Course 2023-2024 in Financial Risk Management
Tutorial Session 3

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[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Agenda

- Tutorial Session 1: Market Risk
- Tutorial Session 2: Credit Risk
- **Tutorial Session 3: Counterparty Credit Risk and Collateral Risk**
- Tutorial Session 4: Operational Risk & Asset Liability Management Risk
- Tutorial Session 5: Copulas, EVT & Stress Testing

Impact of netting agreements in counterparty credit risk

Question 1

The table below gives the current mark-to-market of 7 OTC contracts between Bank A and Bank B:

	Equity			Fixed income		FX	
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
A	$+10$	-5	$+6$	$+17$	-5	-5	$+1$
B	-11	$+6$	-3	-12	$+9$	$+5$	$+1$

The table should be read as follows: Bank A has a mark-to-market equal to 10 for the contract $\mathcal{C}_1$ whereas Bank B has a mark-to-market equal to -11 for the same contract, Bank A has a mark-to-market equal to -5 for the contract $\mathcal{C}_2$ whereas Bank B has a mark-to-market equal to $+6$ for the same contract, etc.

Impact of netting agreements in counterparty credit risk

Question 1.a

Explain why there are differences between the MtM values of a same OTC contract.

Impact of netting agreements in counterparty credit risk

Let $\mathrm{MtM}_A(\mathcal{C})$ and $\mathrm{MTM}_B(\mathcal{C})$ be the MtM values of Bank A and Bank B for the contract $\mathcal{C}$. We must theoretically verify that:

$$\begin{aligned}\mathrm{MtM}_{A+B}(\mathcal{C}) &= \mathrm{MTM}_A(\mathcal{C}) + \mathrm{MTM}_B(\mathcal{C}) \\ &= 0 \end{aligned} \tag{1}$$

In the case of listed products, the previous relationship is verified. In the case of OTC products, there are no market prices, forcing the bank to use pricing models for the valuation. The MTM value is then a mark-to-model price. Because the two banks do not use the same model with the same parameters, we note a discrepancy between the two mark-to-market prices:

$$\mathrm{MTM}_A(\mathcal{C}) + \mathrm{MTM}_B(\mathcal{C}) \neq 0$$

Impact of netting agreements in counterparty credit risk

For instance, we obtain:

$$\begin{aligned}
\mathrm{MTM}_{A+B}\left(\mathcal{C}_1\right) &= 10 - 11 = -1 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_2\right) &= -5 + 6 = 1 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_3\right) &= 6 - 3 = 3 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_4\right) &= 17 - 12 = 5 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_5\right) &= -5 + 9 = 4 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_6\right) &= -5 + 5 = 0 \\
\mathrm{MTM}_{A+B}\left(\mathcal{C}_7\right) &= 1 + 1 = 2
\end{aligned}$$

Only the contract $\mathcal{C}_6$ satisfies the relationship (1).

Impact of netting agreements in counterparty credit risk

Question 1.b

Calculate the exposure at default of Bank A.

Impact of netting agreements in counterparty credit risk

We have:

$$\mathrm{EAD} = \sum_{i=1}^{7} \max\left(\mathrm{MTM}\left(\mathcal{C}_i\right), 0\right)$$

We deduce that:

$$\begin{aligned}\mathrm{EAD}_A &= 10 + 6 + 17 + 1 = 34 \\ \mathrm{EAD}_B &= 6 + 9 + 5 + 1 = 21\end{aligned}$$

Impact of netting agreements in counterparty credit risk

Question 1.c

Same question if there is a global netting agreement.

Impact of netting agreements in counterparty credit risk

If there is a global netting agreement, the exposure at default becomes:

$$\mathrm{EAD} = \max\left(\sum_{i=1}^{7} \mathrm{MTM}\left(\mathcal{C}_i\right), 0\right)$$

Using the numerical values, we obtain:

$$\begin{aligned}
\mathrm{EAD}_A &= \max\left(10 - 5 + 6 + 17 - 5 - 5 + 1, 0\right) \\
&= \max\left(19, 0\right) \\
&= 19
\end{aligned}$$

and:

$$\begin{aligned}
\mathrm{EAD}_B &= \max\left(-11 + 6 - 3 - 12 + 9 + 5 + 1, 0\right) \\
&= \max\left(-5, 0\right) \\
&= 0
\end{aligned}$$

Impact of netting agreements in counterparty credit risk

Question 1.d

Same question if the netting agreement only concerns equity products.

Impact of netting agreements in counterparty credit risk

If the netting agreement only concerns equity contracts, we have:

$$\mathrm{EAD} = \max\left(\sum_{i=1}^{3} \mathrm{MTM}\left(\mathcal{C}_i\right), 0\right) + \sum_{i=4}^{7} \max\left(\mathrm{MTM}\left(\mathcal{C}_i\right), 0\right)$$

It follows that:

$$\begin{aligned}
\mathrm{EAD}_A &= \max(10 - 5 + 6, 0) + 17 + 1 = 29 \\
\mathrm{EAD}_B &= \max(-11 + 6 - 3, 0) + 9 + 5 + 1 = 15
\end{aligned}$$

Impact of netting agreements in counterparty credit risk

Question 2

In the following, we measure the impact of netting agreements on the exposure at default.

Impact of netting agreements in counterparty credit risk

Question 2.a

We consider a first OTC contract $\mathcal{C}_1$ between Bank A and Bank B. The mark-to-market $\mathrm{MtM}_1(t)$ of Bank A for the contract $\mathcal{C}_1$ is defined as follows:

$$\mathrm{MtM}_1(t) = x_1 + \sigma_1 W_1(t)$$

where $W_1(t)$ is a Brownian motion. Calculate the potential future exposure of Bank A.

Impact of netting agreements in counterparty credit risk

The potential future exposure $e_1(t)$ is defined as follows:

$$e_1(t) = \max\left(x_1 + \sigma_1 W_1(t), 0\right)$$

We deduce that:

$$\begin{aligned}\mathbb{E}\left[e_1(t)\right] &= \int_{-\infty}^{\infty} \max(x, 0)\, f(x)\,\mathrm{d}x \\ &= \int_0^{\infty} x f(x)\,\mathrm{d}x\end{aligned}$$

where $f(x)$ is the density function of $\mathrm{MtM}_1(t)$. As we have $\mathrm{MtM}_1(t) \sim \mathcal{N}\left(x_1, \sigma_1^2 t\right)$, we deduce that:

$$\mathbb{E}\left[e_1(t)\right] = \int_0^{\infty} \frac{x}{\sigma_1\sqrt{2\pi t}} \exp\left(-\frac{1}{2}\left(\frac{x - x_1}{\sigma_1\sqrt{t}}\right)^2\right)\mathrm{d}x$$

Impact of netting agreements in counterparty credit risk

With the change of variable $y = \sigma_1^{-1} t^{-1/2} (x - x_1)$, we obtain:

$$\begin{aligned}
\mathbb{E}\left[e_1(t)\right] &= \int_{\frac{-x_1}{\sigma_1\sqrt{t}}}^{\infty} \frac{x_1 + \sigma_1\sqrt{t}y}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) \mathrm{d}y \\
&= x_1 \int_{\frac{-x_1}{\sigma_1\sqrt{t}}}^{\infty} \phi(y)\,\mathrm{d}y + \sigma_1\sqrt{t} \int_{\frac{-x_1}{\sigma_1\sqrt{t}}}^{\infty} y\phi(y)\,\mathrm{d}y \\
&= x_1 \Phi\left(\frac{x_1}{\sigma_1\sqrt{t}}\right) + \sigma_1\sqrt{t}\Big[-\phi(y)\Big]_{\frac{-x_1}{\sigma_1\sqrt{t}}}^{\infty} \\
&= x_1 \Phi\left(\frac{x_1}{\sigma_1\sqrt{t}}\right) + \sigma_1\sqrt{t}\phi\left(\frac{x_1}{\sigma_1\sqrt{t}}\right)
\end{aligned}$$

because $\phi(-x) = \phi(x)$ and $\Phi(-x) = 1 - \Phi(x)$.

Impact of netting agreements in counterparty credit risk

Question 2.b

We consider a second OTC contract between Bank A and Bank B. The mark-to-market is also given by the following expression:

$$\mathrm{MtM}_2(t) = x_2 + \sigma_2 W_2(t)$$

where $W_2(t)$ is a second Brownian motion that is correlated with $W_1(t)$. Let ρ be this correlation such that $\mathbb{E}[W_1(t) W_2(t)] = \rho t$. Calculate the expected exposure of bank A if there is no netting agreement.

Impact of netting agreements in counterparty credit risk

When there is no netting agreement, we have:

$$e(t) = e_1(t) + e_2(t)$$

We deduce that:

$$\begin{aligned}\mathbb{E}[e(t)] &= \mathbb{E}[e_1(t)] + \mathbb{E}[e_2(t)] \\ &= x_1 \Phi\left(\frac{x_1}{\sigma_1\sqrt{t}}\right) + \sigma_1\sqrt{t}\phi\left(\frac{x_1}{\sigma_1\sqrt{t}}\right) + \\ &\quad x_2 \Phi\left(\frac{x_2}{\sigma_2\sqrt{t}}\right) + \sigma_2\sqrt{t}\phi\left(\frac{x_2}{\sigma_2\sqrt{t}}\right)\end{aligned}$$

Impact of netting agreements in counterparty credit risk

Question 2.c

Same question when there is a global netting agreement between Bank A and Bank B.

Impact of netting agreements in counterparty credit risk

In the case of a netting agreement, the potential future exposure becomes:

$$\begin{aligned} e(t) &= \max\left(\mathrm{MtM}_1(t) + \mathrm{MtM}_2(t), 0\right) \\ &= \max\left(\mathrm{MtM}_{1+2}(t), 0\right) \\ &= \max\left(x_1 + x_2 + \sigma_1 W_1(t) + \sigma_2 W_2(t), 0\right) \end{aligned}$$

We deduce that:

$$\mathrm{MtM}_{1+2}(t) \sim \mathcal{N}\left(x_1 + x_2, \left(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2\right) t\right)$$

Using results of Question 2(a), we finally obtain:

$$\begin{aligned} \mathbb{E}\left[e(t)\right] &= (x_1 + x_2)\,\Phi\left(\frac{x_1 + x_2}{\sqrt{\left(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2\right) t}}\right) + \\ &\quad \sqrt{\left(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2\right) t}\,\phi\left(\frac{x_1 + x_2}{\sqrt{\left(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2\right) t}}\right) \end{aligned}$$

Impact of netting agreements in counterparty credit risk

Question 2.d

Comment on these results.

Impact of netting agreements in counterparty credit risk

We have represented the expected exposure $\mathbb{E}\left[e\left(t\right)\right]$ in Figure 1 when $x_1 = x_2 = 0$ and $\sigma_1 = \sigma_2$. We note that it is an increasing function of the time t and the volatility σ. We also observe that the netting agreement may have a big impact, especially when the correlation is low or negative.

Impact of netting agreements in counterparty credit risk

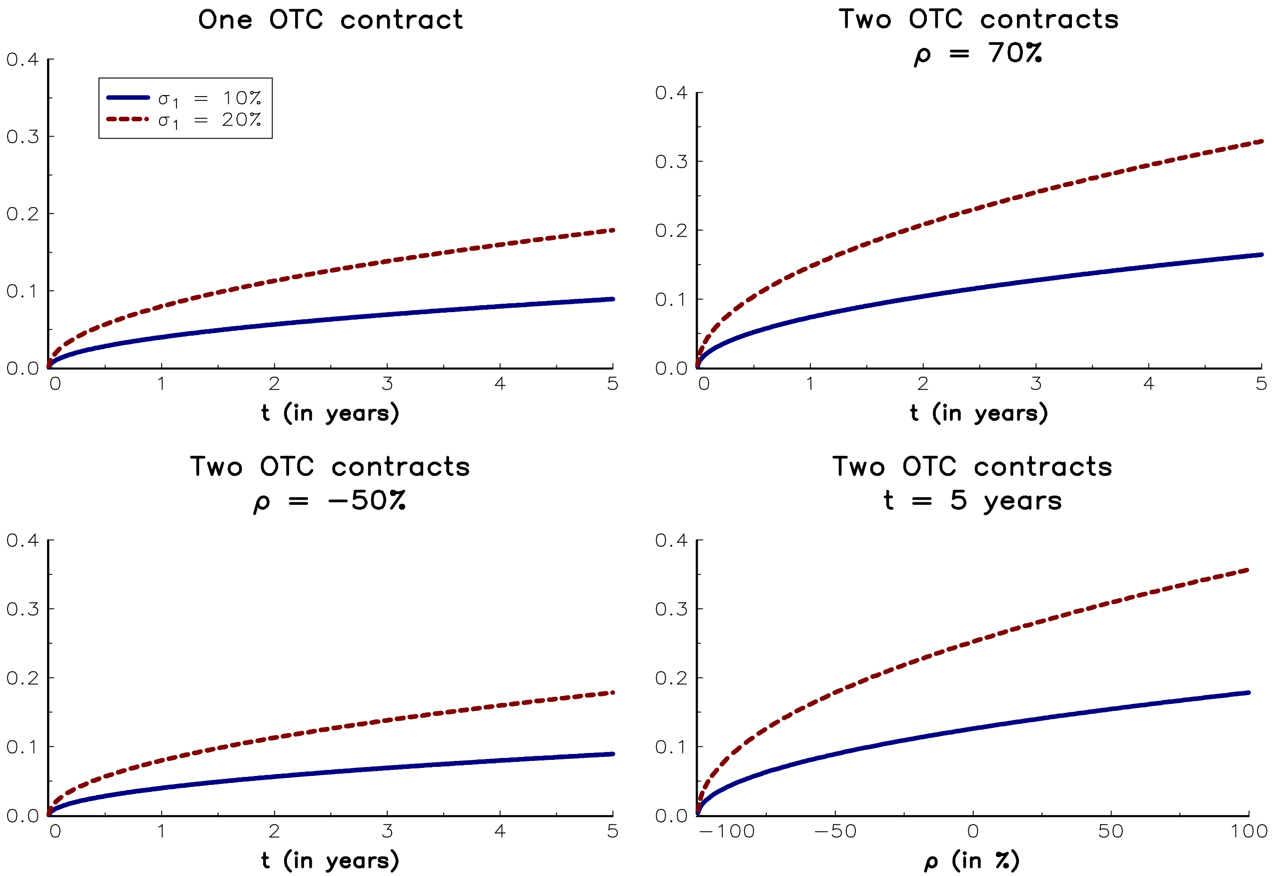

Figure 1: Expected exposure $\mathbb{E}\left[e\left(t\right)\right]$ when there is a netting agreement

Calculation of the CCR capital charge

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N and σ be the notional and the volatility of the underlying contract. We assume that $e(t) = N\sigma\sqrt{t}X$ with $0 \leq X \leq 1$, $\Pr\{X \leq x\} = x^{\gamma}$ and $\gamma > 0$.

Calculation of the CCR capital charge

Question 1

Calculate the peak exposure $\mathrm{PE}_\alpha(t)$, the expected exposure $\mathrm{EE}(t)$ and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

Calculation of the CCR capital charge

We have:

$$\begin{aligned}\mathbf{F}_{[0,t]}(x) &= \Pr\{e(t) \leq x\} \\ &= \Pr\left\{N\sigma\sqrt{t}U \leq x\right\} \\ &= \Pr\left\{U \leq \frac{x}{N\sigma\sqrt{t}}\right\} \\ &= \left(\frac{x}{N\sigma\sqrt{t}}\right)^{\gamma}\end{aligned}$$

with $x \in \left[0, N\sigma\sqrt{t}\right]$. We deduce that:

$$\begin{aligned}\mathrm{PE}_{\alpha}(t) &= \mathbf{F}_{[0,t]}^{-1}(\alpha) \\ &= N\sigma\sqrt{t}\alpha^{1/\gamma}\end{aligned}$$

Calculation of the CCR capital charge

For the expected exposure, we obtain:

$$\begin{aligned}
\mathrm{EE}(t) &= \mathbb{E}[e(t)] \\
&= \int_0^{N\sigma\sqrt{t}} x \frac{\gamma}{\left(N\sigma\sqrt{t}\right)^{\gamma}} x^{\gamma-1} \,\mathrm{d}x \\
&= \frac{\gamma}{\left(N\sigma\sqrt{t}\right)^{\gamma}} \left[\frac{x^{\gamma+1}}{\gamma+1}\right]_0^{N\sigma\sqrt{t}} \\
&= \frac{\gamma}{\gamma+1} N\sigma\sqrt{t}
\end{aligned}$$

Calculation of the CCR capital charge

We deduce that:

$$\mathrm{EEE}(t) = \frac{\gamma}{\gamma + 1} N \sigma \sqrt{t}$$

and:

$$\begin{aligned}
\mathrm{EEPE}(0; t) &= \frac{1}{t} \int_0^t \mathrm{EEE}(s)\, \mathrm{d}s \\
&= \frac{1}{t} \int_0^t \frac{\gamma}{\gamma + 1} N \sigma \sqrt{s}\, \mathrm{d}s \\
&= \frac{\gamma}{\gamma + 1} N \sigma \frac{1}{t} \left[\frac{2}{3} s^{3/2} \right]_0^t \\
&= \frac{2\gamma}{3(\gamma + 1)} N \sigma \sqrt{t}
\end{aligned}$$

Calculation of the CCR capital charge

Question 2

The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N is equal to \$3 mn, the maturity T is one year, the volatility σ is set to 20% and γ is estimated at 2.

Calculation of the CCR capital charge

Question 2.a

Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.

Calculation of the CCR capital charge

When the bank uses an internal model, the regulatory exposure at default is:

$$\mathrm{EAD} = \alpha \times \mathrm{EEPE}\,(0;1)$$

Using the standard value $\alpha = 1.4$, we obtain:

$$\begin{aligned}\mathrm{EAD} &= 1.4 \times \frac{4}{9} \times 3 \times 10^6 \times 0.20 \\ &= \$373\,333\end{aligned}$$

Calculation of the CCR capital charge

Question 2.b

The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[a].

[a] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

Calculation of the CCR capital charge

While the bank uses the FIRB approach, the required capital is:

$$\mathcal{K} = \mathrm{EAD} \times \mathbb{E}\left[\mathrm{LGD}\right] \times \left(\Phi\left(\frac{\Phi^{-1}\left(\mathrm{PD}\right) + \sqrt{\rho}\Phi^{-1}\left(99.9\%\right)}{\sqrt{1-\rho}}\right) - \mathrm{PD}\right)$$

When ρ is equal to 20%, we have:

$$\begin{aligned} \frac{\Phi^{-1}\left(\mathrm{PD}\right) + \sqrt{\rho}\Phi^{-1}\left(99.9\%\right)}{\sqrt{1-\rho}} &= \frac{-2.33 + \sqrt{0.20} \times 3.09}{\sqrt{1-0.20}} \\ &= -1.06 \end{aligned}$$

By using the approximations $-1.06 \simeq 1$ and $\Phi\left(-1\right) \simeq 0.16$, we obtain:

$$\begin{aligned} \mathcal{K} &= 373\,333 \times 0.70 \times \left(0.16 - 0.01\right) \\ &= \$39\,200 \end{aligned}$$

The required capital of this OTC product for counterparty credit risk is then equal to \$39 200.

Calculation of CVA and DVA measures

We consider an OTC contract with maturity T between Bank A and Bank B. We denote by $\mathrm{MtM}(t)$ the risk-free mark-to-market of Bank A. The current date is set to $t = 0$ and we assume that:

$$\mathrm{MtM}(t) = N \cdot \sigma \cdot \sqrt{t} \cdot X$$

where N is the notional of the OTC contract, σ is the volatility of the underlying asset and X is a random variable, which is defined on the support $[-1, 1]$ and whose density function is:

$$f(x) = \frac{1}{2}$$

Calculation of CVA and DVA measures

Question 1

Define the concept of positive exposure $e^{+}(t)$. Show that the cumulative distribution function $\mathbf{F}_{[0,t]}$ of $e^{+}(t)$ has the following expression:

$$\mathbf{F}_{[0,t]}(x) = \mathbb{1}\left\{0 \leq x \leq \sigma\sqrt{t}\right\} \cdot \left(\frac{1}{2} + \frac{x}{2 \cdot N \cdot \sigma \cdot \sqrt{t}}\right)$$

where $\mathbf{F}_{[0,t]}(x) = 0$ if $x \leq 0$ and $\mathbf{F}_{[0,t]}(x) = 1$ if $x \geq \sigma\sqrt{t}$.

Calculation of CVA and DVA measures

The positive exposure $e^{+}(t)$ is the maximum between zero and the mark-to-market value:

$$\begin{aligned} e^{+}(t) &= \max\left(0, \mathrm{MtM}(t)\right) \\ &= \max\left(0, N\sigma\sqrt{t}X\right) \end{aligned}$$

We have:

$$\begin{aligned} \mathbf{F}_{[0,t]}(x) &= \Pr\left\{e^{+}(t) \leq x\right\} \\ &= \Pr\left\{\max\left(0, N\sigma\sqrt{t}X\right) \leq x\right\} \end{aligned}$$

We notice that:

$$\max\left(0, N\sigma\sqrt{t}X\right) = \begin{cases} 0 & \text{if } X \leq 0 \\ N\sigma\sqrt{t}X & \text{otherwise} \end{cases}$$

Calculation of CVA and DVA measures

By assuming that $x \in \left[0, N\sigma\sqrt{t}\right]$, we deduce that:

$$\begin{aligned}
\mathbf{F}_{[0,t]}(x) &= \Pr\left\{e^+(t) \leq x, X \leq 0\right\} + \Pr\left\{e^+(t) \leq x, X > 0\right\} \\
&= \Pr\{0 \leq x, X \leq 0\} + \Pr\left\{N\sigma\sqrt{t}X \leq x, X > 0\right\} \\
&= \frac{1}{2} + \frac{1}{2}\Pr\left\{N\sigma\sqrt{t}U \leq x\right\} \\
&= \frac{1}{2} + \frac{1}{2}\Pr\left\{U \leq \frac{x}{N\sigma\sqrt{t}}\right\}
\end{aligned}$$

where U is the standard uniform random variable. We finally obtain the following expression:

$$\mathbf{F}_{[0,t]}(x) = \frac{1}{2} + \frac{x}{2N\sigma\sqrt{t}}$$

If $x \leq 0$ or $x \geq N\sigma\sqrt{t}$, it is easy to show that $\mathbf{F}_{[0,t]}(x) = 0$ and $\mathbf{F}_{[0,t]}(x) = 1$.

Calculation of CVA and DVA measures

Question 2

Deduce the value of the expected positive exposure $\mathrm{EpE}(t)$.

Calculation of CVA and DVA measures

The expected positive exposure $\mathrm{EpE}(t)$ is defined as follows:

$$\mathrm{EpE}(t) = \mathbb{E}\left[e^{+}(t)\right]$$

Using the expression of $\mathbf{F}_{[0,t]}(x)$, it follows that the density function of $e^{+}(t)$ is equal to:

$$\begin{aligned} f_{[0,t]}(x) &= \frac{\partial\, \mathbf{F}_{[0,t]}(x)}{\partial\, x} \\ &= \frac{1}{2N\sigma\sqrt{t}} \end{aligned}$$

Calculation of CVA and DVA measures

We deduce that:

$$\begin{aligned}
\mathrm{EpE}(t) &= \int_0^{N\sigma\sqrt{t}} x f_{[0,t]}(x)\,\mathrm{d}x \\
&= \int_0^{N\sigma\sqrt{t}} \frac{x}{2N\sigma\sqrt{t}}\,\mathrm{d}x \\
&= \left[\frac{x^2}{4N\sigma\sqrt{t}}\right]_0^{N\sigma\sqrt{t}} \\
&= \frac{N\sigma\sqrt{t}}{4}
\end{aligned}$$

Calculation of CVA and DVA measures

Question 3

We note $\mathcal{R}_B$ the fixed and constant recovery rate of Bank B. Give the mathematical expression of the CVA.

Calculation of CVA and DVA measures

By definition, we have:

$$\mathrm{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \times \int_0^T -B_0(t)\,\mathrm{EpE}(t)\,\mathrm{d}\mathbf{S}_B(t)$$

Calculation of CVA and DVA measures

Question 4

By using the definition of the lower incomplete gamma function $\gamma(s, x)$, show that the CVA is equal to:

$$\mathrm{CVA} = \frac{N \cdot (1 - \mathcal{R}_B) \cdot \sigma \cdot \gamma\left(\frac{3}{2}, \lambda_B T\right)}{4\sqrt{\lambda_B}}$$

when the default time of Bank B is exponential with parameter λ_B and interest rates are equal to zero.

Calculation of CVA and DVA measures

The interest rates are equal to zero meaning that $B_0(t) = 1$. Moreover, we have $\mathbf{S}_B(t) = e^{-\lambda_B t}$. We deduce that:

$$\begin{aligned} \mathrm{CVA} &= (1 - \mathcal{R}_B) \times \int_0^T \frac{N\sigma\sqrt{t}}{4} \lambda_B e^{-\lambda_B t} \, \mathrm{d}t \\ &= \frac{N\lambda_B (1 - \mathcal{R}_B)\sigma}{4} \int_0^T \sqrt{t} e^{-\lambda_B t} \, \mathrm{d}t \end{aligned}$$

The definition of the incomplete gamma function is:

$$\gamma(s, x) = \int_0^x t^{s-1} e^{-t} \, \mathrm{d}t$$

Calculation of CVA and DVA measures

By considering the change of variable $y = \lambda_B t$, we obtain:

$$\begin{aligned}\int_0^T \sqrt{t} e^{-\lambda_B t} \,\mathrm{d}t &= \int_0^{\lambda_B T} \sqrt{\frac{y}{\lambda_B}} e^{-y} \frac{\mathrm{d}y}{\lambda_B} \\ &= \frac{1}{\lambda_B^{3/2}} \int_0^{\lambda_B T} y^{3/2-1} e^{-y} \,\mathrm{d}y \\ &= \frac{\gamma\left(\frac{3}{2}, \lambda_B T\right)}{\lambda_B^{3/2}}\end{aligned}$$

It follows that:

$$\mathrm{CVA} = \frac{N\left(1-\mathcal{R}_B\right)\sigma\gamma\left(\frac{3}{2}, \lambda_B T\right)}{4\sqrt{\lambda_B}}$$

Calculation of CVA and DVA measures

Question 5

Comment on this result.

Calculation of CVA and DVA measures

The CVA is proportional to the notional N of the OTC contract, the loss given default $(1 - \mathcal{R}_B)$ of the counterparty and the volatility σ of the underlying asset. It is an increasing function of the maturity T because we have $\gamma\left(\frac{3}{2}, \lambda_B T_2\right) > \gamma\left(\frac{3}{2}, \lambda_B T_1\right)$ when $T_2 > T_1$. If the maturity is not very large (less than 10 years), the CVA is an increasing function of the default intensity λ_B.

Calculation of CVA and DVA measures

The limit cases are[2]:

$$\lim_{\lambda_B \to \infty} \mathrm{CVA} = \lim_{\lambda_B \to \infty} \frac{N\left(1 - \mathcal{R}_B\right) \sigma \gamma\left(\frac{3}{2}, \lambda_B T\right)}{4\sqrt{\lambda_B}} = 0$$

and:

$$\lim_{T \to \infty} \mathrm{CVA} = \frac{N\left(1 - \mathcal{R}_B\right) \sigma \Gamma\left(\frac{3}{2}\right)}{4\sqrt{\lambda_B}}$$

When the counterparty has a high default intensity, meaning that the default is imminent, the CVA is equal to zero because the mark-to-market value is close to zero. When the maturity is large, the CVA is a decreasing function of the intensity λ_B. Indeed, the probability to observe a large mark-to-market in the future increases when the default time is very far from the current date. We have illustrated these properties in Figure **??** with the following numerical values: $N = \$1$ mn, $\mathcal{R}_B = 40\%$ and $\sigma = 30\%$.

[2] We have $\lim_{x \to \infty} \gamma(s, x) = \Gamma(s)$.

Calculation of CVA and DVA measures

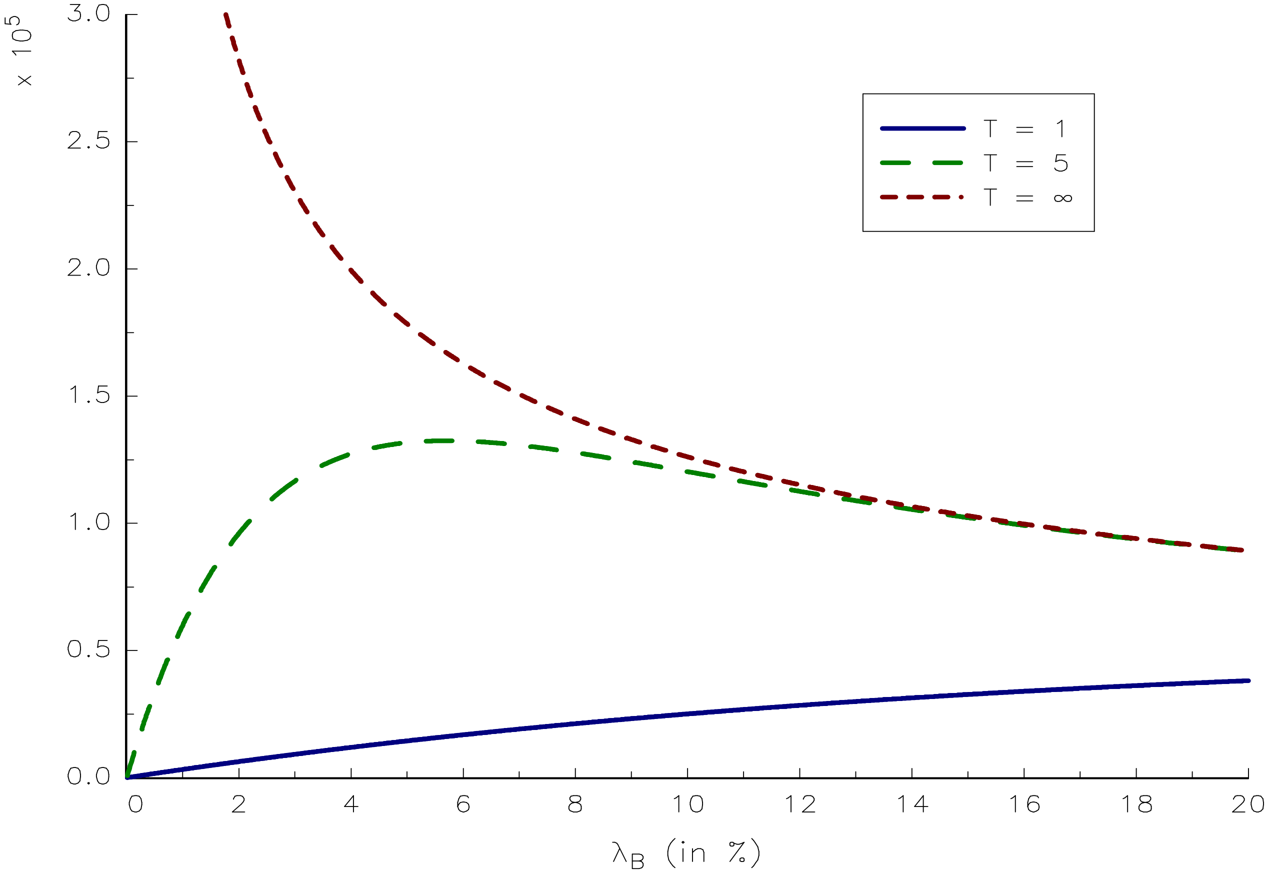

Figure 2: Evolution of the CVA with respect to maturity T and intensity λ_B

Calculation of CVA and DVA measures

Question 6

By assuming that the default time of Bank A is exponential with parameter λ_A, deduce the value of the DVA without additional computations.

Calculation of CVA and DVA measures

We notice that the mark-to-market is perfectly symmetric about 0. We deduce that the expected negative exposure $\mathrm{EnE}(t)$ is equal to the expected positive exposure $\mathrm{EpE}(t)$. It follows that the DVA is equal to:

$$\mathrm{DVA} = \frac{N(1-\boldsymbol{\mathcal{R}}_A)\,\sigma\gamma\left(\frac{3}{2},\lambda_A T\right)}{4\sqrt{\lambda_A}}$$