S]</td>
<td>[G] &gt; [S] &gt; [E]</td>
</tr>
<tr>
<td>Significant market-based ESG indicators ≠ Relevant CRA-based ESG indicators</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High-income countries</th>
<th>Middle-income countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition risk &gt; Physical risk</td>
<td>Physical risk &gt; Transition risk</td>
</tr>
</tbody>
</table>

- [S] matters for middle-income countries, especially for Gender inequality, Working conditions and Migration

- National security, Infrastructure and mobility and International relationships are the relevant [G] metrics

- Fundamental analysis: $R^2_c \approx 70\%$
- Extra-financial analysis: $\Delta R^2_c \approx 13.5\%$

- [E] metrics are second-order variables:
  - Environmental standards
  - Water management
  - Biodiversity
  - Climate change

- Education, Demographic and Human rights are prominent indicators for the [S] pillar

- Government effectiveness, Business environment and R&D dominate the [G] pillar

- Accuracy > 95%
  - AAA, AA, B, CCC > A > BB > BBB
Course 2023-2024 in Sustainable Finance
Lecture 3. Exercise
Equity Portfolio Optimization with ESG Scores

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2024

7 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- **Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns**
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
We consider the CAPM model:

\[ R_i - r = \beta_i (R_m - r) + \varepsilon_i \]

where \( R_i \) is the return of asset \( i \), \( R_m \) is the return of the market portfolio \( w_m \), \( r \) is the risk free asset, \( \beta_i \) is the beta of asset \( i \) with respect to the market portfolio and \( \varepsilon_i \) is the idiosyncratic risk of asset \( i \). We have \( R_m \perp \varepsilon_i \) and \( \varepsilon_i \perp \varepsilon_j \). We note \( \sigma_m \) the volatility of the market portfolio. Let \( \tilde{\sigma}_i \), \( \mu_i \) and \( S_i \) be the idiosyncratic volatility, the expected return and the ESG score of asset \( i \). We use a universe of 6 assets with the following parameter values:

<table>
<thead>
<tr>
<th>Asset ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_i )</td>
<td>0.10</td>
<td>0.30</td>
<td>0.50</td>
<td>0.90</td>
<td>1.30</td>
<td>2.00</td>
</tr>
<tr>
<td>( \tilde{\sigma}_i ) (in %)</td>
<td>17.00</td>
<td>17.00</td>
<td>16.00</td>
<td>10.00</td>
<td>11.00</td>
<td>12.00</td>
</tr>
<tr>
<td>( \mu_i ) (in %)</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>5.50</td>
<td>7.50</td>
<td>11.00</td>
</tr>
<tr>
<td>( S_i )</td>
<td>1.10</td>
<td>1.50</td>
<td>2.50</td>
<td>-1.82</td>
<td>-2.35</td>
<td>-2.91</td>
</tr>
</tbody>
</table>

and \( \sigma_m = 20\% \). The risk-free return \( r \) is set to 1\% and the expected return of the market portfolio \( w_m \) is equal to \( \mu_m = 6\% \).
Question 1

We assume that the CAPM is valid.
Question (a)

Calculate the vector $\mu$ of expected returns.
Using the CAPM, we have:

\[ \mu_i = r + \beta_i (\mu_m - r) \]

For instance, we have:

\[ \mu_1 = 1\% + 0.10 \times (6\% - 1\%) = 1.5\% \]

and:

\[ \mu_2 = 1\% + 0.30 \times 5\% = 2.5\% \]

Finally, we obtain \( \mu = (1.5\%, 2.5\%, 3.5\%, 5.5\%, 7.5\%, 11\%) \)
Question (b)

Compute the covariance matrix $\Sigma$. Deduce the volatility $\sigma_i$ of the asset $i$ and find the correlation matrix $C = (\rho_{i,j})$ between asset returns.
We have:

\[ \Sigma = \sigma_m^2 \beta \beta^T + D \]

where:

\[ D = \text{diag}(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_6^2) \]

The numerical value of \( \Sigma \) is:

\[
\Sigma = \begin{pmatrix}
293 & 12 & 325 \\
12 & 356 & 20 \\
20 & 180 & 60 \\
36 & 108 & 424 \\
52 & 260 & 156 \\
80 & 400 & 720 \\
& 424 & 797 \\
& 1040 & 1744
\end{pmatrix} \times 10^{-4}
\]
We have:

\[ \sigma_i = \sqrt{\Sigma_{i,i}} \]

We obtain:

\[ \sigma = (17.12\%, 18.03\%, 18.87\%, 20.59\%, 28.23\%, 41.76\%) \]

We have:

\[ \rho_{i,j} = \frac{\Sigma_{i,j}}{\sigma_i \sigma_j} \]

We obtain the following correlation matrix expressed in %:

\[
\mathbf{C} = \begin{pmatrix}
100.00 & 3.89 & 6.19 & 10.21 & 10.76 & 11.19 \\
3.89 & 100.00 & 17.64 & 29.09 & 30.65 & 31.88 \\
6.19 & 17.64 & 100.00 & 46.33 & 48.81 & 50.76 \\
10.21 & 29.09 & 46.33 & 100.00 & 80.51 & 83.73 \\
10.76 & 30.65 & 48.81 & 80.51 & 100.00 & 88.21 \\
11.19 & 31.88 & 50.76 & 83.73 & 88.21 & 100.00
\end{pmatrix}
\]
Question (c)

Compute the tangency portfolio $w^*$. Calculate $\mu (w^*)$ and $\sigma (w^*)$. Deduce the Sharpe ratio and the ESG score of the tangency portfolio.
We have:

\[
\mathbf{w}^* = \frac{\Sigma^{-1} (\mu - r\mathbf{1})}{1^\top \Sigma^{-1} (\mu - r\mathbf{1})} = \begin{pmatrix}
0.94% \\
2.81% \\
5.28% \\
24.34% \\
29.06% \\
37.57%
\end{pmatrix}
\]

We deduce:

\[
\mu (\mathbf{w}^*) = \mathbf{w}^\top \mu = 7.9201\%
\]

\[
\sigma (\mathbf{w}^*) = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}^*} = 28.3487\%
\]

\[
\text{SR} (\mathbf{w}^* \mid r) = \frac{7.9201\% - 1\%}{28.3487\%} = 0.2441
\]

\[
\mathcal{S} (\mathbf{w}^*) = \sum_{i=1}^{6} w_i^* \mathcal{S}_i = -2.0347
\]
Question (d)

Compute the beta coefficient $\beta_i (w^*)$ of the six assets with respect to the tangency portfolio $w^*$, and the implied expected return $\tilde{\mu}_i$:

$$\tilde{\mu}_i = r + \beta_i (w^*) (\mu (w^*) - r)$$
We have:

$$\beta_i (w^*) = \frac{e_i^\top \Sigma w^*}{\sigma^2 (w^*)}$$

We obtain:

$$\beta (w^*) = \begin{pmatrix}
0.0723 \\
0.2168 \\
0.3613 \\
0.6503 \\
0.9393 \\
1.4451
\end{pmatrix}$$

The computation of $$\tilde{\mu}_i = r + \beta_i (w^*) (\mu (w^*) - r)$$ gives:

$$\tilde{\mu} = \begin{pmatrix}
1.50\% \\
2.50\% \\
3.50\% \\
5.50\% \\
7.50\% \\
11.00\%
\end{pmatrix}$$
Question (e)

Deduce the market portfolio $w_m$. Comment on these results.
\( \beta_i (w^*) \neq \beta_i (w_m) \) but risk premia are exact

Let us assume that the allocation of \( w_m \) is equal to \( \alpha \) of the tangency portfolio \( w^* \) and \( 1 - \alpha \) of the risk-free asset. We deduce that:

\[
\beta (w_m) = \frac{\sum w_m}{\sigma^2 (w_m)} = \frac{\alpha \sum w^*}{\alpha^2 \sigma^2 (w^*)} = \frac{1}{\alpha} \beta (w^*)
\]

We have:

\[
\alpha = \frac{\beta_i (w^*)}{\beta_i (w_m)} = 72.25\%
\]

The market portfolio \( w_m \) is equal to 72.25% of the tangency portfolio \( w^* \) and 27.75% of the risk-free asset.
• We have:

\[ \mu(w_m) = r + \alpha (\mu(w^*) - r) = 1\% + 72.25\% \times (7.9201\% - 1\%) = 6\% \]

and:

\[ \sigma(w_m) = \alpha \sigma(w^*) = 72.25\% \times 28.3487\% = 20.48\% \]

• We deduce that:

\[ \text{SR}(w_m | r) = \frac{6\% - 1\%}{20.48\%} = 0.2441 \]

• We do not obtain the true value of the Sharpe ratio:

\[ \text{SR}(w_m | r) = \frac{6\% - 1\%}{20\%} = 0.25 \]

• The tangency portfolio has an idiosyncratic risk:

\[ \sqrt{w_m^T \left( \sigma_m^2 \beta \beta^T \right) w_m} = 20\% < \sigma(w_m) = 20.48\% \]
Question 2

We consider long-only portfolios and we also impose a minimum threshold $S^\star$ for the portfolio ESG score:

$$S(w) = w^\top S \geq S^\star$$
Question (a)

Let $\gamma$ be the risk tolerance. Write the mean-variance optimization problem.
We have:

\[
  w^* = \arg \min \frac{1}{2} w^T \Sigma w - \gamma w^T \mu \\
  \text{s.t.} \begin{cases} 
    1_6^T w = 1 \\
    w^T S \geq S^* \\
    0_6 \leq w \leq 1_6
  \end{cases}
\]
Question (b)

Find the QP form of the MVO problem.
The matrix form of the QP problem is:

\[ w^* = \arg \min \frac{1}{2} w^T Q w - w^T R \]

s.t. \[
\begin{cases}
Aw = B \\
Cw \leq D \\
w^- \leq w \leq w^+
\end{cases}
\]

We deduce that \( Q = \Sigma, R = \gamma \mu, A = 1_6^T, B = 1, C = -S^T, D = -S^*, w^- = 0_6 \) and \( w^+ = 1_6 \)
Question (c)

Compare the efficient frontier when (1) there is no ESG constraint ($S^* = -\infty$), (2) we impose a positive ESG score ($S^* = 0$) and (3) the minimum threshold is set to 0.5 ($S^* = 0.5$). Comment on these results.
To compute the efficient frontier, we consider several values of $\gamma \in [-1, 2]$

For each value of $\gamma$, we compute the optimal portfolio $w^*$ and deduce its expected return $\mu(w^*)$ and its volatility $\sigma(w^*)$
Figure 95: Impact of the minimum ESG score on the efficient frontier
Question (d)

For each previous cases, find the tangency portfolio $w^*$ and the corresponding risk tolerance $\gamma^*$. Compute then $\mu (w^*)$, $\sigma (w^*)$, $\text{SR} (w^* \mid r)$ and $\mathcal{S} (w^*)$. Comment on these results.
Let $w^*(\gamma)$ be the MVO portfolio when the risk tolerance is equal to $\gamma$.

By using a fine grid of $\gamma$ values, we can find the optimal value $\gamma^*$ by solving numerically the following optimization problem with the brute force algorithm:

$$\gamma^* = \arg\max_{\gamma} \frac{\mu(w^*(\gamma)) - r}{\sigma(w^*(\gamma))} \quad \text{for } \gamma \in [0, 2]$$

We deduce the tangency portfolio $w^* = w^*(\gamma^*)$. 
Table 67: Impact of the minimum ESG score on the efficient frontier

<table>
<thead>
<tr>
<th>$\mathcal{S}^*$</th>
<th>$-\infty$</th>
<th>0</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*$</td>
<td>1.1613</td>
<td>0.8500</td>
<td>0.8500</td>
</tr>
<tr>
<td></td>
<td>0.9360</td>
<td>9.7432</td>
<td>9.1481</td>
</tr>
<tr>
<td></td>
<td>2.8079</td>
<td>16.3317</td>
<td>19.0206</td>
</tr>
<tr>
<td>$w^*$ (in %)</td>
<td>5.2830</td>
<td>31.0176</td>
<td>40.3500</td>
</tr>
<tr>
<td></td>
<td>24.3441</td>
<td>5.1414</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>29.0609</td>
<td>11.6028</td>
<td>3.8248</td>
</tr>
<tr>
<td></td>
<td>37.5681</td>
<td>26.1633</td>
<td>27.6565</td>
</tr>
<tr>
<td>$\mu (w^*)$ (in %)</td>
<td>7.9201</td>
<td>5.6710</td>
<td>5.3541</td>
</tr>
<tr>
<td>$\sigma (w^*)$ (in %)</td>
<td>28.3487</td>
<td>19.8979</td>
<td>19.2112</td>
</tr>
<tr>
<td>$\bar{SR} (w^* \mid r)$</td>
<td>0.2441</td>
<td>0.2347</td>
<td>0.2266</td>
</tr>
<tr>
<td>$\mathcal{S} (w^*)$</td>
<td>-2.0347</td>
<td>0.0000</td>
<td>0.5000</td>
</tr>
</tbody>
</table>
Question (e)

Draw the relationship between the minimum ESG score $S^*$ and the Sharpe ratio $SR(w^* | r)$ of the tangency portfolio.
We perform the same analysis as previously for several values $S^* \in [-2.5, 2.5]$

We verify that the Sharpe ratio is a decreasing function of $S^*$
Figure 96: Relationship between the minimum ESG score $S^*$ and the Sharpe ratio $\text{SR}(w^* \mid r)$ of the tangency portfolio
Question (f)

We assume that the market portfolio $w_m$ corresponds to the tangency portfolio when $S^* = 0.5$. 
The market portfolio \( w_m \) is then equal to:

\[
w_m = \begin{pmatrix}
9.15 \\
19.02 \\
40.35 \\
0.00 \\
3.82 \\
27.66
\end{pmatrix}
\]

We deduce that:

\[
\mu(w_m) = 5.3541\% \\
\sigma(w_m) = 19.2112\% \\
\text{SR}(w_m | r) = 0.2266 \\
\mathcal{S}(w_m) = 0.5
\]
Question (f).i

Compute the beta coefficient $\beta_i(w_m)$ and the implied expected return $\tilde{\mu}_i(w_m)$ for each asset. Deduce then the alpha return $\alpha_i$ of asset $i$. Comment on these results.
We have:

$$\beta_i(w_m) = \frac{e_i^T \Sigma w_m}{\sigma^2(w_m)}$$

and:

$$\tilde{\mu}_i(w_m) = r + \beta_i(w_m)(\mu(w_m) - r)$$

We deduce that the alpha return is equal to:

$$\alpha_i = \mu_i - \tilde{\mu}_i(w_m)$$

$$= (\mu_i - r) - \beta_i(w_m)(\mu(w_m) - r)$$

We notice that $\alpha_i < 0$ for the first three assets and $\alpha_i > 0$ for the last three assets, implying that:

$$\begin{cases} S_i > 0 \Rightarrow \alpha_i < 0 \\ S_i < 0 \Rightarrow \alpha_i > 0 \end{cases}$$
Table 68: Computation of the alpha return due to the ESG constraint

<table>
<thead>
<tr>
<th>Asset</th>
<th>$\beta_i (w_m)$</th>
<th>$\tilde{\mu}_i (w_m)$ (in %)</th>
<th>$\tilde{\mu}_i (w_m) - r$ (in %)</th>
<th>$\alpha_i$ (in bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1660</td>
<td>1.7228</td>
<td>0.7228</td>
<td>-22.28</td>
</tr>
<tr>
<td>2</td>
<td>0.4321</td>
<td>2.8813</td>
<td>1.8813</td>
<td>-38.13</td>
</tr>
<tr>
<td>3</td>
<td>0.7518</td>
<td>4.2733</td>
<td>3.2733</td>
<td>-77.33</td>
</tr>
<tr>
<td>4</td>
<td>0.8494</td>
<td>4.6984</td>
<td>3.6984</td>
<td>80.16</td>
</tr>
<tr>
<td>5</td>
<td>1.2395</td>
<td>6.3967</td>
<td>5.3967</td>
<td>110.33</td>
</tr>
<tr>
<td>6</td>
<td>1.9955</td>
<td>9.6885</td>
<td>8.6885</td>
<td>131.15</td>
</tr>
</tbody>
</table>
Question (f).ii

We consider the equally-weighted portfolio $w_{ew}$. Compute its beta coefficient $\beta(w_{ew} | w_m)$, its implied expected return $\tilde{\mu}(w_{ew})$ and its alpha return $\alpha(w_{ew})$. Comment on these results.
We have:

\[ \beta (w_{ew} \mid w_m) = \frac{w_{ew}^\top \sum w_m}{\sigma^2 (w_m)} = 0.9057 \]

and:

\[ \tilde{\mu} (w_{ew}) = 1\% + 0.9057 \times (5.3541\% - 1\%) = 4.9435\% \]

We deduce that:

\[ \alpha (w_{ew}) = \mu (w_{ew}) - \tilde{\mu} (w_{ew}) = 5.25\% - 4.9435\% = 30.65 \text{ bps} \]

We verify that:

\[ \alpha (w_{ew}) = \sum_{i=1}^{6} w_{ew,i} \alpha_i = \frac{\sum_{i=1}^{6} \alpha_i}{6} = 30.65 \text{ bps} \]

The equally-weighted portfolio has a positive alpha because:

\[ S (w_{ew}) = -0.33 \ll S (w_m) = 0.50 \]
Question 3

The objective of the investor is twice. He would like to manage the tracking error risk of his portfolio with respect to the benchmark \( b = (15\%, 20\%, 19\%, 14\%, 15\%, 17\%) \) and have a better ESG score than the benchmark. Nevertheless, this investor faces a long-only constraint because he cannot leverage his portfolio and he cannot also be short on the assets.
Question (a)

What is the ESG score of the benchmark?
We have:

$$S(b) = \sum_{i=1}^{6} b_i S_i = -0.1620$$
Question (b)

We assume that the investor’s portfolio is $w = (10\%, 10\%, 30\%, 20\%, 20\%, 10\%)$. Compute the excess score $S(w | b)$, the expected excess return $\mu(w | b)$, the tracking error volatility $\sigma(w | b)$ and the information ratio $IR(w | b)$. Comment on these results.
We have:

\[
\begin{align*}
S (w \mid b) &= (w - b)^\top S = 0.0470 \\
\mu (w \mid b) &= (w - b)^\top \mu = -0.5 \text{ bps} \\
\sigma (w \mid b) &= \sqrt{(w - b)^\top \Sigma (w - b)} = 2.8423\% \\
\text{IR} (w \mid b) &= \frac{\mu (w \mid b)}{\sigma (w \mid b)} = -0.0018
\end{align*}
\]

The portfolio \( w \) is not optimal since it improves the ESG score of the benchmark, but its information ratio is negative. Nevertheless, the expected excess return is close to zero (less than \(-1 \text{ bps}\)).
Question (c)

Same question with the portfolio $w = (10\%, 15\%, 30\%, 10\%, 15\%, 20\%)$. 
We have:

\[
\begin{align*}
S(w | b) &= (w - b)^T S = 0.1305 \\
\mu(w | b) &= (w - b)^T \mu = 29.5 \text{ bps} \\
\sigma(w | b) &= \sqrt{(w - b)^T \Sigma (w - b)} = 2.4949\% \\
\text{IR}(w | b) &= \frac{\mu(w | b)}{\sigma(w | b)} = 0.1182
\end{align*}
\]
Question (d)

In the sequel, we assume that the investor has no return target. In fact, the objective of the investor is to improve the ESG score of the benchmark and control the tracking error volatility. We note $\gamma$ the risk tolerance. Give the corresponding esg-variance optimization problem.
The optimization problem is:

$$w^* = \arg \min \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b)$$

s.t. \[
\begin{cases}
\mathbf{1}_6^T w = 1 \\
\mathbf{0}_6 \leq w \leq \mathbf{1}_6
\end{cases}
\]
Question (e)

Find the matrix form of the corresponding QP problem.
The objective function is equal to:

\[
(*) \quad = \quad \frac{1}{2} \sigma^2 (w \mid b) - \gamma \mathbf{S} (w \mid b)
\]

\[
= \quad \frac{1}{2} (w - b)^\top \Sigma (w - b) - \gamma (w - b)^\top \mathbf{S}
\]

\[
= \quad \frac{1}{2} w^\top \Sigma w - w^\top (\Sigma b + \gamma \mathbf{S}) + \left( \gamma b^\top \mathbf{S} + \frac{1}{2} b^\top \Sigma b \right)
\]

\text{does not depend on } w

We deduce that \( Q = \Sigma \), \( R = \Sigma b + \gamma \mathbf{S} \), \( A = 1_6^\top \), \( B = 1 \), \( w^- = 0_6 \)
and \( w^+ = 1_6 \)
Question (f)

Draw the esg-variance efficient frontier \( (\sigma(w^* \mid b), S(w^* \mid b)) \) where \( w^* \) is an optimal portfolio.
We solve the QP problem for several values of $\gamma \in [0, 5\%]$ and obtain Figure 97.
Figure 97: Efficient frontier of tracking a benchmark with an ESG score objective
Question (g)

Find the optimal portfolio $w^*$ when we target a given tracking error volatility $\sigma^*$. The values of $\sigma^*$ are 0%, 1%, 2%, 3% and 4%.
Using the QP numerical algorithm, we compute the optimal value $\sigma (w | b)$ for $\gamma = 0$ and $\gamma = 5\%$

Then, we apply the bisection algorithm to find the optimal value $\gamma^*$ such that:

$$\sigma (w | b) = \sigma^*$$
Table 69: Solution of the $\sigma$-problem

<table>
<thead>
<tr>
<th>Target $\sigma^*$</th>
<th>0</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*$ (in bps)</td>
<td></td>
<td>0.000</td>
<td>4.338</td>
<td>8.677</td>
<td>13.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.000</td>
<td>15.175</td>
<td>15.350</td>
<td>15.525</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.000</td>
<td>21.446</td>
<td>22.892</td>
<td>24.338</td>
</tr>
<tr>
<td>$w^*$ (in %)</td>
<td></td>
<td>19.000</td>
<td>23.084</td>
<td>27.167</td>
<td>31.251</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.000</td>
<td>9.588</td>
<td>5.176</td>
<td>0.763</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.000</td>
<td>12.656</td>
<td>10.311</td>
<td>7.967</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17.000</td>
<td>18.052</td>
<td>19.104</td>
<td>20.156</td>
</tr>
<tr>
<td>$S(w^* \mid b)$</td>
<td></td>
<td>0.000</td>
<td>0.230</td>
<td>0.461</td>
<td>0.691</td>
</tr>
</tbody>
</table>
Question (h)

Find the optimal portfolio $w^*$ when we target a given excess score $S^*$. The values of $S^*$ are 0, 0.1, 0.2, 0.3 and 0.4.
• Same method as previously with the following equation:

\[ S(w \mid b) = S^* \]

• An alternative approach consists in solving the following optimization problem:

\[ w^* = \arg\min \frac{1}{2} \sigma^2 (w \mid b) \]

s.t. \[ \begin{cases} 1^\top w = 1 \\ S(w \mid b) = S^* \\ 0_6 \leq w \leq 1_6 \end{cases} \]

• We have: \[ Q = \Sigma, \ R = \Sigma b, \ A = \begin{pmatrix} 1_6^\top \\ S^\top \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ S^* + S^\top b \end{pmatrix}, \]

\[ w^- = 0_6 \text{ and } w^+ = 1_6 \]
Table 70: Solution of the $S$-problem

<table>
<thead>
<tr>
<th>Target $S^*$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*$ (in bps)</td>
<td>0.000</td>
<td>1.882</td>
<td>3.764</td>
<td>5.646</td>
<td>7.528</td>
</tr>
<tr>
<td></td>
<td>15.000</td>
<td>15.076</td>
<td>15.152</td>
<td>15.228</td>
<td>15.304</td>
</tr>
<tr>
<td></td>
<td>20.000</td>
<td>20.627</td>
<td>21.255</td>
<td>21.882</td>
<td>22.509</td>
</tr>
<tr>
<td>$w^*$ (in %)</td>
<td>19.000</td>
<td>20.772</td>
<td>22.544</td>
<td>24.315</td>
<td>26.087</td>
</tr>
<tr>
<td></td>
<td>14.000</td>
<td>12.086</td>
<td>10.171</td>
<td>8.257</td>
<td>6.343</td>
</tr>
<tr>
<td></td>
<td>15.000</td>
<td>13.983</td>
<td>12.966</td>
<td>11.949</td>
<td>10.932</td>
</tr>
<tr>
<td></td>
<td>17.000</td>
<td>17.456</td>
<td>17.913</td>
<td>18.369</td>
<td>18.825</td>
</tr>
<tr>
<td>$\sigma (w^*</td>
<td>b) (in %)</td>
<td>0.000</td>
<td>0.434</td>
<td>0.868</td>
<td>1.301</td>
</tr>
</tbody>
</table>
Question (i)

We would like to compare the efficient frontier obtained in Question 3(f) with the efficient frontier when we implement a best-in-class selection or a worst-in-class exclusion. The selection strategy consists in investing only in the best three ESG assets, while the exclusion strategy implies no exposure on the worst ESG asset. Draw the three efficient frontiers. Comment on these results.
For the best-in-class strategy, the optimization problem becomes:

\[ w^* = \arg \min \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b) \]

s.t. \[
\begin{align*}
1^\top w &= 1 \\
0 &= w_i = 0, \quad i = 4, 5, 6 \\
0 &\leq w \leq 1
\end{align*}
\]

The QP form is defined by \( Q = \Sigma, \ R = \Sigma b + \gamma \mathcal{S} \), \( A = 1^\top_6 \), \( B = 1 \), \( w^- = 0_6 \) and \( w^+ = \begin{pmatrix} 1_3 \\ 0_3 \end{pmatrix} \)
For the worst-in-class strategy, the optimization problem becomes:

\[ w^* = \arg \min \frac{1}{2} \sigma^2 (w \mid b) - \gamma S (w \mid b) \]

\[
\begin{align*}
\text{s.t.} & \quad 1^T_6 w = 1 \\
& \quad w_6 = 0 \\
& \quad 0_6 \leq w \leq 1_6
\end{align*}
\]

The QP form is defined by \( Q = \Sigma \), \( R = \Sigma b + \gamma S \), \( A = 1^T_6 \), \( B = 1 \), \( w^- = 0_6 \) and \( w^+ = \left( \begin{array}{c} 1_5 \\ 0 \end{array} \right) \).
- The efficient frontiers are reported in Figure 98
- The exclusion strategy has less impact than the selection strategy
- The selection strategy implies a high tracking error risk
Figure 98: Comparison of the efficient frontiers (ESG integration, best-in-class selection and worst-in-class exclusion)
Question (j)

Which minimum tracking error volatility must the investor accept to implement the best-in-class selection strategy? Give the corresponding optimal portfolio.
We solve the first problem of Question 3(i) with $\gamma = 0$

We obtain:

$$\sigma (w \mid b) \geq 11.17\%$$

The lower bound $\sigma (w^* \mid b) = 11.17\%$ corresponds to the following optimal portfolio:

$$w^* = \begin{pmatrix} 16.31\% \\ 34.17\% \\ 49.52\% \\ 0\% \\ 0\% \\ 0\% \end{pmatrix}$$
Remark

The impact of ESG scores on optimized portfolios depends on their relationship with expected returns, volatilities, correlations, beta coefficients, etc. In the previous exercise, the results are explained because the best-in-class assets are those with the lowest expected returns and beta coefficients while the worst-in-class assets are those with the highest expected returns and beta coefficients. For instance, we obtain a high tracking error risk for the best-in-class selection strategy, because the best-in-class assets have low volatilities and correlations with respect to worst-in-class assets, implying that it is difficult to replicate these last assets with the other assets.
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
The big issue for an investor is:
How to avoid Greenwashing (& ESG washing)?

Greenwash (also greenwashing)

- Activities by a company or an organization that are intended to make people think that it is concerned about the environment, even if its real business actually harms the environment.
- A common form of greenwash is to publicly claim a commitment to the environment while quietly lobbying to avoid regulation.

In finance, greenwashing is understood as making misleading claims about environmental practices, performance or products.

We must distinguish two types of risk:

- Explicit & deliberate greenwashing
  
  **Deliberate greenwashing = mis-selling risk**

- Unintentional greenwashing
  
  **Unintentional greenwashing = misinterpretation risk**
Investment vehicles
- Mutual funds
- ETFs
- Mandates & dedicated funds

Investment strategies
- Thematic strategies (e.g. water, social, wind energy, climate, plastic, etc.)
- ESG-tilted strategies (e.g. exclusion, negative screening, best-in-class, enhanced ESG score, controlled tracking error, etc.)
- Climate strategies (e.g. low carbon, 2°C alignment, activity exclusions\(^9\), etc.)
- Sustainability-linked securities (e.g. green bonds, social bonds, etc.)

Both \(\alpha\) and \(\beta\) management

\(^9\) e.g. coal exploration, oil exploration, electricity generation with a high GHG intensity
### Mutual funds
- Amundi Climate Transition
- Amundi ARI European Credit SRI
- AXA World Funds – Euro Bonds SRI
- CPR Invest Social Impact
- Fidelity U.S. Sustainability Index
- Fidelity Sustainable Water & Waste
- Natixis ESG Dynamic Fund
- Vanguard FTSE Social Index
- Etc.

### ETFs
- Amundi Index MSCI Europe SRI UCITS ETF
- Amundi MSCI Emerging ESG Leaders UCITS ETF
- Amundi EURO ISTOXX Climate Paris Aligned PAB UCITS ETF
- Lyxor New Energy UCITS ETF
- Lyxor World Water UCITS ETF
- SPDR S&P 500 ESG
- First Trust Global Wind Energy ETF
- Invesco S&P 500 ESG UCITS ETF
- Etc.
ESG represents **58% of the net new assets** (NNA) in the European ETF market.

ESG fund assets reach $1,652 bn
- Europe: $1,343 bn (or 81.3%)
- US: $236.4 bn (or 14.3%)
- Asia: $43.1 bn (or 2.6%)

Net flows into sustainable mutual funds and ETFs in Q4 2020: $370 bn (or **+29% of assets**)

Net flows into sustainable mutual funds and ETFs in 2020
- Europe: $273 bn, almost double the total for 2019, almost 5 times more than in 2017
- US: $51.2 bn, more than double the total for 2019, almost 10 times more than in 2018

*Source: Morningstar, Global Sustainable Fund Flows: Q4 2020 in Review (January 2021)*
European sustainable finance labels

- Novethic label (pioneer label in 2009, suspended in 2016)
- FNG label (Germany) — https://fng-siegel.org
- Towards Sustainability label (Belgium) — https://www.towardssustainability.be
- LuxFLAG label (Luxembourg) — https://www.luxflag.org
- Nordic Swan Ecolabel (Nordic countries) — https://www.nordic-ecolabel.org
- Umweltzeichen Ecolabel (Austria) — https://www.umweltzeichen.at/en
- French Greenfin label — https://www.ecologie.gouv.fr/label-greenfin
Remark

According to Novethic (2020), 806 funds had a label at the end of December 2019. Nine months later, this number has increased by 392 and the AUM has be multiplied by 3.2!
“Today it is difficult for consumers, companies and other market actors to make sense of the many environmental labels and initiatives on the environmental performance of products and companies. There are more than 200 environmental labels active in the EU, and more than 450 active worldwide; there are more than 80 widely used reporting initiatives and methods for carbon emissions only. Some of these methods and initiatives are reliable, some not; they are variable in the issues they cover” (European Commission, 2020).

Source: https://ec.europa.eu/environment/eussd/index.htm
Regulation

- EU taxonomy regulation
- Sustainable Finance disclosure regulation (SFDR)
- Climate benchmarks
- Sustainability preferences (MiFID II & IDD)
SFDR

- Article 6: Non-ESG funds (standard funds)
- Article 8: ESG funds (funds that promote E or S characteristics)
- Article 9: Sustainable funds (funds that have a sustainable investment objective: impact investing or reduction of carbon emissions)
New benchmark rules

- Climate transition benchmarks (CTB): high level of decarbonization (−30%), no controversial weapons and tobacco, high positive impact on climate change, etc.
- Paris-aligned benchmarks (PAB): high level of decarbonization (−50%), no controversial weapons and tobacco, no activities in coal, oil and natural gas, global warming below 2°C, etc.

- MSCI Climate Paris Aligned Indexes —
  www.msci.com/esg/climate-paris-aligned-indexes
- FTSE TPI Climate Transition Index Series —
  www.ftserussell.com/products/indices/tpi-climate-transition
- STOXX Climate Transition Benchmark (CTB) and STOXX Paris-Aligned Benchmark (PAB) Indices —
  qontigo.com/solutions/climate-indices
# Sustainable fixed-income products

## Table 71: Sustainable fixed-income market

<table>
<thead>
<tr>
<th>Theme</th>
<th>Label</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSS+</td>
<td>Green</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td></td>
<td>Social</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td></td>
<td>Sustainability</td>
<td>Use of proceeds</td>
</tr>
<tr>
<td>Transition</td>
<td>Sustainability-Linked</td>
<td>Entity KPI-linked</td>
</tr>
<tr>
<td></td>
<td>Transition</td>
<td>Use of proceeds</td>
</tr>
</tbody>
</table>

Source: CBI (2022).
Figure 99: Issuance of GSS securities (in $ bn)

Source: https://www.climatebonds.net/market/data.
Definition

Green bonds are any type of bond instrument where the proceeds or an equivalent amount will be exclusively applied to finance or re-finance, in part or in full, new and/or existing eligible green projects and which are aligned with the four core components of the Green Bond Principles (GBP).


⇒ Green bonds are “regular” bonds¹⁰ aiming at funding projects with positive environmental and/or climate benefits

¹⁰ A regular bond pays regular interest to bondholders
Green Bonds Principles

The 4 core components of the GBP are:

1. Use of proceeds
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

Green Bonds Principles

The use of proceeds includes:

- Renewable energy
- Energy efficiency
- Pollution prevention (e.g. GHG control, soil remediation, waste recycling)
- Sustainable management of living natural resources (e.g. sustainable agriculture, sustainable forestry, restoration of natural landscapes)
- Terrestrial and aquatic biodiversity conservation (e.g. protection of coastal, marine and watershed environments)
- Clean transportation
- Sustainable water management
- Climate change adaptation
- Eco-efficient products
- Green buildings
With respect to the **process for project evaluation and selection** (component 2), the issuer of a green bond should clearly communicate:

- the environmental sustainability objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the “balance sheet” and the allocation of funds\(^\text{11}\)
- An external review (not mandatory but highly recommended)

\(^{11}\)The proceeds should be credited to a sub-account
The **reporting** (component 4) must be based on the following pillars:

- Transparency
- Description of the projects, allocated amounts and expected impacts
- Qualitative performance indicators
- Quantitative performance measures (e.g. energy capacity, electricity generation, GHG emissions reduced/avoided, number of people provided with access to clean power, decrease in water use, reduction in the number of cars required)
Standardization is strongly required by investors and regulators

- Green Bond Principles\textsuperscript{12} (ICMA, 2021)
- Climate Bonds Standard\textsuperscript{13} (CBI, 2019)
- EU Green Bond Standard (2021)
- China Green Bond Principles (PBOC, CBIRC, July 2022)
- Asean Green Bond Standards (ACMF, 2018)

\textsuperscript{12}The first version is published in January 2014
\textsuperscript{13}The first version is published in November 2011
### Types of debt instruments

#### Asset-linked bond structures
- Regular bond
- Revenue bond
- Project bond
- Green loans

#### Asset-backed bond structures
- Securitized bond
- Project bond
- ABS/MBS/CLO/CDO
- Covered bond
Certification

- Second party opinion provided by ESG rating agencies (ISS, Sustainalytics, Vigeo-Eiris);
- Certification by specialized green bond entities (CBI, CICERO, DNV);
- Green bond assessment by statistical rating organizations (Moody’s, S&P).
Examples

- Solar bond by the City of San Francisco in 2001
- Equity-linked climate awareness bond by the European Investment Bank (EIB) in 2007
- First green bond issued by the World Bank (in collaboration with Skandinaviska Enskilda Banken) in November 2008
- First corporate green bonds: French utility company EDF ($1.8 bn) and Swedish real estate company Vasakronan ($120 bn)
- Toyota introduced the auto industry’s first-ever asset-backed green bond in 2014 ($1.75 bn)
- The Commonwealth of Massachusetts issued the first municipal green bond in 2013 ($100 mn)
- The first sovereign green are: Poland in December 2016 ($1 bn) and France\textsuperscript{14} in January 2017 ($10 bn)

\textsuperscript{14}Green OAT 1.75\% 25 June 2039.
The green bond market

Green bond issuers
- Sovereigns (agencies, municipals, governments)
- Multilateral development banks (MDB)
- Energy and utility companies
- Banks
- Other corporates

Green bond investors
- Pension funds
- Sovereign wealth funds
- Insurance companies
- Asset managers
- Retail investors (e.g. employee savings plans)

Strong imbalance between supply and demand
The green bond market

Figure 100: Issuance and notional outstanding of green debt by market type

Source: https://www.climatebonds.net/market/data.
The green bond market

Figure 101: Issuance and notional outstanding of green debt by region

Source: https://www.climatebonds.net/market/data.
The green bond market

**Figure 102:** Issuance and notional outstanding of green debt by use of proceeds

Source: https://www.climatebonds.net/market/data.
The green bond market

Figure 103: Issuance and notional outstanding of green debt by issuer type

Source: https://www.climatebonds.net/market/data.
How to investing in green bonds

Example of green bond funds:

- Allianz IG green bond fund
- Amundi RI impact green bonds
- AXA WF ACT green bonds
- BNP Paribas green bond
- Calvert green bond fund
- Mirova global green bond fund
- TIAA-CREF green bond fund
- Etc.
How to investing in green bonds

List of green bond indices:

- Bloomberg Barclays MSCI Global Green Bond Index
- S&P Green Bond Index
- Solactive Green Bond Index
- ChinaBond China Climate-Aligned Bond Index:
- ICE BofA Green Index

⇒ ETF and index funds
“I show that investors respond positively to the issuance announcement, a response that is stronger for first-time issuers and bonds certified by third parties. The issuers improve their environmental performance post-issuance (i.e., higher environmental ratings and lower CO₂ emissions) and experience an increase in ownership by long-term and green investors. Overall, the findings are consistent with a signaling argument – by issuing green bonds, companies credibly signal their commitment toward the environment.” (Flammer, 2021, page 499).
The economics of green bonds

Green bonds = second-best instrument
The green bond premium

**Definition**
- The green bond premium (or greenium) is the difference in pricing between green bonds and regular bonds.
- The greenium is defined as:

\[ g = y(GB) - y(CB) \]

where \( y(GB) \) is the yield (or return) of the green bond and \( y(CB) \) is the yield (or return) of the conventional twin bond.
The green bond premium

- From the issuer’s point of view, a green bond issuance is more expensive than a conventional issuance due to the need for external review, regular reporting and impact assessments.
- From the investor’s point of view, there is no fundamental difference between a green bond and a conventional bond, meaning that one should consider a negative green bond premium as a market anomaly.
The green bond premium

Green twin bonds

- Introduced in 2020 by Germany
- Issuance of a green and conventional bond at the same time with the same characteristics
- Investors may swap the green bond with the conventional bond any time, but not vice-versa
- Liquidity of the green bond market ↗
Examples of twin bonds:

- On 3 September 2020, the 10-year German green bond with a coupon of 0.00% was priced 1 basis point below the 10-year conventional German bond.

- On 19 January 2022, Denmark issued a 10-year green bond with the same maturity, interest payment dates and coupon rate as the conventional 2031 Danish bond. The effective yield of the green bond was 5 basis points below the twin regular bond.
The green bond premium

**Example #1**

We consider a 10-year green bond $GB_1$ whose current price is equal to 91.35. The corresponding conventional twin bond is a 20-year regular bond, whose remaining maturity is exactly equal to ten years and its price is equal to 90.07%. We assume that the two bonds have the same coupon level, which is equal to 4%.
The green bond premium

Computation of the greenium with the current yield:

- We have:
  \[ y(\text{GB}) = \frac{4}{91.35} = 4.379\% \]
  and:
  \[ y(\text{CB}) = \frac{4}{90.07} = 4.441\% \]
- We deduce that the greenium is equal to:
  \[ g = 4.441\% - 4.379\% = -6.2 \text{ bps} \]
The green bond premium

Computation of the greenium with the yield to maturity:

- We solve the equation:

\[
\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 91.35
\]

and find:

\[y(GB) = 5\%
\]

- We solve the equation:

\[
\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 90.07
\]

and find:

\[y(CB) = 5.169\%
\]

- We deduce that the greenium is equal to:

\[g = 5\% - 5.169\% = -16.9 \text{ bps}
\]
Figure 104: Greenium in bps of the German green (twin) bond (DBR 0% 15/08/2030)

Source: ICE (2022).
The green bond premium

What about the greenium when the green bond is not a twin bond?

⇒ We must distinguish primary and secondary markets
The green bond premium

- In the primary market, the greenium is negative ($\approx 5 - 10$ bps on average)
- How to measure the persistence of the greenium in the secondary market?
The green bond premium

There are two approaches:

1. **Bottom-up approach**
   - Compares the green bond of an issuer with a synthetic conventional bond of the same issuer
   - Same characteristics in terms of currency, seniority and duration

2. **Top-down approach**
   - Compare a green bond index portfolio to a conventional bond index portfolio
   - Same characteristics in terms of currency, sector, credit quality and maturity
The green bond premium

Bottom-up approach

1. We filter all the conventional bonds, which has the same issuer, the same currency, and the same seniority of the green bond GB.

2. We select the two conventional bonds $CB_1$ and $CB_2$ which are the nearest in terms of modified duration:

$$|MD(GB) - MD(CB_j)|_{j\neq 1,2} \geq \sup_{j=1,2} |MD(GB) - MD(CB_j)|$$

3. We perform the linear interpolation/extrapolation of the two yields $y(CB_1)$ and $y(CB_2)$:

$$y(CB) = y(CB_1) + \frac{MD(GB) - MD(CB_1)}{MD(CB_2) - MD(CB_1)} (y(CB_2) - y(CB_1))$$
Example #2

- We consider a green bond, whose modified duration is 8 years. Its yield return is equal to 132 bps.

- We can surround the green bond by two conventional bonds with modified duration 7 and 9.5 years. The yield is respectively equal to 125 and 148 bps.

- The interpolated yield is equal to:

\[ y(CB) = 125 + \frac{8 - 7}{9.5 - 7} (148 - 125) \]

\[ = 134.2 \text{ bps} \]

- It follows that the greenium is equal to $-2.2$ bps:

\[ g = 132 - 134.2 = -2.2 \text{ bps} \]
The green bond premium

Top-down approach

- We consider a portfolio $w = (w_1, \ldots, w_n)$ of green bonds.
- We perform a clustering analysis by considering the 4-uplets $(\text{Currency} \times \text{Sector} \times \text{Credit quality} \times \text{Maturity})$
- Let $(C_h, S_j, R_k, M_l)$ be an observation for the 4-uplet (e.g. EUR, Financials, AAA, 1Y-3Y). We compute its weight:

  $$\omega_{h,j,k,l} = \sum_{i \in (C_h, S_j, R_k, M_l)} w_i$$

- The greenium is then defined as the weighted excess yield:

  $$g = \sum_{h,j,k,l} \omega_{h,j,k,l} (y_{h,j,k,l} (\text{GB}) - y_{h,j,k,l} (\text{CB}))$$
The green bond premium

Main result (Ben Slimane et al., 2020)
The greenium is negative between $-5$ and $-2$ bps on average

Other results:
- Differences between sectors, currencies, maturities, regions and ratings
- Transatlantic divided between US and Europe
- The volatility of green bond portfolios are lower than the volatility of conventional bond portfolios $\Rightarrow$ identical Sharpe ratio since the last four years
- Time-varying property of the greenium
Figure 105: Evolution of the greenium (in bps)

Source: Ben Slimane et al. (2020)
Bond issuers have a competitive advantage to finance their environmental projects using green bonds instead of conventional bonds.

Another premium? the “green bond issuer premium”
Social bonds

Definition

Social Bonds are any type of bond instrument where the proceeds, or an equivalent amount, will be exclusively applied to finance or re-finance in part or in full new and/or existing eligible social projects and which are aligned with the four core components of the Social Bond Principles (SBP).

Source: ICMA (2021), https://www.icmagroup.org/sustainable-finance
Social Bonds Principles (SBP)

The 4 core components of the SBP are:

1. Use of proceeds
   - Eligible social project categories
   - Target populations
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

The **eligible social projects categories** (component 1) are:

- Affordable basic infrastructure (e.g. clean drinking water, sanitation, clean energy)
- Access to essential services (e.g. health, education)
- Affordable housing (e.g. sustainable cities)
- Employment generation (e.g. pandemic crisis)
- Food security and sustainable food systems (e.g. nutritious and sufficient food, resilient agriculture)
- Socioeconomic advancement and empowerment (e.g. income inequality, gender inequality)
- Etc.
Social Bonds Principles

The **target populations** (component 1) are:

- Living below the poverty line
- Excluded and/or marginalised populations/communities
- People with disabilities
- Migrants and/or displaced persons
- Undereducated
- Unemployed
- Women and/or sexual and gender minorities
- Aging populations and vulnerable youth
- Etc.
With respect to the **process for project evaluation and selection** (component 2), the issuer of a social bond should clearly communicate:

- the social objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the “*balance sheet*” and the allocation of funds
- An external review (not mandatory but highly recommended)

---

15 The proceeds should be credited to a sub-account
The **reporting** (component 4) must be based on the following pillars:

- Transparency
- Description of the projects, allocated amounts and expected impacts
- Qualitative performance indicators
- Quantitative performance measures (e.g. number of beneficiaries)
Social bond market

Figure 106: Issuance of social bonds

Source: https://www.climatebonds.net/market/data.
Instituto de Crédito Oficial (Spanish state-owned bank, March 2020) “The Social Bond proceeds under ICO’s Second – Floor facilities will be allocated to loans to finance small, medium and micro enterprises with an emphasis on employment creation or employment retention in: (1) specific economically underperforming regions of Spain; (2) specific municipalities of Spain facing depopulation; (3) regions affected by a natural disaster. [...] The target populations are SMEs in line with European Union’s standards.”
Examples

- Pepper Money (non-bank lender in Australia and New Zealand, April 2022)

  “The positive social impact of a Pepper Money eligible social project derives from its direct contribution to improving access to financial services and socio-economic empowerment, by using proprietary systems to make flexible loan solutions available to applicants who are not served by traditional banks. [...] Pepper Money is seeking to achieve positive social outcomes for a target population of Australians that lack access to essential financial services and experience inequitable access to and lack of control over assets. Pepper Money directly aims to address the positive social outcome of home ownership for borrowers who may have complexity in their income streams, gaps in their loan documentation or have adverse credit history. Traditionally, this cohort has been underserved by banks that rely on inflexible algorithmic loan application processing.”
Danone (French multinational food-products corporation, March 2018)

“The eligible project categories are: (1) research & innovation for advanced medical nutrition (target populations: infants, pregnant women, patients and elderly people with specific nutritional needs), (2) social inclusiveness (target populations: farmers, excluded and/or marginalised populations and/or communities, people living under the poverty line, rural communities in developing countries), (3) responsible farming and agriculture (target populations: milk producers, farmers), etc.”
Examples

- Korian (European care group, October 2021)
  “The proceeds of any instrument issued under the framework will be used [...] to provide services, solutions, and technologies that will enable Korian to meet at least one of its social objectives: (1) to increase and improve long-term care nursing home capacity for dependent older adults; (2) to increase and improve medical capacity for people in need of medical support; (3) to increase and improve access to alternative, nonmedical services, technologies, and housing solutions that facilitate the retention of older adults’ autonomy; and (4) to improve the daily provision of care to and foster a safer living environment for its patients. [...] Furthermore, Korian’s target populations are older adults, which Korian defines as being over 65 years of age, and those who are dependent on others for some degree of care, which is defined by the health authorities or insurance system of the respective country.”
JASSO (Japan Student Services Organization, July 2022)  
“The social project categories concern the financing of the ‘Category 2 Scholarship Loans’ (interest-bearing scholarship loans that have to be repaid) while the target population is made up of students with financial difficulties.”
Sustainability bonds

Sustainability bond = GBP + SBP

Remark

According to CBI, the cumulative issuance of sustainability bonds reaches $620 bn at the end of June 2022
### Sustainability-linked bond (SLB)

- **Two principles:**
  - = a sustainability bond (green/social)
  - + a step up coupon if the KPI is not satisfied

  ⇒ forward-looking performance-based instrument

- The financial characteristics of the bond depends on whether the issuer achieves predefined ESG objectives

- Those objectives are:
  1. measured through predefined Key Performance Indicators (KPI)
  2. assessed against predefined Sustainability Performance Targets (SPT)
ENEL General Purpose SDG Linked Bond

- SDG: 7 (affordable and clean energy), 13 (climate action), 9 (industry, innovation and infrastructure) and 11 (sustainable cities and communities)
- SDG 7 target: renewables installed capacity as of December 31, 2021 $\geq$ 55% (confirmed by external verifier)
- One time step up coupon of 25 bps if SDG 7 is not achieved
- On April 2022, the independent report produced by KPMG certifies that “the renewables installed capacity percentage as of December 31, 2021 is equal to 57.5%”.

H&M sustainability-linked bond

- 18 February 2021
- €500 mn
- Maturity of 8.5 years
- The annual coupon rate is 25 bps
- The objectives to achieve by 2025 are:
  - **KPI₁** Increase the share of recycled materials used to 30% \( (SPT₁) \)
  - **KPI₂** Reduce emissions from the Group’s own operations (scopes 1+2) by 20% \( (SPT₂) \)
  - **KPI₃** Reduce scope 3 emissions from fabric production, garment manufacturing, raw materials and upstream transport by 10% \( (SPT₃) \)
- The global KPI is equal to \( 40\% \times KPI₁ + 20\% \times KPI₂ + 40\% \times KPI₃ \)
- The step-up of the coupons can consequently be 0%, 20%, 40%, 60%, 80% or 100% of the total step-up rate
According to Berrada et al. (2022), “the SLB market has grown strongly since its inception. [...] Bloomberg identifies a total of 434 outstanding bonds flagged as ‘sustainability-linked’ as of February 2022. In contrast, in 2018, there was only a single SLB. The amount raised through the single 2018 SLB issue was $0.22 bn, whereas the total amount raised through all SLBs issued in 2021 was approximately $160 bn”.

- The large majority of SLB issues address exclusively E issues (65%) or a combination of E, S and G issues (17%) or E and G issues (3%)
- The most frequent KPI concerns GHG emissions (40 %), followed by the issuer’s global ESG score (14 %)
Transition bonds

- Financial instruments to support the transition of an issuer, which has significant current carbon emissions
- Fund projects such as renewable energy developments, energy efficiency upgrades, etc.
- The final objective of the bond issuer is always to reduce their carbon emissions
- For example, transition bonds can be used to switch diesel powered ships to natural gas or to implement carbon capture and storage.
Sustainable real assets
Course 2023-2024 in Sustainable Finance
Lecture 5. Impact Investing

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2024

---

16 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
**Agenda**

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Motivations

Principle

- Financial risks $\Rightarrow$ financial performance (return, volatility, Sharpe ratio, etc.)
- Extra-financial risks $\Rightarrow$ financial performance (return, volatility, Sharpe ratio, etc.)
- Extra-financial risks $\Rightarrow$ extra-financial performance (ESG KPIs)

What is the final motivation of the ESG investor?

Financial performance or/and extra-financial performance?
### Definition

The key elements of impact investing are:

1. **Intentionality**
   The intention of an investor to generate a positive and measurable social and environmental impact

2. **Additionality**
   Fulfilling a positive impact beyond the provision of private capital

3. **Measurement**
   Being able to account for in a transparent way on the financial, social and environmental performance of investments

*Source: Eurosif (2019)*

The investor must be able to measure its impact from a quantitative point of view.
Figure 107: Global Impact Investing Network (GIIN)

https://thegiin.org
The example of social impact bonds

Social impact bond (SIB) = pay-for-success bond (≈ call option)

The Peterborough SIB

- On 18 March 2010, the UK Secretary of State for Justice announced a six-year SIB pilot scheme that will see around 3000 short term prisoners from Peterborough prison, serving less than 12 months, receiving intensive interventions both in prison and in the community.
- Funding from investors will be initially used to pay for the services.
- If reoffending is not reduced by at least 7.5%, the investors will receive no recompense.
The example of sustainability-linked bonds

Sustainability-linked\textsuperscript{17} (SLB) = \textbf{pay-for-failure bond} (\approx \text{cap option})

\textbf{Risk taker}

SIB: investor viewpoint \ne SLB: issuer viewpoint

\textsuperscript{17}See the examples of ENEL and H&M previously
Thematics funds

Biodiversity risk

Motivations
Sustainable development goals (SDGs)
The challenge of reporting

Measurement tools

Impact assessment and metrics

- Avoided CO2 emissions in tons per $M invested
- Amount of clean water produced by the project
- Number of children who are less obese
- Land management
- Affordable housing
- Job creation
- Construction of student housing
The sustainable development goals are a collection of 17 interlinked global goals designed to be a “blueprint to achieve a better and more sustainable future for all”

https://sdgs.un.org
Sustainable development goals (SDGs)

Figure 108: The map of sustainable development goals
Figure 109: Mapping the SDGs across E, S and G
Sustainable development goals (SDGs)

Figure 110: Examples of sovereign SDG reports

The challenge of reporting

- Impact reporting and investment standards (IRIS) proposed by GIIN
- EU taxonomy on sustainable finance
- Non-financial reporting directive 2014/95/EU (NFRD)
- Carbon accounting
The challenge of reporting
Biodiversity risk
Thierry Roncalli

* Amundi Asset Management

* University of Paris-Saclay

March 2024

---

18 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Stewardship vs. engagement

Voting $\subset$ Engagement $\subset$ Stewardship
Stewardship vs. engagement

Figure 111: Difference between stewardship and engagement reports

“It guides investors on how to implement the PRI’s Principle 2, which sets out signatories’ commitment to stewardship, stating: we will be active owners and incorporate ESG issues into our ownership policies and practices. [...] The PRI defines stewardship as the use of influence by institutional investors to maximise overall long-term value including the value of common economic, social and environmental assets, on which returns and clients’ and beneficiaries’ interests depend.” (PRI, 2021).
Active ownership ≈ Engagement ≈ Shareholder activism

“investors who, dissatisfied with some aspect of a company’s management or operations, try to bring about change within the company without a change in control” Gillan and Starks (2000).
Definition

- Conflicting interests between shareholders and management (separation between ownership and control)
- Stakeholder theory (Freeman, 2004)

Milton Friedman (1970)
“the social responsibility of business is to increase its profits”

Peter Drucker (1954)
“leaders in every single institution and in every single sector . . . have two responsibilities. They are responsible and accountable for the performance of their institutions, and that requires them and their institutions to be concentrated, focused, limited. They are responsible also, however, for the community as a whole”
Shareholder activism can take various forms

1. Engage behind the scene with management and the board
2. Propose resolutions (shareholder proposals)
3. Vote (form coalition/express dissent/call back lent shares)
4. Voice displeasure publicly (in the media)
5. Initiate a takeover (acquire a sizable equity share)
6. Exit (sell shares, take an offsetting bet)

Source: Bekjarovski and Brière (2018)
“Behind the curtain engagement involves private communication between activist shareholders and the firm’s board or management, that tends to precede public measures such as vote, shareholder proposals and voice. In a sense, the existence of other forms of public activism can be taken as a signal that behind the scene engagements were unsuccessful. When it comes to environmental and social issues, writing to the board or management is a common method though which shareholders can express concern and attempt to influence corporate policy behind the curtain; alternatively, face to face meetings with management or non-executive directors are a more common behind the scene engagement method when it comes to governance.” Bekjarovski and Brière (2018).
Three families of engagement:

- **on-going engagement**, where the goal for investors is to explain their ESG policy and collect information from the company. For instance, they can encourage companies to adopt best ESG practices, alert companies on ESG risks or better understand sectorial ESG challenges;

- **engagement for influence** (or protest), where the goal is to express dissatisfaction with respect to some ESG issues, make recommendations to the firm and measure/control ESG progress of companies;

- **pre-AGM engagement**, where the goal is to discuss with companies any resolution items that the investor may vote against.
Engage behind the scenes

The three steps of identification are:

1. List of engagement issues
2. Screening of companies
3. List of targeted companies

The different stages of engagement tracking are:

1. Issues are raised to the company;
2. Issues are acknowledged by the company;
3. The company develops a strategy to address the issues;
4. The company implements changes and the issues are resolved;
5. The company did not solve the issues and the engagement failed.
Propose resolutions

According to the SEC (Securities Exchange Act Rule 14a-8, §240):

“a shareholder proposal or resolution is a recommendation or requirement that the company and/or its board of directors take action, which the shareholder intend to present at a meeting of the company’s shareholders. The proposal should state as clearly as possible the course of action that the shareholder believes the company should follow. If the proposal is placed on the company’s proxy card, the company must also provide in the form of proxy means for shareholders to specify by boxes a choice between approval or disapproval, or abstention.”
Propose resolutions

Threshold criteria:

- US: $2000 + No-action letter
- France, Germany and UK: 5% of the capital
- Italy: 2.5% of the capital
- Netherlands: 0.33%
- Spain: 3% of the capital

⇒ Collective shareholder proposals

Shareholder resolution = Escalation
Propose resolutions

Some figures (Russell 300 & 2022 proxy season)

- 98% of proposals are filed by the management, while less than 2% corresponds to shareholder resolutions;
- Only 60% of shareholder resolutions are voted; The other 40% are omitted, not presented, withdrawn or pending;
- The average number of proposals per company is around two;
- The proponents of shareholder resolutions are concentrated on a small number of investors or organisations (15 proponents were responsible of 75% of shareholder proposals);
- The repartition of shareholder proposals voted in 2022 was the following: 11% related to E issues, 41% related to S issues and 48% related to G issues.
Vote

- Historical perspectives
- Importance of voting associations and NGOs
- US > Europe
- The concept of proxy voting
  - Institutional Shareholder Services (ISS)
  - Glass Lewis
- Say on Pay (2002)
  - Support rate for Russell 3000 companies: 87% in 2022 (from 15.4% to 99%)
  - Results for Germany, France and Spain
- Say on Climate (2020)
Figure 112: Average support rate of shareholder proposals (Russell 3000 companies)

Source: PwC’s Governance Insights Center (2022).
Some figures with Russell 3000 companies

- 555 shareholder resolutions have been voted
- Only 82 have received majority support
- This means that one shareholder resolution was adopted for 37 companies!

What is the efficiency of vote? ≟ What is the impact of vote?
Figure 113: Pass rate of shareholder proposals (Russell 3000 companies)

Voice

- 1970: Publication of the book *Exit, Voice, and Loyalty: Responses to Decline in Firms, Organizations, and States* by the economist Albert Hirschman
- Exist-voice model: exist **versus** voice or exit **and** voice
- Voice as a form of escalation
- Impact of collaborative engagement (e.g., Climate Action 100+)
- Increasing involvement of NGOs in the debate on engagement and greenwashing
Initiate a takeover

⇒ Hedge funds
Exit

- **Exit** refers to the process of selling off investments in a particular company or industry.
- **Divestment** is a more general term that implies a significant exposure reduction.
- Divestment: Final step in an escalation strategy?
Figure 114: What kinds of institutions are divesting from fossil fuel?

Source: https://divestmentdatabase.org.
Case study: the Cambridge University endowment fund

“A dilemma faced by an increasing number of investors is whether to divest from environmentally damaging businesses or whether to enter into a dialogue with them. This predicament now has its epicentre in Cambridge, England, where the ancient University of Cambridge faces great pressure from students and staff to respond to the threat of climate breakdown. Having already received two reports on its approach to responsible investment, the university has appointed a new chief investment officer (CIO) who, alongside University Council and the wider university community, needs to consider the question of whether to divest from or to engage with fossil-fuel firms.” Chambers et al. (2020).
Case study: Church of England Pensions Board

In 2020, they engaged with 21 companies. At the end of the process, 12 companies were supposed to make sufficient progress, while 9 companies were added to the list of restricted investments. These divestments totalled £32.23 mn (wrt £3.7 bn of assets under management).
Case study: The Universities Superannuation Scheme (USS)

- USS manage about £90 bn
- In 2020, they excluded certain sectors: tobacco manufacturing; thermal coal mining (coal to be burned for electricity generation), specifically where they made up more than 25% of revenues, and certain controversial weapons
- The first exclusion was announced in May 2020
- Two years after, divestment from these sectors is completed
- Ethics for USS ⇒ USS should extend its divestment policy
Individual vs. collaborative engagement
The role of institutional investors
Impact of active ownership
“The company sets the agenda for the annual shareholder meeting;

The custodian confirms the identity of the shareholders and the number of shares eligible for voting – often for a specific date ahead of the meeting (record date);

Shareholders receive the meeting materials from the company (may be before or after the record date);

Shareholders procuring proxy advisory services receive voting recommendations;

Shareholders instruct the custodian on how to vote, often through a proxy voting service provider, within a deadline ahead of the shareholder meeting (cut-off date);

Voting takes place at the shareholder meeting;

Shareholders receive confirmation from the service provider that their voting instructions have been carried out.”
Proxy voting
Voting policy
Figure 115: Voting Matters series of ShareAction

2019 2020 2021 2022

Source: https://shareaction.org.
### Table 72: Statistics of success rate shareholder resolutions

<table>
<thead>
<tr>
<th>Year</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of resolutions</td>
<td>64</td>
<td>102</td>
<td>144</td>
<td>249</td>
</tr>
<tr>
<td>Resolutions with majority support</td>
<td>3</td>
<td>15</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>Success rate (in %)</td>
<td>4.7</td>
<td>14.7</td>
<td>20.1</td>
<td>14.9</td>
</tr>
<tr>
<td>Average support rate (in %)</td>
<td>28.2</td>
<td>29.9</td>
<td>32.9</td>
<td>29.9</td>
</tr>
<tr>
<td>10% percentile of support rate (in %)</td>
<td>6.5</td>
<td>9.2</td>
<td>7.2</td>
<td>9.4</td>
</tr>
<tr>
<td>25% percentile of support rate (in %)</td>
<td>17.0</td>
<td>13.1</td>
<td>12.0</td>
<td>13.5</td>
</tr>
<tr>
<td>75% percentile of support rate (in %)</td>
<td>37.7</td>
<td>42.6</td>
<td>42.8</td>
<td>40.3</td>
</tr>
<tr>
<td>90% percentile of support rate (in %)</td>
<td>41.8</td>
<td>55.2</td>
<td>81.2</td>
<td>57.6</td>
</tr>
<tr>
<td>Average support rate (in %)</td>
<td>28.2</td>
<td>35.8</td>
<td>41.8</td>
<td>31.6</td>
</tr>
</tbody>
</table>

Asset managers

Figure 116: Histogram (in %) of support rates

Table 73: Average support rate in % for ESG resolutions

<table>
<thead>
<tr>
<th>Topic</th>
<th>Method</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>Arithmetic</td>
<td>45.8</td>
<td>57.4</td>
<td>58.9</td>
<td>65.0</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>32.7</td>
<td>42.1</td>
<td>47.6</td>
<td>46.5</td>
</tr>
<tr>
<td>Environment</td>
<td>Arithmetic</td>
<td>45.8</td>
<td>61.0</td>
<td>66.0</td>
<td>64.8</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>32.7</td>
<td>44.7</td>
<td>55.8</td>
<td>48.8</td>
</tr>
<tr>
<td>Social</td>
<td>Arithmetic</td>
<td>53.3</td>
<td>55.2</td>
<td>62.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td>39.0</td>
<td>43.7</td>
<td>44.3</td>
<td></td>
</tr>
<tr>
<td>Pay &amp; politics</td>
<td>Arithmetic</td>
<td></td>
<td></td>
<td></td>
<td>71.5</td>
</tr>
<tr>
<td></td>
<td>Weighted</td>
<td></td>
<td></td>
<td></td>
<td>47.8</td>
</tr>
</tbody>
</table>

Figure 117: Arithmetic average support rate in % per country and year

Figure 118: Weighted average support rate in % per country and year

### Table 74: Best performers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM</th>
<th>Overall</th>
<th>E</th>
<th>S</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Achmea IM</td>
<td>Netherlands</td>
<td>251</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>Impax AM</td>
<td>UK</td>
<td>56</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>BNP PAM</td>
<td>France</td>
<td>761</td>
<td>99</td>
<td>97</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>MN</td>
<td>Netherlands</td>
<td>193</td>
<td>99</td>
<td>97</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>Candriam</td>
<td>Luxembourg</td>
<td>180</td>
<td>98</td>
<td>97</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>PGGM</td>
<td>Netherlands</td>
<td>331</td>
<td>97</td>
<td>93</td>
<td>100</td>
<td>97</td>
</tr>
<tr>
<td>7</td>
<td>Man</td>
<td>UK</td>
<td>149</td>
<td>96</td>
<td>98</td>
<td>94</td>
<td>98</td>
</tr>
<tr>
<td>8</td>
<td>Robeco</td>
<td>Netherlands</td>
<td>228</td>
<td>95</td>
<td>94</td>
<td>94</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>Aviva Investors</td>
<td>UK</td>
<td>363</td>
<td>93</td>
<td>88</td>
<td>96</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>Amundi AM</td>
<td>France</td>
<td>2,348</td>
<td>93</td>
<td>93</td>
<td>92</td>
<td>98</td>
</tr>
<tr>
<td>11</td>
<td>Nordea AM</td>
<td>Finland</td>
<td>333</td>
<td>91</td>
<td>93</td>
<td>89</td>
<td>90</td>
</tr>
<tr>
<td>12</td>
<td>Aegon AM</td>
<td>Netherlands</td>
<td>466</td>
<td>90</td>
<td>85</td>
<td>94</td>
<td>90</td>
</tr>
<tr>
<td>13</td>
<td>Federated Hermes</td>
<td>UK</td>
<td>672</td>
<td>89</td>
<td>88</td>
<td>87</td>
<td>90</td>
</tr>
<tr>
<td>14</td>
<td>Pictet AM</td>
<td>Switzerland</td>
<td>284</td>
<td>88</td>
<td>85</td>
<td>90</td>
<td>91</td>
</tr>
<tr>
<td>15</td>
<td>Legal &amp; General</td>
<td>Switzerland</td>
<td>1,923</td>
<td>86</td>
<td>84</td>
<td>84</td>
<td>98</td>
</tr>
</tbody>
</table>

Source: ShareAction (2023) & Author’s calculations.
### Table 75: Worst performers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM</th>
<th>Overall</th>
<th>E</th>
<th>S</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>Goldman Sachs AM</td>
<td>US</td>
<td>2,218</td>
<td>35</td>
<td>56</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>60</td>
<td>Baillie Gifford</td>
<td>UK</td>
<td>455</td>
<td>31</td>
<td>29</td>
<td>29</td>
<td>45</td>
</tr>
<tr>
<td>61</td>
<td>SSGA</td>
<td>US</td>
<td>4,140</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>22</td>
</tr>
<tr>
<td>62</td>
<td>BlackRock</td>
<td>US</td>
<td>10,014</td>
<td>24</td>
<td>28</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>63</td>
<td>T. Rowe Price</td>
<td>US</td>
<td>1,642</td>
<td>17</td>
<td>26</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>64</td>
<td>Fidelity Investments</td>
<td>US</td>
<td>4,520</td>
<td>17</td>
<td>23</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>65</td>
<td>Vanguard</td>
<td>US</td>
<td>8,274</td>
<td>10</td>
<td>12</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>66</td>
<td>Dimensional Fund Advisors</td>
<td>US</td>
<td>679</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>67</td>
<td>Santander AM</td>
<td>Spain</td>
<td>220</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>68</td>
<td>Walter Scott &amp; Partners</td>
<td>UK</td>
<td>95</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: ShareAction (2023) & Author’s calculations.
Table 76: Ranking of the 25 largest asset managers (2022, overall)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>AUM 2019</th>
<th>Overall 2019</th>
<th>Overall 2020</th>
<th>Overall 2021</th>
<th>Overall 2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>BlackRock</td>
<td>US</td>
<td>10,014</td>
<td>7</td>
<td>12</td>
<td>40</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>Vanguard</td>
<td>US</td>
<td>8,274</td>
<td>8</td>
<td>14</td>
<td>26</td>
<td>10</td>
</tr>
<tr>
<td>23</td>
<td>Fidelity Investments</td>
<td>US</td>
<td>4,520</td>
<td>9</td>
<td>31</td>
<td>29</td>
<td>17</td>
</tr>
<tr>
<td>21</td>
<td>SSGA</td>
<td>US</td>
<td>4,140</td>
<td>26</td>
<td>35</td>
<td>32</td>
<td>29</td>
</tr>
<tr>
<td>18</td>
<td>J.P. Morgan AM</td>
<td>US</td>
<td>2,742</td>
<td>7</td>
<td>43</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>16</td>
<td>Capital Group</td>
<td>US</td>
<td>2,716</td>
<td>3</td>
<td>8</td>
<td>28</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>Amundi AM</td>
<td>France</td>
<td>2,348</td>
<td>66</td>
<td>89</td>
<td>93</td>
<td>93</td>
</tr>
<tr>
<td>20</td>
<td>Goldman Sachs AM</td>
<td>US</td>
<td>2,218</td>
<td>37</td>
<td>45</td>
<td>47</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>Legal &amp; General</td>
<td>UK</td>
<td>1,923</td>
<td>82</td>
<td>96</td>
<td>77</td>
<td>86</td>
</tr>
<tr>
<td>24</td>
<td>T. Rowe Price</td>
<td>US</td>
<td>1,642</td>
<td>5</td>
<td>22</td>
<td>31</td>
<td>17</td>
</tr>
<tr>
<td>15</td>
<td>Invesco</td>
<td>US</td>
<td>1,611</td>
<td>34</td>
<td>37</td>
<td>37</td>
<td>47</td>
</tr>
<tr>
<td>12</td>
<td>Morgan Stanley IM</td>
<td>US</td>
<td>1,566</td>
<td></td>
<td></td>
<td>55</td>
<td>64</td>
</tr>
<tr>
<td>14</td>
<td>Wellington Management</td>
<td>US</td>
<td>1,426</td>
<td>10</td>
<td>51</td>
<td>44</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>Northern Trust AM</td>
<td>US</td>
<td>1,348</td>
<td>21</td>
<td>70</td>
<td>60</td>
<td>83</td>
</tr>
<tr>
<td>13</td>
<td>Nuveen AM</td>
<td>US</td>
<td>1,271</td>
<td>62</td>
<td>63</td>
<td>56</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>UBS AM</td>
<td>Switzerland</td>
<td>1,216</td>
<td>90</td>
<td>79</td>
<td>75</td>
<td>83</td>
</tr>
<tr>
<td>4</td>
<td>DWS</td>
<td>Germany</td>
<td>1,055</td>
<td>74</td>
<td>66</td>
<td>85</td>
<td>86</td>
</tr>
<tr>
<td>10</td>
<td>AXA IM</td>
<td>France</td>
<td>1,009</td>
<td>79</td>
<td>71</td>
<td>55</td>
<td>73</td>
</tr>
<tr>
<td>6</td>
<td>Schroders</td>
<td>UK</td>
<td>991</td>
<td>56</td>
<td>62</td>
<td>73</td>
<td>85</td>
</tr>
<tr>
<td>17</td>
<td>AllianceBernstein</td>
<td>US</td>
<td>779</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Allianz Gf</td>
<td>Germany</td>
<td>766</td>
<td>89</td>
<td>81</td>
<td>77</td>
<td>86</td>
</tr>
<tr>
<td>1</td>
<td>BNP PAM</td>
<td>France</td>
<td>761</td>
<td>48</td>
<td>72</td>
<td>98</td>
<td>99</td>
</tr>
<tr>
<td>19</td>
<td>Columbia Threadneedle</td>
<td>US</td>
<td>754</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Manulife IM</td>
<td>Canada</td>
<td>723</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>APG AM</td>
<td>Netherlands</td>
<td>721</td>
<td>72</td>
<td>70</td>
<td>59</td>
<td>72</td>
</tr>
</tbody>
</table>

Figure 119: Evolution of the support rate in % per asset manager

Main findings

1. “49 additional resolutions would have received majority support if the largest asset managers had voted in favour of them.

2. Voting performance has been stagnant in the US and the UK compared to 2021, while European asset managers have shown a large improvement.

3. Asset managers across the board are hesitant to back action-oriented resolutions, which would have the most transformative impact on environmental and social issues.”
Figure 120: Ranking of the 36 say on climate resolutions with respect to the support rate in %

Source: ShareAction (2023) & Author’s calculations.
Asset managers

3 case studies of Say on Climate resolutions

- Electricité de France or EDF (French energy company): 99.9%
- Barclays (British bank): 80.8%
- Woodside Energy Group Ltd. (Australian energy company): 51.03%
Asset owners
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Scientific evidence of global warming
From the Holocene to the Anthropocene?
The physics of climate change

Course 2022-2023 in Sustainable Finance
Lecture 8. Awareness of Climate Change Impacts

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2024

20The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Lecture 1: Introduction
Lecture 2: ESG Scoring
Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
Lecture 4: Sustainable Financial Products
Lecture 5: Impact Investing
Lecture 6: Engagement & Voting Policy
Lecture 7: Extra-financial Accounting
Lecture 8: Awareness of Climate Change Impacts
Lecture 9: The Ecosystem of Climate Change
Lecture 10: Economic Models & Climate Change
Lecture 11: Climate Risk Measures
Lecture 12: Transition Risk Modeling
Lecture 13: Climate Portfolio Construction
Lecture 14: Physical Risk Modeling
Lecture 15: Climate Stress Testing & Risk Management
Greenhouse effect

Figure 121: A glass house
The founding text of the greenhouse effect is a French scientific publication written by Joseph Fourier in 1824: *Remarques générales sur les températures du globe terrestre et des espaces planétaires*:

“The heat of the Earth’s surface derives from three sources, which must first be distinguished:

1. The Earth is heated by the sun’s rays, whose uneven distribution produces the diversity of climates.
2. The Earth’s temperature depends on the common temperature of planetary spaces, as it is exposed to the irradiation of the innumerable stars that surround the solar system on all sides.
3. The earth has retained within its mass some of the primitive heat it contained when the planets were formed.”

---

21 General Remarks on Global and Planetary Temperatures.
In 1859, Tyndall showed that water vapor has a high heat absorption capacity. He went on to show that carbon dioxide and other gases could also absorb and radiate heat. Tyndall is the first scientist to prove that greenhouse gases exist and are responsible for the greenhouse effect.

Three years earlier, in 1856 and 1857, the American scientist Eunice Newton Foote had published two research papers with experiments showing that water vapor and carbon dioxide absorb heat from solar radiation.

Svante Arrhenius was the first scientist to calculate the effect of a change in atmospheric CO₂ on ground temperature (1896) ⇒ climate sensitivity.
Figure 122: Diagram showing how the greenhouse effect works

Figure 123: The pioneers of the greenhouse effect

- Joseph Fourier (1766-1830)
- Eunice Newton Foote (1819-1888)
- John Tyndall (1820-1893)
- Svante Arrhenius (1859-1927)
### Table 77: List of greenhouse gases

<table>
<thead>
<tr>
<th>Greenhouse gas</th>
<th>Formula</th>
<th>Kyoto Protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water vapor</td>
<td>H₂O</td>
<td></td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>✓</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>✓</td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>N₂O</td>
<td>✓</td>
</tr>
<tr>
<td>Ozone</td>
<td>O₃</td>
<td></td>
</tr>
<tr>
<td><strong>Fluorinated or F-gases</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sulfur hexafluoride</td>
<td>SF₆</td>
<td>✓</td>
</tr>
<tr>
<td>Nitrogen trifluoride</td>
<td>NF₃</td>
<td>✓</td>
</tr>
<tr>
<td>Chlorofluorocarbons</td>
<td>CFCs (CFC-11, CFC-12, etc.)</td>
<td></td>
</tr>
<tr>
<td>Hydrofluorocarbons</td>
<td>HFCs (HFC-23, HFC-32, etc.)</td>
<td>✓</td>
</tr>
<tr>
<td>Hydrochlorofluorocarbons</td>
<td>HCFCs (HCFC-12, etc.)</td>
<td></td>
</tr>
<tr>
<td>Perfluorocarbons</td>
<td>PFCs (CF₄, C₂F₆, etc.)</td>
<td>✓</td>
</tr>
</tbody>
</table>
Callendar Effect

“By fuel combustion man has added about 150 000 million tons of carbon dioxide to the air during the past half century. The author estimates from the best available data that approximately three quarters of this has remained in the atmosphere. The radiation absorption coefficients of carbon dioxide and water vapour are used to show the effect of carbon dioxide on sky radiation. From this the increase in mean temperature, due to the artificial production of carbon dioxide, is estimated to be at the rate of 0.003°C per year at the present time. The temperature observations at zoo meteorological stations are used to show that world temperatures have actually increased at an average rate of 0.005°C per year during the past half century.” (Callendar, 1938, page 223).
Collected papers on global warming by David Archer and Raymond Pierrehumbert

- 1824
  On the Temperatures of the Terrestrial Sphere and Interplanetary Space (Fourier)
- 1861
  On the Absorption and Radiation of Heat by Gases and Vapours, and on the Physical Connexion of Radiation, Absorption, and Conduction (Tyndall)
- 1896
  On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground (Arrhenius)
- 1938
  The Artificial Production of Carbon Dioxide and its Influence on Temperature (Callendar)
- 1956
  The Influence of the 15\(\mu\) Carbon-dioxide Band on the Atmospheric Infra-red Cooling Rate (Plass)
- 1957
  Carbon Dioxide Exchange Between Atmosphere and Ocean and the Question of an Increase of Atmospheric CO\(_2\) during the Past Decades (Revelle and Suess)
- 1958
  Distribution of Matter in the Sea and Atmosphere: Changes in the Carbon Dioxide Content of the Atmosphere and Sea due to Fossil Fuel Combustion (Bolin and Eriksson)
- 1960
  The Concentration and Isotopic Abundances of Carbon Dioxide in the Atmosphere (Keeling)
Collected papers on global warming by David Archer and Raymond Pierrehumbert

- 1967
  Thermal Equilibrium of the Atmosphere with a Given Distribution of Relative Humidity (Manabe and Wetherald)

- 1969
  The Effect of Solar Radiation Variations on the Climate of the Earth (Budyko)
  A Global Climatic Model Based on the Energy Balance of the Earth-Atmosphere System (Sellers)

- 1970
  Is Carbon Dioxide from Fossil Fuel Changing Man’s Environment? (Keeling)

- 1972
  Man-Made Carbon Dioxide and the Greenhouse Effect (Sawyer)

- 1975
  The Effects of Doubling the CO₂ Concentration on the Climate of a General Circulation Model (Manabe and Wetherald)

- 1977
  Changes of Land Biota and Their Importance for the Carbon Cycle (Bolin)
  Neutralization of Fossil Fuel CO₂ by Marine Calcium Carbonate (Broecker and Takahashi)

- 1979
  Carbon Dioxide and Climate: A Scientific Assessment (Charney, Arakawa, Baker et al.)

- 1984
  Climate Sensitivity: Analysis of Feedback Mechanisms (Hansen, Lacis, Rind et al.)
Global warming

Figure 124: Keeling curve: monthly mean CO₂ concentration in Mauna Loa

Charney Report (1979)

On July 23-27, 1979, Jule Charney formed a study group to “assess the scientific basis for projection of possible future climatic changes resulting from man-made releases of carbon dioxide into the atmosphere”. The study group had 13 members, including eminent scientists Akio Arakawa, Bert Bolin, Henry Stommel, etc. The report examined the results of five global climate models that simulate the climatic response to an increase in atmospheric CO2: three by Manabe and his colleagues at NOAA’s Geophysical Fluid Dynamics Laboratory and two by James Hansen and his colleagues at NASA’s Goddart Institute for Space Studies. The report estimated a climate sensitivity of 3°C, with an error of ±1.5°C.
Figure 125: The fathers of the concept of global warming

Guy Stewart Callendar (1898-1964)
Roger Revelle (1909-1991)
Charles David Keeling (1928-2005)
Wallace Broecker (1931-2019)
Testimony of James Hansen to the US Senate (1988)

“Mr. Chairman and committee members, thank you for the opportunity to present the results of my research on the greenhouse effect which has been carried out with my colleagues at the NASA Goddard Institute for Space Studies. I would like to draw three main conclusions. Number one, the earth is warmer in 1988 than at any time in the history of instrumental measurements. Number two, the global warming is now large enough that we can ascribe with a high degree of confidence a cause and effect relationship to the greenhouse effect. And number three, our computer climate simulations indicate that the greenhouse effect is already large enough to begin to affect the probability of extreme events such as summer heat waves.”
In 1988, the United Nations Environment Programme (UNEP) and the World Meteorological Organization (WMO) established the Intergovernmental Panel on Climate Change (IPCC) to provide policymakers with regular scientific assessments of climate change, its impacts and potential future risks, and to recommend options for adaptation and mitigation.
The Anthropocene is a proposed geological epoch that dates from the beginning of significant human impacts on Earth’s geology and ecosystems, including but not limited to human-induced climate change.

The term Anthropocene was popularized by Paul Crutzen and Eugene Stoermer in 2000 (Crutzen and Stoermer, 2000).
**Figure 126: Geologic time scale**

**INTERNATIONAL CHRONOSTRATIGRAPHIC CHART**

Source: International Commission on Stratigraphy (2023),
Geological history of the climate

Table 78: Units of time

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>(in year)</th>
<th>Symbol</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kyr/kyr</td>
<td>Thousand/Kilo years</td>
<td>$10^3$</td>
<td>ka</td>
<td>Kiloannus</td>
</tr>
<tr>
<td>Myr/yr</td>
<td>Mega/Million years</td>
<td>$10^6$</td>
<td>Ma</td>
<td>Megaannus</td>
</tr>
<tr>
<td>Gyr/yr</td>
<td>Giga/Billion years</td>
<td>$10^9$</td>
<td>Ga</td>
<td>Gigaannus</td>
</tr>
</tbody>
</table>
To estimate temperatures during the Precambrian, scientists use indirect methods, including geochemical proxies (chemical properties of rocks and minerals), paleontological studies (type and distribution of fossils and sedimentary rocks), and general circulation models.

One of the most common methods is the clumped isotope thermometer.

The ratio of the two carbon isotopes $^{12}\text{C}$ and $^{13}\text{C}$ is:

$$\delta^{13}\text{C} = 1000 \times \left( \frac{^{13}\text{C}}{^{12}\text{C}_{\text{sample}}} / \frac{^{13}\text{C}}{^{12}\text{C}_{\text{standard}}} - 1 \right)$$

The unit of $\delta^{13}\text{C}$ is parts per thousand.

- Faint young Sun paradox
- Snowball Earth hypothesis
Figure 127: Carbon isotopic evolution of marine carbonate

Source: Shields and Veizer (2002, Figure 2, page 5) & https://earthref.org/ERDA/48.
Geological history of the climate

Figure 128: Neoproterozoic carbon isotope data compilation

Source: Cox et al. (2016, Figure 2, page 90).
Figure 129: Cosmic calendar

Temperature scales

- Three different scales are commonly used to measure temperature: Celsius, Kelvin, and Fahrenheit
- Their symbols are °C, K, and °F
- The relationships between the Celsius and Kelvin scales are $T_\text{°C} = T_\text{K} - 273.15$ and $T_\text{K} = T_\text{°C} + 273.15$.
- For Celsius and Fahrenheit, we have $T_\text{°C} = \frac{5}{9}(T_\text{°F} - 32)$ and $T_\text{°F} = \frac{9}{5}T_\text{°F} + 32$
- Absolute zero is $-273.15^\circ C$, $0 K$ and $-459.67^\circ F$, implying that $T \geq -273.15^\circ C$, $T \geq 0 K$, and $T \geq -459.67^\circ F$
- The melting point (at standard pressure) is obtained at temperatures of $0^\circ C$, $273.15 K$ and $32^\circ F$
- The boiling point of water corresponds to temperatures of $100^\circ C$, $373.15 K$ and $212^\circ F$
Palaeoclimatic change since the Phanerozoic

Figure 130: Earth temperature since 500 Myr BP (°C vs. 1960-1990 average)
### Table 79: Recovered deep and very deep ice cores

<table>
<thead>
<tr>
<th></th>
<th>Greenland</th>
<th>Antarctica</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site 2</td>
<td>1956 305 m</td>
<td>Byrd Station 1957-1958 307 m</td>
</tr>
<tr>
<td>Site 2</td>
<td>1957 411 m</td>
<td>Little America 1958-1959 264 m</td>
</tr>
<tr>
<td>Camp Century</td>
<td>1961-1966 1387 m</td>
<td>Byrd Station 1966-1968 2164 m</td>
</tr>
<tr>
<td>Dye 3</td>
<td>1971 372 m</td>
<td>Vostock 1990-1998 3623 m</td>
</tr>
<tr>
<td>Milcent</td>
<td>1973 398 m</td>
<td>Dome Fuji 1994-1997 2503 m</td>
</tr>
<tr>
<td>Crete</td>
<td>1974 405 m</td>
<td>Vostock 2005-2007 3658 m</td>
</tr>
<tr>
<td>Dye 3</td>
<td>1979-1981 2037 m</td>
<td>Dome Fuji 2003-2007 3035 m</td>
</tr>
<tr>
<td>GRIP</td>
<td>1989-1992 3029 m</td>
<td>Dome C 1999-2005 3270 m</td>
</tr>
<tr>
<td>GISP 2</td>
<td>1989-1993 3057 m</td>
<td>Kohnen Station 2001-2006 2774 m</td>
</tr>
<tr>
<td>NGRIP</td>
<td>1996-2004 3090 m</td>
<td>WAIS 2006-2011 3405 m</td>
</tr>
</tbody>
</table>
Figure 131: Greenland deep drilling sites
Palaeoclimate during the Phanerozoic

Figure 132: Antarctica deep drilling sites
Table 80: Isotopes of chemical elements

<table>
<thead>
<tr>
<th>Element</th>
<th>Stable isotopes</th>
<th>Unstable isotopes</th>
<th>Major isotope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>$^1$H and $^2$H</td>
<td>$^3$H – $^7$H</td>
<td>Protium (99.98%)</td>
</tr>
<tr>
<td>Carbon</td>
<td>$^{12}$C and $^{13}$C</td>
<td>$^{8}$C – $^{11}$C and $^{14}$C – $^{22}$C</td>
<td>Carbon-12 (98.90%)</td>
</tr>
<tr>
<td>Oxygen</td>
<td>$^{16}$O and $^{18}$O</td>
<td>$^{17}$O and $^{19}$O – $^{27}$O</td>
<td>Oxygen-16 (99.76%)</td>
</tr>
<tr>
<td>Uranium</td>
<td></td>
<td>$^{232}$U – $^{242}$U</td>
<td>Uranium-238 (99.27%)</td>
</tr>
</tbody>
</table>
Palaeoclimate during the Phanerozoic

- Analysis of trapped air bubbles in ice cores provides a direct record of the composition of the atmosphere at the time the ice formed.
- Dansgaard (1964) defined the relative deviation $\delta$ of the heavy isotope content as follows:

$$\delta = 1000 \times \left( \frac{R_{\text{sample}} - R_{\text{standard}}}{R_{\text{standard}}} \right)$$

where $R$ is the absolute content.

- $\delta$ is measured in ‰.

- As the chemical formula of water is $\text{H}_2\text{O}$, climate reconstruction from ice cores is based on the analysis of hydrogen and oxygen.
  - In the case of hydrogen, the common isotope is $^1\text{H}$, while the heavy isotope is $^2\text{H}$ (also called the deuterium or D).
  - The ratio $R$ is then $\frac{^2\text{H}}{^1\text{H}}$ and the relative variation is written as $\delta^\text{D}$.
  - In the case of oxygen, the common isotope is $^{16}\text{O}$, while the heavy isotope is $^{18}\text{O}$.
  - The ratio $R$ is then $\frac{^{18}\text{O}}{^{16}\text{O}}$ and the relative variation is written as $\delta^{18}\text{O}$.
Palaeoclimate during the Phanerozoic

Figure 133: Part of an ice core at WAIS Divide Field Camp

Source: Eli Duke, Antarctica: WAIS Divide Field Camp (Flickr),
www.flickr.com/photos/80547277@N00/9518403333.
The aim of ice core analysis is to estimate the temperature function $t \mapsto \mathcal{T}(t)$ with respect to the time age $t$.

The raw analysis provides two measurements: the depth $d$ of the ice core drilling and the isotope ratio measure $\delta$.

We therefore observe the isotope function $d \mapsto \delta(d)$.

To obtain the temperature function, we proceed in two steps:

1. First, we transform the depth $d$ of the ice core drilling into the time age $t$:
   $$ t = \varphi_t(d) $$

2. We then estimate the temperature $\mathcal{T}$ associated with the isotope ratio $\delta(d)$:
   $$ \mathcal{T} = \varphi_{\mathcal{T}}(\delta(d)) $$

Combining the two previous equations gives the desired parametric function $t \mapsto \mathcal{T}(t)$. 
Figure 134: Isotopic reconstruction of Vostok ice cores

Source: Petit et al. (1999) & Author’s calculations.
Palaeoclimate during the Phanerozoic

Figure 135: Temperature reconstruction of Vostok ice cores

Source: Petit et al. (1999).
Palaeoclimate during the Phanerozoic

Figure 136: Gas concentration of Vostok ice cores

Source: Petit et al. (1999).
Palaeoclimatic during the Phanerozoic

Figure 137: Evolution of the atmospheric CO\(_2\) during the last 420 million years

Source: Foster et al. (2017, Figure 1, page 3).
Temperature anomaly

We define the temperature anomaly at time $t$ as the difference between the temperature at time $t$ and the temperature for a reference period:

$$\Delta T(t) = T(t) - T_{\text{Base}}$$

where $T_{\text{Base}}$ is the reference temperature. It is generally the average of the temperature of the reference period:

$$T_{\text{Base}} = \frac{\sum_{j \in \text{Base}} T(j)}{n_{\text{Base}}}$$

For example, the reference temperature can be the average temperature of the 20th century (from 1901 to 2000) or the pre-industrial period.
Figure 138: Global average land-ocean temperature anomaly relative to 1961-1990 average

Source: Morice et al. (2021).
**Figure 139:** Average land-ocean temperature anomaly in the northern and southern hemispheres relative to the 1961-1990 average

Source: Morice et al. (2021).
**Table 81:** Linear projection of land-ocean temperature anomaly (in °C)

<table>
<thead>
<tr>
<th>Year</th>
<th>HadCRUT5 Global</th>
<th>HadCRUT5 Northern</th>
<th>HadCRUT5 Southern</th>
<th>NOAAGlobalTemp v5.1 Global</th>
<th>NOAAGlobalTemp v5.1 Northern</th>
<th>NOAAGlobalTemp v5.1 Southern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2050</td>
<td>1.4336</td>
<td>1.9595</td>
<td>0.9078</td>
<td>1.4576</td>
<td>1.9894</td>
<td>0.9247</td>
</tr>
<tr>
<td>2075</td>
<td>1.9288</td>
<td>2.6540</td>
<td>1.2035</td>
<td>1.9185</td>
<td>2.6715</td>
<td>1.1642</td>
</tr>
<tr>
<td>2100</td>
<td>2.4239</td>
<td>3.3486</td>
<td>1.4992</td>
<td>2.3795</td>
<td>3.3536</td>
<td>1.4038</td>
</tr>
<tr>
<td>Slope</td>
<td>0.0198</td>
<td>0.0278</td>
<td>0.0118</td>
<td>0.0184</td>
<td>0.0273</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

Source: NOAAGlobalTemp v5.1 & Author’s calculations.

**Table 82:** Linear projection of land and ocean temperature anomalies (in °C)

<table>
<thead>
<tr>
<th>Year</th>
<th>Global Land</th>
<th>Global Ocean</th>
<th>Northern Land</th>
<th>Northern Ocean</th>
<th>Southern Land</th>
<th>Southern Ocean</th>
</tr>
</thead>
<tbody>
<tr>
<td>2050</td>
<td>2.4212</td>
<td>1.0238</td>
<td>2.8388</td>
<td>1.3482</td>
<td>1.4719</td>
<td>0.7972</td>
</tr>
<tr>
<td>2075</td>
<td>3.2386</td>
<td>1.3243</td>
<td>3.8176</td>
<td>1.8061</td>
<td>1.9222</td>
<td>0.9875</td>
</tr>
<tr>
<td>2100</td>
<td>4.0560</td>
<td>1.6247</td>
<td>4.7964</td>
<td>2.2641</td>
<td>2.3725</td>
<td>1.1779</td>
</tr>
<tr>
<td>Slope</td>
<td>0.0327</td>
<td>0.0120</td>
<td>0.0392</td>
<td>0.0183</td>
<td>0.0180</td>
<td>0.0076</td>
</tr>
</tbody>
</table>

Source: NOAAGlobalTemp v5.1 & Author’s calculations.
**Figure 140: Average temperature anomaly (land-ocean, land and ocean)**

Source: Vose et al. (2021).
Figure 141: Projection of temperature anomaly by 2100 (in °C)

According to Friedlingstein et al. (2022), the global carbon budget has five main components:

1. Fossil fuel combustion and oxidation from all energy and industrial processes, including cement production and carbonation ($CE_{Industry}$)
2. Emissions from land-use change ($CE_{Land}$)
3. The growth rate of atmospheric CO$_2$ concentration ($CE_{AT}$)
4. The uptake of CO$_2$ by the oceans (ocean sink) ($CS_{Ocean}$)
5. The uptake of CO$_2$ by the land (land sink) ($CS_{Land}$)

From a theoretical point of view, we have the following identity:

\[
CE_{AT} = (CE_{Industry} + CE_{Land}) - (CS_{Ocean} + CS_{Land})
\]

Positive emissions

Negative emissions

The estimated budget imbalance is equal to:

\[
CB_{Imbalance} = CE_{Industry} + CE_{Land} - (CE_{AT} + CS_{Ocean} + CS_{Land})
\]
In 2021, the authors estimate the following figures expressed in gigatonnes of carbon: $CE_{\text{Industry}}^* = 10.13$, $CE_{\text{Land}} = 1.08$, $CE_{\text{AT}} = 5.23$, $CS_{\text{Ocean}} = 2.88$, $CS_{\text{Land}} = 3.45$, $CS_{\text{Cement}} = 0.23$, and $CB_{\text{Imbalance}} = -0.58$

Expressed in gigatonnes of CO$_2$ we obtain: $CE_{\text{Industry}}^* = 37.12$, $CE_{\text{Land}} = 3.94$, $CE_{\text{AT}} = 19.14$, $CS_{\text{Ocean}} = 10.55$, $CS_{\text{Land}} = 12.64$, $CS_{\text{Cement}} = 0.84$, and $CB_{\text{Imbalance}} = -2.12$

Anthropogenic CO$_2$ emissions are therefore 36.28 GtCO$_2$ for industrial processes ($CE_{\text{Industry}}^*$) and 3.94 GtCO$_2$ for land-use change

Since the total is 40.22 GtCO$_2$, 26.23% and 31.43% of the total anthropogenic CO$_2$ emissions have been absorbed by oceans and land, respectively, while 47.60% remain in the atmosphere
Figure 142: Annual CO$_2$ emissions (in GtCO$_2$)

Source: Friedlingstein et al. (2022) & Author’s calculations.
Figure 143: Cumulative CO$_2$ emissions and carbon sinks (in GtCO$_2$)

Source: Friedlingstein et al. (2022) & Author’s calculations.
Anthropogenic GHG emissions

Figure 144: Cumulative CO₂ budget imbalance in atmosphere (in GtCO₂)

Source: Friedlingstein et al. (2022) & Author’s calculations.
The airborne fraction is the ratio of the atmospheric CO$_2$ growth rate to total anthropogenic emissions. For the period 1750-2021, this ratio is equal to:

$$\text{AF} = \frac{CE_{\text{AT}}}{CE_{\text{Industry}} + CE_{\text{Land}}} = \frac{1076}{1717 + 742} = 43.8\%$$

This means that 43.8% of anthropogenic emissions have not been absorbed by natural carbon sinks.

“The observed stability of the airborne fraction over the 1960-2020 period indicates that the ocean and land CO$_2$ sinks have on average been removing about 55% of the anthropogenic emissions.” (Friedlingstein et al., 2022, page 4834).
## Table 83: Breakdown of anthropogenic CO$_2$ emission by energy source (in %)

<table>
<thead>
<tr>
<th>Energy</th>
<th>1850</th>
<th>1900</th>
<th>1950</th>
<th>1975</th>
<th>2000</th>
<th>2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>100.00</td>
<td>95.85</td>
<td>65.10</td>
<td>34.74</td>
<td>36.18</td>
<td>40.11</td>
</tr>
<tr>
<td>Oil</td>
<td>0.00</td>
<td>3.55</td>
<td>26.79</td>
<td>47.64</td>
<td>40.42</td>
<td>33.23</td>
</tr>
<tr>
<td>Gas</td>
<td>0.00</td>
<td>0.60</td>
<td>6.09</td>
<td>13.17</td>
<td>18.61</td>
<td>20.38</td>
</tr>
<tr>
<td>Cement</td>
<td>0.00</td>
<td>0.00</td>
<td>1.13</td>
<td>1.99</td>
<td>2.87</td>
<td>4.33</td>
</tr>
<tr>
<td>Flaring</td>
<td>0.00</td>
<td>0.00</td>
<td>0.81</td>
<td>2.18</td>
<td>1.05</td>
<td>1.12</td>
</tr>
<tr>
<td>Other</td>
<td>0.00</td>
<td>0.00</td>
<td>0.08</td>
<td>0.29</td>
<td>0.86</td>
<td>0.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Energy</th>
<th>Coal</th>
<th>1850</th>
<th>97.58</th>
<th>85.24</th>
<th>63.49</th>
<th>50.00</th>
<th>46.29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>0.00</td>
<td>2.05</td>
<td>12.13</td>
<td>27.43</td>
<td>35.10</td>
<td>35.02</td>
<td></td>
</tr>
<tr>
<td>Gas</td>
<td>0.00</td>
<td>0.37</td>
<td>2.15</td>
<td>6.88</td>
<td>11.72</td>
<td>14.52</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
<td>1.17</td>
<td>1.77</td>
<td>2.56</td>
<td></td>
</tr>
<tr>
<td>Flaring</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
<td>0.87</td>
<td>1.03</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>0.00</td>
<td>0.00</td>
<td>0.06</td>
<td>0.16</td>
<td>0.39</td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

Source: Friedlingstein *et al.* (2022) & Author’s calculations.
Figure 145: Energy source breakdown of anthropogenic cumulative CO₂ (in %)

Source: Friedlingstein et al. (2022) & Author’s calculations.
Anthropogenic GHG emissions

Figure 146: Share of CO₂ emissions by region (in % of total)

Source: Friedlingstein et al. (2022).
Figure 147: Share of cumulative CO$_2$ emissions by region (in % of total)

Source: Friedlingstein et al. (2022).
**Anthropogenic GHG emissions**

**Figure 148:** Share of CO$_2$ emissions by country (in % of total)

Source: Friedlingstein *et al.* (2022).
Figure 149: Share of cumulative CO$_2$ emissions by country (in % of total)

Source: Friedlingstein et al. (2022).
Anthropogenic GHG emissions

Kaya identity

The Kaya identity is defined as:

\[
\text{Anthropogenic CO}_2 \text{ emissions} = \text{Population} \times \frac{\text{GDP}}{\text{Population}} \times \frac{\text{Energy}}{\text{GDP}} \times \frac{\text{CO}_2 \text{ emissions}}{\text{Energy}}
\]

Using the notations of Kaya and Yokobori (1997), this identity is generally expressed as:

\[
F = P \times \frac{G}{P} \times \frac{E}{G} \times \frac{F}{E}
\]

Therefore, the key drivers of anthropogenic CO\(_2\) emissions include four main factors:

- the population (demographics)
- the GDP per capita (economics)
- the energy intensity of the GDP (engineering)
- the carbon intensity (physics)
Figure 150: Key drivers of the Kaya identity

Source: Friedlingstein et al. (2022).
Anthropogenic GHG emissions

Figure 151: CO₂ emissions per capita (in tCO₂ per person)

Source: Friedlingstein et al. (2022).
Anthropogenic GHG emissions

Figure 152: GHG emissions (in GtCO$_2$e)

Source: Jones et al. (2023) & Author’s calculations.
### Table 84: 2021 greenhouse gas emissions (in GtCO₂e)

<table>
<thead>
<tr>
<th></th>
<th>CH₄</th>
<th>CO₂</th>
<th>N₂O</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry</td>
<td>6.23</td>
<td>37.11</td>
<td>0.79</td>
</tr>
<tr>
<td>Land</td>
<td>3.95</td>
<td>4.00</td>
<td>2.18</td>
</tr>
<tr>
<td>Total</td>
<td>10.18</td>
<td>41.12</td>
<td>2.97</td>
</tr>
</tbody>
</table>

(61.2%) (90.3%) (26.7%) (38.8%) (9.7%) (73.3%) (18.8%) (75.8%) (5.5%)

Source: Jones et al. (2023) & Author’s calculations.
Figure 153: Planetary boundaries

Source: Richardson et al. (2023, Figure 1, page 4).
### Table 85: Current status of planetary boundaries

<table>
<thead>
<tr>
<th>No</th>
<th>Earth process system</th>
<th>Control variable</th>
<th>(m_{1750})</th>
<th>(m_{\text{Boundary}})</th>
<th>(m_{\text{upper}})</th>
<th>(m_{2023})</th>
<th>Crossed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Climate change</td>
<td>Atmospheric (\text{CO}_2) concentration (ppm)</td>
<td>280</td>
<td>350</td>
<td>450</td>
<td>417</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Atmospheric radiative forcing (W/m(^2))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Genetic diversity (E/MSY)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Biodiversity loss</td>
<td>Functional integrity (% HANPP)</td>
<td>1.9</td>
<td>10</td>
<td>100</td>
<td>&gt; 100</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>Stratospheric ozone depletion</td>
<td>Stratospheric (\text{O}_3) concentration (DU)</td>
<td>290</td>
<td>276</td>
<td>261</td>
<td>284.6</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>Ocean acidification</td>
<td>Carbonate ion concentration ((\Omega_{\text{arg}}))</td>
<td>3.44</td>
<td>2.752</td>
<td>2.75</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Phosphate flow (TgP/yr) – global</td>
<td>0</td>
<td>11</td>
<td>100</td>
<td>22.6</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Phosphate flow (TgP/yr) – regional</td>
<td>0</td>
<td>6.2</td>
<td>11.2</td>
<td>17.5</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nitrogen flow (TgN/yr)</td>
<td>0</td>
<td>62</td>
<td>82</td>
<td>190</td>
<td>✓</td>
</tr>
<tr>
<td>5</td>
<td>Biogeochemical flows</td>
<td>Area of forested land (%) – global</td>
<td>100</td>
<td>75</td>
<td>54</td>
<td>60</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>% area remaining – tropical</td>
<td>100</td>
<td>85</td>
<td>60</td>
<td>58.6</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>% area remaining – temperate</td>
<td>100</td>
<td>50</td>
<td>30</td>
<td>41.1</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>% area remaining – boreal</td>
<td>100</td>
<td>85</td>
<td>60</td>
<td>63.5</td>
<td>✓</td>
</tr>
<tr>
<td>6</td>
<td>Land-use change</td>
<td>Blue water (%)</td>
<td>9.4</td>
<td>10.2</td>
<td>50</td>
<td>18.2</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Green water (%)</td>
<td>9.8</td>
<td>11.1</td>
<td>50</td>
<td>15.8</td>
<td>✓</td>
</tr>
<tr>
<td>7</td>
<td>Freshwater change</td>
<td>Inter-hemispheric difference (AOD)</td>
<td>0.03</td>
<td>0.10</td>
<td>0.25</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Atmospheric aerosol loading</td>
<td>Synthetic chemicals (%)</td>
<td>0</td>
<td>0</td>
<td>NA</td>
<td>&gt; 0</td>
<td>✓</td>
</tr>
<tr>
<td>9</td>
<td>Novel entities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Richardson et al. (2023, Table 1, pages 4-5).
First law of thermodynamics

“Temperature [...] is a measure of the energy contained in the movement of molecules. Therefore, to understand how the temperature is maintained, one must consider the energy balance that is formally stated in the first law of thermodynamics. The basic global energy balance of Earth is between energy coming from the Sun and energy returned to space by Earth’s radiative emission. The generation of energy in the interior of Earth has a negligible influence on its energy budget.” (Hartmann, 2016, page 25).

- Energy cannot be created or destroyed (law of conservation of energy)
- The total amount of energy in a closed system is conserved
- Energy and forcing are two interchangeable terms, meaning that $F_{\text{solar}} := \mathcal{E}_{\text{solar}}, F_{\text{thermal}} := \mathcal{E}_{\text{infrared}},$ etc.
The total amount of electromagnetic energy emitted by the Sun, also called the solar luminosity is \( L_\odot = 3.828 \times 10^{26} \) watts.

Total solar irradiance (TSI) is defined as:

\[
S_d = \frac{L_\odot}{4\pi d^2}
\]

where \( d \) is the distance of the sphere from the Sun in meters.

For the Earth, the distance is between 147.1 and 152.1 million kilometers.

Using a mean value of 149.6 million kilometers, we get:

\[
S_0 = \frac{3.828 \times 10^{26}}{4\pi (149.6 \times 10^9)^2} = 1372.11 \text{ W/m}^2
\]

A direct measurement by astrophysicists gives 1368 W/m\(^2\).
Planck radiation law and spectral density of electromagnetic radiation

In physics, Planck’s law describes the spectral distribution of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature. The expression for the spectral density function is:

\[ B_\nu (\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp \left( \frac{h\nu}{kT} \right) - 1} \]

where \( h = 6.62607015 \times 10^{-34} \text{ J Hz}^{-1} \) is Planck’s constant, \( c = 299792458 \text{ m s}^{-1} \) is the speed of light in a vacuum, \( k = 1.380649 \times 10^{-23} \text{ J K}^{-1} \) is Boltzmann’s constant, \( T \) is the temperature measured in Kelvin, and \( \nu \) is the frequency in hertz.
Planck law can be written in terms of the wavelength $\lambda$:

$$
\lambda = \frac{c}{\nu} \quad \left( \sim \frac{\text{m s}^{-1}}{\text{Hz}} = \text{m} \right)
$$

and we have:

- **Ultraviolet** wavelength: less than 380 nm
- Visible wavelength: 380 nm to 780 nm
- **Infrared** wavelength: greater than 780 nm
Planck law

Figure 154: Spectral density function $B_\lambda (\lambda, T)$ (in $10^{12}$ W/m$^2$ m$^{-1}$)

![Spectral density function graph]

- $T = 5000$ K
- $T = 3000$ K

UV, Visible, Infrared bands.
Planck law

**Figure 155:** Comparison of the radiation spectra of sunlight and the Earth’s surface (in $10^{12} \text{W/m}^2 \text{m}^{-1}$)

![Graph showing comparison of sunlight and Earth surface radiation spectra](image)

- **Ultraviolet** Sunlight $T = 5525 \text{K}$
- **Visible**
- **Shortwave Infrared**
- **Longwave Infrared** Earth surface $T = 288 \text{K}$

- $\lambda$ (in microns)
Stefan-Boltzmann law

The Stefan-Boltzmann law describes the relationship between the total amount of radiation $\mathcal{E}$ emitted by a body and its temperature $T$:

$$\mathcal{E} = \varepsilon \sigma T^4$$

where:

- $\varepsilon \in [0, 1]$ is the emissivity of the body
- $\sigma = 5.67 \times 10^{-8}$ W/m$^2$ K$^{-4}$ is the Stefan-Boltzmann constant
- For an ideal black body, we have $\varepsilon = 1$
Effective temperature of stars

- The radius of the Sun $R_\odot$ is about 696,342 kilometers.
- The solar irradiance at the photosphere is equal to:
  \[
  S_\odot = \frac{L_\odot}{4\pi R_\odot^2} = \frac{3.828 \times 10^{26}}{4\pi (696,342 \times 10^3)^2} = 62,822,741 \text{ W/m}^2
  \]
- If we assume that the sun is a perfect black body ($\varepsilon \approx 0.96$), we have:
  \[
  \sigma T_\odot^4 = S_\odot \iff T_\odot = \sqrt[4]{\frac{S_\odot}{\sigma}}
  \]
- We have:
  \[
  T_\odot = \sqrt[4]{\frac{62,822,741}{5.67 \times 10^{-8}}} = 5,769 \text{ K}
  \]
Remark

The previous analysis can be extended to other stars. Let $R_{\text{star}}$ be the stellar radius of the star. Since we have $L_{\text{star}} = 4\pi R_{\text{star}}^2 S_{\text{star}}$ and $S_{\text{star}} = \mathcal{E} = \sigma T^4$, we get:

$$T_{\text{star}} = \sqrt[4]{\frac{L_{\text{star}}}{4\pi R_{\text{star}}^2 \sigma}}$$

$T_{\text{star}}$ is defined as the temperature of a black body radiating the same amount of energy per unit area as the star. It may differ from the actual temperature of a star, which depends on its kinetic energy.
Incoming solar radiation

- The incoming solar radiation is equal to:
  \[ F_{\text{solar}} = \frac{1}{4} \left(1 - \alpha_p\right) S_0 \]
  
  where \( \alpha_p \) is the planetary albedo, which measures the amount of reflected sunlight
  
  - \( \alpha_p \) is equal to zero for a perfect black body
  - \( \alpha_p \) one for a perfect white body

Remark

The ratio \( \frac{1}{4} \) comes from the fact that no point on the planet receives the sun’s energy continuously during a full day. On average, we can show that a point on the planet receives \( \frac{1}{4} \) of the solar energy, which is the ratio of the projected area of the sphere \( (\text{Area} = \pi r^2) \) divided by the surface area of the sphere \( (\text{Area} = 4\pi r^2) \)
Figure 156: Incoming solar radiation
Incoming solar radiation

In the case of the Earth, we have $\alpha_p \approx 0.29$ and:

$$F_{\text{solar}} = \frac{1}{4} (1 - 0.29) \times 1368 = 242.82 \text{ W/m}^2$$
**Interpretation**

Consider a room with a surface area of $x$ square meters and receiving an energy $\mathcal{E}$ expressed in watts. The radiation per square meter received by this room is equal to $\mathcal{E}/x$. If the room receives the same equivalent solar radiation $F_{\text{solar}}$, the energy must be equal to:

$$\mathcal{E} = x \cdot F_{\text{solar}}$$

Using a standard 200 watt lamp, we can calculate the number of lamps required to achieve the same equivalent solar radiation. The results are shown below:

<table>
<thead>
<tr>
<th>$x$ (in $m^2$)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{E}$ (in watts)</td>
<td>242.8</td>
<td>1 214</td>
<td>2 428</td>
<td>4 856</td>
<td>12 141</td>
<td>24 282</td>
</tr>
<tr>
<td># lights</td>
<td>1.2</td>
<td>9</td>
<td>12</td>
<td>24</td>
<td>61</td>
<td>121</td>
</tr>
</tbody>
</table>

For a room of 20 $m^2$, we need 24 lights.
Zero-order model of the atmosphere

Figure 157: Zero-order model

\[
\begin{align*}
\text{Incoming solar radiation:} & \quad \frac{1}{4} S_0 \\
\text{Albedo:} & \quad \frac{1}{4} \alpha_p S_0 \\
\text{Earth:} & \quad \frac{1}{4} (1 - \alpha_p) S_0 \\
\text{Infrared radiation:} & \quad \sigma T_e^4 \\
\end{align*}
\]
Effective temperature of the Earth

- The Earth receives the incoming solar radiation $F_{\text{solar}}$, while the black body radiation is given by the Stefan-Boltzmann law.
- We deduce that:

$$\sigma T_e^4 = \frac{1}{4} (1 - \alpha_p) S_0 \iff T_e = \sqrt[4]{\frac{(1 - \alpha_p) S_0}{4\sigma}}$$

- The numerical calculation gives:

$$T_e = \sqrt[4]{\frac{(1 - 0.29) \times 1368}{4 \times 5.67 \times 10^{-8}}} = 255.81 \text{ K} = 255.81^\circ \text{C} - 273.15^\circ \text{C} = -17.34^\circ \text{C}$$

- The effective temperature of the Earth is then close to $-17.34^\circ \text{C}$.
Effective temperature of the Earth

Without greenhouse gases, the surface temperature of the Earth should be equal to the effective temperature. However, we observe:

\[ T_s \approx +15^\circ C \gg T_e \approx -17^\circ C \]
“Since solar radiation is mostly visible and near infrared, and Earth emits primarily thermal infrared radiation, the atmosphere may affect solar and terrestrial radiation very differently. [...] Since the atmospheric layer absorbs all of the energy emitted by the surface below it and emits like a blackbody, the only radiation emitted to space is from the atmosphere in this model.” (Hartman, 2016, pages 32-32).
Impact of the greenhouse effect

Figure 158: Zero-order model with greenhouse effect

\[ \frac{1}{4} (1 - \alpha_p) S_0 \]
Impact of the greenhouse effect

- The energy balance for the Earth’s surface is:

\[ F_{\text{solar}} + \sigma T_a^4 = \sigma T_s^4 \]

while the radiation balance for the atmosphere verifies:

\[ \sigma T_s^4 = 2\sigma T_a^4 \]

- It follows that:

\[ F_{\text{solar}} + \sigma T_a^4 = 2\sigma T_a^4 \]

or (we have \( \sigma T_e^4 = \frac{1}{4} (1 - \alpha_p) S_0 := F_{\text{solar}} \)):

\[ F_{\text{solar}} = \sigma T_a^4 = \sigma T_e^4 \]

- We conclude that:

\[
\begin{cases}
  T_a = T_e \\
  T_s = \sqrt{2} T_e
\end{cases}
\]
Impact of the greenhouse effect

- Using the previous numerical values, we obtain:

\[
\begin{align*}
T_a &= 255.81 \text{ K} = -17.34 \text{°C} \\
T_s &= 304.22 \text{ K} = 31.07 \text{°C}
\end{align*}
\]

- We find that the surface temperature is warmer than the observed global mean surface temperature.

- This is because the assumption that the atmosphere absorbs all the heat radiated from the surface is not true.
Another way to illustrate the greenhouse effect is to estimate the reflection parameter $\gamma_p$, which measures the net thermal radiation of the atmosphere with respect to the black body energy.

We deduce that the balance $E_{\text{net}}$ is:

$$E_{\text{net}} = F_{\text{solar}} - \sigma T_s^4 - \gamma_p \sigma T_s^4$$

Solving the equation $E_{\text{net}} = 0$ gives:

$$\gamma_p = 1 - \frac{F_{\text{solar}}}{\sigma T_s^4} = 1 - \left(1 - \alpha_p \right) \frac{S_0}{4\sigma T_s^4}$$

Using a surface temperature of $15^\circ C$, we get $\gamma_p = 0.3788$

$\Rightarrow$ Only 62% of the infrared radiation goes into space and 38% stays on the surface.
“A layer of atmosphere that is almost opaque for longwave radiation can be crudely approximated as a blackbody that absorbs all terrestrial radiation that is incident upon it and emits like a blackbody at its temperature. For an atmosphere with a large infrared optical depth, the radiative transfer process can be represented with a series of blackbodies arranged in vertical layers. Two layers centered at 0.5 km and 2.0 km altitudes provide a simple approximation for Earth’s atmosphere.” (Hartman, 2016, page 71).
Two-layer model of the atmosphere

Figure 159: Two-layer model

\[
\frac{1}{4} (1 - \alpha_p) S_0
\]

Energy balance models
Climate sensitivity and feedback
Tipping points
Two-layer model of the atmosphere

- We have:
  \[
  \begin{align*}
  F_{\text{solar}} + \sigma T_1^4 &= \sigma T_s^4 \\
  \sigma T_2^4 + \sigma T_s^4 &= 2\sigma T_1^4 \\
  \sigma T_1^4 &= 2\sigma T_2^4
  \end{align*}
  \]

- By replacing \( F_{\text{solar}} \) by \( \sigma T_e^4 \) and dividing the equations by \( \sigma \), we get:
  \[
  \begin{align*}
  T_s^4 &= 3 T_e^4 \\
  T_1^4 &= 2 T_e^4 \\
  T_2^4 &= 1 T_e^4
  \end{align*}
  \]

- The solution is then equal to:
  \[
  \begin{align*}
  T_s &= \sqrt[4]{3} T_e = 336.67 \text{ K} = 63.52^\circ \text{C} \\
  T_1 &= \sqrt[4]{2} T_e = 304.22 \text{ K} = 31.07^\circ \text{C} \\
  T_2 &= \sqrt[4]{1} T_e = 255.81 \text{ K} = -17.34^\circ \text{C}
  \end{align*}
  \]
Multi-layer model of the atmosphere

- Let \( n \) be the total number of layers and \( T_k \) be the temperature at layer \( k \).
- We have:

\[
\begin{align*}
T_s &= T_0 = \sqrt{n+1} T_e \\
T_k &= \sqrt{n+1 - k} T_e \quad \text{for } k = 0, 1, \ldots, n
\end{align*}
\]

- The temperature decreases with the layer index:

\[
\frac{\partial T_k}{\partial k} = -\frac{1}{4} (n+1-k)^{-3/4} T_e \leq 0
\]

Table 86: Layers of the Earth’s atmosphere

<table>
<thead>
<tr>
<th>Index</th>
<th>Layer</th>
<th>Altitude (in Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Troposphere</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Stratosphere</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>Mesosphere</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>Thermosphere</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>Exosphere</td>
<td>6200</td>
</tr>
</tbody>
</table>
Emissivity model of the atmosphere

Figure 160: One-layer model with atmospheric emissivity

\[ \frac{1}{4} (1 - \alpha_p) S_0 \]

\[ (1 - \varepsilon) \sigma T_s^4 \]

\[ \varepsilon \sigma T_a^4 \]
The balance at the top of the atmosphere is:

\[ F_{\text{solar}} - (1 - \varepsilon) \sigma T_s^4 - \varepsilon \sigma T_a^4 = 0 \]

The balance of the atmosphere is:

\[ \varepsilon \sigma T_s^4 - 2\varepsilon \sigma T_a^4 = 0 \]

The balance at the surface is:

\[ F_{\text{solar}} + \varepsilon \sigma T_a^4 - \sigma T_s^4 = 0 \]

The first equation is equivalent to:

\[ (1 - \varepsilon) \sigma T_s^4 + \varepsilon \sigma T_a^4 = F_{\text{solar}} = \sigma T_e^4 \]

Using the second equation, it follows that:

\[ \sigma T_e^4 = (1 - \varepsilon) \sigma T_s^4 + \frac{1}{2} \varepsilon \sigma T_s^4 = \left(1 - \frac{1}{2} \varepsilon\right) T_s^4 \]
We conclude that:

\[ T_s = \sqrt[4]{\frac{2}{2 - \varepsilon} T_e} \]

and:

\[ T_a = \sqrt[4]{\frac{1}{2 - \varepsilon} T_e} \]

Therefore, for a given temperature \( T_s^* \), we can find the unique value of the emissivity:

\[ \varepsilon^* = 2 - 2 \left( \frac{T_e}{T_s^*} \right)^4 \]

Since \( T_s^* \approx 15^\circ C \), the emissivity of the atmosphere is 78%. This model then predicts an atmospheric temperature of \(-30.8^\circ C\).
Emissivity model of the atmosphere

Figure 161: Relationship between atmospheric emissivity and temperature
More generally, the Earth’s energy balance is the sum of net shortwave radiation and net longwave radiation:

$$\mathcal{E}_{\text{net}} = \mathcal{E}_{\text{short down}} - \mathcal{E}_{\text{short up}} + \mathcal{E}_{\text{long down}} - \mathcal{E}_{\text{long up}}$$

where $\mathcal{E}_{\text{short/long down}}$ is shortwave/longwave downward radiation and $\mathcal{E}_{\text{short/long up}}$ is shortwave/longwave upward radiation.

In the previous model, we have $\mathcal{E}_{\text{short net}} = \frac{1}{4} (1 - \alpha_p) S_0$ and $\mathcal{E}_{\text{long net}} = \varepsilon \sigma T_a^4 - \sigma T_s^4$.
Emissivity model of the atmosphere

Figure 162: Earth's Energy Budget

All values are fluxes in Wm$^{-2}$ and are average values based on ten years of data.
Specific heat capacity

**Definition**

The specific heat capacity $c$ of a substance is the heat capacity $C$ of the substance divided by the mass of the substance:

$$c = \frac{C}{M} = \frac{1}{M} \frac{\Delta \mathcal{E}}{\Delta T}$$  \hspace{1cm} (1)

where:

- $\Delta \mathcal{E}$ is the amount of heat required to raise the temperature of the substance by $\Delta T$
- $M$ is the mass of the substance in kilograms (kg)
- $\Delta \mathcal{E}$ is the change in energy in joules (J)
- $\Delta T$ is the change in temperature in Kelvin (K)
- $c$ is the specific heat capacity in joules per kilogram per Kelvin ($\text{J kg}^{-1} \text{K}^{-1}$)
The specific heat capacity of water is $4186 \text{ J kg}^{-1} \text{ K}^{-1}$.

From Equation (1), we deduce that:

$$\Delta \mathcal{E} = M c \Delta T$$ (2)

In this case, the amount of energy required to raise the temperature of $1 \text{ m}^3$ of water by $10^\circ \text{C}$ is equal to:

$$\Delta \mathcal{E} = 10^3 \times 4186 \times 10 = 4186000 \text{ J}$$
The specific heat capacity of air is about $1000 \, \text{J kg}^{-1} \, \text{K}^{-1}$.

The mass of the atmosphere is $5.148 \times 10^{18} \, \text{kg}$.

The amount of energy required to raise the temperature of the atmosphere by $1^\circ\text{C}$ is:

$$\Delta \mathcal{E} = (5.148 \times 10^{18} \, \text{kg}) \times (1000 \, \text{J kg}^{-1} \, \text{K}^{-1}) \times 1 \, \text{K}$$

$$\Delta \mathcal{E} = 5.148 \times 10^{21} \, \text{J}$$
Specific heat capacity

• We can write Equation (1) as follows:

\[ \Delta T = \frac{\Delta E}{Mc} \quad (3) \]

• This equation gives the change in temperature for a change in energy. For example, adding 1000 joules of energy to one liter of water will increase its temperature by about 0.239°C:

\[ \Delta T = \frac{1000}{1 \times 4186} = 0.239 \]
Specific heat capacity

- Equations (1)–(3) can be modified by scaling the mass $M$ of the substance to standardize the required energy $\Delta \mathcal{E}$.
- A possible scaling factor can be the surface area:

\[
m = \frac{M}{\text{Area}}
\]

and we get:

\[
\Delta \mathcal{E} = mc \Delta T
\]

(4)

- For the atmosphere, we have:

\[
m = \frac{M}{\text{Area}} = \frac{5.148 \times 10^{18} \text{ kg}}{510.0645 \times 10^6 \times 10^6 \text{ m}^2} = 1.0093 \times 10^4 \text{ kg m}^{-2}
\]

because the radius $r$ of the Earth is 6371 km and the surface of the Earth is approximately $\text{Area} = 4\pi r^2 = 510.0645$ million $\text{km}^2$. 
In some climate modeling textbooks, Equation (4) is expressed using other formulas for the mass of the atmosphere per unit area. For example, \( m \) can be replaced by the product of height \( h \) and density \( \rho \), or by the ratio of pressure \( p \) to gravitational acceleration \( g \). However, all these quantities are equivalent because we have:

\[
m = h\rho = (8.2 \times 10^3 \text{ m}) \times (1.225 \text{ kg m}^{-3}) = 1.0045 \times 10^3 \text{ kg m}^{-2}
\]

and:

\[
m = \frac{p}{g} = \frac{101325 \text{ Pa}}{9.81 \text{ m s}^{-2}} = \frac{101325 \text{ m}^{-1} \text{ kg s}^{-2}}{9.81 \text{ m s}^{-2}} = 1.0329 \times 10^4 \text{ kg m}^{-2}
\]

where \( h = 8.2 \text{ km} \) is the height of the atmosphere, \( \rho = 1.225 \text{ kg m}^{-3} \) is the density of the atmosphere, \( p = 101325 \text{ Pa} \) is the standard atmospheric pressure at sea level on Earth, and \( g = 9.81 \text{ m s}^{-2} \) is the acceleration due to gravity at the Earth’s surface.

Therefore, we obtain the following equivalent formulas:

\[
mc \Delta T = h\rho c \Delta T = \frac{p}{g} c \Delta T = \Delta \mathcal{E}
\]
Radiative relaxation timescale

- We transform Equation (4) into a differential equation:

\[ mc \frac{dT}{dt} = \frac{dE}{dt} \]

- For a black body, we have:

\[ F_{\text{solar}} - \sigma T^4 = 0 \]

- We deduce that:

\[ E = F_{\text{solar}} - \sigma T^4 \]

\[ = \sigma T_e^4 - \sigma T^4 \]
Radiative relaxation timescale

- Let us assume that $T = T_e + \Delta T$. It follows that:

$$mc \frac{d\Delta T}{dt} = -4\sigma T_e^3 \Delta T$$

because:

$$\frac{\partial}{\partial \Delta T} (\sigma T_e^4 - \sigma T^4) = -4\sigma T_e^3$$

- Let $\tau_e$ be the radiative relaxation timescale defined as:

$$\tau_e = \frac{mc}{4\sigma T_e^3}$$

- We have:

$$\begin{cases} 
\frac{d\Delta T}{dt} = -\frac{1}{\tau_e} \Delta T \\
\Delta T(0) = \Delta T_0
\end{cases}$$
The solution of this ordinary differential equation is well-known and we get:

$$\Delta T (t) = \exp\left(-\frac{t}{\tau_e}\right) \Delta T_0$$

$\Delta T (t)$ gives the impulse response of an initial temperature shock of $\Delta T_0$

Because $\tau_e > 0$, we conclude that the system is stable:

$$\lim_{t \to \infty} \Delta T (t) = 0$$

We note that the equation for $\Delta T (t)$ describes an exponential survival function with parameter $\tau_e^{-1}$

We deduce that the radiative relaxation timescale $\tau_e$ is the mean lifetime
Numerical value of $\tau_e$

Using the previously obtained values for the atmosphere ($m = 1.0093 \times 10^4 \text{ kg m}^{-2}$, $c = 1000 \text{ J kg}^{-1} \text{ K}^{-1}$ and $T_e = 255.81 \text{ K}$), the radiative relaxation timescale is equal to 31 days:

$$
\tau_e = \frac{(1.0093 \times 10^4 \text{ kg m}^{-2}) \times (1000 \text{ J kg}^{-1} \text{ K}^{-1})}{4 \times (5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) \times (255.81 \text{ K})^3} \\
= 2 658 427 \text{ J W}^{-1} \\
= \frac{2 658 427 \text{ s}}{24 \times 3 600 \text{ s}} \\
= 30.77 \text{ days}
$$
Radiative relaxation timescale

Figure 163: Impulse response function for $\Delta T_0 = +1^\circ C$ and a black body
Radiative relaxation timescale

- For a gray body, we found that:

\[ \sigma T_e^4 = (2 - \varepsilon) \sigma T_a^4 = \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^4 \]

- We deduce that:

\[ mc \frac{dT_a}{dt} = \frac{d}{dt} \left( \sigma T_e^4 - (2 - \varepsilon) \sigma T_a^4 \right) \]

and:

\[ mc \frac{dT_s}{dt} = \frac{d}{dt} \left( \sigma T_e^4 - \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^4 \right) \]

- We obtain:

\[ \Delta T_a(t) = \exp \left( -\frac{t}{\tau_a} \right) \Delta T_0 \quad \text{and} \quad \Delta T_s(t) = \exp \left( -\frac{t}{\tau_s} \right) \Delta T_0 \]

where:

\[ \tau_a = \frac{mc}{4 (2 - \varepsilon) \sigma T_e^3} \quad \text{and} \quad \tau_s = \frac{mc}{2 (2 - \varepsilon) \sigma T_e^3} \]
Radiative relaxation timescale

Figure 164: Impulse response function for $\Delta T_0 = +1^\circ C$ and a gray body$^{24}$

![Graph showing impulse response function with different relaxation timescales]

$^{24}$We consider an emissivity value of 65%
“[…] Climate forcing is a change to the climate system that can be expected to change the climate. Examples would be doubling the CO₂, increasing the total solar irradiance (TSI) by 2%, introducing volcanic aerosols into the stratosphere, etc. Climate forcings are usually quantified in terms of how many \( \text{W/m}^2 \) they change the energy balance when imposed. For example, instantaneously doubling the CO₂ changes the energy balance at the top of the atmosphere by about 4 \( \text{W/m}^2 \). A feedback process is a response of the climate system to surface warming that then alters the energy balance in such a way as to change the temperature response to the forcing. A positive feedback makes the forced response bigger, and a negative feedback makes it smaller. Classic examples of positive feedbacks are ice-albedo feedback and water-vapor feedback. When it warms, ice melts, and this reduces Earth’s albedo and causes further warming. When it cools, ice grows, and this increases Earth’s albedo, causing further cooling […].” Hartmann (2016, page 294).
Mathematical definition of climate sensitivity and feedback

- We assume that some extra energy $\Delta F$ is added to the system.
- $\Delta F$ is called the radiative forcing and is measured in $\text{W/m}^2$.
- The climate response is generally measured as the change in the surface temperature $\Delta T_s$.

**Definition of the climate sensitivity**

The climate sensitivity is defined as:

$$\phi := \frac{\Delta T_s}{\Delta F}$$

Using differential notation, we have:

$$\phi := \frac{dT_s}{dF}$$

implying that:

$$dT_s = \phi dF$$
Feedback mechanism

- We assume that the perturbation \( dF \) depends on the temperature \( T_s \) and some exogenous factors \( x_i \):

\[
dF = \frac{\partial F}{\partial T_s} dT_s + \sum_{i=1}^{n} \frac{\partial F}{\partial x_i} dx_i
\]

- We conclude that:

\[
\left(1 - \phi \frac{\partial F}{\partial T_s}\right) dT_s = \phi \sum_{i=1}^{n} \frac{\partial F}{\partial x_i} dx_i
\]

- We then get a feedback mechanism, because the temperature dynamics depend on the factors \( x_i \), but also on the temperature response.
Climate feedback

- For the one-layer model with emissivity, we have $\mathcal{E} = 0$ where:

$$\mathcal{E} = F_{\text{solar}} - \left(\frac{2 - \varepsilon}{2}\right)\sigma T_s^4$$

$$= \frac{1}{4} (1 - \alpha_p) S_0 - \left(\frac{2 - \varepsilon}{2}\right)\sigma T_s^4$$

- The first-order Taylor Series expansion of $\mathcal{E} = 0$ gives:

$$\Delta \mathcal{E} = \frac{1}{4} (1 - \alpha_p) \Delta S_0 - \frac{1}{4} S_0 \Delta \alpha_p + \frac{1}{2} \sigma T_s^4 \Delta \varepsilon - 4 \left(\frac{2 - \varepsilon}{2}\right)\sigma T_s^3 \Delta T_s$$

- Remember that each perturbation $\Delta y$ depends on the temperature $T_s$ and some exogenous factors $x_i$:

$$\Delta y = \frac{\partial y}{\partial T_s} \Delta T_s + \sum_{i=1}^{n} \frac{\partial y}{\partial x_i} \Delta x_i$$
Climate feedback

We have:

\[
\Delta \mathcal{E} = \frac{1}{4} (1 - \alpha_p) \Delta S_0 - \frac{1}{4} S_0 \left( \frac{\partial \alpha_p}{\partial T_s} \Delta T_s + \sum_{i=1}^{n} \frac{\partial \alpha_p}{\partial x_i} \Delta x_i \right) + \\
\frac{1}{2} \sigma T_s^4 \left( \frac{\partial \varepsilon}{\partial T_s} \Delta T_s + \sum_{i=1}^{n} \frac{\partial \varepsilon}{\partial x_i} \Delta x_i \right) - 4 \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^3 \Delta T_s
\]

We deduce that:

\[
\Delta \mathcal{E} = \lambda \Delta T_s + \sum_{i=0}^{n} \Delta F_i
\]  

(5)

where \( \Delta F_0 := \Delta F_{\text{solar}} = \frac{1}{4} (1 - \alpha_p) \Delta S_0 \),

\[
\Delta F_i = \left( -\frac{1}{4} S_0 \frac{\partial \alpha_p}{\partial x_i} + \frac{1}{2} \sigma T_s^4 \frac{\partial \varepsilon}{\partial x_i} \right) \Delta x_i \quad \text{for } i \geq 1
\]

\[
\lambda = -\frac{1}{4} S_0 \frac{\partial \alpha_p}{\partial T_s} + \frac{1}{2} \sigma T_s^4 \frac{\partial \varepsilon}{\partial T_s} - 4 \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^3
\]
Since $\Delta \mathcal{E}$ and $\Delta F_i$ are measured in $\text{W/m}^2$ and $\Delta T_s$ is measured in Kelvin, we deduce that $\lambda$ is measured in $\text{W m}^{-2} \text{K}^{-1}$.

We transform $\Delta \mathcal{E}$ into $\Delta T_s$ by considering the heat capacity $c$ expressed in $\text{W m}^{-2} \text{K}^{-1}$ s:

$$c \frac{d\Delta T_s}{dt} = \lambda \Delta T_s + \Delta F$$  \hspace{1cm} (6)

where $\Delta F = \sum_{i=0}^{n} F_i$.

The climate feedback parameter $\lambda$ can be positive or negative, and we have the following mathematical properties:

- If $\lambda > 0$, the system is unstable;
- If $\lambda < 0$, the system is stable and the equilibrium is reached when:

$$\Delta T_s = \Delta T_s^* = - \frac{\Delta F}{\lambda} = -\phi \Delta F$$  \hspace{1cm} (7)
Climate feedback

We see that the climate feedback parameter can be decomposed into three components:

\[ \lambda = \lambda_0 + \lambda_{\alpha p} + \lambda_{\varepsilon} \]

where:

1. \( \lambda_0 \) is the Planck feedback or the black body response:
   \[ \lambda_0 = -4 \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^3 \]

2. \( \lambda_{\alpha p} \) is the surface albedo feedback:
   \[ \lambda_{\alpha p} = -\frac{1}{4} S_0 \frac{\partial \alpha_p}{\partial T_s} \]

3. \( \lambda_{\varepsilon} \) is the emissivity feedback:
   \[ \lambda_{\varepsilon} = \frac{1}{2} \sigma T_s^4 \frac{\partial \varepsilon}{\partial T_s} \]
Since $\varepsilon < 1$, the Planck feedback is negative, meaning that it stabilizes the climate and counteracts global warming. In fact, as the Earth warms, it emits more thermal radiation into space, and this increased longwave radiation acts as a natural cooling mechanism. Conversely, as the Earth cools, it emits less thermal radiation into space, and this decreased longwave radiation acts as a natural warming mechanism.

Earlier we found that $\varepsilon = 0.78$. So the estimate of $\lambda_0$ is$^{25}$:

$$\begin{align*}
\lambda_0 &= -4 \left( \frac{2 - 0.78}{2} \right) \times (5.67 \times 10^{-8}) \times (273.15 + 15)^3 \\
&= -3.310 \text{ W m}^{-2} \text{ K}^{-1}
\end{align*}$$

$^{25}$For comparison, the most recent estimate is $-3.22 \text{ W m}^{-2} \text{ K}^{-1}$, while the 90% confidence interval is from $-3.4$ to $-3.0 \text{ W m}^{-2} \text{ K}^{-1}$ (IPCC, 2021, Chapter 7, page 968)
Albedo feedback

- The sign of the surface albedo feedback depends on the sign of \( \frac{\partial \alpha_p}{\partial T_s} \).
- Two main factors play a role in determining \( \alpha_p \):
  
  "The albedo of the planet for solar radiation is primarily determined by the clouds and surface, with the main variable component of the latter being the ice/snow cover." (Hansen et al., 1984, page 166).

- If the Earth were completely covered in ice, its albedo \( \alpha_p \) would be about 84\%, meaning it would reflect most of the sunlight that hits it.
- If the Earth were covered by a dark green forest canopy, the albedo would be about 14\%.
- Cloud albedo feedback occurs because changes in cloud cover, cloud altitude or cloud properties affect the amount of reflected shortwave radiation. This feedback can be either positive or negative.
According to IPCC AR6, the value of the global surface albedo feedback is estimated to be \(0.35 \text{ W m}^{-2} \text{ K}^{-1}\), with a 90% confidence interval from 0.10 to 0.60 \(\text{W m}^{-2} \text{ K}^{-1}\).

This means that:

\[
\frac{\partial \alpha_p}{\partial T_s} = -\frac{4 \lambda \alpha_p}{S_0} = -\frac{4 \times 0.35}{1368} = -1.023 \times 10^{-3}
\]
Ice-albedo feedback modeling (Sellers model)

- If $\alpha_p = f_{\text{albedo}}(T_s)$, then $\lambda_{\alpha_p} = -\frac{1}{4} S_0 f'_{\text{albedo}}(T_s)$
- Sellers (1969) suggested:

$$\alpha_p = \begin{cases} 
  b(\phi) - 0.009 T_s & \text{if } T_s \leq 283.16 \text{ K} \\
  b(\phi) - 2.548 & \text{if } T_s \geq 283.16 \text{ K}
\end{cases}$$

where $b(\phi)$ is an estimated coefficient that depends on the latitude $\phi$

- On average we have $\overline{b(\phi)} = 2.8811$
- We deduce that:

$$\lambda_{\alpha_p} = \frac{1}{4} \times 1368 \times 0.009 = 3.078 \text{ W m}^{-2} \text{ K}^{-1}$$

- It is obvious that this positive feedback has been overestimated. The reason is that snow and sea ice cover about 10% of the Earth’s surface. Therefore, we get $\lambda_{\alpha_p} \approx 0.3 \text{ W m}^{-2} \text{ K}^{-1}$
Ice-albedo feedback modeling (Sellers model)
Budyko (1969) assumed that:

\[ \alpha_p = \begin{cases} 
\alpha_{\text{cold}} & \text{if } T_s \leq T_{\text{cold}} \\
\alpha_{\text{warm}} + (\alpha_{\text{cold}} - \alpha_{\text{warm}}) \left( \frac{T_{\text{warm}} - T_s}{T_{\text{warm}} - T_{\text{cold}}} \right)^\eta & \text{if } T_{\text{cold}} \leq T_s \leq T_{\text{warm}} \\
\alpha_{\text{warm}} & \text{if } T_s \geq T_{\text{warm}}
\end{cases} \]

where \( \eta \geq 1 \)

It follows that:

\[ \lambda_{\alpha_p} (T_s) = \frac{1}{4} \eta S_0 (\alpha_{\text{cold}} - \alpha_{\text{warm}}) \left( \frac{T_{\text{warm}} - T_s}{T_{\text{warm}} - T_{\text{cold}}} \right)^{\eta-1} \cdot 1 \{ T_{\text{cold}} \leq T_s \leq T_{\text{warm}} \} \]

Using the values \( \alpha_{\text{cold}} = 0.7, T_{\text{cold}} = 260 \text{ K}, \alpha_{\text{warm}} = 0.3, T_{\text{warm}} = 295 \text{ K} \) and \( \eta = 2 \), we get \( \lambda_{\alpha_p} (282 \text{ K}) = 2.90 \text{ W m}^{-2} \text{ K}^{-1} \), \( \lambda_{\alpha_p} (288 \text{ K}) = 1.56 \text{ W m}^{-2} \text{ K}^{-1} \) and \( \lambda_{\alpha_p} (293 \text{ K}) = 0.45 \text{ W m}^{-2} \text{ K}^{-1} \)
The emissivity feedback is the sum of several components:

\[ \lambda_\varepsilon = \lambda_{\text{water vapor}} + \lambda_{\text{lapse rate}} + \lambda_{\text{cloud longwave}} + \cdots \]
The water vapor feedback, also known as the specific humidity feedback, is the most important positive and destabilizing feedback. It can be described as follows:

“ [...] As the temperature increases, the amount of water vapor in saturated air increases. Since water vapor is the principal greenhouse gas, increasing water vapor content will increase the greenhouse effect of the atmosphere and raise the surface temperature even further.” (Hartmann, 2016, page 297).

According to IPCC AR6, the value $\lambda_{\text{water vapor}}$ of the water vapor feedback is assessed to be $1.85 \text{ W m}^{-2} \text{ K}^{-1}$.
The lapse rate mechanism describes the relationship between temperature and altitude in the atmosphere:

\[ \Gamma = -\frac{\partial T}{\partial z} \in [4 \text{ K km}^{-1}, 10 \text{ K km}^{-1}] \]

where \( z \) is the altitude in kilometers.

- On average, the lapse rate is about 6.5°C per kilometer.
- According to IPCC AR6, the average value of \( \lambda_{\text{lapse rate}} \) is 
\(-0.50 \text{ W m}^{-2} \text{ K}^{-1}\).
Emissivity feedback (clouds)

The case of cloud feedbacks is more complicated because it involves several mechanisms:

1. high cloud altitude
2. tropical high cloud amount
3. subtropical marine low cloud
4. land cloud
5. midlatitude cloud amount
6. extratropical cloud optical depth
7. Arctic cloud

According to IPCC AR6, the value $\lambda_{\text{cloud}}$ of the net cloud feedback is estimated to be $0.42 \text{ W m}^{-2} \text{ K}^{-1}$.

One of the difficulties is to decompose the cloud feedback into shortwave and longwave feedbacks, since the global surface albedo feedback already includes shortwave cloud mechanisms. Assuming that $2/3$ of the cloud feedback is longwave radiation, we get:

$$\lambda_e \approx 1.85 - 0.50 + \frac{2}{3} \times 0.42 = 1.63 \text{ W m}^{-2} \text{ K}^{-1}$$
We have:

\[
\lambda = \lambda_0 + \lambda_{\alpha_p} + \lambda_{\varepsilon} \\
= -3.31 + 0.35 + 1.63 \\
= -1.33 \text{ W m}^{-2} \text{ K}^{-1}
\]

This value is obtained with a simple one-layer model with emissivity.
Total feedback (stochastic analysis)

- We assume that $\tilde{\lambda} \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda^2)$
- As before, we decompose the feedback as a sum of individual feedbacks:
  \[ \tilde{\lambda} = \sum_{i=1}^{n} \tilde{\lambda}_i \]
  where $\tilde{\lambda}_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$
- Assuming that the individual feedbacks are independent, we have:
  \[
  \begin{align*}
  \mu_\lambda &= \sum_{i=1}^{n} \mu_i \\
  \sigma_\lambda &= \sqrt{\sum_{i=1}^{n} \sigma_i^2}
  \end{align*}
  \]
Total feedback (stochastic analysis)

<table>
<thead>
<tr>
<th>Feedback mechanism</th>
<th>$\mu_i$</th>
<th>$\sigma_i$</th>
<th>Shortwave</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-cloud altitude</td>
<td>+0.20</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Tropical marine low cloud</td>
<td>+0.25</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>Tropical anvil cloud area</td>
<td>−0.20</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Land cloud amount</td>
<td>+0.08</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Middle-latitude marine low-cloud amount</td>
<td>+0.12</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>High-latitude low-cloud optical depth</td>
<td>+0.00</td>
<td>0.10</td>
<td>✓</td>
</tr>
<tr>
<td>Planck feedback</td>
<td>−3.20</td>
<td>0.10</td>
<td>✓</td>
</tr>
<tr>
<td>Water vapor + lapse rate</td>
<td>+1.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Surface albedo</td>
<td>+0.30</td>
<td>0.15</td>
<td>✓</td>
</tr>
<tr>
<td>Total cloud</td>
<td>+0.45</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Stratospheric</td>
<td>+0.00</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Atmospheric composition changes</td>
<td>+0.00</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Climate feedback parameter</td>
<td>−1.30</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

Source: Sherwood et al. (2020, Table 1, page 18).
Figure 165: Probability density function of individual cloud feedbacks
Using the parameters of the individual cloud feedbacks, the distribution of the total cloud feedback is Gaussian with:

\[
\mu_{\text{total cloud}} = 0.20 + 0.25 - 0.20 + 0.08 + 0.12 + 0.00 \\
= 0.45 \text{ W m}^{-2} \text{ K}^{-1}
\]

and:

\[
\sigma_{\text{total cloud}} = \sqrt{0.10^2 + 0.16^2 + 0.20^2 + 0.08^2 + 0.12^2 + 0.10^2} \\
= 0.3262 \text{ W m}^{-2} \text{ K}^{-1}
\]
Total feedback (positive feedback)

**Figure 166:** Probability density function of positive feedbacks

![Probability density function of positive feedbacks](image)
Total feedback (positive feedback)

The aggregate positive feedback is the sum of the five main positive feedback components:

\[ \mu_{\text{positive}} = 1.15 + 0.30 + 0.45 + 0.00 + 0.00 = 1.90 \, \text{W} \, \text{m}^{-2} \, \text{K}^{-1} \]

and:

\[ \sigma_{\text{positive}} = \sqrt{0.15^2 + 0.15^2 + 0.33^2 + 0.10^2 + 0.15^2} = 0.4317 \, \text{W} \, \text{m}^{-2} \, \text{K}^{-1} \]
Total feedback

Figure 167: Comparison of $\tilde{\lambda}_{\text{Planck}}$ and $\tilde{\lambda}_{\text{positive}}$
Total feedback

- If we aggregate the positive feedback with the Planck feedback, the climate feedback parameter is \( \lambda \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda^2) \) where:

  \[
  \mu_\lambda = -1.30 \text{ W m}^{-2} \text{ K}^{-1} \\
  \sigma_\lambda = 0.44 \text{ W m}^{-2} \text{ K}^{-1}
  \]

- The probability that the climate feedback is positive is small:

  \[
  \Pr\{\lambda \geq 0\} = \Pr\left\{\frac{\lambda - \mu_\lambda}{\sigma_\lambda} \geq -\frac{\mu_\lambda}{\sigma_\lambda}\right\} \\
  = 1 - \Phi\left(-\frac{\mu_\lambda}{\sigma_\lambda}\right) \\
  = 1 - \Phi\left(-\frac{1.30}{0.44}\right) \\
  = 0.16\%
  \]
Total feedback

Figure 168: Probability density function of the climate feedback parameter

\[
\lambda \text{ (in } \text{W m}^{-2} \text{ K}^{-1})
\]
Definition of ECS

We recall that the definition of equilibrium is:

$$\Delta T_s^* = - \frac{\Delta F}{\lambda} = -\phi \Delta F$$

Earth’s equilibrium climate sensitivity (ECS) is the long-term global mean surface temperature change due to a specific value of radiative forcing corresponding to a doubling of CO$_2$ in the atmosphere:

$$\text{ECS} := \Delta T_{2 \times \text{CO}_2} = - \frac{\Delta F_{2 \times \text{CO}_2}}{\lambda}$$

where ECS (or $\Delta T_{2 \times \text{CO}_2}$) is the equilibrium climate sensitivity, $\lambda$ is the climate sensitivity parameter and $\Delta F_{2 \times \text{CO}_2}$ is the radiative forcing resulting from a doubling of the atmospheric carbon dioxide concentration.
We assume $\Delta F = 4 \text{ W/m}^2$ and $\lambda = -1.33 \text{ W m}^{-2} \text{ K}^{-1}$.

The equilibrium warming is then equal to $3\degree C$:

$$\Delta T^* = -\frac{4 \text{ W/m}^2}{-1.33 \text{ W m}^{-2} \text{ K}^{-1}} = 3.0075 \text{ K}$$

Remember that the dynamics of the temperature change is given by the ordinary differential equation:

$$c \frac{d\Delta T_s}{dt} = \lambda \Delta T_s + \Delta F$$

We assume that the solution has the following form:

$$\Delta T_s (t) = e^{At} B + C$$

We deduce that:

$$\frac{d\Delta T_s}{dt} = Ae^{At} B = A (\Delta T_s (t) - C)$$
The identification of the parameters results in:

\[
\begin{align*}
    A &= \frac{\lambda}{c} \\
    -AC &= \frac{\Delta F}{c} \\
    B + C &= \Delta T_s(0)
\end{align*}
\]

The solutions are then

\[A = c^{-1}\lambda,\]

\[C = -\frac{\Delta F}{Ac} = -\frac{\Delta F}{\lambda} = \Delta T_s^*\]

and:

\[B = \Delta T_s(0) - C = \Delta T_s(0) - \Delta T_s^*\]

Finally, we conclude that:

\[\Delta T_s(t) = \exp\left(-\frac{t}{\tau}\right) (\Delta T_s(0) - \Delta T_s^*) + \Delta T_s^*\]

where:

\[\tau = -\frac{c}{\lambda}\]
Again, we obtain that the equation for $\Delta T_s(t)$ describes an exponential survival function with parameter $\tau^{-1}$.

Using a specific heat capacity of $c = 4 \times 10^8 \text{ J m}^{-2} \text{ K}^{-1}$, we get:

$$\tau = -\frac{c}{\lambda} = -\frac{4 \times 10^8 \text{ J m}^{-2} \text{ K}^{-1}}{-1.33 \text{ W m}^{-2} \text{ K}^{-1}} = \frac{4 \times 10^8 \text{ W m}^{-2} \text{ K}^{-1} \text{s}}{1.33 \text{ W m}^{-2} \text{ K}^{-1}} = 3.0075 \times 10^8 \text{ s}$$

The relaxation time $\tau$ of the climate system is then equal to 3,480.92 days or 9.53 years.

Due to the exponential distribution, $\tau$ is also the mean lifetime.

Since we have:

$$\Delta T_s(t) - \Delta T_s^* = \exp\left(-\frac{t}{\tau}\right) (\Delta T_s(0) - \Delta T_s^*)$$

the half-life $t_{1/2}$ is defined as the solution of the equation

$$\exp\left(-\frac{t_{1/2}}{\tau}\right) = \frac{1}{2} \quad \text{or} \quad t_{1/2} = \tau \ln(2) = 6.61 \text{ years}$$

The equilibrium is $$\lim_{t \to \infty} \Delta T_s(t) = \Delta T_s^* = 3.0075^\circ \text{C}.$$
Figure 169: Surface temperature dynamics after a radiative forcing of 4 W/m²
($\lambda = -1.33 \text{ W m}^{-2} \text{ K}^{-1}$)
Stochastic equilibrium temperature modeling

Remark
The previous analysis assumes that the climate feedback parameter is certain. In fact, it is stochastic, which means that the equilibrium temperature and the temperature dynamics are stochastic.
If \( \tilde{\lambda} \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda^2) \), the equilibrium temperature is equal to:

\[
\Delta \tilde{T}_s^* = -\frac{\Delta F}{\tilde{\lambda}} = \frac{1}{\xi}
\]

where:

\[
\xi = -\frac{\tilde{\lambda}}{\Delta F} \sim \mathcal{N}(\mu_\xi, \sigma_\xi^2) \equiv \mathcal{N}\left(-\frac{\mu_\lambda}{\Delta F}, \frac{\sigma_\lambda^2}{\Delta F^2}\right)
\]

We deduce that \( \Delta \tilde{T}_s^* \) follows a reciprocal normal distribution:

\[
f(x) = \frac{1}{\sigma_\xi x^2 \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x^{-1} - \mu_\xi}{\sigma_\xi}\right)^2}
\]

Using the previous calibration of the climate feedback parameter \( \tilde{\lambda} \sim \mathcal{N}(-1.30, 0.44^2) \), the distribution of \( \Delta \tilde{T}_s^* \) is right skewed and has an excess of kurtosis.
Figure 170: Probability density function of the equilibrium temperature $(\Delta F = 4 \text{ W/m}^2, \mu = -1.30 \text{ W m}^{-2} \text{ K}^{-1} \text{ and } \sigma = 0.44 \text{ W m}^{-2} \text{ K}^{-1})$
The cumulative distribution function and the exceedance probability are equal to:

<table>
<thead>
<tr>
<th>Temperature $\theta$</th>
<th>2°C</th>
<th>3°C</th>
<th>4°C</th>
<th>5°C</th>
<th>7°C</th>
<th>10°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr \left{ \Delta \tilde{T}_s^* \geq \theta \right}$</td>
<td>94.26%</td>
<td>52.86%</td>
<td>24.61%</td>
<td>12.63%</td>
<td>4.73%</td>
<td>1.88%</td>
</tr>
</tbody>
</table>

The probability of observing an equilibrium temperature greater than 4°C and 7°C is close to 25% and 5%.
The mean lifetime follows a reciprocal normal distribution:

\[ \tilde{\tau} = -\frac{c}{\lambda} \sim \mathcal{R}\mathcal{N} \left( -\frac{\mu\lambda}{c}, \frac{\sigma^2}{c} \right) \]
Figure 171: Probability density function of the relaxation time
\( (\mu_\lambda = -1.30 \text{ W m}^{-2} \text{ K}^{-1} \text{ and } \sigma_\lambda = 0.44 \text{ W m}^{-2} \text{ K}^{-1}) \)
Figure 172: Monte Carlo simulation of the surface temperature dynamics after a radiative forcing of $4 \text{ W/m}^2$ ($\mu_\lambda = -1.30 \text{ W m}^{-2} \text{ K}^{-1}$ and $\sigma_\lambda = 0.44 \text{ W m}^{-2} \text{ K}^{-1}$)
According to IPCC AR6, the total anthropogenic ERF\(^\text{26}\) over the industrial era (1750-2019) is 2.72 W/m\(^2\).

The contribution of anthropogenic greenhouse gas emissions is 3.84 W/m\(^2\) and the combined effect of all radiative feedbacks (including Planck feedback) is estimated to be −1.16 W/m\(^2\).

The best estimate of the ECS is 3°C.

The likely range is 2.5°C to 4°C (with a probability of 66%), and the very likely range is 2°C to 5°C (with a probability of 90%).

\(^{26}\)Effective radiative forcing (ERF) is a measure of the change in radiative flux at the top of the atmosphere and does not include the Planck feedback.
Equilibrium climate sensitivity estimation

**Figure 173**: Anthropogenic effective radiative forcing (ERF) from 1750 to 2019 by contributing forcing agents

Source: IPCC (2021, Figure 7.6, Chapter 7, page 959).
**Table 88: Effective radiative forcing from 1750 to 2019**

<table>
<thead>
<tr>
<th>Forcing agent</th>
<th>1800</th>
<th>1850</th>
<th>1900</th>
<th>1950</th>
<th>2000</th>
<th>2010</th>
<th>2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>0.070</td>
<td>0.140</td>
<td>0.346</td>
<td>0.648</td>
<td>1.561</td>
<td>1.854</td>
<td>2.156</td>
</tr>
<tr>
<td>CH₄</td>
<td>0.025</td>
<td>0.049</td>
<td>0.119</td>
<td>0.245</td>
<td>0.509</td>
<td>0.518</td>
<td>0.544</td>
</tr>
<tr>
<td>N₂O</td>
<td>0.004</td>
<td>0.007</td>
<td>0.032</td>
<td>0.069</td>
<td>0.157</td>
<td>0.181</td>
<td>0.208</td>
</tr>
<tr>
<td>Other GHG</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.010</td>
<td>0.375</td>
<td>0.392</td>
<td>0.408</td>
</tr>
<tr>
<td>O₃</td>
<td>0.015</td>
<td>0.030</td>
<td>0.081</td>
<td>0.167</td>
<td>0.399</td>
<td>0.443</td>
<td>0.474</td>
</tr>
<tr>
<td>H₂O (stratospheric)</td>
<td>0.002</td>
<td>0.005</td>
<td>0.011</td>
<td>0.022</td>
<td>0.047</td>
<td>0.048</td>
<td>0.050</td>
</tr>
<tr>
<td>Contrails</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.005</td>
<td>0.039</td>
<td>0.044</td>
<td>0.058</td>
</tr>
<tr>
<td>Aerosol</td>
<td>−0.018</td>
<td>−0.078</td>
<td>−0.346</td>
<td>−0.708</td>
<td>−1.221</td>
<td>−1.266</td>
<td>−1.058</td>
</tr>
<tr>
<td>Black carbon on snow</td>
<td>0.002</td>
<td>0.006</td>
<td>0.020</td>
<td>0.032</td>
<td>0.069</td>
<td>0.085</td>
<td>0.080</td>
</tr>
<tr>
<td>Land use</td>
<td>−0.011</td>
<td>−0.031</td>
<td>−0.084</td>
<td>−0.144</td>
<td>−0.194</td>
<td>−0.197</td>
<td>−0.200</td>
</tr>
<tr>
<td>Total (anthropogenic)</td>
<td>0.089</td>
<td>0.128</td>
<td>0.179</td>
<td>0.346</td>
<td>1.739</td>
<td>2.103</td>
<td>2.720</td>
</tr>
<tr>
<td>Volcanic</td>
<td>0.183</td>
<td>0.194</td>
<td>0.198</td>
<td>0.182</td>
<td>0.175</td>
<td>0.137</td>
<td>0.140</td>
</tr>
<tr>
<td>Solar</td>
<td>−0.043</td>
<td>0.008</td>
<td>−0.037</td>
<td>0.057</td>
<td>0.110</td>
<td>−0.008</td>
<td>−0.022</td>
</tr>
<tr>
<td>Total (natural)</td>
<td>0.140</td>
<td>0.202</td>
<td>0.160</td>
<td>0.239</td>
<td>0.285</td>
<td>0.129</td>
<td>0.118</td>
</tr>
<tr>
<td>Total</td>
<td>0.229</td>
<td>0.330</td>
<td>0.339</td>
<td>0.585</td>
<td>2.025</td>
<td>2.232</td>
<td>2.838</td>
</tr>
</tbody>
</table>

Source: IPCC (2021, Figure 7.6, Chapter 7, page 959).
Equilibrium climate sensitivity estimation

**Figure 174:** Contribution to effective radiative forcing (ERF) (a) and global mean surface air temperature (GSAT) change (b) from component emissions between 1750 to 2019 based on CMIP6 models.

Source: IPCC (2021, Figure 6.12, Chapter 6, page 854).
Normal ratio distribution

- We assume that $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{N}(\mu_y, \sigma_y^2)$
- $Z = X / Y \sim \mathcal{N}_R \mathcal{D}(\mu_x, \sigma_x^2, \mu_y, \sigma_y^2)$
- The CDF of $Z$ is:
  \[
  F_z(z) = \Phi( - \frac{\mu_y z - \mu_x}{\sigma_x \sigma_y a(z)}, -\frac{\mu_y}{\sigma_y}, \rho_z) + \Phi( \frac{\mu_y z - \mu_x}{\sigma_x \sigma_y a(z)}, \frac{\mu_y}{\sigma_y}, \rho_z)
  \]
- The PDF of $Z$ is:
  \[
  f_z(z) = \frac{b(z)}{\sigma_x \sigma_y \sqrt{2} \pi a^3(z)} \left( 2\Phi \left( \frac{b(z)}{a(z)} \right) - 1 \right) \exp \left( \frac{b^2(z) - ca^2(z)}{2a^2(z)} \right) + \frac{1}{\sigma_x \sigma_y a^2(z) \pi} \exp \left( - \frac{c}{2} \right)
  \]
- $a(z) = \sqrt{\frac{1}{\sigma_x^2} z^2 + \frac{1}{\sigma_y^2}}$, $b(z) = \frac{\mu_x}{\sigma_x^2} z + \frac{\mu_y}{\sigma_y^2}$, $c = \frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2}$ and
- $\rho_z = \frac{z}{\sigma_x a(z)}$
Equilibrium climate sensitivity estimation

We now have all the information we need to calculate the equilibrium climate sensitivity
Equilibrium climate sensitivity estimation

- Using the parameters from Sherwood et al. (2020)
  \[ \lambda \sim \mathcal{N}(-1.30, 0.44^2) \text{ and } \Delta F_{2 \times \text{CO}_2} \sim \mathcal{N}(4.00, 0.30^2), \]
  we get:

  \[
  \text{ECS} = - \frac{\Delta F_{2 \times \text{CO}_2}}{\lambda} = - \frac{\mathcal{N}(4.00, 0.30^2)}{\mathcal{N}(-1.30, 0.44^2)} = \mathcal{NRD}(-4.00, 0.30^2, -1.30, 0.44^2)
  \]

  where \(\mathcal{NRD}\) is the normal ratio distribution

- The exceedance probability is equal to:

<table>
<thead>
<tr>
<th>Temperature (\theta)</th>
<th>2°C</th>
<th>3°C</th>
<th>4°C</th>
<th>5°C</th>
<th>7°C</th>
<th>10°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Pr}{\text{ECS} \geq \theta})</td>
<td>93.24%</td>
<td>52.79%</td>
<td>24.92%</td>
<td>12.85%</td>
<td>4.81%</td>
<td>1.91%</td>
</tr>
</tbody>
</table>

- We have a probability about 25% to observe a global warming greater than 4°C
Equilibrium climate sensitivity estimation

Figure 175: Probability density function of the equilibrium climate sensitivity ($\Delta F = +4.00 \text{ W/m}^2$, $\sigma (\Delta F) = 0.30 \text{ W/m}^2$, $\mu_\lambda = -1.30 \text{ W m}^{-2} \text{ K}^{-1}$ and $\sigma_\lambda = 0.44 \text{ W m}^{-2} \text{ K}^{-1}$)
An example of unstable equilibrium

- In the Budyko model, we have at equilibrium:

\[ \mathcal{E} = \frac{1}{4} (1 - \alpha_p(T_s)) S_0 - \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^4 = 0 \]

where:

\[ \alpha_p(T_s) = \begin{cases} 
\alpha_{\text{cold}} & \text{if } T_s \leq T_{\text{cold}} \\
\alpha_{\text{warm}} + (\alpha_{\text{cold}} - \alpha_{\text{warm}}) \left( \frac{T_{\text{warm}} - T_s}{T_{\text{warm}} - T_{\text{cold}}} \right)^\eta & \text{if } T_{\text{cold}} \leq T_s \leq T_{\text{warm}} \\
\alpha_{\text{warm}} & \text{if } T_s \geq T_{\text{warm}} 
\end{cases} \]

- The two forcing functions are

\[ \begin{aligned}
F_{\text{solar}}(T_s) &= \frac{1}{4} (1 - \alpha_p((T_s))) S_0 \\
F_{\text{blackbody}}(T_s) &= \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^4 
\end{aligned} \]
An example of unstable equilibrium

**Figure 176:** Equilibrium states of the Budyko ice-albedo model
An example of unstable equilibrium

There are three equilibrium states:

- $T_1^* = 233.38 \text{ K}$
- $T_2^* = 268.43 \text{ K}$
- $T_3^* = 288.13 \text{ K}$
An example of unstable equilibrium

The total feedback is equal to:

$$\lambda(T_s) = \lambda_0(T_s) + \lambda_{\alpha_p}(T_s)$$

where:

$$\lambda_0(T_s) = -4 \left( \frac{2 - \varepsilon}{2} \right) \sigma T_s^3$$

and:

$$\lambda_{\alpha_p}(T_s) = \frac{1}{4} \eta S_0 (\alpha_{\text{cold}} - \alpha_{\text{warm}}) \frac{(T_{\text{warm}} - T_s)^{\eta-1}}{(T_{\text{warm}} - T_{\text{cold}})^{\eta}} \cdot 1 \{T_{\text{cold}} \leq T_s \leq T_{\text{warm}}\}$$

<table>
<thead>
<tr>
<th>State</th>
<th>Temperature</th>
<th>$\lambda_0(T_s)$</th>
<th>$\lambda_{\alpha_p}(T_s)$</th>
<th>$\lambda(T_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1^*$</td>
<td>233.38 K</td>
<td>$-39.77^\circ$C</td>
<td>$-1.76$</td>
<td>0.00</td>
</tr>
<tr>
<td>$T_2^*$</td>
<td>268.43 K</td>
<td>$-4.72^\circ$C</td>
<td>$-2.68$</td>
<td>6.75</td>
</tr>
<tr>
<td>$T_3^*$</td>
<td>288.13 K</td>
<td>14.98$^\circ$C</td>
<td>$-3.31$</td>
<td>0.45</td>
</tr>
</tbody>
</table>
An example of unstable equilibrium

- $T_1^*$ and $T_3^*$ are two stable equilibria because $\lambda$ is negative, but $T_2^*$ is an unstable equilibrium.
- To illustrate this instability, we consider the dynamics of the temperature:

$$c \frac{dT_s}{dt} = F_{\text{Solar}}(T_s) - F_{\text{Blackbody}}(T_s) + \Delta F$$

where $c = 4 \times 10^8 \text{ J m}^{-2} \text{ K}^{-1}$ is the heat capacity and $\Delta F$ is the perturbation.
- The fourth panel shows a small perturbation $\Delta F = \pm 0.1 \text{ W/m}^2$. 
An example of unstable equilibrium

Figure 177: Equilibrium states of the Budyko ice-albedo model
The concept of a tipping point emerged from stability analysis and bifurcation.

The term was popularized in 2000 by Malcolm Gladwell in his book “The Tipping Point: How Little Things Can Make a Big Difference”, which explored the concept in sociological change.

In climate change, it was popularized in the late 2000s by Lenton et al. (2008).

In common parlance, a tipping point is a change...
“The term tipping point commonly refers to a critical threshold at which a tiny perturbation can qualitatively alter the state or development of a system.” (Lenton et al., 2008, page 1786).

“A climate tipping point occurs when a small change in forcing triggers a strongly nonlinear response in the internal dynamics of part of the climate system, qualitatively changing its future state.” (Lenton et al., 2011, page 201).
“Tipping points refer to critical thresholds in a system that, when exceeded, can lead to a significant change in the state of the system, often with an understanding that the change is irreversible.” (IPCC, 2018, page 262).

A tipping point is “a hypothesized critical threshold when global or regional climate changes from one stable state to another stable state. The tipping point event may be irreversible.” (IPCC, 2021, page 1463).
Bifurcation theory

Figure 178: Stable equilibria
Figure 179: Unstable equilibrium
Bifurcation theory

Figure 180: Irreversible tipping point
Bifurcation theory

- We consider the dynamical system of the form:

\[
\frac{dx}{dt} = f(x, \mu)
\]

where \( \mu \) is the parameter set

- If \( f(x, \mu) > 0 \), then \( x \) increases with time, while if \( f(x, \mu) < 0 \), then \( x \) decreases with time

- A fixed point \( x^* \) is then a value of \( x \) such that the system does not change with time, i.e. \( f(x^*, \mu) = 0 \)

- Let \( x = x^* + \Delta x \) where \( \Delta x \) is small. We have:

\[
\frac{dx}{dt} = f(x^* + \Delta x, \mu) = f(x^*, \mu) + \frac{\partial f(x^*, \mu)}{\partial x} \Delta x + O(\Delta x^2)
\]

\[
= f(x^*, \mu) + \lambda \Delta x + O(\Delta x^2)
\]

where \( \lambda \) is the feedback of the system
Since we have:

$$\frac{dx}{dt} = \frac{d (x^* + \Delta x)}{dt} = \frac{d\Delta x}{dt}$$

and:

$$f (x^* + \Delta x, \mu) \approx f (x^*, \mu) + \lambda \Delta x = \lambda \Delta x$$

We deduce that:

$$\frac{d\Delta x}{dt} = \lambda \Delta x$$

- If $\lambda > 0$, then any small perturbation of the fixed point grows exponentially (the fixed point is unstable)
- If $\lambda < 0$, then any small perturbation of the fixed point decays exponentially (the fixed point is stable)
- The relaxation timescale is equal to:

$$\tau = \frac{1}{| \partial_x f (x^*, \mu) |}$$
Bifurcation theory

Example #1

We consider the dynamical system

\[ \frac{dx}{dt} = x^2 + \mu \]
Bifurcation theory

- We have $\partial_x f(x, \mu) = 2x$
- The fixed points are solutions of the equation $x^2 = -\mu$
- If $\mu > 0$, there are no fixed points
- If $\mu = 0$, the fixed point is $x^* = 0$ and is not stable
- If $\mu < 0$, there are two fixed points. $x_1^* = -\sqrt{-\mu}$ is stable while $x_2^* = \sqrt{-\mu}$ is unstable
The stability analysis evaluates the behavior of $f(x^* + \varepsilon, \mu)$ and checks if the point $x^* + \Delta x$ converges to the point $x^*$

We can consider a second approach to stability where we evaluate the behavior of $f(x^*, \mu + \varepsilon)$, i.e. we apply the perturbation not directly to $x$ but to the control parameter

We say that the value $\mu^*$ is a bifurcation value if $f(x, \mu^*)$ is not structurally stable
In the previous example, the dynamical system exhibits a bifurcation that occurs at $\mu = 0$

This type of bifurcation is called a saddle-node or fold bifurcation, because fixed points are created or destroyed
**Bifurcation theory**

**Figure 181: Bifurcation diagram**

- **Saddle-node**
  - Stable
  - Unstable

- **Transcritical**

- **Supercritical pitchfork**

- **Subcritical pitchfork**
Bifurcation theory

- A transcritical bifurcation occurs when fixed points exchange stability for a critical value of $\mu$.
- For example, the system $\frac{dx(t)}{dt} = \mu x - x^2$ has a transcritical bifurcation at $\mu = 0$. 
Bifurcation theory

Figure 182: Bifurcation diagram

Saddle-node

Transcritical

Supercritical pitchfork

Subcritical pitchfork
Bifurcation theory

A pitchfork bifurcation occurs when the system is unchanged and exhibits symmetry when $x = -x$. There are two forms of pitchfork bifurcation:

1. In a supercritical pitchfork bifurcation, a stable fixed point becomes unstable at a critical value of $\mu$. A canonical example is:

\[
\frac{dx}{dt} = \mu x - x^3
\]

2. In a subcritical pitchfork bifurcation, an unstable fixed point becomes stable at a critical value of $\mu$. A canonical example is:

\[
\frac{dx}{dt} = \mu x + x^3 - x^5
\]
Bifurcation theory

**Figure 183: Bifurcation diagram**

- **Saddle-node**
  - Stable
  - Unstable

- **Transcritical**

- **Supercritical pitchfork**

- **Subcritical pitchfork**
Hysteresis

Definition

In bifurcation theory, hysteresis refers to a phenomenon where the behavior of the system depends on the history of its past states.
Hysteresis

Consider the previous subcritical pitchfork bifurcation and suppose the system is at equilibrium $x^* = 0$

- If we increase $\mu$, the system jumps to one of the stable branches
- If we decrease $\mu$, the system does not return to its past equilibrium, but stays on the stable branch
- However, if we decrease $\mu$ even more, the system jumps to its past equilibrium $x^* = 0$ when $\mu$ reaches $\mu^* = -0.25$
- The path of the system has formed a hysteresis loop
Bifurcation theory

Figure 184: Bifurcation diagram

Saddle-node

Transcritical

Supercritical pitchfork

Subcritical pitchfork
The Rössler model is a system of ordinary differential equations that exhibits chaotic dynamics:

\[
\begin{align*}
\frac{dx(t)}{dt} &= -(y(t) + z(t)) \\
\frac{dy(t)}{dt} &= x(t) + ay(t) \\
\frac{dz(t)}{dt} &= b + x(t)z(t) - cz(t)
\end{align*}
\]

where \(a\), \(b\) and \(c\) are three parameters that control the behavior of the system.

- The Rössler attractor is a three-dimensional surface described by \((x(t), y(t), z(t))\).
- The attractor is very sensitive to the initial conditions \((x(0), y(0), z(0))\) and the set of parameters \((a, b, c)\).
Chaos theory

Figure 185: Rössler attractor
Application to the Budyko ice-albedo model

- We use the default values $S_0 = 1368 \text{ W/m}^2$, $\varepsilon = 78\%$, $\eta = 3$, $\alpha_{\text{cold}} = 0.7$, $T_{\text{cold}} = 260 \text{ K}$, $\alpha_{\text{warm}} = 0.3$ and $T_{\text{warm}} = 295 \text{ K}$
- We have three fixed points: $T_{1}^* = 233.38 \text{ K}$, $T_{2}^* = 268.43 \text{ K}$, and $T_{3}^* = 288.13 \text{ K}$
- $T_{1}^*$ and $T_{3}^*$ are stable
- $T_{3}^*$ is unstable
Application to the Budyko ice-albedo model

- To perform the bifurcation analysis of the Budyko ice-albedo model, we consider the range \( T_s \in [200 \, \text{K}, 300 \, \text{K}] \) and divide the range into ten intervals.

- Using the bisection algorithm, we solve the equation for each interval of \( T_s \) and each value of the parameter of interest:

\[
\frac{1}{4} \left(1 - \alpha_p ((T_s))\right) S_0 - \left(\frac{2 - \varepsilon}{2}\right) \sigma T_s^4 = 0
\]

- We collect all fixed points \( \{T_1^*, T_2^*, \ldots\} \)

- For each fixed point we calculate the feedback:

\[
\lambda (T_s) = \frac{1}{4} \eta S_0 (\alpha_{\text{cold}} - \alpha_{\text{warm}}) \frac{(T_{\text{warm}} - T_s)^{\eta-1}}{(T_{\text{warm}} - T_{\text{cold}})^{\eta}}.
\]

\[
\mathbb{1} \{T_{\text{cold}} \leq T_s \leq T_{\text{warm}}\} - 4 \left(\frac{2 - \varepsilon}{2}\right) \sigma T_s^3
\]
Application to the Budyko ice-albedo model

Figure 186: Bifurcation of the Budyko ice-albedo model

- The graphs illustrate the behavior of the Budyko ice-albedo model under different conditions, showing stable and unstable states as parameters vary.

- The graphs depict how the temperature of the system changes with respect to different variables, highlighting critical points of bifurcation.

- The model is used to study tipping points in climate change, where small changes in parameters can lead to significant shifts in the system's state.

Thierry Roncalli
Course 2022-2023 in Sustainable Finance
The relaxation timescale is equal to:

$$\tau = \frac{c}{|\lambda(T_s)|}$$

where $c = 4 \times 10^8 \text{ J m}^{-2} \text{ K}^{-1}$ is the heat capacity
Application to the Budyko ice-albedo model

Figure 187: Relaxation timescale of the Budyko ice-albedo model (in years)
The AMOC is characterised by its north-south flow strength, measured in Sverdrups (Sv). In Figure 2b, the blue time series shows that a small but long-lasting overshoot allows the flow strength to recover after a strong initial warming. Therefore, the system is bistable, with a desirable upper stable branch (representing contemporary conditions) and an undesirable lower stable system state coexisting. Importantly, beyond the threshold trajectory is able to cross the unstable state (black dashed curve) above the threshold, only the AMOC 'off' state remains. Instead of an instantaneous transition, there can be another reason for tipping to be postponed, such as the collapse of ice sheets and disruption to monsoons. Other transitions may take much longer, and in these slow onset cases, ice sheet loss can be another reason for tipping to be postponed.

Figure 188: Bifurcation and overshooting tipping points

(a) Slow/fast tipping onset elements
(b) AMOC overshooting

Source: Ritchie et al. (2021, Figures 1c & 3c, pages 518 & 520.)
Climate tipping elements

Figure 189: Geographical distribution of global and regional tipping elements

Source: Armstrong McKay et al. (2022).
Table 89: Threshold, timescale, and impact estimates for the global and regional tipping elements

<table>
<thead>
<tr>
<th>Category</th>
<th>#</th>
<th>Climate tipping element</th>
<th>Tipping point</th>
<th>Threshold (°C)</th>
<th>Timescale</th>
<th>Maximum impact (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>1</td>
<td>Greenland ice sheet collapse</td>
<td>1.5°C</td>
<td>0.8-3.0</td>
<td>10 kyr</td>
<td>0.13°C (0.5-3.0)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>West Antarctic ice sheet collapse</td>
<td>1.5°C</td>
<td>1.0-3.0</td>
<td>2 kyr</td>
<td>0.05°C (1.0)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Labrador-Imringer seas collapse</td>
<td>1.8°C</td>
<td>1.1-3.8</td>
<td>10 yr</td>
<td>-0.50°C (-3.0)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>East Antarctic subglacial basins collapse</td>
<td>3.0°C</td>
<td>2.0-6.0</td>
<td>2 kyr</td>
<td>0.05°C (0.4-2.0)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Amazon rainforest dieback</td>
<td>3.5°C</td>
<td>2.0-6.0</td>
<td>100 yr</td>
<td>0.20°C (0.6-1.2)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Boreal permafrost collapse</td>
<td>4.0°C</td>
<td>3.0-6.0</td>
<td>50 yr</td>
<td>0.40°C (1.4-5.0)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>AMOC collapse</td>
<td>4.0°C</td>
<td>1.4-8.0</td>
<td>50 yr</td>
<td>-0.50°C (-4/-10)</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Arctic winter sea ice collapse</td>
<td>6.3°C</td>
<td>4.5-8.7</td>
<td>20 yr</td>
<td>0.60°C (0.6-1.2)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>East Antarctic ice sheet collapse</td>
<td>7.5°C</td>
<td>5.0-10.0</td>
<td>&gt; 10 kyr</td>
<td>0.60°C (2.0)</td>
</tr>
<tr>
<td>Regional</td>
<td>10</td>
<td>Low-latitude coral reefs die-off</td>
<td>1.5°C</td>
<td>1.0-2.0</td>
<td>10 yr</td>
<td>0.04°C (1.0-2.0)</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Boreal permafrost arrest</td>
<td>1.5°C</td>
<td>1.0-2.3</td>
<td>200 yr</td>
<td>0.08°C (0.5-100)</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>Barents sea ice arrest</td>
<td>1.6°C</td>
<td>1.5-1.7</td>
<td>25 yr</td>
<td>-0.18°C (-0.5/-2)</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>Mountain glaciers arrest</td>
<td>2.0°C</td>
<td>1.5-3.0</td>
<td>200 yr</td>
<td>-0.16°C (0.5-1.0)</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>Sahel and West African monsoon greening</td>
<td>2.8°C</td>
<td>2.0-3.5</td>
<td>50 yr</td>
<td>-0.16°C (0.5-1.0)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>Boreal forest (southern) die-off</td>
<td>4.0°C</td>
<td>1.4-5.0</td>
<td>100 yr</td>
<td>-0.18°C (-0.5/-2)</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>Boreal forest (northern) expansion</td>
<td>4.0°C</td>
<td>1.5-7.2</td>
<td>100 yr</td>
<td>0.14°C (0.5-1.0)</td>
</tr>
</tbody>
</table>

Source: Armstrong McKay et al. (2022, Table 1, page 3).
1. Greenland ice sheet

The Greenland ice sheet is the second largest ice sheet in the world. It covers 80% of the surface of Greenland. Its melting would increase sea level rise, possibly up to 7.4 meters, accelerate ocean acidification, and have a potentially positive feedback effect on climate change.
2. West Antarctic ice sheet

The West Antarctic ice sheet is a large ice sheet in Antarctica. It sits on a bedrock that is mostly below sea level and has formed a deep subglacial basin due to the weight of the ice sheet, which can be up to 4 kilometers thick in places. Its collapse could raise global sea levels, possibly up to 3 meters.
3. Labrador-Irminger seas

The Labrador and Irminger seas are located in the subpolar North Atlantic, between Canada, Greenland and Iceland. These seas are characterized by cold and salty waters that generate deep convection. This deep convection and the regulation of ocean salinity influence the circulation of the Atlantic meridional overturning circulation (AMOC). The collapse of the deep convection system in the Labrador and Irminger seas would affect the overall circulation in the North Subpolar Gyre.
4. East Antarctic subglacial basins

East Antarctic subglacial basins are large, ice-filled depressions in the bedrock adjacent to the East Antarctic ice sheet. They also serve as reservoirs for meltwater. Certain subglacial basins, such as Wilkes, Aurora, and Recovery, are more susceptible to a situation called marine ice sheet instability (MISI). As the ice shelf at the edge of the ice sheet retreats, warm ocean water can flow into the deeper basin, further destabilizing the ice shelf. This process can create a self-perpetuating cycle of ice melt and ice shelf retreat.
5. Amazon rainforest

The Amazon rainforest, also known as the Amazon jungle or Amazonia, is the largest rainforest in the world. It spans nine countries: Brazil, Peru, Colombia, Ecuador, Bolivia, Venezuela, Guyana, Suriname, and French Guiana. It contains the largest and most biodiverse area of tropical rainforest in the world. The Amazon rainforest acts as a massive carbon sink, absorbing and storing vast amounts of carbon dioxide through the process of photosynthesis. If the forest were to be subject to widespread deforestation or degradation, the stored carbon could be released back into the atmosphere, contributing to increased greenhouse gas concentrations. The Amazon also plays a critical role in the Earth’s water cycle, influencing regional and global weather patterns. Its dense vegetation effectively captures rainwater and slowly releases it into streams and rivers, helping to maintain stable water levels, prevent flooding, and provide a steady source of fresh water.
6. Boreal permafrost

Boreal permafrost is a permanently frozen layer of soil and rock that underlies much of the world’s boreal forest. It is found in Siberia, Alaska, Northern Canada, and the Tibetan Plateau. Permafrost forms when the soil temperature remains below 0°C for at least two consecutive years. Boreal permafrost contains large amounts of organic carbon stored in the form of dead plant material that could not decompose due to the cold temperatures. It is also one of the largest reservoirs of methane. Rising temperatures may cause the boreal permafrost to thaw, releasing large amounts of carbon and methane into the atmosphere.
7. AMOC

The Atlantic meridional overturning circulation (AMOC) is a large, complex system of ocean currents that transports warm water from the tropics to the North Atlantic and cold water from the North Atlantic to the subtropics. It is also known as the Gulf Stream system. A weakening of the AMOC could have complex and regionally specific effects on temperatures. On a global scale, it could result in less warm water reaching higher latitudes, leading to cooler sea surface temperatures in the North Atlantic and warmer temperatures in the Southern Hemisphere.
8. Arctic winter sea ice

Arctic winter sea ice is the maximum extent of sea ice that forms in the Arctic Ocean during the winter months. It helps to regulate global temperatures by reflecting sunlight back into space. This albedo reflection helps to cool the Arctic. As sea ice melts, more sunlight is absorbed by the ocean, causing a further warming trend\(^a\). However, the impact of the albedo effect remains controversial.

\(^a\)Moreover, the warming Arctic has the potential to release methane from permafrost.
9. East Antarctic ice sheet

The East Antarctic ice sheet is the largest and thickest ice sheet on Earth. A complete collapse would raise the global sea levels by 50 meters. However, the East Antarctic ice sheet is generally considered to be more stable than the West Antarctic ice sheet, due to its higher elevation and more remote location.
10. Low-latitude coral reefs

Low-latitude coral reefs occur in the Atlantic, Indian, and Pacific Oceans, most notably in the Philippines, Indonesia, and Australia. They require warm, sunny weather and unpolluted water. Therefore, coral reefs can be affected by climate change, although their impact on climate change is more limited.
11. Boreal permafrost

We have already seen that the boreal permafrost is a global tipping element, but it is also a regional tipping element. In fact, an abrupt thaw of the boreal permafrost would have devastating consequences for the region, affecting infrastructure (roads, buildings, transportation), the environment (flooding, forests, vegetation), and living conditions and health.
12. Barents sea ice

Barents sea ice is found in the Barents sea, an arm of the Arctic Ocean between Norway and Russia. The sea ice forms during the winter months and melts during the summer months. This regional tipping element is strongly related to two global tipping elements: Labrador-Irminger seas and AMOC.
13. Mountain glaciers

Mountain glaciers are large masses of ice that form on mountains at high altitudes. They are formed from compacted snow that has accumulated over many years. The melting of mountain glaciers would have a major regional impact on human life.
14. Sahel and West African monsoon

The West African monsoon is a seasonal wind pattern that affects the Sahel, bringing moisture from the Atlantic Ocean during the rainy season and drying out the region during the dry season. It is responsible for the region’s agriculture and supports the livelihoods of millions of people. Changes in rainfall can affect vegetation, agriculture and people.
15. Boreal forest (southern)

The boreal forest, also known as the taiga, is a biome that surrounds the Arctic region. Countries with significant areas of boreal forest include Canada, Russia, Sweden, Norway and Finland. The southern edge of the boreal forest is the boundary between the boreal forest and temperate forests or grasslands. The risk could be an abrupt die-off.
16. Boreal forest (northern)

The northern edge of the boreal forest is typically found at higher latitudes, closer to the Arctic Circle. The change could be an abrupt expansion into a tundra forest characterised by treeless landscapes and permafrost.
Cascading tipping points and climate domino effects

**Figure 190:** Double fold bifurcation

\[ \left( -\sqrt{\frac{4}{27}}, +\sqrt{\frac{1}{3}} \right) \]

Source: Klose et al. (2020, Figure 1, page 3).
Scientific evidence of global warming
From the Holocene to the Anthropocene?
The physics of climate change
Energy balance models
Climate sensitivity and feedback
Tipping points

Cascading tipping points and climate domino effects

**Figure 191:** Convergence to the equilibrium

![Graph showing convergence to equilibrium](image)

- Blue line: \( \tau_i = 1 \) year
- Red line: \( \tau_i = 10 \) years
- Green line: \( \tau_i = 50 \) years

\[ x_i(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\tau_i} & 0 \leq t < \tau_i \\ 1 & t \geq \tau_i \end{cases} \]
Figure 192: Master-slave bifurcation

Source: Klose et al. (2020, Figure 2, page 6).
Cascading tipping points and climate domino effects

**Figure 193:** Equilibria of the slave subsystem

- **Stable lower branch**
- **Stable upper branch**
- **Unstable branch**
Scientific evidence of global warming
From the Holocene to the Anthropocene?
The physics of climate change
Energy balance models
Climate sensitivity and feedback
Tipping points

Cascading tipping points and climate domino effects

Figure 194: Interactions between climate tipping elements and their roles in tipping cascades

Source: Wunderling et al. (2021, Figure 1, page 603).
Course 2023-2024 in Sustainable Finance
Lecture 9. The Ecosystem of Climate Change

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2024

---

The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- **Lecture 9: The Ecosystem of Climate Change**
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
The United Nations Environment Programme (UNEP) and the World Meteorological Organization (WMO) established the Intergovernmental Panel on Climate Change in 1988.

The members of the IPCC are then governments (95 as of January 2024), not scientists, academic institutions, or NGOs.

Its original mandate was to “prepare a comprehensive review and recommendations on the state of knowledge of the science of climate change, the social and economic impacts of climate change, and possible response strategies and elements for inclusion in a possible future international climate convention.”
Working Group I (WGI)
Assesses the scientific aspects of the climate system that underpin past, present and future climate change.

Working Group II (WGII)
Assesses climate change impacts, adaptation options and vulnerabilities on human and natural systems.

Working Group III (WGIII)
Focuses on mitigation of climate change, assessment of methods to limit and reduce greenhouse gas emissions, and removal of greenhouse gases from the atmosphere.
Table 90: List of IPCC Assessment Reports

<table>
<thead>
<tr>
<th>Date</th>
<th>Symbol</th>
<th>Title of the report</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>FAR/AR1</td>
<td>First IPCC Assessment Report</td>
</tr>
<tr>
<td>1995</td>
<td>SAR/AR2</td>
<td>Second Assessment Report</td>
</tr>
<tr>
<td>2001</td>
<td>TAR/AR3</td>
<td>Third Assessment Report</td>
</tr>
<tr>
<td>2007</td>
<td>AR4</td>
<td>Fourth Assessment Report</td>
</tr>
<tr>
<td>2014</td>
<td>AR5</td>
<td>Fifth Assessment Report</td>
</tr>
<tr>
<td>2023</td>
<td>AR6</td>
<td>Sixth Assessment Report</td>
</tr>
</tbody>
</table>
The three main conclusions of the AR6 Synthesis Report are:

1. Unequivocal human influence
   The report reaffirms that human activities are unequivocally causing climate change. The report states that global surface temperatures have reached 1.1°C above pre-industrial levels.

2. Widespread and intensifying impacts
   The report emphasizes that climate change is already having a profound impact on the planet, affecting ecosystems, human health, and infrastructure. These impacts are projected to worsen in the future, even with moderate warming.

3. Urgency of mitigation and adaptation
   The report underscores the urgency of taking action to address climate change. Limiting global warming to 1.5°C will require rapid and far-reaching changes in energy systems, food production and land management. These changes are possible and would bring significant environmental and human health benefits.
### Table 91: Some well-known climate research institutions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Climate research institutions</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>AARI</td>
<td>Arctic and Antarctic Research Institute</td>
<td>Russia</td>
</tr>
<tr>
<td>AWI</td>
<td>Alfred Wegener Institute for Polar and Marine Research</td>
<td>Germany</td>
</tr>
<tr>
<td>CICERO</td>
<td>Center for International Climate and Environmental Research</td>
<td>Norway</td>
</tr>
<tr>
<td>CIMA</td>
<td>Centro de Investigaciones del Mar y la Atmósfera</td>
<td>Argentina</td>
</tr>
<tr>
<td>CIRED</td>
<td>Centre international de recherche sur l’environnement et le développement</td>
<td>France</td>
</tr>
<tr>
<td>CMCC</td>
<td>Euro-Mediterranean Center on Climate Change</td>
<td>Italy</td>
</tr>
<tr>
<td>GISS</td>
<td>NASA Goddard Institute for Space Studies</td>
<td>US</td>
</tr>
<tr>
<td>IAP</td>
<td>Institute of Atmospheric Physics Chinese Academy of Sciences</td>
<td>China</td>
</tr>
<tr>
<td>IIASA</td>
<td>International Institute for Applied Systems Analysis</td>
<td>Austria</td>
</tr>
<tr>
<td>IPSL</td>
<td>Institut Pierre-Simon Laplace (LSCE)</td>
<td>France</td>
</tr>
<tr>
<td>INECC</td>
<td>National Institute of Ecology and Climate Change</td>
<td>Mexico</td>
</tr>
<tr>
<td>JAMSTEC</td>
<td>Japan Agency for Marine-Earth Science and Technology</td>
<td>Japan</td>
</tr>
<tr>
<td>MOHC</td>
<td>Met Office Hadley Centre for Climate Science and Services</td>
<td>UK</td>
</tr>
<tr>
<td>NCAR</td>
<td>National Center for Atmospheric Research</td>
<td>US</td>
</tr>
<tr>
<td>NERC</td>
<td>Natural Environment Research Council</td>
<td>UK</td>
</tr>
<tr>
<td>NIES</td>
<td>National Institute for Environmental Studies</td>
<td>Japan</td>
</tr>
<tr>
<td>NOAA</td>
<td>National Oceanic and Atmospheric Administration</td>
<td>US</td>
</tr>
<tr>
<td>PBL</td>
<td>PBL Netherlands Environmental Assessment Agency</td>
<td>Netherlands</td>
</tr>
<tr>
<td>PCMDI</td>
<td>Program for Climate Model Diagnosis &amp; Intercomparison (LLNL)</td>
<td>US</td>
</tr>
<tr>
<td>PIK</td>
<td>Potsdam Institute for Climate Impact Research</td>
<td>Germany</td>
</tr>
<tr>
<td>PNNL</td>
<td>Pacific Northwest National Laboratory</td>
<td>US</td>
</tr>
<tr>
<td>REKLIM</td>
<td>Helmholtz Climate Initiative Regional Climate</td>
<td>Germany</td>
</tr>
<tr>
<td>SEI</td>
<td>Stockholm Environment Institute</td>
<td>Sweden</td>
</tr>
<tr>
<td>SIO</td>
<td>Scripps Institution of Oceanography</td>
<td>US</td>
</tr>
</tbody>
</table>
National Center for Atmospheric Research (NCAR)

The National Center for Atmospheric Research is a federally funded research and development center managed by the University Corporation for Atmospheric Research (UCAR) and funded by the National Science Foundation (NSF). NCAR was established in 1960, and its founding director was Walter Orr Roberts. Its headquarters are located in Boulder, Colorado. NCAR’s annual budget was $173 million in 2017. The center is organized into eight laboratories with approximately 1500 members. Notable scientists include or were Guy Brasseur, Clara Deser, Brant Foote, Marika M. Holland, Paul R. Julian, Jean-Francois Lamarque, David M. Lawrence, Gerald A. Meehl, Joanne Simpson, Kevin E. Trenberth, Warren M. Washington, and Tom M. L. Wigley.
Potsdam Institute for Climate Impact Research (PIK)

The Potsdam Institute for Climate Impact Research is a government-funded research institute in Potsdam, Germany. It was founded in 1991 by Hans Joachim Schellnhuber, who is now Director of IIASA. With a network of about 400 researchers, PIK’s mission is to address scientific issues in the fields of climate risks and sustainable development. The current directors of the institute are Ottmar Edenhofer, who also serves as chief economist, and Johan Rockström, former director of the Stockholm Resilience Center. In 2022, the Institute received about 13.3 million euros in institutional funding. Additional project funding from external sources amounted to approximately 18.2 million euros. The institute boasts many renowned researchers, including Elmar Kriegler, Christoph Müller, Ottmar Edenhofer, Alexander Popp, Stefan Rahmstorf and Johan Rockström.
The International Institute for Applied Systems Analysis is an independent international research institute located in Laxenburg near Vienna, Austria. IIASA was founded by a charter signed on October 4, 1972 by representatives of the Soviet Union, the United States, and ten other countries from the Eastern and Western blocs. IIASA brings together experts from various fields to study complex issues such as climate change, energy, and sustainable development. In 2022, IIASA's annual budget was 24.4 million euros, of which just under half came from the Institute's national and regional member organizations (Austria, Brazil, China, Egypt, Finland, Germany, India, Iran, Israel, Japan, South Korea, Norway, Russia, Slovakia, Sub-Saharan Africa, Sweden, Ukraine, United Kingdom, United States, Vietnam). IIASA has about 500 researchers from 50 countries. Since December 2023, the Director General is Hans Joachim Schellnhuber. Among the researchers who work, have worked or have visited the research center, we can mention George Dantzig, Shinichiro Fujimori, Petr Havlik, Leonid Kantorovich, Tjalling Koopmans, Nebojsa Nakicenovic, William D. Nordhaus, Michael Obersteiner, Howard Raiffa, Keywan Riahi, Joeri Rogelj and Thomas Schelling.
National Oceanic and Atmospheric Administration (NOAA)

The National Oceanic and Atmospheric Administration is an agency of the United States Department of Commerce (DOC) responsible for monitoring and managing the nation’s weather, climate, and oceans. NOAA was established in 1970. The scope of NOAA is vast: weather forecasting, oceanography, fisheries management, satellite operations. As a result, it employs 12,000 people worldwide, while the number of NOAA scientists and engineers is about 6,500. The Office of Oceanic and Atmospheric Research (OAR) is the primary research arm of NOAA. It is responsible for conducting a wide range of research on the Earth’s atmosphere, oceans, and coasts. One of OAR’s goals is to understand the causes and effects of climate change. The most prominent affiliated research center is the Geophysical Fluid Dynamics Laboratory (GFDL), a joint program of Princeton University and NOAA. GFDL researchers include Thomas L. Delworth, Larry W. Horowitz, Thomas R. Knutson, Vaishali Naik, and Venkatachalam Ramaswamy.
The Institut Pierre-Simon Laplace is a French university research institute that brings together 10 laboratories with about 1500 members. The institute was founded in 1991 by Gérard Mégie, and one of its directors was Jean Jouzel from 2001 to 2008. The 10 laboratories are (1) the Centre d’Enseignement et de Recherche en Environnement Atmosphérique (CEREA), (2) Géosciences Paris-Sud (GEOPS), (3) the Laboratoire Atmosphères, Milieux, Observations spatiales (LATMOS), (4) the team TASQ of the Laboratoire d’Études du Rayonnement et de la Matière en Astrophysique et Atmosphères (LERMA), (5) the Laboratoire Inter-universitaire des Systèmes Atmosphériques (LISA), (6) the Laboratoire de Météorologie Dynamique (LMD), (7) the Laboratoire d’Océanographie et du Climat Expérimentation et Approches Numériques (LOCEAN), (8) the Laboratoire des Sciences du Climat et de l’Environnement (LSCE), (9) the research center Milieux Environnementaux, Transferts et Interactions dans les hydrosystèmes et les Sol (METIS), and (10) the team Surface & Réservoirs of the Laboratoire de Géologie de l’ENS. IPSL is placed under the supervision of Centre National de la Recherche Scientifique (CNRS), Sorbonne Université (SU), Université Versailles Saint-Quentin (UVSQ), École Polytechnique, Commissariat à l’Énergie Atomique et aux Énergies Alternatives (CEA), Institut de Recherche pour le Développement (IRD) and École Nationale des Ponts et Chaussées (ENPC). Researchers include Sandrine Bony, Laurent Bopp, Olivier Boucher, Pascale Braconnot, Philippe Ciais, Jean Jouzel, Pierre Friedlingstein, Valérie Masson-Delmotte, Robert Vautard, and Nicolas Viovy.
### Table 92: Top 30 climate scientists on the Reuters Hot List

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Gender</th>
<th>Institution</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Keywan Riahi</td>
<td>M</td>
<td>International Institute for Applied Systems Analysis</td>
<td>Austria</td>
</tr>
<tr>
<td>2</td>
<td>Anthony A. Leiserowitz</td>
<td>M</td>
<td>Yale University</td>
<td>United States</td>
</tr>
<tr>
<td>3</td>
<td>Pierre Friedlingstein</td>
<td>M</td>
<td>University of Exeter</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>4</td>
<td>Detlef Peter Van Vuuren</td>
<td>M</td>
<td>Utrecht University</td>
<td>Netherlands</td>
</tr>
<tr>
<td>5</td>
<td>James E. Hansen</td>
<td>M</td>
<td>Columbia University</td>
<td>United States</td>
</tr>
<tr>
<td>6</td>
<td>Petr Havlik</td>
<td>M</td>
<td>International Institute for Applied Systems Analysis</td>
<td>Austria</td>
</tr>
<tr>
<td>7</td>
<td>Edward Wile Maibach</td>
<td>M</td>
<td>George Mason University</td>
<td>United States</td>
</tr>
<tr>
<td>8</td>
<td>Josep G. Canadell</td>
<td>M</td>
<td>Commonwealth Scientific and Industrial Research Organisation</td>
<td>Australia</td>
</tr>
<tr>
<td>9</td>
<td>Sonia Isabelle Seneviratne</td>
<td>F</td>
<td>ETH Zurich</td>
<td>Switzerland</td>
</tr>
<tr>
<td>10</td>
<td>Mario Herrero</td>
<td>M</td>
<td>Commonwealth Scientific and Industrial Research Organisation</td>
<td>Australia</td>
</tr>
<tr>
<td>11</td>
<td>David B. Lobell</td>
<td>M</td>
<td>Stanford University</td>
<td>United States</td>
</tr>
<tr>
<td>12</td>
<td>Carlos Manuel Duarte</td>
<td>M</td>
<td>King Abdullah University of Science and Technology</td>
<td>Saudi Arabia</td>
</tr>
<tr>
<td>13</td>
<td>Kevin E. Trenberth</td>
<td>M</td>
<td>National Center for Atmospheric Research</td>
<td>United States</td>
</tr>
<tr>
<td>14</td>
<td>Stephen A. Sitch</td>
<td>M</td>
<td>University of Exeter</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>14</td>
<td>Glen P. Peters</td>
<td>M</td>
<td>Center for International Climate and Environmental Research</td>
<td>Norway</td>
</tr>
<tr>
<td>16</td>
<td>Ove I. Hoegh-Guldberg</td>
<td>M</td>
<td>University of Queensland</td>
<td>Australia</td>
</tr>
<tr>
<td>17</td>
<td>Richard Arthur Betts</td>
<td>M</td>
<td>Met Office</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>18</td>
<td>Michael G. Oppenheimer</td>
<td>M</td>
<td>Princeton University</td>
<td>United States</td>
</tr>
<tr>
<td>18</td>
<td>William Neil Adger</td>
<td>M</td>
<td>University of Exeter</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>20</td>
<td>William Wai Lung Cheung</td>
<td>M</td>
<td>University of British Columbia</td>
<td>Canada</td>
</tr>
</tbody>
</table>
### Table 93: Top 30 climate scientists on the Reuters Hot List

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Gender</th>
<th>Institution</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Christopher B. Field</td>
<td>M</td>
<td>Stanford University</td>
<td>United States</td>
</tr>
<tr>
<td>23</td>
<td>Shinichiro Fujimori</td>
<td>M</td>
<td>Kyoto University</td>
<td>Japan</td>
</tr>
<tr>
<td>23</td>
<td>Elmar Kriegler</td>
<td>M</td>
<td>Potsdam Institute for Climate Impact Research</td>
<td>Germany</td>
</tr>
<tr>
<td>25</td>
<td>Yadvinder Singh Malhi</td>
<td>M</td>
<td>University of Oxford</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>26</td>
<td>Ken Caldeira</td>
<td>M</td>
<td>Carnegie Institution for Science’s Department of Global Ecology</td>
<td>United States</td>
</tr>
<tr>
<td>27</td>
<td>Chris D. Thomas</td>
<td>M</td>
<td>University of York</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>28</td>
<td>Stéphane Hallegatte</td>
<td>M</td>
<td>World Bank</td>
<td>United States</td>
</tr>
<tr>
<td>28</td>
<td>Andy P. Haines</td>
<td>M</td>
<td>London School of Hygiene &amp; Tropical Medicine</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>30</td>
<td>Michael Obersteiner</td>
<td>M</td>
<td>International Institute for Applied Systems Analysis</td>
<td>Austria</td>
</tr>
<tr>
<td>40</td>
<td>Philippe Ciais</td>
<td>M</td>
<td>Laboratoire des Sciences du Climat et de l'Environnement</td>
<td>France</td>
</tr>
<tr>
<td>75</td>
<td>Pete Smith</td>
<td>M</td>
<td>University of Aberdeen</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>164</td>
<td>Richard S. J. Tol</td>
<td>M</td>
<td>VU Amsterdam</td>
<td>Netherlands</td>
</tr>
<tr>
<td>173</td>
<td>William D. Nordhaus</td>
<td>M</td>
<td>Yale University</td>
<td>United States</td>
</tr>
<tr>
<td>240</td>
<td>Phil D. Jones</td>
<td>M</td>
<td>University of East Anglia</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>338</td>
<td>Filippo Giorgi</td>
<td>M</td>
<td>International Centre for Theoretical Physics</td>
<td>Italy</td>
</tr>
<tr>
<td>639</td>
<td>Klaus Hasselmann</td>
<td>M</td>
<td>Max Planck Institute for Meteorology</td>
<td>Germany</td>
</tr>
<tr>
<td>755</td>
<td>Syukuro Manabe</td>
<td>M</td>
<td>Princeton University</td>
<td>United States</td>
</tr>
</tbody>
</table>

The top five names of the Reuters Hot List are:

- **Keywan Riahi**, Director of the Energy, Climate and Environment (ECE) Program at IIASA and one of the principal developers of the Representative Concentration Pathway (RCP) and Shared Socio-economic Pathway (SSP) concepts

- **Anthony A. Leiserowitz**, Professor at Yale University, who studies public perceptions of climate change

- **Pierre Friedlingstein**, Chair in Mathematical Modeling of the Climate System at the University of Exeter, coordinator of the annual publication of the Global Carbon Budget

- **Detlef P. Van Vuuren**: Professor at Utrecht University, Project Leader of the IMAGE Integrated Assessment Team at PBL Netherlands Environmental Assessment Agency, one of the main developers of the RCP and SSP concepts

- **James E. Hansen**: one of the world’s most influential climate scientists and the director of NASA Goddard Institute for Space Studies from 1981 to 2013
### Table 94: Top 20 climate research institutions on the Reuters Hot List

<table>
<thead>
<tr>
<th>Institution</th>
<th>Count</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potsdam Institute for Climate Impact Research (PIK)</td>
<td>14</td>
<td>Germany</td>
</tr>
<tr>
<td>University of Reading</td>
<td>13</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>Institute of Atmospheric Physics (CAS)</td>
<td>13</td>
<td>China</td>
</tr>
<tr>
<td>Utrecht University</td>
<td>12</td>
<td>Netherlands</td>
</tr>
<tr>
<td>Met Office (MOHC)</td>
<td>12</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>National Center for Atmospheric Research (NCAR)</td>
<td>11</td>
<td>United States</td>
</tr>
<tr>
<td>Columbia University</td>
<td>10</td>
<td>United States</td>
</tr>
<tr>
<td>ETH Zurich</td>
<td>10</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Laboratoire des Sciences du Climat et de l’Environnement (IPSL)</td>
<td>10</td>
<td>France</td>
</tr>
<tr>
<td>International Institute for Applied Systems Analysis (IIASA)</td>
<td>9</td>
<td>Austria</td>
</tr>
<tr>
<td>University of Melbourne</td>
<td>9</td>
<td>Australia</td>
</tr>
<tr>
<td>University of Leeds</td>
<td>9</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>Geophysical Fluid Dynamics Laboratory (NOAA)</td>
<td>9</td>
<td>United States</td>
</tr>
<tr>
<td>Max Planck Institute for Meteorology (MPIM)</td>
<td>9</td>
<td>Germany</td>
</tr>
<tr>
<td>Pacific Northwest National Laboratory (PNNL)</td>
<td>8</td>
<td>United States</td>
</tr>
<tr>
<td>Lamont-Doherty Earth Observatory (Columbia University)</td>
<td>8</td>
<td>United States</td>
</tr>
<tr>
<td>Massachusetts Institute of Technology</td>
<td>8</td>
<td>United States</td>
</tr>
<tr>
<td>University of Washington</td>
<td>8</td>
<td>United States</td>
</tr>
<tr>
<td>Wageningen University &amp; Research</td>
<td>8</td>
<td>Netherlands</td>
</tr>
<tr>
<td>University of Tokyo</td>
<td>8</td>
<td>Japan</td>
</tr>
<tr>
<td>University of Bremen</td>
<td>8</td>
<td>Germany</td>
</tr>
</tbody>
</table>
**Table 95: Top 15 country on the Reuters Hot List**

<table>
<thead>
<tr>
<th>Country</th>
<th>Count</th>
<th>Frequency</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>292</td>
<td>29.3%</td>
<td>29.3%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>112</td>
<td>11.3%</td>
<td>40.6%</td>
</tr>
<tr>
<td>Germany</td>
<td>91</td>
<td>9.1%</td>
<td>49.7%</td>
</tr>
<tr>
<td>China</td>
<td>87</td>
<td>8.7%</td>
<td>58.5%</td>
</tr>
<tr>
<td>Australia</td>
<td>74</td>
<td>7.4%</td>
<td>65.9%</td>
</tr>
<tr>
<td>France</td>
<td>40</td>
<td>4.0%</td>
<td>69.9%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>39</td>
<td>3.9%</td>
<td>73.9%</td>
</tr>
<tr>
<td>Canada</td>
<td>37</td>
<td>3.7%</td>
<td>77.6%</td>
</tr>
<tr>
<td>Japan</td>
<td>30</td>
<td>3.0%</td>
<td>80.6%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>28</td>
<td>2.8%</td>
<td>83.4%</td>
</tr>
<tr>
<td>Spain</td>
<td>22</td>
<td>2.2%</td>
<td>85.6%</td>
</tr>
<tr>
<td>Italy</td>
<td>20</td>
<td>2.0%</td>
<td>87.6%</td>
</tr>
<tr>
<td>Norway</td>
<td>13</td>
<td>1.3%</td>
<td>88.9%</td>
</tr>
<tr>
<td>Denmark</td>
<td>12</td>
<td>1.2%</td>
<td>90.2%</td>
</tr>
<tr>
<td>South Korea</td>
<td>12</td>
<td>1.2%</td>
<td>91.4%</td>
</tr>
<tr>
<td>Austria</td>
<td>10</td>
<td>1.0%</td>
<td>92.4%</td>
</tr>
</tbody>
</table>
Figure 195: Location of the top 1000 climate scientists
### Table 96: Top 30 leading institutions (Nature index, 2016 ranking)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Academy of Sciences (CAS), China</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Helmholtz Association of German Research Centres, Germany</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>French National Centre for Scientific Research (CNRS), France</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>National Aeronautics and Space Administration (NASA), United States of America (USA)</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Swiss Federal Institute of Technology Zurich (ETH Zurich), Switzerland</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>U.S. Geological Survey (USGS), United States of America (USA)</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td>12</td>
<td>7</td>
<td>9</td>
<td>17</td>
<td>31</td>
</tr>
<tr>
<td>University of Colorado Boulder (CU-Boulder), United States of America (USA)</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>14</td>
<td>20</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>University of California, Berkeley (UC Berkeley), United States of America (USA)</td>
<td>8</td>
<td>19</td>
<td>12</td>
<td>9</td>
<td>17</td>
<td>18</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>Stanford University, United States of America (USA)</td>
<td>8</td>
<td>8</td>
<td>13</td>
<td>22</td>
<td>9</td>
<td>15</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>California Institute of Technology (Caltech), United States of America (USA)</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>7</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>National Oceanic and Atmospheric Administration (NOAA), United States of America (USA)</td>
<td>11</td>
<td>7</td>
<td>7</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Columbia University in the City of New York (CU), United States of America (USA)</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>11</td>
<td>11</td>
<td>13</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>University of Washington (UW), United States of America (USA)</td>
<td>13</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>17</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>University of California, San Diego (UC San Diego), United States of America (USA)</td>
<td>14</td>
<td>14</td>
<td>18</td>
<td>26</td>
<td>32</td>
<td>24</td>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td>The University of Tokyo (UTokyo), Japan</td>
<td>15</td>
<td>15</td>
<td>20</td>
<td>23</td>
<td>20</td>
<td>31</td>
<td>29</td>
<td>50</td>
</tr>
<tr>
<td>Peking University (PKU), China</td>
<td>16</td>
<td>16</td>
<td>19</td>
<td>13</td>
<td>19</td>
<td>11</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>The University of Texas at Austin (UT Austin), United States of America (USA)</td>
<td>17</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>32</td>
<td>27</td>
</tr>
<tr>
<td>The University of Queensland (UQ), Australia</td>
<td>18</td>
<td>57</td>
<td>62</td>
<td>37</td>
<td>43</td>
<td>26</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Nanjing University (NJU), China</td>
<td>19</td>
<td>18</td>
<td>11</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Massachusetts Institute of Technology (MIT), United States of America (USA)</td>
<td>20</td>
<td>21</td>
<td>17</td>
<td>20</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>University of Minnesota (UMN), United States of America (USA)</td>
<td>21</td>
<td>22</td>
<td>27</td>
<td>30</td>
<td>39</td>
<td>41</td>
<td>42</td>
<td>52</td>
</tr>
<tr>
<td>University of California, Los Angeles (UCLA), United States of America (USA)</td>
<td>22</td>
<td>17</td>
<td>15</td>
<td>21</td>
<td>29</td>
<td>28</td>
<td>26</td>
<td>23</td>
</tr>
<tr>
<td>University of Maryland, College Park (UMCP), United States of America (USA)</td>
<td>23</td>
<td>32</td>
<td>55</td>
<td>40</td>
<td>53</td>
<td>66</td>
<td>77</td>
<td>80</td>
</tr>
<tr>
<td>Woods Hole Oceanographic Institution (WHOI), United States of America (USA)</td>
<td>24</td>
<td>34</td>
<td>50</td>
<td>30</td>
<td>37</td>
<td>51</td>
<td>56</td>
<td>67</td>
</tr>
<tr>
<td>The Pennsylvania State University (Penn State), United States of America (USA)</td>
<td>25</td>
<td>27</td>
<td>29</td>
<td>25</td>
<td>25</td>
<td>30</td>
<td>28</td>
<td>56</td>
</tr>
<tr>
<td>University of Michigan (U-M), United States of America (USA)</td>
<td>26</td>
<td>20</td>
<td>23</td>
<td>35</td>
<td>27</td>
<td>35</td>
<td>50</td>
<td>73</td>
</tr>
<tr>
<td>University of Toronto (U of T), Canada</td>
<td>27</td>
<td>38</td>
<td>34</td>
<td>29</td>
<td>34</td>
<td>34</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Georgia Institute of Technology (Georgia Tech), United States of America (USA)</td>
<td>28</td>
<td>50</td>
<td>27</td>
<td>72</td>
<td>55</td>
<td>76</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>National Institute of Geophysics and Volcanology (INGV), Italy</td>
<td>29</td>
<td>35</td>
<td>45</td>
<td>69</td>
<td>97</td>
<td>88</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>Princeton University, United States of America (USA)</td>
<td>30</td>
<td>67</td>
<td>42</td>
<td>44</td>
<td>46</td>
<td>47</td>
<td>52</td>
<td>43</td>
</tr>
</tbody>
</table>

Source: Nature (2023), [www.nature.com/nature-index/annual-tables](http://www.nature.com/nature-index/annual-tables).
Table 97: Top 30 leading institutions (Nature index, 2023 ranking)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Academy of Sciences (CAS), China</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Nanjing University (NJU), China</td>
<td>19</td>
<td>18</td>
<td>11</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Helmholtz Association of German Research Centres, Germany</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Peking University (PKU), China</td>
<td>16</td>
<td>16</td>
<td>19</td>
<td>13</td>
<td>11</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>University of Chinese Academy of Sciences (UCAS), China</td>
<td>77</td>
<td>24</td>
<td>18</td>
<td>13</td>
<td>10</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>French National Centre for Scientific Research (CNRS), France</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Sun Yat-sen University (SYSU), China</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>65</td>
</tr>
<tr>
<td>Tongji University, China</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>42</td>
</tr>
<tr>
<td>National Aeronautics and Space Administration (NASA), USA</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Swiss Federal Institute of Technology Zurich (ETH Zurich), Switzerland</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Zhejiang University (ZJU), China</td>
<td>75</td>
<td>73</td>
<td>53</td>
<td>36</td>
<td>40</td>
<td>25</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>University of Science and Technology of China (USTC), China</td>
<td>74</td>
<td>39</td>
<td>31</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>China University of Geosciences (CUG), China</td>
<td>44</td>
<td>37</td>
<td>31</td>
<td>18</td>
<td>14</td>
<td>19</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>China Meteorological Administration (CMA), China</td>
<td>100</td>
<td>95</td>
<td>70</td>
<td>54</td>
<td>23</td>
<td>25</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>California Institute of Technology (Caltech), United States of America (USA)</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>7</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Ministry of Natural Resources (MNR), China</td>
<td>75</td>
<td>59</td>
<td>44</td>
<td>38</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>National Oceanic and Atmospheric Administration (NOAA), USA</td>
<td>11</td>
<td>7</td>
<td>7</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Tsinghua University, China</td>
<td>53</td>
<td>36</td>
<td>30</td>
<td>28</td>
<td>26</td>
<td>21</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Southern University of Science and Technology (SUSTech), China</td>
<td>9</td>
<td>8</td>
<td>13</td>
<td>22</td>
<td>9</td>
<td>15</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>Stanford University, United States of America (USA)</td>
<td>13</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>17</td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>University of Washington (UW), United States of America (USA)</td>
<td>12</td>
<td>12</td>
<td>16</td>
<td>11</td>
<td>13</td>
<td>23</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>University of California, Los Angeles (UCLA), United States of America (USA)</td>
<td>22</td>
<td>17</td>
<td>15</td>
<td>21</td>
<td>29</td>
<td>28</td>
<td>26</td>
<td>23</td>
</tr>
<tr>
<td>University of California, Berkeley (UC Berkeley), United States of America (USA)</td>
<td>8</td>
<td>19</td>
<td>12</td>
<td>9</td>
<td>17</td>
<td>18</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>Wuhan University (WHU), China</td>
<td>95</td>
<td>55</td>
<td>58</td>
<td>67</td>
<td>42</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>University of Colorado Boulder (CU-Boulder), United States of America (USA)</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>14</td>
<td>20</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>The University of Texas at Austin (UT Austin), United States of America (USA)</td>
<td>17</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>32</td>
<td>27</td>
</tr>
<tr>
<td>Harbin Institute of Technology (HIT), China</td>
<td>67</td>
<td>48</td>
<td>54</td>
<td>45</td>
<td>49</td>
<td>49</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>University of California, San Diego (UC San Diego), United States of America (USA)</td>
<td>14</td>
<td>14</td>
<td>18</td>
<td>26</td>
<td>32</td>
<td>24</td>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td>Nanjing University of Information Science and Technology (NUIST), China</td>
<td>100</td>
<td>77</td>
<td>49</td>
<td>46</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Nature (2023), www.nature.com/nature-index/annual-tables.
Scientific journals

General scientific journals
- Nature
- Nature Communications
- Science
- Science Advances
- American Economic Journal
- Proceedings of the National Academy of Sciences

Figure 196: Volume 626, Issue 7999, 15 February 2024
Specialized journals

- Atmospheric and Environmental Sciences
  - Atmospheric Chemistry and Physics; Bulletin of the American Meteorological Society; Climate Dynamics; Climate in the Past; Earth System Dynamics; Earth System Science Data; Earth’s Future; Environmental Research Letters; Journal of Advances in Modeling Earth Systems; Journal of Climate; Journal of Geophysical Research: Atmospheres; Geophysical Research Letters; Geoscientific Model Development; Global Change Biology; Journal of the Atmospheric Sciences; npj Climate and Atmospheric Science; Quarterly Journal of the Royal Meteorological Society; Reviews of Geophysics; Tellus.

- Climate change
  - WIREs Climate Change; Climatic Change; Current Climate Change Reports; Global Environmental Change; Global and Planetary Change; Nature Climate Change; Nature Sustainability.

- Economics
  - Climate Change Economics; Climate Policy; Ecological Economics; Environmental and Resource Economics; Environmental Modeling & Assessment; Journal of Environmental Economics and Management; Resource and Energy Economics; Review of Environmental Economics and Policy.

- Energy

- Specialized topics
  - Arctic, Antarctic, and Alpine Research; Artic Ice Journal; Cryosphere; Frontiers in Earth Science: Cryospheric Science; Polar Science; Quaternary Science Reviews; Water Research.
Earth Summit

- Creation of the United Nations Environment Programme (UNEP)
- June 1992: United Nations Conference on Environment and Development (UNCED) or the Earth Summit (Rio de Janeiro, Brazil)
Earth Summit

- Rio Declaration on Environment and Development signed by 170 countries
- Forest Principles (also known as the Rio Forest Principles)
- Agenda 21 ➔ Sustainable Development Goals (SDGs)
  1. Social and economic dimensions.
  2. Conservation and management of resources for development
  3. Strengthening the role of major groups
  4. Means of implementation
- The so-called Rio Conventions
  - UN Framework Convention on Climate Change (UNFCCC)
- United Nations
  - Framework Convention on Climate Change
  - Convention on Biological Diversity (CBD)
  - United Nations Convention to Combat Desertification (UNCCD)
  - Launch of the UNEP Finance Initiative (UNEP-FI)
UNFCCC

United Nations Framework Convention on Climate Change
The UNFCCC is the international treaty (or a convention) adopted at the Earth Summit. It consists of 26 articles and two annexes. The objective of the UNFCCC is defined in Article 2:

“The ultimate objective of this Convention and any related legal instruments that the Conference of the Parties may adopt is to achieve, in accordance with the relevant provisions of the Convention, stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time-frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner.”
Three types of Parties:

- **Annex I Parties**
- **Annex II Parties** (subset of Annex I Parties corresponding to developed countries)
- **Non-Annex I Parties**

⇒ All Parties (including Non-Annex I Parties) must publish national inventories of anthropogenic emissions and implement climate change mitigation programs

---

28In 1992, Annex I Parties were Australia, Austria, Belarus*, Belgium, Bulgaria*, Canada, Czechoslovakia*, Denmark, European Economic Community, Estonia*, Finland, France, Germany, Greece, Hungary*, Iceland, Ireland, Italy, Japan, Latvia*, Lithuania*, Luxembourg, Netherlands, New Zealand, Norway, Poland*, Portugal, Romania*, Russian Federation*, Spain, Sweden, Switzerland, Turkey, Ukraine*, United Kingdom of Great Britain and Northern Ireland, United States of America. The symbol * indicated countries in transition.
### Table 98: Chronological list of the meetings of the Conference of the Parties

<table>
<thead>
<tr>
<th>Year</th>
<th>COP</th>
<th>CMP</th>
<th>CMA</th>
<th>City</th>
<th>Country</th>
<th>Treaty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td></td>
<td></td>
<td></td>
<td>Rio de Janero</td>
<td>Brazil</td>
<td>Convention</td>
</tr>
<tr>
<td>1995</td>
<td>1</td>
<td></td>
<td></td>
<td>Berlin</td>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>2</td>
<td></td>
<td></td>
<td>Geneva</td>
<td>Switzerland</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>3</td>
<td></td>
<td></td>
<td>Kyoto</td>
<td>Japan</td>
<td>Kyoto Protocol</td>
</tr>
<tr>
<td>1998</td>
<td>4</td>
<td></td>
<td></td>
<td>Buenos Aires</td>
<td>Argentina</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>5</td>
<td></td>
<td></td>
<td>Bonn</td>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>6</td>
<td>1</td>
<td></td>
<td>The Hague</td>
<td>Netherlands</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>6</td>
<td>2</td>
<td></td>
<td>Bonn</td>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>7</td>
<td></td>
<td></td>
<td>Marrakech</td>
<td>Morocco</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>8</td>
<td></td>
<td></td>
<td>New Delhi</td>
<td>India</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>9</td>
<td></td>
<td></td>
<td>Milan</td>
<td>Italy</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>10</td>
<td></td>
<td></td>
<td>Buenos Aires</td>
<td>Argentina</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>11</td>
<td>1</td>
<td></td>
<td>Montreal</td>
<td>Canada</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>12</td>
<td>2</td>
<td></td>
<td>Nairobi</td>
<td>Kenya</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>13</td>
<td>3</td>
<td></td>
<td>Bali</td>
<td>Indonesia</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>14</td>
<td>4</td>
<td></td>
<td>Poznań</td>
<td>Poland</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>15</td>
<td>5</td>
<td></td>
<td>Copenhagen</td>
<td>Denmark</td>
<td></td>
</tr>
</tbody>
</table>
### Conference of the Parties (COP)

**Table 99: Chronological list of the meetings of the Conference of the Parties**

<table>
<thead>
<tr>
<th>Year</th>
<th>COP</th>
<th>CMP</th>
<th>CMA</th>
<th>City</th>
<th>Country</th>
<th>Treaty</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>16</td>
<td>6</td>
<td></td>
<td>Cancún</td>
<td>Mexico</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>17</td>
<td>7</td>
<td></td>
<td>Durban</td>
<td>South Africa</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>18</td>
<td>8</td>
<td></td>
<td>Doha</td>
<td>Qatar</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>19</td>
<td>9</td>
<td></td>
<td>Warsaw</td>
<td>Poland</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>20</td>
<td>10</td>
<td></td>
<td>Lima</td>
<td>Peru</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>21</td>
<td>11</td>
<td></td>
<td>Paris</td>
<td>France</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>22</td>
<td>12</td>
<td>1-1</td>
<td>Marrakech</td>
<td>Morocco</td>
<td>Paris Agreement</td>
</tr>
<tr>
<td>2017</td>
<td>23</td>
<td>13</td>
<td>1-2</td>
<td>Bonn</td>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>24</td>
<td>14</td>
<td>1-3</td>
<td>Katowice</td>
<td>Poland</td>
<td></td>
</tr>
<tr>
<td>2019</td>
<td>25</td>
<td>15</td>
<td>2</td>
<td>Madrid</td>
<td>Spain</td>
<td></td>
</tr>
<tr>
<td>2021</td>
<td>26</td>
<td>16</td>
<td>3</td>
<td>Glasgow</td>
<td>United Kingdom</td>
<td></td>
</tr>
<tr>
<td>2022</td>
<td>27</td>
<td>17</td>
<td>4</td>
<td>Sharm El Sheikh</td>
<td>Egypt</td>
<td></td>
</tr>
<tr>
<td>2023</td>
<td>28</td>
<td>18</td>
<td>5</td>
<td>Dubai</td>
<td>United Arab Emirates</td>
<td></td>
</tr>
<tr>
<td>2024</td>
<td>29</td>
<td>19</td>
<td>6</td>
<td>Baku</td>
<td>Azerbaijan</td>
<td></td>
</tr>
</tbody>
</table>
Conference of the Parties (COP)

Important COPs

- COP18: Doha (2012) ⇒ Second commitment of the Kyoto Protocol
- COP26: Glasgow (2021) ⇒ GFANZ & Net Zero
- COP28: Dubai (2023) ⇒ “Beginning of the end” of the fossil fuel era
Kyoto Protocol

- Berlin Mandate
- IPCC published the Revised 1996 IPCC Guidelines for National Greenhouse Gas Inventories
- COP3 (Kyoto, 1997)
- The Kyoto Protocol entered into force on 16 February 2005
- The Protocol’s first commitment period started in 2008 and ended in 2012
- At COP18 in Doha, a second commitment extended the first one from 2013 to 2020

⇒ Launch of the EU ETS in 2005
Table 100: Quantified first commitment under the Kyoto Protocol to limit or reduce emissions (% of base year)

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>+10%</td>
<td>Iceland</td>
</tr>
<tr>
<td>+8%</td>
<td>Australia</td>
</tr>
<tr>
<td>+1%</td>
<td>Norway</td>
</tr>
<tr>
<td>0%</td>
<td>New Zealand, Russian Federation, Ukraine</td>
</tr>
<tr>
<td>−5%</td>
<td>Croatia</td>
</tr>
<tr>
<td>−6%</td>
<td>Canada, Hungary, Japan, Poland</td>
</tr>
<tr>
<td>−7%</td>
<td>United States of America</td>
</tr>
<tr>
<td>−8%</td>
<td>Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, European Community, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Liechtenstein, Lithuania, Luxembourg, Monaco, Netherlands, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom of Great Britain and Northern Ireland</td>
</tr>
</tbody>
</table>
Table 101: List of greenhouse gases under the Kyoto Protocol and the Doha Amendment

<table>
<thead>
<tr>
<th>Greenhouse gas</th>
<th>Symbol</th>
<th>Kyoto Protocol</th>
<th>Doha Amendment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>N₂O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Hydrofluorocarbons</td>
<td>HFCs</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Perfluorocarbons</td>
<td>PFCs</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sulphur hexafluoride</td>
<td>SF₆</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Nitrogen trifluoride</td>
<td>NF₃</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
Table 102: Quantified second commitment under the Kyoto Protocol (Doha Amendment) to limit or reduce emissions (% of base year)

<table>
<thead>
<tr>
<th>R</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.5%</td>
<td>Australia</td>
</tr>
<tr>
<td>−5%</td>
<td>Kazakhstan</td>
</tr>
<tr>
<td>−12%</td>
<td>Belarus</td>
</tr>
<tr>
<td>−16%</td>
<td>Liechtenstein, Norway, Switzerland</td>
</tr>
<tr>
<td>−20%</td>
<td>Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, European Union, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom of Great Britain and Northern Ireland</td>
</tr>
<tr>
<td>−22%</td>
<td>Monaco</td>
</tr>
<tr>
<td>−24%</td>
<td>Ukraine</td>
</tr>
</tbody>
</table>
What is the track record of the Kyoto Protocol?

- Nordhaus and Boyer (1999) wrote that “[…] the benefit-cost ratio of the Kyoto Protocol is 1/7. Additionally, the emissions strategy is highly cost-ineffective, with the global temperature reduction achieved at a cost almost 8 times the cost of a strategy which is cost-effective in terms of where and when efficiency.”

- Tol (1999) concluded that “[…] the agreements of the Kyoto Protocol are not readily reconciled with economic rationality.”

- Etc.

- BUT...

**Total EU emissions have been reduced about 19% below base year levels**
The **Paris Agreement** is an international treaty with the following goals:

- Keep a global temperature rise this century well below 2°C above the pre-industrial levels
- Pursue efforts to limit the temperature increase to 1.5°C
- Increase the ability of countries to deal with the impacts of climate change
- Make finance flows consistent with low GHG emissions and climate-resilient pathways

⇒ Nationally determined contribution (NDC)
Figure 197: IPCC Special Report on Global Warming of 1.5°C (SR15)
Figure 198: The Economist, Say goodbye to 1.5°C, November 5, 2022
Nationally determined contribution

NDC Registry: https://unfccc.int/NDCREG
### Nationally determined contribution

#### First Party: Afghanistan

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base year</strong></td>
<td>2005</td>
</tr>
<tr>
<td><strong>Target years</strong></td>
<td>2020 to 2030</td>
</tr>
<tr>
<td><strong>Contribution type</strong></td>
<td>Conditional</td>
</tr>
<tr>
<td><strong>Sectors</strong></td>
<td>Energy, natural resource management, agriculture, waste management and mining</td>
</tr>
<tr>
<td><strong>Gases covered</strong></td>
<td>Carbon dioxide (CO₂), methane (CH₄), and nitrous oxide (N₂O)</td>
</tr>
<tr>
<td><strong>Target</strong></td>
<td>There will be a 13.6% reduction in GHG emissions by 2030 compared to a business-as-usual (BAU) 2030 scenario, conditional on external support</td>
</tr>
<tr>
<td><strong>Financial needs</strong></td>
<td>Total: $17.405 bn</td>
</tr>
</tbody>
</table>
Last Part: Zimbabwe (2015 INDC)

- Business-as-usual (BAU) scenario
  “The INDC BAU baseline focused solely on per capita energy emissions. Zimbabwean per capita energy emissions were projected to be $1.06 \text{tCO}_2\text{e}$ in 2020, $2.57 \text{tCO}_2\text{e}$ in 2025 and $3.31 \text{tCO}_2\text{e}$ in 2030 under business-as-usual.”

- Emission reduction target
  “The INDC emission reduction target was a 33% reduction in energy-related emissions per capita compared to BAU by 2030, conditional on international support. In the mitigation scenario, energy-related emissions per capita were projected to be $2.21 \text{tCO}_2\text{e}$ in 2030.”
Last Part: Zimbabwe (2021 Revised NDC)

- Business-as-usual (BAU) scenario
  "Updated to include all IPCC sectors. National total emissions in the base data period ranged between 25.24 MtCO$_2$e in 2011 and 41.66 MtCO$_2$e in 2015. Emissions in 2017 were 35.84 MtCO$_2$e. National total emissions per capita in the base data period ranged between 2.03 tCO$_2$e in 2011 and 2.98 tCO$_2$e in 2015. Emissions per capita in 2017 were 2.45 tCO$_2$e. The NDC revision process incorporated impacts of COVID-19 on emissions trends and macroeconomic parameters, including GDP, which fed into the updated baseline."

- Emission reduction target
  "The updated target is a 40% reduction in economy-wide GHG emissions per capita compared to BAU by 2030, conditional on international support. In the mitigation scenario, economy-wide emissions per capita are projected to be 2.3 tCO$_2$e in 2030."
Nationally determined contribution

(1) GHG emissions in 2022 in MtCO₂e, (2) share in %, (3) cumulative share in %, (4) GHG emissions per capita in 2022 in tCO₂e, (5) GHG emissions per GDP in 2022 in kgCO₂e, (6) GHG emissions growth between 2010 and 2022 in %, (7) NDC reduction rate, (8) base year (BAU = business-as-usual), (9) target year, (10) mitigation type (AER = absolute emissions reduction, CIR = carbon intensity reduction, RER = relative emissions reduction).

Table 103: 2022 GHG emissions and NDCs of top emitting countries

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>China</td>
<td>15685</td>
<td>29.16</td>
<td>29.16</td>
<td>10.95</td>
<td>0.61</td>
<td>35.62</td>
<td>65</td>
<td>2005</td>
<td>2030</td>
<td>CIR</td>
</tr>
<tr>
<td>2</td>
<td>United States</td>
<td>6017</td>
<td>11.19</td>
<td>40.35</td>
<td>17.90</td>
<td>0.28</td>
<td>−10.01</td>
<td>50</td>
<td>2005</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>3</td>
<td>India</td>
<td>3943</td>
<td>7.33</td>
<td>47.68</td>
<td>2.79</td>
<td>0.39</td>
<td>38.85</td>
<td>45</td>
<td>2005</td>
<td>2030</td>
<td>CIR</td>
</tr>
<tr>
<td>4</td>
<td>Russia</td>
<td>2580</td>
<td>4.80</td>
<td>52.48</td>
<td>17.99</td>
<td>0.64</td>
<td>14.90</td>
<td>30</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>5</td>
<td>Brazil</td>
<td>1310</td>
<td>2.44</td>
<td>54.91</td>
<td>6.05</td>
<td>0.40</td>
<td>11.27</td>
<td>50</td>
<td>2005</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>6</td>
<td>Indonesia</td>
<td>1241</td>
<td>2.31</td>
<td>57.22</td>
<td>4.47</td>
<td>0.36</td>
<td>50.00</td>
<td>32</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>7</td>
<td>Japan</td>
<td>1183</td>
<td>2.20</td>
<td>59.42</td>
<td>9.41</td>
<td>0.23</td>
<td>−10.85</td>
<td>46</td>
<td>2013</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>8</td>
<td>Iran</td>
<td>952</td>
<td>1.77</td>
<td>61.19</td>
<td>11.20</td>
<td>0.70</td>
<td>17.94</td>
<td>32</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>9</td>
<td>Mexico</td>
<td>820</td>
<td>1.52</td>
<td>62.71</td>
<td>5.99</td>
<td>0.33</td>
<td>6.64</td>
<td>22</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>10</td>
<td>Saudi Arabia</td>
<td>811</td>
<td>1.51</td>
<td>64.22</td>
<td>22.64</td>
<td>0.45</td>
<td>30.84</td>
<td>34</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
</tbody>
</table>
### Table 104: 2022 GHG emissions and NDCs of top emitting countries

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Germany</td>
<td>784</td>
<td>1.46</td>
<td>65.68</td>
<td>9.49</td>
<td>0.17</td>
<td>-16.68</td>
<td>55</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>12</td>
<td>Canada</td>
<td>757</td>
<td>1.41</td>
<td>67.09</td>
<td>19.79</td>
<td>0.40</td>
<td>4.22</td>
<td>40</td>
<td>2005</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>13</td>
<td>International Shipping</td>
<td>751</td>
<td>1.40</td>
<td>68.48</td>
<td>7.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>South Korea</td>
<td>726</td>
<td>1.35</td>
<td>69.83</td>
<td>14.01</td>
<td>0.31</td>
<td>7.79</td>
<td>40</td>
<td>2018</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>15</td>
<td>Turkey</td>
<td>688</td>
<td>1.28</td>
<td>71.11</td>
<td>8.09</td>
<td>0.24</td>
<td>61.73</td>
<td>21</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>16</td>
<td>Australia</td>
<td>571</td>
<td>1.06</td>
<td>72.17</td>
<td>21.98</td>
<td>0.43</td>
<td>-5.06</td>
<td>43</td>
<td>2005</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>17</td>
<td>Pakistan</td>
<td>546</td>
<td>1.02</td>
<td>73.19</td>
<td>2.53</td>
<td>0.42</td>
<td>36.49</td>
<td>15</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>18</td>
<td>South Africa</td>
<td>535</td>
<td>0.99</td>
<td>74.18</td>
<td>8.91</td>
<td>0.66</td>
<td>-9.11</td>
<td>40</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>19</td>
<td>Vietnam</td>
<td>489</td>
<td>0.91</td>
<td>75.09</td>
<td>4.88</td>
<td>0.44</td>
<td>58.37</td>
<td>9</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>20</td>
<td>Thailand</td>
<td>464</td>
<td>0.86</td>
<td>75.95</td>
<td>6.67</td>
<td>0.37</td>
<td>8.76</td>
<td>20</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>21</td>
<td>France</td>
<td>430</td>
<td>0.80</td>
<td>76.75</td>
<td>6.50</td>
<td>0.14</td>
<td>-16.37</td>
<td>55</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>22</td>
<td>United Kingdom</td>
<td>427</td>
<td>0.79</td>
<td>77.54</td>
<td>6.27</td>
<td>0.14</td>
<td>-28.29</td>
<td>68</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>23</td>
<td>International Aviation</td>
<td>426</td>
<td>0.79</td>
<td>78.34</td>
<td></td>
<td></td>
<td>8.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Nigeria</td>
<td>408</td>
<td>0.76</td>
<td>79.09</td>
<td>1.88</td>
<td>0.38</td>
<td>11.08</td>
<td>20</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>25</td>
<td>Poland</td>
<td>401</td>
<td>0.75</td>
<td>79.84</td>
<td>10.62</td>
<td>0.29</td>
<td>-4.13</td>
<td>55</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>26</td>
<td>Italy</td>
<td>395</td>
<td>0.73</td>
<td>80.57</td>
<td>6.70</td>
<td>0.15</td>
<td>-21.62</td>
<td>55</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>27</td>
<td>Argentina</td>
<td>383</td>
<td>0.71</td>
<td>81.29</td>
<td>8.27</td>
<td>0.37</td>
<td>10.60</td>
<td>10</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>28</td>
<td>Egypt</td>
<td>378</td>
<td>0.70</td>
<td>81.99</td>
<td>3.55</td>
<td>0.27</td>
<td>19.16</td>
<td>33</td>
<td>2020</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>29</td>
<td>Iraq</td>
<td>368</td>
<td>0.68</td>
<td>82.67</td>
<td>8.41</td>
<td>0.90</td>
<td>75.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Malaysia</td>
<td>354</td>
<td>0.66</td>
<td>83.33</td>
<td>10.50</td>
<td>0.37</td>
<td>26.48</td>
<td>45</td>
<td>2005</td>
<td>2030</td>
<td>CIR</td>
</tr>
</tbody>
</table>
### Table 105: 2022 GHG emissions and NDCs of top emitting countries

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>Kazakhstan</td>
<td>332</td>
<td>0.62</td>
<td>83.95</td>
<td>17.33</td>
<td>0.65</td>
<td>3.50</td>
<td>15</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>32</td>
<td>Spain</td>
<td>329</td>
<td>0.61</td>
<td>84.56</td>
<td>7.08</td>
<td>0.17</td>
<td>13.07</td>
<td>55</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>33</td>
<td>Taiwan</td>
<td>308</td>
<td>0.57</td>
<td>85.13</td>
<td>12.86</td>
<td>0.19</td>
<td>1.57</td>
<td>31</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>34</td>
<td>United Arab Emirates</td>
<td>295</td>
<td>0.55</td>
<td>85.68</td>
<td>29.33</td>
<td>0.42</td>
<td>31.21</td>
<td>7</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>35</td>
<td>Algeria</td>
<td>284</td>
<td>0.53</td>
<td>86.21</td>
<td>6.38</td>
<td>0.57</td>
<td>36.01</td>
<td>7</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>36</td>
<td>Bangladesh</td>
<td>281</td>
<td>0.52</td>
<td>86.73</td>
<td>1.62</td>
<td>0.26</td>
<td>29.87</td>
<td>7</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>37</td>
<td>Philippines</td>
<td>265</td>
<td>0.49</td>
<td>87.22</td>
<td>2.35</td>
<td>0.27</td>
<td>50.56</td>
<td>3</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>38</td>
<td>Uzbekistan</td>
<td>227</td>
<td>0.42</td>
<td>87.65</td>
<td>6.67</td>
<td>0.79</td>
<td>9.25</td>
<td>35</td>
<td>2010</td>
<td>2030</td>
<td>CIR</td>
</tr>
<tr>
<td>39</td>
<td>Colombia</td>
<td>216</td>
<td>0.40</td>
<td>88.05</td>
<td>4.23</td>
<td>0.27</td>
<td>18.88</td>
<td>51</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>40</td>
<td>Ukraine</td>
<td>209</td>
<td>0.39</td>
<td>88.43</td>
<td>4.84</td>
<td>0.55</td>
<td>47.32</td>
<td>65</td>
<td>1990</td>
<td>2030</td>
<td>AER</td>
</tr>
<tr>
<td>41</td>
<td>Qatar</td>
<td>195</td>
<td>0.36</td>
<td>88.80</td>
<td>67.38</td>
<td>0.74</td>
<td>41.86</td>
<td>25</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>42</td>
<td>Ethiopia</td>
<td>192</td>
<td>0.36</td>
<td>89.15</td>
<td>1.63</td>
<td>0.66</td>
<td>53.40</td>
<td>14</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>43</td>
<td>Venezuela</td>
<td>170</td>
<td>0.32</td>
<td>89.47</td>
<td>4.99</td>
<td>1.02</td>
<td>35.03</td>
<td>20</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>44</td>
<td>Myanmar</td>
<td>169</td>
<td>0.31</td>
<td>89.78</td>
<td>3.04</td>
<td>0.76</td>
<td>42.10</td>
<td>44</td>
<td>BAU</td>
<td>2030</td>
<td>RER</td>
</tr>
<tr>
<td>45</td>
<td>Kuwait</td>
<td>168</td>
<td>0.31</td>
<td>90.10</td>
<td>37.96</td>
<td>0.80</td>
<td>35.24</td>
<td>7</td>
<td>BAU</td>
<td>2035</td>
<td>RER</td>
</tr>
</tbody>
</table>

**Total**: 53,786  6.76  0.39  14.46
Kaya decomposition

- Population-based decomposition

\[
\text{GHG emissions} = \text{Population} \times \frac{\text{GHG emissions}}{\text{Population}}
\]

- GDP-based decomposition

\[
\text{GHG emissions} = \text{GDP} \times \frac{\text{GHG emissions}}{\text{GDP}}
\]

- Simplified Kaya identity:

\[
\text{GHG emissions} = \text{GDP} \times \frac{\text{Energy}}{\text{GDP}} \times \frac{\text{GHG emissions}}{\text{Energy}}
\]
Figure 199: Growth of greenhouse gas emissions in % for developed countries (base year = 1990)
Figure 200: Growth of greenhouse gas emissions in % for developing countries (base year = 1990)
Figure 201: Per capita GHG emissions (1990 vs. 2022)
Figure 202: GHG emissions per GDP (1990 vs. 2022)
Figure 203: The CAT thermometer

CAT warming projections
Global temperature increase by 2100
December 2023 Update

European Green Deal (December 2019)

Adopted by the EU Council in June 2021, the European Climate Law includes the following components:

- Set a 2030 target to reduce net greenhouse gas emissions by at least 55% compared to 1990 levels;
- Establish a system to monitor progress and take further action if necessary;
- Establish the European Scientific Advisory Board on Climate Change to provide independent scientific advice;
- Define the EU-wide greenhouse gas emission reduction path for 2030-2050;
- Provide predictability for investors and other economic actors;
- Ensure that the transition to climate neutrality is irreversible;
- Commit to negative emissions after 2050.
Communication on the European Green Deal, 14 July 2021

EUROPEAN GREEN DEAL

REACHING OUR 2030 CLIMATE TARGETS

Source: https://twitter.com/EU_Commission/status/1415289818565816321.
In July 2021, the European Commission presented the Fit for 55 package

First quantitative target of the European Climate Change Law, namely to reduce Europe’s net greenhouse gas emissions by at least 55% by 2030

The package consists of several legislative proposals

As of 1 January 2024, 12 proposals have been adopted
Fit for 55 package

1. Reform of the emissions trading system (ETS, April 2023)
2. New emissions trading system for building and road transport fuels (April 2023)
3. Social climate fund (April 2023)
4. Revision of the effort sharing regulation (ESR, March 2023)
5. Revision of the regulation on land use, forestry and agriculture (LULUCF, March 2023)
6. Revised regulation on CO$_2$ emissions from new cars and vans (March 2023)
7. Carbon border adjustment mechanism (CBAM, April 2023)
8. Renewable energy directive (RED III, October 2023)
9. Energy efficiency directive (EED, July 2023)
10. Alternative fuels infrastructure regulation (AFIR)
11. ReFuelEU aviation regulation
12. FuelEU maritime regulation
Table 106: National reduction targets under the ESR (revised 2023 vs. original 2018)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>36.0%</td>
<td>48.0%</td>
<td>France</td>
<td>37.0%</td>
<td>47.5%</td>
<td>Malta</td>
<td>19.0%</td>
<td>19.0%</td>
</tr>
<tr>
<td>Belgium</td>
<td>35.0%</td>
<td>47.0%</td>
<td>Germany</td>
<td>38.0%</td>
<td>50.0%</td>
<td>Netherlands</td>
<td>36.0%</td>
<td>48.0%</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>0.0%</td>
<td>10.0%</td>
<td>Greece</td>
<td>16.0%</td>
<td>22.7%</td>
<td>Poland</td>
<td>7.0%</td>
<td>17.7%</td>
</tr>
<tr>
<td>Croatia</td>
<td>7.0%</td>
<td>16.7%</td>
<td>Hungary</td>
<td>7.0%</td>
<td>17.7%</td>
<td>Portugal</td>
<td>17.0%</td>
<td>28.7%</td>
</tr>
<tr>
<td>Cyprus</td>
<td>24.0%</td>
<td>32.0%</td>
<td>Ireland</td>
<td>30.0%</td>
<td>42.0%</td>
<td>Romania</td>
<td>2.0%</td>
<td>12.7%</td>
</tr>
<tr>
<td>Czechia</td>
<td>14.0%</td>
<td>26.0%</td>
<td>Italy</td>
<td>33.0%</td>
<td>43.7%</td>
<td>Slovakia</td>
<td>12.0%</td>
<td>22.7%</td>
</tr>
<tr>
<td>Denmark</td>
<td>39.0%</td>
<td>50.0%</td>
<td>Latvia</td>
<td>6.0%</td>
<td>17.0%</td>
<td>Slovenia</td>
<td>15.0%</td>
<td>27.0%</td>
</tr>
<tr>
<td>Estonia</td>
<td>13.0%</td>
<td>24.0%</td>
<td>Lithuania</td>
<td>9.0%</td>
<td>21.0%</td>
<td>Spain</td>
<td>26.0%</td>
<td>37.7%</td>
</tr>
<tr>
<td>Finland</td>
<td>39.0%</td>
<td>50.0%</td>
<td>Luxembourg</td>
<td>40.0%</td>
<td>50.0%</td>
<td>Sweden</td>
<td>40.0%</td>
<td>50.0%</td>
</tr>
</tbody>
</table>

In addition to the previous 12 adopted proposals, three new proposals have already been provisionally agreed by the co-legislators:

- Energy performance of buildings directive (EPBD)
- EU methane regulation for the energy sector
- Updated EU rules to decarbonise gas markets and promote hydrogen

**Remark**

One proposal did not materialise: revision of the Energy Taxation Directive (ETD)
Figure 204: What is included in the Fit for 55 package?

The US Environmental Protection Agency (EPA) has implemented the following regulations since 2020:

- An 85% reduction in the production and consumption of hydrofluorocarbons over the next 15 years, beginning in 2021
- Vehicle emissions standards, including stricter GHG emissions standards for cars and light trucks beginning in 2023
- Aviation GHG standards for commercial aircraft and large business jets
- Renewable Fuel Standard (RFS) program, a federal program that requires a certain amount of renewable fuel to be blended into transportation fuel each year
- Regulation of power plant emissions through the Acid Rain Program (ARP) and Mercury and Air Toxics Standards (MATS)

These new regulations complement existing regulations such as the Clean Air Act (CAA) of 1970, the federal law that regulates air emissions from stationary and mobile sources, including power plants, factories, and vehicles.
- In August 2022, the United States passed the Inflation Reduction Act (IRA)
- In December 2023, the EPA issued a new methane rule to reduce methane emissions from oil and natural gas operations
- In 2022 and 2023, some states also increased their renewable portfolio standards (RPS)
**France**

- Energy Transition Act of 2015 (Loi relative à la transition énergétique pour la croissance verte du 17 août 2015)
- Energy and Climate Act of 2019 (Loi relative à l’énergie et au climat du 8 novembre 2019)
- Mobility Orientation Act of 2019 (Loi d’orientation des mobilités du 24 décembre 2019)
- Anti-Waste Act for a Circular Economy of 2020 (Loi relative à la lutte contre le gaspillage et à l’économie circulaire du 10 février 2020)
- National low carbon strategy (Stratégie nationale bas-carbone du 21 avril 2020 or SNBC)
- Climate and Resilience Act of 2021 (Loi climat et résilience du 22 août 2021)
- Environmental Regulation (Règlementation environnementale RE2020)
Germany

- Klimaschutzgesetz (climate change act)
- Klimaschutzprogramm 2030 (climate action programme 2030)
- Erneuerbare-Energien-Gesetz (EEG or renewable energy sources act)
- Energieeinsparverordnung (EnEV or energy saving ordinance)
- Energiewirtschaftsgesetz (EnWG or energy industry act)
United Kingdom

- Climate Change Act 2008
- UK ETS
- UK CBAM
Other countries


- China: renewable energy law (2005), energy conservation law (2007), carbon emissions trading management regulations (2021), 14th five-year plan for green development (2021-2025)

Course 2023-2024 in Sustainable Finance
Lecture 10. Economic Models & Climate Change

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2024

---

29 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
“There is no Plan B, because there is no Planet B“
Ban Ki-moon, UN Secretary-General, September 2014

Is it a question of climate-related issues?
In fact, it is more an economic growth issue

“The Golden Rule of Accumulation: A Fable for Growthmen“
Edmund Phelps, American Economic Review, 1961
Nobel Prize in Economics, 2006
Sustainable growth and climate change

Environmental

Economy

Social

Sustainability

Governance

Growth

Adam Smith (1776)

An Inquiry into the Nature and Causes of

The Wealth of Nations
Limits of economic models
The Solow growth model

The model

- Production function:

\[ Y(t) = F(K(t), A(t)L(t)) \]

where \( K(t) \) is the capital, \( L(t) \) is the labor and \( A(t) \) is the knowledge factor.

- Law of motion for the capital per unit of effective labor

\[ k(t) = \frac{K(t)}{(A(t)L(t))} \]

\[ \frac{dk(t)}{dt} = sf(k(t)) - (gL + gA + \delta_K)k(t) \]

where \( s \) is the saving rate, \( \delta_K \) is the depreciation rate of capital and \( g_A \) and \( g_L \) are the productivity and labor growth rates.
Limits of economic models
The golden rule

Golden rule with the Cobb-Douglas production and Hicks neutrality

The equilibrium to respect the ‘fairness’ between generations is:

\[ k^* = \left( \frac{s}{gL + gA + \delta_K} \right)^{\frac{1}{1 - \alpha}} \]

“Each generation in a boundless golden age of natural growth will prefer the same investment ratio, which is to say the same natural growth path” (Phelps, 1961, page 640).

“By a golden age I shall mean a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital-output ratio is stationary over time” (Phelps, 1961, page 639).
What is economic growth and what is the balanced growth path?

- There is a saving rate that maximizes consumption over time and between generations ("the fair rate to preserve future generations")
- Economic growth corresponds to the exponential growth of capital and output to answer the needs of the growing population
- Introducing human and natural capitals add constraints and therefore reduce growth!

Economic growth $\Rightarrow \{\text{productivity} \uparrow \text{and labor} \uparrow \text{maximization of } \text{consumption-based utility} \text{ function}\}$
What are the effects of environmental constraints on growth?

Introducing a decreasing natural capital (Romer, 2006)

The balanced growth path $g^*_Y$ is equal to:

$$g^*_Y = g_L + g_A - \frac{g_L + g_A + \delta_{N_c} \vartheta}{1 - \alpha}$$

where $\delta_{N_c}$ is the depreciation rate of natural capital and $\vartheta$ is the elasticity of output with respect to (normalized) natural capital $N_c(t)$.

"The static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline" (Hotteling, 1931, page 138-139)

Accounting for environment... changes the definition of economic growth.
Preferences modeling (Ramsey model)

- $\rho$ is the discount rate (time preference)
- $c(t)$ is the consumption per capita and $u$ is the CRRA utility function:

$$u(c(t)) = \begin{cases} 
\frac{1}{1-\theta}c(t)^{1-\theta} & \text{if } \theta > 0, \quad \theta \neq 1 \\
\ln c(t) & \text{if } \theta = 1
\end{cases}$$

where $\theta$ is the risk aversion parameter
- Maximization of the welfare function:

$$\int_0^\infty e^{-\rho t} u(c(t)) \, dt$$
Does the golden rule of saving rates hold in a Keynesian approach with discounted maximization of consumption?

- **There is still time to avoid the worst impacts of climate change, if we take strong action now** (Stern, 2007)
- **I got it wrong on climate change – it’s far, far worse** (Stern, 2013)

The value of a loss in 100 years almost disappears... while it is only the next generation!
### How many planets do we need?

To achieve the current levels of consumption for the world population, we need:

<table>
<thead>
<tr>
<th>Country</th>
<th>Planets Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qatar</td>
<td>9.0</td>
</tr>
<tr>
<td>Sweden</td>
<td>4.0</td>
</tr>
<tr>
<td>China</td>
<td>2.4</td>
</tr>
<tr>
<td>India</td>
<td>0.8</td>
</tr>
<tr>
<td>Qatar</td>
<td>5.1</td>
</tr>
<tr>
<td>Sweden</td>
<td>4.0</td>
</tr>
<tr>
<td>Germany</td>
<td>3.0</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>0.4</td>
</tr>
<tr>
<td>US</td>
<td>5.1</td>
</tr>
<tr>
<td>France</td>
<td>2.8</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.6</td>
</tr>
<tr>
<td>Yemen</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**World:** 1.75 planets

Source: [https://overshoot.footprintnetwork.org/how-many-earths-or-countries-do-we-need](https://overshoot.footprintnetwork.org/how-many-earths-or-countries-do-we-need)
Limits of economic models
Fairness between generations

Keynes

“In the long run, we are all dead”

John Maynard Keynes\textsuperscript{a}, A Tract on Monetary Reform, 1923.

\textsuperscript{a}“Men will not always die quietly”, The Economic Consequences of the Peace, 1919.

Carney

“The Tragedy of the Horizon”

Mark Carney, Chairman of the Financial Stability Board, 2015

⇒ Back to the Golden Rule and the Fable for Growthmen...
Integrated assessment models (IAMs)

Main categories

- **Optimization models**
  The inputs of these models are parameters and assumptions about the structure of the relationships between variables. The outputs provided by optimization process are scenarios depending on a set of constraints.

- **Evaluation models**
  Based on exogenous scenarios, the outputs provide results from partial equilibriums between variables.

Three main components of IAMs

1. Economic growth relationships
2. Dynamics of climate emissions
3. Objective function
Figure 206: Economic models of climate risk

- Production
  - Industry and business generate CO₂ emissions

- Climate change
  - Change in radiative warming; ocean currents; sea level rise; etc.

- Impact and damages
  - Losses on the entire economy

- Objective
  - Measures and tax policies to control CO₂ emissions
Modeling framework

- Economic module
  1. Production function \( \rightarrow \) GDP
  2. Impact of the climate risk on GDP (damage losses, mitigation and adaptation costs)
  3. The climate loss function depends on the temperature

- Climate module
  1. Dynamics of GHG emissions
  2. Modeling of Atmospheric and lower ocean temperatures

- Optimal control problem
  1. Maximization of the utility function
  2. We can test many variants
The most famous IAM is the Dynamic Integrated model of Climate and the Economy (or DICE) developed by William Nordhaus\textsuperscript{30}.

\textsuperscript{30}2018 Nobel Laureate
The **gross production** $Y(t)$ is given by a Cobb-Douglas function:

$$Y(t) = A(t) K(t)^\gamma L(t)^{1-\gamma}$$

where:
- $A(t)$ is the total productivity factor
- $K(t)$ is the capital input
- $L(t)$ is the labor input
- $\gamma \in ]0, 1[$ measures the elasticity of the capital factor

- Climate change impacts the **net output**:

$$Q(t) = \Omega_{\text{climate}}(t) Y(t) \leq Y(t)$$

- Classical identities $Q(t) = C(t) + I(t)$ and $I(t) = s(t) Q(t)$
The dynamics of the state variables are:

\[
\begin{align*}
A(t) &= (1 + g_A(t)) A(t - 1) \\
K(t) &= (1 - \delta_K) K(t - 1) + I(t) \\
L(t) &= (1 + g_L(t)) L(t - 1)
\end{align*}
\]

We have:

\[
\begin{align*}
g_A(t) &= \frac{1}{1 + \delta_A} g_A(t - 1) \\
g_L(t) &= \frac{1}{1 + \delta_L} g_L(t - 1)
\end{align*}
\]
Example #1

The world population was equal to 7.725 billion in 2019 and 7.805 billion in 2020. At the beginning of the 1970s, we estimate that the annual growth rate was equal to 2.045%. According to the United Nations, the global population could surpass 10 billion by 2100.
In 2020, the annual growth rate was equal to:

\[ g_L(2020) = \frac{L(2020)}{L(2019)} - 1 = \frac{7.805}{7.725} - 1 = 1.036\% \]

Since we have \( g_L(t) = \left(\frac{1}{1 + \delta_L}\right)^{t-t_0} g_L(t_0) \), we deduce that:

\[ \delta_L = \left(\frac{g_L(t_0)}{g_L(t)}\right)^{1/(t-t_0)} - 1 \]

An estimate of \( \delta_L \) is then:

\[ \delta_L = \left(\frac{g_L(1970)}{g_L(2020)}\right)^{1/30} - 1 = 2.292\% \]
Figure 207: Evolution of the labor input $L(t)$
**Figure 208: Projection of the world population**

Economic module

Labor input

- AR(1) model:

\[ g_L(t) = \phi g_L(t - 1) + \varepsilon(t) \]

We have

\[ \hat{\delta}_L = \frac{(1 - \hat{\phi})}{\hat{\phi}} \]

- Log-linear model:

\[ \ln g_L(t) = \beta_0 + \beta_1 (t - t_0) + \varepsilon(t) \]

We have:

\[ \hat{\delta}_L = e^{-\hat{\beta}_1} - 1 \]
Figure 209: Population growth rate

### Table 107: Average productivity growth rate (in %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>1.02</td>
<td>0.07</td>
<td>−0.23</td>
<td>1.02</td>
<td>0.36</td>
<td>0.13</td>
</tr>
<tr>
<td>BRA</td>
<td>2.39</td>
<td>2.05</td>
<td>−1.04</td>
<td>−1.12</td>
<td>−0.17</td>
<td>−1.63</td>
</tr>
<tr>
<td>CAN</td>
<td>2.18</td>
<td>0.38</td>
<td>−0.25</td>
<td>0.21</td>
<td>−0.21</td>
<td>0.40</td>
</tr>
<tr>
<td>CHN</td>
<td>−0.03</td>
<td>−0.06</td>
<td>−0.04</td>
<td>−0.41</td>
<td>2.24</td>
<td>−0.35</td>
</tr>
<tr>
<td>FRA</td>
<td>3.59</td>
<td>1.63</td>
<td>1.12</td>
<td>0.61</td>
<td>−0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>DEU</td>
<td>2.33</td>
<td>1.63</td>
<td>0.75</td>
<td>1.52</td>
<td>0.01</td>
<td>0.74</td>
</tr>
<tr>
<td>IND</td>
<td>2.37</td>
<td>−1.22</td>
<td>1.06</td>
<td>1.04</td>
<td>0.70</td>
<td>1.89</td>
</tr>
<tr>
<td>ITA</td>
<td>3.71</td>
<td>1.66</td>
<td>−0.19</td>
<td>−0.20</td>
<td>−1.32</td>
<td>−0.34</td>
</tr>
<tr>
<td>JPN</td>
<td>4.05</td>
<td>0.77</td>
<td>1.09</td>
<td>−0.22</td>
<td>−0.15</td>
<td>0.69</td>
</tr>
<tr>
<td>ZAF</td>
<td>2.37</td>
<td>0.30</td>
<td>−0.84</td>
<td>−1.11</td>
<td>0.50</td>
<td>−1.20</td>
</tr>
<tr>
<td>GBR</td>
<td>0.50</td>
<td>0.72</td>
<td>0.75</td>
<td>0.42</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>USA</td>
<td>1.00</td>
<td>0.42</td>
<td>0.46</td>
<td>0.73</td>
<td>0.65</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Source: Penn World Table 10.01 (Feenstra et al., 2015) & Author’s calculations.
Figure 210: Total factor productivity index (base 100 = 1960)

Source: Penn World Table 10.01 (Feenstra et al., 2015) & Author’s calculations.
Economic module
Total factor productivity

Figure 211: Dynamics of the TFP growth rate*

*We use the following calibration rule: $\delta_A = \sqrt[d]{d} - 1$
Economic module
Investment, capital stock and gross output

- Penn World Table/IMF’s ICSD
- In 2019, we obtain \( I(2019) = \$30.625 \text{ tn}, K(2019) = \$318.773 \text{ tn} \) and \( Y(2019) = \$124.418 \text{ tn} \)
- We also have:
  \[
  \delta_K(t) = \frac{K(t-1) - K(t) + I(t)}{K(t-1)}
  \]
  and we obtain \( \delta_K(2019) = 6.25\% \)
- To calibrate the initial value of \( A(t) \), we inverse the Coob-Douglas function:
  \[
  A(2019) = \frac{Y(t)}{K(t)^\gamma L(t)^{1-\gamma}} = \frac{124.418}{318.773^{0.30} \times 7.725^{0.70}} = 5.276
  \]
- The saving rate \( s(t) \) is exogenous

Thierry Roncalli

Course 2023-2024 in Sustainable Finance
Figure 212: Historical estimates of $I(t)$, $K(t)$, $Y(t)$ and $\delta_K(t)$

Source: IMF Investment and Capital Stock Dataset (2021) & Author’s calculations.
Economic module

**Figure 213:** Simulation of the DICE macroeconomic module

- $I(t)$ (in $\text{tn}$)
- $K(t)$ (in $\text{tn}$)
- $L(t)$ (in bn)
- $Y(t)$ (in $\text{tn}$)

$s(t) = 25\%$

$s(t) = 15\%$
The survival function is given by:

\[ \Omega_{\text{climate}}(t) = \Omega_D(t) \Omega_\Lambda(t) = \frac{1}{1 + D(t) (1 - \Lambda(t))} \]

where:

- \( D(t) \geq 0 \) is the climate damage function (physical risk)
- \( \Lambda(t) \geq 0 \) is the mitigation or abatement cost (transition risk)
The cost $D(t)$ resulting from natural disasters depends on the atmospheric temperature $T\text{AT}(t)$:

$$D(t) = \psi_1 T\text{AT}(t) + \psi_2 T\text{AT}(t)^2$$

The abatement cost function depends on the control variable $\mu(t)$:

$$\Lambda(t) = \theta_1(t) \mu(t)^{\theta_2}$$

The global impact of climate change is equal to:

$$\Omega_{\text{climate}}(t) = \frac{1 - \theta_1(t) \mu(t)^{\theta_2}}{1 + \psi_1 T\text{AT}(t) + \psi_2 T\text{AT}(t)^2}$$
Economic module
Cost function of climate change

Figure 214: Loss function due to climate damage costs

Weitzman
Hanemann-Weitzman
Pindyck
Hanemann
Newbold-Marten
Nordhaus

\( L_D (T_{AT}) \) (in %)

\( T_{AT} \) (in °C)
Economic module
Cost function of climate change

Figure 215: Abatement cost function
The total GHG emissions depend on the production $Y(t)$ and the land use emissions $CE_{Land}(t)$:

$$CE(t) = CE_{Industry}(t) + CE_{Land}(t)$$
$$= (1 - \mu(t))\sigma(t)Y(t) + CE_{Land}(t)$$

- $\sigma(t)$ is the anthropogenic carbon intensity of the economy:

$$\sigma(t) = (1 + g_{\sigma}(t))\sigma(t - 1)$$

where:

$$g_{\sigma}(t) = \frac{1}{1 + \delta_{\sigma}}g_{\sigma}(t - 1)$$
Figure 216: Physical carbon pump

Source: ocean-climate.org.
We have:

\[
\begin{align*}
CC_{AT}(t) &= \phi_{1,1}CC_{AT}(t-1) + \phi_{1,2}CC_{UP}(t-1) + \phi_1 CE(t) \\
CC_{UP}(t) &= \phi_{2,1}CC_{AT}(t-1) + \phi_{2,2}CC_{UP}(t-1) + \phi_{2,3}CC_{LO}(t-1) \\
CC_{LO}(t) &= \phi_{3,2}CC_{UP}(t-1) + \phi_{3,3}CC_{LO}(t-1)
\end{align*}
\]

The dynamics of \( CC = (CC_{AT}, CC_{UP}, CC_{LO}) \) is a VAR(1) process:

\[
CC(t) = \Phi_{CC}CC(t-1) + B_{CC}CE(t)
\]

Carbon cycle diffusion matrix

We have:

\[
\Phi_{CC} = \begin{pmatrix}
91.20\% & 3.83\% & 0 \\
8.80\% & 95.92\% & 0.03\% \\
0 & 0.25\% & 99.97\%
\end{pmatrix}
\]
Figure 217: Impulse response analysis ($\Delta C E = -1 \text{ GtCO}_2\text{e}$)
We have:

\[ F_{RAD}(t) = \frac{\eta}{\ln 2} \ln \left( \frac{CC_{AT}(t)}{CC_{AT}(1750)} \right) + F_{EX}(t) \]

where:

- \( F_{RAD}(t) \) is the change in total radiative forcing of GHG emissions since 1750 (expressed in W/m\(^2\))
- \( \eta \) is the temperature forcing parameter
- \( F_{EX}(t) \) is the exogenous forcing (other GHG emissions)
The climate system for temperatures is characterized by a two-layer system:

\[
\begin{align*}
    \mathcal{T}_{AT}(t) &= \mathcal{T}_{AT}(t-1) + \xi_1 (\mathcal{F}_{RAD}(t) - \xi_2 \mathcal{T}_{AT}(t-1) - \\
                      &\quad \xi_3 (\mathcal{T}_{AT}(t-1) - \mathcal{T}_{LO}(t-1))) \\
    \mathcal{T}_{LO}(t) &= \mathcal{T}_{LO}(t-1) + \xi_4 (\mathcal{T}_{AT}(t-1) - \mathcal{T}_{LO}(t-1))
\end{align*}
\]

Let \( \mathcal{T} = (\mathcal{T}_{AT}, \mathcal{T}_{LO}) \) be the temperature vector. We have:

\[
\mathcal{T}(t) = \Xi_T \mathcal{T}(t-1) + B_T \mathcal{F}_{RAD}(t)
\]
Table 108: Output of the DICE climate module ($Y(t) = Y(t_0), \mu(t) = \mu(t_0)$)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$CE(t)$</th>
<th>$\sigma(t)$</th>
<th>$CC_{AT}(t)$</th>
<th>$F_{RAD}(t)$</th>
<th>$T_{AT}(t)$</th>
<th>$T_{LO}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>36.91</td>
<td>0.55</td>
<td>830.4</td>
<td>2.14</td>
<td>0.800</td>
<td>0.007</td>
</tr>
<tr>
<td>2015</td>
<td>36.25</td>
<td>0.55</td>
<td>825.7</td>
<td>2.14</td>
<td>0.900</td>
<td>0.027</td>
</tr>
<tr>
<td>2020</td>
<td>36.06</td>
<td>0.56</td>
<td>821.9</td>
<td>2.14</td>
<td>0.986</td>
<td>0.048</td>
</tr>
<tr>
<td>2025</td>
<td>35.97</td>
<td>0.57</td>
<td>818.9</td>
<td>2.14</td>
<td>1.061</td>
<td>0.072</td>
</tr>
<tr>
<td>2030</td>
<td>35.98</td>
<td>0.57</td>
<td>816.6</td>
<td>2.15</td>
<td>1.127</td>
<td>0.097</td>
</tr>
<tr>
<td>2035</td>
<td>36.05</td>
<td>0.58</td>
<td>814.9</td>
<td>2.16</td>
<td>1.186</td>
<td>0.122</td>
</tr>
<tr>
<td>2040</td>
<td>36.18</td>
<td>0.58</td>
<td>813.9</td>
<td>2.18</td>
<td>1.238</td>
<td>0.149</td>
</tr>
<tr>
<td>2045</td>
<td>36.36</td>
<td>0.59</td>
<td>813.3</td>
<td>2.20</td>
<td>1.286</td>
<td>0.176</td>
</tr>
<tr>
<td>2050</td>
<td>36.58</td>
<td>0.59</td>
<td>813.3</td>
<td>2.23</td>
<td>1.329</td>
<td>0.204</td>
</tr>
<tr>
<td>2055</td>
<td>36.82</td>
<td>0.60</td>
<td>813.6</td>
<td>2.26</td>
<td>1.370</td>
<td>0.232</td>
</tr>
<tr>
<td>2060</td>
<td>37.09</td>
<td>0.61</td>
<td>814.4</td>
<td>2.29</td>
<td>1.408</td>
<td>0.261</td>
</tr>
<tr>
<td>2065</td>
<td>37.39</td>
<td>0.61</td>
<td>815.4</td>
<td>2.32</td>
<td>1.445</td>
<td>0.289</td>
</tr>
<tr>
<td>2070</td>
<td>37.70</td>
<td>0.62</td>
<td>816.8</td>
<td>2.35</td>
<td>1.480</td>
<td>0.318</td>
</tr>
<tr>
<td>2075</td>
<td>38.02</td>
<td>0.62</td>
<td>818.4</td>
<td>2.39</td>
<td>1.514</td>
<td>0.347</td>
</tr>
<tr>
<td>2080</td>
<td>38.36</td>
<td>0.63</td>
<td>820.3</td>
<td>2.43</td>
<td>1.547</td>
<td>0.376</td>
</tr>
<tr>
<td>2085</td>
<td>38.71</td>
<td>0.64</td>
<td>822.4</td>
<td>2.46</td>
<td>1.580</td>
<td>0.406</td>
</tr>
<tr>
<td>2090</td>
<td>39.06</td>
<td>0.64</td>
<td>824.7</td>
<td>2.50</td>
<td>1.612</td>
<td>0.435</td>
</tr>
<tr>
<td>2095</td>
<td>39.43</td>
<td>0.65</td>
<td>827.1</td>
<td>2.55</td>
<td>1.645</td>
<td>0.464</td>
</tr>
<tr>
<td>2100</td>
<td>39.80</td>
<td>0.66</td>
<td>829.7</td>
<td>2.59</td>
<td>1.677</td>
<td>0.494</td>
</tr>
</tbody>
</table>
Climate module

Figure 218: Simulation of the DICE climate module

- $Y(t)$ (in $\text{tn}$)
- $\mu(t)$
- $T_{AT}(t)$ (in $\circ\text{C}$)
- $T_{LO}(t)$ (in $\circ\text{C}$)

Legend:
- $g_Y = 0\%$, $g_\mu = 0\%$
- $g_Y = 1\%$, $g_\mu = 0.5\%$
- $g_Y = 2\%$, $g_\mu = 1\%$
- $g_Y = 3\%$, $g_\mu = 8\%$
Climate module

Figure 219: The nightmare climate-economic scenario ($g_Y = 0\%$, $\mu(t) = 0$)
The optimal control problem

Optimization problem

- The social welfare function $W$ is equal to:

$$W(s(t), \mu(t)) = \sum_{t=t_0+1}^{T} \frac{L(t)U(c(t))}{(1 + \rho)^{t-t_0}}$$

where $\rho$ is the (generational) discount rate and $c(t) = C(t)/L(t)$ is the consumption per capita.

- $U(c) = (c^{1-\alpha} - 1) / (1 - \alpha)$ is the CRRA utility function.

- The optimal control problem is then given by:

$$\left(s^*(t), \mu^*(t)\right) = \arg \max W(s(t), \mu(t))$$

s.t. \quad \left\{ \begin{array}{l}
\text{DICE Equations} \\
\mu(t) \in [0, 1] \\
s(t) \in [0, 1]
\end{array} \right.$$
The important variables are:

- $T_{AT}(t)$ — Atmospheric temperature
- $\mu(t)$ — Control rate (mitigation policies)
- $CE(t)$ — Total emissions of GHG
- $SCC(t)$ — Social cost of carbon
“The most important single economic concept in the economics of climate change is the social cost of carbon (SCC). This term designates the economic cost caused by an additional tonne of carbon dioxide emissions or its equivalent. In a more precise definition, it is the change in the discounted value of economic welfare from an additional unit of COtwo-equivalent emissions. The SCC has become a central tool used in climate change policy, particularly in the determination of regulatory policies that involve greenhouse gas emissions.” (Nordhaus, 2017).
Social cost of carbon (SCC)

Mathematical definition

- The social cost of carbon is then defined as:

\[
SCC(t) = \frac{\frac{\partial W(t)}{\partial CE(t)}}{\frac{\partial W(t)}{\partial C(t)}} = \frac{\partial C(t)}{\partial CE(t)}
\]

- It is expressed in $/tCO_2$
Social cost of carbon (SCC)

**Figure 220:** Optimal welfare scenario (DICE 2013R)

Social cost of carbon (SCC)

**Figure 221:** 2°C scenario (DICE 2013R)

Social cost of carbon (SCC)

Figure 222: Optimal welfare scenario (DICE 2016R)

Social cost of carbon (SCC)

**Figure 223:** 2°C scenario (DICE 2016R)

The tragedy of the horizon
In 2013, the DICE model suggested to reduce drastically CO$_2$ emissions...

Since 2016, the 2°C trajectory is no longer feasible! (minimum $\approx 2.6^\circ$C)

For many models, we now have:

$$P(\Delta T > 2^\circ C) > 95\%$$
### Table 109: Global SCC under different scenario assumptions (in $/tCO_2$)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2050</th>
<th>CAGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>31.2</td>
<td>37.3</td>
<td>44.0</td>
<td>51.6</td>
<td>102.5</td>
<td>3.46%</td>
</tr>
<tr>
<td>Optimal</td>
<td>30.7</td>
<td>36.7</td>
<td>43.5</td>
<td>51.2</td>
<td>103.6</td>
<td>3.54%</td>
</tr>
<tr>
<td>2.5°C-max</td>
<td>184.4</td>
<td>229.1</td>
<td>284.1</td>
<td>351.0</td>
<td>1006.2</td>
<td>4.97%</td>
</tr>
<tr>
<td>2.5°C-mean</td>
<td>106.7</td>
<td>133.1</td>
<td>165.1</td>
<td>203.7</td>
<td>543.3</td>
<td>4.76%</td>
</tr>
</tbody>
</table>

In 2007, Nicholas Stern published a report called *The Economics of Climate Change: The Stern Review*

The Stern Review called for sharp and immediate action to stabilize greenhouse gases because:

"the benefits of strong, early action on climate change outweighs the costs"

The Stern Review proposes to use $\rho = 0.10\%$
Figure 224: Discounted value of $10
The time (or generational) discount rate $\rho$ is also called the pure rate of time preference.

It is related to the Ramsey rule:

$$ r = \rho + \alpha g $$

where:

- $r$ is the real interest rate
- $g = \partial c(t)/c(t)$ is the growth rate of per capita consumption
- $\alpha$ is the consumption elasticity of the utility function
Social cost of carbon (SCC)
The Stern-Nordhaus controversy

We report the computations done by Dasgupta (2008):

<table>
<thead>
<tr>
<th>Model</th>
<th>$\rho$</th>
<th>$\alpha$</th>
<th>$g_c$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cline (1992)</td>
<td>0.0%</td>
<td>1.5</td>
<td>1.3%</td>
<td>2.05%</td>
</tr>
<tr>
<td>Nordhaus (2007)</td>
<td>3.0%</td>
<td>1.0</td>
<td>1.3%</td>
<td>4.30%</td>
</tr>
<tr>
<td>Stern (2007)</td>
<td>0.1%</td>
<td>1.0</td>
<td>1.3%</td>
<td>1.40%</td>
</tr>
</tbody>
</table>
### Social cost of carbon (SCC)

**The Stern-Nordhaus controversy**

<table>
<thead>
<tr>
<th>Discount rate</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2050</th>
<th>CAGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern</td>
<td>197.4</td>
<td>266.5</td>
<td>324.6</td>
<td>376.2</td>
<td>629.2</td>
<td>3.37%</td>
</tr>
<tr>
<td>Nordhaus</td>
<td>30.7</td>
<td>36.7</td>
<td>43.5</td>
<td>51.2</td>
<td>103.6</td>
<td>3.54%</td>
</tr>
<tr>
<td>2.5%</td>
<td>128.5</td>
<td>140.0</td>
<td>152.0</td>
<td>164.6</td>
<td>235.7</td>
<td>1.75%</td>
</tr>
<tr>
<td>3%</td>
<td>79.1</td>
<td>87.3</td>
<td>95.9</td>
<td>104.9</td>
<td>156.6</td>
<td>1.97%</td>
</tr>
<tr>
<td>4%</td>
<td>36.3</td>
<td>40.9</td>
<td>45.8</td>
<td>51.1</td>
<td>81.7</td>
<td>2.34%</td>
</tr>
<tr>
<td>5%</td>
<td>19.7</td>
<td>22.6</td>
<td>25.7</td>
<td>29.1</td>
<td>49.2</td>
<td>2.65%</td>
</tr>
</tbody>
</table>

Some models

- AIM __________________________ RCP 6.0
- DICE/RICE
- FUND
- GCAM
- IMACLIM (CIRED)
- IMAGE _________________________ RCP 2.6
- MESSAGE _________________________ RCP 8.5
- MiniCAM ________________________ RCP 4.5
- PAGE
- REMIND
- RESPONSE (CIRED)
- WITCH
Some models

Table 111: Main integrated assessment models

<table>
<thead>
<tr>
<th>Model</th>
<th>Reference</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stylized simple models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DICE</td>
<td>Nordhaus and Sztorc (2013)</td>
<td>Dynamic Integrated Climate-Economy</td>
</tr>
<tr>
<td>FUND</td>
<td>Anthoff and Tol (2014)</td>
<td>Climate Framework for Uncertainty, Negotiation and Distribution</td>
</tr>
<tr>
<td><strong>Complex models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GCAM</td>
<td>Calvin et al. (2019)</td>
<td>Global Change Assessment Model</td>
</tr>
<tr>
<td>GLOBIOM</td>
<td>Havlik et al. (2018)</td>
<td>Global Biosphere Management Model</td>
</tr>
<tr>
<td>IMACLIM-R</td>
<td>Sassi et al. (2010)</td>
<td>Integrated Model to Assess Climate Change</td>
</tr>
<tr>
<td>IMAGE</td>
<td>Stehfest et al. (2014)</td>
<td>Integrated Model to Assess the Greenhouse Effect</td>
</tr>
<tr>
<td>MAGICCC</td>
<td>Meinshausen et al. (2011)</td>
<td>Model for the Assessment of Greenhouse Gas Induced Climate Change</td>
</tr>
<tr>
<td>MAgPIE</td>
<td>Dietrich et al. (2019)</td>
<td>Model of Agricultural Production and its Impact on the Environment</td>
</tr>
<tr>
<td>REMIND</td>
<td>Aboumahboub et al. (2020)</td>
<td>R Egional Model of INvestments and Development</td>
</tr>
<tr>
<td>WITCH</td>
<td>Bosetti et al. (2006)</td>
<td>World Induced Technical Change Hybrid</td>
</tr>
</tbody>
</table>

Source: Grubb et al. (2021) & Author’s research.
## Stylized IAMs

### The Leaders
- DICE
- FUND
- PAGE

⇒ SCC: PAGE ≻ DICE ≻ FUND
Figure 225: Histogram of the 150,000 US Government SCC estimates for 2020 with a 3% discount rate
Stylized IAMs
The liability/fairness question

Liability

Aristotle (384 BC – 322 BC)

Karl Marx and Friedrich Engels (1848)
The Communist Manifesto

\[ \text{Environment} \rightarrow \text{Social} \rightarrow \text{Growth} \rightarrow \text{Environment} \]
Stylized IAMs

The liability/fairness question

Fairness

Du Contrat Social
Stylized IAMs
Climate risk and inequalities

Three types of inequalities

- Spatial (or regional) inequalities
- Social (or intra-generation) inequalities
- Time (or inter-generation) inequalities

⇒ These issues are highly related to liability risks:

“[…] liability risks stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible” (Mark Carney, 2018)

- Regional inequalities ⇒ lack of cooperation between countries (e.g., Glasgow COP 26)
- Social inequalities ⇒ climate action postponing (e.g., carbon tax in France)
The Regional Integrated model of Climate and the Economy (RICE) model is a sub-regional neoclassical climate economy model (Nordhaus and Yang, 1996).

⇒ Sub-regional problem of welfare:
  - Each region of the world has a different utility functions
  - The big issue is how the most developed regions can finance the transition to a low-carbon economy of the less developed regions

**Both spacial and time (inter-generation) inequalities**
The **Nested Inequalities Climate-Economy** (NICE) model integrates distributional differences of income (Dennig et al., 2015)

“ [...] If the distribution of damage is less skewed to high income than the distribution of consumption, then weak or no climate policy will result in sufficiently large damages on the lower economic strata to eventually stop their welfare levels from improving, and instead cause them to decline” (Dennig et al., 2015)

Both social (intra-generation) and time (inter-generation) inequalities
Figure 226: Linkages between the major systems in GCAM

Source: Calvin et al. (2019).
Complex IAMs

Figure 227: The main land use sectors of GLOBIOM

Source: https://iiasa.github.io/GLOBIOM.
Figure 228: Overview of the IIASA IAM framework

Figure 229: The Remind-MAgPIE framework

Criticisms of integrated assessment models

“IAM-based analyses of climate policy create a perception of knowledge and precision that is illusory and can fool policymakers into thinking that the forecasts the models generate have some kind of scientific legitimacy” (Pindyck, 2017)

- Certain inputs, such as the discount rate, are arbitrary
- There is a lot of uncertainty about climate sensitivity and the temperature trajectory
- Modeling damage functions is arbitrary
- IAMs are unable to consider tail risk
Figure 230: Scenario evaluation

- Climate scenario (input)
- Evaluation Process
- Economic scenario (output)
The representative concentration pathways (RCPs) — IPCC AR5
The IEA scenarios
The 1.5°C scenarios — SR15
The scenarios for the future published — IPCC AR6
RCP 2.6: GHG emissions start declining by 2020 and go to zero by 2100 (IMAGE)
RCP 4.5: GHG emissions peak around 2040, and then decline (MiniCAM)
RCP 6.0: GHG emissions peak around 2080, and then decline (AIM)
RCP 8.5: GHG emissions continue to rise throughout the 21st century (MESSAGE)
Climate scenarios
The RCP scenarios

Figure 231: Total radiative forcing (in W/m²)

Source: https://tntcat.iiasa.ac.at/RcpDb.
Climate scenarios

The RCP scenarios

Figure 232: Greenhouse gas concentration trajectory

Source: https://tntcat.iiasa.ac.at/RcpDb.
Climate scenarios

The RCP scenarios

Figure 233: Greenhouse gas emissions trajectory

Source: https://tntcat.iiasa.ac.at/RcpDb.
Figure 234: Total GHG emissions trajectory (in GtCO₂e)

Source: https://tntcat.iiasa.ac.at/RcpDb.
Figure 235: Direct CO₂ emissions (in Gt)

Climate scenarios
The 1.5°C scenarios

Figure 236: IPCC 1.5°C scenarios of CO₂ emissions

Source: https://data.ene.iaasa.ac.at/iamc-1.5c-explorer.
Climate scenarios
The 1.5°C scenarios

Figure 237: Confidence interval of the average IPCC 1.5°C scenario

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Figure 238: IPCC 1.5°C scenarios of the global mean temperature

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
**Climate scenarios**

The 1.5°C scenarios

**Figure 239:** Confidence interval of the exceedance probability \( \Pr \{ T > 1.5^\circ C \} \)

![Confidence interval of the exceedance probability](https://data.ene.iiasa.ac.at/iamc-1.5c-explorer)

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Climate scenarios

The 1.5°C scenarios

Figure 240: Confidence interval of the exceedance probability $\Pr \{ T > 2^\circ C \}$

Source: https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.
Climate scenarios
The AR6 scenarios

The new dataset contains 188 models, 1,389 scenarios, 244 countries and regions, and 1,791 variables, which can be split into six main categories:

- **Agriculture**: agricultural demand, crop, food, livestock, production, etc.
- **Capital cost**: coal, electricity, gas, hydro, hydrogen, nuclear, etc.
- **Energy**: capacity, efficiency, final energy, lifetime, OM cost, primary/secondary energy, etc.
- **GHG impact**: carbon sequestration, concentration, emissions, forcing, temperature, etc.
- **Natural resources**: biodiversity, land cover, water consumption, etc.
- **Socio-economic variables**: capital formation, capital stock, consumption, discount rate, employment, expenditure, export, food demand, GDP, Gini coefficient, import, inequality, interest rate, investment, labour supply, policy cost, population, prices, production, public debt, government revenue, taxes, trade, unemployment, value added, welfare, etc.
Climate scenarios
The AR6 scenarios

Figure 241: Histogram of some AR6 output variables by 2100

- Surface temperature (in °C)
- Carbon sequestration (CCS) (in bn)
- Electricity capacity (in $ tn)
- Kyoto gases (in GtCO$_2$e)

Source: https://data.ene.iiasa.ac.at/ar6.
Figure 242: Histogram of some AR6 output variables by 2100

Agricultural demand (in GtCO₂e)

Agricultural production (in $ tn)

Land cover, cropland (in K$/tCO₂)

Water withdrawal, irrigation (in $ tn)

Source: https://data.ene.iiasa.ac.at/ar6.
“The SSP narratives [are] a set of five qualitative descriptions of future changes in demographics, human development, economy and lifestyle, policies and institutions, technology, and environment and natural resources. [...] Development of the narratives drew on expert opinion to (1) identify key determinants of the challenges [to mitigation and adaptation] that were essential to incorporate in the narratives and (2) combine these elements in the narratives in a manner consistent with scholarship on their inter-relationships. The narratives are intended as a description of plausible future conditions at the level of large world regions that can serve as a basis for integrated scenarios of emissions and land use, as well as climate impact, adaptation and vulnerability analyses.” (O’Neill et al., 2017)
Figure 243: The shared socioeconomic pathways

Source: O’Neill et al. (2017).
Figure 244: The shared socioeconomic pathways

- **SSP1**: Sustainability (Taking the Green Road)
  - Low challenges for both mitigation and adaptation, rapid development

- **SSP2**: Middle of the Road
  - Moderate challenges for mitigation and adaptation

- **SSP3**: Regional Rivalry (A Rocky Road)
  - High challenges for both mitigation and adaptation — Concerns about competitiveness/security and regional conflicts pushing countries to focus on regional issues

- **SSP4**: Inequality (A Road Divided)
  - Low challenges for mitigation, high for adaptation — Unequal investment in human capital, concentration of power in a small business elite

- **SSP5**: Fossil-fueled Development (Taking the Highway)
  - High challenges for mitigation, low for adaptation

Source: O’Neill et al. (2017).
The mitigation/adaptation trade-off is obviously an environmental issue, but the SSPs encompass other environmental narratives, e.g. land use, energy efficiency and green economy.

The social dimension is the central theme of SSPs, and concerns demography, wealth, inequality & poverty, health, education, employment, and more generally the evolution of society. This explains that SSPs and SDGs are highly interconnected.

Finally, the governance dimension is present though two major themes: international fragmentation or cooperation, and the political/economic system, including corruption, stability, rule of law, etc.
Shared socioeconomic pathways

- SSP1: IMAGE (PBL)
- SSP2: MESSAGE-GLOBIOM (IIASA)
- SSP3: AIM/CGE (NIES)
- SSP4: GCAM (PNNL)
- SSP5: REMIND-MAGPIE (PIK) and WITCH-GLOBIOM (FEEM)
Figure 245: SSP demography projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Shared socioeconomic pathways

**Figure 246: SSP economic projections**

- **GDP (in $ tn)**
- **GDP per capita (in K$)**
- **GDP growth (in %)**
- **GDP/Energy (in $ bn/EJ)**

Source: [https://tntcat.iiasa.ac.at/SspDb](https://tntcat.iiasa.ac.at/SspDb).
Figure 247: SSP environmental projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 248: SSP land use projections

Source: https://tntcat.iiasa.ac.at/SspDb.
Shared socioeconomic pathways

Figure 249: Example of SSP regional differences

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 250: Gini coefficient projections by 2100

Source: https://tntcat.iiasa.ac.at/SspDb.
Figure 251: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)
Overview

The NGFS scenarios explore a set of six scenarios which are consistent with the NGFS framework (see figure) published in the First NGFS Comprehensive Report covering the following dimensions:

- **Orderly** scenarios assume climate policies are introduced early and become gradually more stringent. Both physical and transition risks are relatively subdued.
- **Disorderly** scenarios explore higher transition risk due to policies being delayed or divergent across countries and sectors. For example, carbon prices are typically higher for a given temperature outcome.
- **Hot house world** scenarios assume that some climate policies are implemented in some jurisdictions, but globally efforts are insufficient to halt significant global warming. The scenarios result in severe physical risk including irreversible impacts like sea-level rise.

Objectives and framework

The NGFS scenarios explore the impacts of climate change and climate policy with the aim of providing a common reference framework.

Figure 252: NGFS scenarios framework (2022)
NGFS scenarios

- **Orderly scenarios**
  - #1 Net zero 2050 (NZ)
  - #2 Below 2°C (B2D)

- **Disorderly scenarios**
  - #3 Divergent net zero (DNZ)
  - #4 Delayed transition (DT)

- **Hot house world scenarios**
  - #5 Nationally determined contributions (NDC)
  - #6 Current policies (CP)
Scenarios

- Net Zero 2050
- Below 2°C
- Divergent Net Zero
- Delayed Transition
- Nationally Determined Contributions (NDCs)
- Current Policies

Physical risk
- Policy ambition
- Policy reaction
- Technology change
- Carbon dioxide removal

Transition risk
- Regional policy variation

Colour coding indicates whether the characteristic makes the scenario more or less severe from a macro-/financial risk perspective:
- Higher risk
- Moderate risk
- Lower risk

Figure 253: Physical and transition risk level of NGFS scenarios
### Variables (economic)
- Central bank intervention rate
- Domestic demand
- Effective exchange rate
- Exchange rate
- Exports (goods and services)
- Gross Domestic Product (GDP)
- Gross domestic income
- Imports (goods and services)
- Inflation rate
- Long term & real interest rates
- Trend output for capacity utilisation
- Unemployment

### Variables (energy)
- Coal price
- Gas price
- Oil price
- Quarterly consumption of coal
- Quarterly consumption of gas
- Quarterly consumption of oil
- Quarterly consumption of renewables
- Total energy consumption

### Models (IPCC)
- Meta-model: NiGEM 1.21
- Sub-models:
  - GCAM 5.3
  - MESSAGE-GLOBIOM 1.1
  - REMIND-MAgPIE 2.1-4.2

### 6 scenarios
- Net Zero 2050 (NZ)
- Below 2°C (B2D)
- Divergent Net Zero (DNZ)
- Delayed Transition (DT)
- Notionally Determined Contribution (NDC)
- Current Policies (CP)
### Table 112: Impact of climate change on the GDP loss by 2050 (GCAM)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−3.09</td>
<td>−5.64</td>
<td>−2.35</td>
<td>−3.28</td>
<td>−5.15</td>
<td>−2.56</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−0.75</td>
<td>−3.66</td>
<td>−1.78</td>
<td>−0.89</td>
<td>−0.88</td>
<td></td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.84</td>
<td>−5.64</td>
<td>−6.00</td>
<td>−5.05</td>
<td>−6.03</td>
<td>−3.44</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td>−6.03</td>
<td>−5.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 113: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−2.05</td>
<td>−5.26</td>
<td>−1.55</td>
<td>−2.64</td>
<td>−4.78</td>
<td>−1.59</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−1.46</td>
<td></td>
<td>−10.00</td>
<td>−10.77</td>
<td>−1.39</td>
<td>−3.26</td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.51</td>
<td>−5.26</td>
<td>−11.53</td>
<td>−13.37</td>
<td>−6.16</td>
<td>−4.84</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td></td>
<td></td>
<td>−11.57</td>
<td>−13.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 114: Impact of climate change on the GDP loss by 2050 (REMIND-MAgPIE)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chronic physical risk</td>
<td>−2.24</td>
<td>−6.05</td>
<td>−1.67</td>
<td>−2.65</td>
<td>−5.41</td>
<td>−1.76</td>
</tr>
<tr>
<td>Transition risk</td>
<td>−0.78</td>
<td></td>
<td>−3.01</td>
<td>−1.95</td>
<td>−0.33</td>
<td>−1.46</td>
</tr>
<tr>
<td>Combined risk</td>
<td>−3.02</td>
<td>−6.05</td>
<td>−4.68</td>
<td>−4.59</td>
<td>−5.73</td>
<td>−3.21</td>
</tr>
<tr>
<td>Combined + business confidence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−4.70</td>
</tr>
</tbody>
</table>

Thierry Roncalli

Course 2023-2024 in Sustainable Finance
Table 115: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

<table>
<thead>
<tr>
<th>Risk</th>
<th>B2D</th>
<th>CP</th>
<th>DNZ</th>
<th>DT</th>
<th>NDC</th>
<th>NZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>-1.50</td>
<td>-7.29</td>
<td>-5.44</td>
<td>-8.76</td>
<td>-6.78</td>
<td>-1.38</td>
</tr>
<tr>
<td>Australia</td>
<td>-4.11</td>
<td>-3.90</td>
<td>-11.03</td>
<td>-11.74</td>
<td>-5.77</td>
<td>-5.19</td>
</tr>
<tr>
<td>Brazil</td>
<td>-4.43</td>
<td>-5.92</td>
<td>-13.15</td>
<td>-15.90</td>
<td>-6.67</td>
<td>-6.65</td>
</tr>
<tr>
<td>Canada</td>
<td>-1.02</td>
<td>-2.37</td>
<td>-15.07</td>
<td>-18.12</td>
<td>-4.33</td>
<td>-4.87</td>
</tr>
<tr>
<td>China</td>
<td>-2.33</td>
<td>-4.97</td>
<td>-5.13</td>
<td>-6.73</td>
<td>-4.67</td>
<td>-2.76</td>
</tr>
<tr>
<td>Developing Europe</td>
<td>-0.28</td>
<td>-3.11</td>
<td>-0.56</td>
<td>-7.38</td>
<td>-2.73</td>
<td>0.39</td>
</tr>
<tr>
<td>Europe</td>
<td>-1.02</td>
<td>-2.84</td>
<td>-9.64</td>
<td>-11.02</td>
<td>-4.01</td>
<td>-1.62</td>
</tr>
<tr>
<td>France</td>
<td>-1.15</td>
<td>-2.80</td>
<td>-8.35</td>
<td>-9.48</td>
<td>-3.68</td>
<td>-1.56</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.77</td>
<td>-2.38</td>
<td>-8.58</td>
<td>-9.38</td>
<td>-3.63</td>
<td>-1.21</td>
</tr>
<tr>
<td>India</td>
<td>-3.45</td>
<td>-8.61</td>
<td>-16.43</td>
<td>-17.74</td>
<td>-8.71</td>
<td>-3.86</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.15</td>
<td>-3.69</td>
<td>-9.23</td>
<td>-12.88</td>
<td>-4.85</td>
<td>-0.89</td>
</tr>
<tr>
<td>Japan</td>
<td>-1.26</td>
<td>-4.14</td>
<td>-7.16</td>
<td>-10.05</td>
<td>-4.61</td>
<td>-1.40</td>
</tr>
<tr>
<td>Latam</td>
<td>-4.35</td>
<td>-6.10</td>
<td>-12.70</td>
<td>-14.58</td>
<td>-6.97</td>
<td>-5.74</td>
</tr>
<tr>
<td>Russia</td>
<td>-12.18</td>
<td>-2.26</td>
<td>-23.46</td>
<td>-23.80</td>
<td>-7.54</td>
<td>-17.11</td>
</tr>
<tr>
<td>South Africa</td>
<td>-2.02</td>
<td>-5.06</td>
<td>-7.24</td>
<td>-9.16</td>
<td>-5.38</td>
<td>-3.04</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.11</td>
<td>-3.49</td>
<td>-3.23</td>
<td>-7.57</td>
<td>-3.33</td>
<td>0.12</td>
</tr>
<tr>
<td>Spain</td>
<td>-2.41</td>
<td>-3.81</td>
<td>-12.49</td>
<td>-12.89</td>
<td>-5.41</td>
<td>-3.30</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.32</td>
<td>-2.25</td>
<td>-9.47</td>
<td>-10.35</td>
<td>-2.18</td>
<td>2.30</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.86</td>
<td>-1.90</td>
<td>-6.50</td>
<td>-8.05</td>
<td>-2.56</td>
<td>-1.33</td>
</tr>
<tr>
<td>United States</td>
<td>-2.67</td>
<td>-4.38</td>
<td>-15.37</td>
<td>-17.66</td>
<td>-6.31</td>
<td>-4.36</td>
</tr>
<tr>
<td>World</td>
<td>-3.51</td>
<td>-5.26</td>
<td>-11.53</td>
<td>-13.37</td>
<td>-6.16</td>
<td>-4.84</td>
</tr>
</tbody>
</table>
Figure 254: GDP impact by 2050 (% change from baseline) — Delayed transition scenario
NGFS scenarios

Figure 255: GDP impact by 2050 (% change from baseline) — Net zero 2050 scenario
Figure 256: Impact of climate scenarios on economics (% change from baseline) — China
Figure 257: Impact of climate scenarios on economics (% change from baseline) — United States
Figure 258: Impact of climate scenarios on economics (% change from baseline) — France

- Inflation
- Unemployment
- Private investment
- Productivity
- Equity prices
- Public investment

Legend:
- CP
- NZ
- DT
Figure 259: Impact of climate scenarios on economics (% change from baseline) — United Kingdom
The NGFS scenarios explore a set of seven scenarios which are consistent with the NGFS framework (see figure). The scenarios cover the following dimensions:

- **Orderly** scenarios assume climate policies are introduced early and become gradually more stringent. Both physical and transition risks are relatively subdued.
- **Disorderly** scenarios explore higher transition risks due to policies being delayed or divergent across countries and sectors. For example, (shadow) carbon prices are typically higher for a given temperature outcome.
- **Hot house world** scenarios assume that some climate policies are implemented in some jurisdictions, but globally efforts are insufficient to halt significant global warming. The scenarios result in severe physical risk including irreversible impacts like sea-level rise.
- **Too-little-too-late** scenarios assume that a late and uncoordinated transition fails to limit physical risks. This quadrant is explored for the first time in this vintage.

Objectives and framework

The NGFS scenarios explore the impacts of climate change and the transition with the aim of providing a common reference framework.

Positioning of scenarios is approximate, based on an assessment of physical and transition risks out to 2100.

Notes: (*) Shadow carbon prices are defined as the marginal abatement cost of an incremental ton of greenhouse gas emissions. Prices are influenced by the stringency of policy as well as how technology costs will evolve.
**NGFS scenarios (2023)**

Figure 261: Physical and transition risk level of NGFS scenarios (2023)

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Scenario</th>
<th>Physical risk</th>
<th>Transition risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End of century (peak) warming – model average</td>
<td>Policy reaction</td>
<td>Technology change</td>
</tr>
<tr>
<td>Orderly</td>
<td>Low Demand</td>
<td>1.4 °C (1.6 °C)</td>
<td>Immediate</td>
</tr>
<tr>
<td></td>
<td>Net Zero 2050</td>
<td>1.4 °C (1.6 °C)</td>
<td>Immediate</td>
</tr>
<tr>
<td></td>
<td>Below 2 °C</td>
<td>1.7 °C (1.8 °C)</td>
<td>Immediate and smooth</td>
</tr>
<tr>
<td>Disorderly</td>
<td>Delayed Transition</td>
<td>1.7 °C (1.8 °C)</td>
<td>Delayed</td>
</tr>
<tr>
<td></td>
<td>Hot house world</td>
<td>2.4 °C (2.4 °C)</td>
<td>NDCs</td>
</tr>
<tr>
<td></td>
<td>Nationally Determined Contributions (NDCs)</td>
<td>2.4 °C (2.4 °C)</td>
<td>NDCs</td>
</tr>
<tr>
<td></td>
<td>Current Policies</td>
<td>2.9 °C (2.9 °C)</td>
<td>None – current policies</td>
</tr>
<tr>
<td></td>
<td>Too-little-too-late</td>
<td>2.3 °C (2.3 °C)</td>
<td>Delayed and Fragmented</td>
</tr>
</tbody>
</table>

Thierry Roncalli

Course 2023-2024 in Sustainable Finance
The input-output (IO) model was first introduced by Leontief (1936, 1941)

It quantifies the interdependencies between different sectors in a single or multi-regional economy, based on the product flows between sectors

The underlying idea is to model the linkages between sectors and to describe the relationships from each of the producer/seller sectors to each of the purchaser/buyer sectors
The demand-pull quantity model

- $n$ different sectors
- $Z_{i,j}$ is the value of transactions from sector $i$ to sector $j$:
  - It is the output that sector $i$ sells to sector $j$
  - It is the input of sector $i$ required by sector $j$ for its production (or output)
- $y_i$ is the final demand for products sold by sector $i$
- $x_i$ is the total production of sector $i$
The demand-pull quantity model

- We have the following accounting identity:
  \[ x_i = \sum_{j=1}^{n} Z_{i,j} + y_i \]
  - Supply
  - Demand

  \[ z_i = \sum_{j=1}^{n} Z_{i,j} \] represents intermediate demand

- The interdependence relation between sectors is usually expressed as a ratio between \( Z_{i,j} \) and \( x_j \):
  \[ A_{i,j} = \frac{Z_{i,j}}{x_j} \]

- \( A = (A_{i,j}) = Z \text{ diag } (x)^{-1} \) is the input-output matrix of the technical coefficients \( A_{i,j} \)
In a matrix form, we have $x = Z \mathbf{1}_n + y$ and $Z = A \text{diag} (x) = A \odot x^T$, and we deduce that:

$$x = Ax + y$$

where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$

Assuming that final demand is exogenous, technical coefficients are fixed and output is endogenous, we obtain:

$$x = (I_n - A)^{-1} y$$

$L = (I_n - A)^{-1}$ is known as the Leontief inverse (or multiplier) matrix and represents the amount of total output from sector $i$ that is required by sector $j$ to satisfy its final demand.
This economy has four sectors: energy, materials, industrials and services. In this economy, businesses in the energy sector buy $500 of goods and services from other businesses in the energy sector, $500 of goods and services from the materials sector, $250 of goods and services from the industrials sector, and $100 of goods and services from the services sector. The final demand for goods and services produced in the energy sector is equal to $850, while the total output of this sector is equal to $5,000.
The demand-pull quantity model

We deduce that the matrix of technical coefficients is equal to:

\[ A = Z \text{diag}(x)^{-1} = \begin{pmatrix} 10\% & 20\% & 20\% & 10\% \\ 10\% & 10\% & 20\% & 5\% \\ 5\% & 20\% & 30\% & 10\% \\ 2\% & 5\% & 10\% & 35\% \end{pmatrix} \]

It follows that the multiplier matrix is equal to:

\[ \mathcal{L} = (I_4 - A)^{-1} = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix} \]

We verify that:

\[ x = \mathcal{L}y = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix} \begin{pmatrix} 850 \\ 875 \\ 3300 \\ 7025 \end{pmatrix} = \begin{pmatrix} 5000 \\ 4000 \\ 8000 \\ 12500 \end{pmatrix} \]
Suppose we have a variation in final demand. We obtain $\Delta x = \mathcal{L}\Delta y$. For instance, an increase of $10 in the final demand for services implies:

$$\Delta x = \mathcal{L}\Delta y = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 2.8842 \\ 1.8907 \\ 3.0444 \\ 16.0872 \end{pmatrix}$$

This means that energy production increases by $2.88, materials production increases by $1.89, and so on.
The cost-push price model

- $m$ is number of primary inputs (e.g., labor, capital, etc.)

- $V = (V_{k,j})$ is the value added matrix where $V_{k,j}$ represents the amount of primary input $k$ required to produce the output of sector $j$

- Since the total input of each sector is equal to its total output, we have:

$$x_j = \sum_{i=1}^{n} Z_{i,j} + \sum_{k=1}^{m} V_{k,j}$$

- Therefore, $v_j = \sum_{k=1}^{m} V_{k,j} = x_j - \sum_{i=1}^{n} Z_{i,j}$ represents the other expenditure of sector $j$ or the total primary inputs used in sector $j$

- We have $v = (v_1, \ldots, v_n) = V^\top 1_m$
The cost-push price model

- Let \( p = (p_1, \ldots, p_n) \) and \( \psi = (\psi_1, \ldots, \psi_m) \) be the vector of sector prices and primary inputs.
- \( p_j \) and \( \psi_k \) are the prices per unit of sector \( j \) and primary input \( k \).
- The interdependence relationship between primary inputs and sectors is expressed as the ratio between \( V_{k,j} \) and \( x_j \):

\[
B_{k,j} = \frac{V_{k,j}}{x_j}
\]

- \( B = (B_{k,j}) \equiv V \text{ diag}(\mathbf{x})^{-1} \) is the input-output matrix of the technical coefficients \( B_{k,j} \).
- The value of the output must be equal to the value of its inputs:

\[
\underbrace{p_j x_j}_{\text{Value of the output}} = \sum_{i=1}^{n} Z_{i,j} p_i + \sum_{k=1}^{m} V_{k,j} \psi_k
\]

Value of the inputs
The cost-push price model

- We deduce that:

\[
p_j = \sum_{i=1}^{n} \frac{Z_{i,j}}{x_j} p_i + \sum_{k=1}^{m} \frac{V_{k,j}}{x_j} \psi_k
\]

\[
= \sum_{i=1}^{n} A_{i,j} p_i + \sum_{k=1}^{m} B_{k,j} \psi_k
\]

- In a matrix form, we get:

\[
p = A^\top p + B^\top \psi
\]

- \( \nu = B^\top \psi \) is the vector of value added ratios
- Finally, the output prices are equal to:

\[
p = \left( I_n - A^\top \right)^{-1} \nu
\]

- \( \tilde{C} = \left( I_n - A^\top \right)^{-1} \) is known as the dual inverse matrix and represents the amount of costs from sector \( j \) that are passed on to sector \( i \)
Comprehensive input-output model

- Demand-pull quantity model + cost-push price model:

\[
\begin{align*}
\mathbf{x} &= (I_n - A)^{-1} \mathbf{y} \\
\mathbf{v} &= \mathbf{V}^\top \mathbf{1}_m \\
\mathbf{u} &= \mathbf{B}^\top \mathbf{\psi} \\
\mathbf{p} &= (I_n - A^\top)^{-1} \mathbf{v} \\
\mathbf{x}^\top \mathbf{v} &= \mathbf{y}^\top \mathbf{p}
\end{align*}
\]

- Equilibrium \( \Rightarrow \) the total value of the revenues \( \mathbf{y}^\top \mathbf{p} \) is equal to the total value of costs \( \mathbf{x}^\top \mathbf{v} \)
In this model, $A$, $B$ and $V$ are the model parameters, $\psi$, $v$ and $y$ are the exogenous variables, and $x$ and $p$ are the endogenous variables. By changing the model parameters or the exogenous variables, we can measure the impacts $\Delta y$ and $\Delta v$ on the quantities and prices in the economy.

Demand-pull quantity model
- We have:

\[
x = (I_n - A)^{-1} y = \mathcal{L} y
\]

where $\mathcal{L} = (I_n - A)^{-1}$ is the Leontief inverse matrix

Cost-push price model
- We have:

\[
p = (I_n - A^\top)^{-1} v = \tilde{\mathcal{L}} v
\]

where $\tilde{\mathcal{L}} = (I_n - A^\top)^{-1} = \mathcal{L}^\top$ is the dual inverse matrix
### Example #3

<table>
<thead>
<tr>
<th></th>
<th>Energy</th>
<th>Materials</th>
<th>To Industrials</th>
<th>Services</th>
<th>Final Demand</th>
<th>Total Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>500</td>
<td>800</td>
<td>1600</td>
<td>1250</td>
<td>850</td>
<td>5000</td>
</tr>
<tr>
<td>Materials</td>
<td>500</td>
<td>400</td>
<td>1600</td>
<td>625</td>
<td>875</td>
<td>4000</td>
</tr>
<tr>
<td>Industrials</td>
<td>250</td>
<td>800</td>
<td>2400</td>
<td>1250</td>
<td>3300</td>
<td>8000</td>
</tr>
<tr>
<td>Services</td>
<td>100</td>
<td>200</td>
<td>800</td>
<td>4375</td>
<td>7025</td>
<td>12500</td>
</tr>
<tr>
<td>Value added</td>
<td>Labour</td>
<td>3000</td>
<td>800</td>
<td>1000</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Capital</td>
<td>650</td>
<td>1000</td>
<td>600</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>5000</td>
<td>4000</td>
<td>8000</td>
<td>12000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The energy sector has a labour consumption of $3000 and a total output of $5000. By construction, the income of the sector is equal to the output of the sector. We deduce that the capital item (capital interest and net profit) is equal to $650.
The Cost-push price model

We have:

\[ V = \begin{pmatrix} 3000 & 800 & 1000 & 3000 \\ 650 & 1000 & 600 & 2000 \end{pmatrix} \]

and:

\[ v = V^T 1_2 = x - Z^T 1_4 = \begin{pmatrix} 3650 \\ 1800 \\ 1600 \\ 5000 \end{pmatrix} \]

We deduce that:

\[ B = \begin{pmatrix} 0.60 & 0.20 & 0.125 & 0.24 \\ 0.13 & 0.25 & 0.075 & 0.16 \end{pmatrix} \]
Since we have a monetary input-output table, the labour and capital costs are equal to the monetary unit, i.e. one dollar ($\psi_1 = \psi_2 = 1$). It follows that:

$$\nu = B^T \mathbf{1}_2 = \begin{pmatrix} 0.73 \\ 0.45 \\ 0.20 \\ 0.40 \end{pmatrix}$$

The interpretation is as follows. For the energy sector, intermediate consumption is 27% and value added is 73%. For the other three sectors, the value added ratios are 45%, 20% and 40% respectively.
Finally, we obtain:

$$\tilde{L} = (I_n - A^\top)^{-1} = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix}$$

and:

$$p = \tilde{L}v = 1_4$$

We check that the prices in a monetary input-output table are normalized to one dollar. In this basic economy, the total final demand $y^\top p$ is equal to $12050$, which is equal to the total value added $x^\top v$.
Suppose we have a variation in the labour/capital costs. We obtain \( \Delta p = \tilde{L} \Delta \nu \). For example, a 10\% increase in costs in the energy sector means that the price of energy increases by 11.88\%, the price of materials by 3.89\%, and so on:

\[
\Delta p = \tilde{L} \Delta \nu = \begin{pmatrix}
1.1881 & 0.1678 & 0.1430 & 0.0715 \\
0.3894 & 1.2552 & 0.4110 & 0.1718 \\
0.4919 & 0.4336 & 1.6303 & 0.2993 \\
0.2884 & 0.1891 & 0.3044 & 1.6087
\end{pmatrix}
\begin{pmatrix}
0.10 \\
0.00 \\
0.00 \\
0.00
\end{pmatrix}
\]

\[
= \begin{pmatrix}
11.88\% \\
3.89\% \\
4.92\% \\
2.88\%
\end{pmatrix}
\]
Inflation

- The definition of a price index is:

\[ \mathcal{P}I = \sum_{j=1}^{n} \alpha_j p_j = \alpha^\top p \]

where \( \alpha = (\alpha_1, \ldots, \alpha_n) \) is the weights of the basket of items

1. Producer price index (PPI): \( \alpha_j \propto x_j \)
2. Consumer price index (CPI): \( \alpha_j \propto y_j \)

- The inflation rate between two dates \( t_0 \) and \( t_1 \) is:

\[ \pi = \frac{\mathcal{P}I (t_1) - \mathcal{P}I (t_0)}{\mathcal{P}I (t_0)} = \frac{\alpha^\top (I_n - A^\top)^{-1} \Delta v (t_0, t_1)}{\alpha^\top (I_n - A^\top)^{-1} v (t_0)} \]

- We can simplify this formula because
  \( p(t_0) = (I_n - A^\top)^{-1} v(t_0) = 1_n \) and \( 1_n^\top \alpha = 1 \):

\[ \pi = \alpha^\top (I_n - A^\top)^{-1} \Delta v \]
Looking at the previous example, a 10% increase in energy costs will cause the producer price index to rise by 5.10% and the consumer price index by 4.15%.
Neumann series

A Neumann series is $S := \sum_{k=0}^{\infty} T^k$ where $T$ is a bounded linear operator and $T^k = T^{k-1} \circ T = T \circ T^{k-1}$. If the Neumann series converges, then $\text{Id} - T$ is invertible and its inverse is the Neumann series:

$$(\text{Id} - T)^{-1} = S = \sum_{k=0}^{\infty} T^k$$

If $A$ is an invertible matrix, we conclude that:

$$(I_n - A)^{-1} = \sum_{k=0}^{\infty} A^k$$

and $\lim_{k=0} \|A^k\| = 0$. This result generalizes the geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots \quad \text{where} \ |x| < 1$$
Mathematical properties

- We have:

\[(I_n - A^\top)^{-1} = (I_n - A)^{-1} = ((I_n - A)^{-1})^\top\]

- The matrix \(\mathcal{L}\) admits the following Neumann series:

\[
\mathcal{L} = (I_n - A)^{-1} = I_n + A + A^2 + A^3 + \ldots
\]

\[
= \sum_{k=0}^{\infty} A^k
\]

- The multiplier matrix \(\mathcal{L}\) is nonsingular

- \(\mathcal{L} \succeq I_n\) because \(A^k \succeq 0_{n,n}\) for \(k \geq 1\)
We also get the following decomposition:

\[ x = \sum_{k=0}^{\infty} A^k y = y + Ay + A^2 y + \ldots = \sum_{k=0}^{\infty} y^{(k)} \]

where:
- \( y^{(0)} = y \) is the final demand (or zero-tier intermediate demand)
- \( y^{(1)} = Ay \) is the first-tier intermediate demand
- \( y^{(2)} = A^2 y \) is the second-tier intermediate demand
- \( y^{(k)} = A^k y \) is the \( k \)th-tier intermediate demand

We have:

\[ \frac{\partial x}{\partial y} = (I_n - A)^{-1} \equiv \mathcal{L} \preceq I_n \]
Mathematical properties

Remark

The matrix $\mathcal{L}$ is also called the multiplier matrix because it is an analogy to Keynesian consumption theory and the effect of a change in aggregate demand on the output.
### Table 116: Tier decomposition of the output (Example #2)

<table>
<thead>
<tr>
<th></th>
<th>$k$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(k)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>850.0</td>
<td>1622.5</td>
<td>1066.0</td>
<td>630.8</td>
<td>362.1</td>
<td>205.2</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>875.0</td>
<td>1183.8</td>
<td>805.1</td>
<td>486.3</td>
<td>282.1</td>
<td>160.7</td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td>3300.0</td>
<td>1910.0</td>
<td>1175.8</td>
<td>695.1</td>
<td>400.0</td>
<td>227.1</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>7025.0</td>
<td>2849.5</td>
<td>1280.0</td>
<td>627.1</td>
<td>325.9</td>
<td>175.4</td>
<td></td>
</tr>
<tr>
<td>$y(0:k)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>850.0</td>
<td>2472.5</td>
<td>3538.5</td>
<td>4169.2</td>
<td>4531.3</td>
<td>4736.5</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>875.0</td>
<td>2058.8</td>
<td>2863.9</td>
<td>3350.1</td>
<td>3632.2</td>
<td>3792.9</td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td>3300.0</td>
<td>5210.0</td>
<td>6385.8</td>
<td>7080.9</td>
<td>7480.9</td>
<td>7708.0</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>7025.0</td>
<td>9874.5</td>
<td>11154.0</td>
<td>11781.0</td>
<td>12107.5</td>
<td>12283.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$k$</th>
<th>10</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(k)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>11.45</td>
<td>0.03</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>9.01</td>
<td>0.03</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td>12.69</td>
<td>0.04</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>9.29</td>
<td>0.03</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$y(0:k)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>4985.42</td>
<td>4999.96</td>
<td>5000.00</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>3988.52</td>
<td>3999.97</td>
<td>4000.00</td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td>7983.83</td>
<td>7999.95</td>
<td>8000.00</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>12488.18</td>
<td>12499.96</td>
<td>12500.00</td>
<td></td>
</tr>
</tbody>
</table>
A multi-regional input-output table (MRIO) involves several regions.

The two best known MRIO databases are:

1. GTAP (global trade analysis project):
   www.gtap.agecon.purdue.edu
2. WIOD (world input-output database):
   www.rug.nl/ggdc/valuechain/wiod

Other MRIO databases: OECD
Multi-regional input-output analysis

The structure of the WIOD database is:

<table>
<thead>
<tr>
<th></th>
<th>$Z$</th>
<th>$y$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>$(56 \times 44) \times (56 \times 44)$</td>
<td>$(56 \times 44) \times (5 \times 44)$</td>
<td>$(56 \times 44) \times 1$</td>
</tr>
<tr>
<td>Value added</td>
<td>$V$</td>
<td>$6 \times (56 \times 44)$</td>
<td>$6 \times (5 \times 44)$</td>
</tr>
<tr>
<td>Output</td>
<td>$w$</td>
<td>$1 \times (56 \times 44)$</td>
<td>$0_{6,5 \times 44}$</td>
</tr>
</tbody>
</table>

- 44 regions
- 56 sectors
Figure 262: Sparsity pattern of the input-output matrix $A$
The WIOD database

**Figure 263:** Spectrum of the matrix $A$
Figure 264: Frobenious norm of the matrix $A^k$
Application to environmental problems

EEIO = MRIO + GHG emissions
Production-based vs. consumption-based inventory

- $C(x) = \begin{pmatrix} C_{g,j}(x) \end{pmatrix}$ is the pollution output matrix where $C_{g,j}(x)$ is the total amount of the $g^{th}$ pollutant generated by the output of the $j^{th}$ sector.
- $D(y) = C(x) \text{diag}(x)^{-1} = \begin{pmatrix} D_{g,j}(y) \end{pmatrix}$ is the matrix of direct impact coefficients where $D_{g,j}(y) = c_{g,j}(y)/x_j$ is the amount of the $g^{th}$ pollutant generated by 1$ of the output of the $j^{th}$ sector.
- $\varpi = (\varpi_1, \ldots, \varpi_m)$ is the vector of pollution level:

$$\varpi = D(x)x = D(x) \begin{pmatrix} I_n - A \end{pmatrix}^{-1} y = D(y)y$$

where $D(y) = D(x) \begin{pmatrix} I_n - A \end{pmatrix}^{-1}$ is the pollutant multiplier matrix with respect to the final demand $y$.
- $D(y)$ also measures the product carbon footprint (PCF).
Production-based vs. consumption-based inventory

- Because $\varpi_g = (D(y)y)_g = \sum_{j=1}^{n} D_{g,j}^{(y)} y_j$, we deduce that the total contribution of sector $j$ to the $g^{th}$ pollutant is equal to:

$$C_{g,j}^{(x)} = \frac{\partial \varpi_g}{\partial y_j} y_j = D_{g,j}^{(y)} y_j$$

- Again, we can decompose the pollutant level according to the $k^{th}$ tier:

$$\varpi = D(y)y = \sum_{k=0}^{\infty} D^{(x)} A^k y = \sum_{k=0}^{\infty} \varpi^{(k)}$$

where:

- $\varpi^{(0)} = D^{(x)} y$ is the pollutant level due to the final demand (or the zero-tier pollutant level)
- $\varpi^{(1)} = D^{(x)} Ay$ is the pollutant level due to the first-tier supply chain
- $\varpi^{(k)} = D^{(x)} A^k y$ is the $k^{th}$-tier pollutant level

- The matrix $D^{(y)}(k) = D^{(x)} A^k$ is called the $k^{th}$-tier multiplier matrix and satisfies the identity $D^{(y)}(k) = \sum_{k=0}^{\infty} D^{(y)}(k)$
**Example #4**

We consider three products, whose input-output table is given below:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P₁</td>
<td>P₂</td>
<td>P₃</td>
</tr>
<tr>
<td>P₁</td>
<td>100</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>P₂</td>
<td>250</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>P₃</td>
<td>25</td>
<td>200</td>
<td>75</td>
</tr>
<tr>
<td>Value added</td>
<td>625</td>
<td>1350</td>
<td>125</td>
</tr>
<tr>
<td>Total outlays</td>
<td>1000</td>
<td>2000</td>
<td>500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GHG</th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>50</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>CH₄</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Intermediate production of $100$ of $P₁$, $300$ of $P₂$, and $100$ of $P₃$ is required to produce $500$ of $P₁$. This environmentally-extended input-output table has two additional rows corresponding to the GHG emissions. For instance, the production of $P₁$ causes $50$ kgCO₂ and $3$ kgCH₄.
The matrix of technical coefficients is equal to:

\[ A = Z \text{diag}(x)^{-1} = \begin{pmatrix} 10.0\% & 15.0\% & 20.0\% \\ 25.0\% & 7.5\% & 40.0\% \\ 2.5\% & 10.0\% & 15.0\% \end{pmatrix} \]

It follows that the matrix of multipliers is equal to:

\[ \mathcal{L} = (I_3 - A)^{-1} = \begin{pmatrix} 1.1871 & 0.2346 & 0.3897 \\ 0.3539 & 1.2090 & 0.6522 \\ 0.0766 & 0.1491 & 1.2647 \end{pmatrix} \]
Production-based vs. consumption-based inventory

The direct impact matrix is equal to the GHG emissions divided by the output:

\[
D^{(x)} = \begin{pmatrix}
\frac{50}{1000} & \frac{20}{2000} & \frac{5}{500} \\
\frac{3}{1000} & \frac{1}{2000} & \frac{0}{500}
\end{pmatrix} = \begin{pmatrix}
0.05 & 0.01 & 0.01 \\
0.003 & 0.0005 & 0
\end{pmatrix}
\]

The unit of \(D^{(x)}\) is expressed in kilogram of the gas per dollar. Åš

For instance, the GHG intensities of the product \(P_1\) are equal to 0.05 kgCO\(_2\)/$ and 0.003 kgCH\(_4\)/$
Finally, we obtain:

\[ D^{(y)} = D^{(x)} \mathcal{L} = \begin{pmatrix} 0.0637 & 0.0253 & 0.0387 \\ 0.0037 & 0.0013 & 0.0015 \end{pmatrix} \]

- \( D^{(x)} \) corresponds to the production-based inventory
- \( D^{(y)} \) corresponds to the consumption-based inventory

This gives us the following decomposition:

\[ C^{(y)} = \begin{pmatrix} 31.83 & 35.44 & 7.73 \\ 1.87 & 1.83 & 0.30 \end{pmatrix} \neq \begin{pmatrix} 50 & 20 & 5 \\ 3 & 1 & 0 \end{pmatrix} = C^{(x)} \]

**Remark**

The two contribution matrices are different. For instance, while the production of \( P_1 \) is responsible of 50 kgCO\(_2\), the final consumption of \( P_1 \) is responsible of only 31.83 kgCO\(_2\), meaning that 18.17 kgCO\(_2\) are emitted by \( P_1 \) for the other two products.
The carbon footprint is evaluated in CO$_2$e.

We have:

\[
CI_{\text{total}} = \mathcal{L}^\top CI_1 \\
= (I_n - A^\top)^{-1} CI_1 \\
= \tilde{\mathcal{L}} CI_1
\]

where:

- $CI_1 = CI_{\text{direct}}$ is the vector of direct carbon intensities
- $CI_{\text{total}}$ is the vector of direct plus indirect carbon intensities
The indirect carbon intensities are given by:

\[
CI_{\text{indirect}} = CI_{\text{total}} - CI_1 \\
= \left( (I_n - A^\top)^{-1} - I_n \right) CI_{\text{direct}}
\]

We can decompose \( CI_{\text{indirect}} \) using the Neumann series:

\[
CI_{\text{indirect}} = A^\top CI_1 + (A^\top)^2 CI_1 + \ldots + (A^\top)^k CI_1 + \ldots
\]

We have:

\[
CI_{\text{total}} = CI_1 + A^\top CI_1 + (A^\top)^2 CI_1 + \ldots + (A^\top)^k CI_1 + \ldots
\]
Example #5

<table>
<thead>
<tr>
<th>From</th>
<th>Energy</th>
<th>Materials</th>
<th>Industrials</th>
<th>Services</th>
<th>Final Demand</th>
<th>Total Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>500</td>
<td>800</td>
<td>1600</td>
<td>1250</td>
<td>850</td>
<td>5000</td>
</tr>
<tr>
<td>Materials</td>
<td>500</td>
<td>400</td>
<td>1600</td>
<td>625</td>
<td>875</td>
<td>4000</td>
</tr>
<tr>
<td>Industrials</td>
<td>250</td>
<td>800</td>
<td>2400</td>
<td>1250</td>
<td>3300</td>
<td>8000</td>
</tr>
<tr>
<td>Services</td>
<td>100</td>
<td>200</td>
<td>800</td>
<td>4375</td>
<td>7025</td>
<td>12500</td>
</tr>
</tbody>
</table>

The carbon emissions, expressed in ktCO$_2$e, are as follows: 500 for the energy sector, 200 for the materials sector, 200 for the industrials sector and 125 for the services sector.
The vector of Scope 1 carbon intensities is equal to:

\[ CI_1 = \text{diag}(x)^{-1} CE_1 = \begin{pmatrix} 500/5000 \\ 200/4000 \\ 200/8000 \\ 125/12500 \end{pmatrix} \times 10^3 = \begin{pmatrix} 100 \\ 50 \\ 25 \\ 10 \end{pmatrix} \]

We have:

\[ CI_{\text{total}} = \tilde{L} CI_1 \]

\[ = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix} \begin{pmatrix} 100 \\ 50 \\ 25 \\ 10 \end{pmatrix} \]

\[ = \begin{pmatrix} 131.49 \\ 113.69 \\ 114.62 \\ 61.99 \end{pmatrix} \]
# Estimation of first-tier and indirect emissions

**Table 117:** Direct and indirect carbon intensities (Example #5)

<table>
<thead>
<tr>
<th>Sector</th>
<th>CI\textsubscript{1}</th>
<th>CI\textsubscript{total} ,(in\ tCO\textsubscript{2}e/$\text{mn}$)</th>
<th>CI\textsubscript{direct}</th>
<th>CI\textsubscript{indirect}</th>
<th>CI\textsubscript{direct} ,(in\ %)</th>
<th>CI\textsubscript{indirect} ,(in\ %)</th>
<th>CI\textsubscript{total} ,(in\ %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>100.00</td>
<td>131.49</td>
<td>100.00</td>
<td>31.49</td>
<td>76.05%</td>
<td>23.95%</td>
<td>1.31</td>
</tr>
<tr>
<td>Materials</td>
<td>50.00</td>
<td>113.69</td>
<td>50.00</td>
<td>63.69</td>
<td>43.98%</td>
<td>56.02%</td>
<td>2.27</td>
</tr>
<tr>
<td>Industrials</td>
<td>25.00</td>
<td>114.62</td>
<td>25.00</td>
<td>89.62</td>
<td>21.81%</td>
<td>78.19%</td>
<td>4.58</td>
</tr>
<tr>
<td>Services</td>
<td>10.00</td>
<td>61.99</td>
<td>10.00</td>
<td>51.99</td>
<td>16.13%</td>
<td>83.87%</td>
<td>6.20</td>
</tr>
</tbody>
</table>
Integrated assessment models
Scenarios
Environmentally-extended input-output model
Input-output analysis
Estimation of indirect emissions
Taxation, pass-through and price dynamics

Estimation of first-tier and indirect emissions

Table 118: Tier decomposition of carbon intensities (Example #5)

<table>
<thead>
<tr>
<th>Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>16.45</td>
<td>6.99</td>
<td>3.60</td>
<td>1.97</td>
<td>1.09</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Materials</td>
<td>30.50</td>
<td>14.97</td>
<td>8.13</td>
<td>4.47</td>
<td>2.48</td>
<td>0.14</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Industrials</td>
<td>38.50</td>
<td>22.79</td>
<td>12.58</td>
<td>6.96</td>
<td>3.88</td>
<td>0.21</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Services</td>
<td>18.50</td>
<td>13.50</td>
<td>8.45</td>
<td>4.98</td>
<td>2.86</td>
<td>0.16</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\[ \mathbf{CI}(k) = (\mathbf{A}^T)^k \mathbf{CI}_1 \]

\[ \mathbf{CI}(1-k) = \sum_{h=1}^{k} (\mathbf{A}^T)^h \mathbf{CI}_1 \]

We denote by \( \mathbf{CI}(k) = (\mathbf{A}^T)^k \mathbf{CI}_1 \) the indirect carbon intensity when considering the \( k^{\text{th}} \) tier, and \( \mathbf{CI}(1-k) = \sum_{h=1}^{k} (\mathbf{A}^T)^h \mathbf{CI}_1 \) the cumulative indirect carbon intensity for the first \( k \) tiers.

Thierry Roncalli
Course 2023-2024 in Sustainable Finance
1054 / 1665
Estimation of first-tier and indirect emissions

The estimation of total emissions use the following identities:

\[
\frac{CE_{\text{total}}}{CE_1} = \frac{CI_{\text{total}}}{CI_1} \iff CE_{\text{total}} = CI_{\text{total}} \odot \frac{CE_1}{CI_1} = x \odot CI_{\text{total}}
\]

Therefore, the indirect emissions are given by:

\[
CE_{\text{indirect}} = CE_{\text{total}} - CE_{\text{direct}} = (CI_{\text{total}} - CI_1) \odot \frac{CE_1}{CI_1}
\]

Table 119: Decomposition of carbon emissions (Example #5)

<table>
<thead>
<tr>
<th>Sector</th>
<th>CE\text{direct}</th>
<th>CE\text{indirect} (in ktCO}_2\text{e})</th>
<th>CE\text{total}</th>
<th>CE\text{direct}</th>
<th>CE\text{indirect} (in %)</th>
<th>CE\text{total}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>500</td>
<td>157.44</td>
<td>657.44</td>
<td>48.78</td>
<td>8.85</td>
<td>23.45</td>
</tr>
<tr>
<td>Materials</td>
<td>200</td>
<td>254.76</td>
<td>454.76</td>
<td>19.51</td>
<td>14.32</td>
<td>16.22</td>
</tr>
<tr>
<td>Industrials</td>
<td>200</td>
<td>716.97</td>
<td>916.97</td>
<td>19.51</td>
<td>40.30</td>
<td>32.70</td>
</tr>
<tr>
<td>Services</td>
<td>125</td>
<td>649.92</td>
<td>774.92</td>
<td>12.20</td>
<td>36.53</td>
<td>27.64</td>
</tr>
<tr>
<td>Total</td>
<td>1025</td>
<td>1779.10</td>
<td>2804.10</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Summary of formulas

- We have:

\[
\begin{align*}
CI_{\text{total}} & = (I_n - A^T)^{-1} CI_1 \\
CE_{\text{total}} & = x \odot CI_{\text{total}} \\
CI_{(k)} & = (A^T)^k CI_1 \\
CE_{(k)} & = x \odot CI_{(k)}
\end{align*}
\]

- If we want to aggregate the results such that \( i \in \Omega \), we have:

\[
CE_{\text{total}} (\Omega) = \sum_{i \in \Omega} CE_{\text{total},i} = \omega^T CE_{\text{total}}
\]

where \( \omega = (\omega_i) \) is a vector of dimension \( n \times 1 \) with \( \omega_i = 1 \) if \( i \in \Omega \) and \( \omega_i = 0 \) otherwise.

- The carbon intensity (WACI) of \( \Omega \) is equal to:

\[
CI_{\text{total}} (\Omega) = \frac{\sum_{i \in \Omega} CE_{\text{total},i}}{\sum_{i \in \Omega} x_i} = \frac{\sum_{i \in \Omega} x_i CI_{\text{total},i}}{\sum_{i \in \Omega} x_i} = \sum_{i \in \Omega} w_i CI_{\text{total},i}
\]

where \( w_i = \left( \sum_{j \in \Omega} x_j \right)^{-1} x_i \) is the weight of item \( i \) in the set \( \Omega \).
### Comparison of upstream emissions between Exiobase, Trucost and WIOD

#### Table 120: Ratio of upstream carbon emissions (global analysis)

<table>
<thead>
<tr>
<th>Tier</th>
<th>WIOD 2014</th>
<th>Exiobase 2014</th>
<th>Exiobase 2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>m(k)</td>
<td>m(0-k)</td>
<td>c(0-k)</td>
<td>m(k)</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>31.8%</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>0.77</td>
<td>56.1%</td>
<td>0.72</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>71.9%</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>82.1%</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>88.6%</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>92.7%</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>0.08</td>
<td>95.4%</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>0.05</td>
<td>97.0%</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td>98.1%</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>0.02</td>
<td>98.8%</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>99.2%</td>
<td>0.01</td>
</tr>
<tr>
<td>∞</td>
<td>0.00</td>
<td>100.0%</td>
<td>2.76</td>
</tr>
</tbody>
</table>

*Note: The table compares the ratio of upstream carbon emissions between WIOD, Exiobase, and Exiobase for different tiers.*
Comparison of Exiobase, Trucost and WIOD

**Figure 265:** Multiplication coefficient $m_{(0-1)}$ and $m_{(0-\infty)}$ (global analysis)
Comparison of Exiobase, Trucost and WIOD

Figure 266: Multiplying coefficient $m_{(0-\infty)}$ (country analysis, WIOD 2014)
Comparison of upstream emissions between Exiobase, Trucost and WIOD

Table 121: Direct + indirect carbon intensities of GICS sectors (MSCI World index, May 2023)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Exiobase 2022</th>
<th>Trucost 2021</th>
<th>WIOD 2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>66</td>
<td>78</td>
<td>102</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>168</td>
<td>209</td>
<td>219</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>437</td>
<td>387</td>
<td>277</td>
</tr>
<tr>
<td>Energy</td>
<td>1373</td>
<td>796</td>
<td>757</td>
</tr>
<tr>
<td>Financials</td>
<td>83</td>
<td>55</td>
<td>83</td>
</tr>
<tr>
<td>Health Care</td>
<td>108</td>
<td>120</td>
<td>167</td>
</tr>
<tr>
<td>Industrials</td>
<td>276</td>
<td>277</td>
<td>307</td>
</tr>
<tr>
<td>Information Technology</td>
<td>110</td>
<td>138</td>
<td>131</td>
</tr>
<tr>
<td>Materials</td>
<td>791</td>
<td>973</td>
<td>747</td>
</tr>
<tr>
<td>Real Estate</td>
<td>128</td>
<td>134</td>
<td>138</td>
</tr>
<tr>
<td>Utilities</td>
<td>1872</td>
<td>1833</td>
<td>1889</td>
</tr>
<tr>
<td><strong>MSCI World</strong></td>
<td><strong>299</strong></td>
<td><strong>281</strong></td>
<td><strong>278</strong></td>
</tr>
</tbody>
</table>
### Table 122: Breakdown of the portfolio intensity by GICS sector (MSCI World Index, May 2023)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Exiobase 2022</th>
<th>Trucost 2021</th>
<th>WIOD 2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>1.5%</td>
<td>1.9%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>5.9%</td>
<td>7.8%</td>
<td>8.4%</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>11.6%</td>
<td>10.9%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Energy</td>
<td>22.9%</td>
<td>14.1%</td>
<td>13.6%</td>
</tr>
<tr>
<td>Financials</td>
<td>4.2%</td>
<td>2.9%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Health Care</td>
<td>4.8%</td>
<td>5.7%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.2%</td>
<td>10.8%</td>
<td>12.1%</td>
</tr>
<tr>
<td>Information Technology</td>
<td>7.5%</td>
<td>10.0%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Materials</td>
<td>11.7%</td>
<td>15.3%</td>
<td>11.9%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>1.1%</td>
<td>1.2%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Utilities</td>
<td>18.6%</td>
<td>19.4%</td>
<td>20.2%</td>
</tr>
</tbody>
</table>
Comparison of Exiobase, Trucost and WIOD

Figure 267: Total carbon intensity $\mathbf{CI}_{\text{total}}$ by GICS sector (MSCI World Index, May 2023)
Imported and exported carbon emissions

- Let $\mathbf{CI}_1 = (\mathbf{CI}_{1,1}, \ldots, \mathbf{CI}_{n,1})$ be the vector of carbon intensities evaluated in CO$_2$e, where $\mathbf{CI}_{j,1}$ measures the direct emission intensity of sector $j$.
- The vector of consumption-based carbon intensities is equal to:

$$\mathbf{CI}^{(y)} = \tilde{\mathcal{L}} \mathbf{CI}_1 := \mathbf{CI}_{\text{total}}$$
Let $y = (y_{j,r})$ be the $n \times p$ matrix, where $y_{j,r}$ is the final demand of the $j^{th}$ sector and the $r^{th}$ region.

We have:

$$CE^{(y)} = CI^{(y)} \odot y$$

where $CE^{(y)}$ is the $n \times p$ matrix of carbon emissions.
The consumption-based carbon emissions of the \( r \)th region is then equal to:

\[
\text{CE}^{(y,r)} = 1_n^\top \left( \text{CE}^{(y)} e_r \right)
\]

while the imported and exported carbon emissions of the \( r \)th region are:

\[
\text{CE}_{\text{imported}}^{(y,r)} = \sum_{j \not\in r} \left( \text{CE}^{(y)} e_r \right)_j
\]

and:

\[
\text{CE}_{\text{exported}}^{(y,r)} = \sum_{j \in r} \sum_{k \not\in r} \left( \text{CE}^{(y)} e_r \right)_j
\]
Figure 268: CO₂ emissions embedded in trade, 2020

This is measured as emissions exported or imported as a percentage of domestic production emissions. Positive values (red) represent net importers of CO₂. Negative values (blue) represent net exporters of CO₂.

Source: https://ourworldindata.org/consumption-based-co2.
### Table 123: Top importing and exporting countries by carbon emissions (in MtCO$_2$e, 2018)

<table>
<thead>
<tr>
<th>Rank</th>
<th>ISO</th>
<th>Country</th>
<th>Balance</th>
<th>Rank</th>
<th>ISO</th>
<th>Country</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>USA</td>
<td>United States</td>
<td>−752.10</td>
<td>1</td>
<td>CHN</td>
<td>China</td>
<td>895.45</td>
</tr>
<tr>
<td>2</td>
<td>JPN</td>
<td>Japan</td>
<td>−160.62</td>
<td>2</td>
<td>RUS</td>
<td>Russian Federation</td>
<td>343.48</td>
</tr>
<tr>
<td>3</td>
<td>DEU</td>
<td>Germany</td>
<td>−128.73</td>
<td>3</td>
<td>ZAF</td>
<td>South Africa</td>
<td>122.47</td>
</tr>
<tr>
<td>4</td>
<td>GBR</td>
<td>United Kingdom</td>
<td>−123.77</td>
<td>4</td>
<td>IND</td>
<td>India</td>
<td>106.10</td>
</tr>
<tr>
<td>5</td>
<td>FRA</td>
<td>France</td>
<td>−111.65</td>
<td>5</td>
<td>TWN</td>
<td>Chinese Taipei</td>
<td>77.03</td>
</tr>
<tr>
<td>6</td>
<td>ITA</td>
<td>Italy</td>
<td>−80.09</td>
<td>6</td>
<td>SGP</td>
<td>Singapore</td>
<td>62.19</td>
</tr>
<tr>
<td>7</td>
<td>HKG</td>
<td>Hong Kong, China</td>
<td>−70.14</td>
<td>7</td>
<td>KOR</td>
<td>Korea</td>
<td>54.35</td>
</tr>
<tr>
<td>8</td>
<td>CHE</td>
<td>Switzerland</td>
<td>−44.53</td>
<td>8</td>
<td>CAN</td>
<td>Canada</td>
<td>53.12</td>
</tr>
<tr>
<td>9</td>
<td>PHL</td>
<td>Philippines</td>
<td>−40.49</td>
<td>9</td>
<td>VNM</td>
<td>Viet Nam</td>
<td>52.31</td>
</tr>
<tr>
<td>10</td>
<td>SWE</td>
<td>Sweden</td>
<td>−29.67</td>
<td>10</td>
<td>MYS</td>
<td>Malaysia</td>
<td>46.52</td>
</tr>
</tbody>
</table>

Imported and exported carbon emissions

Figure 269: Total production- and consumption-based CO₂ emitted by OECD and non-OECD countries (in GtCO₂e)
Figure 270: Decomposition of OECD imported emissions (in GtCO$_2$e)
Figure 271: Net exported emissions (in MtCO$_2$e)
Imported and exported carbon emissions

Figure 272: Net exported emissions (in MtCO$_2$e)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The absolute amount of the carbon tax for sector $j$ is equal to:

$$T_{\text{direct},j} = \tau_j C\mathcal{E}_{1,j}$$

where $\tau_j$ is the nominal carbon tax expressed in $$/tCO_2e$ and $C\mathcal{E}_{1,j}$ is the Scope 1 emissions of the sector.

We deduce that the carbon tax rate is equal to:

$$t_{\text{direct},j} = \frac{T_{\text{direct},j}}{x_j} = \frac{\tau_j C\mathcal{E}_{1,j}}{x_j} = \tau_j C\mathcal{I}_{1,j}$$

Note that $t_{\text{direct},j}$ has no unit and is equal to the product of the tax and the Scope 1 carbon intensity.

The input-output model implies that:

$$p_j x_j = \sum_{i=1}^{n} Z_{i,j} p_i + \sum_{k=1}^{m} V_{k,j} \psi_k + T_{\text{direct},j}$$
We deduce that:

\[ p_j = \sum_{i=1}^{n} A_{i,j} p_i + \sum_{k=1}^{m} B_{k,j} \psi_k + t_{\text{direct},j} = \sum_{i=1}^{n} A_{i,j} p_i + v_j + t_{\text{direct},j} \]

It follows that:

\[ p = (I_n - A^\top)^{-1} (v + t_{\text{direct}}) \]

where \( t_{\text{direct}} = (t_{\text{direct},1}, \ldots, t_{\text{direct},n}) \) is the vector of direct tax rates.

**Remark**

We recover the **cost-push price model**, where the vector \( v \) of value added ratios is replaced by \( v + t_{\text{direct}} \). Because \( \Delta v = t_{\text{direct}} \), the vector of price changes due to the carbon tax is equal to:

\[ \Delta p = (I_n - A^\top)^{-1} t_{\text{direct}} \]
Value added approach
Impact on the price index

• The definition of a price index is:

  \[ \mathcal{PI} = \sum_{i=1}^{n} \alpha_i p_i = \alpha^\top p \]

  where \( \alpha = (\alpha_1, \ldots, \alpha_n) \) are the weights of the items in the basket

• We deduce that the inflation rate is:

  \[ \pi = \frac{\Delta \mathcal{PI}}{\mathcal{PI}^-} = \frac{\mathcal{PI} - \mathcal{PI}^-}{\mathcal{PI}^-} = \frac{\alpha^\top (l_n - A^\top)^{-1} t_{\text{direct}}}{\alpha^\top (l_n - A^\top)^{-1} \nu} \]

  We can simplify this formula because \( p^- = (l_n - A^\top)^{-1} \nu = 1_n \) and \( 1_n^\top \alpha = 1 \). Finally, we have:

  \[ \pi = \alpha^\top (l_n - A^\top)^{-1} t_{\text{direct}} \]
The total tax cost is equal to:

\[ T_{\text{total}} = x \odot \Delta p = x \odot (I_n - A^\top)^{-1} t_{\text{direct}} \]

while the direct tax cost is \( T_{\text{direct}} = x \odot t_{\text{direct}} \).

We can show that the total tax cost is greater than the direct tax cost for all the sectors:

\[ T_{\text{total},j} \geq T_{\text{direct},j} \]

Since the total cost to the economy is equal to

\[ \text{Cost}_{\text{total}} = \sum_{j=1}^{n} T_{\text{total},j} = x^\top (I_n - A^\top)^{-1} t_{\text{direct}}, \]

the tax incidence is then equal to:

\[ \mathcal{T} = \frac{\text{Cost}_{\text{total}}}{1_n^\top x} = \frac{x^\top (I_n - A^\top)^{-1} t_{\text{direct}}}{1_n^\top x} \]
Remark

In some research papers, we can find two formulas that seem to be intuitive:

\[ T'_\text{total} = (I_n - A^\top)^{-1} T_{\text{direct}} \]

and:

\[ T''_{\text{total}} = \tau \odot C E_{\text{total}} \]

The two previous equations are generally wrong.
Let us denote by \( f(\tau) \) the function \( f \) that depends on the vector \( \tau = (\tau_1, \ldots, \tau_n) \) of carbon taxes.

The functions \( \Delta p, \pi, T_{\text{total}}, Cost_{\text{total}} \) and \( TI \) are homogeneous\(^{31}\) and additive\(^{32}\).

Let \( \lambda \geq 0 \) be a positive scalar. We have:

\[
\Delta p(\lambda \tau) = (I_n - A^\top)^{-1} t_{\text{direct}}(\lambda \tau) = \lambda (I_n - A^\top)^{-1} t_{\text{direct}}(\tau) = \lambda \Delta p(\tau)
\]

\(^{31}\)This means that \( f(\lambda \tau) = \lambda f(\tau) \).

\(^{32}\)We have \( f(\tau + \tau') = f(\tau) + f(\tau') \).
If the tax is uniform $\tau = \tau_1n$, the vector of total tax amount is the product of the tax by the total emissions:

$$T_{\text{total}}(\tau_1n) = \tau C\mathcal{E}_{\text{total}}$$

The tax incidence for a given sector is then proportional to the direct plus indirect carbon emissions of the sector.

At the global level, the tax incidence is equal to the carbon tax multiplied by the total carbon intensity of the world:

$$\mathcal{T}\mathcal{I}(\tau_1n) = \frac{1_n^T \tau C\mathcal{E}_{\text{total}}}{1_n^T x} = \tau C\mathcal{I}_{\text{total}}$$
Value added approach

Example #6

**Table 124**: Environmentally extended monetary input-output table

<table>
<thead>
<tr>
<th>Sector</th>
<th>Z</th>
<th>y</th>
<th>x</th>
<th>CE1</th>
<th>CI1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>500</td>
<td>800</td>
<td>1600</td>
<td>1250</td>
<td>500</td>
</tr>
<tr>
<td>Materials</td>
<td>500</td>
<td>400</td>
<td>1600</td>
<td>625</td>
<td>875</td>
</tr>
<tr>
<td>Industrials</td>
<td>250</td>
<td>800</td>
<td>2400</td>
<td>1250</td>
<td>3300</td>
</tr>
<tr>
<td>Services</td>
<td>100</td>
<td>200</td>
<td>800</td>
<td>4375</td>
<td>7025</td>
</tr>
<tr>
<td>Value added</td>
<td>3650</td>
<td>1800</td>
<td>1600</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>5000</td>
<td>4000</td>
<td>8000</td>
<td>12500</td>
<td></td>
</tr>
</tbody>
</table>

The values of \(Z_{i,j}\), \(y_j\), \(x_j\) and \(V_{1,j}\) are in $ mn. The carbon emissions are expressed in \(\text{ktCO}_2\text{e}\), while the carbon intensities are expressed in \(\text{tCO}_2\text{e}/\$ \text{mn}\).
Value added approach

- We have:

\[ A = Z \text{diag}^{-1}(x) = \begin{pmatrix} 0.10 & 0.20 & 0.20 & 0.10 \\ 0.10 & 0.10 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.30 & 0.10 \\ 0.02 & 0.05 & 0.10 & 0.35 \end{pmatrix} \]

and:

\[ \tilde{\mathcal{L}} = (I_4 - A^\top)^{-1} = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix} \]

- Then, we calculate the vector \( \nu \) of value added ratios:

\[ \nu = \begin{pmatrix} 3650/5000 \\ 1800/4000 \\ 1600/8000 \\ 5000/12500 \end{pmatrix} = \begin{pmatrix} 0.73 \\ 0.45 \\ 0.20 \\ 0.40 \end{pmatrix} \]

- We check that \( p^- = \tilde{\mathcal{L}} \nu = 1_4 \)
Value added approach

- We introduce a differentiated carbon tax: $\tau_1 = $200/tCO$_2$e and $\tau_2 = \tau_3 = \tau_4 = $100/tCO$_2$e
- The direct tax costs are 100, 20, 20 and 12.5 million dollars for Energy, Materials, Industrials and Services respectively
- We deduce that the vector of carbon tax rates is $t_{\text{direct}} = (2.00\%, 0.50\%, 0.25\%, 0.10\%)$
- It follows that:

$$p = (I_n - A^\top)^{-1} (\nu + t_{\text{direct}}) = \begin{pmatrix}
1.0250 \\
1.0153 \\
1.0164 \\
1.0091 \\
\end{pmatrix}$$

- If we assume that the basket of goods and services is $\alpha = (10\%, 20\%, 30\%, 40\%)$, the price index $PI$ is 1.0141
- The inflation rate $\pi$ is 1.410\%
Table 125: Total carbon costs (in $ mn) (differentiated tax)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$T_{\text{direct}}$</th>
<th>$T_{\text{total}}$</th>
<th>$T'_{\text{total}}$</th>
<th>$T''_{\text{total}}$</th>
<th>$C\mathcal{E}_{\text{direct}}$</th>
<th>$C\mathcal{E}_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>100.00</td>
<td>125.15</td>
<td>125.92</td>
<td>131.49</td>
<td>500.00</td>
<td>657.44</td>
</tr>
<tr>
<td>Materials</td>
<td>20.00</td>
<td>61.05</td>
<td>74.41</td>
<td>45.48</td>
<td>200.00</td>
<td>454.76</td>
</tr>
<tr>
<td>Industrials</td>
<td>20.00</td>
<td>131.05</td>
<td>94.21</td>
<td>91.70</td>
<td>200.00</td>
<td>916.97</td>
</tr>
<tr>
<td>Services</td>
<td>12.50</td>
<td>113.54</td>
<td>58.82</td>
<td>77.49</td>
<td>125.00</td>
<td>774.92</td>
</tr>
<tr>
<td>Sum</td>
<td>152.50</td>
<td>430.79</td>
<td>353.36</td>
<td>346.15</td>
<td>1,025.00</td>
<td>2,804.10</td>
</tr>
</tbody>
</table>

Table 126: Total carbon costs (in $ mn) (uniform tax of $100/tCO_2e)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$T_{\text{direct}}$</th>
<th>$T_{\text{total}}$</th>
<th>$T'_{\text{total}}$</th>
<th>$T''_{\text{total}}$</th>
<th>$C\mathcal{E}_{\text{direct}}$</th>
<th>$C\mathcal{E}_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>50.00</td>
<td>65.74</td>
<td>66.51</td>
<td>65.74</td>
<td>500.00</td>
<td>657.44</td>
</tr>
<tr>
<td>Materials</td>
<td>20.00</td>
<td>45.48</td>
<td>54.94</td>
<td>45.48</td>
<td>200.00</td>
<td>454.76</td>
</tr>
<tr>
<td>Industrials</td>
<td>20.00</td>
<td>91.70</td>
<td>69.62</td>
<td>91.70</td>
<td>200.00</td>
<td>916.97</td>
</tr>
<tr>
<td>Services</td>
<td>12.50</td>
<td>77.49</td>
<td>44.40</td>
<td>77.49</td>
<td>125.00</td>
<td>774.92</td>
</tr>
<tr>
<td>Sum</td>
<td>102.50</td>
<td>280.41</td>
<td>235.47</td>
<td>280.41</td>
<td>1,025.00</td>
<td>2,804.10</td>
</tr>
</tbody>
</table>
Pass-through rate

“cost pass-through describes what happens when a business changes the price of the production or services it sells following a change in the cost of producing them” RBB Economics (2014).

⇒ A pass-through rate is related to the supply and demand elasticity:

- In the case of competition, the pass-through rate \( \phi \in [0, 100\%] \) is:

\[
\phi = \frac{\frac{d p}{d \tau}}{\frac{d p}{d \tau} - \frac{d q}{d \tau}} = \frac{\text{price sensitivity of supply}}{\text{price sensitivity of supply} - \text{price sensitivity of demand}}
\]

- In a monopolistic situation, the previous formula becomes \( \phi \geq 50\% \):

\[
\phi = \frac{1}{2 + \text{elasticity of the slope of inverse demand}}
\]
Figure 273: Demand curvature

(a) Linear demand
- Constant slope as price increases

(b) Concave demand
- Flatter slope as price increases

(c) Convex demand
- Steeper slope as price increases

(d) Extreme cases
- Perfectly inelastic
- Perfectly elastic

Source: RBB Economics (2014, Figure 2, page 16).
Table 127: Pass-through rates (in %) for intensive sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity, gas and steam</td>
<td>100%</td>
</tr>
<tr>
<td>Petroleum refining</td>
<td>100%</td>
</tr>
<tr>
<td>Base metals</td>
<td>78%</td>
</tr>
<tr>
<td>Mining</td>
<td>78%</td>
</tr>
<tr>
<td>Waste/wastewater</td>
<td>78%</td>
</tr>
<tr>
<td>Land transport</td>
<td>78%</td>
</tr>
<tr>
<td>Fishery</td>
<td>75%</td>
</tr>
<tr>
<td>Non-metallic minerals</td>
<td>60%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>50%</td>
</tr>
<tr>
<td>Chemicals</td>
<td>40%</td>
</tr>
<tr>
<td>Maritime transport</td>
<td>30%</td>
</tr>
<tr>
<td>Aviation</td>
<td>30%</td>
</tr>
<tr>
<td>Paper</td>
<td>10%</td>
</tr>
</tbody>
</table>

Source: Sautel et al. (2022, page 35).
Pass-through integration
Recurrence formula without pass-through

- We have:

\[
\Delta p = \tilde{L} \Delta \upsilon = \sum_{k=0}^{\infty} (A^T)^k \Delta \upsilon = \sum_{k=0}^{\infty} \Delta p(k)
\]

where \( \Delta p(k) = (A^T)^k \Delta \upsilon \) is the price impact at the \( k^{th} \) tier.

- In fact, \( \Delta p(k) \) satisfies the following recurrence relation:

\[
\begin{align*}
\Delta p(k) &= A^T \Delta p(k-1) \\
\Delta p(0) &= \Delta \upsilon
\end{align*}
\]

- If we consider the price \( p_j \) of sector \( j \), we have \( \Delta p(0)_j = \Delta \upsilon_j \) and:

\[
\Delta p(k)_j = \sum_{i=1}^{n} A_{i,j} \Delta p(k-1)_i
\]
Pass-through integration
cascading effect of the carbon tax

- In the zeroth round, it induces an additional cost $\Delta v_j$, which is fully passed on to the price $p_j$ of the sector.
- The new price is then $p_j + \Delta p(0), j = p_j + \Delta v_j$.
- In the first round, sector $j$ faces new additional costs due to the price increase of intermediate consumption:

$$\Delta p(1), j = \sum_{i=1}^{n} A_{i,j} \Delta p(0), i = \sum_{i=1}^{n} A_{i,j} \Delta v_i$$

- The iteration process continues and we have at the second round:

$$\Delta p(2), j = \sum_{i=1}^{n} A_{i,j} \Delta p(1), i = \sum_{i=1}^{n} \sum_{k=1}^{n} A_{i,j} A_{k,i} \Delta v_k$$
Pass-through integration
Recurrence formula with pass-through

- We introduce the pass-through mechanism
- We have $\Delta p(0)_{j} = \phi_{j} \Delta v_{j}$ where $\phi_{j}$ denotes the pass-through rate of sector $j$
- In the first round, we have:

$$
\Delta p(1)_{j} = \sum_{i=1}^{n} A_{i,j} (\phi_{i} \Delta p(0)_{i}) = \sum_{i=1}^{n} A_{i,j} (\phi_{i} \Delta v_{i})
$$

- More generally, the recurrence relation is:

$$
\Delta p(k)_{j} = \sum_{i=1}^{n} A_{i,j} \phi_{i} \Delta p(k-1)_{i}
$$
Pass-through integration

Recurrence formula with pass-through

- Let $\phi = (\phi_1, \ldots, \phi_n)$ and $\Phi = \text{diag}(\phi)$ be the pass-through vector and matrix.
- The recurrence matrix form is:

\[
\begin{cases}
\Delta p(k) = A^\top \Phi \Delta p(k-1) \\
\Delta p(0) = \Phi \Delta \upsilon
\end{cases}
\]

- We deduce that:

\[
\Delta p = \sum_{k=0}^{\infty} (A^\top \Phi)^k \Phi \Delta \upsilon = (I_n - A^\top \Phi)^{-1} \Phi \Delta \upsilon = \tilde{\mathcal{L}}(\phi) \Delta \upsilon
\]

where $\tilde{\mathcal{L}}(\phi) = (I_n - A^\top \Phi)^{-1} \Phi$
Finally, we have:

$$\Delta p = \tilde{\mathcal{L}}(\phi) \Delta \upsilon := (I_n - A^\top \Phi)^{-1} \Phi \Delta \upsilon$$

Since $A$ is a substochastic matrix and $\Phi$ is a positive diagonal matrix, we verify that

$$\phi' \succeq \phi \Rightarrow \tilde{\mathcal{L}}(\phi') \succeq \tilde{\mathcal{L}}(\phi)$$

The lower bound is then reached when $\phi = 0_n$

The upper bound is reached when $\phi = 1_n$
Application to the carbon tax

- We have $\Delta v = t_{\text{direct}}$
- The cost paid by producers is:
  \[
  T_{\text{producer}} = x \odot (l_n - \Phi) \cdot t_{\text{direct}}
  \]
  \[
  = x \odot (1_n - \phi) \odot t_{\text{direct}}
  \]
  \[
  = (1_n - \phi) \odot T_{\text{direct}}
  \]
- The cost paid by consumers (downstream of the value chain) is:
  \[
  T_{\text{consumer}} = T_{\text{downstream}} = x \odot \tilde{L}(\phi) \cdot t_{\text{direct}}
  \]
- We deduce that:
  \[
  T_{\text{total}} = T_{\text{producer}} + T_{\text{consumer}} = x \odot \left( l_n - \Phi + \tilde{L}(\phi) \right) \cdot t_{\text{direct}}
  \]
- If $\phi_j = 100\%$, we have $\tilde{L}(1_n) = \tilde{L}$ and $\Delta p = \tilde{L} \cdot t_{\text{direct}}$
- If $\phi_j = 0\%$, we have $\tilde{L}(0_n) = 0_{n,n}$, $\Delta p = 0_n$, $T_{\text{producer}} = T_{\text{direct}}$ but $T_{\text{consumer}} = 0_n$
Remark

The functions $\Delta p$, $\pi$, $T_{\text{total}}$, $Cost_{\text{total}}$ and $TI$ remain homogeneous and additive with respect to $\tau$. We can also show that:

\[ \phi' \succeq \phi \Rightarrow T_{\text{total}}(\tau, \phi') \succeq T_{\text{total}}(\tau, \phi) \]

The effects of the tax is maximum when $\phi = 1_n$ and minimum when $\phi = 0_n$. If we consider a uniform pass-through, the total cost of the carbon tax is an increasing function of the pass-through rate.
Example #7

Table 128: Environmentally extended monetary input-output table

<table>
<thead>
<tr>
<th>Sector</th>
<th>$Z_{i,j}$</th>
<th>$z_{j}$</th>
<th>$x_{j}$</th>
<th>$CE_{1}$</th>
<th>$CI_{1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>500</td>
<td>800</td>
<td>1600</td>
<td>1250</td>
<td>500</td>
</tr>
<tr>
<td>Materials</td>
<td>500</td>
<td>400</td>
<td>1600</td>
<td>625</td>
<td>875</td>
</tr>
<tr>
<td>Industrials</td>
<td>250</td>
<td>800</td>
<td>2400</td>
<td>1250</td>
<td>3300</td>
</tr>
<tr>
<td>Services</td>
<td>100</td>
<td>200</td>
<td>800</td>
<td>4375</td>
<td>7025</td>
</tr>
<tr>
<td>Value added</td>
<td>3650</td>
<td>1800</td>
<td>1600</td>
<td>5000</td>
<td>125</td>
</tr>
<tr>
<td>Income</td>
<td>5000</td>
<td>4000</td>
<td>8000</td>
<td>12500</td>
<td></td>
</tr>
</tbody>
</table>

- The values of $Z_{i,j}$, $y_j$, $x_j$ and $V_{1,j}$ are in $\text{\$ mn}$
- The carbon emissions are expressed in $\text{ktCO}_2\text{e}$, while the carbon intensities are expressed in $\text{tCO}_2\text{e}/\text{\$ mn}$
- The carbon tax is differentiated: $\tau_1 = $200/\text{tCO}_2\text{e}$ and $\tau_2 = \tau_3 = \tau_4 = $100/\text{tCO}_2\text{e}$
Figure 274: Producer and consumer cost contributions (uniform pass-through)
**Application to the carbon tax**

**Figure 275:** Producer and consumer cost contributions ($\phi_2 = \phi_3 = \phi_4 = 0\%$)

![Graph showing producer and consumer cost contributions for different values of $\phi_1$]
The direct cost is:

$$
Cost_{\text{direct}} = \sum_{j=1}^{n} T_{\text{direct}}
$$

The total cost is:

$$
Cost_{\text{total}} = \sum_{j=1}^{n} T_{\text{total}}
$$

If the carbon tax is set at $100/\text{tCO}_2\text{e}$, the direct cost is $4.8 \text{ tn}$, while the total cost is $6.1 \text{ tn}$ if $\phi = 50\%$ and $13.3 \text{ tn}$ if $\phi = 100\%$
These correspond to 2.8%, 3.6% and 7.8% of the world GDP respectively.

If we apply a carbon tax of $500/\text{tCO}_2\text{e}$, these costs become $24.2$, $30.4$ and $66.4 \text{ tn}$ respectively.

The relationship between total costs and the pass-through parameter is cubic:

$$
\frac{Cost_{\text{total}}(\tau, \phi 1_n)}{Cost_{\text{direct}}(\tau, \phi 1_n)} \approx 1 + m(1-\infty)\phi^3
$$
Empirical results (Desnos et al., 2023)

**Figure 276:** World economic cost in $ tn (global analysis, uniform tax, Exiobase 2022)
Empirical results (Desnos et al., 2023)

- Producer price index (PPI): the basket weights are proportional to the output ($\alpha_j \propto x_j$)
- Consumer price index (CPI): the basket weights are proportional to the final demand ($\alpha_j \propto y_j$)
- For a carbon tax of $500/\text{tCO}_2\text{e}$ and a pass-through rate of 100%, the PPI inflation rate is close to 40%, while the CPI inflation rate reaches 30%
- The dispersion of the inflation rates explained by three factors (basket composition, value chain impact and direct carbon emissions of the country):

$$
\pi = \alpha^\top \cdot \hat{\mathcal{L}}(\phi) \cdot t_{\text{direct}}
$$

Basket \quad Value chain \quad Scope 1
Empirical results (Desnos et al., 2023)

**Figure 277:** World inflation rate in % (global analysis, uniform tax, Exiobase 2022)
Empirical results (Desnos et al., 2023)

Figure 278: Production inflation rate in % (global analysis, uniform tax, $\tau = 100/\text{tCO}_2\text{e}$, $\phi = 100\%$, Exiobase 2022)
Empirical results (Desnos et al., 2023)

Regional taxation
Empirical results (Desnos et al., 2023)

Figure 279: Cost breakdown (EU, uniform tax, $\phi = 50\%$, Exiobase 2022)
### Table 129: Domestic and foreign impacts (in $ bn) of a regional tax (uniform taxation, $\phi = 100\%$, Exiobase 2022)

<table>
<thead>
<tr>
<th>Carbon tax</th>
<th>Domestic impact</th>
<th></th>
<th>Foreign impact</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EU</td>
<td>USA</td>
<td>China</td>
<td>EU</td>
</tr>
<tr>
<td>$100/{t \text{CO}_2e}$</td>
<td>792</td>
<td>886</td>
<td>4,710</td>
<td>104</td>
</tr>
<tr>
<td>$250/{t \text{CO}_2e}$</td>
<td>1,979</td>
<td>2,215</td>
<td>11,774</td>
<td>261</td>
</tr>
<tr>
<td>$500/{t \text{CO}_2e}$</td>
<td>3,959</td>
<td>4,430</td>
<td>23,549</td>
<td>521</td>
</tr>
</tbody>
</table>
Empirical results (Desnos et al., 2023)

Table 130: Fifteen most affected foreign countries (uniform tax, \( \tau = 100/t{\text{CO}}_2e \), \( \phi = 100\% \), Exiobase 2022)

<table>
<thead>
<tr>
<th>Rank</th>
<th>EU tax</th>
<th>US tax</th>
<th>Chinese tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ROW 25.25%</td>
<td>CHN 24.74%</td>
<td>ROW 36.89%</td>
</tr>
<tr>
<td>2</td>
<td>CHN 23.62%</td>
<td>ROW 18.60%</td>
<td>USA 12.95%</td>
</tr>
<tr>
<td>3</td>
<td>USA 11.45%</td>
<td>CAN 9.35%</td>
<td>KOR 8.87%</td>
</tr>
<tr>
<td>4</td>
<td>GBR 8.77%</td>
<td>MEX 8.51%</td>
<td>IND 6.91%</td>
</tr>
<tr>
<td>5</td>
<td>CHE 4.32%</td>
<td>KOR 6.89%</td>
<td>JPN 6.44%</td>
</tr>
<tr>
<td>6</td>
<td>KOR 4.05%</td>
<td>JPN 5.05%</td>
<td>DEU 3.61%</td>
</tr>
<tr>
<td>7</td>
<td>IND 3.67%</td>
<td>IND 4.28%</td>
<td>MEX 2.19%</td>
</tr>
<tr>
<td>8</td>
<td>JPN 3.31%</td>
<td>DEU 2.80%</td>
<td>FRA 1.88%</td>
</tr>
<tr>
<td>9</td>
<td>TUR 2.62%</td>
<td>BRA 2.51%</td>
<td>GBR 1.83%</td>
</tr>
<tr>
<td>10</td>
<td>TWN 2.08%</td>
<td>GBR 2.34%</td>
<td>BRA 1.75%</td>
</tr>
<tr>
<td>11</td>
<td>CAN 2.06%</td>
<td>FRA 1.63%</td>
<td>IDN 1.74%</td>
</tr>
<tr>
<td>12</td>
<td>RUS 1.96%</td>
<td>TWN 1.59%</td>
<td>CAN 1.62%</td>
</tr>
<tr>
<td>13</td>
<td>BRA 1.90%</td>
<td>IRL 1.47%</td>
<td>ITA 1.59%</td>
</tr>
<tr>
<td>14</td>
<td>MEX 1.70%</td>
<td>ITA 1.43%</td>
<td>AUS 1.31%</td>
</tr>
<tr>
<td>15</td>
<td>NOR 1.46%</td>
<td>NLD 1.23%</td>
<td>TUR 1.11%</td>
</tr>
</tbody>
</table>
“The total additional cost of introducing a price of €250 per tonne of CO₂ to be paid by French emitting installations is €57.6 billion, or about 2.5 points of GDP. Of this total, €7 billion corresponds to purchases by foreign operators and investments by French and foreign operators. [...] Of this €50.3 billion, French companies would ultimately bear 57% of the additional costs, or about €28.7 billion. The rest would be passed on to final demand, i.e. 21.6 billion euros.” (Sautel et al., 2022, page 39).
Pass-through modeling

It is common to assume that the pass-through rate follows a beta distribution, as it is a parameter between 0 and 1:

$$\phi \sim B(\alpha, \beta)$$

**Table 131**: Probabilistic characterization of the four pass-through types

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Highly-elastic</th>
<th>High-elastic</th>
<th>Medium-elastic</th>
<th>Low-elastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3.0</td>
<td>4.0</td>
<td>14.0</td>
<td>12.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>12.0</td>
<td>6.0</td>
<td>6.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Moments</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_\phi$</td>
<td>20%</td>
<td>40%</td>
<td>70%</td>
<td>95%</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>10%</td>
<td>15%</td>
<td>10%</td>
<td>6%</td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_\phi (2.5%)$</td>
<td>5%</td>
<td>14%</td>
<td>49%</td>
<td>79%</td>
</tr>
<tr>
<td>$Q_\phi (97.5%)$</td>
<td>43%</td>
<td>70%</td>
<td>87%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Pass-through modeling

Figure 280: Probability density function of pass-through rates

![Probability density function of pass-through rates](image-url)
Figure 281: Probability density function of pass-through rates
Impact of a global carbon tax

- The total cost vector is $T_{\text{total}} = x \otimes \left( I_n - \text{diag}(\phi) + \tilde{L}(\phi) \right) t_{\text{direct}}$
- The pass-through rate of the sector is constant and equal to the mean of the corresponding beta distribution: $\phi = \mu_{\phi}$
- We consider two decompositions:

$$
\begin{align*}
T_{\text{producer}} &= x \otimes \left( I_n - \phi \right) \otimes t_{\text{direct}} \\
T_{\text{downstream}} &= x \otimes \tilde{L}(\phi) t_{\text{direct}}
\end{align*}
$$

and:

$$
\begin{align*}
T_{\text{direct}} &= x \otimes t_{\text{direct}} \\
T_{\text{indirect}} &= T_{\text{total}} - T_{\text{direct}} = x \otimes \left( \tilde{L}(\phi) - \text{diag}(\phi) \right) t_{\text{direct}}
\end{align*}
$$

- Government revenue is equal to the direct cost of the carbon tax:

$$R_{\text{government}} = T_{\text{direct}} = x \otimes t_{\text{direct}}$$

- Inflation rates are calculated using the following formula:

$$\pi = \alpha^\top \Delta p = \alpha^\top \tilde{L}(\phi) t_{\text{direct}}$$

- $\alpha_{\text{ppi}} = x / \left( 1_n^\top x \right)$ and $\alpha_{\text{cpi}} = y / \left( 1_n^\top y \right)$
Impact of a global carbon tax

Table 132: Economic impact of a global carbon tax ($100/tCO₂e)

<table>
<thead>
<tr>
<th>Region</th>
<th>Cost</th>
<th>Revenue</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{total}$</td>
<td>$T_{direct}$</td>
<td>$T_{indirect}$</td>
</tr>
<tr>
<td>World</td>
<td>5.01%</td>
<td>2.82%</td>
<td>2.18%</td>
</tr>
<tr>
<td>AUS</td>
<td>4.63%</td>
<td>2.93%</td>
<td>1.70%</td>
</tr>
<tr>
<td>AUT</td>
<td>2.08%</td>
<td>0.91%</td>
<td>1.17%</td>
</tr>
<tr>
<td>BEL</td>
<td>1.77%</td>
<td>0.94%</td>
<td>0.82%</td>
</tr>
<tr>
<td>BGR</td>
<td>7.07%</td>
<td>3.94%</td>
<td>3.12%</td>
</tr>
<tr>
<td>BRA</td>
<td>5.22%</td>
<td>3.78%</td>
<td>1.44%</td>
</tr>
<tr>
<td>CAN</td>
<td>3.74%</td>
<td>2.25%</td>
<td>1.49%</td>
</tr>
<tr>
<td>CHE</td>
<td>0.75%</td>
<td>0.30%</td>
<td>0.45%</td>
</tr>
<tr>
<td>CHN</td>
<td>7.47%</td>
<td>3.44%</td>
<td>4.03%</td>
</tr>
<tr>
<td>CYP</td>
<td>5.05%</td>
<td>3.94%</td>
<td>1.11%</td>
</tr>
<tr>
<td>CZE</td>
<td>4.47%</td>
<td>2.13%</td>
<td>2.34%</td>
</tr>
<tr>
<td>DEU</td>
<td>1.99%</td>
<td>1.10%</td>
<td>0.89%</td>
</tr>
<tr>
<td>DNK</td>
<td>1.47%</td>
<td>0.98%</td>
<td>0.49%</td>
</tr>
<tr>
<td>ESP</td>
<td>2.25%</td>
<td>1.15%</td>
<td>1.11%</td>
</tr>
<tr>
<td>FIN</td>
<td>2.80%</td>
<td>1.36%</td>
<td>1.44%</td>
</tr>
<tr>
<td>FRA</td>
<td>1.39%</td>
<td>0.79%</td>
<td>0.60%</td>
</tr>
<tr>
<td>GBR</td>
<td>1.53%</td>
<td>0.88%</td>
<td>0.65%</td>
</tr>
<tr>
<td>GRC</td>
<td>6.39%</td>
<td>4.61%</td>
<td>1.78%</td>
</tr>
<tr>
<td>HRV</td>
<td>3.57%</td>
<td>2.18%</td>
<td>1.38%</td>
</tr>
<tr>
<td>HUN</td>
<td>3.41%</td>
<td>1.83%</td>
<td>1.58%</td>
</tr>
<tr>
<td>IDN</td>
<td>11.38%</td>
<td>6.83%</td>
<td>4.55%</td>
</tr>
<tr>
<td>IRL</td>
<td>1.47%</td>
<td>0.95%</td>
<td>0.52%</td>
</tr>
</tbody>
</table>
## Impact of a global carbon tax

Table 133: Economic impact of a global carbon tax ($100/t\text{CO}_2\text{e}$)

<table>
<thead>
<tr>
<th>Region</th>
<th>$T_{total}$</th>
<th>$T_{direct}$</th>
<th>$T_{indirect}$</th>
<th>$T_{producer}$</th>
<th>$T_{downstream}$</th>
<th>$T_{net}$</th>
<th>Revenue</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>5.01%</td>
<td>2.82%</td>
<td>2.18%</td>
<td>0.93%</td>
<td>4.08%</td>
<td>2.18%</td>
<td>2.82%</td>
<td>4.08%</td>
</tr>
<tr>
<td>ITA</td>
<td>2.22%</td>
<td>0.93%</td>
<td>1.29%</td>
<td>0.28%</td>
<td>1.93%</td>
<td>1.29%</td>
<td>0.93%</td>
<td>1.93%</td>
</tr>
<tr>
<td>JPN</td>
<td>2.85%</td>
<td>1.38%</td>
<td>1.47%</td>
<td>0.32%</td>
<td>2.53%</td>
<td>1.47%</td>
<td>1.38%</td>
<td>2.53%</td>
</tr>
<tr>
<td>KOR</td>
<td>4.23%</td>
<td>1.61%</td>
<td>2.61%</td>
<td>0.38%</td>
<td>3.85%</td>
<td>2.61%</td>
<td>1.61%</td>
<td>3.85%</td>
</tr>
<tr>
<td>LTU</td>
<td>4.06%</td>
<td>2.41%</td>
<td>1.65%</td>
<td>1.00%</td>
<td>3.06%</td>
<td>1.65%</td>
<td>2.41%</td>
<td>3.06%</td>
</tr>
<tr>
<td>LUX</td>
<td>1.15%</td>
<td>0.51%</td>
<td>0.64%</td>
<td>0.35%</td>
<td>0.80%</td>
<td>0.64%</td>
<td>0.51%</td>
<td>0.80%</td>
</tr>
<tr>
<td>LVA</td>
<td>3.43%</td>
<td>2.15%</td>
<td>1.28%</td>
<td>1.07%</td>
<td>2.36%</td>
<td>1.28%</td>
<td>2.15%</td>
<td>2.36%</td>
</tr>
<tr>
<td>MEX</td>
<td>5.59%</td>
<td>3.60%</td>
<td>1.99%</td>
<td>1.02%</td>
<td>4.57%</td>
<td>1.99%</td>
<td>3.60%</td>
<td>4.57%</td>
</tr>
<tr>
<td>MLT</td>
<td>1.82%</td>
<td>0.64%</td>
<td>1.18%</td>
<td>0.17%</td>
<td>1.65%</td>
<td>1.18%</td>
<td>0.64%</td>
<td>1.65%</td>
</tr>
<tr>
<td>NLD</td>
<td>2.25%</td>
<td>1.14%</td>
<td>1.12%</td>
<td>0.51%</td>
<td>1.74%</td>
<td>1.12%</td>
<td>1.14%</td>
<td>1.74%</td>
</tr>
<tr>
<td>NOR</td>
<td>1.81%</td>
<td>1.31%</td>
<td>0.51%</td>
<td>0.58%</td>
<td>1.23%</td>
<td>0.51%</td>
<td>1.31%</td>
<td>1.23%</td>
</tr>
<tr>
<td>POL</td>
<td>5.84%</td>
<td>3.44%</td>
<td>2.40%</td>
<td>0.98%</td>
<td>4.86%</td>
<td>2.40%</td>
<td>3.44%</td>
<td>4.86%</td>
</tr>
<tr>
<td>PRT</td>
<td>3.77%</td>
<td>2.13%</td>
<td>1.64%</td>
<td>0.70%</td>
<td>3.07%</td>
<td>1.64%</td>
<td>2.13%</td>
<td>3.07%</td>
</tr>
<tr>
<td>ROU</td>
<td>4.10%</td>
<td>2.19%</td>
<td>1.91%</td>
<td>0.69%</td>
<td>3.42%</td>
<td>1.91%</td>
<td>2.19%</td>
<td>3.42%</td>
</tr>
<tr>
<td>RUS</td>
<td>12.79%</td>
<td>8.55%</td>
<td>4.24%</td>
<td>1.44%</td>
<td>11.34%</td>
<td>4.24%</td>
<td>8.55%</td>
<td>11.34%</td>
</tr>
<tr>
<td>SVK</td>
<td>3.29%</td>
<td>1.62%</td>
<td>1.66%</td>
<td>0.42%</td>
<td>2.87%</td>
<td>1.66%</td>
<td>1.62%</td>
<td>2.87%</td>
</tr>
<tr>
<td>SVN</td>
<td>2.79%</td>
<td>1.51%</td>
<td>1.28%</td>
<td>0.47%</td>
<td>2.32%</td>
<td>1.28%</td>
<td>1.51%</td>
<td>2.32%</td>
</tr>
<tr>
<td>SWE</td>
<td>1.21%</td>
<td>0.59%</td>
<td>0.62%</td>
<td>0.21%</td>
<td>1.00%</td>
<td>0.62%</td>
<td>0.59%</td>
<td>1.00%</td>
</tr>
<tr>
<td>TUR</td>
<td>5.78%</td>
<td>3.73%</td>
<td>2.05%</td>
<td>1.39%</td>
<td>4.39%</td>
<td>2.05%</td>
<td>3.73%</td>
<td>4.39%</td>
</tr>
<tr>
<td>TWN</td>
<td>5.16%</td>
<td>2.21%</td>
<td>2.95%</td>
<td>0.75%</td>
<td>4.41%</td>
<td>2.95%</td>
<td>2.21%</td>
<td>4.41%</td>
</tr>
<tr>
<td>USA</td>
<td>2.17%</td>
<td>1.40%</td>
<td>0.78%</td>
<td>0.34%</td>
<td>1.83%</td>
<td>0.78%</td>
<td>1.40%</td>
<td>1.83%</td>
</tr>
<tr>
<td>ROW</td>
<td>7.55%</td>
<td>5.14%</td>
<td>2.40%</td>
<td>1.87%</td>
<td>5.68%</td>
<td>2.40%</td>
<td>5.14%</td>
<td>5.68%</td>
</tr>
</tbody>
</table>
Course 2023-2024 in Sustainable Finance
Lecture 11. Climate Risk Measures

Thierry Roncalli

*Amundi Asset Management

*University of Paris-Saclay

March 2024

33 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
How to define the carbon footprint?

Wackernagel and Rees (1996) published the seminal book on the ecological footprint:

“the carbon footprint stands for a certain amount of gaseous emissions that are relevant to climate change and associated with human production or consumption activities”

Wiedmann and Minx (2008) proposed this definition:

“The carbon footprint is a measure of the exclusive total amount of carbon dioxide emissions that is directly and indirectly caused by an activity or is accumulated over the life stages of a product”
The carbon footprint is measured in carbon dioxide equivalent (CO\textsubscript{2}e) ⇒ a common unit

We have:

\[
\text{equivalent mass of CO}_2 = \text{mass of the gas} \times \text{gwp of the gas}
\]

Examples (IPCC, AR5, 2013):
- 1 kg of methane corresponds to 28 kg of CO\textsubscript{2}
- 1 kg of nitrous oxide corresponds to 265 kg of CO\textsubscript{2}

The carbon footprint is equal to:

\[
m = \sum_{i=1}^{n} m_i \cdot \text{gwp}_i
\]

The units are: kgCO\textsubscript{2}e, tCO\textsubscript{2}e, ktCO\textsubscript{2}e, MtCO\textsubscript{2}e and GtCO\textsubscript{2}e
Example #1

We consider a company $A$ that emits 3017 tonnes of $CO_2$, 10 tonnes of $CH_4$ and 1.8 tonnes of $N_2O$. For the company $B$, the GHG emissions are respectively equal to 2302 tonnes of $CO_2$, 32 tonnes of $CH_4$ and 3.0 tonnes of $N_2O$.

The mass of $CO_2$ equivalent for companies $A$ and $B$ is equal to:

$$m_A = 3017 \times 1 + 10 \times 28 + 1.8 \times 265 = 3774 \text{ tCO}_2\text{e}$$

and:

$$m_B = 2302 \times 1 + 32 \times 28 + 3.0 \times 265 = 3993 \text{ tCO}_2\text{e}$$
According to IPCC (2007), GWP is defined as “the cumulative radiative forcing, both direct and indirect effects, over a specified time horizon resulting from the emission of a unit mass of gas related to some reference gas”.

- Each gas differs in their capacity to absorb the energy (radiative efficiency) and how long it stays in the atmosphere (lifetime)
- The impact of a gas on global warming depends on the combination of radiative efficiency and lifetime
The mathematics of GWP

- The mathematical definition of the global warming potential is:

\[
gwp_i(t) = \frac{A_{\text{gwp}i}(t)}{A_{\text{gwp}0}(t)} = \frac{\int_0^t RF_i(s) \, ds}{\int_0^t RF_0(s) \, ds} = \frac{\int_0^t A_i(s) S_i(s) \, ds}{\int_0^t A_0(s) S_0(s) \, ds}
\]

where \( A_i(t) \) is the radiative efficiency value of gas \( i \), \( S_i(t) \) is the decay function and \( i = 0 \) is the reference gas (e.g., CO\(_2\)).

- We assume that:

\[
S_i(t) = \sum_{j=1}^m a_{i,j} e^{-\lambda_{i,j} t}
\]

where \( \sum_{j=1}^m a_{i,j} = 1 \)

- We obtain:

\[
gwp_i(t) = \frac{A_i \sum_{j=1}^m a_{i,j} \lambda_{i,j}^{-1} (1 - e^{-\lambda_{i,j} t})}{A_0 \sum_{j=1}^m a_{0,j} \lambda_{0,j}^{-1} (1 - e^{-\lambda_{0,j} t})}
\]
Estimation of the global warming potential

- Carbon dioxide
  - $A_{CO_2} = 1.76 \times 10^{-18}$
  - The impulse response function is:
    $$S_{CO_2}(t) = 0.2173 + 0.2240 \cdot \exp \left( -\frac{t}{394.4} \right) + 0.2824 \cdot \exp \left( -\frac{t}{36.54} \right) + 0.2763 \cdot \exp \left( -\frac{t}{4.304} \right)$$

- Methane
  - $A_{CH_4} = 2.11 \times 10^{-16}$
  - The impulse response function is:
    $$S_{CH_4}(t) = \exp \left( -\frac{t}{12.4} \right)$$
Estimation of the global warming potential

Figure 282: Fraction of gas remaining in the atmosphere

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

Remark

- The decay function is a survival function
- The density function is equal to $f_i(t) = -\partial_t S_i(t)$
- Let $\tau_i$ be random time that the gas remains in the atmosphere
- In the case of the exponential distribution $E(\lambda)$, we have

\[
S_i(t) = e^{-\lambda t} \\
 f_i(t) = \lambda e^{-\lambda t} \\
 \mathbb{E}[\tau_i] = \frac{1}{\lambda}
\]

$\Rightarrow$ The survival function of the CH$_4$ gas is exponential with a mean time equal to 12.4 years ($\lambda = 1/12.4$)
Estimation of the global warming potential

In the general case, the probability density function is equal to:

\[ f_i (t) = -\partial_t S_i (t) = \sum_{j=1}^{m} a_{i,j} \lambda_{i,j} e^{-\lambda_{i,j} t} \]

The mean time \( T_i \) is given by:

\[
T_i := \mathbb{E} [\tau_i] = \int_{0}^{\infty} s f_i (s) \, ds \\
= \sum_{j=1}^{m} a_{i,j} \int_{0}^{\infty} \lambda_{i,j} s e^{-\lambda_{i,j} s} \, ds \\
= \sum_{j=1}^{m} \frac{a_{i,j}}{\lambda_{i,j}}
\]

Remark

We have \( T_{\text{CH}_4} = 12.4 \) years, but \( T_{\text{CO}_2} = \infty \)
Estimation of the global warming potential

Figure 283: Probability density function of the random time

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

Remark
- $f_i(t)$ is an exponential mixture distribution where $m$ is the number of mixture components
- $\mathcal{E}(\lambda_{i,j})$ is the probability distribution associated with the $j^{th}$ component
- $a_{i,j}$ is the mixture weight of the $j^{th}$ component

We have:

$$T_i = \mathbb{E}[\tau_i] = \sum_{j=1}^{m} a_{i,j} \mathbb{E}[\tau_{i,j}] = \sum_{j=1}^{m} a_{i,j} T_{i,j}$$

For the CO$_2$ gas, the exponential mixture distribution is defined by the following parameters:

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{i,j}$</td>
<td>0.2173</td>
<td>0.2240</td>
<td>0.2824</td>
<td>0.2763</td>
</tr>
<tr>
<td>$\lambda_{i,j} \times 10^3$</td>
<td>0.00</td>
<td>2.535</td>
<td>27.367</td>
<td>232.342</td>
</tr>
<tr>
<td>$T_{i,j}$ (in years)</td>
<td>$\infty$</td>
<td>394.4</td>
<td>36.54</td>
<td>4.304</td>
</tr>
</tbody>
</table>
Estimation of the global warming potential

Figure 284: Survival function

We have $$S_{CO_2}(\infty) = 21.73\%$$!
Estimation of the global warming potential

**Figure 285**: Absolute global warming potential

Source: Kleinberg (2020) & Author’s calculations.
Figure 286: Global warming potential for methane

Source: Kleinberg (2020) & Author’s calculations.
Estimation of the global warming potential

We have:

- \( \text{Agwp}_{\text{CO}_2} (\infty) = \infty \)
- \( \text{Agwp}_{\text{CH}_4} (\infty) = A_{\text{CH}_4} \times T_{\text{CH}_4} \approx 2.11 \times 12.4 = 26.164 \)
- The instantaneous global warming potential of the methane is equal to:
  \[
  gwp_{\text{CH}_4} (0) = \frac{A_{\text{CH}_4}}{A_{\text{CO}_2}} = \frac{2.11 \times 10^{-16}}{1.76 \times 10^{-18}} \approx 119.9
  \]
- After 100 years, we obtain:
  \[
  gwp_{\text{CH}_4} (100) = 28.3853
  \]
  This is the IPCC value!
- Because of the persistant regime of the carbon dioxyde, we have
  \( gwp_{\text{CH}_4} (\infty) = 0 \)
- We have:
  \[
  gwp_{\text{CH}_4} (t) \leq 1 \iff t \geq 6382 \text{ years}
  \]
## Estimation of the global warming potential

**Table 134: GWP values for 100-year time horizon**

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>AR2</th>
<th>AR4</th>
<th>AR5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide</td>
<td>CO$_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Methane</td>
<td>CH$_4$</td>
<td>21</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>N$_2$O</td>
<td>310</td>
<td>298</td>
<td>265</td>
</tr>
<tr>
<td>Sulphur hexafluoride</td>
<td>SF$_6$</td>
<td>23 900</td>
<td>22 800</td>
<td>23 500</td>
</tr>
<tr>
<td>Hydrofluorocarbons (HFC)</td>
<td>CHF$_3$</td>
<td>11 700</td>
<td>14 800</td>
<td>12 400</td>
</tr>
<tr>
<td></td>
<td>CH$_2$F$_2$</td>
<td>650</td>
<td>675</td>
<td>677</td>
</tr>
<tr>
<td></td>
<td>Etc.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perfluorocarbons (PFC)</td>
<td>CF$_4$</td>
<td>6 500</td>
<td>7 390</td>
<td>6 630</td>
</tr>
<tr>
<td></td>
<td>C$_2$F$_6$</td>
<td>9 200</td>
<td>12 200</td>
<td>11 100</td>
</tr>
<tr>
<td></td>
<td>Etc.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consolidation accounting at the company level

Two approaches:

1. Equity share approach
2. Control approach
   1. Financial control
   2. Operational control
### Table 135: Percent of reported GHG emissions under each consolidation method

<table>
<thead>
<tr>
<th>Accounting categories</th>
<th>GHG accounting based on</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>equity share</td>
<td>financial control</td>
</tr>
<tr>
<td>Wholly owned asset</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Group companies/subsidiaries</td>
<td>OWNR</td>
<td>100%</td>
</tr>
<tr>
<td>Associated/affiliated compa-</td>
<td>OWNR</td>
<td>0%</td>
</tr>
<tr>
<td>nies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint ventures/partnerships</td>
<td>OWNR</td>
<td>OWNR</td>
</tr>
<tr>
<td>Fixed asset investments</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Franchises</td>
<td>OWNR</td>
<td>100%</td>
</tr>
</tbody>
</table>

Source: GHG Protocol (2004, Table 1, page 19).

OWNR = Ownership ratio
For each company, the brown number corresponds to the carbon emissions in tCO$_2$e. The three figures at the right or left of the node correspond respectively to the equity share, the financial control and the operational control.
Consolidation accounting at the company level

- Equity share approach:
  \[ CE_A = 827 + 100\% \times 135 + 90\% \times 261 + 45\% \times 220 + 0\% \times 1385 + \\
  90\% \times 75\% \times 63 + 90\% \times 50\% \times 179 + 45\% \times 33\% \times 37 \]
  \[ = 1424.4 \text{tCO}_2\text{e} \]

- Financial control approach:
  \[ CE_A = 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1385 + \\
  100\% \times 100\% \times 63 + 100\% \times 50\% \times 179 + 100\% \times 0\% \times 37 \]
  \[ = 1595.50 \text{tCO}_2\text{e} \]

- Operational control approach:
  \[ CE_A = 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1385 + \\
  100\% \times 100\% \times 63 + 100\% \times 0\% \times 179 + 100\% \times 0\% \times 37 \]
  \[ = 1506.00 \text{tCO}_2\text{e} \]
Scope 1, 2 and 3 of carbon emissions

GHG Protocol (www.ghgprotocol.org/corporate-standard)

- Scope 1 denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- Scope 2 corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- Scope 3 are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories\(^a\):
  - Upstream scope 3 emissions are defined as indirect carbon emissions related to purchased goods and services.
  - Downstream scope 3 emissions are defined as indirect carbon emissions related to sold goods and services.

\(^a\)The upstream value chain includes all activities related to the suppliers whereas the downstream value chain refers to post-manufacturing activities.
### Table 136: Examples of CDP reporting ($CE$ in $tCO_2e$, year 2020)

<table>
<thead>
<tr>
<th>Scope</th>
<th>Category</th>
<th>Sub-category</th>
<th>Amazon</th>
<th>Danone</th>
<th>ENEL</th>
<th>Pfizer</th>
<th>Netflix</th>
<th>Walmart</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Location-based (2a)</td>
<td>Purchased goods and services</td>
<td>16 683 423</td>
<td>19 920 918</td>
<td>2 526 537</td>
<td>765 208</td>
<td>130 200 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Capital goods</td>
<td>13 202 065</td>
<td>191 894</td>
<td>116 366</td>
<td>645 328</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fuel and energy related activities</td>
<td>1 248 847</td>
<td>283 764</td>
<td>1 061 268</td>
<td>203 093</td>
<td>12 287</td>
<td>3 327 874</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream transportation and distribution</td>
<td>8 563 695</td>
<td>321 558</td>
<td>112 358</td>
<td>723 558</td>
<td>64 693</td>
<td>342 577</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waste generated in operations</td>
<td>16 628</td>
<td>152 789</td>
<td>3 161</td>
<td>14 940</td>
<td></td>
<td>869 927</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Business travel</td>
<td>313 043</td>
<td></td>
<td>35 128</td>
<td>41 439</td>
<td>37 439</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Employee commuting</td>
<td>306 033</td>
<td></td>
<td>48 414</td>
<td>19 116</td>
<td>3 500 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream leased assets</td>
<td>1 223 903</td>
<td></td>
<td>30 522</td>
<td>131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Market-based (2b)</td>
<td>Purchased goods and services</td>
<td>16 683 423</td>
<td>19 920 918</td>
<td>2 526 537</td>
<td>765 208</td>
<td>130 200 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Capital goods</td>
<td>13 202 065</td>
<td>191 894</td>
<td>116 366</td>
<td>645 328</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fuel and energy related activities</td>
<td>1 248 847</td>
<td>283 764</td>
<td>1 061 268</td>
<td>203 093</td>
<td>12 287</td>
<td>3 327 874</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream transportation and distribution</td>
<td>8 563 695</td>
<td>321 558</td>
<td>112 358</td>
<td>723 558</td>
<td>64 693</td>
<td>342 577</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waste generated in operations</td>
<td>16 628</td>
<td>152 789</td>
<td>3 161</td>
<td>14 940</td>
<td></td>
<td>869 927</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Business travel</td>
<td>313 043</td>
<td></td>
<td>35 128</td>
<td>41 439</td>
<td>37 439</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Employee commuting</td>
<td>306 033</td>
<td></td>
<td>48 414</td>
<td>19 116</td>
<td>3 500 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upstream leased assets</td>
<td>1 223 903</td>
<td></td>
<td>30 522</td>
<td>131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Downstream</td>
<td>Downstream transportation and distribution</td>
<td>2 785 676</td>
<td>1 627 090</td>
<td></td>
<td>7 295</td>
<td>5 099</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Processing of sold products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use of sold products</td>
<td>1 426 543</td>
<td>1 885 548</td>
<td>46 524 860</td>
<td></td>
<td>952</td>
<td>32 211 000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>End-of-life treatment of sold products</td>
<td>0</td>
<td>782 649</td>
<td></td>
<td></td>
<td></td>
<td>130</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Downstream leased assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>349</td>
<td>130 000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Franchises</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Investments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36 839</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>Scope 1 + 2a</td>
<td>18 642 924</td>
<td>1 533 064</td>
<td>50 245 685</td>
<td>1 206 037</td>
<td>59 468</td>
<td>18 268 299</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 1 + 2b</td>
<td>14 888 227</td>
<td>1 147 564</td>
<td>53 110 954</td>
<td>1 196 981</td>
<td>31 024</td>
<td>16 426 836</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 3 upstream</td>
<td>41 557 637</td>
<td>20 679 029</td>
<td>1 176 787</td>
<td>3 774 086</td>
<td>1 019 240</td>
<td>138 923 145</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 3 downstream</td>
<td>4 212 219</td>
<td>4 295 287</td>
<td>46 524 860</td>
<td>44 134</td>
<td>1 301</td>
<td>32 346 229</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 3</td>
<td>45 769 856</td>
<td>24 974 316</td>
<td>47 701 647</td>
<td>3 818 220</td>
<td>1 020 541</td>
<td>171 269 374</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 1 + 2a + 3</td>
<td>64 412 780</td>
<td>26 507 380</td>
<td>97 947 332</td>
<td>5 024 257</td>
<td>1 080 009</td>
<td>189 537 673</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scope 1 + 2b + 3</td>
<td>60 658 083</td>
<td>26 121 880</td>
<td>100 812 601</td>
<td>5 015 201</td>
<td>1 051 565</td>
<td>187 696 210</td>
</tr>
</tbody>
</table>

Source: CDP database as of 01/07/2022 & Author’s computation.
Scope 1, 2 and 3 of carbon emissions

CDP questionnaire for corporates

- www.cdp.net/en/guidance/guidance-for-companies
- HTML, Word and PDF formats
- 129 pages and 16 sections: $SC_1$ (§C6.1), $SC_2$ (§C6.3) and $SC_3$ emissions (§C6.5) — emissions intensities (§C6.10)
Computation of scope 1 emissions

- We allocate the activities to the three scopes
- Then, we apply an emission factor to each activity and each gas:

\[ E_{g,h} = A_h \cdot \mathcal{E} \mathcal{F}_{g,h} \]

where \( A_h \) is the \( h^{\text{th}} \) activity rate (also called activity data) and \( \mathcal{E} \mathcal{F}_{g,h} \) is the emission factor for the \( h^{\text{th}} \) activity and the \( g^{\text{th}} \) gas

- \( A_h \) can be measured in volume, weight, distance, duration, surface, etc.
- \( E_{g,h} \) is expressed in tonne
- \( \mathcal{E} \mathcal{F}_{g,h} \) is measured in tonne per activity unit

- For each gas, we calculate the total emissions:

\[ E_g = \sum_{h=1}^{n_A} E_{g,h} = \sum_{h=1}^{n_A} A_h \cdot \mathcal{E} \mathcal{F}_{g,h} \]

- Finally, we estimate the carbon emissions by applying the right GWP:

\[ CE = \sum_{g=1}^{n_G} \text{gwp}_g \cdot E_g \]
The choice of data inputs is codified by IPCC (2019):

- Tier 1 methods use global default emission factors;
- Tier 2 methods use country-level or region-specific emission factors;
- Tier 3 methods use directly monitored or site-specific emission factors.

⇒ IPCC Emission Factor Database, National Inventory Reports (NIRs), country emission factor databases, etc.

France

- The database of emission factors is managed by ADEME (Agence de l’Environnement et de la Maîtrise de l’Energie)
- It contains about 5,300 validated emission factors
**GHG inventory document of Enel (2021)**

- **Scope 1 emissions expressed in ktCO$_2$e:**

<table>
<thead>
<tr>
<th></th>
<th>CO$_2$</th>
<th>CH$_4$</th>
<th>N$_2$O</th>
<th>NF$_3$</th>
<th>SF$_6$</th>
<th>HFCs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity power generation</td>
<td>50643.54</td>
<td>385.25</td>
<td>98.14</td>
<td>0.014</td>
<td>31.15</td>
<td>10.22</td>
<td>51168.32</td>
</tr>
<tr>
<td>Electricity distribution</td>
<td>208.33</td>
<td>0.24</td>
<td>0.45</td>
<td></td>
<td>111.62</td>
<td></td>
<td>320.64</td>
</tr>
<tr>
<td>Real estate</td>
<td>79.87</td>
<td>0.22</td>
<td>1.24</td>
<td></td>
<td></td>
<td></td>
<td>81.30</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>50931.72</td>
<td>385.71</td>
<td>99.83</td>
<td>0.014</td>
<td>142.77</td>
<td>10.22</td>
<td>51750.26</td>
</tr>
</tbody>
</table>

- The scope 1 emissions of Enel is equal to 51.75 MtCO$_2$e
### Table 137: Examples of emission factors (EFDB, IPCC)

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Gas</th>
<th>Region</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron and steel production</td>
<td>Integrated facility</td>
<td>CO₂</td>
<td>Canada</td>
<td>1.6</td>
<td>t/tonne</td>
</tr>
<tr>
<td>Iron and steel production</td>
<td>Electrode consumption from steel produced in electric arc furnaces</td>
<td>CO₂</td>
<td>Global</td>
<td>5.0</td>
<td>kg/tonne</td>
</tr>
<tr>
<td>Iron and steel production</td>
<td>Steel processing (rolling mills)</td>
<td>N₂O</td>
<td>Global</td>
<td>40</td>
<td>g/tonne</td>
</tr>
<tr>
<td>Manufacture of solid fuels</td>
<td>Metallurgical coke production</td>
<td>CO₂</td>
<td>Global</td>
<td>0.56</td>
<td>t/tonne</td>
</tr>
<tr>
<td>Manufacture of solid fuels</td>
<td>Crude oil</td>
<td>CH₄</td>
<td>Global</td>
<td>0.1</td>
<td>g/tonne</td>
</tr>
<tr>
<td>Fuel combustion activities</td>
<td>Natural gas</td>
<td>CO₂</td>
<td>Global</td>
<td>20</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td>Fuel combustion activities</td>
<td>Ethane</td>
<td>CO₂</td>
<td>Global</td>
<td>15.3</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td>Fuel combustion activities</td>
<td>Ethane</td>
<td>CO₂</td>
<td>Global</td>
<td>16.8</td>
<td>tCarbon/TeraJoule</td>
</tr>
<tr>
<td>Fuel combustion activities</td>
<td>Ethane</td>
<td>CH₄</td>
<td>Global</td>
<td>0.9</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Cement production</td>
<td>Cement production</td>
<td>CO₂</td>
<td>Global</td>
<td>0.4985</td>
<td>t/tonne</td>
</tr>
<tr>
<td>Cement production</td>
<td>Enteric fermentation</td>
<td>CH₄</td>
<td>Global</td>
<td>18</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Horses</td>
<td>Manure management (annual average temperature is less than 15°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>1.4</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Horses</td>
<td>Manure management (annual average temperature is between 15°C and 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>2.1</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Buffalo</td>
<td>Enteric fermentation</td>
<td>CH₄</td>
<td>Global</td>
<td>55</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Poultry</td>
<td>Manure management (annual average temperature is less than 15°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.078</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Poultry</td>
<td>Manure management (annual average temperature is between 15°C and 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.117</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Poultry</td>
<td>Manure management (annual average temperature is greater than 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.157</td>
<td>kg/head/year</td>
</tr>
<tr>
<td>Poultry</td>
<td>Manure management (annual average temperature is greater than 25°C)</td>
<td>CH₄</td>
<td>Developed countries</td>
<td>0.023</td>
<td>kg/head/year</td>
</tr>
</tbody>
</table>

Source: EFDB, www.ipcc-nggip.iges.or.jp/EFDB.
Scope 2 emissions

Definition

Scope 2 is “an indirect emission category that includes GHG emissions from the purchased or acquired electricity, steam, heat, or cooling consumed” (GHG Protocol, 2015):

- **Electricity**
  People use electricity for operating machines, lighting, heating, cooling, electric vehicle charging, computers, electronics, public transportation systems, etc.

- **Steam**
  Industries use steam for mechanical work, heating, propulsion, driven turbines in electric power plants, etc.

- **Heat**
  Buildings use heat to control inside temperature and heat water, while the industrial sector uses heat for washing, cooking, sterilizing, drying, etc. Heat may be produced from electricity, solar heat processes or thermal combustion.

- **Cooling**
  It is produced from electricity or though the processes of forced air, conduction, convection, etc.
**Scope 2 emissions**

**Figure 288:** Energy production and consumption from owned/operated generation

**Figure 289:** Direct line energy transfer

## Chapter 5: Identifying Scope 2 Emissions and Setting the Scope 2 Boundary

Electricity generators report any emissions from generation in scope 1, but most renewable or nuclear technology would report “zero” emissions from this generation. A grid operator or utility dispatches these generation units throughout the day on the basis of contracts, cost, and other factors. Because it is a shared network as opposed to a direct line, consumers may not be able to identify the specific power plant producing the energy they are using at any given time. Use of specified generation on the grid can only be determined contractually. Energy on the grid moves to the nearest point it can be used, and multiple regions can exchange power depending on the capacity and needs of these regions. Steam, heat, and cooling can also be delivered through a grid, often called a district energy system. Such systems provide energy to multiple consumers, though they often have only one generation facility and serve a more limited geographic area than electricity grids.

4. If some consumed electricity comes from owned/operated equipment, and some is purchased from the grid (Figure 5.4). Some companies own, operate, or host energy generation sources such as solar panels or fuel cells on the premises of their building or in close proximity to where the energy is consumed. This arrangement is often termed “distributed generation” or “on-site” consumption, as it consists of generation units across decentralized locations (often Figure 5.3 electricity distribution on a grid).

---

**Figure 290: Electricity production on a grid**

Source: GHG Protocol (2015, Figure 5.4, page 38).
Figure 291: Facility consuming both energy generated on-site and purchased from the grid

Source: GHG Protocol (2015, Figure 5.3, page 37).
Scope 2 emissions are calculated using activity data and emission factors expressed in MWh and $\text{tCO}_2e$/MWh:

\[
CE = \sum_s A_s \cdot \mathcal{E} \cdot \mathcal{F}_s
\]

where:
- $A_s$ is the amount of purchased electricity for the energy generation source $s$
- $\mathcal{E} \cdot \mathcal{F}_s$ is the emission factor of the source $s$
Computation of scope 2 emissions

Example #2
We consider a company, whose electricity consumption is equal to 2000 MWh per year. The electricity comes from two sources: 60% from a direct line with an electricity supplier (source $S_1$) and 40% from the country grid (source $S_2$). The emission factors are respectively equal to 200 and 350 gCO$_2$e/kWh.
The electricity consumption from source $S_1$ is equal to $60\% \times 2\,000 = 1\,200$ MWh or $1\,200\,000$ kWh

We deduce that the carbon emissions from this source is:

$$CE(S_1) = (1.2 \times 10^6) \times 200 = 240 \times 10^6 \text{ gCO}_2\text{e} = 240 \text{ tCO}_2\text{e}$$

For the second source, we obtain:

$$CE(S_2) = (0.8 \times 10^6) \times 350 = 280 \times 10^6 \text{ gCO}_2\text{e} = 280 \text{ tCO}_2\text{e}$$

We deduce that the Scope 2 carbon emissions of this company is equal to $520 \text{ tCO}_2\text{e}$
Scope 2 emissions accounting

Two main methods:

- **Location-based method**
  In this approach, the company uses the average emission factor of the region or the country. For instance, if the electricity consumption is located in France, the company can use the emission intensity of the French energy mix;

- **Market-based method**
  This approach reflects the GHG emissions from the electricity that the company has chosen in the market. This means that the scope 2 carbon emissions will depend on the scope 1 carbon intensity of the electricity supplier.
Scope 2 emission factors

**Figure 292**: Emission factor in $g\text{CO}_2\text{e}/\text{kWh}$ of electricity generation (European Union, 1990 – 1992)

### Table 138: Emission factor in gCO$_2$e/kWh of electricity generation in the world

<table>
<thead>
<tr>
<th>Region</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>484</td>
<td>Australia</td>
<td>531</td>
<td>Germany</td>
<td>354</td>
<td>Portugal</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>539</td>
<td>Canada</td>
<td>128</td>
<td>India</td>
<td>637</td>
<td>Russia</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>280</td>
<td>China</td>
<td>544</td>
<td>Iran</td>
<td>492</td>
<td>Spain</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>North America</td>
<td>352</td>
<td>Costa Rica</td>
<td>33</td>
<td>Italy</td>
<td>226</td>
<td>Switzerland</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>South America</td>
<td>204</td>
<td>Cuba</td>
<td>575</td>
<td>Japan</td>
<td>479</td>
<td>United Kingdom</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>World</td>
<td>442</td>
<td>France</td>
<td>58</td>
<td>Norway</td>
<td>26</td>
<td>United States</td>
<td>380</td>
<td></td>
</tr>
</tbody>
</table>

Source: [https://ourworldindata.org/grapher/carbon-intensity-electricity](https://ourworldindata.org/grapher/carbon-intensity-electricity)
Example #3

We consider a French bank, whose activities are mainly located in France and the Western Europe. Below, we report the energy consumption (in MWh) by country:

<table>
<thead>
<tr>
<th>Country</th>
<th>Consumption (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>125,807</td>
</tr>
<tr>
<td>Germany</td>
<td>71,890</td>
</tr>
<tr>
<td>Italy</td>
<td>197,696</td>
</tr>
<tr>
<td>Netherlands</td>
<td>18,152</td>
</tr>
<tr>
<td>Spain</td>
<td>61,106</td>
</tr>
<tr>
<td>UK</td>
<td>124,010</td>
</tr>
<tr>
<td>France</td>
<td>1,132,261</td>
</tr>
<tr>
<td>Ireland</td>
<td>125,807</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>33,069</td>
</tr>
<tr>
<td>Portugal</td>
<td>12,581</td>
</tr>
<tr>
<td>Switzerland</td>
<td>73,148</td>
</tr>
<tr>
<td>World</td>
<td>37,742</td>
</tr>
</tbody>
</table>
Computation of scope 2 emissions

If we consider a Tier 1 approach, we can estimate the scope 2 emissions of the bank by computing the total activity data and multiplying by the global emission factor.

Since we have twelve sources, we obtain:

\[ A = \sum_{s=1}^{12} A_s = 125\,807 + 1\,132\,261 + \ldots + 37\,742 = 2\,013\,269 \text{ MWh} \]

and:

\[ CE = A \cdot EF_{\text{World}} = (2\,013\,269 \times 10^3) \times 442 = 889\,864\,898\,000 \text{ gCO}_2\text{e} \]
\[ = 889.86 \text{ ktCO}_2\text{e} \]
Another Tier 1 approach is to consider the emission factor of the European Union, because the rest of the world represents less than 2% of the electricity consumption. Using $EF_{EU} = 275$, we obtain $CE = 553.65$ ktCO$_2$e.
The third approach uses a Tier 2 method by considering the emission factor of each country.

We use the previous figures and the following emission factors: Belgium (143); Ireland (402); Luxembourg (68) and Netherlands (331).

We deduce that:

\[
CE = \sum_{s=1}^{12} A_s \cdot EF_s
\]

\[
= (125807 \times 143 + 1132261 \times 58 + \ldots \\
+ 124010 \times 270 + 37742 \times 442) \times \frac{10^3}{10^9}
\]

\[
= 278.85 \text{ ktCO}_2\text{e}
\]

⇒ The estimated scope 2 emissions of this bank are sensitive to the approach.
Example #4

We consider a Norwegian company, whose current electricity consumption is equal to 1351 Mwh. 60% of the electricity comes from the Norwegian hydroelectricity and the GO system guarantees that this green electricity emits 1 gCO$_2$e/kWh.

If we assume that the remaining 40% of the electricity consumption comes from the Norwegian grid$^{34}$, the market based scope 2 emissions of this company are equal to:

$$CE = \frac{10^6 \times 60\% \times 1 + 10^6 \times 40\% \times 26}{10^6}$$

$$= 11 \text{ ktCO}_2\text{e}$$

$^{34}$The emission factor for Norway is 26 gCO$_2$e/kWh.
## Computation of scope 2 emissions

### Table 139: Emission factor in gCO$_2$e/KWh from electricity supply technologies (IPCC, 2014; UNECE, 2022)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Characteristic</th>
<th>IPCC</th>
<th>UNECE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind</td>
<td>Onshore</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Offshore</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Nuclear</td>
<td></td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>Hydro power</td>
<td></td>
<td>24</td>
<td>11</td>
</tr>
<tr>
<td>Solar power</td>
<td>CSP</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Rooftop (PV)</td>
<td>41</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Utility/ground (PV)</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>Geothermal</td>
<td></td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Biomass</td>
<td>Dedicated</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>Gas</td>
<td>CCS</td>
<td>169</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>Combined cycle</td>
<td>490</td>
<td>430</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>CCS</td>
<td>161</td>
<td>350</td>
</tr>
<tr>
<td>Coal</td>
<td>CCS</td>
<td>820</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td>70–290</td>
<td>190–470</td>
</tr>
</tbody>
</table>

CSP: concentrated solar power; PV: photovoltaic power; CCS: carbon capture and storage; PC: pulverized coal.
Reporting of scope 2 emissions

GHG inventory document of Enel (2021)

- The scope 2 emissions expressed in ktCO$_2$e are:

<table>
<thead>
<tr>
<th></th>
<th>Electricity purchased from the grid</th>
<th>Losses on the distribution grid</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location-based</td>
<td>1336.67</td>
<td>2966.52</td>
<td>4303.18</td>
</tr>
<tr>
<td>Market-based</td>
<td>2351.00</td>
<td>4763.15</td>
<td>7114.15</td>
</tr>
</tbody>
</table>
### Table 140: Statistics of CDP scope 2 emissions (2020)

<table>
<thead>
<tr>
<th></th>
<th>$CE_{loc} = 0$</th>
<th>$CE_{loc} = CE_{mkt} = 0$</th>
<th>$CE_{mkt} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0.89%</td>
<td>0.39%</td>
<td>8.78%</td>
</tr>
<tr>
<td>$CE_{loc} &gt; CE_{mkt}$</td>
<td>70.43%</td>
<td>9.48%</td>
<td>20.09%</td>
</tr>
<tr>
<td>Mean variation ratio</td>
<td>+43.89%</td>
<td>0.00%</td>
<td>−22.04%</td>
</tr>
</tbody>
</table>

Source: CDP database as of 01/07/2022 & Author’s computation.
Scope 3 categories

**Upstream**
1. Purchased goods and services
2. Capital goods
3. Fuel and energy related activities
4. Upstream transportation and distribution
5. Waste generated in operations
6. Business travel
7. Employee commuting
8. Upstream leased assets
9. Other upstream

**Downstream**
1. Downstream transportation and distribution
2. Processing of sold products
3. Use of sold products
4. End-of-life treatment of sold products
5. Downstream leased assets
6. Franchises
7. Investments
8. Other downstream
Scope 3 emissions are all the indirect emissions in the company’s value chain, apart from indirect emissions which are reported in scope 2:

- **Purchased goods and services (not included in categories 2-8)**
  Extraction, production, and transportation of goods and services purchased or acquired by the company

- **Capital goods**
  Extraction, production, and transportation of capital goods purchased or acquired by the company

- **Fuel- and energy-related activities (not included in scopes 1 or 2)**
  Extraction, production, and transportation of fuels and energy purchased or acquired by the company

- **Upstream transportation and distribution**
  Transportation and distribution of products purchased by the company between the company’s tier 1 suppliers and its own operations; Transportation and distribution services purchased by the company, including inbound logistics, outbound logistics (e.g., sold products), and transportation and distribution between the company’s own facilities
Scope 3 emissions

- **Waste generated in operations**
  Disposal and treatment of waste generated in the company’s operations

- **Business travel**
  Transportation of employees for business-related activities

- **Employee commuting**
  Transportation of employees between their homes and their work sites

- **Upstream leased assets**
  Operation of assets leased by the company (lessee)
Scope 3 emissions

1. **Downstream transportation and distribution**
   Transportation and distribution of products sold by the company between the company’s operations and the end consumer (if not paid for by the company)

2. **Processing of sold products**
   Processing of intermediate products sold by downstream companies (e.g., manufacturers)

3. **Use of sold products**
   End use of goods and services sold by the company

4. **End-of-life treatment of sold products**
   Waste disposal and treatment of products sold by the company at the end of their life
Scope 3 emissions

- **Downstream leased assets**
  Operation of assets owned by the company (lessor) and leased to other entities

- **Franchises**
  Operation of franchises reported by franchisor

- **Investments**
  Operation of investments (including equity and debt investments and project finance)
### Table 141: Scope 3 emission factors for business travel and employee commuting (United States)

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>CO₂ (kg/unit)</th>
<th>CH₄ (g/unit)</th>
<th>N₂O (g/unit)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger car</td>
<td>0.332</td>
<td>0.0070</td>
<td>0.0070</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Light-duty truck</td>
<td>0.454</td>
<td>0.0120</td>
<td>0.0090</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>0.183</td>
<td>0.0700</td>
<td>0.0070</td>
<td>vehicle-mile</td>
</tr>
<tr>
<td>Intercity rail (northeast corridor)</td>
<td>0.058</td>
<td>0.0055</td>
<td>0.0007</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Intercity rail (other routes)</td>
<td>0.150</td>
<td>0.0117</td>
<td>0.0038</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Intercity rail (national average)</td>
<td>0.113</td>
<td>0.0092</td>
<td>0.0026</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Commuter rail</td>
<td>0.139</td>
<td>0.0112</td>
<td>0.0028</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Transit rail (subway, tram)</td>
<td>0.099</td>
<td>0.0084</td>
<td>0.0012</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Bus</td>
<td>0.056</td>
<td>0.0210</td>
<td>0.0009</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (short haul, &lt; 300 miles)</td>
<td>0.207</td>
<td>0.0064</td>
<td>0.0066</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (medium haul, 300-2300 miles)</td>
<td>0.129</td>
<td>0.0006</td>
<td>0.0041</td>
<td>passenger-mile</td>
</tr>
<tr>
<td>Air travel (long haul, &gt; 2300 miles)</td>
<td>0.163</td>
<td>0.0006</td>
<td>0.0052</td>
<td>passenger-mile</td>
</tr>
</tbody>
</table>


These factors are intended for use in the distance-based method defined in the Scope 3 Calculation Guidance. If fuel data are available, then the fuel-based method should be used.
Table 142: Examples of monetary scope 3 emission factors

<table>
<thead>
<tr>
<th>Category</th>
<th>S3E</th>
<th>ADEME</th>
<th>Category</th>
<th>S3E</th>
<th>ADEME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2500</td>
<td>2300</td>
<td>Air transport</td>
<td>1970</td>
<td>1190</td>
</tr>
<tr>
<td>Construction</td>
<td>810</td>
<td>360</td>
<td>Education</td>
<td>310</td>
<td>120</td>
</tr>
<tr>
<td>Financial intermediation</td>
<td>140</td>
<td>110</td>
<td>Health and Social Work</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>560</td>
<td>320</td>
<td>Rubber and plastics</td>
<td>1270</td>
<td>800</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>300</td>
<td>170</td>
<td>Textiles</td>
<td>1100</td>
<td>600</td>
</tr>
</tbody>
</table>

Two methods for measuring the carbon footprint of an investment portfolio:

- Financed emissions approach
- Ownership approach
The investor calculates the carbon emissions that are financed across both equity and debt.

EVIC is used to estimate the value of the enterprise. It is “the sum of the market capitalization of ordinary and preferred shares at fiscal year end and the book values of total debt and minorities interests” (TEG, 2019).

Let $W$ be the wealth invested in the company, the financed emissions are equal to:

$$CE(W) = \frac{W}{EVIC} \cdot CE$$

In the case of a portfolio $(W_1, \ldots, W_n)$ where $W_i$ is the wealth invested in company $i$, we have:

$$CE(W) = \sum_{i=1}^{n} CE_i(W_i) = \sum_{i=1}^{n} \frac{W_i}{EVIC_i} \cdot CE_i$$

$CE(W)$ is expressed in tCO$_2$e.
We break down the carbon emissions between the stockholders of the company

We have:

$$CE(W) = \sum_{i=1}^{n} \frac{W_i}{MV_i} \cdot CE_i = \sum_{i=1}^{n} \omega_i \cdot CE_i$$

where:

- $MV_i$ is the market value of company $i$
- $\omega_i$ is the ownership ratio of the investor
Let \( W = \sum_{i=1}^{n} W_i \) be the portfolio value.

The portfolio weight of asset \( i \) is given by:

\[
 w_i = \frac{W_i}{W}
\]

We deduce that:

\[
 \bar{w}_i = \frac{W_i}{MV_i} = \frac{w_i \cdot W}{MV_i}
\]

It follows that:

\[
 CE(W) = \sum_{i=1}^{n} \frac{w_i \cdot W}{MV_i} CE_i = W \left( \sum_{i=1}^{n} \frac{w_i \cdot CE_i}{MV_i} \right) = W \left( \sum_{i=1}^{n} w_i \cdot CI_{i}^{MV} \right)
\]

where \( CI_{i}^{MV} \) is the market value-based carbon intensity:

\[
 CI_{i}^{MV} = \frac{CE_i}{MV_i}
\]

\( CE(W) \) is generally computed with \( W = \$1 \text{ mn} \) and is expressed in \( \text{tCO}_2 \text{e} \) (per \$ \text{ mn invested})
Remark

The ownership approach is valid only for equity portfolios. To compute the market value (or the total market capitalization), we use the following approximation:

\[ MV = \frac{MC}{FP} \]

where \( MC \) and \( FP \) are the free float market capitalisation and percentage of the company.
Example #5

We consider a $100 mn investment portfolio with the following composition: $63.1 mn in company $A$, $16.9 mn in company $B$ and $20.0 mn in company $C$. The data are the following:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Market capitalization (in $ bn)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>31/12/2021</td>
</tr>
<tr>
<td>$A$</td>
<td>12.886</td>
</tr>
<tr>
<td>$B$</td>
<td>7.005</td>
</tr>
<tr>
<td>$C$</td>
<td>3.271</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Debt (in $ bn)</th>
<th>$FP$ (in %)</th>
<th>$SC_{1-2}$ (in ktCO$_2$e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1.112</td>
<td>99.8</td>
<td>756.144</td>
</tr>
<tr>
<td>$B$</td>
<td>0.000</td>
<td>39.3</td>
<td>23.112</td>
</tr>
<tr>
<td>$C$</td>
<td>0.458</td>
<td>96.7</td>
<td>454.460</td>
</tr>
</tbody>
</table>
As of 31 January 2023, the EVIC value for company A is equal to:

$$\text{EVIC}_A = \frac{10,356}{0.998} + 1,112 = 11,489 \text{ mn}$$

We deduce that the financed emissions are equal to:

$$CE_A (63.1 \text{ mn}) = \frac{63.1}{11,489} \times 756.144 = 4.153 \text{ ktCO}_2e$$
If we assume that the investor has no bond in the portfolio, we can use the ownership approach:

\[ \omega_A = \frac{63.1}{10625/0.998} = 59.2695 \text{ bps} \]

The carbon emissions of the investment in company A is then equal to:

\[ CE_A (\$63.1 \text{ mn}) = 59.2695 \times 10^{-4} \times 756.144 = 4.482 \text{ ktCO}_2\text{e} \]
Finally, we obtain the following results:

<table>
<thead>
<tr>
<th></th>
<th>Financed emissions</th>
<th>Carbon emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>4.153</td>
<td>4.482</td>
</tr>
<tr>
<td>Company B</td>
<td>0.023</td>
<td>0.022</td>
</tr>
<tr>
<td>Company C</td>
<td>2.356</td>
<td>2.530</td>
</tr>
<tr>
<td>Portfolio</td>
<td>6.532</td>
<td>7.034</td>
</tr>
</tbody>
</table>
Figure 293: 2019 carbon emissions per GICS sector in GtCO$_2$e (scopes 1 & 2)

Table 143: Breakdown (in %) of carbon emissions in 2019

<table>
<thead>
<tr>
<th>Sector</th>
<th>$SC_1$</th>
<th>$SC_2$</th>
<th>$SC_{1–2}$</th>
<th>$SC_{3}^{up}$</th>
<th>$SC_{3}^{down}$</th>
<th>$SC_3$</th>
<th>$SC_{1–3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>0.1</td>
<td>5.1</td>
<td>0.8</td>
<td>1.5</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>1.7</td>
<td>9.7</td>
<td>2.9</td>
<td>14.1</td>
<td>10.2</td>
<td>10.8</td>
<td>9.1</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>2.3</td>
<td>6.7</td>
<td>2.9</td>
<td>18.6</td>
<td>1.6</td>
<td>4.4</td>
<td>4.1</td>
</tr>
<tr>
<td>Energy</td>
<td>15.0</td>
<td>8.5</td>
<td>14.0</td>
<td>14.1</td>
<td>40.1</td>
<td>36.0</td>
<td>31.2</td>
</tr>
<tr>
<td>Financials</td>
<td>0.7</td>
<td>1.8</td>
<td>0.9</td>
<td>2.6</td>
<td>1.8</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.3</td>
<td>1.7</td>
<td>0.5</td>
<td>2.6</td>
<td>0.2</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.2</td>
<td>8.9</td>
<td>10.0</td>
<td>15.6</td>
<td>24.2</td>
<td>22.8</td>
<td>20.0</td>
</tr>
<tr>
<td>Information Technology</td>
<td>0.6</td>
<td>6.8</td>
<td>1.5</td>
<td>4.9</td>
<td>2.3</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Materials</td>
<td>29.8</td>
<td>40.7</td>
<td>31.4</td>
<td>20.2</td>
<td>13.5</td>
<td>14.6</td>
<td>18.2</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.3</td>
<td>2.8</td>
<td>0.6</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Utilities</td>
<td>39.0</td>
<td>7.3</td>
<td>34.4</td>
<td>4.7</td>
<td>4.8</td>
<td>4.8</td>
<td>11.2</td>
</tr>
<tr>
<td>Total (in GtCO$_2$e)</td>
<td>15.1</td>
<td>2.6</td>
<td>17.6</td>
<td>10.3</td>
<td>53.7</td>
<td>64.0</td>
<td>81.6</td>
</tr>
</tbody>
</table>

Figure 294: 2019 carbon emissions per GICS sector in GtCO$_2$e (scopes 1, 2 & 3 upstream)

Figure 295: 2019 carbon emissions per GICS sector in GtCO$_2$e (scopes 1, 2 & 3)

Figure 296: Sector contribution in %

Figure 297: Histogram of 2019 carbon emissions (logarithmic scale, tCO$_2$e)

Definition

Negative emissions, also known as carbon dioxide removal or CDR, is the process of removing CO$_2$ from the atmosphere.

There are two main categories of negative emissions:

- **Natural climate solutions**
  Examples include forest restoration and afforestation, reducing soil disturbance, etc.

- **Negative emission technologies (NET)**
  Examples are direct air capture with carbon storage (DACCS), bioenergy with carbon capture and storage (BECCS), enhanced weathering, ocean fertilization, etc.
Afforestation is the process of creating a new forest (planting trees in an area where there was no forest in the past), while reforestation is the process of planting trees in areas where there was forest before.

Reducing soil disturbance is the practice of minimizing disturbance to the soil surface and structure, such as using minimum tillage or planting certain crops that protect the soil.

DACCS special filters to capture $\text{CO}_2$ directly from the air, while the captured $\text{CO}_2$ is then stored underground or used in other applications.
BECCS involves capturing and storing the CO₂ emissions from burning biomass, such as wood or grasses.

Enhanced weathering involves the application of finely ground minerals, such as olivine or basalt, to land surfaces. When these minerals react with atmospheric CO₂, they form harmless minerals and carbonates, trapping the carbon in a stable mineral form. The goal is to accelerate the natural process of weathering.

Ocean fertilization involves adding nutrients to the ocean, which can stimulate the growth of phytoplankton in the ocean, which then absorbs CO₂ through photosynthesis.
“[...] (1) Physical greenhouse gases are removed from the atmosphere. (2) The removed gases are stored out of the atmosphere in a manner intended to be permanent. (3) Upstream and downstream greenhouse gas emissions associated with the removal and storage process, such as biomass origin, energy use, gas fate, and co-product fate, are comprehensively estimated and included in the emission balance. (4) The total quantity of atmospheric greenhouse gases removed and permanently stored is greater than the total quantity of greenhouse gases emitted to the atmosphere.” (Tanzer and Ramírez, 2019, page 1216).
There are two general types of DAC processes:

- DAC with liquid solvents (L-DAC)
- DAC with solid sorbents (S-DAC)
Direct air capture

In an L-DAC process, there are four stages: absorption, regeneration, purification and separation:

\[
\begin{align*}
2\text{KOH} + \text{CO}_2 & \rightarrow \text{H}_2\text{O} + \text{K}_2\text{CO}_3 \\
\text{CaO} + \text{H}_2\text{O} & \rightarrow \text{Ca(OH)}_2 \\
\text{K}_2\text{CO}_3 + \text{Ca(OH)}_2 & \rightarrow 2\text{KOH} + \text{CaCO}_3 \\
\text{CaCO}_3 & \rightarrow \text{CaO} + \text{CO}_2
\end{align*}
\]

The goal is to use the liquid solvent KOH to react with atmospheric carbon dioxide CO₂ to produce pure CO₂ and calcium oxide CaO.

In an S-DAC process, solid materials or sorbents, such as porous polymers or metal-organic frameworks, are used to adsorb CO₂.
The costs associated with DAC technology include the initial investment to build the DAC system (e.g., air contractor, causticizer, calciner, and slaker), the price of solvents and sorbents, the electricity needs to perform the chemical reactions, and the cost of storage.

- The current price of removing a tonne of CO\textsubscript{2} is around $1000.
- The carbon efficiency of the best DAC plans is less than 70%.
Direct air capture

An example of DAC companies: Climeworks

Climeworks (https://climeworks.com) is a Swiss company founded in 2009 as a spin-off from ETH Zurich. It specializes in DAC technology and has established itself as a pioneer in this field with two other companies: Carbon Engineering (Canada) and Global Thermostat (USA). In September 2021, Climeworks inaugurates the world’s first large-scale direct air capture and storage plant “Orca” in Iceland, with a capacity to capture 4,000 tonnes of CO₂ per year. The storage of CO₂ is carried out by the company Carbfix, which injects it deep underground, where it mineralizes and turns into stone. In June 2022, Climeworks announces a second, newest and largest direct air capture and storage facility, “Mammoth”, also in Iceland. It will have a nominal CO₂ capture capacity of up to 36,000 tonnes per year when fully operational.
Avoided emissions often incorrectly referred to as Scope 4 emissions.

- This is the difference between the total, attributional, life-cycle GHG inventories of a company’s product (the assessed product) and an alternative (or reference) product that provides an equivalent function:

\[
\AE = \CE \text{ (reference product)} - \CE \text{ (assessed product)}
\]

- Avoided emissions can be positive ($\AE \geq 0$) or negative ($\AE < 0$).
Avoided emissions

Electric car

- An electric car emits CO$_2$, especially when we consider the life cycle of the batteries, but electric cars do not emit greenhouse gases from burning gasoline.
- The reference product is the gasoline-powered car.
- The assessed product is the electric car.
- There are two issues in calculating avoided emissions:
  1. which car should we choose to represent the gasoline car or the reference product?
  2. what is the use of the electric car?
- The avoided emissions depend on many factors, such as the carbon intensity of the electricity, recycling assumptions, etc.
Cap-and-trade systems
These systems place a limit on the total amount of GHG emissions that can be released from a given region or industry. Companies are allocated a certain number of carbon credits (emission allowances) and can buy or sell credits to meet their emissions targets. These government-regulated schemes make up the compliance carbon market.

Voluntary carbon markets
These markets are not regulated by the government, and companies can voluntarily buy carbon credits to offset their emissions. Voluntary carbon markets are often used to offset emissions from activities not covered by cap-and-trade systems. In this case, the avoided emissions from a carbon offset (e.g., through the use of negative emission technologies) must be counted on the balance sheet of the buyer, not the seller, who is the developer of the project.
Figure 298: Voluntary carbon market size by volume of traded carbon credits

Source: Ecosystem Marketplace (2023, Figure 2, page 8).
Efficiency of carbon dioxide removal

\[ \eta(t) = \frac{\text{CO}_2^{\text{stored}}(t) - \text{CO}_2^{\text{leaked}}(t)}{\text{CO}_2^{\text{stored}}(t)} \]

Table 144: Summary of key features for each CDR pathway

<table>
<thead>
<tr>
<th>CDR</th>
<th>( \eta(100) )</th>
<th>( \eta(1000) )</th>
<th>Timing</th>
<th>Permanence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Afforestation</td>
<td>63 to 99%</td>
<td>31 to 95%</td>
<td>Decades</td>
<td>Very low</td>
</tr>
<tr>
<td>Reforestation</td>
<td>63 to 99%</td>
<td>31 to 95%</td>
<td>Decades</td>
<td>Very low</td>
</tr>
<tr>
<td>BECCS</td>
<td>52 to 87%</td>
<td>78 to 87%</td>
<td>Immediate to decades</td>
<td>High/very high</td>
</tr>
<tr>
<td>Biochar</td>
<td>20 to 39%</td>
<td>−3 to 5%</td>
<td>Immediate</td>
<td>Low/very low</td>
</tr>
<tr>
<td>DACCS</td>
<td>−5 to 90%</td>
<td>−5 to 90%</td>
<td>Immediate</td>
<td>Very high</td>
</tr>
<tr>
<td>Enhanced weathering</td>
<td>17 to 92%</td>
<td>51 to 92%</td>
<td>Immediate to decades</td>
<td>High/very high</td>
</tr>
</tbody>
</table>

Source: Chiquier et al. (2022, Table 1, page 4400).
Figure 299: Perceived CO$_2$ emissions of a simplified steel production system when viewed from different system boundaries

Source: Tanzer and Ramírez (2019, Figure 2, page 1214).
Carbon intensity

- Carbon emissions = absolute carbon footprint in an absolute value
- Carbon intensity = relative carbon footprint

⇒ we normalize the carbon emissions by a size or activity unit
We can measure the carbon footprint of:

- countries by tCO$_2$e per capita
- watching television by CO$_2$e emissions per viewer-hour
- washing machines by kgCO$_2$e per wash
- cars by kgCO$_2$e per kilometer driven
- companies by ktCO$_2$e per $1$ mn revenue
- etc.
Product carbon footprint (PCF)

- The product carbon footprint measures the relative carbon emissions of a product throughout its life cycle.
- Life cycle assessment (LCA), distinguishes two methods:
  - **Cradle-to-gate** refers to the carbon footprint of a product from the moment it is produced (including the extraction of raw materials) to the moment it enters the store.
  - **Cradle-to-grave** covers the entire life cycle of a product, including the use-phase and recycling.
### Physical intensity ratios

**Table 145: Examples of product carbon footprint (in kgCO₂e per unit)**

<table>
<thead>
<tr>
<th>Product</th>
<th>Category</th>
<th>Cradle-to-gate</th>
<th>Cradle-to-grave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screen</td>
<td>21.5 inches</td>
<td>222</td>
<td>236</td>
</tr>
<tr>
<td></td>
<td>23.8 inches</td>
<td>248</td>
<td>265</td>
</tr>
<tr>
<td>Computer</td>
<td>Laptop</td>
<td>156</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>Desktop</td>
<td>169</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>High performance</td>
<td>295</td>
<td>394</td>
</tr>
<tr>
<td>Smartphone</td>
<td>Classical</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>5 inches</td>
<td>33</td>
<td>32</td>
</tr>
<tr>
<td>Oven</td>
<td>Built-in electric</td>
<td>187</td>
<td>319</td>
</tr>
<tr>
<td></td>
<td>Professional (combi steamer)</td>
<td>734</td>
<td>12 676</td>
</tr>
<tr>
<td>Washing machine</td>
<td>Capacity 5kg</td>
<td>248</td>
<td>468</td>
</tr>
<tr>
<td></td>
<td>Capacity 7kg</td>
<td>275</td>
<td>539</td>
</tr>
<tr>
<td>Shirt</td>
<td>Coton</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Viscose</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Balloon</td>
<td>Football</td>
<td>3.4</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>Basket-ball</td>
<td>3.6</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Corporate carbon footprint (CCF)

- Extension of the PCF to companies
- The CCF of a cement manufacturer is measured by the amount of GHG emissions per tonne of cement
- The CCF of airlines is measured by the amount of GHG emissions per RPK (revenue passenger kilometers, which is calculated by multiplying the number of paying passengers by the distance traveled)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport sector (aviation)</td>
<td>CO₂e/RPK</td>
<td>Revenue passenger kilometers</td>
</tr>
<tr>
<td>Transport sector (shipping)</td>
<td>CO₂e/RTK</td>
<td>Revenue tonne kilometers</td>
</tr>
<tr>
<td>Industry (cement)</td>
<td>CO₂e/t cement</td>
<td>Tonne of cement</td>
</tr>
<tr>
<td>Industry (steel)</td>
<td>CO₂e/t steel</td>
<td>Tonne of steel</td>
</tr>
<tr>
<td>Electricity</td>
<td>CO₂e/MWh</td>
<td>Megawatt hour</td>
</tr>
<tr>
<td>Buildings</td>
<td>CO₂e/SQM</td>
<td>Square meter</td>
</tr>
</tbody>
</table>
Problem

- How to aggregate carbon footprint?
- Portfolio managers use monetary intensity ratios, which are defined as:

\[ CI = \frac{CE}{Y} \]

where \( CE \) is the company’s carbon emissions and \( Y \) is a monetary variable measuring its activity
For instance, we can use revenues, sales, etc. to normalize carbon emissions:

\[
\begin{align*}
CI_{\text{Revenue}} &= \frac{CE}{\text{Revenue}} \\
CI_{\text{Sales}} &= \frac{CE}{\text{Sales}} \\
CI_{\text{EVIC}} &= \frac{CE}{\text{EVIC}} \\
CI_{\text{MV}} &= \frac{CE}{\text{MV}}
\end{align*}
\]

Remark

The previous carbon emission metrics based on EVIC and market value can be viewed as carbon intensity metrics.
If we consider the EVIC-based approach, the carbon intensity of the portfolio is given by:

\[ \mathcal{C}I^{\text{EVIC}}(w) = \frac{\mathcal{C}E^{\text{EVIC}}(W)}{W} \]

\[ = \frac{1}{W} \sum_{i=1}^{n} \frac{W_i}{\text{EVIC}_i} \cdot \mathcal{C}E_i \]

\[ = \sum_{i=1}^{n} \frac{W_i}{W} \cdot \frac{\mathcal{C}E_i}{\text{EVIC}_i} \]

\[ = \sum_{i=1}^{n} w_i \cdot \mathcal{C}I_i^{\text{EVIC}} \]

where \( w = (w_1, \ldots, w_n) \) is the vector of portfolio weights.

In a similar way, we obtain:

\[ \mathcal{C}I^{\text{MV}}(w) = \sum_{i=1}^{n} w_i \cdot \mathcal{C}I_i^{\text{MV}} \]
We consider the revenue-based carbon intensity (also called the economic carbon intensity).

The carbon intensity of the portfolio is:

\[
\text{CI}^{\text{Revenue}} (w) = \frac{CE (w)}{Y (w)}
\]

where:

- \( CE (w) \) measures the carbon emissions of the portfolio:

\[
CE (w) = \sum_{i=1}^{n} W_i \cdot \frac{CE_i}{MV_i} = W \sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot CE_i
\]

- \( Y (w) \) is the total revenue of the portfolio:

\[
Y (w) = \sum_{i=1}^{n} W_i \cdot \frac{Y_i}{MV_i} = W \sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot Y_i
\]
We deduce that:

\[
CI^{\text{Revenue}}(w) = \frac{\sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot CE_i}{\sum_{i=1}^{n} \frac{w_i}{MV_i} \cdot Y_i} = \sum_{i=1}^{n} w_i \cdot \omega_i \cdot CI^{\text{Revenue}}_i
\]

where \( \omega_i \) is the ratio between the revenue per market value of company \( i \) and the weighted average revenue per market value of the portfolio:

\[
\omega_i = \frac{Y_i}{MV_i} \frac{1}{\sum_{k=1}^{n} w_k \cdot \frac{Y_k}{MV_k}}
\]

We conclude that:

\[
CI^{\text{Revenue}}(w) \neq \sum_{i=1}^{n} w_i \cdot CI^{\text{Revenue}}_i
\]
In order to avoid the previous problem, we generally use the weighted average carbon intensity (WACI) of the portfolio:

\[ C\!I^{\text{Revenue}}(w) = \sum_{i=1}^{n} w_i \cdot C\!I_i^{\text{Revenue}} \]

This method is the standard approach in portfolio management.
Carbon intensity is always additive when we consider a given issuer:

\[
CI_{i}(SC_{1-3}) = \frac{CE_{i}(SC_{1}) + CE_{i}(SC_{2}) + CE_{i}(SC_{3})}{Y_{i}}
\]

\[
= CI_{i}(SC_{1}) + CI_{i}(SC_{2}) + CI_{i}(SC_{3})
\]
Example #6

We assume that $cE_1 = 5 \times 10^6$ CO$_2$e, $Y_1 = 0.2 \times 10^6$, $MV_1 = 10 \times 10^6$, $cE_2 = 50 \times 10^6$ CO$_2$e, $Y_2 = 4 \times 10^6$ and $MV_2 = 10 \times 10^6$. We invest $W = 10$ mn.
Illustration

- We deduce that:

\[
CI_1 = \frac{5 \times 10^6}{0.2 \times 10^6} = 25.0 \text{ tCO}_2\text{e}/\$ \text{ mn}
\]

and

\[
CI_2 = 12.5 \text{ tCO}_2\text{e}/\$ \text{ mn}
\]

- We have:

\[
\begin{align*}
CE(w) &= W \left( w_1 \frac{CE_1}{MV_1} + w_2 \frac{CE_2}{MV_2} \right) \\
Y(w) &= W \left( w_1 \frac{Y_1}{MV_1} + w_2 \frac{Y_2}{MV_2} \right) \\
CI(w) &= w_1 CI_1 + w_2 CI_2
\end{align*}
\]
We obtain the following results:

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$CE(w)$ $(\times 10^6 \text{ CO}_2\text{e})$</th>
<th>$Y(w)$ $(\times $10^6)$</th>
<th>$\frac{CE(w)}{Y(w)}$</th>
<th>$CI(w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100%</td>
<td>50.00</td>
<td>4.00</td>
<td>12.50</td>
<td>12.50</td>
</tr>
<tr>
<td>10%</td>
<td>90%</td>
<td>45.50</td>
<td>3.62</td>
<td>12.57</td>
<td>13.75</td>
</tr>
<tr>
<td>20%</td>
<td>80%</td>
<td>41.00</td>
<td>3.24</td>
<td>12.65</td>
<td>15.00</td>
</tr>
<tr>
<td>30%</td>
<td>70%</td>
<td>36.50</td>
<td>2.86</td>
<td>12.76</td>
<td>16.25</td>
</tr>
<tr>
<td>50%</td>
<td>50%</td>
<td>27.50</td>
<td>2.10</td>
<td>13.10</td>
<td>18.75</td>
</tr>
<tr>
<td>70%</td>
<td>30%</td>
<td>18.50</td>
<td>1.34</td>
<td>13.81</td>
<td>21.25</td>
</tr>
<tr>
<td>80%</td>
<td>20%</td>
<td>14.00</td>
<td>0.96</td>
<td>14.58</td>
<td>22.50</td>
</tr>
<tr>
<td>90%</td>
<td>10%</td>
<td>9.50</td>
<td>0.58</td>
<td>16.38</td>
<td>23.75</td>
</tr>
<tr>
<td>100%</td>
<td>0%</td>
<td>5.00</td>
<td>0.20</td>
<td>25.00</td>
<td>25.00</td>
</tr>
</tbody>
</table>

We notice that the weighted average carbon intensity can be very different than the economic carbon intensity.
Remark

For sovereign issuers, the economic carbon intensity is measured in mega-tonnes of CO$_2$e per million dollars of GDP while the physical carbon intensity unit is tCO$_2$e per capita.
Figure 300: Histogram of 2019 carbon intensities (logarithmic scale, tCO$_2$e/$\text{mn}$)

### Table 146: Examples of 2019 carbon emissions and intensities

<table>
<thead>
<tr>
<th>Company</th>
<th>Carbon emissions (in tCO₂e)</th>
<th>Revenue (in $ mn)</th>
<th>Intensity (in tCO₂e/$ mn)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( SC_1 )</td>
<td>( SC_2 )</td>
<td>( SC_{up} )</td>
</tr>
<tr>
<td>Airbus</td>
<td>576 705</td>
<td>386 674</td>
<td>12 284 183</td>
</tr>
<tr>
<td>Allianz</td>
<td>46 745</td>
<td>224 315</td>
<td>3 449 234</td>
</tr>
<tr>
<td>Alphabet</td>
<td>111 283</td>
<td>5 118 152</td>
<td>7 142 566</td>
</tr>
<tr>
<td>Amazon</td>
<td>5 700 000</td>
<td>5 500 000</td>
<td>20 054 722</td>
</tr>
<tr>
<td>Apple</td>
<td>50 549</td>
<td>8 621 127</td>
<td>27 624 282</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>64 829</td>
<td>280 789</td>
<td>1 923 307</td>
</tr>
<tr>
<td>Boeing</td>
<td>611 001</td>
<td>871 000</td>
<td>9 878 431</td>
</tr>
<tr>
<td>BP</td>
<td>49 199 999</td>
<td>5 200 000</td>
<td>103 840 194</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>905 000</td>
<td>926 000</td>
<td>15 197 607</td>
</tr>
<tr>
<td>Danone</td>
<td>722 122</td>
<td>944 877</td>
<td>28 969 780</td>
</tr>
<tr>
<td>Enel</td>
<td>69 981 891</td>
<td>5 365 386</td>
<td>8 726 973</td>
</tr>
<tr>
<td>Exxen</td>
<td>111 000 000</td>
<td>9 000 000</td>
<td>107 282 831</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>81 655</td>
<td>692 299</td>
<td>3 101 582</td>
</tr>
<tr>
<td>Juventus</td>
<td>6 665</td>
<td>15 739</td>
<td>35 842</td>
</tr>
<tr>
<td>LVMH</td>
<td>67 613</td>
<td>262 609</td>
<td>11 853 749</td>
</tr>
<tr>
<td>Microsoft</td>
<td>113 414</td>
<td>3 556 553</td>
<td>5 977 488</td>
</tr>
<tr>
<td>Nestle</td>
<td>3 291 303</td>
<td>3 206 495</td>
<td>61 262 078</td>
</tr>
<tr>
<td>Netflix</td>
<td>38 481</td>
<td>145 443</td>
<td>1 900 283</td>
</tr>
<tr>
<td>NVIDIA</td>
<td>2 767</td>
<td>65 048</td>
<td>2 756 353</td>
</tr>
<tr>
<td>PepsiCo</td>
<td>3 552 415</td>
<td>1 556 523</td>
<td>32 598 029</td>
</tr>
<tr>
<td>Pfizer</td>
<td>734 638</td>
<td>762 840</td>
<td>4 667 225</td>
</tr>
<tr>
<td>Roche</td>
<td>288 157</td>
<td>32 649</td>
<td>5 812 735</td>
</tr>
<tr>
<td>Samsung Electronics</td>
<td>5 067 000</td>
<td>10 998 000</td>
<td>33 554 245</td>
</tr>
<tr>
<td>TotalEnergies</td>
<td>40 909 135</td>
<td>3 596 127</td>
<td>49 817 293</td>
</tr>
<tr>
<td>Toyota</td>
<td>2 522 987</td>
<td>5 227 844</td>
<td>66 148 020</td>
</tr>
<tr>
<td>Volkswagen</td>
<td>4 494 066</td>
<td>5 973 894</td>
<td>65 335 372</td>
</tr>
<tr>
<td>Walmart</td>
<td>6 101 641</td>
<td>13 057 352</td>
<td>40 651 079</td>
</tr>
</tbody>
</table>

Table 147: Examples of 2019 carbon intensities

<table>
<thead>
<tr>
<th>Company</th>
<th>Intensity (in tCO₂e/$ mn)</th>
<th>$S_C^1$</th>
<th>$S_C^2$</th>
<th>$S_C^{up}$</th>
<th>$S_C^{down}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon</td>
<td></td>
<td>20.5</td>
<td>19.6</td>
<td>71.5</td>
<td>37.2</td>
</tr>
<tr>
<td><strong>Apple</strong></td>
<td></td>
<td><strong>0.2</strong></td>
<td>3.3</td>
<td><strong>106.2</strong></td>
<td>21.0</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td></td>
<td>0.8</td>
<td>3.6</td>
<td>24.6</td>
<td>0.0</td>
</tr>
<tr>
<td>BP</td>
<td></td>
<td>177.7</td>
<td>18.8</td>
<td>375.1</td>
<td>2104.5</td>
</tr>
<tr>
<td>Caterpillar</td>
<td></td>
<td>16.8</td>
<td>17.2</td>
<td>282.5</td>
<td>7472.0</td>
</tr>
<tr>
<td><strong>Danone</strong></td>
<td></td>
<td>25.5</td>
<td>33.4</td>
<td><strong>1023.4</strong></td>
<td><strong>157.7</strong></td>
</tr>
<tr>
<td>Exxon</td>
<td></td>
<td>434.3</td>
<td>35.2</td>
<td>419.8</td>
<td>2324.6</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td></td>
<td>0.7</td>
<td>6.0</td>
<td>26.8</td>
<td>133.6</td>
</tr>
<tr>
<td>LVMH</td>
<td></td>
<td>1.1</td>
<td>4.4</td>
<td>197.3</td>
<td>15.7</td>
</tr>
<tr>
<td>Microsoft</td>
<td></td>
<td>0.9</td>
<td>28.3</td>
<td>47.5</td>
<td>31.8</td>
</tr>
<tr>
<td><strong>Nestle</strong></td>
<td></td>
<td>35.3</td>
<td>34.4</td>
<td><strong>657.6</strong></td>
<td><strong>363.9</strong></td>
</tr>
<tr>
<td>Pfizer</td>
<td></td>
<td>14.2</td>
<td>14.7</td>
<td>90.2</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Samsung Electronics</strong></td>
<td></td>
<td><strong>25.6</strong></td>
<td><strong>55.6</strong></td>
<td>169.7</td>
<td>308.4</td>
</tr>
<tr>
<td>Volkswagen</td>
<td></td>
<td>15.9</td>
<td>21.1</td>
<td>231.0</td>
<td>1254.9</td>
</tr>
<tr>
<td>Walmart</td>
<td></td>
<td>11.9</td>
<td>25.4</td>
<td>79.0</td>
<td>62.9</td>
</tr>
</tbody>
</table>

**Statistics**

**Table 148**: Carbon intensity in tCO₂e/$ mn per GICS sector and sector contribution in % (MSCI World, June 2022)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$b_i$ (in %)</th>
<th>Carbon intensity</th>
<th>Risk contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_{1-2}$</td>
<td>$SC_{1-3}^{up}$</td>
</tr>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>23</td>
<td>65</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>28</td>
<td>55</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>632</td>
<td>698</td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>111</td>
<td>130</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>7</td>
<td>23</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>478</td>
<td>702</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>22</td>
<td>101</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>1.744</td>
<td>1.794</td>
</tr>
<tr>
<td>MSCI World</td>
<td></td>
<td>130</td>
<td>163</td>
</tr>
<tr>
<td>MSCI World EW</td>
<td></td>
<td>168</td>
<td>211</td>
</tr>
</tbody>
</table>

Let \( b = (b_1, \ldots, b_n) \) be the weights of the assets that belong to a benchmark.

Its weighted average carbon intensity is given by:

\[
CI(b) = \sum_{i=1}^{n} b_i \cdot CI_i
\]

where \( CI_i \) is the carbon intensity of asset \( i \).

If we focus on the carbon intensity for a given sector, we use the following formula:

\[
CI(S_{\text{sector}_j}) = \frac{\sum_{i \in S_{\text{sector}_j}} b_i \cdot CI_i}{\sum_{i \in S_{\text{sector}_j}} b_i}
\]
The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period \([t_0, t]\).

From a mathematical point of view, it corresponds to the signed area of the region bounded by the function \(CE(t)\):

\[
CB(t_0, t) = \int_{t_0}^{t} CE(s) \, ds
\]
Example #7

Below, we report the historical data of carbon emissions from 2010 to 2020. Moreover, the company has announced his carbon targets for the years until 2050.

Table 149: Carbon emissions in MtCO$_2$e

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CE(t)</td>
<td>4.800</td>
<td>4.950</td>
<td>5.100</td>
<td>5.175</td>
<td>5.175</td>
<td>5.175</td>
<td>5.175</td>
<td>5.100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
<th>2025*</th>
<th>2030*</th>
<th>2035*</th>
<th>2040*</th>
<th>2050*</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE(t)</td>
<td>5.025</td>
<td>4.950</td>
<td>4.875</td>
<td>4.200</td>
<td>3.300</td>
<td>1.500</td>
<td>0.750</td>
<td>0.150</td>
</tr>
</tbody>
</table>

The asterisk * indicates that the company has announced a carbon target for this year.
Carbon budget

Figure 301: Past, expected and net carbon budgets (Example #7)
Computation of the carbon budget
Numerical solution

- We consider the equally-spaced partition
  \{[t_0, t_0 + \Delta t], \ldots, [t - \Delta t, t]\} of \[t_0, t\]
- Let \( m = \frac{t - t_0}{\Delta t} \) be the number of intervals
- We set \( CE_k = CE(t_0 + k\Delta t) \)
- The right Riemann approximation is:

\[
CB(t_0, t) = \int_{t_0}^{t} CE(s) \, ds \approx \sum_{k=1}^{m} CE(t_0 + k\Delta t) \Delta t = \Delta t \sum_{k=1}^{m} CE_k
\]

- The left Riemann sum is:

\[
CB(t_0, t) \approx \Delta t \sum_{k=0}^{m-1} CE_k
\]

- The midpoint rule is:

\[
CB(t_0, t) \approx \Delta t \sum_{k=1}^{m} CE\left(t_0 + \frac{k}{2} \Delta t\right)
\]
If we use a constant linear reduction rate \( R(t_0, t) = R(t - t_0) \), we obtain the following analytical expression:

\[
CB(t_0, t) = \int_{t_0}^{t} (CE(t_0) - R(s - t_0)) \, ds = (t - t_0) CE(t_0) - \frac{(t - t_0)^2}{2} R
\]

In the case of a constant compound reduction rate:

\[
CE(t) = (1 - R)^{(t-t_0)} CE(t_0)
\]

we obtain:

\[
CB(t_0, t) = CE(t_0) \int_{t_0}^{t} (1 - R)^{(s-t_0)} \, ds = \frac{(1 - R)^{(t-t_0)} - 1}{\ln(1 - R)} CE(t_0)
\]
If we assume that $CE(t) = e^{-R(t-t_0)}CE(t_0)$, we have:

$$CB(t_0, t) = CE(t_0) \left[ -\frac{e^{-R(s-t_0)}}{R} \right]^t_{t_0} = CE(t_0) \frac{1 - e^{-R(t-t_0)}}{R}$$

**Remark**

*If the carbon emissions increase at a positive growth rate $g$, we set $R = -g$.****
Carbon budget and global warming

**Figure 302:** Probability to reach 1.5°C

The remaining carbon budget $CB(2019, t)$ is:

- 580 GtCO$_2$e for a 50% probability of limiting warming to 1.5°C
- 420 GtCO$_2$e for a 66% probability
- 300 GtCO$_2$e for a 83% probability

IPCC (2018)
If we assume that $CE(t) = \beta_0 + \beta_1 t$, we deduce that:

$$CB(t_0, t) = \int_{t_0}^{t} (\beta_0 + \beta_1 s) \, ds$$

$$= \left[ \beta_0 s + \frac{1}{2} \beta_1 s^2 \right]_{t_0}^{t}$$

$$= \beta_0 (t - t_0) + \frac{1}{2} \beta_1 (t^2 - t_0^2)$$

We can extend this formula to a piecewise linear function:
Table 150: IEA NZE scenario (in GtCO₂e)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>12.4</td>
<td>13.3</td>
<td>13.5</td>
<td>13.6</td>
<td>13.3</td>
<td>13.3</td>
<td>13.5</td>
<td>14.0</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>Buildings</td>
<td>2.89</td>
<td>2.81</td>
<td>2.78</td>
<td>2.9</td>
<td>2.84</td>
<td>2.87</td>
<td>2.91</td>
<td>2.95</td>
<td>2.98</td>
<td>3.01</td>
</tr>
<tr>
<td>Transport</td>
<td>7.01</td>
<td>7.13</td>
<td>7.18</td>
<td>7.37</td>
<td>7.5</td>
<td>7.72</td>
<td>7.88</td>
<td>8.08</td>
<td>8.25</td>
<td>8.29</td>
</tr>
<tr>
<td>Industry</td>
<td>8.06</td>
<td>8.47</td>
<td>8.57</td>
<td>8.71</td>
<td>8.78</td>
<td>8.71</td>
<td>8.56</td>
<td>8.52</td>
<td>8.72</td>
<td>8.9</td>
</tr>
<tr>
<td>Other</td>
<td>1.87</td>
<td>1.89</td>
<td>1.91</td>
<td>1.96</td>
<td>1.87</td>
<td>1.89</td>
<td>1.89</td>
<td>1.92</td>
<td>1.92</td>
<td>1.91</td>
</tr>
<tr>
<td>Gross emissions</td>
<td>32.2</td>
<td>33.3</td>
<td>33.7</td>
<td>34.4</td>
<td>34.5</td>
<td>34.5</td>
<td>34.5</td>
<td>35.0</td>
<td>35.9</td>
<td>35.9</td>
</tr>
<tr>
<td>BECCS/DACCS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Net emissions</td>
<td>32.2</td>
<td>33.3</td>
<td>33.7</td>
<td>34.4</td>
<td>34.5</td>
<td>34.5</td>
<td>34.5</td>
<td>35.0</td>
<td>35.9</td>
<td>35.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
<th>2035</th>
<th>2040</th>
<th>2045</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity</td>
<td>13.5</td>
<td>10.8</td>
<td>5.82</td>
<td>2.12</td>
<td>-0.08</td>
<td>-0.31</td>
<td>-0.37</td>
</tr>
<tr>
<td>Buildings</td>
<td>2.86</td>
<td>2.43</td>
<td>1.81</td>
<td>1.21</td>
<td>0.69</td>
<td>0.32</td>
<td>0.12</td>
</tr>
<tr>
<td>Transport</td>
<td>7.15</td>
<td>7.23</td>
<td>5.72</td>
<td>4.11</td>
<td>2.69</td>
<td>1.5</td>
<td>0.69</td>
</tr>
<tr>
<td>Industry</td>
<td>8.48</td>
<td>8.14</td>
<td>6.89</td>
<td>5.25</td>
<td>3.48</td>
<td>1.8</td>
<td>0.52</td>
</tr>
<tr>
<td>Other</td>
<td>1.91</td>
<td>1.66</td>
<td>0.91</td>
<td>0.09</td>
<td>-0.46</td>
<td>-0.82</td>
<td>-0.96</td>
</tr>
<tr>
<td>Gross emissions</td>
<td>33.9</td>
<td>30.3</td>
<td>21.5</td>
<td>13.7</td>
<td>7.77</td>
<td>4.3</td>
<td>1.94</td>
</tr>
<tr>
<td>BECCS/DACCS</td>
<td>0</td>
<td>-0.06</td>
<td>-0.32</td>
<td>-0.96</td>
<td>-1.46</td>
<td>-1.8</td>
<td>-1.94</td>
</tr>
<tr>
<td>Net emissions</td>
<td>33.9</td>
<td>30.2</td>
<td>21.1</td>
<td>12.8</td>
<td>6.32</td>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: IEA (2021, Figure 2.3, page 55)
Net zero emissions scenario (IEA)

Figure 303: CO₂ emissions by sector in the IEA NZE scenario (in GtCO₂e)

Source: IEA (2021) & Author’s calculations
Net zero emissions scenario (IEA)

### Table 151: Carbon budget in the IEA NZE scenario (in GtCO$_2$e)

<table>
<thead>
<tr>
<th></th>
<th>Electricity</th>
<th>Buildings</th>
<th>Transport</th>
<th>Industry</th>
<th>Other</th>
<th>Gross emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>74.4</td>
<td>50.2</td>
<td>43.7</td>
<td>16.2</td>
<td>10.8</td>
<td>195.4</td>
</tr>
<tr>
<td>2030</td>
<td>115.9</td>
<td>87.8</td>
<td>76.0</td>
<td>26.8</td>
<td>17.3</td>
<td>324.9</td>
</tr>
<tr>
<td>2040</td>
<td>140.9</td>
<td>140.0</td>
<td>117.6</td>
<td>39.1</td>
<td>18.8</td>
<td>466.6</td>
</tr>
<tr>
<td>2045</td>
<td>139.9</td>
<td>153.2</td>
<td>128.1</td>
<td>41.6</td>
<td>15.6</td>
<td>496.8</td>
</tr>
<tr>
<td>2050</td>
<td>138.2</td>
<td>159.0</td>
<td>133.6</td>
<td>42.7</td>
<td>11.2</td>
<td>512.4</td>
</tr>
</tbody>
</table>

Source: IEA (2021) & Author’s calculations
Linear trend model

- The linear trend model is defined by:

\[ CE(t) = \beta_0 + \beta_1 t + u(t) \]

where \( u(t) \sim N(0, \sigma_u^2) \)

- OLS estimation

- The projected carbon trajectory is given by:

\[ CE^{Trend}(t) = \hat{CE}(t) = \hat{\beta}_0 + \hat{\beta}_1 t \]
Carbon trend
Linear trend model

- We have:
  \[ \hat{CE}(0) = \hat{\beta}_0 \]
- Base year: \( t_0 \)
- The linear trend model becomes:
  \[ CE(t) = \beta'_0 + \beta'_1 (t - t_0) + u(t) \]
- We have the following relationships:
  \[
  \begin{cases}
  \beta'_0 = \beta_0 + \beta_1 t_0 \\
  \beta'_1 = \beta_1
  \end{cases}
  \]
Example #8

Below, we report the evolution of scope 1 + 2 carbon emissions for company A:

<table>
<thead>
<tr>
<th>Year</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE(t)$</td>
<td>57.8</td>
<td>58.4</td>
<td>57.9</td>
<td>55.1</td>
<td>51.6</td>
<td>48.3</td>
<td>47.1</td>
</tr>
<tr>
<td>$CE(t)$</td>
<td>46.1</td>
<td>44.4</td>
<td>42.7</td>
<td>41.4</td>
<td>40.2</td>
<td>41.9</td>
<td>45.0</td>
</tr>
</tbody>
</table>
Carbon trend

Linear trend model

We obtain the following estimates:

- $\hat{\beta}_0 = 2,970.43$, $\hat{\beta}_1 = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2007$, $\hat{\beta}_0' = 57.85$, $\hat{\beta}_1' = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2020$, $\hat{\beta}_0' = 38.99$, $\hat{\beta}_1' = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- The two estimated models are coherent:

$$
C\mathcal{E}^{Trend}(t) = 38.99 - 1.4512 \times (t - 2020)
$$

$$
= 2,970.43 - 1.4512 \times t
$$

- We have:

$$
C\mathcal{E}^{Trend}(2025) = 38.99 - 1.4512 \times 5 = 31.73 \text{ MtCO}_2\text{e}
$$

- We have $C\mathcal{E}(2020) = 45.0 \gg \widehat{C\mathcal{E}}(2020) = 38.99$
- The rescaled model has the following expression:

$$
C\mathcal{E}^{Trend}(t) = 45 - 1.4512 \times (t - 2020)
$$
Carbon trend
Linear trend model

Figure 304: Linear carbon trend (Example #8)
Log-linear trend model

- The log-linear trend model is:

\[ \ln C\mathcal{E}(t) = \gamma_0 + \gamma_1 (t - t_0) + v(t) \]

- Let \( Y(t) = \ln C\mathcal{E}(t) \) be the logarithmic transform of the carbon emissions

- OLS estimation using \( Y(t) \)
Carbon trend

Log-linear trend model

- We have:
  \[
  \hat{CE}(t) = \exp\left(\hat{Y}(t)\right) = \exp(\hat{\gamma}_0 + \hat{\gamma}_1(t - t_0)) = \hat{CE}(t_0) \exp(\hat{\gamma}_1(t - t_0))
  \]
  where \(\hat{CE}(t_0) = \exp(\hat{\gamma}_0)\)

- The mathematical expectation of \(CE(t)\) is equal to:
  \[
  \mathbb{E}[CE(t)] = \mathbb{E}\left[e^{Y(t)}\right] = \mathbb{E}\left[\mathcal{LN}(\gamma_0 + \gamma_1(t - t_0), \sigma_v^2)\right]
  = \exp\left(\gamma_0 + \gamma_1(t - t_0) + \frac{1}{2}\sigma_v^2\right)
  = \hat{CE}(t_0) \exp(\hat{\gamma}_1(t - t_0))
  \]
  where \(\hat{CE}(t_0) = \exp(\hat{\gamma}_0 + \frac{1}{2}\hat{\sigma}_v^2)\)

- The rescaled log-linear trend model is:
  \[
  CE^{Trend}(t) = CE(t_0) \exp(\hat{\gamma}_1(t - t_0))
  \]
Interpretation of the slope

- $\beta_1$ is the absolute variation of carbon emissions:

$$\frac{\partial CE(t)}{\partial t} = \beta_1$$

implies that the relative variation of carbon emissions is:

$$\frac{\partial CE(t)}{CE(t)} = \frac{\beta_1}{CE(t)}$$

- $\gamma_1$ is the relative variation of carbon emissions:

$$\frac{\partial CE(t)}{CE(t)} = \frac{\partial \ln CE(t)}{\partial t} = \gamma_1$$
Example #8:

- We obtain the following results: \( \hat{\gamma}_0 = 3.6800, \hat{\gamma}_1 = -2.95\% \) and \( \hat{\sigma}_v = 0.0520 \)

- \( \hat{CE}(2020) = 39.65 \text{ MtCO}_2\text{e} \) without the correction of the variance bias

- \( \hat{CE}(2020) = 39.70 \text{ MtCO}_2\text{e} \) with the correction of the variance bias
**Figure 305: Log-linear carbon trend (Example #8)**

![Log-linear carbon trend graph](image-url)
Example #9

We consider several historical trajectories of scope 1 carbon emissions:

<table>
<thead>
<tr>
<th>Year</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>2011</td>
<td>11.1</td>
<td>10.2</td>
<td>9.9</td>
<td>9.5</td>
</tr>
<tr>
<td>2012</td>
<td>10.5</td>
<td>10.5</td>
<td>9.5</td>
<td>9.0</td>
</tr>
<tr>
<td>2013</td>
<td>12.5</td>
<td>11.0</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>2014</td>
<td>13.0</td>
<td>10.8</td>
<td>9.3</td>
<td>8.3</td>
</tr>
<tr>
<td>2015</td>
<td>14.8</td>
<td>10.8</td>
<td>8.8</td>
<td>8.1</td>
</tr>
<tr>
<td>2016</td>
<td>16.0</td>
<td>13.0</td>
<td>8.7</td>
<td>7.7</td>
</tr>
<tr>
<td>2017</td>
<td>16.5</td>
<td>12.5</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>2018</td>
<td>17.0</td>
<td>13.5</td>
<td>9.0</td>
<td>7.0</td>
</tr>
<tr>
<td>2019</td>
<td>17.5</td>
<td>13.6</td>
<td>8.0</td>
<td>6.1</td>
</tr>
<tr>
<td>2020</td>
<td>19.8</td>
<td>13.6</td>
<td>8.2</td>
<td>6.2</td>
</tr>
</tbody>
</table>
Linear vs. log-linear trend model

Figure 306: Log-linear vs. linear carbon trend (Example #9)
Stochastic trend model

- The linear trend model can be written as:

\[
\begin{cases}
  y(t) = \mu(t) + u(t) \\
  \mu(t) = \mu(t - 1) + \beta_1
\end{cases}
\]

where \( u(t) \sim \mathcal{N}(0, \sigma_u^2) \)

- We have \( y(t) = \beta_0 + \beta_1 t + u(t) \) where \( \beta_0 = \mu(t_0) - \beta_1 t_0 \)

- The local linear trend model is defined as:

\[
\begin{cases}
  y(t) = \mu(t) + u(t) \\
  \mu(t) = \mu(t - 1) + \beta_1 (t - 1) + \eta(t) \\
  \beta_1(t) = \beta_1(t - 1) + \zeta(t)
\end{cases}
\]

where \( \eta(t) \sim \mathcal{N}(0, \sigma_\eta^2) \) and \( \zeta(t) \sim \mathcal{N}(0, \sigma_\zeta^2) \)

- The stochastic trend \( \mu(t) \) and slope \( \beta_1(t) \) are estimated with KF
Example #8

- We estimate the parameters \((\sigma_u, \sigma_\eta, \sigma_\zeta)\) by maximizing the Whittle log-likelihood function.
- We obtain \(\hat{\sigma}_u = 0.7022, \hat{\sigma}_\eta = 0.7019\) and \(\hat{\sigma}_\zeta = 0.8350\).
- The standard deviation of the stochastic slope variation \(\beta_1(t) - \beta_1(t - 1)\) is then equal to 0.8350 MtCO\(_2\)e.
### Table 153: Kalman filter estimation of the stochastic trend (Example #8)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$C\varepsilon(t)$</th>
<th>$\hat{\beta}_1(t)$ (RLS)</th>
<th>$\hat{\beta}_1(t)$ (KF)</th>
<th>$\mu(t)$ (KF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>57.80</td>
<td>0.0000</td>
<td>57.80</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>58.40</td>
<td>0.2168</td>
<td>58.25</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>57.90</td>
<td>0.0500</td>
<td>-0.0441</td>
<td>58.00</td>
</tr>
<tr>
<td>2010</td>
<td>55.10</td>
<td>-0.8600</td>
<td>-1.3941</td>
<td>55.56</td>
</tr>
<tr>
<td>2011</td>
<td>51.60</td>
<td>-1.5700</td>
<td>-2.6080</td>
<td>52.01</td>
</tr>
<tr>
<td>2012</td>
<td>48.30</td>
<td>-2.0200</td>
<td>-3.1288</td>
<td>48.47</td>
</tr>
<tr>
<td>2013</td>
<td>47.10</td>
<td>-2.0929</td>
<td>-2.2977</td>
<td>46.82</td>
</tr>
<tr>
<td>2014</td>
<td>46.10</td>
<td>-2.0321</td>
<td>-1.5508</td>
<td>45.85</td>
</tr>
<tr>
<td>2015</td>
<td>44.40</td>
<td>-1.9817</td>
<td>-1.5029</td>
<td>44.38</td>
</tr>
<tr>
<td>2016</td>
<td>42.70</td>
<td>-1.9406</td>
<td>-1.5887</td>
<td>42.73</td>
</tr>
<tr>
<td>2017</td>
<td>41.40</td>
<td>-1.8891</td>
<td>-1.4655</td>
<td>41.36</td>
</tr>
<tr>
<td>2018</td>
<td>40.20</td>
<td>-1.8329</td>
<td>-1.3202</td>
<td>40.15</td>
</tr>
<tr>
<td>2019</td>
<td>41.90</td>
<td>-1.6824</td>
<td>0.1339</td>
<td>41.41</td>
</tr>
<tr>
<td>2020</td>
<td>45.00</td>
<td>-1.4512</td>
<td>1.7701</td>
<td>44.45</td>
</tr>
</tbody>
</table>
We have:

\[
CM_{\text{Long}}(t) = \frac{\hat{\beta}_1(t)}{CE(t)}
\]

or:

\[
CM_{\text{Long}}(t) = \hat{\gamma}_1(t)
\]
### Table 154: Statistics (in %) of carbon momentum $CM_{\text{Long}}^L(t)$ (MSCI World index, 1995 – 2021, linear trend)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Carbon emissions</th>
<th>Carbon intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_{1-2}$</td>
</tr>
<tr>
<td>Median</td>
<td>0.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Negative</td>
<td>49.9</td>
<td>41.1</td>
</tr>
<tr>
<td>Positive</td>
<td>50.1</td>
<td>58.9</td>
</tr>
<tr>
<td>$&lt; -10%$</td>
<td>23.4</td>
<td>15.8</td>
</tr>
<tr>
<td>$&lt; -5%$</td>
<td>32.1</td>
<td>22.2</td>
</tr>
<tr>
<td>$&gt; +5%$</td>
<td>22.9</td>
<td>27.5</td>
</tr>
<tr>
<td>$&gt; +10%$</td>
<td>9.2</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Source: Trucost database (2022) & Authors’ calculations.
Table 155: Statistics (in %) of carbon momentum $C_M^{Long} (t)$ (MSCI World index, 1995 – 2021, log-linear trend)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Carbon emissions $SC_1$</th>
<th>$SC_{1-2}$</th>
<th>$SC_{1-3}^{up}$</th>
<th>Carbon intensity $SC_1$</th>
<th>$SC_{1-2}$</th>
<th>$SC_{1-3}^{up}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>-0.1</td>
<td>1.7</td>
<td>2.8</td>
<td>-3.6</td>
<td>-1.9</td>
<td>-1.2</td>
</tr>
<tr>
<td>Negative</td>
<td>50.6</td>
<td>40.3</td>
<td>29.0</td>
<td>76.3</td>
<td>69.0</td>
<td>75.8</td>
</tr>
<tr>
<td>Positive</td>
<td>49.4</td>
<td>59.7</td>
<td>71.0</td>
<td>23.7</td>
<td>31.0</td>
<td>24.2</td>
</tr>
<tr>
<td>$&lt; -10%$</td>
<td>13.6</td>
<td>8.0</td>
<td>2.8</td>
<td>20.8</td>
<td>12.3</td>
<td>2.1</td>
</tr>
<tr>
<td>$&lt; -5%$</td>
<td>26.6</td>
<td>16.9</td>
<td>7.5</td>
<td>42.3</td>
<td>29.0</td>
<td>8.4</td>
</tr>
<tr>
<td>$&gt; +5%$</td>
<td>29.8</td>
<td>35.9</td>
<td>37.1</td>
<td>9.0</td>
<td>10.1</td>
<td>4.0</td>
</tr>
<tr>
<td>$&gt; +10%$</td>
<td>16.9</td>
<td>19.4</td>
<td>19.2</td>
<td>4.0</td>
<td>4.1</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Source: Trucost database (2022) & Authors’ calculations.
The **PAC** framework

- Participation
- Ambition
- Credibility
The \( \mathcal{PAC} \) framework requires three time series:

- The historical pathway of carbon emission
- The reduction targets announced by the company
  \[
  \mathcal{CT} = \{ \mathcal{R}^{\text{Target}} (t_0, t_k), k = 1, \ldots, n_T \}
  \]
- The market-based sector scenario associated to the company that defines the decarbonization pathway
  \[
  \mathcal{CS} = \{ \mathcal{R}^{\text{Scenario}} (t_0, t_k), k = 1, \ldots, n_S \}
  \]
The PAC framework

<table>
<thead>
<tr>
<th>Year</th>
<th>Electricity</th>
<th>Industry</th>
<th>Transport</th>
<th>Buildings</th>
<th>Other</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>20.0</td>
<td>4.0</td>
<td>-1.1</td>
<td>15.0</td>
<td>13.1</td>
<td>10.6</td>
</tr>
<tr>
<td>2030</td>
<td>56.9</td>
<td>18.8</td>
<td>20.0</td>
<td>36.7</td>
<td>52.4</td>
<td>36.6</td>
</tr>
<tr>
<td>2035</td>
<td>84.3</td>
<td>38.1</td>
<td>42.5</td>
<td>57.7</td>
<td>95.3</td>
<td>59.6</td>
</tr>
<tr>
<td>2040</td>
<td>100.0</td>
<td>59.0</td>
<td>62.4</td>
<td>75.9</td>
<td>100.0</td>
<td>77.1</td>
</tr>
<tr>
<td>2045</td>
<td>100.0</td>
<td>78.8</td>
<td>79.0</td>
<td>88.8</td>
<td>100.0</td>
<td>87.3</td>
</tr>
<tr>
<td>2050</td>
<td>100.0</td>
<td>93.9</td>
<td>90.3</td>
<td>95.8</td>
<td>100.0</td>
<td>94.3</td>
</tr>
</tbody>
</table>

Source: IEA (2021) & Author’s calculations.

Table 156: Reduction rates of the IEA NZE scenario (base year = 2020)
The **PAC** framework

### The 3 questions of the **PAC** framework

1. Is the trend of the issuer in line with the scenario?
2. Is the commitment of the issuer to fight climate change ambitious?
3. Is the target setting of the company relevant and robust, or is it a form of greenwashing?
Example #10

- We consider Example #8
- Company A has announced the following targets:
  1. $R^{Target}(2020, 2025) = 40\%$
  2. $R^{Target}(2020, 2030) = 50\%$
  3. $R^{Target}(2020, 2035) = 75\%$
  4. $R^{Target}(2020, 2040) = 80\%$
  5. $R^{Target}(2020, 2050) = 90\%$
- Company A is an utility corporation $\Rightarrow$ we use the IEA NZE scenario for the sector Electricity
The **PAC** framework

### Table 157: Comparison of carbon budgets (Example #10, base year = 2020)

<table>
<thead>
<tr>
<th>Year</th>
<th>Trend (linear)</th>
<th>Trend (log-linear)</th>
<th>Target</th>
<th>Scenario (global)</th>
<th>Scenario (electricity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025</td>
<td>207</td>
<td>209</td>
<td>180</td>
<td>213</td>
<td>203</td>
</tr>
<tr>
<td>2030</td>
<td>377</td>
<td>390</td>
<td>304</td>
<td>385</td>
<td>341</td>
</tr>
<tr>
<td>2035</td>
<td>512</td>
<td>546</td>
<td>388</td>
<td>502</td>
<td>407</td>
</tr>
<tr>
<td>2040</td>
<td>610</td>
<td>680</td>
<td>439</td>
<td>573</td>
<td>425</td>
</tr>
<tr>
<td>2045</td>
<td>671</td>
<td>796</td>
<td>478</td>
<td>613</td>
<td>425</td>
</tr>
<tr>
<td>2050</td>
<td>697</td>
<td>896</td>
<td>506</td>
<td>634</td>
<td>425</td>
</tr>
</tbody>
</table>
Figure 307: Carbon trend, targets and NZE scenario of company A

Source: IEA (2021) & Author’s calculations.
Assessment of the \( \mathcal{PAC} \) pillars

Figure 308: Illustration of the participation, ambition and credibility pillars

<table>
<thead>
<tr>
<th>Participation</th>
<th>Ambition</th>
<th>Credibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) positive</td>
<td>(a) positive</td>
<td>(a) positive</td>
</tr>
<tr>
<td>(b) negative</td>
<td>(b) negative</td>
<td>(b) negative</td>
</tr>
<tr>
<td>(c) mixed</td>
<td>(c) mixed</td>
<td>(c) mixed</td>
</tr>
</tbody>
</table>
Temperature scoring system

Figure 309: The PAC scoring system

(a) Model student?

(b) Black sheep?

(c) Shy child?

(d) Greenwashing?
Figure 310: Carbon emissions, trend, targets and NZE scenario (Company B)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 311: Carbon emissions, trend, targets and NZE scenario (Company C)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 312: Carbon emissions, trend, targets and NZE scenario (Company D)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 313: Carbon emissions, trend, targets and NZE scenario (median analysis, global universe)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Figure 314: Carbon emissions, trend, targets and NZE scenario (median analysis, sector universe)

Source: CDP database (2021), IEA (2021) & Leguenedal et al. (2022)
Greenness measures

- Brown intensity: $BI$
- Green intensity: $GI$
- We have $BI \in [0, 1]$, $GI \in [0, 1]$ and $0 \leq BI + GI \leq 1$
- Most of the time, we have $BI + GI \neq 1$


Figure 315: Several taxonomies

(a) Green activities

(b) Brown activities

(c) All activities
Definition

The EU taxonomy for sustainable activities is “a classification system, establishing a list of environmentally sustainable economic activities.”
These economic activities must have a substantive contribution to at least one of the following six environmental objectives:

- climate change mitigation
- climate change adaptation
- sustainable use and protection of water and marine resources
- transition to a circular economy
- pollution prevention and control
- protection and restoration of biodiversity and ecosystem
A business activity must also meet two other criteria to qualify as sustainable:

- The activity must do no significant harm to the other environmental objectives (DNSH constraint)
- It must comply with minimum social safeguards (MS constraint)
Figure 316: EU taxonomy for sustainable activities

1a. SC  
Substantially contribute to at least one of the six objectives

1b. TSC  
Comply with Technical Screening Criteria

2. DNSH  
Do No Significant Harm to any other five objectives

3. MS  
Comply with Minimum (Social) Safeguards
We have:

\[ GI = \frac{GR}{TR} \cdot (1 - \mathcal{P}) \cdot 1 \{ S \geq S^* \} \]

where:

- \( GR \) is the green revenue deduced from the six environmentally sustainable objectives
- \( TR \) is the total revenue
- \( \mathcal{P} \) is the penalty coefficient reflecting the DNSH constraint
- \( S \) is the minimum safeguard score
- \( S^* \) is the threshold
The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

\[ \text{GRS} = \frac{GR}{TR} \]

By construction, we have \(0 \leq \text{GRS} \leq 1\)

This measure is then impacted by the DNSH coefficient

The two extreme cases are:

\[ \begin{cases} 
  P = 1 & \Rightarrow \text{GI} = \text{GRS} \\
  P = 0 & \Rightarrow \text{GI} = 0 
\end{cases} \]

We have \(0 \leq \text{GI} = \text{GRS} \cdot (1 - P) \leq \text{GRS}\)

The indicator function \(1 \{s \geq s^*\}\) is a binary all-or-nothing variable:

\(S < S^* \Rightarrow \text{GI} = 0\)
Example #11

We consider a company in the hydropower sector which has five production sites. Below, we indicate the power density efficiency, the GHG emissions, the DNSH compliance with respect to the biodiversity and the corresponding revenue:

<table>
<thead>
<tr>
<th>Site</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency (in Watt per $m^2$)</td>
<td>3.2</td>
<td>3.5</td>
<td>3.3</td>
<td>5.6</td>
<td>4.2</td>
</tr>
<tr>
<td>GHG emissions (in gCO$_2$e per kWh)</td>
<td>35</td>
<td>103</td>
<td>45</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>Biodiversity DNSH compliance</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Revenue (in $ mn)</td>
<td>103</td>
<td>256</td>
<td>89</td>
<td>174</td>
<td>218</td>
</tr>
</tbody>
</table>
Green revenue share

- The total revenue is equal to:
  \[ TR = 103 + 256 + 89 + 174 + 218 = 840 \text{ mn} \]

- The fourth site does not pass the technical screening, because the power density is above 5 Watt per \( m^2 \)

- The second site does not also comply because it has a GHG emissions greater than 100 gCO\(_2\)e per kWh

- We deduce that the green revenue is equal to:
  \[ GR = 103 + 89 + 218 = 410 \text{ mn} \]

- We conclude that the green revenue share is equal to 48.8%

- According to the EU green taxonomy, the green intensity is lower because the last site is close to a biodiversity area and has a negative impact:
  \[ GI = \frac{103 + 89}{840} = 22.9\% \]
### Table 158: Statistics in % of green revenue share (MSCI ACWI IMI, June 2022)

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency $F(x)$</th>
<th>0</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
<th>Max</th>
<th>Mean</th>
<th>Avg</th>
<th>Wgt</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td>9.82</td>
<td>1.47</td>
<td>0.96</td>
<td>0.75</td>
<td>0.00</td>
<td>0.00</td>
<td>2.85</td>
<td>100.00</td>
<td>1.36</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td>14.10</td>
<td>1.45</td>
<td>0.65</td>
<td>0.31</td>
<td>0.00</td>
<td>1.25</td>
<td>6.12</td>
<td>100.00</td>
<td>1.39</td>
<td>3.50</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td>4.84</td>
<td>1.68</td>
<td>1.02</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>1.16</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td>4.79</td>
<td>0.30</td>
<td>0.10</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>99.69</td>
<td>0.32</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td></td>
<td>1.00</td>
<td>0.39</td>
<td>0.20</td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>98.47</td>
<td>0.26</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td>4.75</td>
<td>0.28</td>
<td>0.11</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>99.98</td>
<td>0.29</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>27.85</td>
<td>5.82</td>
<td>3.17</td>
<td>1.68</td>
<td>0.42</td>
<td>11.82</td>
<td>30.36</td>
<td>100.00</td>
<td>4.78</td>
<td>5.24</td>
<td></td>
</tr>
</tbody>
</table>

Source: MSCI (2022) & Barahhou (2022)

$F(x) = \Pr \{ GRS > x \}$, $Q(\alpha) = \inf \{ x : \Pr \{ GRS \leq x \} \geq \alpha \}$, arithmetic average $n^{-1} \sum_{i=1}^{n} GRS_i$ and weighted mean $GRS(b) = \sum_{i=1}^{n} b_i GRS_i$.
• The green revenue share of the MSCI World index is equal to 5.24%.
• The green revenue share of the Bloomberg Global Investment Grade Corporate Bond index is equal to 3.49%.
• Alessi and Battiston (2022) estimated “a greenness of about 2.8% for EU financial markets.”
Green capex
Green-to-brown ratio
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Lecture 1: Introduction
Lecture 2: ESG Scoring
Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
Lecture 4: Sustainable Financial Products
Lecture 5: Impact Investing
Lecture 6: Engagement & Voting Policy
Lecture 7: Extra-financial Accounting
Lecture 8: Awareness of Climate Change Impacts
Lecture 9: The Ecosystem of Climate Change
Lecture 10: Economic Models & Climate Change
Lecture 11: Climate Risk Measures
Lecture 12: Transition Risk Modeling
Lecture 13: Climate Portfolio Construction
Lecture 14: Physical Risk Modeling
Lecture 15: Climate Stress Testing & Risk Management
Climate transition risk

Definition

- Transition risks arise from the sudden shift towards a low-carbon economy.
- Such transitions could mean that some sectors of the economy face big shifts in asset values or higher costs of doing business.

“It’s not that policies stemming from deals like the Paris Climate Agreement are bad for our economy — in fact, the risk of delaying action altogether would be far worse. Rather, it’s about the speed of transition to a greener economy — and how this affects certain sectors and financial stability” (Bank of England, 2021)
The carbon footprint approach assumes that the **climate-related market risk** of a company is measured by its **current carbon intensity**

...But the **market perception** of the climate change may be different
Carbon price

Two main pricing systems:
- Carbon tax
- Emissions trading system (ETS)

Underlying idea
- A high carbon tax impacts the creditworthiness of corporates
- This impact is different from one issuer to another one
- Identifying for each company the carbon price that would lead the default probability in the Merton model to exceed a certain threshold
Climate policy

- Carbon pricing: main tool to implement public policies to reduce CO₂ emissions

“Carbon pricing is an instrument that captures the external costs of greenhouse gas (GHG) emissions — the costs of emissions that the public pays for, such as damage to crops, health care costs from heat waves and droughts, and loss of property from flooding and sea level rise — and ties them to their sources through a price, usually in the form of a price on the carbon dioxide emitted.” (World Bank, 2021)

- Can take several forms: carbon tax, emissions trading system, carbon credit
- Underlying idea: the biggest GHG emitters must pay higher tax or face higher costs
- Goal: transform deeply their activities ⇒ lower their emissions
Economic theory of negative externalities

Prices vs. Quantities

- Weitzman (1974)
- Laffont (1977)
- Pizer (2002)
- McKibbin and Wilcoxen (2002)
- Hepburn (2006)
- Tang et al. (2018)
“Choosing appropriate policy instruments is an important part of successful regulation. Once objectives are agreed and suitable targets adopted, policy-makers can employ command-and-control regulation and/or economic instruments, and choose between fixing a price or a quantity.” (Hepburn, 2006, page 226).
Optimal taxation
Economic theory of quotas
Uncertainty
<table>
<thead>
<tr>
<th>Irreversibility</th>
<th>Regulation policy</th>
<th>Economic theory of negative externalities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity valuation</td>
<td>Carbon pricing</td>
</tr>
<tr>
<td></td>
<td>Financing and technological risks</td>
<td>Emissions trading system</td>
</tr>
</tbody>
</table>

Thierry Roncalli

Course 2023-2024 in Sustainable Finance

1284 / 1665
Cost-benefit analysis versus option theory
Two forms of carbon pricing

External carbon pricing
- Managed by governments
- Examples: carbon tax, emission trading system (ETS)
- Reflect the price of a tonne of CO$_2$e emitted.

Internal carbon pricing
- Managed by corporates
- Examples: internal carbon fee, shadow price, implicit price

There are 68 explicit carbon pricing mechanisms as of 1\textsuperscript{st} August 2022, with the following breakdown: 32 emission trading systems and 36 carbon taxes.
Two forms of carbon pricing

Figure 317: Map of explicit carbon prices around the world in 2022

Prices given in USD/tCO2e:
- Established Emissions Trading Scheme
- Established Carbon Tax
- Positive development (%)
- Negative development (%)
- No percentage displayed = no price change

Source: I4CE - Institute for Climate Economics, with data from ICAP, World Bank, government officials and public information, September 2022.
“Present value of the economic impact of an additional tonne of CO₂ emitted”

Introduced by Nordhaus in 1991:

$$SCC(t) = -\frac{\partial W}{\partial W} = -\frac{\partial W}{\partial C(t)}$$

where $W$ denotes the social welfare function, $CE(t)$ is the total GHG emissions at time $t$ and $C(t)$ is the consumption at time $t$.

The term $\frac{\partial W}{\partial CE(t)} \leq 0$ is the marginal social welfare with respect to GHG emissions, while $\frac{\partial W}{\partial C(t)} \geq 0$ is the marginal utility of consumption.
Social cost of carbon

**Figure 318:** Histogram of the 150,000 US Government SCC estimates for 2020 with a 3% discount rate

**Figure 319:** Probability distribution of the SCC (log-normal)
Implied carbon prices

- Sweden
- Switzerland
- Finland
- Liechtenstein
- France
- British Columbia
- New Zealand (ETS)
- Norway
- Canada
- Denmark
- EU ETS
- Iceland
- New Brunswick
- Latvia
- Ireland
- New England Territories
- Quebec (ETS)
- RCC1 (ETS)
- Slovenia
- Portugal
- Spain
- Spain ETS
- United Kingdom
- Canada (ETS)
- Chile
- ETS
- Switzerland
- Singapore
- Nova Scotia (ETS)
- Alberta (ETS)
- Colombia
- Japan
- China ETS
- Korea (ETS)
- South Africa (ETS)
- Ukraine
- Poland
- Shanghai (ETS)
We assume a flat carbon tax:
\[ \text{Cost} = C_T \cdot C_E \]

Utilities, Energy, Materials and Industrials are the most impacted sectors ⇒ Cost over Dividend ratio of 210.64, 81.64, 94.05 and 30.45

Communication Services, Information Technology, Real Estate and Health Care are less impacted ⇒ Cost over Dividend ratio of 0.93, 1.42, 0.99 and 1.80
Carbon tax: a high heterogeneity between and within sectors

**Figure 321:** Boxplot of the Cost/Dividend ratio in % (MSCI World index, December 2021, $\mathcal{C} = 100/t\text{CO}_2$)

**Figure 322:** Boxplot of the Cost/Dividend ratio in % (MSCI World index, December 2021, $\mathcal{C} = 100/t\text{CO}_2$)
Emissions trading system
Definition
Market where corporates and countries can trade carbon emissions to meet their targets. A global amount of emissions that can be traded by the different entities.

Figure 323: EU ETS carbon allowance price

- Cap-and-trade system
- Remained for a long time below €30/tCO\textsubscript{2e}, sharp decline after the Global Financial Crisis from €30 in 2008 to a mere €2.75 in April 2013
- Carbon price went from €34 in January 2021 to nearly €100 in February 2023
Probability distribution of the EU ETS carbon price

**Figure 324:** Volatility of carbon price

**Figure 325:** PDF of the carbon price
Carbon pricing
Stranded assets
This figure presents the energy generation breakdown for some countries. We can distinguish countries that rely on hydroelectric power (Brazil, Norway), nuclear (France, Switzerland) and mixed solutions (Canada, Germany, Spain, USA). 

(*) Each grid circle represents 20% of energy generation. The scale of the radar chart is then 40% for Canada, Germany, Spain and USA, 60% for China, France and Switzerland, 80% for Brazil and 100% for Norway.
Financing side of the net-zero transition
The main transformation involves the power sector in two directions:

- Massive electrification of the world economy
- Greening electricity to achieve clean power generation

**Table 159: The 2050 net-zero scenarios**

<table>
<thead>
<tr>
<th>Production (TWh)</th>
<th>Energy Intensity (g/kWh)</th>
<th>Carbon Emissions (GtCO₂e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 000</td>
<td>500</td>
<td>15</td>
</tr>
<tr>
<td>100 000</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>
### How to achieve net zero emissions

#### Table 160: Emission factor in gCO₂e/kWh of electricity generation in the world

<table>
<thead>
<tr>
<th>Region</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
<th>EF</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>484</td>
<td>Australia</td>
<td>531</td>
<td>Germany</td>
<td>354</td>
<td>Portugal</td>
</tr>
<tr>
<td>Asia</td>
<td>539</td>
<td>Canada</td>
<td>128</td>
<td>India</td>
<td>637</td>
<td>Russia</td>
</tr>
<tr>
<td>Europe</td>
<td>280</td>
<td>China</td>
<td>544</td>
<td>Iran</td>
<td>492</td>
<td>Spain</td>
</tr>
<tr>
<td>North America</td>
<td>352</td>
<td>Costa Rica</td>
<td>33</td>
<td>Italy</td>
<td>226</td>
<td>Switzerland</td>
</tr>
<tr>
<td>South America</td>
<td>204</td>
<td>Cuba</td>
<td>575</td>
<td>Japan</td>
<td>479</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>World</td>
<td>442</td>
<td>France</td>
<td>58</td>
<td>Norway</td>
<td>26</td>
<td>United States</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thierry Roncalli  
Course 2023-2024 in Sustainable Finance  
1301 / 1665
How to achieve net zero emissions

**Figure 326:** Emission factor in $g\text{CO}_2\text{e}/\text{kWh}$ of electricity generation (European Union, 1990-2022)
Transforming the global value chain into a net zero economy

Figure 327: Global value chain in 2017

Figure 328: Global value chain in 2030

Figure 329: Global value chain in 2050

Funding requirements

Figure 330: Net zero capital investments

- Power generation (37.7%)
- Power networks (25.9%)
- Power storage (5.8%)
- Industry (2.0%)
- Hydrogen (2.3%)
- Transport (8.1%)
- Removals (3.9%)

Source: Energy Transitions Commission (2023a) & Ben Slimane et al. (2023b).
Funding requirements

**Table 161:** Importance of GICS sectors in net zero investing

<table>
<thead>
<tr>
<th>Sector</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td></td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td></td>
</tr>
<tr>
<td>Consumer Staples</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td></td>
</tr>
<tr>
<td>Health Care</td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td></td>
</tr>
<tr>
<td>Information Technology</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td></td>
</tr>
</tbody>
</table>

We obtain the following five clusters from the most important to the least important:

1. Utilities;
2. Materials, Industrials;
3. Consumer Discretionary, Real Estate;
4. Energy, Information Technology, Consumer Staples, Health Care;
5. Financials, Consumer Services.
## Table 162: Mineral requirements for clean energy technologies

<table>
<thead>
<tr>
<th>Technology</th>
<th>Aluminium</th>
<th>Chromium</th>
<th>Copper</th>
<th>Cobalt</th>
<th>Graphite</th>
<th>Lithium</th>
<th>Neodymium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bioenergy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity Networks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EVs and Battery storage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geothermal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrogen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydropower</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nuclear</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solar PV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: IEA (2022, page 45).
Table 163: Mineral requirements for clean energy technologies

<table>
<thead>
<tr>
<th>Technology</th>
<th>Nickel</th>
<th>Platinum</th>
<th>Polysilicon</th>
<th>REEs</th>
<th>Silver</th>
<th>Steel</th>
<th>Uranium</th>
<th>Zinc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bioenergy</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>CSP</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Electricity Networks</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>EVs and Battery storage</td>
<td>●</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Geothermal</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Hydropower</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Nuclear</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Solar PV</td>
<td>○</td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Wind</td>
<td>●</td>
<td>○</td>
<td>○</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

Source: IEA (2022, page 45).
Material and resource requirements

**Figure 331**: Demand and primary supply in 2030

Source: Energy Transitions Commission (2023a).
Total investment: 3.5 trillion per year to 2050
### Power ($2,400 bn)
- Total electricity supply from around 30,000 TWh today to over 100,000 TWh by mid-century
- Extension of transmission and distribution networks from about 70 million km to up to 200 million km
- Green hydrogen production of 500-800 Mt per year

### Buildings ($500 bn)
- Need to retrofit older buildings and create new carbon-efficient buildings
- $500 bn per year invested in the buildings sector: incorporate new green technologies ($230 bn), purchase renewable heat ($130 bn) and install new heat pumps ($150 bn)
Mobility ($280 bn)

- The largest part of the transition from ICE (internal combustion engines) to EVs will require $130 bn per year to develop charging and refuelling facilities
- $70 bn will be spent on sustainable aircraft manufacturing facilities and aircraft batteries
- $40 bn will be spent on greening the shipping system through new infrastructure, vessels and investments

Sustainable agriculture and land use requirements ($50 bn)

- The demand for wind and solar farms is far greater than the previous demand based on the fossil fuel system, but still far less than the demand for agriculture
- Agriculture is the largest driver of deforestation
Sector analysis

Hydrogen ($80 bn)
- $80 bn investment will be allocated to the production and distribution of hydrogen
- $40 bn will be used to produce green and blue hydrogen and to recycle grey hydrogen
- $40 bn will help build pipelines, refuelling stations, exchange terminals and storage capacity

Industry ($70 bn)
- $10 bn will be used to decarbonise steel
- $10 bn for cement plants
- $40 bn to fully develop and integrate CCS and other decarbonisation technologies
- $10 bn to deploy low-carbon technologies in smelters and refineries
Waste management and circular economy ($135 bn)

- Waste is generated at every stage of the transition, from food waste from agriculture to waste from solar panels, wind farms or even mining
- The energy transition will generate up to 13 billion tonnes of waste from all materials by 2050

Water management ($25 bn)

- Global water consumption will be 4 000 billion m$^3$ per year in 2050, of which 70% is used for agriculture (2 800 billion m$^3$), 58 billion m$^3$ for clean energy production and 37 billion m$^3$ for fossil fuels
- For clean energy production, water is used for nuclear power generation (14 billion m$^3$ per year), hydrogen production by electrolysis (11 billion m$^3$ per year), carbon capture (19–29 billion m$^3$ per year) and cleaning solar panels
- Global energy use in the water sector expected to double by 2040
Narrow definition of the satellite investment portfolio

Table 164: Main sub-industries of the net zero satellite portfolio (GICS level 4)

<table>
<thead>
<tr>
<th>Code</th>
<th>Industry Description</th>
<th>Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>15102010</td>
<td>Construction Materials</td>
<td>Materials</td>
</tr>
<tr>
<td>15104010</td>
<td>Aluminum</td>
<td>Materials</td>
</tr>
<tr>
<td>15104020</td>
<td>Diversified Metals &amp; Mining</td>
<td>Materials</td>
</tr>
<tr>
<td>15104025</td>
<td>Copper</td>
<td>Materials</td>
</tr>
<tr>
<td>15104040</td>
<td>Precious Metals &amp; Minerals</td>
<td>Materials</td>
</tr>
<tr>
<td>15104045</td>
<td>Silver</td>
<td>Materials</td>
</tr>
<tr>
<td>15104050</td>
<td>Steel</td>
<td>Materials</td>
</tr>
<tr>
<td>20102010</td>
<td>Building Products</td>
<td>Capital Goods</td>
</tr>
<tr>
<td>20103010</td>
<td>Construction &amp; Engineering</td>
<td>Industrials</td>
</tr>
<tr>
<td>20104010</td>
<td>Electrical Components &amp; Equipment</td>
<td>Capital Goods</td>
</tr>
<tr>
<td>20104020</td>
<td>Heavy Electrical Equipment</td>
<td>Capital Goods</td>
</tr>
<tr>
<td>20106010</td>
<td>Construction Machinery &amp; Heavy Transportation Eqpt.</td>
<td>Industrials</td>
</tr>
<tr>
<td>20106015</td>
<td>Agricultural &amp; Farm Machinery</td>
<td>Industrials</td>
</tr>
<tr>
<td>20201050</td>
<td>Environmental &amp; Facilities Services</td>
<td>Commercial &amp; Professional Services</td>
</tr>
<tr>
<td>20304010</td>
<td>Rail Transportation</td>
<td>Transportation</td>
</tr>
<tr>
<td>20305010</td>
<td>Airport Services</td>
<td>Transportation</td>
</tr>
<tr>
<td>20305020</td>
<td>Highways &amp; Railtracks</td>
<td>Transportation</td>
</tr>
<tr>
<td>20305030</td>
<td>Marine Ports &amp; Services</td>
<td>Transportation</td>
</tr>
<tr>
<td>25101010</td>
<td>Automotive Parts &amp; Equipment</td>
<td>Automobiles &amp; Components</td>
</tr>
<tr>
<td>25102010</td>
<td>Automobile Manufacturers</td>
<td>Consumer Discretionary</td>
</tr>
<tr>
<td>25201010</td>
<td>Consumer Electronics</td>
<td>Consumer Discretionary</td>
</tr>
<tr>
<td>25201030</td>
<td>Homebuilding</td>
<td>Consumer Discretionary</td>
</tr>
<tr>
<td>25201040</td>
<td>Household Appliances</td>
<td>Consumer Discretionary</td>
</tr>
<tr>
<td>30202010</td>
<td>Agricultural Products</td>
<td>Services Food, Beverage &amp; Tobacco</td>
</tr>
<tr>
<td>55101010</td>
<td>Electric Utilities</td>
<td>Utilities</td>
</tr>
<tr>
<td>55103010</td>
<td>Multi-Utilities</td>
<td>Utilities</td>
</tr>
<tr>
<td>55104010</td>
<td>Water Utilities</td>
<td>Utilities</td>
</tr>
<tr>
<td>55105020</td>
<td>Renewable Electricity</td>
<td>Utilities</td>
</tr>
<tr>
<td>60201030</td>
<td>Real Estate Development</td>
<td>Real Estate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Real Estate</td>
</tr>
</tbody>
</table>
Figure 332: Narrow specification of the satellite investment universe

<table>
<thead>
<tr>
<th>Sector</th>
<th>Industry Group</th>
<th>Industry</th>
<th>Sub-industry</th>
<th>Satellite</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 165: What’s on track (energy system overview)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Efficiency</td>
<td>on track</td>
</tr>
<tr>
<td>Behavioural Changes</td>
<td>more efforts needed</td>
</tr>
<tr>
<td>Electrification</td>
<td>on track</td>
</tr>
<tr>
<td>Renewables</td>
<td>on track</td>
</tr>
<tr>
<td>Bioenergy</td>
<td>on track</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>on track</td>
</tr>
<tr>
<td>Carbon Capture, Utilisation and Storage</td>
<td>not on track</td>
</tr>
<tr>
<td>Innovation</td>
<td>on track</td>
</tr>
<tr>
<td>International Collaboration</td>
<td>on track</td>
</tr>
<tr>
<td>Digitalisation</td>
<td>on track</td>
</tr>
</tbody>
</table>

- on track
- more efforts needed
- not on track

Source: IEA (2023).
### Table 166: What’s on track (sector analysis)

<table>
<thead>
<tr>
<th>Transversal Technologies &amp; Infrastructure</th>
<th>Electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂ Transport and Storage</td>
<td>Coal</td>
</tr>
<tr>
<td>CO₂ Capture and Utilisation</td>
<td>Natural Gas</td>
</tr>
<tr>
<td>Bioenergy with Carbon Capture and Storage</td>
<td>Solar PV</td>
</tr>
<tr>
<td>Direct Air Capture</td>
<td>Wind</td>
</tr>
<tr>
<td>Electrolysers</td>
<td>Hydroelectricity</td>
</tr>
<tr>
<td>District Heating</td>
<td>Demand Response</td>
</tr>
<tr>
<td>Data Centres and Transmission Networks</td>
<td>Nuclear Power</td>
</tr>
<tr>
<td>Transport</td>
<td>Grid-scale Storage</td>
</tr>
<tr>
<td>Cars and Vans</td>
<td>Smart Grids</td>
</tr>
<tr>
<td>Trucks and Buses</td>
<td>Energy</td>
</tr>
<tr>
<td>Rail</td>
<td>Oil &amp; Natural Gas Supply</td>
</tr>
<tr>
<td>Aviation</td>
<td>Methane Abatement</td>
</tr>
<tr>
<td>International Shipping</td>
<td>Gas Flaring</td>
</tr>
<tr>
<td>Electric Vehicles</td>
<td>Biofuels</td>
</tr>
<tr>
<td>Industry</td>
<td>Buildings</td>
</tr>
<tr>
<td>Steel</td>
<td>Heating</td>
</tr>
<tr>
<td>Chemicals</td>
<td>Space Cooling</td>
</tr>
<tr>
<td>Cement</td>
<td>Lighting</td>
</tr>
<tr>
<td>Aluminium</td>
<td>Appliances &amp; Equipment</td>
</tr>
<tr>
<td>Paper</td>
<td>Building Envelopes</td>
</tr>
<tr>
<td>Light Industry</td>
<td>Heat Pumps</td>
</tr>
</tbody>
</table>
Public vs. private investments

Figure 333: Public investment – relative difference in % compared with the baseline scenario

Figure 334: Private investment – relative difference in % compared with the baseline scenario

Public vs. private investments

What are the narratives

- Net zero emissions scenario ⇒ Huge cost
- This cost mainly concerns the Utilities sector

- Utilities ⇒ Huge capex ⇒ ROE ↓
- Private investors are reluctant to finance the utilities sector
- Private investment ↓

- A strong increase of public investment
- Debt ↑ ⇒ Interest rates ↑
- Investors prefer to invest in sovereign bonds than financing directly net zero

Vase communication between public investment and private investment
The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- **Lecture 13: Climate Portfolio Construction**
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Definition

We have:

\[ x^* = \arg \min \frac{1}{2} x^\top Q x - x^\top R \]

s.t. \[
\begin{cases}
Ax = B \\
Cx \leq D \\
x^- \leq x \leq x^+
\end{cases}
\]

where \( x \) is a \( n \times 1 \) vector, \( Q \) is a \( n \times n \) matrix, \( R \) is a \( n \times 1 \) vector, \( A \) is a \( n_A \times n \) matrix, \( B \) is a \( n_A \times 1 \) vector, \( C \) is a \( n_C \times n \) matrix, \( D \) is a \( n_C \times 1 \) vector, and \( x^- \) and \( x^+ \) are two \( n \times 1 \) vectors.
A quadratic form is a polynomial with terms all of degree two

\[ QF(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j} x_i x_j = x^\top A x \]

**Canonical form**

\[ QF(x_1, \ldots, x_n) = \frac{1}{2} (x^\top A x + x^\top A^\top x) = \frac{1}{2} x^\top (A + A^\top) x = \frac{1}{2} x^\top Q x \]

**Generalized quadratic form**

\[ QF(x; Q, R, c) = \frac{1}{2} x^\top Q x - x^\top R + c \]
Quadratic form
Main properties

1. \( \varphi \cdot QF(w; Q, R, c) = QF(w; \varphi Q, \varphi R, \varphi c) \)
2. \( QF(x; Q_1, R_1, c_1) + QF(x; Q_2, R_2, c_2) = QF(x; Q_1 + Q_2, R_1 + R_2, c_1 + c_2) \)
3. \( QF(x - y; Q, R, c) = QF(x; Q, R + Qy, \frac{1}{2}y^T Q y + y^T R + c) \)
4. \( QF(x - y; Q, R, c) = QF(y; Q, Qx - R, \frac{1}{2}x^T Q x - x^T R + c) \)
5. \( \frac{1}{2} \sum_{i=1}^n q_i x_i^2 = QF(x; D(q), 0_n, 0) \) where \( q = (q_1, \ldots, q_n) \) is a \( n \times 1 \) vector and \( D(q) = \text{diag}(q) \)
6. \( \frac{1}{2} \sum_{i=1}^n q_i (x_i - y_i)^2 = QF(x; D(q), D(q) y, \frac{1}{2}y^T D(q) y) \)
7. \( \frac{1}{2} (\sum_{i=1}^n q_i x_i)^2 = QF(x; T(q), 0_n, 0) \) where \( T(q) = qq^T \)
8. \( \frac{1}{2} (\sum_{i=1}^n q_i (x_i - y_i))^2 = QF(x; T(q), T(q) y, \frac{1}{2}y^T T(q) y) \)
We note $\omega = (\omega_1, \ldots, \omega_n)$ where $\omega_i = 1 \{i \in \Omega\}$

1. $\frac{1}{2} \sum_{i \in \Omega} q_i x_i^2 = QF \left( x; D(\omega \circ q), 0_n, 0 \right)$

2. $\frac{1}{2} \sum_{i \in \Omega} q_i (x_i - y_i)^2 = QF \left( x; D(\omega \circ q), D(\omega \circ q)y, \frac{1}{2} y^T D(\omega \circ q)y \right)$

3. $\frac{1}{2} (\sum_{i \in \Omega} q_i x_i)^2 = QF \left( x; T(\omega \circ q), 0_n, 0 \right)$

4. $\frac{1}{2} (\sum_{i \in \Omega} q_i (x_i - y_i))^2 = QF \left( x; T(\omega \circ q), T(\omega \circ q)y, \frac{1}{2} y^T T(\omega \circ q)y \right)$

5. $D(\omega \circ q) = \text{diag}(\omega \circ q) = D(\omega) D(q)$

6. $T(\omega \circ q) = (\omega \circ q)(\omega \circ q)^T = (\omega \omega^T) \circ q q^T = T(\omega) \circ T(q)$
Equity portfolio
Basic optimization problems

Mean-variance optimization

The long-only mean-variance optimization problem is given by:

\[ w^\star = \arg \min \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu \]

s.t. \[
\begin{align*}
1_n^\top w &= 1 \\
0_n &\leq w \leq 1_n
\end{align*}
\]

where:
- \( \gamma \) is the risk-tolerance coefficient
- the equality constraint is the budget constraint (\( \sum_{i=1}^n w_i = 1 \))
- the bounds correspond to the no short-selling restriction (\( w_i \geq 0 \))

**QP form**

\[ Q = \Sigma, \ R = \gamma \mu, \ A = 1_n^\top, \ B = 1, \ w^- = 0_n \text{ and } w^+ = 1 \]
Equity portfolio
Basic optimization problems

Tracking error optimization

The tracking error optimization problem is defined as:

\[ w^* = \arg \min w^\top \Sigma w - w^\top (\gamma \mu + \Sigma b) \]

s.t. \[ \begin{align*}
1^\top w &= 1 \\
0_n &\leq w \leq 1_n
\end{align*} \]

\[ Q = \Sigma, \quad R = \gamma \mu + \Sigma b, \quad A = 1_n^\top, \quad B = 1, \quad w^- = 0_n \text{ and } w^+ = 1 \]

\[ \Rightarrow \text{Portfolio replication: } R = \Sigma b \]
Specification of the constraints

Sector weight constraint

- We have
  \[ s_j^- \leq \sum_{i \in \text{Sector}_j} w_i \leq s_j^+ \]
  
- \( s_j \) is the \( n \times 1 \) sector-mapping vector: \( s_{i,j} = 1 \{ i \in \text{Sector}_j \} \)

- We notice that:
  \[ \sum_{i \in \text{Sector}_j} w_i = s_j^\top w \]

- We deduce that:
  \[ s_j^- \leq \sum_{i \in \text{Sector}_j} w_i \leq s_j^+ \iff \begin{cases} s_j^- \leq s_j^\top w \\ s_j^\top w \leq s_j^+ \end{cases} \iff \begin{cases} -s_j^\top w \leq -s_j^- \\ s_j^\top w \leq s_j^+ \end{cases} \]

**QP form**

\[ \begin{pmatrix} -s_j^\top \\ s_j \end{pmatrix} w \leq \begin{pmatrix} -s_j^- \\ s_j^+ \end{pmatrix} \]
General constraint:
\[ \sum_{i=1}^{n} w_i S_i \geq S^* \iff -S^\top w \leq -S^* \]

QP form
- \[ C = -S^\top \]
- \[ D = -S^* \]
Specification of the constraints
Score constraint

- Sector-specific constraint:

\[
\sum_{i \in \text{Sector}_j} w_i S_i \geq S_j^* \iff \sum_{i=1}^{n} 1 \{ i \in \text{Sector}_j \} \cdot w_i S_i \geq S_j^* \\
\iff \sum_{i=1}^{n} s_{i,j} w_i S_i \geq S_j^* \\
\iff \sum_{i=1}^{n} w_i \cdot (s_{i,j} S_i) \geq S_j^* \\
\iff (s_j \circ S)^\top w \geq S_j^*
\]

QP form

- \( C = - (s_j \circ S)^\top \)
- \( D = -S_j^* \)
Example #1

- The capitalization-weighted equity index is composed of 8 stocks.
- The weights are equal to 23%, 19%, 17%, 13%, 9%, 8%, 6% and 5%.
- The ESG score, carbon intensity and sector of the eight stocks are the following:

<table>
<thead>
<tr>
<th>Stock</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>-1.20</td>
<td>0.80</td>
<td>2.75</td>
<td>1.60</td>
<td>-2.75</td>
<td>-1.30</td>
<td>0.90</td>
<td>-1.70</td>
</tr>
<tr>
<td>$CT$</td>
<td>125</td>
<td>75</td>
<td>254</td>
<td>822</td>
<td>109</td>
<td>17</td>
<td>341</td>
<td>741</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Example #1 (Cont’d)

- The stock volatilities are equal to 22%, 20%, 25%, 18%, 35%, 23%, 13% and 29%
- The correlation matrix is given by:

\[
C = \begin{pmatrix}
100 & 100 \\
80 & 100 \\
70 & 75 & 100 \\
60 & 65 & 80 & 100 \\
70 & 50 & 70 & 85 & 100 \\
50 & 60 & 70 & 80 & 60 & 100 \\
70 & 50 & 70 & 75 & 80 & 50 & 100 \\
60 & 65 & 70 & 75 & 65 & 70 & 80 & 100
\end{pmatrix}
\]
We have:

$$w^* = \operatorname{arg\ min} \frac{1}{2} w^T Q w - w^T R$$

subject to

$$\begin{cases} Aw = B \\ Cw \leq D \\ w^- \leq w \leq w^+ \end{cases}$$
Equity portfolios
Objective function

Using $\Sigma_{i,j} = C_{i,j} \sigma_i \sigma_j$, we obtain:

$$Q = \Sigma = 10^{-4} \times$$

$$
\begin{pmatrix}
484.00 & 352.00 & 385.00 & 237.60 & 539.00 & 253.00 & 200.20 & 382.80 \\
352.00 & 400.00 & 375.00 & 234.00 & 350.00 & 276.00 & 130.00 & 377.00 \\
385.00 & 375.00 & 625.00 & 360.00 & 612.50 & 402.50 & 227.50 & 507.50 \\
237.60 & 234.00 & 360.00 & 324.00 & 535.50 & 331.20 & 175.50 & 391.50 \\
539.00 & 350.00 & 612.50 & 535.50 & 1225.00 & 483.00 & 364.00 & 659.75 \\
253.00 & 276.00 & 402.50 & 331.20 & 483.00 & 529.00 & 149.50 & 466.90 \\
200.20 & 130.00 & 227.50 & 175.50 & 364.00 & 149.50 & 169.00 & 301.60 \\
382.80 & 377.00 & 507.50 & 391.50 & 659.75 & 466.90 & 301.60 & 841.00 \\
\end{pmatrix}
$$
We have:

\[ R = \sum b = \begin{pmatrix} 3.74 \\ 3.31 \\ 4.39 \\ 3.07 \\ 5.68 \\ 3.40 \\ 2.02 \\ 4.54 \end{pmatrix} \times 10^{-2} \]
The portfolio is long-only

**QP form**

- \( w^- = 0_8 \)
- \( w^+ = 1_8 \)
The budget constraint \( \sum_{i=1}^{8} w_i = 1 \Rightarrow \) a first linear equation
\[ A_0 w = B_0 \]

QP form
- \( A_0 = 1_8^T \)
- \( B_0 = 1 \)
We can impose the sector neutrality of the portfolio meaning that:

$$\sum_{i \in \text{Sector}_j} w_i = \sum_{i \in \text{Sector}_j} b_i$$

The sector neutrality constraint can be written as:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} w = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

**QP form**

- $A_1 = s_1^T = (1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0)$
- $A_2 = s_2^T = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1)$
- $B_1 = s_1^T b = \sum_{i \in \text{Sector}_1} b_i$
- $B_2 = s_2^T b = \sum_{i \in \text{Sector}_2} b_i$
We can impose a relative reduction of the benchmark carbon intensity:

\[
CI(w) \leq (1 - R) CI(b) \iff C_1 w \leq D_1
\]

QP form

- \( C_1 = CI^T \) (because \( CI(w) = CI^T w \))
- \( D_1 = (1 - R) CI(b) \)

We can impose an absolute increase of the benchmark ESG score:

\[
S(w) \geq S(b) + \Delta S^*
\]

Since \( S(w) = S^T w \), we deduce that \( C_2 w \leq D_2 \)

QP form

- \( C_2 = -S^T \)
- \( D_2 = -(S(b) + \Delta S^*) \)
### Equity portfolios

#### Combination of constraints

<table>
<thead>
<tr>
<th>Set of constraints</th>
<th>Carbon intensity</th>
<th>ESG score</th>
<th>Sector neutrality</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_1$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>#2</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$C_2$</td>
<td>$D_2$</td>
</tr>
<tr>
<td>#3</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>$A_0$</td>
<td>$B_0$</td>
<td>$\begin{bmatrix} C_1 \ C_2 \end{bmatrix}$</td>
<td>$\begin{bmatrix} D_1 \ D_2 \end{bmatrix}$</td>
</tr>
<tr>
<td>#4</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>$\begin{bmatrix} A_0 \ A_1 \ A_2 \end{bmatrix}$</td>
<td>$\begin{bmatrix} B_0 \ B_1 \ B_2 \end{bmatrix}$</td>
<td>$\begin{bmatrix} C_1 \ C_2 \end{bmatrix}$</td>
<td>$\begin{bmatrix} D_1 \ D_2 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
**Table 167:** $\mathcal{R} = 30\%$ and $\Delta S^* = 0.50$ (Example #1)

<table>
<thead>
<tr>
<th>Weights (in %)</th>
<th>Benchmark</th>
<th>Set #1</th>
<th>Set #2</th>
<th>Set #3</th>
<th>Set #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>23.00</td>
<td>18.17</td>
<td>25.03</td>
<td>8.64</td>
<td>12.04</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>24.25</td>
<td>14.25</td>
<td>29.27</td>
<td>23.76</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>16.92</td>
<td>21.95</td>
<td>26.80</td>
<td>30.55</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>2.70</td>
<td>27.30</td>
<td>1.48</td>
<td>2.25</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>9.00</td>
<td>12.31</td>
<td>3.72</td>
<td>10.63</td>
<td>8.51</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>11.23</td>
<td>1.34</td>
<td>6.30</td>
<td>10.20</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>11.28</td>
<td>1.68</td>
<td>16.87</td>
<td>12.69</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>3.15</td>
<td>4.74</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Benchmark</th>
<th>Set #1</th>
<th>Set #2</th>
<th>Set #3</th>
<th>Set #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma (w^*</td>
<td>b)$ (in %)</td>
<td>0.00</td>
<td>0.50</td>
<td>1.18</td>
<td>1.90</td>
</tr>
<tr>
<td>$\mathcal{CI} (w^*)$</td>
<td>261.72</td>
<td>183.20</td>
<td>367.25</td>
<td>183.20</td>
<td>183.20</td>
</tr>
<tr>
<td>$\mathcal{R} (w^*</td>
<td>b)$ (in %)</td>
<td>30.00</td>
<td>-40.32</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>$S (w^*)$</td>
<td>0.17</td>
<td>0.05</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>$S (w^*) - S (b)$</td>
<td>-0.12</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$w^* (Sector_1)$ (in %)</td>
<td>57.00</td>
<td>66.00</td>
<td>44.67</td>
<td>65.41</td>
<td>57.00</td>
</tr>
<tr>
<td>$w^* (Sector_2)$ (in %)</td>
<td>43.00</td>
<td>34.00</td>
<td>55.33</td>
<td>34.59</td>
<td>43.00</td>
</tr>
</tbody>
</table>
The carbon intensity of the $j^{th}$ sector within the portfolio $w$ is:

$$CI(w; \text{Sector}_j) = \sum_{i \in \text{Sector}_j} \tilde{w}_i CI_i$$

where $\tilde{w}_i$ is the normalized weight in the sector bucket:

$$\tilde{w}_i = \frac{w_i}{\sum_{k \in \text{Sector}_j} w_k}$$

Another expression of $CI(w; \text{Sector}_j)$ is:

$$CI(w; \text{Sector}_j) = \frac{\sum_{i \in \text{Sector}_j} w_i CI_i}{\sum_{i \in \text{Sector}_j} w_i} = \frac{(s_j \circ CI)^\top w}{s_j^\top w}$$
If we consider the constraint $\mathbf{CI} (w; \text{Sector}_j) \leq \mathbf{CI}_j^\star$, we obtain:

\begin{align*}
\forall & \quad \Leftrightarrow \quad \mathbf{CI} (w; \text{Sector}_j) \leq \mathbf{CI}_j^\star \\
\Leftrightarrow & \quad (\mathbf{s}_j \circ \mathbf{CI})^\top w \leq \mathbf{CI}_j^\star (\mathbf{s}_j^\top w) \\
\Leftrightarrow & \quad ((\mathbf{s}_j \circ \mathbf{CI}) - \mathbf{CI}_j^\star \mathbf{s}_j)^\top w \leq 0 \\
\Leftrightarrow & \quad (\mathbf{s}_j \circ (\mathbf{CI} - \mathbf{CI}_j^\star))^\top w \leq 0
\end{align*}

QP form

\begin{itemize}
  \item $C = (\mathbf{s}_j \circ (\mathbf{CI} - \mathbf{CI}_j^\star))^\top$
  \item $D = 0$
\end{itemize}
Equity portfolios
Dealing with constraints on relative weights

Example #2

- Example #1
- We would like to reduce the carbon footprint of the benchmark by 30%
- We impose the sector neutrality
Equity portfolios
Dealing with constraints on relative weights

QP form

\[ A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \]

\[ B = \begin{pmatrix} 100 \% \\ 57 \% \\ 43 \% \end{pmatrix} \]

\[ C = \begin{pmatrix} 125 & 75 & 254 & 822 & 109 & 17 & 341 & 741 \end{pmatrix} \]

\[ D = 183.2040 \]
The optimal solution is:

\[ w^* = (21.54\%, 18.50\%, 21.15\%, 3.31\%, 10.02\%, 15.26\%, 6.94\%, 3.27\%) \]

- \( \sigma (w^* \mid b) = 112 \text{ bps} \)
- \( CI (w^*) = 183.20 \text{ vs. } CI (b) = 261.72 \)

**BUT**

\[
\begin{align*}
CI (w^*; Sector_1) &= 132.25 \\
CI (w^*; Sector_2) &= 250.74
\end{align*}
\]

\[
\begin{align*}
CI (b; Sector_1) &= 128.54 \\
CI (b; Sector_2) &= 438.26
\end{align*}
\]

The global reduction of 30% is explained by:

- an increase of 2.89% of the carbon footprint for the first sector
- a decrease of 42.79% of the carbon footprint for the second sector
We impose $R_1 = 20%$

**QP form**

\[
C = \begin{pmatrix}
  s_1 \circ (CI - (1 - R_1) CI(b; Sector_1)) \\
  125 & 75 & 254 & 822 & 109 & 17 & 341 & 741 \\
  22.1649 & -27.8351 & 0 & 0 & 6.1649 & 0 & 238.1649 & 0
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
  183.2040 \\
  0
\end{pmatrix}
\]
Solving the new QP problem gives the following optimal portfolio:

\[ w^* = (22.70\%, 22.67\%, 19.23\%, 5.67\%, 11.39\%, 14.50\%, 0.24\%, 3.61\%) \]

- \[ \sigma (w^* | b) = 144 \text{ bps} \]
- \[ CI (w^*) = 183.20 \]
- \[ CI (w^*; Sector_1) = 102.84 \text{ (reduction of } 20\%) \]
- \[ CI (w^*; Sector_2) = 289.74 \text{ (reduction of } 33.89\%) \]
Risk measure of a bond portfolio

- We consider a zero-coupon bond, whose price and maturity date are $B(t, T)$ and $T$:

$$B_t(t, T) = e^{-(r(t)+s(t))(T-t)+L(t)}$$

where $r(t)$, $s(t)$ and $L(t)$ are the interest rate, the credit spread and the liquidity premium.

- We deduce that:

$$\text{d} \ln B(t, T) = -(T-t) \text{d}r(t) - (T-t) \text{d}s(t) + \text{d}L(t)$$

$$= -D \text{d}r(t) - (D s(t)) \frac{\text{d}s(t)}{s(t)} + \text{d}L(t)$$

$$= -D \text{d}r(t) - DTS(t) \frac{\text{d}s(t)}{s(t)} + \text{d}L(t)$$

where:

- $D = T - t$ is the remaining maturity (or duration)
- $DTS(t)$ is the duration-times-spread factor
If we assume that $r(t)$, $s(t)$ and $L(t)$ are independent, the risk of the defaultable bond is equal to:

$$\sigma^2 (d \ln B(t, T)) = D^2 \sigma^2 (dr(t)) + DTS(t)^2 \sigma^2 \left( \frac{ds(t)}{s(t)} \right) + \sigma^2 (dL(t))$$

Three risk components

$$\sigma^2 (d \ln B(t, T)) = D^2 \sigma_r^2 + DTS(t)^2 \sigma_s^2 + \sigma_L^2$$

⇒ The historical volatility of a bond price is not a relevant risk measure
Bond portfolio optimization
Without a benchmark

- Duration risk:
  \[ MD(w) = \sum_{i=1}^{n} w_i \cdot MD_i \]

- DTS risk:
  \[ DTS(w) = \sum_{i=1}^{n} w_i \cdot DTS_i \]

- Clustering approach = generalization of the sector approach, e.g. (EUR, Financials, AAA to A-, 1Y-3Y)

- We have:
  \[ MD_j(w) = \sum_{i \in Sector_j} w_i \cdot MD_i \]

  and:
  \[ DTS_j(w) = \sum_{i \in Sector_j} w_i \cdot DTS_i \]
Objective function without a benchmark

We have:

\[ w^* = \arg \min_w \frac{\varphi_{MD}}{2} \sum_{j=1}^{n_{sector}} (MD_j(w) - MD_j^*)^2 + \]
\[ \frac{\varphi_{DTS}}{2} \sum_{j=1}^{n_{sector}} (DTS_j(w) - DTS_j^*)^2 - \gamma \sum_{i=1}^{n} w_i C_i \]

where:

- \( \varphi_{MD} \geq 0 \) and \( \varphi_{DTS} \geq 0 \) indicate the relative weight of each risk component
- \( C_i \) is the expected carry of bond \( i \) and \( \gamma \) is the risk-tolerance coefficient
Bond portfolio optimization
Without a benchmark

**QP form**

\[
\begin{align*}
    w^* &= \arg \min QF(w; Q, R, c) \\
    \text{s.t.} \quad &
    \begin{cases}
        1_n^T w = 1 \\
        0_n \leq w \leq 1_n
    \end{cases}
\end{align*}
\]

where \( QF(w; Q, R, c) \) is the quadratic form of the objective function
We have:

\[
\frac{1}{2} \left( \text{MD}_j (w) - \text{MD}^*_j \right)^2 = \frac{1}{2} \left( \sum_{i \in \text{Sector}_j} w_i \text{MD}_i - \text{MD}^*_j \right)^2 \\
= \frac{1}{2} \left( \sum_{i=1}^{n} s_{i,j} w_i \text{MD}_i - \text{MD}^*_j \right)^2 \\
= \frac{1}{2} \left( \sum_{i=1}^{n} s_{i,j} \text{MD}_i w_i \right)^2 - w^\top (s_j \circ \text{MD}) \text{MD}^*_j + \frac{1}{2} \text{MD}^*_j^2 \\
= \mathcal{QF} \left( w; \mathcal{T} (s_j \circ \text{MD}) , (s_j \circ \text{MD}) \text{MD}^*_j , \frac{1}{2} \text{MD}^*_j^2 \right)
\]

where \( \text{MD} = (\text{MD}_1, \ldots, \text{MD}_n) \) is the vector of modified durations and \( \mathcal{T} (u) = uu^\top \).
We deduce that:

\[
\frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} (\text{MD}_j(w) - \text{MD}_j^*)^2 = QF(w; Q_{\text{MD}}, R_{\text{MD}}, c_{\text{MD}})
\]

where:

\[
\begin{align*}
Q_{\text{MD}} &= \sum_{j=1}^{n_{\text{sector}}} T(s_j \circ \text{MD}) \\
R_{\text{MD}} &= \sum_{j=1}^{n_{\text{sector}}} (s_j \circ \text{MD}) \text{MD}_j^* \\
c_{\text{MD}} &= \frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} \text{MD}_j^2
\end{align*}
\]
In a similar way, we have:

\[
\frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} (DTS_j(w) - DTS_j^*)^2 = QF(w; Q_{DTS}, R_{DTS}, c_{DTS})
\]

where:

\[
\begin{cases}
Q_{DTS} = \sum_{j=1}^{n_{\text{sector}}} T(s_j \circ DTS) \\
R_{MD} = \sum_{j=1}^{n_{\text{sector}}} (s_j \circ DTS) DTS_j^* \\
c_{DTS} = \frac{1}{2} \sum_{j=1}^{n_{\text{sector}}} DTS_j^{*2}
\end{cases}
\]
Bond portfolio optimization
Without a benchmark

We have:

\[-\gamma \sum_{i=1}^{n} w_i C_i = \gamma QF(w; 0_{n,n}, C, 0) = QF(w; 0_{n,n}, \gamma C, 0)\]

where $C = (C_1, \ldots, C_n)$ is the vector of expected carry values
Bond portfolio optimization
Without a benchmark

Quadratic form of the objective function

The function to optimize is:

\[ QF(w; Q, R, c) = \varphi_{MD} QF(w; Q_{MD}, R_{MD}, c_{MD}) + \]
\[ \varphi_{DTS} QF(w; Q_{DTS}, R_{DTS}, c_{DTS}) + \]
\[ QF(w; 0_{n,n}, \gamma C, 0) \]

where:

\[
\begin{aligned}
Q &= \varphi_{MD} Q_{MD} + \varphi_{DTS} Q_{DTS} \\
R &= \gamma C + \varphi_{MD} R_{MD} + \varphi_{DTS} R_{DTS} \\
c &= \varphi_{MD} c_{MD} + \varphi_{DTS} c_{DTS}
\end{aligned}
\]
The MD- and DTS-based tracking error variances are equal to:

\[ R_{MD} (w \mid b) = \sigma_{MD}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in S_{ector_j}} (w_i - b_i) MD_i \right)^2 \]

and:

\[ R_{DTS} (w \mid b) = \sigma_{DTS}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in S_{ector_j}} (w_i - b_i) DTS_i \right)^2 \]

This means that \( MD_j^* = \sum_{i \in S_{ector_j}} b_i \) MD and \( DTS_j^* = \sum_{i \in S_{ector_j}} b_i \) DTS.

The active share risk is defined as:

\[ R_{AS} (w \mid b) = \sigma_{AS}^2 (w \mid b) = \sum_{i=1}^{n} (w_i - b_i)^2 \]
Objective function with a benchmark

The optimization problem becomes:

$$w^* = \arg \min \frac{1}{2} \mathcal{R}(w \mid b) - \gamma \sum_{i=1}^{n} (w_i - b_i) C_i$$

s.t. \[ \left\{ \begin{array}{l} 1_n^\top w = 1 \\ 0_n \leq w \leq 1_n \end{array} \right. \]

where the synthetic risk measure is equal to:

$$\mathcal{R}(w \mid b) = \varphi_{AS} \mathcal{R}_{AS}(w \mid b) + \varphi_{MD} \mathcal{R}_{MD}(w \mid b) + \varphi_{DTS} \mathcal{R}_{DTS}(w \mid b)$$
We can show that

\[ w^{*} = \text{arg min } QF (w ; Q (b) , R (b) , c (b)) \]

s.t. \[
\begin{cases}
1^T w = 1 \\
0_n \leq w \leq 1_n
\end{cases}
\]

where:

\[
\begin{align*}
Q (b) &= \varphi_{AS} Q_{AS} (b) + \varphi_{MD} Q_{MD} (b) + \varphi_{DTS} Q_{DTS} (b) \\
R (b) &= \gamma C + \varphi_{AS} R_{AS} (b) + \varphi_{MD} R_{MD} (b) + \varphi_{DTS} R_{DTS} (b) \\
c (b) &= \gamma b^T C + \varphi_{AS} c_{AS} (b) + \varphi_{MD} c_{MD} (b) + \varphi_{DTS} c_{DTS} (b)
\end{align*}
\]

\[
\begin{align*}
Q_{AS} (b) &= I_n, \quad R_{AS} (b) = b, \quad c_{AS} (b) = \frac{1}{2} b^T b, \quad Q_{MD} (b) = Q_{MD}, \\
R_{MD} (b) &= Q_{MD} b = R_{MD}, \quad c_{MD} (b) = \frac{1}{2} b^T Q_{MD} b = c_{MD}, \\
Q_{DTS} (b) &= Q_{DTS}, \quad R_{DTS} (b) = Q_{DTS} b = R_{DTS}, \text{ and} \\
c_{DTS} (b) &= \frac{1}{2} b^T Q_{DTS} b = c_{DTS}
\end{align*}
\]
Example #3

We consider an investment universe of 9 corporate bonds with the following characteristics:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$</td>
<td>21</td>
<td>19</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$CI_i$</td>
<td>111</td>
<td>52</td>
<td>369</td>
<td>157</td>
<td>18</td>
<td>415</td>
<td>17</td>
<td>253</td>
<td>900</td>
</tr>
<tr>
<td>$MD_i$</td>
<td>3.16</td>
<td>6.48</td>
<td>3.54</td>
<td>9.23</td>
<td>6.40</td>
<td>2.30</td>
<td>8.12</td>
<td>7.96</td>
<td>5.48</td>
</tr>
<tr>
<td>$DTS_i$</td>
<td>107</td>
<td>255</td>
<td>75</td>
<td>996</td>
<td>289</td>
<td>45</td>
<td>620</td>
<td>285</td>
<td>125</td>
</tr>
<tr>
<td>$Sector$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

We impose that $0.25 \times b_i \leq w_i \leq 4 \times b_i$. We have $\varphi_{AS} = 100$, $\varphi_{MD} = 25$ and $\varphi_{DTS} = 0.001$.

\(^a\)The units are: $b_i$ in %, $CI_i$ in tCO\(_2\)e/$ mn, MD\(_i\) in years and DTS\(_i\) in bps
The optimization problem is defined as:

\[
 w^\star (\mathcal{R}) = \arg\min \frac{1}{2} w^\top Q (b) w - w^\top R (b)
\]

s.t. \[
\begin{align*}
1_9^\top w &= 1 \\
\mathcal{C}^\top w &\leq (1 - \mathcal{R}) \mathcal{C}^\top (b) \\
b &\leq w \leq 4b \\
\frac{b}{4} &\leq w \leq 4b
\end{align*}
\]

where \( \mathcal{R} \) is the reduction rate
Bond portfolio optimization
With a benchmark

Since the bonds are ordering by sectors, \( Q(b) \) is a block diagonal matrix:

\[
Q(b) = \begin{pmatrix}
Q_1 & 0_{3\times3} & 0_{3\times3} \\
0_{3\times3} & Q_2 & 0_{3\times3} \\
0_{3\times3} & 0_{3\times3} & Q_3
\end{pmatrix} \times 10^3
\]

where:

\[
Q_1 = \begin{pmatrix}
0.3611 & 0.5392 & 0.2877 \\
0.5392 & 1.2148 & 0.5926 \\
0.2877 & 0.5926 & 0.4189
\end{pmatrix}, \quad Q_2 = \begin{pmatrix}
3.2218 & 1.7646 & 0.5755 \\
1.7646 & 1.2075 & 0.3810 \\
0.5755 & 0.3810 & 0.2343
\end{pmatrix}
\]

and:

\[
Q_3 = \begin{pmatrix}
2.1328 & 1.7926 & 1.1899 \\
1.7926 & 1.7653 & 1.1261 \\
1.1899 & 1.1261 & 0.8664
\end{pmatrix}
\]

\[
R(b) = (2.243, 4.389, 2.400, 6.268, 3.751, 1.297, 2.354, 2.120, 1.424) \times 10^2
\]
### Table 168: Weights in % of optimized bond portfolios (Example #3)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>21.00</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>6.00</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>$w^*$ (10%)</td>
<td>21.92</td>
<td>19.01</td>
<td>15.53</td>
<td>11.72</td>
<td>11.68</td>
<td>7.82</td>
<td>6.68</td>
<td>4.71</td>
<td>0.94</td>
</tr>
<tr>
<td>$w^*$ (30%)</td>
<td>26.29</td>
<td>20.24</td>
<td>10.90</td>
<td>10.24</td>
<td>16.13</td>
<td>3.74</td>
<td>9.21</td>
<td>2.50</td>
<td>0.75</td>
</tr>
<tr>
<td>$w^*$ (50%)</td>
<td>27.48</td>
<td>23.97</td>
<td>4.00</td>
<td>6.94</td>
<td>22.70</td>
<td>2.00</td>
<td>11.15</td>
<td>1.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

### Table 169: Risk statistics of optimized bond portfolios (Example #3)

| Portfolio | $\text{AS}_{\text{sector}}$ (in %) | MD ($w$) (in years) | DTS ($w$) (in bps) | $\sigma_{\text{AS}} (w | b)$ (in %) | $\sigma_{\text{MD}} (w | b)$ (in years) | $\sigma_{\text{DTS}} (w | b)$ (in bps) | $CI (w)$ gCO$_2$e/$\$ |
|-----------|-----------------------------------|---------------------|-------------------|-------------------------------|---------------------------------|---------------------------------|---------------------|
| $b$       | 0.00                             | 5.43                | 290.18            | 0.00                          | 0.00                            | 0.00                            | 184.39              |
| $w^*$ (10%) | 3.00                           | 5.45                | 293.53            | 2.62                          | 0.02                            | 3.80                            | 165.95              |
| $w^*$ (30%) | 14.87                          | 5.58                | 303.36            | 10.98                         | 0.10                            | 14.49                           | 129.07              |
| $w^*$ (50%) | 28.31                          | 5.73                | 302.14            | 21.21                         | 0.19                            | 30.11                           | 92.19               |
Figure 335: Relationship between the reduction rate and the tracking risk (Example #3)
Advanced optimization problems
Large bond universe

- QP: $n \leq 5,000$ (the dimension of $Q$ is $n \times n$)
- LP: $n \gg 10^6$

Some figures as of 31/01/2023
- MSCI World Index (DM): $n = 1,508$ stocks
- MSCI World IMI (DM): $n = 5,942$ stocks
- MSCI World AC (DM + EM): $n = 2,882$ stocks
- MSCI World AC IMI (DM + EM): $n = 7,928$ stocks
- Bloomberg Global Aggregate Total Return Index: $n = 28,799$ securities
- ICE BOFA Global Broad Market Index: $n = 33,575$ securities

- Trick: $\mathcal{L}_2$-norm risk measures $\Rightarrow \mathcal{L}_1$-norm risk measures
We replace the synthetic risk measure by:

\[ D(w \mid b) = \varphi'_{AS} D_{AS}(w \mid b) + \varphi'_{MD} D_{MD}(w \mid b) + \varphi'_{DTS} D_{DTS}(w \mid b) \]

where:

\[ D_{AS}(w \mid b) = \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| \]

\[ D_{MD}(w \mid b) = \sum_{j=1}^{n_{sector}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \right| \]

\[ D_{DTS}(w \mid b) = \sum_{j=1}^{n_{sector}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \right| \]
Advanced optimization problems
Large bond universe

The optimization problem becomes:

\[ w^* = \arg\min \quad \mathcal{D}(w \mid b) - \gamma \sum_{i=1}^{n} (w_i - b_i) C_i \]

s.t. \[ \begin{cases} 1_n^T w = 1 \\ 0_n \leq w \leq 1_n \end{cases} \]
**Absolute value trick**

If $c_i \geq 0$, then:

$$\min \sum_{i=1}^{n} c_i |f_i(x)| + g(x) \iff \begin{cases} 
\min \sum_{i=1}^{n} c_i \tau_i + g(x) \\
\text{s.t.} \begin{cases} 
|f_i(x)| \leq \tau_i \\
\tau_i \geq 0
\end{cases}
\end{cases}$$

The problem becomes linear:

$$|f_i(x)| \leq \tau_i \iff -\tau_i \leq f_i(x) \land f_i(x) \leq \tau_i$$
Advanced optimization problems

Large bond universe

Linear programming

The standard formulation of a linear programming problem is:

\[ x^* = \arg \min_{x} c^\top x \]
\[ \text{s.t.} \begin{cases} Ax = b \\ Cx \leq D \\ x^- \leq x \leq x^+ \end{cases} \]

where \( x \) is a \( n \times 1 \) vector, \( c \) is a \( n \times 1 \) vector, \( A \) is a \( n_A \times n \) matrix, \( B \) is a \( n_A \times 1 \) vector, \( C \) is a \( n_C \times n \) matrix, \( D \) is a \( n_C \times 1 \) vector, and \( x^- \) and \( x^+ \) are two \( n \times 1 \) vectors.
Advanced optimization problems

Large bond universe

We have:

\[
w^* = \arg\min w \in \mathbb{R}^n \frac{1}{2} \varphi'_{AS} \sum_{i=1}^{n} \tau_{i,w} + \varphi'_{MD} \sum_{j=1}^{n_{sector}} \tau_{j,MD} + \varphi'_{DTS} \sum_{j=1}^{n_{sector}} \tau_{j,DTS} -
\]

\[
\gamma \sum_{i=1}^{n} (w_i - b_i) C_i
\]

\[
\begin{cases}
1^T_n w = 1 \\
0_n \leq w \leq 1_n \\
|w_i - b_i| \leq \tau_{i,w} \\
\left|\sum_{i \in S_{sector_j}} (w_i - b_i) MD_i\right| \leq \tau_{j,MD} \\
\left|\sum_{i \in S_{sector_j}} (w_i - b_i) DTS_i\right| \leq \tau_{j,DTS} \\
\tau_{i,w} \geq 0, \tau_{j,MD} \geq 0, \tau_{j,DTS} \geq 0
\end{cases}
\]
Advanced optimization problems

Large bond universe

\[ |w_i - b_i| \leq \tau_{i,w} \iff \begin{cases} w_i - \tau_{i,w} \leq b_i \\ -w_i - \tau_{i,w} \leq -b_i \end{cases} \]
Advanced optimization problems

Large bond universe

\[(*) \iff \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \cdot \text{MD}_i \right| \leq \tau_{j,\text{MD}} \]

\[\iff -\tau_{j,\text{MD}} \leq \sum_{i \in \text{Sector}_j} (w_i - b_i) \cdot \text{MD}_i \leq \tau_{j,\text{MD}} \]

\[\iff -\tau_{j,\text{MD}} + \sum_{i \in \text{Sector}_j} b_i \cdot \text{MD}_i \leq \sum_{i \in \text{Sector}_j} w_i \cdot \text{MD}_i \leq \tau_{j,\text{MD}} + \sum_{i \in \text{Sector}_j} b_i \cdot \text{MD}_i \]

\[\iff -\tau_{j,\text{MD}} + \text{MD}_j^* \leq (s_j \circ \text{MD})^T w \leq \tau_{j,\text{MD}} + \text{MD}_j^* \]

\[\iff \begin{cases} (s_j \circ \text{MD})^T w - \tau_{j,\text{MD}} \leq \text{MD}_j^* \\ - (s_j \circ \text{MD})^T w - \tau_{j,\text{MD}} \leq -\text{MD}_j^* \end{cases} \]
Advanced optimization problems

Large bond universe

\[ \sum_{i \in \text{Sector}_j} (w_i - b_i) DTS_i \leq \tau_{j,\text{DTS}} \Leftrightarrow \left\{ \begin{array}{l} (s_j \circ DTS)^	op w - \tau_{j,\text{DTS}} \leq DTS_j^* \\ -(s_j \circ DTS)^	op w - \tau_{j,\text{DTS}} \leq -DTS_j^* \end{array} \right. \]
Advanced optimization problems

LP formulation

- $x$ is a vector of dimension $n_x = 2 \times (n + n_{sector})$:

\[
x = \begin{pmatrix}
  w \\
  \tau_w \\
  \tau_{MD} \\
  \tau_{DTS}
\end{pmatrix}
\]
The vector $c$ is equal to:

$$
c = \begin{pmatrix}
-\gamma C \\
\frac{1}{2} \varphi_{AS}^T \mathbf{1}_n \\
\varphi_{MD}^T \mathbf{1}_{n_{Sector}} \\
\varphi_{DTS}^T \mathbf{1}_{n_{Sector}}
\end{pmatrix}
$$
The linear equality constraint $Ax = B$ is defined by:

$$A = \begin{pmatrix} 1_n \top & 0_n \top & 0_{nSectors} \top & 0_{nSectors} \top \end{pmatrix}$$

and:

$$B = 1$$
The linear inequality constraint $Cx \leq D$ is defined by:

$$C = \begin{pmatrix}
  I_n & -I_n & 0_n, n_{\text{sector}} & 0_n, n_{\text{sector}} \\
  -I_n & -I_n & 0_n, n_{\text{sector}} & 0_n, n_{\text{sector}} \\
  C_{\text{MD}} & 0_{n_{\text{sector}}, n} & -I_n, n_{\text{sector}} & -I_n, n_{\text{sector}} \\
  -C_{\text{MD}} & 0_{n_{\text{sector}}, n} & -I_n, n_{\text{sector}} & -I_n, n_{\text{sector}} \\
  C_{\text{DTS}} & 0_{n_{\text{sector}}, n} & 0_{n_{\text{sector}}, n_{\text{sector}}} & 0_{n_{\text{sector}}, n_{\text{sector}}} \\
  -C_{\text{DTS}} & 0_{n_{\text{sector}}, n} & 0_{n_{\text{sector}}, n_{\text{sector}}} & 0_{n_{\text{sector}}, n_{\text{sector}}}
\end{pmatrix}$$

$$D = \begin{pmatrix}
  b \\
  -b \\
  \text{MD}^* \\
  -\text{MD}^* \\
  \text{DTS}^* \\
  -\text{DTS}^*
\end{pmatrix}$$
Advanced optimization problems

LP formulation

- $C_{MD}$ and $C_{DTS}$ are two $n_{Sector} \times n$ matrices, whose elements are:
  \[(C_{MD})_{j,i} = s_{i,j} \text{MD}_i\]
  and:
  \[(C_{DTS})_{j,i} = s_{i,j} \text{DTS}_i\]
- We have:
  \[\text{MD}^* = (\text{MD}_1^*, \ldots, \text{MD}_{n_{Sector}}^*)\]
  and
  \[\text{DTS}^* = (\text{DTS}_1^*, \ldots, \text{DTS}_{n_{Sector}}^*)\]
Advanced optimization problems

LP formulation

- The bounds are:
  \[ x^- = 0_{n_x} \]

  and:
  \[ x^+ = \infty \cdot 1_{n_x} \]
Advanced optimization problems

LP formulation

Additional constraints:

\[
\begin{align*}
A'w &= B' \\
C'w &\leq D'
\end{align*}
\quad \iff 
\begin{align*}
\begin{pmatrix}
A' & 0_{n_A,n_x-n}
\end{pmatrix}x &= B' \\
\begin{pmatrix}
C' & 0_{n_A,n_x-n}
\end{pmatrix}x &\leq D'
\end{align*}
\]
Advanced optimization problems
Large bond universe

Toy example

We consider a toy example with four corporate bonds:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$ (in %)</td>
<td>35</td>
<td>15</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$CI_i$ (in tCO$_2$e/$ mn)</td>
<td>117</td>
<td>284</td>
<td>162.5</td>
<td>359</td>
</tr>
<tr>
<td>MD$_i$ (in years)</td>
<td>3.0</td>
<td>5.0</td>
<td>2.0</td>
<td>6.0</td>
</tr>
<tr>
<td>DTS$_i$ (in bps)</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

We would like to reduce the carbon footprint by 20%, and we set $\varphi^{'}_{AS} = 100$, $\varphi^{'}_{MD} = 25$ and $\varphi^{'}_{DTS} = 1$.
We have \( n = 4 \), \( n_{\text{sector}} = 2 \) and:

\[
\mathbf{x} = \begin{pmatrix}
  w_1, w_2, w_3, w_4, \\
  \tau_{W_1}, \tau_{W_2}, \tau_{W_3}, \tau_{W_4}, \\
  \tau_{MD_1}, \tau_{MD_2}, \\
  \tau_{DTS_1}, \tau_{DTS_2}
\end{pmatrix}
\]

Since the vector \( C \) is equal to \( \mathbf{0}_4 \), we obtain:

\[
c = (0, 0, 0, 0, 50, 50, 50, 50, 25, 25, 1, 1)
\]
The equality system $A x = B$ is defined by:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and:

$$B = 1$$
The inequality system \( Cx \leq D \) is given by:

\[
C = \begin{pmatrix}
3 & 5 & 0 & 0 & -l_4 & -l_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 6 & 0 & 0 & -1 & 0 & 0 & 0 \\
-3 & -5 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -2 & -6 & 0 & 0 & 0 & -1 & 0 & 0 \\
100 & 150 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 200 & 250 & 0 & 0 & 0 & 0 & 0 & -1 \\
-100 & -150 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -200 & -250 & 0 & 0 & 0 & 0 & 0 & -1 \\
117 & 284 & 162.5 & 359 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

and:

\[
D = (0.35, 0.15, 0.2, 0.3, -0.35, -0.15, -0.2, -0.3, \ldots, 1.8, 2.2, -1.8, -2.2, 57.5, 115, -57.5, -115, 179)
\]
The last row of $Cx \leq D$ corresponds to the carbon footprint constraint.

We have:

$$C\mathcal{I}(b) = 223.75 \text{ tCO}_2\text{e}/\text{ mn}$$

and:

$$(1 - R)C\mathcal{I}(b) = 0.80 \times 223.75 = 179.00 \text{ tCO}_2\text{e}/\text{ mn}$$
We solve the LP program, and we obtain the following solution:

\[
\begin{align*}
    \mathbf{w}^* &= (47.34\%, 0\%, 33.3\%, 19.36\%) \\
    \mathbf{\tau}_{w^*} &= (12.34\%, 15\%, 13.3\%, 10.64\%) \\
    \mathbf{\tau}_{MD}^* &= (0.3798, 0.3725) \\
    \mathbf{\tau}_{DTS}^* &= (10.1604, 0)
\end{align*}
\]
**Advanced optimization problems**

**Large bond universe**

- **Interpretation of** $\tau_w^*$:

\[
\begin{align*}
  w^* \pm \tau_w^* &= \begin{pmatrix}
    47.34\% \\
    0.00\% \\
    33.30\% \\
    19.36\%
  \end{pmatrix}
  \begin{pmatrix}
    - \\
    + \\
    - \\
    +
  \end{pmatrix}
  \begin{pmatrix}
    12.34\% \\
    15.00\% \\
    13.30\% \\
    10.64\%
  \end{pmatrix}
  = \begin{pmatrix}
    35\% \\
    15\% \\
    20\% \\
    30\%
  \end{pmatrix} = b
\end{align*}
\]

- **Interpretation of** $\tau_{MD}^*$:

\[
\left( \sum_{i \in \text{Sector}_1} w_i^* \text{MD}_i \right) \pm \tau_{MD}^* = \begin{pmatrix}
  1.42 \\
  1.83
\end{pmatrix}
  \begin{pmatrix}
    + \\
    +
  \end{pmatrix}
  \begin{pmatrix}
    0.38 \\
    0.37
  \end{pmatrix}
  = \begin{pmatrix}
    1.80 \\
    2.20
  \end{pmatrix} = \begin{pmatrix}
    \text{MD}_1^* \\
    \text{MD}_2^*
  \end{pmatrix}
\]

- **Interpretation of** $\tau_{DTS}^*$:

\[
\left( \sum_{i \in \text{Sector}_1} w_i^* \text{DTS}_i \right) \pm \tau_{DTS}^* = \begin{pmatrix}
  47.34 \\
  115.00
\end{pmatrix}
  \begin{pmatrix}
    + \\
    +
  \end{pmatrix}
  \begin{pmatrix}
    10.16 \\
    0.00
  \end{pmatrix}
  = \begin{pmatrix}
    57.50 \\
    115.00
  \end{pmatrix} = \begin{pmatrix}
    \text{DTS}_1^* \\
    \text{DTS}_2^*
  \end{pmatrix}
\]
Advanced optimization problems
Large bond universe

Example #4 (Example #3 again)

We consider an investment universe of 9 corporate bonds with the following characteristics:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$</td>
<td>21</td>
<td>19</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$CI_i$</td>
<td>111</td>
<td>52</td>
<td>369</td>
<td>157</td>
<td>18</td>
<td>415</td>
<td>17</td>
<td>253</td>
<td>900</td>
</tr>
<tr>
<td>MD$_i$</td>
<td>3.16</td>
<td>6.48</td>
<td>3.54</td>
<td>9.23</td>
<td>6.40</td>
<td>2.30</td>
<td>8.12</td>
<td>7.96</td>
<td>5.48</td>
</tr>
<tr>
<td>DTS$_i$</td>
<td>107</td>
<td>255</td>
<td>75</td>
<td>996</td>
<td>289</td>
<td>45</td>
<td>620</td>
<td>285</td>
<td>125</td>
</tr>
<tr>
<td>Sector</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

We impose that $0.25 \times b_i \leq w_i \leq 4 \times b_i$ and assume that

$\varphi'_{AS} = \varphi_{AS} = 100$, $\varphi'_{MD} = \varphi_{MD} = 25$ and $\varphi'_{DTS} = \varphi_{DTS} = 0.001$

$^a$The units are: $b_i$ in %, $CI_i$ in tCO$_2$e/$mn$, MD$_i$ in years and DTS$_i$ in bps
Table 170: Weights in % of optimized bond portfolios (Example #4)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>21.00</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>6.00</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>$w^* (10%)$</td>
<td>21.70</td>
<td>19.00</td>
<td>16.00</td>
<td>12.00</td>
<td>11.00</td>
<td>8.00</td>
<td>7.46</td>
<td>4.00</td>
<td>0.84</td>
</tr>
<tr>
<td>$w^* (30%)$</td>
<td>34.44</td>
<td>19.00</td>
<td>4.00</td>
<td>11.65</td>
<td>11.98</td>
<td>6.65</td>
<td>7.52</td>
<td>4.00</td>
<td>0.75</td>
</tr>
<tr>
<td>$w^* (50%)$</td>
<td>33.69</td>
<td>19.37</td>
<td>4.00</td>
<td>3.91</td>
<td>24.82</td>
<td>2.00</td>
<td>10.46</td>
<td>1.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 171: Risk statistics of optimized bond portfolios (Example #4)

| Portfolio | $A_{S_{sector}}$ (in %) | MD (w) (in years) | DTS (w) (in bps) | $\sigma_{AS} (w | b)$ (in %) | $\sigma_{MD} (w | b)$ (in years) | $\sigma_{DTS} (w | b)$ (in bps) | $C_I (w)$ gCO2e/$ |
|-----------|--------------------------|-------------------|-----------------|-----------------|-----------------|-----------------|----------------|
| $b$       | 0.00                     | 5.43              | 290.18          | 0.00            | 0.00            | 0.00            | 184.39         |
| $w^* (10\%)$ | 2.16                     | 5.45              | 297.28          | 2.16            | 0.02            | 7.10            | 165.95         |
| $w^* (30\%)$ | 15.95                    | 5.43              | 300.96          | 15.95           | 0.00            | 13.20           | 129.07         |
| $w^* (50\%)$ | 31.34                    | 5.43              | 268.66          | 31.34           | 0.00            | 65.12           | 92.19          |
Threshold approach

The optimization problem is:

\[
\begin{align*}
    w^* & = \arg \min \frac{1}{2} (w - b)^\top \Sigma (w - b) \\
    \text{s.t.} & \quad 1_n^\top w = 1 \\
    & \quad w \in \Omega \\
    & \quad 0_n \leq w \leq 1_n \\
    & \quad CI(w) \leq (1 - R) CI(b)
\end{align*}
\]
Equity portfolios

Order-statistic approach

- $C\mathcal{I}_{i:n}$ is the order statistics of $(C\mathcal{I}_1, \ldots, C\mathcal{I}_n)$:

  \[
  \min C\mathcal{I}_i = C\mathcal{I}_{1:n} \leq C\mathcal{I}_{2:n} \leq \cdots \leq C\mathcal{I}_{i:n} \leq \cdots \leq C\mathcal{I}_{n:n} = \max C\mathcal{I}_i
  \]

- The carbon intensity bound $C\mathcal{I}^{(m,n)}$ is defined as:

  \[
  C\mathcal{I}^{(m,n)} = C\mathcal{I}_{n-m+1:n}
  \]

  where $C\mathcal{I}_{n-m+1:n}$ is the $(n - m + 1)$-th order statistic of $(C\mathcal{I}_1, \ldots, C\mathcal{I}_n)$

- Exclusion process:

  \[
  C\mathcal{I}_i \geq C\mathcal{I}^{(m,n)} \Rightarrow w_i = 0
  \]
Order-statistic approach (Cont’d)

The optimization problem is:

\[ w^* = \arg\min \frac{1}{2} (w - b)^\top \Sigma (w - b) \]

s.t. \[
\begin{align*}
1_n^\top w &= 1 \\
ne w &\in \Omega \\
0_n &\leq w \leq 1 \\
\{ CI &< CI^{(m,n)} \}
\end{align*}
\]
Naive approach

We re-weight the remaining assets:

\[
    w_i^* = \frac{1 \left\{ CI_i < CI^{(m,n)} \right\} \cdot b_i}{\sum_{k=1}^{n} 1 \left\{ CI_k < CI^{(m,n)} \right\} \cdot b_k}
\]
Example #5

We consider a capitalization-weighted equity index, which is composed of eight stocks. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in tCO$_2$e/$\text{mn}$) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the market one-factor model: the beta $\beta_i$ of each stock is equal to 0.30, 1.80, 0.85, 0.83, 1.47, 0.94, 1.67 and 1.08, the idiosyncratic volatilities $\tilde{\sigma}_i$ are respectively equal to 10%, 5%, 6%, 12%, 15%, 4%, 8% and 7%, and the estimated market volatility $\sigma_m$ is 18%.
The covariance matrix is:

\[ \Sigma = \beta \beta^T \sigma_m^2 + D \]

where:

- \( \beta \) is the vector of beta coefficients
- \( \sigma_m^2 \) is the variance of the market portfolio
- \( D = \text{diag} (\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_n^2) \) is the diagonal matrix of idiosyncratic variances
Table 172: Optimal decarbonization portfolios (Example #5, threshold approach)

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>$CI_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>20.54</td>
<td>21.14</td>
<td>21.86</td>
<td>22.58</td>
<td>22.96</td>
<td>100.5</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>19.33</td>
<td>19.29</td>
<td>18.70</td>
<td>18.11</td>
<td>17.23</td>
<td>97.2</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>15.67</td>
<td>12.91</td>
<td>8.06</td>
<td>3.22</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>12.28</td>
<td>10.95</td>
<td>8.74</td>
<td>6.53</td>
<td>3.36</td>
<td>352.3</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>12.26</td>
<td>12.60</td>
<td>13.07</td>
<td>13.53</td>
<td>14.08</td>
<td>27.1</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>11.71</td>
<td>16.42</td>
<td>22.57</td>
<td>28.73</td>
<td>34.77</td>
<td>54.2</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>6.36</td>
<td>6.69</td>
<td>7.00</td>
<td>7.30</td>
<td>7.59</td>
<td>78.6</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>1.86</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
<tr>
<td>$\sigma(w^*</td>
<td>b)$</td>
<td>0.00</td>
<td>30.01</td>
<td>61.90</td>
<td>104.10</td>
<td>149.65</td>
<td>196.87</td>
</tr>
<tr>
<td>$CI(w)$</td>
<td>160.57</td>
<td>144.52</td>
<td>128.46</td>
<td>112.40</td>
<td>96.34</td>
<td>80.29</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{R}(w</td>
<td>b)$</td>
<td>0.00</td>
<td>10.00</td>
<td>20.00</td>
<td>30.00</td>
<td>40.00</td>
<td>50.00</td>
</tr>
</tbody>
</table>

The reduction rate and the weights are expressed in % whereas the tracking error volatility is measured in bps.
**Table 173:** Optimal decarbonization portfolios (Example #5, order-statistic approach)

<table>
<thead>
<tr>
<th>( w^*_m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>( C\mathcal{I}_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w^*_1 )</td>
<td>20.00</td>
<td>20.40</td>
<td>22.35</td>
<td>26.46</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>( w^*_2 )</td>
<td>19.00</td>
<td>19.90</td>
<td>20.07</td>
<td>20.83</td>
<td>7.57</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>( w^*_3 )</td>
<td>17.00</td>
<td>17.94</td>
<td>21.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>( w^*_4 )</td>
<td>13.00</td>
<td>13.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>( w^*_5 )</td>
<td>12.00</td>
<td>12.12</td>
<td>12.32</td>
<td>12.79</td>
<td>13.04</td>
<td>14.26</td>
<td>18.78</td>
<td>100.00</td>
<td>27.1</td>
</tr>
<tr>
<td>( w^*_6 )</td>
<td>8.00</td>
<td>10.04</td>
<td>17.14</td>
<td>32.38</td>
<td>74.66</td>
<td>75.12</td>
<td>81.22</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>( w^*_7 )</td>
<td>6.00</td>
<td>6.37</td>
<td>6.70</td>
<td>7.53</td>
<td>4.73</td>
<td>10.62</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>( w^*_8 )</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
</tbody>
</table>

\( \sigma (w^*|b) \) | 0.00 | 0.37 | 1.68 | 2.25 | 3.98 | 4.04 | 4.30 | 15.41 |
\( C\mathcal{I} (w) \) | 160.57 | 145.12 | 113.48 | 73.78 | 55.08 | 52.93 | 49.11 | 27.10 |
\( R (w|b) \) | 0.00 | 9.62 | 29.33 | 54.05 | 65.70 | 67.04 | 69.42 | 83.12 |

The reduction rate, the weights and the tracking error volatility are expressed in %
### Table 174: Optimal decarbonization portfolios (Example #5, naive approach)

<table>
<thead>
<tr>
<th>( m )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
<th>( 7 )</th>
<th>( CI_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>20.00</td>
<td>21.05</td>
<td>24.39</td>
<td>30.77</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>19.00</td>
<td>20.00</td>
<td>23.17</td>
<td>29.23</td>
<td>42.22</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>17.00</td>
<td>17.89</td>
<td>20.73</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>13.00</td>
<td>13.68</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>12.00</td>
<td>12.63</td>
<td>14.63</td>
<td>18.46</td>
<td>26.67</td>
<td>46.15</td>
<td>60.00</td>
<td>100.00</td>
<td>27.1</td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>8.00</td>
<td>8.42</td>
<td>9.76</td>
<td>12.31</td>
<td>17.78</td>
<td>30.77</td>
<td>40.00</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>( w_7^* )</td>
<td>6.00</td>
<td>6.32</td>
<td>7.32</td>
<td>9.23</td>
<td>13.33</td>
<td>23.08</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>( w_8^* )</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>426.7</td>
</tr>
</tbody>
</table>

\[ \sigma (w^* | b) \] 0.00 | 0.39 | 1.85 | 3.04 | 9.46 | 8.08 | 8.65 | 15.41 |

\[ CI (w) \] 160.57 | 146.57 | 113.95 | 78.26 | 68.38 | 47.32 | 37.94 | 27.10 |

\[ R (w | b) \] 0.00 | 8.72 | 29.04 | 51.26 | 57.41 | 70.53 | 76.37 | 83.12 |

The reduction rate, the weights and the tracking error volatility are expressed in %.
Figure 336: Efficient decarbonization frontier (Example #5)
Figure 337: Efficient decarbonization frontier of the interpolated naive approach (Example #5)
Example #6

We consider a debt-weighted bond index, which is composed of eight bonds. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in $\text{tCO}_2\text{e} / \text{m} \text{n}$) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the modified duration which is respectively equal to 3.1, 6.6, 7.2, 5, 4.7, 2.1, 8.1 and 2.6 years, and the duration-times-spread factor, which is respectively equal to 100, 155, 575, 436, 159, 145, 804 and 365 bps. There are two sectors. Bonds #1, #3, #4 and #8 belong to $\text{Sector}_1$ while Bonds #2, #5, #6 and #7 belong to $\text{Sector}_2$. 
Table 175: Optimal decarbonization portfolios (Example #6, threshold approach)

|        | 0   | 10  | 20  | 30  | 40  | 50  | CI
|--------|-----|-----|-----|-----|-----|-----|----
| $w_1^*$ | 20.00 | 21.62 | 23.93 | 26.72 | 30.08 | 33.44 | 100.5
| $w_2^*$ | 19.00 | 18.18 | 16.98 | 14.18 | 7.88  | 1.58  | 97.2
| $w_3^*$ | 17.00 | 18.92 | 21.94 | 22.65 | 16.82 | 11.00 | 250.4
| $w_4^*$ | 13.00 | 11.34 | 5.35  | 0.00  | 0.00  | 0.00  | 352.3
| $w_5^*$ | 12.00 | 13.72 | 16.14 | 21.63 | 33.89 | 46.14 | 27.1
| $w_6^*$ | 8.00  | 9.60  | 10.47 | 10.06 | 7.21  | 4.36  | 54.2
| $w_7^*$ | 6.00  | 5.56  | 5.19  | 4.75  | 4.11  | 3.48  | 78.6
| $w_8^*$ | 5.00  | 1.05  | 0.00  | 0.00  | 0.00  | 0.00  | 426.7
| AS$_{sector}$ | 0.00 | 6.87 | 15.49 | 24.07 | 31.97 | 47.58 |
| MD ($w$)  | 5.48 | 5.49 | 5.45 | 5.29 | 4.90 | 4.51 |
| DTS ($w$) | 301.05 | 292.34 | 282.28 | 266.12 | 236.45 | 206.78 |
| $\sigma_{AS} (w \mid b)$ | 0.00 | 5.57 | 12.31 | 19.82 | 30.04 | 43.58 |
| $\sigma_{MD} (w \mid b)$ | 0.00 | 0.01 | 0.04 | 0.17 | 0.49 | 0.81 |
| $\sigma_{DTS} (w \mid b)$ | 0.00 | 8.99 | 19.29 | 35.74 | 65.88 | 96.01 |
| CI ($w$)  | 160.57 | 144.52 | 128.46 | 112.40 | 96.34 | 80.29 |
| $\mathcal{R} (w \mid b)$ | 0.00 | 10.00 | 20.00 | 30.00 | 40.00 | 50.00 |
### Table 176: Optimal decarbonization portfolios (Example #6, order-statistic approach)

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$CI_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^*_1$</td>
<td>20.00</td>
<td>20.83</td>
<td>24.62</td>
<td>64.64</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.5</td>
</tr>
<tr>
<td>$w^*_2$</td>
<td>19.00</td>
<td>18.60</td>
<td>18.13</td>
<td>21.32</td>
<td>3.32</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>97.2</td>
</tr>
<tr>
<td>$w^*_3$</td>
<td>17.00</td>
<td>17.79</td>
<td>26.30</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>250.4</td>
</tr>
<tr>
<td>$w^*_4$</td>
<td>13.00</td>
<td>14.53</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>352.3</td>
</tr>
<tr>
<td>$w^*_5$</td>
<td>12.00</td>
<td>12.89</td>
<td>13.96</td>
<td>6.00</td>
<td>36.57</td>
<td>41.27</td>
<td>41.27</td>
<td>100.0</td>
<td>27.1</td>
</tr>
<tr>
<td>$w^*_6$</td>
<td>8.00</td>
<td>9.74</td>
<td>11.85</td>
<td>0.00</td>
<td>60.11</td>
<td>58.73</td>
<td>58.73</td>
<td>0.00</td>
<td>54.2</td>
</tr>
<tr>
<td>$w^*_7$</td>
<td>6.00</td>
<td>5.62</td>
<td>5.15</td>
<td>8.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>78.6</td>
</tr>
<tr>
<td>$w^*_8$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\text{AS}_{\text{sector}}$</td>
<td>0.00</td>
<td>5.78</td>
<td>19.72</td>
<td>49.00</td>
<td>76.68</td>
<td>80.00</td>
<td>80.00</td>
<td>88.00</td>
<td></td>
</tr>
<tr>
<td>$\text{MD} (w)$</td>
<td>5.48</td>
<td>5.52</td>
<td>5.54</td>
<td>4.77</td>
<td>3.27</td>
<td>3.17</td>
<td>3.17</td>
<td>4.70</td>
<td></td>
</tr>
<tr>
<td>$\text{DTS} (w)$</td>
<td>301.05</td>
<td>295.08</td>
<td>284.71</td>
<td>171.82</td>
<td>150.45</td>
<td>150.78</td>
<td>150.78</td>
<td>159.00</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{AS}} (w \mid b)$</td>
<td>0.00</td>
<td>5.73</td>
<td>17.94</td>
<td>50.85</td>
<td>66.96</td>
<td>68.63</td>
<td>68.63</td>
<td>95.33</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{MD}} (w \mid b)$</td>
<td>0.00</td>
<td>0.03</td>
<td>0.04</td>
<td>0.63</td>
<td>2.66</td>
<td>2.64</td>
<td>2.64</td>
<td>3.21</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{DTS}} (w \mid b)$</td>
<td>0.00</td>
<td>6.21</td>
<td>16.87</td>
<td>128.04</td>
<td>197.22</td>
<td>197.29</td>
<td>197.29</td>
<td>199.22</td>
<td></td>
</tr>
<tr>
<td>$CI (w)$</td>
<td>160.57</td>
<td>147.94</td>
<td>122.46</td>
<td>93.63</td>
<td>45.72</td>
<td>43.02</td>
<td>43.02</td>
<td>27.10</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{R} (w \mid b)$</td>
<td>0.00</td>
<td>7.87</td>
<td>23.74</td>
<td>41.69</td>
<td>71.53</td>
<td>73.21</td>
<td>73.21</td>
<td>83.12</td>
<td></td>
</tr>
</tbody>
</table>
Decarbonization scenario per sector:

$$CI(w; \text{Sector}_j) \leq (1 - R_j) CI(b; \text{Sector}_j)$$

We have:

$$(s_j \circ (CI - CI^*_j))^\top w \leq 0$$

where $CI^*_j = (1 - R_j) CI(b; \text{Sector}_j)$
Portfolio optimization in practice
Portfolio decarbonization
Net-zero investing
Equity and bond portfolios
Sector-specific constraints
Empirical results

Sector-specific constraints
Sector scenario

QP form

\[
C = \begin{pmatrix}
(s_1 \circ (CI - CI^*))^T \\
\vdots \\
(s_j \circ (CI - CI^*))^T \\
\vdots \\
(s_{n_{\text{sector}}} \circ (CI - CI^*_n_{\text{sector}}))^T
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
(1 - R_1) CI (b; \text{Sector}_1) \\
\vdots \\
(1 - R_j) CI (b; \text{Sector}_j) \\
\vdots \\
(1 - R_{n_{\text{sector}}}) CI (b; \text{Sector}_{n_{\text{sector}}})
\end{pmatrix}
\]
### Table 177: Carbon intensity and threshold in tCO$_2$e/$ mn per GICS sector (MSCI World, 2030)

<table>
<thead>
<tr>
<th>Sector</th>
<th>CI (b; Sector$_j$)</th>
<th>R$_j$ (in %)</th>
<th>CI$_j^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SC_1$</td>
<td>$SC_{1-2}$</td>
<td>$SC_{1-3}^{up}$</td>
</tr>
<tr>
<td>Communication Services</td>
<td>2</td>
<td>28</td>
<td>134</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>23</td>
<td>65</td>
<td>206</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>28</td>
<td>55</td>
<td>401</td>
</tr>
<tr>
<td>Energy</td>
<td>632</td>
<td>698</td>
<td>1,006</td>
</tr>
<tr>
<td>Financials</td>
<td>13</td>
<td>19</td>
<td>52</td>
</tr>
<tr>
<td>Health Care</td>
<td>10</td>
<td>22</td>
<td>120</td>
</tr>
<tr>
<td>Industrials</td>
<td>111</td>
<td>130</td>
<td>298</td>
</tr>
<tr>
<td>Information Technology</td>
<td>7</td>
<td>23</td>
<td>112</td>
</tr>
<tr>
<td>Materials</td>
<td>478</td>
<td>702</td>
<td>1,113</td>
</tr>
<tr>
<td>Real Estate</td>
<td>22</td>
<td>101</td>
<td>167</td>
</tr>
<tr>
<td>Utilities</td>
<td>1,744</td>
<td>1,794</td>
<td>2,053</td>
</tr>
<tr>
<td>MSCI World</td>
<td>130</td>
<td>163</td>
<td>310</td>
</tr>
</tbody>
</table>
Sector-specific constraints

Sector and weight deviation constraints (equity portfolio)

1. Asset weight deviation constraint:
   \[ \Omega := C_1 \left( m^-_w, m^+_w \right) = \left\{ w : m^-_w b \leq w \leq m^+_w b \right\} \]

2. Sector weight deviation constraint:
   \[ \Omega := C_2 \left( m^-_s, m^+_s \right) = \left\{ \forall j : m^-_s \sum_{i \in \text{Sector}_j} b_i \leq \sum_{i \in \text{Sector}_j} w_i \leq m^+_s \sum_{i \in \text{Sector}_j} b_i \right\} \]

3. \( C_2 (m_s) = C_2 \left( 1/m_s, m_s \right) \)

4. \( C_3 \left( m^-_w, m^+_w, m_s \right) = C_1 \left( m^-_w, m^+_w \right) \cap C_2 \left( m_s \right) \)
Sector-specific constraints

Sector and weight deviation constraints (bond portfolio)

1. **Modified duration constraint:**

\[ \Omega := C'_1 = \{ w : \text{MD} (w) = \text{MD} (b) \} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i) \text{MD}_i = 0 \right\} \]

2. **DTS constraint**

\[ \Omega := C'_2 = \{ w : \text{DTS} (w) = \text{DTS} (b) \} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i) \text{DTS}_i = 0 \right\} \]

3. **Maturity/rating buckets:**

\[ \Omega := \left\{ w : \sum_{i \in \text{Bucket}_j} (x_i - b_i) = 0 \right\} \]

- **C'_3:** Bucket_j is the j^{th} maturity bucket, e.g., 0–1, 1–3, 3–5, 5–7, 7–10 and 10+
- **C'_4:** Bucket_j is the j^{th} rating category, e.g., AAA–AA (AAA, AA+, AA and AA−), A (A+, A and A−) and BBB (BBB+, BBB, BBB−)
Two types of sectors:

- High climate impact sectors (HCIS):
  
  “sectors that are key to the low-carbon transition” (TEG, 2019)

- Low climate impact sectors (LCIS)

Let $\mathcal{HCIS}(w) = \sum_{i \in \text{HCIS}} w_i$ be the HCIS weight of portfolio $w$:

$$\mathcal{HCIS}(w) \geq \mathcal{HCIS}(b)$$
### Sector-specific constraints

#### HCIS constraint

**Table 178:** Weight and carbon intensity when applying the HCIS filter (MSCI World, June 2022)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index $b^*_j$</th>
<th>HCIS $b^*_j'$</th>
<th>$\mathbf{SC}_1$</th>
<th>$\mathbf{SC}_{1-2}$</th>
<th>$\mathbf{SC}_{1-3}^{up}$</th>
<th>$\mathbf{SC}_{1-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\mathbf{CI}$</td>
<td>$\mathbf{CI}'$</td>
<td>$\mathbf{CI}$</td>
<td>$\mathbf{CI}'$</td>
</tr>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>0.00</td>
<td>2</td>
<td>28</td>
<td>134</td>
<td>172</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>8.01</td>
<td>23</td>
<td>65</td>
<td>206</td>
<td>189</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>7.80</td>
<td>28</td>
<td>55</td>
<td>401</td>
<td>401</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>4.99</td>
<td>632</td>
<td>698</td>
<td>1006</td>
<td>1006</td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>0.00</td>
<td>13</td>
<td>19</td>
<td>52</td>
<td>244</td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>9.98</td>
<td>10</td>
<td>13</td>
<td>120</td>
<td>141</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>7.96</td>
<td>111</td>
<td>132</td>
<td>298</td>
<td>332</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>10.67</td>
<td>7</td>
<td>12</td>
<td>112</td>
<td>165</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>4.28</td>
<td>478</td>
<td>478</td>
<td>1113</td>
<td>1113</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>2.90</td>
<td>22</td>
<td>22</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>3.21</td>
<td>1744</td>
<td>1744</td>
<td>2053</td>
<td>2053</td>
</tr>
<tr>
<td>MSCI World</td>
<td>100.00</td>
<td>59.79</td>
<td>130</td>
<td>210</td>
<td>163</td>
<td>252</td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Empirical results (equity portfolios)

**Figure 338:** Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $SC_{1-2}$)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Empirical results (equity portfolios)

**Figure 339:** Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $SC_{1-3}$)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Equity portfolios

Barahhou et al. (2022) consider the basic optimization problem:

\[
\begin{align*}
    w^* &= \arg\min w \left( \frac{1}{2} (w - b)^\top \Sigma (w - b) \right) \\
    \text{s.t.} \quad &
    \begin{cases}
        CI(w) \leq (1 - R) CI(b) \\
        w \in \Omega_0 \cap \Omega
    \end{cases}
\end{align*}
\]

What is the impact of constraints $\Omega_0 \cap \Omega$?
Figure 340: Impact of the carbon scope on the tracking error volatility (MSCI World, June 2022, $C_0$ constraint)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
**Table 179**: Sector allocation in % (MSCI World, June 2022, scope $\mathcal{SC}_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td>7.95</td>
<td>8.15</td>
<td>8.42</td>
<td>8.78</td>
<td>9.34</td>
<td>10.13</td>
<td>12.27</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td>10.69</td>
<td>10.69</td>
<td>10.65</td>
<td>10.52</td>
<td>10.23</td>
<td>9.62</td>
<td>6.74</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>7.80</td>
<td>7.69</td>
<td>7.48</td>
<td>7.11</td>
<td>6.35</td>
<td>5.03</td>
<td>1.77</td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>4.14</td>
<td>3.65</td>
<td>3.10</td>
<td>2.45</td>
<td><strong>1.50</strong></td>
<td><strong>0.49</strong></td>
<td><strong>0.00</strong></td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td>14.53</td>
<td>15.17</td>
<td>15.94</td>
<td>16.90</td>
<td><strong>18.39</strong></td>
<td><strong>20.55</strong></td>
<td><strong>28.62</strong></td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td>14.74</td>
<td>15.09</td>
<td>15.50</td>
<td>16.00</td>
<td>16.78</td>
<td>17.77</td>
<td>17.69</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>9.28</td>
<td>9.01</td>
<td>8.71</td>
<td>8.36</td>
<td>7.79</td>
<td>7.21</td>
<td>6.03</td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td>21.68</td>
<td>22.03</td>
<td>22.39</td>
<td>22.88</td>
<td>23.51</td>
<td>24.12</td>
<td>24.02</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>3.78</td>
<td>3.46</td>
<td>3.06</td>
<td>2.56</td>
<td><strong>1.85</strong></td>
<td><strong>1.14</strong></td>
<td><strong>0.24</strong></td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td>3.12</td>
<td>3.27</td>
<td>3.41</td>
<td>3.57</td>
<td>3.72</td>
<td>3.71</td>
<td>2.51</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td>2.28</td>
<td>1.79</td>
<td>1.36</td>
<td>0.90</td>
<td><strong>0.54</strong></td>
<td><strong>0.24</strong></td>
<td><strong>0.12</strong></td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)

Portfolio decarbonization = strategy **long on Financials** and **short on Energy, Materials and Utilities**
Figure 341: Impact of $C_1$ constraint on the tracking error volatility (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Figure 342: Impact of $C_2$ and $C_3$ constraints (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
Equity portfolios

**Figure 343:** Tracking error volatility with $C_3 (0, 10, 2)$ constraint (MSCI World, June 2022)

Source: MSCI (2022), Trucost (2022) & Barahhou et al. (2022)
**First approach**

- The carbon footprint contribution of the \( m \) worst performing assets is:

\[
CFC^{(m,n)} = \sum_{i=1}^{n} \frac{1 \{ CI_i \geq CI^{(m,n)} \} \cdot b_i CI_i}{CI(b)}
\]

where \( CI^{(m,n)} = CI_{n-m+1:n} \) is the \( (n - m + 1) \)-th order statistic
Equity portfolios

Figure 344: Carbon footprint contribution $CFC^{(m,n)}$ in % (MSCI World, June 2022, first approach)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Second approach

- Another definition:

\[
CFC^{(m,n)} = \frac{\sum_{i=1}^{n} \mathbb{1} \left\{ CIC_i \geq CIC^{(m,n)} \right\} \cdot b_i CIC_i}{Cl(b)}
\]

where \( CIC_i = b_i CIC \) and \( CIC^{(m,n)} = CIC_{n-m+1:n} \)

- Weight contribution:

\[
WFC^{(m,n)} = \sum_{i=1}^{n} \mathbb{1} \left\{ CIC_i \geq CIC^{(m,n)} \right\} \cdot b_i
\]
Figure 345: Carbon footprint contribution $CFC^{(m,n)}$ in % (MSCI World, June 2022, second approach)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
### Table 180: Carbon footprint contribution $CFC^{(m,n)}$ in % (MSCI World, June 2022, second approach, $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$m$</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Staples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>0.78</td>
<td>1.37</td>
<td>2.44</td>
<td>2.93</td>
<td>4.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>3.75</td>
<td>4.44</td>
<td>4.92</td>
<td>5.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.61</td>
<td>17.35</td>
<td>23.78</td>
<td>29.56</td>
<td>31.78</td>
<td>33.02</td>
<td>33.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td>0.72</td>
<td>1.53</td>
<td>1.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Health Care</td>
<td>0.21</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrials</td>
<td>2.16</td>
<td>5.59</td>
<td>7.13</td>
<td>8.70</td>
<td>9.48</td>
<td>13.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information Technology</td>
<td>0.98</td>
<td>1.58</td>
<td>1.94</td>
<td>2.15</td>
<td>3.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>4.08</td>
<td>4.08</td>
<td>5.81</td>
<td>7.31</td>
<td>8.81</td>
<td>9.59</td>
<td>10.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td>0.81</td>
<td>3.20</td>
<td>3.89</td>
<td>5.24</td>
<td>7.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4.08</td>
<td>16.15</td>
<td>26.06</td>
<td>40.21</td>
<td>54.66</td>
<td>63.94</td>
<td>70.29</td>
<td>82.70</td>
<td></td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
### Table 181: Weight contribution $\nu^C(m,n)$ in % (MSCI World, June 2022, second approach, $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>$b_j$ (in %)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.08</td>
<td></td>
<td>0.08</td>
<td>3.03</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>10.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.58</td>
<td>1.79</td>
<td>2.44</td>
<td>4.51</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>7.80</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>1.90</td>
<td>2.50</td>
<td>2.84</td>
<td>3.84</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>4.99</td>
<td>1.71</td>
<td>2.25</td>
<td>2.96</td>
<td>3.62</td>
<td>3.99</td>
<td>4.33</td>
<td>4.65</td>
<td></td>
</tr>
<tr>
<td>Financials</td>
<td>13.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.74</td>
<td>1.17</td>
<td>2.33</td>
</tr>
<tr>
<td>Health Care</td>
<td>14.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.95</td>
<td>1.34</td>
</tr>
<tr>
<td>Industrials</td>
<td>9.90</td>
<td>0.06</td>
<td>0.32</td>
<td>0.70</td>
<td>0.96</td>
<td>1.20</td>
<td>4.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information Technology</td>
<td>21.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.16</td>
<td>4.70</td>
<td>8.42</td>
<td>8.78</td>
</tr>
<tr>
<td>Materials</td>
<td>4.28</td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
<td>0.47</td>
<td>0.88</td>
<td>1.10</td>
<td>1.40</td>
<td>1.87</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Utilities</td>
<td>3.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.31</td>
<td>0.86</td>
<td>1.04</td>
<td>1.31</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.29</td>
<td>2.71</td>
<td>3.30</td>
<td>5.49</td>
<td>14.50</td>
<td>21.32</td>
<td>26.63</td>
<td>41.24</td>
</tr>
</tbody>
</table>

Source: MSCI (2022), Trucost (2022) & Author’s calculations
The order-statistic optimization problem is:

\[ w^* = \arg \min \frac{1}{2} (w - b)^\top \Sigma (w - b) \]

s.t. \[ \begin{cases} 1_n^\top w = 1 \\ 0_n \leq w \leq w^{(m,n)} \end{cases} \]

where the upper bound \( w^{(m,n)} \) is equal to \( 1 \left\{ CI < CI^{(m,n)} \right\} \) for the first ordering approach and \( 1 \left\{ CIC < CIC^{(m,n)} \right\} \) for the second ordering approach.

Thierry Roncalli
Course 2023-2024 in Sustainable Finance

1429 / 1665
The naive method is:

$$w^*_i = \frac{e_i b_i}{\sum_{k=1}^{n} e_k b_k}$$

where $e_i$ is defined as $1 \{ CI_i < CI^{(m,n)} \}$ for the first ordering approach and $1 \{ CIC_i < CIC^{(m,n)} \}$ for the second ordering approach.
Figure 346: Tracking error volatility (MSCI World, June 2022, $SC_{1-3}$, first ordering method)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
Figure 347: Tracking error volatility (MSCI World, June 2022, $SC_{1-3}$, second ordering method)

Source: MSCI (2022), Trucost (2022) & Author’s calculations
The optimization problem is:

\[ w^* = \arg \min \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| + 50 \sum_{j=1}^{\text{nSector}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \right| \text{DTS}_i \]

s.t. \[ \{ \begin{aligned} & \mathcal{C} \mathcal{I}(w) \leq (1 - \mathcal{R}) \mathcal{C} \mathcal{I}(b) \\ & w \in \mathcal{C}_0 \cap \mathcal{C}_1' \cap \mathcal{C}_3' \cap \mathcal{C}_4' \end{aligned} \]
Figure 348: Impact of the carbon scope on the active share in % (ICE Global Corp., June 2022)

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
**Figure 349:** Impact of the carbon scope on the DTS risk in bps (ICE Global Corp., June 2022)

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
Table 182: Sector allocation in % (ICE Global Corp., June 2022, scope $\mathbf{SC}_{1-3}$)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Index</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Services</td>
<td>7.34</td>
<td>7.35</td>
<td>7.34</td>
<td>7.37</td>
<td>7.43</td>
<td>7.43</td>
<td>7.31</td>
<td>7.30</td>
</tr>
<tr>
<td>Consumer Discretionary</td>
<td>5.97</td>
<td>5.97</td>
<td>5.96</td>
<td>5.94</td>
<td>5.93</td>
<td>5.46</td>
<td>4.48</td>
<td>3.55</td>
</tr>
<tr>
<td>Consumer Staples</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.04</td>
<td>6.02</td>
<td>5.39</td>
<td>4.06</td>
</tr>
<tr>
<td>Energy</td>
<td>6.49</td>
<td>5.49</td>
<td>4.42</td>
<td>3.84</td>
<td>3.69</td>
<td>3.23</td>
<td>2.58</td>
<td>2.52</td>
</tr>
<tr>
<td>Financials</td>
<td>33.91</td>
<td>34.64</td>
<td>35.66</td>
<td>35.96</td>
<td>36.09</td>
<td>37.36</td>
<td>38.86</td>
<td>39.00</td>
</tr>
<tr>
<td>Health Care</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.50</td>
<td>7.52</td>
<td>7.48</td>
</tr>
<tr>
<td>Information Technology</td>
<td>5.57</td>
<td>5.57</td>
<td>5.59</td>
<td>5.59</td>
<td>5.60</td>
<td>5.60</td>
<td>5.52</td>
<td>5.27</td>
</tr>
<tr>
<td>Materials</td>
<td>3.44</td>
<td>3.43</td>
<td>3.31</td>
<td>3.18</td>
<td>3.12</td>
<td>2.64</td>
<td>2.25</td>
<td>1.86</td>
</tr>
<tr>
<td>Real Estate</td>
<td>4.76</td>
<td>4.74</td>
<td>4.74</td>
<td>4.74</td>
<td>4.74</td>
<td>4.66</td>
<td>4.61</td>
<td>3.93</td>
</tr>
<tr>
<td>Utilities</td>
<td>10.06</td>
<td>9.89</td>
<td>9.82</td>
<td>9.64</td>
<td>8.52</td>
<td>8.04</td>
<td>7.92</td>
<td>6.88</td>
</tr>
</tbody>
</table>

Source: ICE (2022), Trucost (2022) & Barahhou et al. (2022)
Choice of the decarbonization scenario

Figure 350: CO₂ emissions by sector in the IEA NZE scenario (in GtCO₂e)

The carbon emissions/intensity approach

A decarbonization scenario is defined as a function that relates a decarbonization rate to a time index \( t \):

\[
 f : \mathbb{R}^+ \rightarrow [0, 1] \\
 t \mapsto \mathcal{R}(t_0, t)
\]

where \( t_0 \) is the base year and \( \mathcal{R}(t_0, t_0^-) = 0 \)

Two choices

1. Carbon emissions

\[
 CE(t) = (1 - \mathcal{R}(t_0, t)) CE(t_0)
\]

2. Carbon intensity (CTB/PAB)

\[
 CI(t) = (1 - \Delta \mathcal{R})^{t-t_0} (1 - \mathcal{R}^-) CI(t_0)
\]

where \( \Delta \mathcal{R} = 7\% \) and \( \mathcal{R}^- \) takes the values 30%/50% (CTB/PAB)
The carbon emissions/intensity approach

Figure 351: IEA, NZAOA, CTB and PAB decarbonization pathways

IEA = International Energy Agency, NZAOA = Net Zero Asset Owners Alliance, CTB = Climate Transition Benchmark, PAB = Paris Aligned Benchmark
### The carbon emissions/intensity approach

**Table 183:** IEA, NZAOA, CTB and PAB decarbonization rates (baseline = 2020)

<table>
<thead>
<tr>
<th>Year</th>
<th>CTB</th>
<th>PAB</th>
<th>NZE Scenario</th>
<th>NZAOA Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R−</td>
<td>ΔR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>30%</td>
<td>7%</td>
<td>0.0%</td>
<td>10.0%</td>
</tr>
<tr>
<td>2021</td>
<td>34.9%</td>
<td>7%</td>
<td>1.7%</td>
<td>14.0%</td>
</tr>
<tr>
<td>2022</td>
<td>39.5%</td>
<td>7%</td>
<td>3.9%</td>
<td>18.0%</td>
</tr>
<tr>
<td>2023</td>
<td>43.7%</td>
<td>7%</td>
<td>6.7%</td>
<td>22.0%</td>
</tr>
<tr>
<td>2024</td>
<td>47.6%</td>
<td>7%</td>
<td>9.9%</td>
<td>26.0%</td>
</tr>
<tr>
<td>2025</td>
<td>51.3%</td>
<td>7%</td>
<td>13.6%</td>
<td>30.0%</td>
</tr>
<tr>
<td>2026</td>
<td>54.7%</td>
<td>7%</td>
<td>17.8%</td>
<td>34.0%</td>
</tr>
<tr>
<td>2027</td>
<td>57.9%</td>
<td>7%</td>
<td>22.3%</td>
<td>38.0%</td>
</tr>
<tr>
<td>2028</td>
<td>60.8%</td>
<td>7%</td>
<td>27.2%</td>
<td>42.0%</td>
</tr>
<tr>
<td>2029</td>
<td>63.6%</td>
<td>7%</td>
<td>32.1%</td>
<td>46.0%</td>
</tr>
<tr>
<td>2030</td>
<td>66.1%</td>
<td>7%</td>
<td>37.1%</td>
<td>50.0%</td>
</tr>
<tr>
<td>2035</td>
<td>76.4%</td>
<td>7%</td>
<td>60.2%</td>
<td>70.3%</td>
</tr>
<tr>
<td>2040</td>
<td>83.6%</td>
<td>7%</td>
<td>77.2%</td>
<td>89.6%</td>
</tr>
<tr>
<td>2045</td>
<td>88.6%</td>
<td>7%</td>
<td>87.6%</td>
<td>95.2%</td>
</tr>
<tr>
<td>2050</td>
<td>92.1%</td>
<td>7%</td>
<td>94.6%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2023)
The carbon emissions/intensity approach

Figure 352: Sectoral decarbonization pathways

Electricity ≻ Buildings ≻ Transport ≻ Industry
The carbon budget approach

A NZE scenario is defined by the following constraints:

\[
\begin{align*}
\text{CB} (t_0, 2050) & \leq \text{CB}^+ \text{ GtCO}_2e \\
\text{CE} (2050) & \approx 0 \text{ GtCO}_2e
\end{align*}
\]

where \( t_0 \) is the base date and \( \text{CB}^+ \) is the maximum carbon budget.

**IPCC SR15**

- \( t_0 = 2019 \) and \( \text{CB}^+ = 580 \text{ GtCO}_2e \): there is a 50\% probability of limiting the global warming to 1.5°C
- \( t_0 = 2019 \) and \( \text{CB}^+ = 420 \text{ GtCO}_2e \): the probability is 66\%
- \( t_0 = 2019 \) and \( \text{CB}^+ = 300 \text{ GtCO}_2e \): the probability is 83\%
The carbon budget approach

If we have:

\[ \mathcal{CE}(t) = (1 - \Delta R)^{t - t_0} (1 - R^-) \mathcal{CE}(t_0) \]

we obtain:

\[ \mathcal{CB}(t_0, t) = \left( \frac{(1 - \Delta R)^{t - t_0} - 1}{\ln(1 - \Delta R)} \right) (1 - R^-) \mathcal{CE}(t_0) \]

Table 184: Carbon budget \( \mathcal{CB}(2020, 2050) \) (in GtCO\(_2\)e) when defining the decarbonization pathway of carbon emissions and assuming that \( \mathcal{CE}(2020) = 36 \) GtCO\(_2\)e

<table>
<thead>
<tr>
<th>( \Delta R )</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^- )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>551</td>
<td>496</td>
<td>441</td>
<td>386</td>
<td>276</td>
<td>138</td>
</tr>
<tr>
<td>10%</td>
<td>491</td>
<td>442</td>
<td>393</td>
<td>344</td>
<td>245</td>
<td>123</td>
</tr>
<tr>
<td>20%</td>
<td>440</td>
<td>396</td>
<td>352</td>
<td>308</td>
<td>220</td>
<td>110</td>
</tr>
<tr>
<td>30%</td>
<td>396</td>
<td>357</td>
<td>317</td>
<td>277</td>
<td>198</td>
<td>99</td>
</tr>
<tr>
<td>50%</td>
<td>359</td>
<td>323</td>
<td>287</td>
<td>251</td>
<td>180</td>
<td>90</td>
</tr>
<tr>
<td>75%</td>
<td>327</td>
<td>294</td>
<td>262</td>
<td>229</td>
<td>164</td>
<td>82</td>
</tr>
</tbody>
</table>
Dynamic decarbonization and portfolio alignment

We have:

\[ CI(t, w) \leq (1 - R(t_0, t)) CI(t_0, b(t_0)) \]

where:

- \( t_0 \) is the base year
- \( R(t_0, t) \) is the decarbonization pathway of the NZE scenario
- \( CI(t_0, b(t_0)) \) is the carbon intensity of the benchmark at time \( t_0 \)
Dynamic decarbonization and portfolio alignment

Some properties:

- Decarbonizing the aligned portfolio becomes easier over time as the benchmark decarbonizes itself:
  \[ CI(t, b(t)) \ll CI(t_0, b(t_0)) \quad \text{for } t > t_0 \]

- Decarbonizing the aligned portfolio becomes more difficult over time as the benchmark carbonizes itself:
  \[ CI(t, b(t)) \gg CI(t_0, b(t_0)) \quad \text{for } t > t_0 \]

- The aligned portfolio matches the benchmark portfolio if the benchmark is sufficiently decarbonized:
  \[ CI(t, b(t)) \leq (1 - R(t_0, t)) CI(t_0, b(t_0)) \]
The optimization problem becomes:

\[
  w^*(t) = \arg \min \frac{1}{2} (w - b(t))^T \Sigma(t) (w - b(t))
\]

s.t. \[
  \begin{align*}
    & C_I(t, w) \leq (1 - R(t_0, t)) C_I(t_0, b(t_0)) \\
    & w \in \Omega_0 \cap \Omega
  \end{align*}
\]

where:

- \( \Omega_0 = C_0 = \{ w : 1_n^T w = 1, 0_n \leq w \leq 1_n \} \) defines the long-only constraint
- \( \Omega \) is the set of additional constraints
We consider Example #5. We want to align the portfolio with respect to the CTB scenario. To compute the optimal portfolio $w^*(t)$ where $t = t_0 + h$ and $h = 0, 1, 2, \ldots$ years, we assume that the benchmark $b(t)$, the covariance matrix $\Sigma(t)$, and the vector $\text{CI}(t)$ of carbon intensities do not change over time.
First, we compute the mapping function between the time \( t \) and the decarbonization rate \( R(t_0, t) \):

\[
R(t_0, t) = 1 - (1 - 30\%) \times (1 - 7\%)^h
\]

We get \( R(t_0, t_0) = 30\% \), \( R(t_0, t_0 + 1) = 34.90\% \), \( R(t_0, t_0 + 2) = 39.46\% \), and so on.

Second, we solve the optimization problem for the different values of time \( t \).
Table 185: Equity portfolio alignment (Example #7)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$b(t_0)$</th>
<th>$t_0$</th>
<th>$t_0 + 1$</th>
<th>$t_0 + 2$</th>
<th>$t_0 + 3$</th>
<th>$t_0 + 4$</th>
<th>$t_0 + 5$</th>
<th>$t_0 + 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>21.86</td>
<td>22.21</td>
<td>22.54</td>
<td>22.84</td>
<td>23.02</td>
<td>22.92</td>
<td>8.81</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>18.70</td>
<td>18.41</td>
<td>18.15</td>
<td>17.90</td>
<td>17.58</td>
<td>17.04</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>8.06</td>
<td>5.69</td>
<td>3.48</td>
<td>1.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>8.74</td>
<td>7.66</td>
<td>6.65</td>
<td>5.72</td>
<td>4.56</td>
<td>2.70</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>13.07</td>
<td>13.29</td>
<td>13.51</td>
<td>13.70</td>
<td>13.91</td>
<td>14.18</td>
<td>21.22</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>22.57</td>
<td>25.59</td>
<td>28.39</td>
<td>31.00</td>
<td>33.39</td>
<td>35.54</td>
<td>62.31</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>7.00</td>
<td>7.15</td>
<td>7.29</td>
<td>7.42</td>
<td>7.53</td>
<td>7.63</td>
<td>7.66</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The reduction rate and weights are expressed in %, while the tracking error volatility is measured in bps.
For bonds, the tracking error volatility is replaced by the active risk function:

\[
D(w | b) = \varphi \sum_{s=1}^{n_{\text{sector}}} \left| \sum_{i \in s} (w_i - b_i) DTS_i \right| + \frac{1}{2} \sum_{i \in b} |w_i - b_i| + \mathbb{1}_{\Omega_{\text{MD}}} (w)
\]

where:
- DTS\(_i\) and MD\(_i\) are the duration-times-spread and modified duration factors
- \(\Omega_{\text{MD}} = \{ w : \sum_{i=1}^{n} (w_i - b_i) MD_i = 0 \}\)
- \(\mathbb{1}_{\Omega} (w)\) is the convex indicator function
The optimization problem becomes then:

\[
    w^*(t) = \arg \min \mathcal{D}(w \mid b(t)) \\
    \text{s.t. } \begin{cases} 
        C(t, w) \leq (1 - R(t_0, t)) C(t_0, b(t_0)) \\
        w \in \Omega_0 \cap \Omega 
    \end{cases}
\]
Example #8

We consider Example #6. We want to align the portfolio with respect to the CTB scenario. To compute the optimal portfolio $w^*(t)$ where $t = t_0 + h$ and $h = 0, 1, 2, \ldots$ years, we assume that the benchmark, the modified duration and the duration-times-spread factors do not change over time.
Bond portfolios

The corresponding LP problem is:

\[ x^* = \arg \min c^T x \]

\[
\begin{aligned}
Ax &= B \\
Cx &\leq D \\
x^- &\leq x \leq x^+
\end{aligned}
\]

where:

- \( x = (w, \tau_w, \tau_{DTS}) \) is a 18 \times 1 vector
- The 18 \times 1 vector \( c \) is equal to \( \left( 0_8, \frac{1}{2} 1_8, \varphi 1_2 \right) \)
- The equality constraint includes the convex indicator function \( 1_{\Omega_{MD}}(w) \) and is defined by:

\[
Ax = B \iff \begin{pmatrix} 1_8^T & 0_8^T & 0_2^T \\ MD^T & 0_8^T & 0_2^T \end{pmatrix} x = \begin{pmatrix} 1 \\ 5.476 \end{pmatrix}
\]
The inequality constraints are:

\[ \mathbf{C} \mathbf{x} \leq \mathbf{D} \iff \begin{pmatrix} I_8 & -I_8 & 0_{8,2} \\ -I_8 & -I_8 & 0_{8,2} \\ \mathbf{C}_{DTS} & 0_{2,8} & -I_2 \\ -\mathbf{C}_{DTS} & 0_{2,8} & -I_2 \\ \mathbf{C} \mathbf{I}(t)^\top & 0_{1,8} & 0 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} b \\ -b \\ 192.68 \\ 108.37 \\ -192.68 \\ -108.37 \\ 160.574 \times (1 - \mathcal{R}(t_0, t)) \end{pmatrix} \]

where:

\[ \mathbf{C}_{DTS} = \begin{pmatrix} 100 & 0 & 575 & 436 & 0 & 0 & 0 & 0 & 365 \\ 0 & 155 & 0 & 0 & 159 & 145 & 804 & 0 \end{pmatrix} \]

Finally, the bounds are \( \mathbf{x}^- = 0_{18} \) and \( \mathbf{x}^+ = \infty \cdot \mathbf{1}_{18} \)

where:
### Table 186: Bond portfolio alignment (Example #8)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$b(t_0)$</th>
<th>$t_0$</th>
<th>$t_0 + 1$</th>
<th>$t_0 + 2$</th>
<th>$t_0 + 3$</th>
<th>$t_0 + 4$</th>
<th>$t_0 + 5$</th>
<th>$t_0 + 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
<td>20.00</td>
<td>13.98</td>
<td>17.64</td>
<td>16.02</td>
<td>5.02</td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>13.99</td>
<td>17.79</td>
<td>19.00</td>
<td>19.00</td>
<td>19.00</td>
<td>19.00</td>
<td>19.00</td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>25.43</td>
<td>20.96</td>
<td>17.78</td>
<td>17.00</td>
<td>13.64</td>
<td>11.65</td>
<td>4.61</td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>28.97</td>
<td>30.71</td>
<td>35.84</td>
<td>43.52</td>
<td>48.80</td>
<td>53.33</td>
<td>71.37</td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
<td>5.67</td>
<td>6.46</td>
<td>0.92</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>3.61</td>
<td>2.53</td>
<td>1.70</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| $\text{AS}(w)$ | 0.00 | 25.40 | 22.68 | 24.62 | 31.52 | 36.80 | 41.33 | 59.37 |
| $\text{MD}(w)$ | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 |
| $\text{DTS}(w)$ | 301.05 | 274.61 | 248.91 | 230.60 | 220.10 | 204.46 | 197.26 | 174.46 |
| $\mathcal{D}(w | b)$ | 0.00 | 0.39 | 0.49 | 0.60 | 0.72 | 0.85 | 0.99 | 1.57 |
| $\mathcal{CI}(w)$ | 160.57 | 112.40 | 104.53 | 97.22 | 90.41 | 84.08 | 78.20 | 54.40 |
| $\mathcal{R}(w | b)$ | 0.00 | 30.00 | 34.90 | 39.46 | 43.70 | 47.64 | 51.30 | 66.12 |

The reduction rate, weights, and active share metrics are expressed in %, the MD metrics are measured in years, and the DTS metrics are calculated in bps.
Defining a net-zero investment policy

General framework

The set of constraints to be applied must include the transition dimension:

\[ \Omega = \Omega_{\text{alignment}} \cap \Omega_{\text{transition}} \]

where:

\[ \Omega_{\text{alignment}} = \{ w : CI(t, w) \leq (1 - R(t_0, t)) CI(t_0, b(t_0)) \} \]

and:

\[ \Omega_{\text{transition}} = \Omega_{\text{self-decarbonization}} \cap \Omega_{\text{greenness}} \cap \Omega_{\text{exclusion}} \]
Self-decarbonization and endogeneity of the decarbonization pathway

<table>
<thead>
<tr>
<th></th>
<th>Bad case</th>
<th>Mixed case</th>
<th>Good case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective decarbonization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at the beginning of the year $t$</td>
<td>30%</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>at the end of the year $t$</td>
<td>25%</td>
<td>33%</td>
<td>36%</td>
</tr>
<tr>
<td>Self-decarbonization</td>
<td>0%</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>Relabancing requirement</td>
<td>10%</td>
<td>2%</td>
<td>0%</td>
</tr>
</tbody>
</table>

We can specify the self-decarbonization constraint as follows:

$$\Omega_{\text{self-decarbonization}} = \{ w : \mathcal{CM}(t, w) \leq \mathcal{CM}^*(t) \}$$

where:

- $\mathcal{CM}(t, w)$ is the carbon momentum of the portfolio $w$ at time $t$
- $\mathcal{CM}^*(t)$ is the self-decarbonization minimum threshold
Green footprint

The greenness constraint can be written as follows:

\[ \Omega_{\text{greenness}} = \{ w : GI(t, w) \geq GI^*(t) \} \]

where:
- \( GI(t, w) \) is the green intensity of the portfolio \( w \) at time \( t \)
- \( GI^*(t) \) is the minimum threshold

**Remark**

_in general, the absolute measure \( GI^*(t) \) is expressed as a relative value with respect to the benchmark:_

\[ GI^*(t) = (1 + G) GI(t, b(t)) \]

where \( G \) is the minimum growth value. For example, if \( G = 100\% \), we want to improve the green footprint of the benchmark so that the green intensity of the portfolio is at least twice the green intensity of the benchmark.
Net-zero exclusion policy

- Net-zero enemies
- Temperature score (Implied Temperature Rating or ITR)
- Barahhou et al. (2022) suggest excluding issuers whose carbon momentum is greater than a threshold $\mathcal{CM}^+$:

$$\Omega_{\text{exclusion}} = \left\{ w : \mathcal{CM}_i \geq \mathcal{CM}^+ \Rightarrow w_i = 0 \right\}$$
The optimization problem becomes:

\[
    w^*(t) = \arg \min \frac{1}{2} (w - b(t))^\top \Sigma(t) (w - b(t)) \\
    \text{s.t.} \begin{cases} 
        CI(t, w) \leq (1 - R(t_0, t)) CI(t_0, b(t_0)) & \leftarrow \text{Alignment} \\
        CM(t, w) \leq CM^*(t) & \leftarrow \text{Self-decarbonization} \\
        GI(t, w) \geq (1 + G) GI(t, b(t)) & \leftarrow \text{Greenness} \\
        0 \leq w_i \leq 1 \{ CM_i(t) \leq CM^+ \} & \leftarrow \text{Exclusion} \\
        w \in \Omega_0 \cap \Omega & \leftarrow \text{Other constraints}
    \end{cases}
\]
We deduce that the quadratic form is $Q = \Sigma (t), R = \Sigma (t) b (t), A = 1_n^T, B = 1, w^- = 0_n, w^+ = 1 \{ CM (t) \leq CM^+ \}$.

- If we assume that the carbon momentum function is a linear function:

$$CM (t, w) = w^T CM (t) = \sum_{i=1}^{n} w_i CM_i (t)$$

where $CM (t) = (CM_1 (t), \ldots, CM_n (t))$ is the carbon momentum vector, we get:

$$Cw \leq D \iff \begin{pmatrix} CI (t)^T \\ CM (t)^T \\ -GI (t)^T \end{pmatrix} w \leq \begin{pmatrix} (1 - R (t_0, t)) CI (t_0, b (t_0)) \\ CM^* (t) \\ - (1 + G) GI (t, b (t)) \end{pmatrix}$$
If we use an exact calculation of the carbon momentum at the portfolio level, we get:

$$Cw \leq D \iff \begin{pmatrix} CI(t)^T & \zeta^T & -G I(t) \end{pmatrix} w \leq \begin{pmatrix} (1 - R(t_0, t)) CI(t_0, b(t_0)) & 0 \\ - (1 + G) GI(t, b(t)) \end{pmatrix}$$

where $\zeta = (\zeta_1, \ldots, \zeta_n)$ and $\zeta_i = CI_i(t)(CM_i(t) - CM^*(t))$
Example #9

We consider Example #7. The carbon momentum values are equal to $-3.1\%, -1.2\%, -5.8\%, -1.4\%, +7.4\%, -2.6\%, +1.2\%, \text{ and } -8.0\%$. We measure the green intensity by the green revenue share. Its values are equal to $10.2\%, 45.3\%, 7.5\%, 0\%, 0\%, 35.6\%, 17.8\% \text{ and } 3.0\%$. The net-zero investment policy imposes to follow the CTB decarbonization pathway with a self-decarbonization of 3\%, and to improve the green intensity of the benchmark by 100\%.
### Table 187: Net-zero equity portfolio (Example #9)

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( b(t_0) )</th>
<th>( t_0 )</th>
<th>( t_0 + 1 )</th>
<th>( t_0 + 2 )</th>
<th>( t_0 + 3 )</th>
<th>( t_0 + 4 )</th>
<th>( t_0 + 5 )</th>
<th>( t_0 + 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1^* )</td>
<td>20.00</td>
<td>5.26</td>
<td>3.51</td>
<td>1.49</td>
<td>0.00</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>19.00</td>
<td>20.96</td>
<td>17.27</td>
<td>13.00</td>
<td>8.82</td>
<td>4.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_3^* )</td>
<td>17.00</td>
<td>3.35</td>
<td>7.27</td>
<td>11.82</td>
<td>15.02</td>
<td>14.32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_4^* )</td>
<td>13.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_5^* )</td>
<td>12.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_6^* )</td>
<td>8.00</td>
<td>60.06</td>
<td>64.69</td>
<td>70.05</td>
<td>75.37</td>
<td>81.51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_7^* )</td>
<td>6.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_8^* )</td>
<td>5.00</td>
<td>10.37</td>
<td>7.25</td>
<td>3.64</td>
<td>0.79</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma(w^* \mid b(t)) )</td>
<td>0.00</td>
<td>370.16</td>
<td>376.38</td>
<td>398.30</td>
<td>430.94</td>
<td>472.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( CI(t,w) )</td>
<td>160.57</td>
<td>110.85</td>
<td>104.53</td>
<td>97.22</td>
<td>90.41</td>
<td>84.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R(w \mid b(t_0)) )</td>
<td>0.00</td>
<td>30.96</td>
<td>34.90</td>
<td>39.46</td>
<td>43.70</td>
<td>47.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( CM(t,w) )</td>
<td>–1.66</td>
<td>–3.00</td>
<td>–3.00</td>
<td>–3.00</td>
<td>–3.00</td>
<td>–3.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( GI(t,w) )</td>
<td>15.99</td>
<td>31.98</td>
<td>31.98</td>
<td>31.98</td>
<td>31.98</td>
<td>31.98</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The reduction rate, weights, carbon momentum and green intensity are expressed in %, while the tracking error volatility is measured in bps.
The optimization problem becomes:

\[ w^* (t) = \arg \min \mathcal{D}(w | b(t)) \]

subject to:

\[
\begin{align*}
& CI(t, w) \leq (1 - R(t_0, t)) CI(t_0, b(t_0)) & \leftarrow \text{Alignment} \\
& CM(t, w) \leq CM^*(t) & \leftarrow \text{Self-decarbonization} \\
& GI(t, w) \geq (1 + G) GI(t, b(t)) & \leftarrow \text{Greenness} \\
& 0 \leq w_i \leq 1 \{CM_i(t) \leq CM^+\} & \leftarrow \text{Exclusion} \\
& w \in \Omega_0 \cap \Omega & \leftarrow \text{Other constraints}
\end{align*}
\]
We get the same LP form except for the set of inequality constraints $Cx \leq D$:

$$
\begin{pmatrix}
    l_n & -l_n & 0_{n,n_{\text{sector}}} \\
    -l_n & -l_n & 0_{n,n_{\text{sector}}} \\
    C_{\text{DTS}} & 0_{n_{\text{sector}},n} & 0_{n_{\text{sector}}} \\
    -C_{\text{DTS}} & 0_{n_{\text{sector}},n} & 0_{n_{\text{sector}}} \\
    CI(t)^T & 0_{1,n} & 0_{1,n_{\text{sector}}} \\
    CM(t)^T & 0_{1,n} & 0_{1,n_{\text{sector}}} \\
    -GI(t)^T & 0_{1,n} & 0_{1,n_{\text{sector}}}
\end{pmatrix}
\begin{pmatrix}
    x
\end{pmatrix}
\leq
\begin{pmatrix}
    b \\
    -b \\
    DTS^* \\
    -DTS^* \\
    (1 - R(t_0, t)) CI(t_0, b(t_0)) \\
    CM^*(t) \\
    -(1 + G) GI(t, b(t))
\end{pmatrix}
$$

and the upper bound:

$$
x^+ = \left( 1 \{ CM(t) \leq CM^+ \}, \infty \cdot 1_n, \infty \cdot 1_{n_{\text{sector}}} \right)
$$
Example #10

We consider Example #8. The carbon momentum values are equal to $-3.1\%$, $-1.2\%$, $-5.8\%$, $-1.4\%$, $+7.4\%$, $-2.6\%$, $+1.2\%$, and $-8.0\%$. We measure the green intensity by the green revenue share. Its values are equal to $10.2\%$, $45.3\%$, $7.5\%$, $0\%$, $0\%$, $35.6\%$, $17.8\%$ and $3.0\%$. The net-zero investment policy imposes to follow the CTB decarbonization pathway with a self-decarbonization of $2\%$, and to improve the green intensity of the benchmark by $100\%$. 
### Table 188: Net-zero bond portfolio (Example #10)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$b(t_0)$</th>
<th>$t_0$</th>
<th>$t_0 + 1$</th>
<th>$t_0 + 2$</th>
<th>$t_0 + 3$</th>
<th>$t_0 + 4$</th>
<th>$t_0 + 5$</th>
<th>$t_0 + 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^*$</td>
<td>20.00</td>
<td>4.28</td>
<td>13.80</td>
<td>20.48</td>
<td>26.34</td>
<td>19.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_2^*$</td>
<td>19.00</td>
<td>34.78</td>
<td>38.94</td>
<td>42.72</td>
<td>46.23</td>
<td>49.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_3^*$</td>
<td>17.00</td>
<td>21.03</td>
<td>13.86</td>
<td>7.73</td>
<td>2.11</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_4^*$</td>
<td>13.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_5^*$</td>
<td>12.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_6^*$</td>
<td>8.00</td>
<td>39.91</td>
<td>33.40</td>
<td>29.07</td>
<td>25.32</td>
<td>31.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_7^*$</td>
<td>6.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_8^*$</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $w$ | 0.00 | 51.72 | 45.34 | 45.27 | 50.89 | 53.98 |  |
| MD ($w$) | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 | 5.48 |  |
| DTS ($w$) | 301.05 | 236.99 | 202.30 | 173.29 | 146.83 | 141.34 |  |
| $D$ ($w$ | 0.00 | 0.87 | 0.95 | 1.09 | 1.28 | 1.48 |  |
| CI ($w$) | 160.57 | 112.40 | 104.53 | 97.22 | 90.41 | 84.08 |  |
| R ($w$ | 0.00 | 30.00 | 34.90 | 39.46 | 43.70 | 47.64 |  |
| CM ($t, w$) | -1.66 | -2.81 | -2.57 | -2.35 | -2.15 | -2.01 |  |
| GI ($t, w$) | 15.99 | 31.98 | 31.98 | 32.37 | 32.80 | 35.52 |  |

The reduction rate, weights, carbon momentum, green intensity and active share metrics are expressed in %, the MD metrics are measured in years, and the DTS metrics are calculated in bps.
Empirical results

Figure 353: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, June 2022, $C_0$ constraint)

Empirical results

Figure 354: Tracking error volatility of dynamic decarbonized portfolios (MSCI World, June 2022, $C_3 (0, 10, 2)$ constraint)

![Graphs showing tracking error volatility over time for different portfolios (SC1, SC1-2, SC1-up, SC1-3)]

Empirical results

The previous analysis deals only with the decarbonization dimension. Barahhou et al. (2022) then introduced the transition dimension and solved the following optimization problem:

\[
w^* (t) = \arg \min \frac{1}{2} (w - b(t))^\top \Sigma (t) (w - b(t))
\]

subject to

\[
\begin{align*}
CI (t, w) & \leq (1 - \mathcal{R} (t_0, t)) CI (t_0, b(t_0)) \\
CM (t, w) & \leq CM^* (t) \\
GI (t, w) & \geq (1 + G) GI (t, b(t)) \\
w & \in C_0 \cap C_3 (0, 10, 2)
\end{align*}
\]

where \(CM^* (t) = -5\%\) and \(G = 100\%\)
Empirical results

Figure 355: Tracking error volatility of net-zero portfolios (MSCI World, June 2022, \( C_0 \) constraint, \( G = 100\% \), \( CM^* = -5\% \), PAB)

Empirical results

Figure 356: Tracking error volatility of net-zero portfolios (MSCI World, June 2022, $C_3 (0, 10, 2)$ constraint, $G = 100\%$, $CM^* = -5\%$, PAB)

Empirical results

**Figure 357:** Tracking error volatility of net-zero portfolios (MSCI EMU, June 2022, $C_3 (0, 10, 2)$ constraint, $\mathcal{G} = 100\%$, $CM^* = -5\%$, PAB)

Figure 358: Tracking error volatility of net-zero portfolios (MSCI USA, Jun. 2022, \( C_3 (0, 10, 2) \) constraint, \( G = 100\% \), \( \mathcal{CM}^* = -5\% \), PAB)

Empirical results

Figure 359: Radar chart of investment universe shrinkage (MSCI World, June 2022, $C_3 (0, 10, 2)$ constraint, $G = 100\%$, $CM^* = -5\%$, PAB, Scope $SC_{1-3}$)

Empirical results

Figure 360: Impact of momentum exclusion on universe shrinkage (MSCI World, June 2022, $C_3 (0, 10, 2)$ constraint, $G = 100\%, \ CM^* = -5\%,$ PAB, Scope $SC_{1-3}, \ CM^+ = 0\%$)

Empirical results

Figure 361: Duration-times-spread cost of dynamically decarbonized portfolios (Global Corporate, June 2022)

Empirical results

Figure 362: Active share of dynamically decarbonized portfolios (Global Corporate, June 2022)

Empirical results

Figure 363: IEA decarbonization pathways

- Advanced economies (CE)
- Developing economies (CE)
- Advanced economies (CI)
- Developing economies (CI)
**Empirical results**

**Table 189:** First year of country exit from the NZE investment portfolio (GHG/GDP intensity metric)

<table>
<thead>
<tr>
<th>Country</th>
<th>Exit Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2025</td>
</tr>
<tr>
<td>Austria</td>
<td>2029</td>
</tr>
<tr>
<td>Belgium</td>
<td>2028</td>
</tr>
<tr>
<td>Canada</td>
<td>2024</td>
</tr>
<tr>
<td>Chile</td>
<td>2029</td>
</tr>
<tr>
<td>China</td>
<td>2028</td>
</tr>
<tr>
<td>Colombia</td>
<td>2029</td>
</tr>
<tr>
<td>Cyprus</td>
<td>2029</td>
</tr>
<tr>
<td>Czechia</td>
<td>2024</td>
</tr>
<tr>
<td>Denmark</td>
<td>2029</td>
</tr>
<tr>
<td>Estonia</td>
<td>2025</td>
</tr>
<tr>
<td>Finland</td>
<td>2029</td>
</tr>
<tr>
<td>France</td>
<td>2029</td>
</tr>
<tr>
<td>Germany</td>
<td>2029</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>2029</td>
</tr>
<tr>
<td>Hungary</td>
<td>2029</td>
</tr>
<tr>
<td>Indonesia</td>
<td>2024</td>
</tr>
<tr>
<td>Ireland</td>
<td>2029</td>
</tr>
<tr>
<td>Israel</td>
<td>2029</td>
</tr>
<tr>
<td>Italy</td>
<td>2029</td>
</tr>
<tr>
<td>Japan</td>
<td>2029</td>
</tr>
<tr>
<td>Latvia</td>
<td>2028</td>
</tr>
<tr>
<td>Lithuania</td>
<td>2029</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>2029</td>
</tr>
<tr>
<td>Malaysia</td>
<td>2029</td>
</tr>
<tr>
<td>Malta</td>
<td>2029</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2024</td>
</tr>
<tr>
<td>Norway</td>
<td>2029</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2029</td>
</tr>
<tr>
<td>Peru</td>
<td>2029</td>
</tr>
<tr>
<td>Poland</td>
<td>2029</td>
</tr>
<tr>
<td>Portugal</td>
<td>2028</td>
</tr>
<tr>
<td>Romania</td>
<td>2025</td>
</tr>
<tr>
<td>Singapore</td>
<td>2029</td>
</tr>
<tr>
<td>Slovakia</td>
<td>2029</td>
</tr>
<tr>
<td>Slovenia</td>
<td>2028</td>
</tr>
<tr>
<td>South Korea</td>
<td>2024</td>
</tr>
<tr>
<td>Spain</td>
<td>2028</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2029</td>
</tr>
<tr>
<td>Sweden</td>
<td>2024</td>
</tr>
<tr>
<td>Thailand</td>
<td>2029</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2029</td>
</tr>
<tr>
<td>United States</td>
<td>2028</td>
</tr>
</tbody>
</table>

Source: Barahhou et al. (2023, Table 9, page 26.)
Empirical results

**Table 190**: Country exclusion year by intensity metric

<table>
<thead>
<tr>
<th>Metric</th>
<th>GHG GDP</th>
<th>GHG Population</th>
<th>CO₂ (production) GDP</th>
<th>CO₂ (consumption) Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>2028</td>
<td>2031</td>
<td>2027</td>
<td>2031</td>
</tr>
<tr>
<td>France</td>
<td>2029</td>
<td>2032</td>
<td>2027</td>
<td>2031</td>
</tr>
<tr>
<td>Indonesia</td>
<td>2024</td>
<td>2032</td>
<td>2024</td>
<td>2031</td>
</tr>
<tr>
<td>Ireland</td>
<td>2029</td>
<td>2030</td>
<td>2027</td>
<td>2030</td>
</tr>
<tr>
<td>Japan</td>
<td>2029</td>
<td>2032</td>
<td>2027</td>
<td>2031</td>
</tr>
<tr>
<td>United States</td>
<td>2028</td>
<td>2030</td>
<td>2026</td>
<td>2029</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2029</td>
<td>2032</td>
<td>2027</td>
<td>2031</td>
</tr>
<tr>
<td>Sweden</td>
<td>2029</td>
<td>2032</td>
<td>2027</td>
<td>2031</td>
</tr>
</tbody>
</table>

Source: Barahhou et al. (2023, Table 14, page 31.)
The core-satellite approach

The two building block approach

Decarbonizing the portfolio
- Net-zero decarbonization portfolio
- Net-zero transition portfolio
- Dynamic low-carbon portfolio

Financing the transition
- Net-zero contribution portfolio
- Net-zero funding portfolio
- Net-zero transformation portfolio
The core-satellite approach

Decarbonized portfolio
- Carbon intensity
- Decarbonization pathway(s)
- Top-down approach
- Portfolio construction
- Net-zero carbon metrics

Transition portfolio
- Green intensity
- Financing the transition
- Bottom-up approach
- Security selection
- Net-zero transition metrics

\[ 1 - \alpha(t) \quad + \quad \alpha(t) \]
A typical program for the equity bucket looks like this:

\[
w^*(t) = \arg \min \frac{1}{2} (w - b(t))^\top \Sigma(t) (w - b(t))
\]

subject to

\[
\begin{align*}
CI(t, w) &\leq (1 - R(t_0, t)) CI(t_0, b(t_0)) \\
CM(t, w) &\leq CM^*(t) \\
0 \leq w_i \leq 1 \{CM_i(t) \leq CM^+\} \\
w &\in \Omega_0 \cap \Omega
\end{align*}
\]
For the bond bucket, we get a similar optimization problem:

\[ w^*(t) = \arg\min \mathcal{D}(w | b(t)) \]

s.t. \[
\begin{align*}
\mathcal{CI}(t, w) &\leq (1 - R(t_0, t)) \mathcal{CI}(t_0, b(t_0)) \\
\mathcal{CM}(t, w) &\leq \mathcal{CM}^*(t) \\
0 &\leq w_i \leq 1 \{ \mathcal{CM}_i(t) \leq \mathcal{CM}^+ \} \\
\mathcal{D} &\in \Omega_0 \cap \Omega
\end{align*}
\]
The electricity sector scenario in the core portfolio

The constraint to meet a reduction rate for a given sector \( S_{sector_j} \) is:

\[
\frac{\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} w_i C I_i}{\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} w_i} = C I (S_{sector_j}, R_j)
\]

where \( C I (S_{sector_j}, R_j) \) is the carbon intensity target for the given sector:

\[
C I (S_{sector_j}, R_j) = (1 - R_j) \frac{\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} b_i C I_i}{\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} b_i}
\]

We deduce that:

\[
\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} w_i C I_i = C I (S_{sector_j}, R_j) \sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} w_i
\]

which is equivalent to the following constraint:

\[
\sum_{i=1}^{n} \mathbb{1} \{ i \in S_{sector_j} \} w_i (C I_i - C I (S_{sector_j}, R_j)) = 0 \iff (s_j \circ (C I_i - C I_j^*))^\top w = 0
\]

where \( C I_j^* = C I (S_{sector_j}, R_j) \)
Figure 364: TE volatility of decarbonized portfolios (MSCI World, December 2021, $CM^* = -3.5\%$, $CM^+ = 10\%$, IEA NZE electricity sector scenario)

Source: Ben Slimane et al. (2023).
Core portfolio

Figure 365: Active risk of decarbonized portfolios (Global Corporate, December 2021, $CM^* = -3.5\%, CM^+ = 10\%$, IEA NZE electricity sector scenario)

Source: Ben Slimane et al. (2023).
Satellite portfolio

- Green, sustainability and sustainability-linked bonds
- Green stocks
- Green infrastructure
- Sustainable real estate
Figure 366: Narrow specification of the satellite investment universe GICS

<table>
<thead>
<tr>
<th>Sector</th>
<th>Industry Group</th>
<th>Industry</th>
<th>Sub-industry</th>
<th>Satellite</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Ben Slimane et al. (2023).
### Green bonds

**Table 191: GSS+ bond issuance**

<table>
<thead>
<tr>
<th>Year</th>
<th>Green # $ bn</th>
<th>Social # $ bn</th>
<th>Sustainability # $ bn</th>
<th>SLB # $ bn</th>
</tr>
</thead>
<tbody>
<tr>
<td>2022</td>
<td>1784 531.6</td>
<td>542 152.8</td>
<td>614 174.8</td>
<td>382 144.3</td>
</tr>
<tr>
<td>2021</td>
<td>1971 686.1</td>
<td>554 242.1</td>
<td>646 233.2</td>
<td>343 161.5</td>
</tr>
<tr>
<td>2020</td>
<td>1076 291.2</td>
<td>273 172.0</td>
<td>308 154.8</td>
<td>47 16.5</td>
</tr>
<tr>
<td>2019</td>
<td>877 268.0</td>
<td>99 22.2</td>
<td>333 85.2</td>
<td>18 8.9</td>
</tr>
<tr>
<td>2018</td>
<td>582 165.3</td>
<td>48 16.5</td>
<td>52 22.1</td>
<td>1 2.2</td>
</tr>
<tr>
<td>2017</td>
<td>472 160.9</td>
<td>46 11.8</td>
<td>17 9.2</td>
<td>1 0.2</td>
</tr>
<tr>
<td>2016</td>
<td>285 99.7</td>
<td>14 2.2</td>
<td>16 6.6</td>
<td>0 0.0</td>
</tr>
</tbody>
</table>

Source: Bloomberg (2023), GSS+ Instrument Indicator & Author’s calculations.
**Figure 367: Performance and duration of the Bloomberg Global Green Bond and Aggregate indices**

The chart above illustrates the performance of the Bloomberg Global Green Bond and Aggregate indices from 2014 to 2023. The indices are represented by two lines on the graph, with the Global Green Bond index depicted in blue and the Global Aggregate index in red. The performance of the indices is shown on the left y-axis, ranging from 80 to 120, with the years 2014 to 2023 along the x-axis.

The duration of each index is shown in the lower part of the chart, with years 2014 to 2023 along the x-axis and the duration (in years) on the y-axis. The duration values range from approximately 5 to 9 years.
Figure 368: Performance and tracking error volatility of thematic equity indices

Index


One-year tracking error volatility (in %)

The European Commission defines green infrastructure as “a strategically planned network of natural and semi-natural areas with other environmental features, designed and managed to deliver a wide range of ecosystem services, while also enhancing biodiversity”. Green infrastructure is implemented in a variety of sectors, from energy through energy transmission infrastructure, water through natural water retention measures or sustainable urban drainage systems, to the urban landscape with street trees to help sequester carbon or green roofs to help regulate the temperature of buildings. The cost of implementing green infrastructure is in the identification, mapping, planning and creation of the infrastructure, but the environmental, economic and social benefits make it worthwhile. Funds that assess infrastructure needs are emerging in the market and typically invest in owners of sustainable infrastructure assets as well as companies that are leaders in infrastructure investment. In addition to infrastructure funds, investors are also considering direct investments such as green car parks, water infrastructure and flood defences.
Sustainable real estate

- CRREM (Carbon Risk Real Estate Monitor) ⇒ whole-building approach for in-use emissions
- GRESB ⇒ GHG Protocol principles to the real estate industry (corporate approach)
- SBTi Building Guidelines
- PCAF/CRREM/GRESB joint technical guidance ⇒ Accounting and reporting of financed GHG emissions from real estate operations (GHG Protocol)
Let $\alpha_{\text{equity}}$ and $\alpha_{\text{bond}}$ be the proportions of stocks and bonds in the multi-asset portfolio.
Let $\alpha_{\text{satellite}}$ be the weight of the satellite portfolio.

The core allocation is given by the vector $(\alpha_{\text{core\ equity}}, \alpha_{\text{core\ bond}})$, while the satellite allocation is defined by $(\alpha_{\text{satellite\ equity}}, \alpha_{\text{satellite\ bond}})$.

We have the following identities:

\[
\begin{aligned}
\alpha_{\text{equity}} &= (1 - \alpha_{\text{satellite}}) \alpha_{\text{core\ equity}} + \alpha_{\text{satellite}} \alpha_{\text{satellite\ equity}} \\
\alpha_{\text{bond}} &= (1 - \alpha_{\text{satellite}}) \alpha_{\text{core\ bond}} + (1 - \alpha_{\text{satellite}}) \alpha_{\text{satellite\ bond}}
\end{aligned}
\]
In general, the fund manager targets a strategic asset allocation at the portfolio level, *i.e.* the proportions $\alpha_{\text{equity}}$ and $\alpha_{\text{bond}}$ are given.

For example, a defensive portfolio corresponds to a 20/80 constant mix strategy, while the 50/50 allocation is known as a balanced portfolio. Another famous allocation rule is the 60/40 portfolio, which is 60% in stocks and 40% in bonds.

The solution is to calculate the proportion of bonds in the core portfolio relative to the proportion of bonds in the satellite portfolio:

$$\alpha_{\text{core}} \cdot \alpha_{\text{bond}} = \frac{\alpha_{\text{bond}} - \alpha_{\text{satellite}}}{1 - \alpha_{\text{satellite}}}$$
Example #11

We consider a 60/40 constant mix strategy. The satellite portfolio represents 10% of the net zero investments. We assume that the satellite portfolio has 70% exposure to green bonds.
We have $\alpha_{\text{equity}} = 60\%$, $\alpha_{\text{bond}} = 40\%$, $\alpha_{\text{core}} = 90\%$, $\alpha_{\text{satellite}} = 10\%$ and $\alpha_{\text{satellite bond}} = 70\%$. We deduce that:

$$
\alpha_{\text{core}} = \frac{0.40 - 0.10 \times 0.70}{1 - 0.10} = \frac{33}{90} = 36.67\%
$$

$$
\alpha_{\text{bond}} = \frac{0.10 \times 0.70}{1 - 0.10} = \frac{33}{90} = 36.67\%
$$

The core allocation is then (63.33\%, 36.67\%), while the satellite allocation is (30\%, 70\%). We check that:

$$
\begin{cases}
\alpha_{\text{equity}} = 0.90 \times \left(1 - \frac{33}{90}\right) + 0.10 \times 0.30 = 60\% \\
\alpha_{\text{bond}} = 0.90 \times \frac{33}{90} + 0.10 \times 0.70 = 40\%
\end{cases}
$$
### Table 192: Calculating the bond allocation in the core portfolio ($\alpha_{\text{core bond}}$ in %)

<table>
<thead>
<tr>
<th>$\alpha_{\text{satellite}}$</th>
<th>60/40</th>
<th>50/50</th>
<th>20/80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>40.0</td>
<td>50.0</td>
<td>80.0</td>
</tr>
<tr>
<td>1%</td>
<td>39.7</td>
<td>49.7</td>
<td>80.1</td>
</tr>
<tr>
<td>5%</td>
<td>38.4</td>
<td>48.4</td>
<td>80.5</td>
</tr>
<tr>
<td>10%</td>
<td>36.7</td>
<td>47.8</td>
<td>81.1</td>
</tr>
<tr>
<td>15%</td>
<td>34.7</td>
<td>46.5</td>
<td>81.8</td>
</tr>
<tr>
<td>20%</td>
<td>32.5</td>
<td>45.0</td>
<td>82.5</td>
</tr>
<tr>
<td>25%</td>
<td>30.0</td>
<td>43.3</td>
<td>83.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha_{\text{satellite}}$</th>
<th>60/40</th>
<th>50/50</th>
<th>20/80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>40.0</td>
<td>50.0</td>
<td>80.0</td>
</tr>
<tr>
<td>1%</td>
<td>39.7</td>
<td>49.7</td>
<td>80.1</td>
</tr>
<tr>
<td>5%</td>
<td>38.4</td>
<td>48.4</td>
<td>80.5</td>
</tr>
<tr>
<td>10%</td>
<td>36.7</td>
<td>47.8</td>
<td>81.1</td>
</tr>
<tr>
<td>15%</td>
<td>34.7</td>
<td>46.5</td>
<td>81.8</td>
</tr>
<tr>
<td>20%</td>
<td>32.5</td>
<td>45.0</td>
<td>82.5</td>
</tr>
<tr>
<td>25%</td>
<td>30.0</td>
<td>43.3</td>
<td>83.3</td>
</tr>
</tbody>
</table>
The tracking error volatility of the core-satellite portfolio has the following expression:

$$\sigma (w | b) = \sqrt{\tilde{\alpha}^T \tilde{\Sigma} (w | b) \tilde{\alpha}} = \sqrt{(\tilde{\alpha} \circ \tilde{\sigma} (w | b))^T \tilde{\rho} (w | b) (\tilde{\alpha} \circ \tilde{\sigma} (w | b))}$$

where:

- $\tilde{\alpha}$ is the vector of allocation:

$$\tilde{\alpha} = \begin{pmatrix}
(1 - \alpha^{\text{satellite}}) \alpha^{\text{core equity}} \\
(1 - \alpha^{\text{satellite}}) \alpha^{\text{core bond}} \\
\alpha^{\text{satellite}} \alpha^{\text{satellite equity}} \\
\alpha^{\text{satellite}} \alpha^{\text{satellite bond}}
\end{pmatrix}$$

- $\tilde{\rho} (w | b)$ is the correlation matrix of $R (w) - R (b)$

- $\tilde{\sigma} (w | b)$ is the vector of tracking error volatilities:

$$\tilde{\sigma} (w | b) = \begin{pmatrix}
\sigma (w^{\text{core equity}} | b^{\text{equity}}) \\
\sigma (w^{\text{core bond}} | b^{\text{bond}}) \\
\sigma (w^{\text{satellite equity}} | b^{\text{equity}}) \\
\sigma (w^{\text{satellite bond}} | b^{\text{bond}})
\end{pmatrix}$$
Example #12

The tracking error volatilities are 2% for the core equity portfolio, 25 bps for the core bond portfolio, 20% for the satellite equity portfolio, and 3% for the satellite bond portfolio. To define the correlation matrix $\tilde{\rho} (w | b)$, we assume an 80% correlation between the two equity baskets, a 50% correlation between the two bond baskets, and a 0% correlation between the equity and bond baskets. We consider a 60/40 constant mix strategy. The satellite portfolio represents 10% of the net zero portfolio and has 70% exposure to green bonds.
Allocation process

Tracking error risk of the core-satellite portfolio

We compute the tracking error covariance matrix $\tilde{\Sigma}(w \mid b)$ as follows:

- The tracking error variance for the core equity portfolio is $\tilde{\Sigma}_{1,1}(w \mid b) = 0.02^2$
- The tracking error variance for the satellite equity portfolio is $\tilde{\Sigma}_{3,3}(w \mid b) = 0.20^2$
- The tracking error covariance for the two core portfolios is $\tilde{\Sigma}_{1,2}(w \mid b) = 0 \times 0.02 \times 0.0025$
- The tracking error covariance for the core equity portfolio and the satellite equity portfolio is $\tilde{\Sigma}_{1,3}(w \mid b) = 0.80 \times 0.02 \times 0.20$
- Etc.

Finally, we get:

$$
\tilde{\Sigma}(w \mid b) = \begin{pmatrix}
4 & 0 & 32 & 0 \\
0 & 0.0625 & 0 & 0.375 \\
32 & 0 & 400 & 0 \\
0 & 0.375 & 0 & 9
\end{pmatrix} \times 10^{-4}
$$

and $\sigma(w \mid b) = 1.68\%$ because $\tilde{\alpha} = (57\%, 33\%, 3\%, 7\%)$
## Table 193: Estimation of the tracking error volatility of the core-satellite portfolio (in %)

<table>
<thead>
<tr>
<th>$\alpha_{\text{satellite}}$</th>
<th>Bond</th>
<th>Defensive</th>
<th>Balanced</th>
<th>60/40</th>
<th>Dynamic</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0.38</td>
<td>0.62</td>
<td>1.36</td>
<td>1.62</td>
<td>2.15</td>
<td>2.69</td>
</tr>
<tr>
<td>20%</td>
<td>0.63</td>
<td>1.00</td>
<td>2.18</td>
<td>2.60</td>
<td>3.45</td>
<td>4.31</td>
</tr>
<tr>
<td>30%</td>
<td>0.92</td>
<td>1.43</td>
<td>3.11</td>
<td>3.71</td>
<td>4.93</td>
<td>6.16</td>
</tr>
<tr>
<td>Upper bound</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0.53</td>
<td>1.18</td>
<td>2.16</td>
<td>2.49</td>
<td>3.15</td>
<td>3.80</td>
</tr>
<tr>
<td>20%</td>
<td>0.80</td>
<td>1.76</td>
<td>3.20</td>
<td>3.68</td>
<td>4.64</td>
<td>5.60</td>
</tr>
<tr>
<td>30%</td>
<td>1.07</td>
<td>2.34</td>
<td>4.24</td>
<td>4.87</td>
<td>6.13</td>
<td>7.40</td>
</tr>
</tbody>
</table>
Allocation process
Tracking error risk of the core-satellite portfolio

$$\frac{\partial \alpha_{\text{satellite}}(t)}{\partial t} \geq 0$$
Course 2023-2024 in Sustainable Finance
Lecture 13. Exercise — Equity and Bond Portfolio Optimization with Green Preferences

Thierry Roncalli*

*Amundi Asset Management

*University of Paris-Saclay

March 2024

37 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Carbon intensity of the benchmark Equity portfolios Bond portfolios

Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management

Thierry Roncalli
Course 2023-2024 in Sustainable Finance
We consider an investment universe of 8 issuers. In the table below, we report the carbon emissions $C\varepsilon_{i,j}$ (in ktCO$_2$e) of these companies and their revenues $Y_i$ (in $\text{bn}$), and we indicate in the last row whether the company belongs to sector $\text{Sector}_1$ or $\text{Sector}_2$:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C\varepsilon_{i,1}$</td>
<td>75</td>
<td>5000</td>
<td>720</td>
<td>50</td>
<td>2500</td>
<td>25</td>
<td>30000</td>
<td>5</td>
</tr>
<tr>
<td>$C\varepsilon_{i,2}$</td>
<td>75</td>
<td>5000</td>
<td>1030</td>
<td>350</td>
<td>4500</td>
<td>5</td>
<td>2000</td>
<td>64</td>
</tr>
<tr>
<td>$C\varepsilon_{i,3}$</td>
<td>24000</td>
<td>15000</td>
<td>1210</td>
<td>550</td>
<td>500</td>
<td>187</td>
<td>30000</td>
<td>199</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>300</td>
<td>328</td>
<td>125</td>
<td>100</td>
<td>200</td>
<td>102</td>
<td>107</td>
<td>25</td>
</tr>
<tr>
<td>$\text{Sector}$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The benchmark $b$ of this investment universe is defined as:

$$b = (22\%, \ 19\%, \ 17\%, \ 13\%, \ 11\%, 8\%, 6\%, 4\%)$$

In what follows, we consider long-only portfolios.
Question 1

We want to compute the carbon intensity of the benchmark.
Question (a)

Compute the carbon intensities $CI_{i,j}$ of each company $i$ for the scopes 1, 2 and 3.
We have:

\[ CI_{i,j} = \frac{CE_{i,j}}{Y_i} \]

For instance, if we consider the 8\textsuperscript{th} issuer, we have\textsuperscript{38}:

\[ CI_{8,1} = \frac{CE_{8,1}}{Y_8} = \frac{5}{25} = 0.20 \text{ tCO}_2e/\text{ mn} \]

\[ CI_{8,2} = \frac{CE_{8,2}}{Y_8} = \frac{64}{25} = 2.56 \text{ tCO}_2e/\text{ mn} \]

\[ CI_{8,3} = \frac{CE_{8,3}}{Y_8} = \frac{199}{25} = 7.96 \text{ tCO}_2e/\text{ mn} \]

\textsuperscript{38}Because 1 ktCO\textsubscript{2}e/\text{ bn} = 1 tCO\textsubscript{2}e/\text{ mn}.
Since we have:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CEi,1$</td>
<td>75</td>
<td>5000</td>
<td>720</td>
<td>50</td>
<td>2500</td>
<td>25</td>
<td>30000</td>
<td>5</td>
</tr>
<tr>
<td>$CEi,2$</td>
<td>75</td>
<td>5000</td>
<td>1030</td>
<td>350</td>
<td>4500</td>
<td>5</td>
<td>2000</td>
<td>64</td>
</tr>
<tr>
<td>$CEi,3$</td>
<td>24000</td>
<td>15000</td>
<td>1210</td>
<td>550</td>
<td>500</td>
<td>187</td>
<td>30000</td>
<td>199</td>
</tr>
<tr>
<td>$Yi$</td>
<td>300</td>
<td>328</td>
<td>125</td>
<td>100</td>
<td>200</td>
<td>102</td>
<td>107</td>
<td>25</td>
</tr>
</tbody>
</table>

we obtain:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CIi,1$</td>
<td>0.25</td>
<td>15.24</td>
<td>5.76</td>
<td>0.50</td>
<td>12.50</td>
<td>0.25</td>
<td>280.37</td>
<td>0.20</td>
</tr>
<tr>
<td>$CIi,2$</td>
<td>0.25</td>
<td>15.24</td>
<td>8.24</td>
<td>3.50</td>
<td>22.50</td>
<td>0.05</td>
<td>18.69</td>
<td>2.56</td>
</tr>
<tr>
<td>$CIi,3$</td>
<td>80.00</td>
<td>45.73</td>
<td>9.68</td>
<td>5.50</td>
<td>2.50</td>
<td>1.83</td>
<td>280.37</td>
<td>7.96</td>
</tr>
</tbody>
</table>
Question (b)

Deduce the carbon intensities $CI_{i,j}$ of each company $i$ for the scopes $1 + 2$ and $1 + 2 + 3$. 
We have:

\[ \mathcal{CI}_{i,1-2} = \frac{\mathcal{CE}_{i,1} + \mathcal{CE}_{i,2}}{\gamma_i} = \mathcal{CI}_{i,1} + \mathcal{CI}_{i,2} \]

and:

\[ \mathcal{CI}_{i,1-3} = \mathcal{CI}_{i,1} + \mathcal{CI}_{i,2} + \mathcal{CI}_{i,3} \]

We deduce that:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{CI}_{i,1} )</td>
<td>0.25</td>
<td>15.24</td>
<td>5.76</td>
<td>0.50</td>
<td>12.50</td>
<td>0.25</td>
<td>280.37</td>
<td>0.20</td>
</tr>
<tr>
<td>( \mathcal{CI}_{i,1-2} )</td>
<td>0.50</td>
<td>30.49</td>
<td>14.00</td>
<td>4.00</td>
<td>35.00</td>
<td>0.29</td>
<td>299.07</td>
<td>2.76</td>
</tr>
<tr>
<td>( \mathcal{CI}_{i,1-3} )</td>
<td>80.50</td>
<td>76.22</td>
<td>23.68</td>
<td>9.50</td>
<td>37.50</td>
<td>2.12</td>
<td>579.44</td>
<td>10.72</td>
</tr>
</tbody>
</table>
Question (c)

Deduce the weighted average carbon intensity (WACI) of the benchmark if we consider the scope $1 + 2 + 3$. 
We have:

\[ CI (b) = \sum_{i=1}^{8} b_i CI_i \]

\[ = 0.22 \times 80.50 + 0.19 \times 76.2195 + 0.17 \times 23.68 + 0.13 \times 9.50 + \]

\[ 0.11 \times 37.50 + 0.08 \times 2.1275 + 0.06 \times 579.4393 + 0.04 \times 10.72 \]

\[ = 76.9427 \text{ tCO}_2\text{e}/\$ \text{ mn} \]
Question (d)

We assume that the market capitalization of the benchmark portfolio is equal to $10\text{ tn}$ and we invest $1\text{ bn}$. 
Question (d).i

Deduce the market capitalization of each company (expressed in $ bn).
We have:

\[ b_i = \frac{MC_i}{\sum_{k=1}^{8} MC_k} \]

and \( \sum_{k=1}^{8} MC_k = $10 \text{ tn.} \) We deduce that:

\[ MC_i = 10 \times b_i \]

We obtain the following values of market capitalization expressed in $ \text{bn}:

<table>
<thead>
<tr>
<th>Issuer</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC_i</td>
<td>2200</td>
<td>1900</td>
<td>1700</td>
<td>1300</td>
<td>1100</td>
<td>800</td>
<td>600</td>
<td>400</td>
</tr>
</tbody>
</table>
Question (d).ii

Compute the ownership ratio for each asset (expressed in bps).
Let $W$ be the wealth invested in the benchmark portfolio $b$. The wealth invested in asset $i$ is equal to $b_i W$. We deduce that the ownership ratio is equal to:

$$\omega_i = \frac{b_i W}{MC_i} = \frac{b_i W}{b_i \sum_{k=1}^{n} MC_k} = \frac{W}{\sum_{k=1}^{n} MC_k}$$

When we invest in a capitalization-weighted portfolio, the ownership ratio is the same for all the assets. In our case, we have:

$$\omega_i = \frac{1}{10 \times 1000} = 0.01\%$$

The ownership ratio is equal to 1 basis point.
Question (d).iii

Compute the carbon emissions of the benchmark portfolio if we invest $1$ bn and we consider the scope $1 + 2 + 3$.

We assume that the float percentage is equal to $100\%$ for all the 8 companies.
Using the financed emissions approach, the carbon emissions of our investment is equal to:

\[ CE (\$1 \text{ bn}) = 0.01\% \times (75 + 75 + 24000) + 0.01\% \times (5000 + 5000 + 15000) + \ldots + 0.01\% \times (5 + 64 + 199) = 12.3045 \text{ ktCO}_2e \]
Question (d).iv

Compare the (exact) carbon intensity of the benchmark portfolio with the WACI value obtained in Question 1.(c).
We compute the revenues of our investment:

$$Y \ ($1 \text{ bn}) = 0.01\% \sum_{i=1}^{8} Y_i = $0.1287 \text{ bn}$$

We deduce that the exact carbon intensity is equal to:

$$CI \ ($1 \text{ bn}) = \frac{CE \ ($1 \text{ bn})}{Y \ ($1 \text{ bn})} = \frac{12.3045}{0.1287} = 95.6061 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

We notice that the WACI of the benchmark underestimates the exact carbon intensity of our investment by 19.5%:

$$76.9427 < 95.6061$$
Question 2

We want to manage an equity portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $R$. We assume that the volatility of the stocks is respectively equal to 22\%, 20\%, 25\%, 18\%, 40\%, 23\%, 13\% and 29\%. The correlation matrix between these stocks is given by:

\[
\rho = \begin{pmatrix}
100 & 100 & 100 & 100 & 100 & 100 & 100 \\
80 & 100 & 75 & 70 & 65 & 60 & 50 \\
70 & 75 & 100 & 80 & 70 & 65 & 70 \\
60 & 65 & 80 & 100 & 70 & 65 & 65 \\
70 & 50 & 70 & 85 & 70 & 70 & 75 \\
50 & 60 & 70 & 80 & 60 & 60 & 50 \\
70 & 50 & 70 & 75 & 80 & 50 & 100 \\
60 & 65 & 70 & 65 & 70 & 60 & 100
\end{pmatrix}
\]
Question (a)

Compute the covariance matrix $\Sigma$. 
The covariance matrix $\Sigma = (\Sigma_{i,j})$ is defined by:

$$\Sigma_{i,j} = \rho_{i,j} \sigma_i \sigma_j$$

We obtain the following numerical values (expressed in bps):

$$\Sigma = \begin{pmatrix}
484.0 & 352.0 & 385.0 & 237.6 & 616.0 & 253.0 & 200.2 & 382.8 \\
352.0 & 400.0 & 375.0 & 234.0 & 400.0 & 276.0 & 130.0 & 377.0 \\
385.0 & 375.0 & 625.0 & 360.0 & 700.0 & 402.5 & 227.5 & 507.5 \\
237.6 & 234.0 & 360.0 & 324.0 & 612.0 & 331.2 & 175.5 & 391.5 \\
616.0 & 400.0 & 700.0 & 612.0 & 1600.0 & 552.0 & 416.0 & 754.0 \\
253.0 & 276.0 & 402.5 & 331.2 & 552.0 & 529.0 & 149.5 & 466.9 \\
200.2 & 130.0 & 227.5 & 175.5 & 416.0 & 149.5 & 169.0 & 226.2 \\
382.8 & 377.0 & 507.5 & 391.5 & 754.0 & 466.9 & 226.2 & 841.0
\end{pmatrix}$$
Question (b)

Write the optimization problem if the objective function is to minimize the tracking error risk under the constraint of carbon intensity reduction.
The tracking error variance of portfolio \( w \) with respect to benchmark \( b \) is equal to:

\[
\sigma^2 (w \mid b) = (w - b)^\top \Sigma (w - b)
\]

The carbon intensity constraint has the following expression:

\[
\sum_{i=1}^{8} w_i CI_i \leq (1 - R) CI(b)
\]

where \( R \) is the reduction rate and \( CI(b) \) is the carbon intensity of the benchmark. Let \( CI^* = (1 - R) CI(b) \) be the target value of the carbon footprint. The optimization problem is then:

\[
w^* = \arg\min \frac{1}{2} \sigma^2 (w \mid b)
\]

subject to:

\[
\begin{align*}
\sum_{i=1}^{8} w_i CI_i &\leq CI^* \\
\sum_{i=1}^{8} w_i &= 1 \\
0 &\leq w_i \leq 1
\end{align*}
\]

We add the second and third constraints in order to obtain a long-only portfolio.
Question (c)

Give the QP formulation of the optimization problem.
The objective function is equal to:

\[ f(w) = \frac{1}{2} \sigma^2 (w | b) = \frac{1}{2} (w - b)^\top \Sigma (w - b) = \frac{1}{2} w^\top \Sigma w - w^\top \Sigma b + \frac{1}{2} b^\top \Sigma b \]

while the matrix form of the carbon intensity constraint is:

\[ CI^\top w \leq CI^\star \]

where \( CI = (CI_1, \ldots, CI_8) \) is the column vector of carbon intensities. Since \( b^\top \Sigma b \) is a constant and does not depend on \( w \), we can cast the previous optimization problem into a QP problem:

\[
\begin{align*}
    w^* &= \text{arg min } \frac{1}{2} w^\top Q w - w^\top R \\
    \text{s.t.} & \quad A w = B \\
    & \quad C w \leq D \\
    & \quad w^- \leq w \leq w^+ 
\end{align*}
\]

We have \( Q = \Sigma, R = \Sigma b, A = 1_8^\top, B = 1, C = CI^\top, D = CI^\star, w^- = 0_8 \) and \( w^+ = 1_8 \).
Question (d)

\( R \) is equal to 20%. Find the optimal portfolio if we target scope 1 + 2. What is the value of the tracking error volatility?
We have:

\[ CI(b) = 0.22 \times 0.50 + 0.19 \times 30.4878 + \ldots + 0.04 \times 2.76 \]
\[ = 30.7305 \text{ tCO}_2\text{e/}$ \text{mn} \]

We deduce that:

\[ CI^* = (1 - R) CI(b) = 0.80 \times 30.7305 = 24.5844 \text{ tCO}_2\text{e/}$ \text{mn} \]

Therefore, the inequality constraint of the QP problem is:

\[
\begin{pmatrix}
0.50 & 30.49 & 14.00 & 4.00 & 35.00 & 0.29 & 299.07 & 2.76 \\
\vdots \\
0.50 & 30.49 & 14.00 & 4.00 & 35.00 & 0.29 & 299.07 & 2.76 \\
\end{pmatrix}
\begin{pmatrix}
w_1 \\
w_2 \\
\vdots \\
w_7 \\
w_8 \\
\end{pmatrix}
\leq 24.5844
We obtain the following optimal solution:

\[
\begin{pmatrix}
23.4961 \\
17.8129 \\
17.1278 \\
15.4643 \\
10.4037 \\
7.5903 \\
4.0946 \\
4.0104
\end{pmatrix}
\]

The minimum tracking error volatility \( \sigma (w^* | b) \) is equal to 15.37 bps.
Question (e)

Same question if $R$ is equal to 30%, 50%, and 70%.
Table 194: Solution of the equity optimization problem (scope $SC_{1-2}$)

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>0%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>23.4961</td>
<td>24.2441</td>
<td>25.7402</td>
<td>30.4117</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.8129</td>
<td>17.2194</td>
<td>16.0323</td>
<td>9.8310</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>17.1278</td>
<td>17.1917</td>
<td>17.3194</td>
<td>17.8348</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>15.4643</td>
<td>16.6964</td>
<td>19.1606</td>
<td>23.3934</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.4037</td>
<td>10.1055</td>
<td>9.5091</td>
<td>7.1088</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>7.5903</td>
<td>7.3854</td>
<td>6.9757</td>
<td>6.7329</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>4.0946</td>
<td>3.1418</td>
<td>1.2364</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>4.0104</td>
<td>4.0157</td>
<td>4.0261</td>
<td>4.6874</td>
</tr>
<tr>
<td>$\mathcal{C}(w)$</td>
<td>30.7305</td>
<td>24.5844</td>
<td>21.5114</td>
<td>15.3653</td>
<td>9.2192</td>
</tr>
<tr>
<td>$\sigma(w</td>
<td>b)$</td>
<td>0.00</td>
<td>15.37</td>
<td>23.05</td>
<td>38.42</td>
</tr>
</tbody>
</table>
In Table 194, we report the optimal solution $w^*$ (expressed in %) of the optimization problem for different values of $R$. We also indicate the carbon intensity of the portfolio (in tCO$_2$/mn) and the tracking error volatility (in bps). For instance, if $R$ is set to 50%, the weights of assets #1, #3, #4 and #8 increase whereas the weights of assets #2, #5, #6 and #7 decrease. The carbon intensity of this portfolio is equal to 15.3653 tCO$_2$/mn. The tracking error volatility is below 40 bps, which is relatively low.
Question (f)

We target scope 1 + 2 + 3. Find the optimal portfolio if $R$ is equal to 20%, 30%, 50% and 70%. Give the value of the tracking error volatility for each optimized portfolio.
In this case, the inequality constraint $Cw \leq D$ is defined by:

$$
C = CI_1^\top = \begin{pmatrix}
80.5000 \\
76.2195 \\
23.6800 \\
9.5000 \\
37.5000 \\
2.1275 \\
579.4393 \\
10.7200 \\
\end{pmatrix}^\top
$$

and:

$$
D = (1 - R) \times 76.9427
$$
We obtain the results given in Table 195.

**Table 195: Solution of the equity optimization problem (scope $SC_{1-3}$)**

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>0%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>23.9666</td>
<td>24.9499</td>
<td>26.4870</td>
<td>13.6749</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.4410</td>
<td>16.6615</td>
<td>8.8001</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>17.1988</td>
<td>17.2981</td>
<td>19.4253</td>
<td>24.1464</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>16.5034</td>
<td>18.2552</td>
<td>25.8926</td>
<td>41.0535</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.2049</td>
<td>9.8073</td>
<td>7.1330</td>
<td>3.5676</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>7.4169</td>
<td>7.1254</td>
<td>7.0659</td>
<td>8.8851</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>3.2641</td>
<td>1.8961</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>4.0043</td>
<td>4.0065</td>
<td>5.1961</td>
<td>8.6725</td>
</tr>
<tr>
<td>$\mathcal{C}I \left( \bar{w} \right)$</td>
<td>76.9427</td>
<td>61.5541</td>
<td>53.8599</td>
<td>38.4713</td>
<td>23.0828</td>
</tr>
<tr>
<td>$\sigma \left( \bar{w} \left</td>
<td>b \right. \right)$</td>
<td>0.00</td>
<td>21.99</td>
<td>32.99</td>
<td>104.81</td>
</tr>
</tbody>
</table>
Question (g)

Compare the optimal solutions obtained in Questions 2.(e) and 2.(f).
Figure 369: Impact of the scope on the tracking error volatility

\[
\sigma(w | b) \text{ (in %)} \quad \mathcal{R} \text{ (in %)}
\]

- SC_{1-2}
- SC_{1-3}
Figure 370: Impact of the scope on the portfolio allocation (in %)
In Figure 369, we report the relationship between the reduction rate $\mathcal{R}$ and the tracking error volatility $\sigma (w \mid b)$. The choice of the scope has little impact when $\mathcal{R} \leq 45\%$. Then, we notice a high increase when we consider the scope $1 + 2 + 3$. The portfolio’s weights are given in Figure 370. For assets #1 and #3, the behavior is divergent when we compare scopes $1 + 2$ and $1 + 2 + 3$. 
Question 3

We want to manage a bond portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $R$. We use the scope $1 + 2 + 3$. In the table below, we report the modified duration $MD_i$ and the duration-times-spread factor $DTS_i$ of each corporate bond $i$:

<table>
<thead>
<tr>
<th>Asset</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD$_i$ (in years)</td>
<td>3.56</td>
<td>7.48</td>
<td>6.54</td>
<td>10.23</td>
<td>2.40</td>
<td>2.30</td>
<td>9.12</td>
<td>7.96</td>
</tr>
<tr>
<td>DTS$_i$ (in bps)</td>
<td>103</td>
<td>155</td>
<td>75</td>
<td>796</td>
<td>89</td>
<td>45</td>
<td>320</td>
<td>245</td>
</tr>
</tbody>
</table>

$S$ector

1 2 1 1 2 1 2 2
Question 3 (Cont’d)

We remind that the active risk can be calculated using three functions. For the active share, we have:

\[ R_{AS} (w \mid b) = \sigma_{AS}^2 (w \mid b) = \sum_{i=1}^{n} (w_i - b_i)^2 \]

We also consider the MD-based tracking error risk:

\[ R_{MD} (w \mid b) = \sigma_{MD}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in \text{sector}_j} (w_i - b_i) \text{MD}_i \right)^2 \]

and the DTS-based tracking error risk:

\[ R_{DTS} (w \mid b) = \sigma_{DTS}^2 (w \mid b) = \sum_{j=1}^{n_{sector}} \left( \sum_{i \in \text{sector}_j} (w_i - b_i) \text{DTS}_i \right)^2 \]
Question 3 (Cont’d)

Finally, we define the synthetic risk measure as a combination of AS, MD and DTS active risks:

\[ R(w | b) = \varphi_{AS} R_{AS}(w | b) + \varphi_{MD} R_{MD}(w | b) + \varphi_{DTS} R_{DTS}(w | b) \]

where \( \varphi_{AS} \geq 0, \varphi_{MD} \geq 0 \) and \( \varphi_{DTS} \geq 0 \) indicate the weight of each risk. In what follows, we use the following numerical values: \( \varphi_{AS} = 100, \varphi_{MD} = 25 \) and \( \varphi_{DTS} = 1 \). The reduction rate \( R \) of the weighted average carbon intensity is set to 50% for the scope 1 + 2 + 3.
Question (a)

Compute the modified duration $\text{MD} (b)$ and the duration-times-spread factor $\text{DTS} (b)$ of the benchmark.
We have:

\[
MD (b) = \sum_{i=1}^{n} b_i MD_i \\
= 0.22 \times 3.56 + 0.19 \times 7.48 + \ldots + 0.04 \times 7.96 \\
= 5.96 \text{ years}
\]

and:

\[
DTS (b) = \sum_{i=1}^{n} b_i DTS_i \\
= 0.22 \times 103 + 0.19 \times 155 + \ldots + 0.04 \times 155 \\
= 210.73 \text{ bps}
\]
Question (b)

Let $w_{ew}$ be the equally-weighted portfolio. Compute $^a$ $\mathcal{MD}(w_{ew})$, $DTS(w_{ew})$, $\sigma_{AS}(w_{ew} \mid b)$, $\sigma_{MD}(w_{ew} \mid b)$ and $\sigma_{DTS}(w_{ew} \mid b)$.

$^a$ Precise the corresponding unit (years, bps or %) for each metric.
We have:

\[
\begin{align*}
\text{MD}(w_{\text{ew}}) &= 6.20 \text{ years} \\
\text{DTS}(w_{\text{ew}}) &= 228.50 \text{ bps} \\
\sigma_{\text{AS}}(w_{\text{ew}} | b) &= 17.03\% \\
\sigma_{\text{MD}}(w_{\text{ew}} | b) &= 1.00 \text{ years} \\
\sigma_{\text{DTS}}(w_{\text{ew}} | b) &= 36.19 \text{ bps}
\end{align*}
\]
Question (c)

We consider the following optimization problem:

\[
w^* = \arg \min \frac{1}{2} \mathcal{R}_{AS} (w \mid b)
\]
\[
\begin{align*}
\sum_{i=1}^{n} w_i &= 1 \\
\text{MD} (w) &= \text{MD} (b) \\
\text{DTS} (w) &= \text{DTS} (b) \\
\mathcal{CI} (w) &\leq (1 - \mathcal{R}) \mathcal{CI} (b) \\
0 &\leq w_i \leq 1
\end{align*}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( w^* \). Compute \( \text{MD} (w^*) \), \( \text{DTS} (w^*) \), \( \sigma_{AS} (w^* \mid b) \), \( \sigma_{MD} (w^* \mid b) \) and \( \sigma_{DTS} (w^* \mid b) \).
We have:

\[
R_{AS} (w \mid b) = (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 + (w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2
\]

The objective function is then:

\[
f (w) = \frac{1}{2} R_{AS} (w \mid b)
\]

The optimal solution is equal to:

\[
w^* = (17.30\%, 17.41\%, 20.95\%, 14.41\%, 10.02\%, 11.09\%, 0\%, 8.81\%)
\]

The risk metrics are:

\[
\begin{align*}
MD (w^*) &= 5.96 \text{ years} \\
DTS (w^*) &= 210.73 \text{ bps} \\
\sigma_{AS} (w^* \mid b) &= 10.57\% \\
\sigma_{MD} (w^* \mid b) &= 0.43 \text{ years} \\
\sigma_{DTS} (w^* \mid b) &= 15.21 \text{ bps}
\end{align*}
\]
Question (d)

We consider the following optimization problem:

\[
\begin{align*}
    w^* &= \arg \min \frac{\varphi_{AS}}{2} \mathcal{R}_{AS} (w \mid b) + \frac{\varphi_{MD}}{2} \mathcal{R}_{MD} (w \mid b) \\
    \text{s.t.} \quad &\sum_{i=1}^{n} w_i = 1 \\
    &\text{DTS}(w) = \text{DTS}(b) \\
    &\text{CI}(w) \leq (1 - \mathcal{R}) \text{CI}(b) \\
    &0 \leq w_i \leq 1
\end{align*}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( w^* \). Compute \( \text{MD}(w^*) \), \( \text{DTS}(w^*) \), \( \sigma_{AS}(w^* \mid b) \), \( \sigma_{MD}(w^* \mid b) \) and \( \sigma_{DTS}(w^* \mid b) \).
We have\(^{39}\):

\[
R_{\text{MD}} (w | b) = \left( \sum_{i=1,3,4,6} (w_i - b_i) \text{MD}_i \right)^2 + \left( \sum_{i=2,5,7,8} (w_i - b_i) \text{MD}_i \right)^2
\]

\[
= \left( \sum_{i=1,3,4,6} w_i \text{MD}_i - \sum_{i=1,3,4,6} b_i \text{MD}_i \right)^2 + \left( \sum_{i=2,5,7,8} w_i \text{MD}_i - \sum_{i=2,5,7,8} b_i \text{MD}_i \right)^2
\]

\[
= (3.56w_1 + 6.54w_3 + 10.23w_4 + 2.30w_6 - 3.4089)^2 + (7.48w_2 + 2.40w_5 + 9.12w_7 + 7.96w_8 - 2.5508)^2
\]

The objective function is then:

\[
f (w) = \frac{\varphi_{\text{AS}}}{2} R_{\text{AS}} (w | b) + \frac{\varphi_{\text{MD}}}{2} R_{\text{MD}} (w | b)
\]

\(^{39}\)We verify that \(3.4089 + 2.5508 = 5.9597\) years.
The optimal solution is equal to:

\[ w^* = (16.31\%, 18.44\%, 17.70\%, 13.82\%, 11.67\%, 11.18\%, 0\%, 10.88\%) \]

The risk metrics are:

\[
\begin{align*}
&\text{MD}(w^*) = 5.93 \text{ years} \\
&DTS(w^*) = 210.73 \text{ bps} \\
&\sigma_{AS}(w^* | b) = 11.30\% \\
&\sigma_{MD}(w^* | b) = 0.03 \text{ years} \\
&\sigma_{DTS}(w^* | b) = 3.70 \text{ bps}
\end{align*}
\]
Question (e)

We consider the following optimization problem:

\[ \mathbf{w}^\star = \arg \min \frac{1}{2} \mathcal{R}(\mathbf{w} | \mathbf{b}) \]
\[
\text{s.t. } \begin{cases} 
\sum_{i=1}^{n} w_i = 1 \\
\mathcal{C}\mathcal{I}(\mathbf{w}) \leq (1 - \mathcal{R}) \mathcal{C}\mathcal{I}(\mathbf{b}) \\
0 \leq w_i \leq 1
\end{cases}
\]

Give the analytical value of the objective function. Find the optimal portfolio \( \mathbf{w}^\star \). Compute \( \text{MD}(\mathbf{w}^\star) \), \( \text{DTS}(\mathbf{w}^\star) \), \( \sigma_{\text{AS}}(\mathbf{w}^\star | \mathbf{b}) \), \( \sigma_{\text{MD}}(\mathbf{w}^\star | \mathbf{b}) \) and \( \sigma_{\text{DTS}}(\mathbf{w}^\star | \mathbf{b}) \).
We have\(^40\):

\[
\mathcal{R}_{\text{DTS}} (w | b) = \left( \sum_{i=1,3,4,6} (w_i - b_i) \text{DTS}_i \right)^2 + \left( \sum_{i=2,5,7,8} (w_i - b_i) \text{DTS}_i \right)^2 = (103w_1 + 75w_3 + 796w_4 + 45w_6 - 142.49)^2 + (155w_2 + 89w_5 + 320w_7 + 245w_8 - 68.24)^2
\]

The objective function is then:

\[
f (w) = \frac{\varphi_{\text{AS}}}{2} \mathcal{R}_{\text{AS}} (w | b) + \frac{\varphi_{\text{MD}}}{2} \mathcal{R}_{\text{MD}} (w | b) + \frac{\varphi_{\text{DTS}}}{2} \mathcal{R}_{\text{DTS}} (w | b)
\]

\(^40\)We verify that 142.49 + 68.24 = 210.73 bps.
The optimal solution is equal to:

\[ w^* = (16.98\%, 17.21\%, 18.26\%, 13.45\%, 12.10\%, 9.46\%, 0\%, 12.55\%) \]

The risk metrics are:

\[
\begin{align*}
&MD (w^*) = 5.97 \text{ years} \\
&DTS (w^*) = 210.68 \text{ bps} \\
&\sigma_{AS} (w^* | b) = 11.94\% \\
&\sigma_{MD} (w^* | b) = 0.03 \text{ years} \\
&\sigma_{DTS} (w^* | b) = 0.06 \text{ bps}
\end{align*}
\]
Question (f)

Comment on the results obtained in Questions 3.(c), 3.(d) and 3.(e).
Table 196: Solution of the bond optimization problem (scope $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Benchmark</th>
<th>3.(c)</th>
<th>3.(d)</th>
<th>3.(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>17.3049</td>
<td>16.3102</td>
<td>16.9797</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.4119</td>
<td>18.4420</td>
<td>17.2101</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>20.9523</td>
<td>17.6993</td>
<td>18.2582</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>14.4113</td>
<td>13.8195</td>
<td>13.4494</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>10.0239</td>
<td>11.6729</td>
<td>12.1008</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>11.0881</td>
<td>11.1792</td>
<td>9.4553</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>8.8075</td>
<td>10.8769</td>
<td>12.5464</td>
</tr>
<tr>
<td>$\text{MD}(w)$</td>
<td>5.9597</td>
<td>5.9597</td>
<td>5.9344</td>
<td>5.9683</td>
</tr>
<tr>
<td>$\text{DTS}(w)$</td>
<td>210.7300</td>
<td>210.7300</td>
<td>210.7300</td>
<td>210.6791</td>
</tr>
<tr>
<td>$\sigma_{\text{AS}}(w \mid b)$</td>
<td>0.0000</td>
<td>10.5726</td>
<td>11.3004</td>
<td>11.9400</td>
</tr>
<tr>
<td>$\sigma_{\text{MD}}(w \mid b)$</td>
<td>0.0000</td>
<td>0.4338</td>
<td>0.0254</td>
<td>0.0308</td>
</tr>
<tr>
<td>$\sigma_{\text{DTS}}(w \mid b)$</td>
<td>0.0000</td>
<td>15.2056</td>
<td>3.7018</td>
<td>0.0561</td>
</tr>
<tr>
<td>$\mathcal{CI}(w)$</td>
<td>76.9427</td>
<td>38.4713</td>
<td>38.4713</td>
<td>38.4713</td>
</tr>
</tbody>
</table>
Question (g)

How to find the previous solution of Question 3.(e) using a QP solver?
The goal is to write the objective function into a quadratic function:

\[
f (w) = \frac{\varphi_{AS}}{2} R_{AS} (w \mid b) + \frac{\varphi_{MD}}{2} R_{MD} (w \mid b) + \frac{\varphi_{DTS}}{2} R_{DTS} (w \mid b)
\]

\[
= \frac{1}{2} w^\top Q (b) w - w^\top R (b) + c (b)
\]

where:

\[
R_{AS} (w \mid b) = (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 +
(w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2
\]

\[
R_{MD} (w \mid b) = (3.56w_1 + 6.54w_3 + 10.23w_4 + 2.30w_6 - 3.4089)^2 +
(7.48w_2 + 2.40w_5 + 9.12w_7 + 7.96w_8 - 2.5508)^2
\]

\[
R_{DTS} (w \mid b) = (103w_1 + 75w_3 + 796w_4 + 45w_6 - 142.49)^2 +
(155w_2 + 89w_5 + 320w_7 + 245w_8 - 68.24)^2
\]
We use the analytical approach which is described in Section 11.1.2 on pages 332-339. Moreover, we rearrange the universe such that the first fourth assets belong to the first sector and the last fourth assets belong to the second sector. In this case, we have:

\[
\begin{pmatrix}
  w_1, w_3, w_4, w_6, w_2, w_5, w_7, w_8 \\
  \text{\underline{Sector}_1} & \text{\underline{Sector}_2}
\end{pmatrix}
\]
The matrix $Q(b)$ is block-diagonal:

$$Q(b) = \begin{pmatrix} Q_1 & 0_{4,4} \\ 0_{4,4} & Q_2 \end{pmatrix}$$

where the matrices $Q_1$ and $Q_2$ are equal to:

$$Q_1 = \begin{pmatrix} 11025.8400 & 8307.0600 & 82898.4700 & 4839.7000 \\ 8307.0600 & 6794.2900 & 61372.6050 & 3751.0500 \\ 82898.4700 & 61372.6050 & 636332.3225 & 36408.2250 \\ 4839.7000 & 3751.0500 & 36408.2250 & 2257.2500 \end{pmatrix}$$

and:

$$Q_2 = \begin{pmatrix} 25523.7600 & 14243.8000 & 51305.4400 & 39463.5200 \\ 14243.8000 & 8165.0000 & 29027.2000 & 22282.6000 \\ 51305.4400 & 29027.2000 & 104579.3600 & 80214.8800 \\ 39463.5200 & 22282.6000 & 80214.8800 & 61709.0400 \end{pmatrix}$$
The vector \( R(b) \) is defined as follows:

\[
R(b) = \begin{pmatrix}
15001.8621 \\
11261.1051 \\
114306.8662 \\
6616.0617 \\
11073.1996 \\
6237.4080 \\
22424.3824 \\
17230.4092
\end{pmatrix}
\]

Finally, the value of \( c(b) \) is equal to:

\[
c(b) = 12714.3386
\]
Using a QP solver, we obtain the following numerical solution:

\[
\begin{pmatrix}
  w_1 \\
  w_3 \\
  w_4 \\
  w_6 \\
  w_2 \\
  w_5 \\
  w_7 \\
  w_8 \\
\end{pmatrix}
= 
\begin{pmatrix}
  16.9796 \\
  18.2582 \\
  13.4494 \\
  9.4553 \\
  17.2102 \\
  12.1009 \\
  0.0000 \\
  12.5464 \\
\end{pmatrix}
\times 10^{-2}
\]

We observe some small differences (after the fifth digit) because the QP solver is more efficient than a traditional nonlinear solver.
Question 4

We consider a variant of Question 3 and assume that the synthetic risk measure is:

\[ \mathcal{D}(w \mid b) = \varphi_{AS} \mathcal{D}_{AS}(w \mid b) + \varphi_{MD} \mathcal{D}_{MD}(w \mid b) + \varphi_{DTS} \mathcal{D}_{DTS}(w \mid b) \]

where:

\[ \mathcal{D}_{AS}(w \mid b) = \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| \]

\[ \mathcal{D}_{MD}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{MD}_i \right| \]

\[ \mathcal{D}_{DTS}(w \mid b) = \sum_{j=1}^{n_{\text{sector}}} \left| \sum_{i \in \text{Sector}_j} (w_i - b_i) \text{DTS}_i \right| \]
Question (a)

Define the corresponding optimization problem when the objective is to minimize the active risk and reduce the carbon intensity of the benchmark by $\mathcal{R}$. 
The optimization problem is:

\[ w^* = \arg \min D(w \mid b) \]

\[
\begin{aligned}
\text{s.t.} \quad & \begin{cases}
1_8^T w = 1 \\
\mathcal{C} \mathcal{I}^T w \leq (1 - \mathcal{R}) \mathcal{C} \mathcal{I} (b) \\
0_8 \leq w \leq 1_8
\end{cases}
\end{aligned}
\]
Question (b)

Give the LP formulation of the optimization problem.
We use the absolute value trick and obtain the following optimization problem:

\[ \begin{align*}
    w^* &= \arg \min \frac{1}{2} \varphi_{\text{AS}} \sum_{i=1}^{8} \tau_{i,w} + \varphi_{\text{MD}} \sum_{j=1}^{2} \tau_{j,\text{MD}} + \varphi_{\text{DTS}} \sum_{j=1}^{2} \tau_{j,\text{DTS}} \\
    &\quad \text{s.t.} \quad 1^T_8 w = 1 \\
    &\quad \quad 0_8 \leq w \leq 1_8 \\
    &\quad \quad CI^T w \leq (1 - \mathcal{R}) CI (b) \\
    &\quad \quad |w_i - b_i| \leq \tau_{i,w} \\
    &\quad \quad \sum_{i \in \text{sector}_j} (w_i - b_i) \text{MD}_i \leq \tau_{j,\text{MD}} \\
    &\quad \quad \sum_{i \in \text{sector}_j} (w_i - b_i) \text{DTS}_i \leq \tau_{j,\text{DTS}} \\
    &\quad \quad \tau_{i,w} \geq 0, \tau_{j,\text{MD}} \geq 0, \tau_{j,\text{DTS}} \geq 0
\end{align*} \]
We can now formulate this problem as a standard LP problem:

\[ x^* = \arg \min c^\top x \]

\[
\begin{align*}
A x &= B \\
C x &\leq D \\
x^- &\leq x \leq x^+
\end{align*}
\]

where \( x \) is the 20 \( \times \) 1 vector defined as follows:

\[
x = \begin{pmatrix}
w \\
\tau_w \\
\tau_{MD} \\
\tau_{DTS}
\end{pmatrix}
\]
The $20 \times 1$ vector $c$ is equal to:

$$c = \begin{pmatrix}
0_8 \\
\frac{1}{2} \varphi_{AS} \mathbf{1}_8 \\
\varphi_{MD} \mathbf{1}_2 \\
\varphi_{DTS} \mathbf{1}_2
\end{pmatrix}$$

The equality constraint is defined by $A = \begin{pmatrix}
\mathbf{1}_8^T & 0_8^T & 0_2^T & 0_2^T
\end{pmatrix}$ and $B = 1$. The bounds are $x^- = 0_{20}$ and $x^+ = \infty \cdot \mathbf{1}_{20}$. 
For the inequality constraint, we have\(^4\):

\[
C x \leq D \iff \begin{pmatrix}
I_8 & -I_8 & 0_{8,2} & 0_{8,2} \\
-I_8 & -I_8 & 0_{8,2} & 0_{8,2} \\
C_{MD} & 0_{2,8} & -I_2 & 0_{2,2} \\
-C_{MD} & 0_{2,8} & -I_2 & 0_{2,2} \\
C_{DTS} & 0_{2,8} & 0_{2,2} & -I_2 \\
-C_{DTS} & 0_{2,8} & 0_{2,2} & -I_2 \\
CI^\top & 0_{1,8} & 0 & 0
\end{pmatrix} x \leq \begin{pmatrix}
-\ b \\
-\ b \\
-MD^* \\
-MD^* \\
-DTS^* \\
-DTS^* \\
(1 - R) CI (b)
\end{pmatrix}
\]

where:

\[
C_{MD} = \begin{pmatrix}
3.56 & 0.00 & 6.54 & 10.23 & 0.00 & 2.30 & 0.00 & 0.00 \\
0.00 & 7.48 & 0.00 & 0.00 & 2.40 & 0.00 & 9.12 & 7.96
\end{pmatrix}
\]

and:

\[
C_{DTS} = \begin{pmatrix}
103 & 0 & 75 & 796 & 0 & 45 & 0 & 0 \\
0 & 155 & 0 & 0 & 89 & 0 & 320 & 245
\end{pmatrix}
\]

The \(2 \times 1\) vectors \(MD^*\) and \(DTS^*\) are respectively equal to \((3.4089, 2.5508)\) and \((142.49, 68.24)\).

\(^4\) \(C\) is a \(25 \times 8\) matrix and \(D\) is a \(25 \times 1\) vector.
Question (c)
Find the optimal portfolio when $R$ is set to 50%. Compare the solution with this obtained in Question 3.(e).
We obtain the following solution:

\[ w^* = (18.7360, 15.8657, 17.8575, 13.2589, 11, 9.4622, 0, 13.8196) \times 10^{-2} \]
\[ \tau_{w}^* = (3.2640, 3.1343, 0.8575, 0.2589, 0, 1.4622, 6, 9.8196) \times 10^{-2} \]
\[ \tau_{MD} = (0, 0) \]
\[ \tau_{DTS} = (0, 0) \]
Table 197: Solution of the bond optimization problem (scope $SC_{1-3}$)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Benchmark</th>
<th>3.(e)</th>
<th>4.(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>22.0000</td>
<td>16.9796</td>
<td>18.7360</td>
</tr>
<tr>
<td>$w_2$</td>
<td>19.0000</td>
<td>17.2102</td>
<td>15.8657</td>
</tr>
<tr>
<td>$w_3$</td>
<td>17.0000</td>
<td>18.2582</td>
<td>17.8575</td>
</tr>
<tr>
<td>$w_4$</td>
<td>13.0000</td>
<td>13.4494</td>
<td>13.2589</td>
</tr>
<tr>
<td>$w_5$</td>
<td>11.0000</td>
<td>12.1009</td>
<td>11.0000</td>
</tr>
<tr>
<td>$w_6$</td>
<td>8.0000</td>
<td>9.4553</td>
<td>9.4622</td>
</tr>
<tr>
<td>$w_7$</td>
<td>6.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$w_8$</td>
<td>4.0000</td>
<td>12.5464</td>
<td>13.8196</td>
</tr>
<tr>
<td>$\text{MD}(w)$</td>
<td>5.9597</td>
<td>5.9683</td>
<td>5.9597</td>
</tr>
<tr>
<td>$\text{DTS}(w)$</td>
<td>210.7300</td>
<td>210.6791</td>
<td>210.7300</td>
</tr>
<tr>
<td>$\sigma_{AS}(w | b)$</td>
<td>0.0000</td>
<td>11.9400</td>
<td>12.4837</td>
</tr>
<tr>
<td>$\sigma_{MD}(w | b)$</td>
<td>0.0000</td>
<td>0.0308</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_{DTS}(w | b)$</td>
<td>0.0000</td>
<td>0.0561</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{D}_{AS}(w | b)$</td>
<td>0.0000</td>
<td>25.6203</td>
<td>24.7964</td>
</tr>
<tr>
<td>$\mathcal{D}_{MD}(w | b)$</td>
<td>0.0000</td>
<td>0.0426</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{D}_{DTS}(w | b)$</td>
<td>0.0000</td>
<td>0.0608</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{CI}(w)$</td>
<td>76.9427</td>
<td>38.4713</td>
<td>38.4713</td>
</tr>
</tbody>
</table>
In Table 197, we compare the two solutions\textsuperscript{42}. They are very close. In fact, we notice that the LP solution matches perfectly the MD and DTS constraints, but has a higher AS risk $\sigma_{AS} (w \mid b)$. If we note the two solutions $w^* (\mathcal{L}_1)$ and $w^* (\mathcal{L}_2)$, we have:

\[
\begin{align*}
\mathcal{R} (w^* (\mathcal{L}_2) \mid b) &= 1.4524 < \mathcal{R} (w^* (\mathcal{L}_1) \mid b) = 1.5584 \\
\mathcal{D} (w^* (\mathcal{L}_2) \mid b) &= 13.9366 > \mathcal{D} (w^* (\mathcal{L}_1) \mid b) = 12.3982
\end{align*}
\]

There is a trade-off between the $\mathcal{L}_1$- and $\mathcal{L}_2$-norm risk measures. This is why we cannot say that one solution dominates the other.

\textsuperscript{42}The units are the following: % for the weights $w_i$, and the active share metrics $\sigma_{AS} (w \mid b)$ and $\mathcal{D}_{AS} (w \mid b)$; years for the modified duration metrics $\mathcal{MD} (w)$, $\sigma_{MD} (w \mid b)$ and $\mathcal{D}_{MD} (w \mid b)$; bps for the duration-times-spread metrics $\mathcal{DTS} (w)$, $\sigma_{DTS} (w \mid b)$ and $\mathcal{D}_{DTS} (w \mid b)$; tCO$_2$e/$\$ mn for the carbon intensity $\mathcal{DTS} (w)$.
Course 2023-2024 in Sustainable Finance
Lecture 14. Physical Risk Modeling

Thierry Roncalli*

* Amundi Asset Management

* University of Paris-Saclay

March 2024

43 The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Prologue: Global temperatures (1900-2023)

Figure 371: Global temperatures (1900-1904)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 372: Global temperatures (1950-1954)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 373: Global temperatures (1970-1974)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 374: Global temperatures (1980-1984)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 375: Global temperatures (1990-1994)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 376: Global temperatures (2000-2004)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 377: Global temperatures (2010-2014)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 378: Global temperatures (2015-2019)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Prologue: Global temperatures (1950-2023)

Figure 379: Global temperatures (2022)

Source: earthobservatory.nasa.gov/world-of-change/global-temperatures.
Definition

Figure 380: Systemic risk dynamics of climate-related physical risks

- **Climate hazard**
  - Changes in rainfall patterns
  - Drought 2040: 700 million people/yr
  - Agri drought 2040: 32% cropland/yr
  - Major heatwaves 2040: 3.9 billion people/yr
  - Storms and cyclones
  - Wildfires
  - River and coastal flooding 2040: 47 million people/yr

- **Impacts of concern**
  - Pests and diseases
  - Water scarcity
  - Crop failure
  - Too hot to work outside
  - Loss of infrastructure
  - Loss of livelihoods
  - Loss/shifts in ecosystems

- **Consequences**
  - Social unrest
  - Health crises
  - Unemployment and poverty
  - Deaths
  - Food crises
  - GDP loss
  - Business interruption
  - Market destabilization
  - Reduced global trade
  - Protectionism
  - Migration and displacement of people
    - Rural to urban
    - Refugee crisis
    - Forced/unsafe migration
    - Forced immobility (trapped populations)
  - Armed conflict
    - Regional conflicts
    - Rise of extremist groups
    - Police/military intervention
    - Organized crime and violence
    - Conflict between people and states
    - Civil war and war
  - Destabilization of markets
    - Commodity price spikes
    - Fall of asset prices
    - Large-scale asset sell-off
    - Falling stock markets
    - Underfunded pension funds
    - Financial market collapse

Physical risk and insurance companies
Physical risk and investors

Responsible investors have paid more attention to transition risk than to physical risk. But recent events show that the physical risk is also a major concern. This is the financial losses that actually result from climate change, rather than from adapting the economy to avoid them. It includes droughts, floods, storms, etc.
General circulation models

- Community Earth System Model (CESM)
- European Centre Hamburg Model (ECHAM)
- Hadley Centre Global Environment Model (HadGEM)
- Institut Pierre Simon Laplace Climate Model (IPSL-CM)
- Max Planck Institute Earth System Model (MPI-ESM)
- Norwegian Earth System Model (NorESM)
- Coupled Model Intercomparison Project (CMIP Phase 6)
Chronic vs. acute risk
Figure 381: Physical risk modeling
The climate data source is the set $\Theta_s = \{\theta (\lambda, \varphi, z, t)\}$

$\theta = (\theta_1, \ldots, \theta_k)$ is a vector of $k$ climate variables such as temperature, pressure or wind speed.

Each variable $\theta_k$ has four coordinates:

1. Latitude $\lambda$
2. Longitude $\varphi$
3. Height (or altitude) $z$
4. Time $t$

Three types of sources:

1. Meteorological records
2. Reanalysis
3. Historical simulations by a climate model
Figure 382: Slice* of wind speed (07/11/2013, tropical cyclone Haiyan)

Source: MERRA reanalysis, Global Modeling and Assimilation Office, NASA.

*This is a slice of the MERRA-2 reanalysis at a height of 10 meters on 7th November 2013.
The red dot is the location of the eye of the tropical cyclone Haiyan, which affected more than 10 million people in the Philippines.
Event intensity sensitivity

- We first have define the sensitivity of the intensity of extreme events to climate change.
- Let $\mathbb{E}[I(\Theta_s(C))]$ be the expected intensity of the event in the scenario associated with the GHG concentration $C$.
- The sensitivity of the event is equal to:

$$\Delta I(C) = \mathbb{E}[I(\Theta_s(C))] - I(\Theta_s(C_0))$$

where $I(\Theta_s(C_0))$ is the current intensity or the reference intensity in a scenario where climate objectives are met.
- For instance, we know that the maximum wind of tropical cyclones increases by more than 10% in scenarios with a high GHG concentration.
The asset value of the portfolio can then be written as:

\[
\Psi (t) = \sum_{j=1}^{n} x_j \Psi_j (\lambda, \varphi, t)
\]

where \( \Psi_j (\lambda, \varphi, t) \) is the geolocated asset value estimated at time \( t \) and \( x_j \) is the weight of asset \( j \) in the portfolio.

This requires the geolocation of the portfolio.
Statistical modeling of physical risk

Figure 383: Geolocation of world power plants by energy source

Source: Global Power Database version 1.3 (June 2021).
**Statistical modeling of physical risk**

**Vulnerability**

- The damage function $\Omega_j (I) \in [0, 1]$ is the fraction of property loss with respect to the intensity.
- It is generally calibrated on past damages (insurance claims, economic loss, etc.) and disasters.
Market pricing

- The physical risk implied by the concentration scenario $C$ is equal to:

$$
\Delta \text{Loss} (t, C) = \beta \cdot DD (t, C) = \beta \sum_{j=1}^{n} x_j \Psi_j (\lambda, \varphi, t) \Omega_j (\Delta I (t, C))
$$

- $\Delta \text{Loss} (t, C)$ is the relative loss due to the events on the portfolio
- $\beta$ is the transmission factor of the direct damage $DD (t, C)$ on the underlying to the loss of financial value in the investment portfolio
- For example, if the facilities of an energy producer are damaged at 50%, the securities issued by this company will be impacted at 50% $\times \beta$
Climate hazard location
Asset location

Two main modules:

- Simulation and generation of tropical cyclones under a given climate change scenario
- Geolocation of assets, damage modeling and loss estimation
Figure 384: What is a cyclone?

Figure 385: Modeling framework (Module 1)

Source: Le Guenedal et al. (2021).
Figure 386: Sample of storms (ERA-5 climate data)

Source: Le Guenedal et al. (2021).
Physics of cyclones

- Wind pressure relationship (Bloemendaal et al., 2020):
  \[ V = a(P_{\text{env}} - P_c)^b \]

- Maximum potential intensity (Holland, 1997; Emanuel, 1999):
  \[ MPI = f(y, SST, T_{\text{tropo}}, MSLP, RH, P_c) \]

- Maximum pressure drop (Bloemendaal et al., 2020):
  \[ MPD \sim P_{\text{env}} - P_c = A + Be^{C(SST-T_0)} \quad T_0 = 30^\circ C \]

- Pressure incremental variation (James and Mason, 2005):
  \[ \Delta t P_c (t) = c_0 + c_1 \Delta t P_c (t - 1) + c_2 e^{-c_3(P_c(t) - MPI(x,y,t))} + \varepsilon (P_c, t) \]
  \[ \varepsilon (P_c, t) \sim N(0, \sigma_{P_c}^2) \]

- Decay function (Kaplan and DeMaria, 1995):
  \[ V(t_L) = V_b + (R \cdot V_0 - V_b)e^{-\alpha t} - C \]
  where \( C = m \left( \ln \frac{D}{D_0} \right) + b, \quad m = \tilde{c}_1 t_L (t_{0, L} - t_L) \) and \( b = d_1 t_L (t_{0, L} - t_L) \)
Figure 387: Maximum wind speed in m/s (2070-2100)

Source: Le Guenedal et al. (2021).

The cyclone simulation database must be sensitive to the climate change scenario.
Figure 388: GDP decomposition of North America (or physical asset values) (Litpop database)

Source: Le Guenedal et al. (2021).
Applications
Tropical cyclone damage modeling

**Figure 389: The case of Katrina (2005)**

Physical asset values (mUSD)
- 148,413,16
- 0.13534
- 0.00012
- 0.00000

Katrina (2005) wind speed
- 150
- 125
- 100
- 75
- 50
- 25

Source: Le Guenedal et al. (2021).
Applications
Tropical cyclone damage modeling

Figure 390: The grid approach

![Map of North America with colored grid]  

Physical asset values (mUSD)

- 162,754.79
- 403.43
- 1.00
- 0.00

Source: Le Guenedal et al. (2021).
Figure 391: Average global losses

Source: Le Guenedal et al. (2021).
Applications
Tropical cyclone damage modeling

Table 198: Average increase of financial losses per year

<table>
<thead>
<tr>
<th>SSP</th>
<th>RCP 2.6</th>
<th>RCP 4.5</th>
<th>RCP 8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP2</td>
<td>+43%</td>
<td>+153%</td>
<td>+247%</td>
</tr>
<tr>
<td>SSP5</td>
<td>+157%</td>
<td>+360%</td>
<td>+543%</td>
</tr>
</tbody>
</table>

Source: Le Guenedal et al. (2021).

Remark

- There are simulations that lead to annual losses that easily exceed 2 or 3 trillion dollars per year
- 1 Katrina = $180 billion in 2005
Floods
Drought
Water stress
Extreme heat
Wildfire
Crop calendar adjustment

Figure 392: Crop calendar adjustment*

* Differences (days) in simulated average sowing (a) and maturity (b) dates between timely adaptation and no adaptation scenarios for the same climate period (2080-2099, RCP6.0)

Source: Minoli et al. (2022).
Crop yields

Figure 393: Crop yield* (2088 vs. 2022)

Crop yields

Figure 394: Crop yield* (2088 vs. 2022)

Agriculture productivity
Changes in growing seasons
Land management
Infrastructure costs
Insurance costs
<table>
<thead>
<tr>
<th>Modeling of physical risk</th>
<th>Agriculture and food security</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extreme weather modeling</td>
<td>Insurance and economic costs</td>
</tr>
<tr>
<td>Impact of climate-related physical risks</td>
<td>Other risks</td>
</tr>
</tbody>
</table>

Biodiversity risk
Health risk
Migration risk
Productivity risk
<table>
<thead>
<tr>
<th>Water risk</th>
</tr>
</thead>
</table>

Modeling of physical risk
Extreme weather modeling
Impact of climate-related physical risks
Agriculture and food security
Insurance and economic costs
Other risks
Course 2023-2024 in Sustainable Finance
Lecture 15. Climate Stress Testing and Risk Management

Thierry Roncalli*

* Amundi Asset Management
* University of Paris-Saclay

March 2024

\(^{44}\) The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Agenda

- Lecture 1: Introduction
- Lecture 2: ESG Scoring
- Lecture 3: Impact of ESG Investing on Asset Prices and Portfolio Returns
- Lecture 4: Sustainable Financial Products
- Lecture 5: Impact Investing
- Lecture 6: Engagement & Voting Policy
- Lecture 7: Extra-financial Accounting
- Lecture 8: Awareness of Climate Change Impacts
- Lecture 9: The Ecosystem of Climate Change
- Lecture 10: Economic Models & Climate Change
- Lecture 11: Climate Risk Measures
- Lecture 12: Transition Risk Modeling
- Lecture 13: Climate Portfolio Construction
- Lecture 14: Physical Risk Modeling
- Lecture 15: Climate Stress Testing & Risk Management
Transmission channels

Figure 395: Publication of the Basel Committee on climate-related financial risks (2021)
Transmission channels

**Figure 396:** Publication of the Basel Committee on climate-related financial risks (2022, 2023)
Direct and indirect transmission
Credit transmission channel
Market transmission channel
Systemic risk
Financial regulation

Figure 397: Campiglio et al. (2018)

Emanuele Campiglio, Yannis Dafermos, Pierre Monnin, Josh Ryan-Collins, Guido Schotten, Misa Tanaka
Climate change challenges for central banks and financial regulators

Article (Accepted version)
(Refereed)

Original citation:
DOI: 10.1038/s41558-018-0175-0
© 2018 Macmillan Publishers Limited, part of Springer Nature
This version available at: http://eprints.lse.ac.uk/88364
Available in LSE Research Online: June 2018

LSE has developed LSE Research Online so that users may access research output of the School. Copyright © and Moral Rights for the papers on this site are retained by the individual authors and/or other copyright owners. Users may download and/or print one copy of any article(s) in LSE Research Online to facilitate their private study or for non-commercial research. You may not engage in further distribution of the material or use it for any profit-making activities or any commercial gain. You may freely distribute the URL (http://eprints.lse.ac.uk) of the LSE Research Online website.

This document is the author's final accepted version of the journal article. There may be differences between this version and the published version. You are advised to consult the publisher's version if you wish to cite from it.
Figure 398: Impact on the trading book
### Commodity market

- Market risk
- Credit risk
- Stress testing
- Commodity market
- Stock market
- Climate value-at-risk
<table>
<thead>
<tr>
<th>Stock market</th>
<th>Credit risk</th>
<th>Stress testing</th>
<th>Commodity market</th>
<th>Climate value-at-risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market risk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Climate value-at-risk
Figure 399: ESRB (2021), Climate-related Risk and Financial Stability
Mortgage portfolios
Loan portfolios
Bond pricing
Structural models
Default barrier models
CDS pricing
<table>
<thead>
<tr>
<th>Market risk</th>
<th>Credit risk</th>
<th>Stress testing</th>
<th>Mortgage and loan portfolios</th>
<th>Bond portfolios</th>
<th>Capital requirements</th>
</tr>
</thead>
</table>

Bond portfolios
Introducing climate risk into risk-weighted assets
A climate stress-test of the financial system

Stefano Battiston1, Antoine Mandel2, Irene Monasterolo1, Franziska Schütes1 and Gabriele Visentin1

The urgency of estimating the impact of climate risks on the financial system is increasingly recognized among scholars and practitioners. By adopting a network approach to financial dependencies, we develop a sector-based climate stress-test methodology and apply it to large-scale Area capital and asset market simulations. We construct a network of economic activities and of financial institutions and estimate exposure due to physical and transition risk. To our knowledge this is the first attempt to use a network approach to assess the impact of climate policy on the financial system. Our results show a large portion of investors’ equity portfolios, especially for investment and pension funds. Additionally, the portion of capital loss attributed to their exposure to specific sectors is comparable to banks’ capital. Our results suggest that climate-policy timing matters. An early and stable policy framework would allow for smooth asset value adjustments and lead to potential net winners and losers. In contrast, a late and abrupt policy framework could have adverse systemic consequences.

Acknowledgements

We are grateful to all the participants of the Workshop on Modeling and Stress Testing of Financial Systems at the University of Milan, the Basque Foundation for Science and the Global Climate Forum for their helpful feedback. We are also grateful to various practitioners for providing us with valuable insights and feedback. The authors acknowledge support from the Swiss National Science Foundation (grant no. 157502), the NCCR Financial Economics, the EU project ‘India RIS’ and the Swiss National Science Foundation (grant no. P300P1-170956).


References


Earnings’ risk
Figure 401: Climate risk stress test
Banking

- Market risk
- Credit risk
- Stress testing
- Corporates
- Banks
- Insurance companies

Thierry Roncalli

Course 2023-2024 in Sustainable Finance
Figure 402: 2023 ACPR insurance climate exercise

Scenarios and main assumptions of the 2023 ACPR insurance climate exercise
Insurance