Course 2023-2024 in Financial Risk Management
Lecture 7. Asset Liability Management Risk

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[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

1. **Overview**
   The objective of this course is to understand the theoretical and practical aspects of risk management
2. **Prerequisites**
   M1 Finance or equivalent
3. **ECTS**
   4
4. **Keywords**
   Finance, Risk Management, Applied Mathematics, Statistics
5. **Hours**
   Lectures: 36h, Training sessions: 15h, HomeWork: 30h
6. **Evaluation**
   There will be a final three-hour exam, which is made up of questions and exercises
7. **Course website**
   `http://www.thierry-roncalli.com/RiskManagement.html`

Objective of the course

The objective of the course is twofold:

1. knowing and understanding the financial regulation (banking and others) and the international standards (especially the Basel Accords)
2. being proficient in risk measurement, including the mathematical tools and risk models

Class schedule

Course sessions

- September 15 (6 hours, AM+PM)
- September 22 (6 hours, AM+PM)
- September 19 (6 hours, AM+PM)
- October 6 (6 hours, AM+PM)
- October 13 (6 hours, AM+PM)
- October 27 (6 hours, AM+PM)

Tutorial sessions

- October 20 (3 hours, AM)
- October 20 (3 hours, PM)
- November 10 (3 hours, AM)
- November 10 (3 hours, PM)
- November 17 (3 hours, PM)

Class times: Fridays 9:00am-12:00pm, 1:00pm–4:00pm, University of Evry, Room 209 IDF

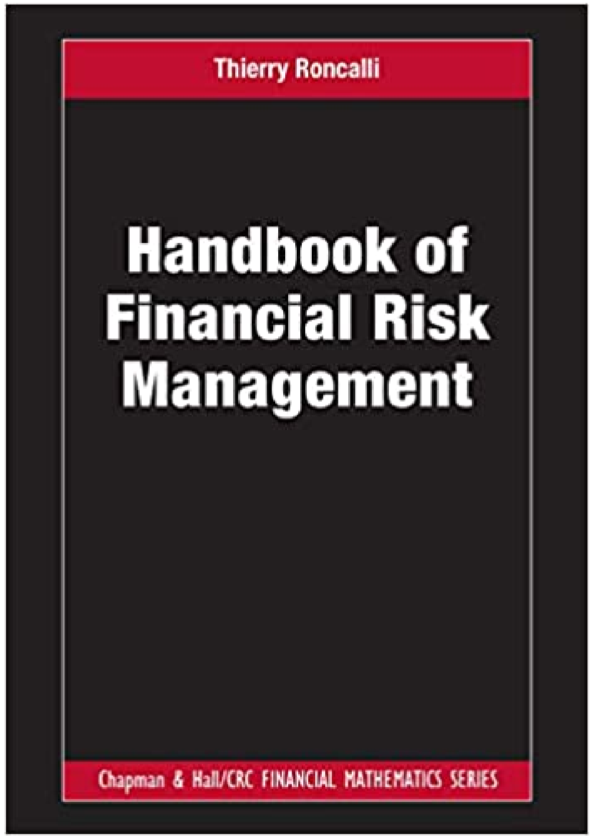
Agenda

Agenda

- Tutorial Session 1: Market Risk
- Tutorial Session 2: Credit Risk
- Tutorial Session 3: Counterparty Credit Risk and Collateral Risk
- Tutorial Session 4: Operational Risk & Asset Liability Management Risk
- Tutorial Session 5: Copulas, EVT & Stress Testing

Textbook

- Roncalli, T. (2020), *Handbook of Financial Risk Management*, Chapman & Hall/CRC Financial Mathematics Series.



Additional materials

- Slides, tutorial exercises and past exams can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagement.html`

- Solutions of exercises can be found in the companion book, which can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagementBook.html`

Agenda

- ALM risk $\Rightarrow$ banking book
- Not only a risk management issue, but also concerns commercial choices and business models
- Several ALM risks: liquidity risk, interest rate risk, embedded option risk, currency risk

The ALM function is located in the finance department, not in the risk management department

Balance sheet

Table: A simplified balance sheet

Assets	Liabilities
Cash	Due to central banks
Loans and leases	Deposits
Mortgages	Deposit accounts
Consumer credit	Savings
Credit cards	Term deposits
Interbank loans	Interbank funding
Investment securities	Short-term debt
Sovereign bonds	Subordinated debt
Corporate bonds	Reserves
Other assets	Equity capital

Income statement

Table: A simplified income statement

	Interest income
$-$	Interest expenses
$=$	Net interest income
$+$	Non-interest income
$=$	Gross income
$-$	Operating expenses
$=$	Net income
$-$	Provisions
$=$	Earnings before tax
$-$	Income tax
$=$	Profit after tax

Accounting standards

Four main systems:

1. US GAAP
2. Japanese combined system
3. Chinese accounting standards
4. International Financial Reporting Standards (IFRS)

$\Rightarrow$ IFRS is implemented in European Union, Australia, Middle East, Russia, South Africa, etc.

Accounting standards

Before 2018	After 2018

IAS 39

- financial assets at fair value through profit and loss (FVTPL)
- available-for-sale financial assets (AFS)
- loans and receivables (L&R);
- held-to-maturity investments (HTM)

IFRS 9

- amortized cost (AC)
- fair value (FV)

ALM committee (ALCO)

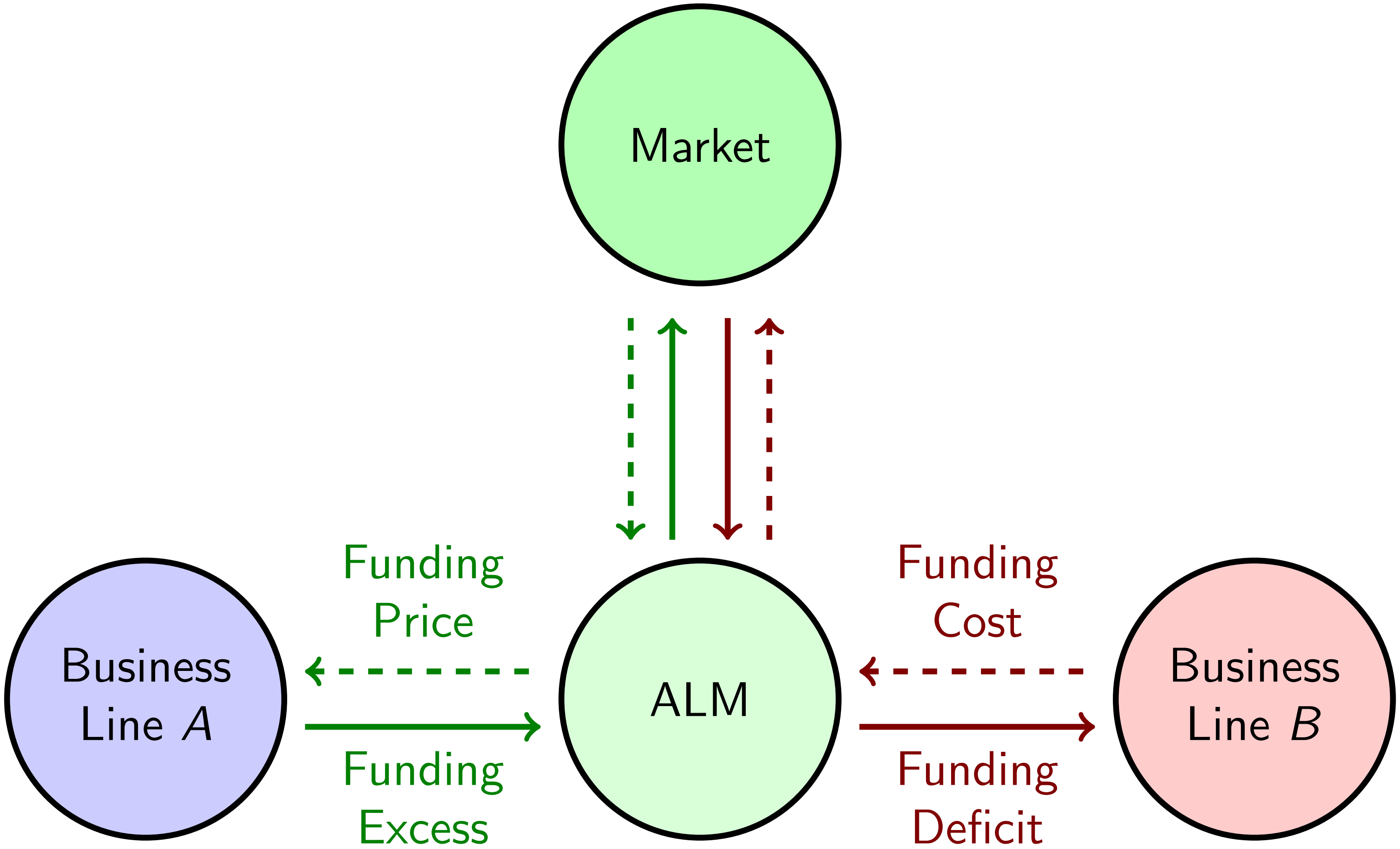


Figure: Internal and external funding transfer

Liquidity gap

- $A(t)$ is the value of assets at time t
- $L(t)$ is the value of liabilities at time t
- Funding ratio
$$\mathcal{FR}(t) = \frac{A(t)}{L(t)}$$
- Funding gap
$$\mathcal{FG}(t) = A(t) - L(t)$$
- Liquidity ratio
$$\mathcal{LR}(t) = \frac{L(t)}{A(t)}$$
- Liquidity gap
$$\mathcal{LG}(t) = L(t) - A(t)$$

Liquidity gap

Example

The assets $A(t)$ are composed of loans that are linearly amortized in a monthly basis during the next year. The amount is equal to 120. The liabilities $L(t)$ are composed of three short-term in fine debt instruments, and the capital. The corresponding debt notional is respectively equal to 65, 10 and 5 whereas the associated remaining maturity is equal to two, seven and twelve months. The amount of capital is stable for the next twelve months and is equal to 40

Liquidity gap

Table: Computation of the liquidity gap

Period	0	1	2	3	4	5	6	7	8	9	10	11	12
Loans	120	110	100	90	80	70	60	50	40	30	20	10	0
Assets	120	110	100	90	80	70	60	50	40	30	20	10	0
Debt #1	65	65	65										
Debt #2	10	10	10	10	10	10	10	10					
Debt #3	5	5	5	5	5	5	5	5	5	5	5	5	5
Debt (total)	80	80	80	15	15	15	15	15	5	5	5	5	5
Equity	40	40	40	40	40	40	40	40	40	40	40	40	40
Liabilities	120	120	120	55	55	55	55	55	45	45	45	45	45
$\mathcal{LG}(t)$	0	10	20	-35	-25	-15	-5	5	5	15	25	35	45

Liquidity gap

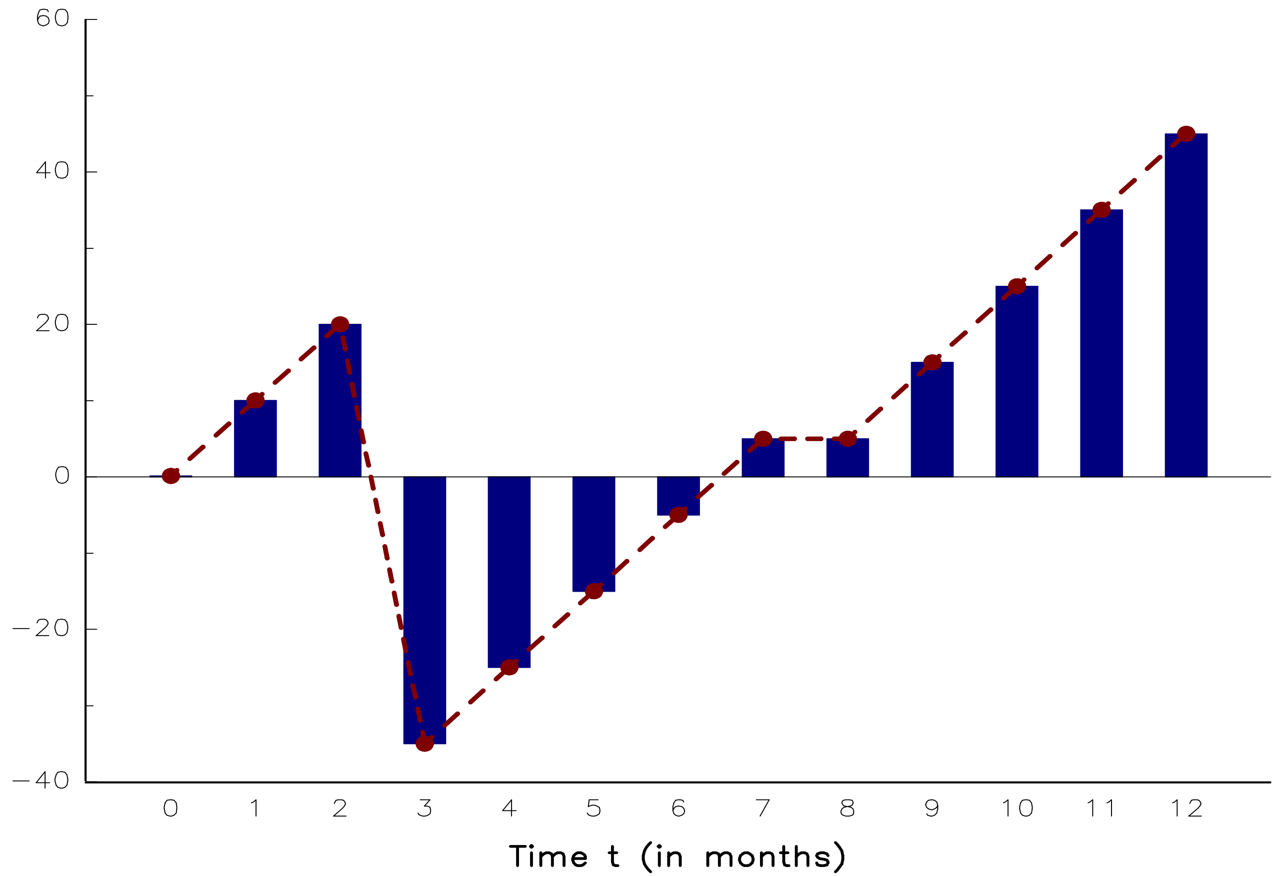


Figure: An example of liquidity gap

Asset and liability amortization

General principles of debt amortization

- The annuity amount $A(t)$ at time t is composed of the interest payment $I(t)$ and the principal payment $P(t)$:

$$A(t) = I(t) + P(t)$$

- The interest payment at time t is equal to the interest rate $i(t)$ times the outstanding principal balance $N(t-1)$:

$$I(t) = i(t) N(t-1)$$

- The outstanding principal balance $N(t)$ equal to

$$N(t) = N(t-1) - P(t)$$

- The outstanding principal balance $N(t)$ is equal to the present value $C(t)$ of forward annuity amounts: $N(t) = C(t)$

Asset and liability amortization

- Constant amortization debt (or linear amortization of the capital): $P(t)$ is constant over time (HFRM, page 379):

$$\begin{aligned} P(t) &= \frac{1}{n} N_0 \\ A(t) &= I(t) + P(t) = \left(\frac{1}{n} + i \left(1 - \frac{t-1}{n} \right) \right) N_0 \end{aligned}$$

- Constant payment debt: the annuity amount $A(t)$ is constant

$$\begin{aligned} A(t) &= A = \frac{i}{1-(1+i)^{-n}} N_0 \\ I(t) &= \left(1 - \frac{1}{(1+i)^{n-t+1}} \right) A \end{aligned}$$

- Bullet repayment debt: the notional is fully repaid at the time of maturity

$$I(t) = iN_0 \text{ and } P(t) = \mathbb{1}\{t = n\} \cdot N_0$$

Asset and liability amortization

Example

We consider a 10-year mortgage, whose notional is equal to \$100. The annual interest rate i is equal to 5%, and we assume annual principal payments

Asset and liability amortization

Table: Repayment schedule of the constant amortization mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	15.00	5.00	10.00	10.00	90.00
2	90.00	14.50	4.50	10.00	20.00	80.00
3	80.00	14.00	4.00	10.00	30.00	70.00
4	70.00	13.50	3.50	10.00	40.00	60.00
5	60.00	13.00	3.00	10.00	50.00	50.00
6	50.00	12.50	2.50	10.00	60.00	40.00
7	40.00	12.00	2.00	10.00	70.00	30.00
8	30.00	11.50	1.50	10.00	80.00	20.00
9	20.00	11.00	1.00	10.00	90.00	10.00
10	10.00	10.50	0.50	10.00	100.00	0.00

Asset and liability amortization

Table: Repayment schedule of the constant payment mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	12.95	5.00	7.95	7.95	92.05
2	92.05	12.95	4.60	8.35	16.30	83.70
3	83.70	12.95	4.19	8.77	25.06	74.94
4	74.94	12.95	3.75	9.20	34.27	65.73
5	65.73	12.95	3.29	9.66	43.93	56.07
6	56.07	12.95	2.80	10.15	54.08	45.92
7	45.92	12.95	2.30	10.65	64.73	35.27
8	35.27	12.95	1.76	11.19	75.92	24.08
9	24.08	12.95	1.20	11.75	87.67	12.33
10	12.33	12.95	0.62	12.33	100.00	0.00

Asset and liability amortization

Table: Repayment schedule of the bullet repayment mortgage

t	$C(t-1)$	$A(t)$	$I(t)$	$P(t)$	$Q(t)$	$N(t)$
1	100.00	5.00	5.00	0.00	0.00	100.00
2	100.00	5.00	5.00	0.00	0.00	100.00
3	100.00	5.00	5.00	0.00	0.00	100.00
4	100.00	5.00	5.00	0.00	0.00	100.00
5	100.00	5.00	5.00	0.00	0.00	100.00
6	100.00	5.00	5.00	0.00	0.00	100.00
7	100.00	5.00	5.00	0.00	0.00	100.00
8	100.00	5.00	5.00	0.00	0.00	100.00
9	100.00	5.00	5.00	0.00	0.00	100.00
10	100.00	105.00	5.00	100.00	100.00	0.00

Asset and liability amortization

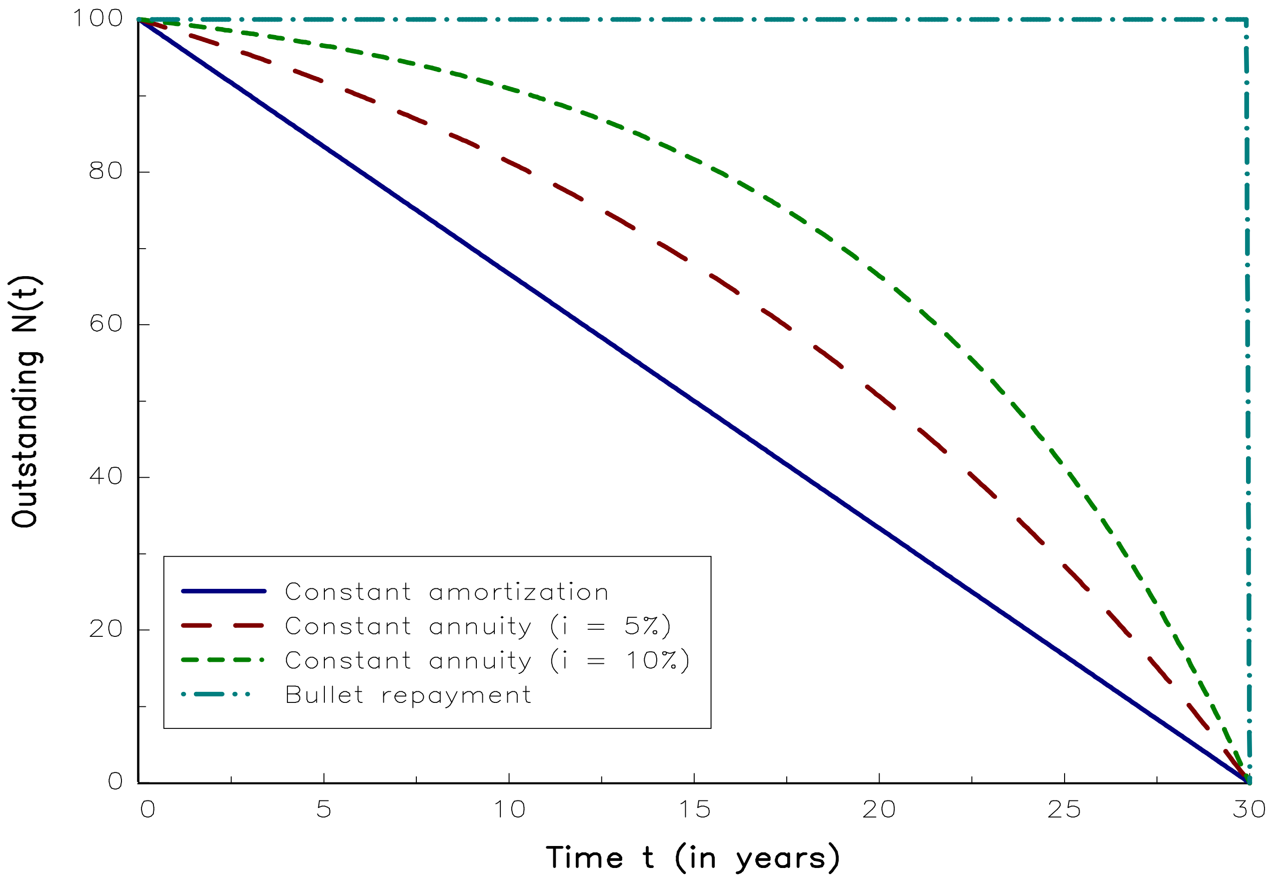


Figure: Amortization schedule of the 30-year mortgage (monthly payments)

Asset and liability amortization

Example

We consider the following balance sheet:

Assets				Liabilities			
Items	Notional	Rate	Mat.	Items	Notional	Rate	Mat.
Loan #1	100	5%	10	Debt #1	120	5%	10
Loan #2	50	8%	16	Debt #2	80	3%	5
Loan #3	40	3%	8	Debt #3	70	4%	10
Loan #4	110	2%	7	Capital #4	30		

All the debt instruments are subject to monthly principal payments

Mixed schedule = constant principal (loan #3 and debt #2), constant annuity (loan #1, loan #2 and debt #1) and bullet repayment (loan #4 and debt #2)

Asset and liability amortization

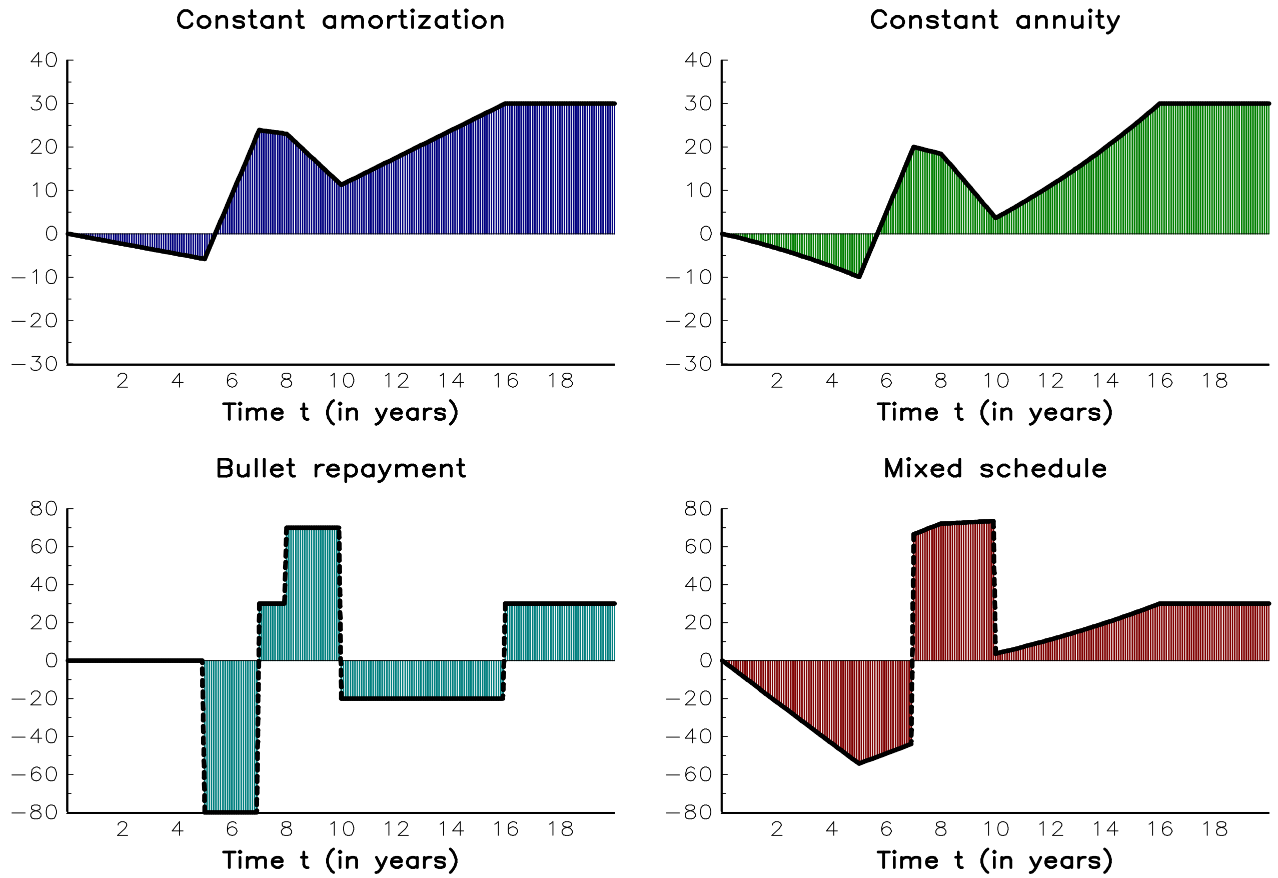


Figure: Impact of the amortization schedule on the liquidity gap

Asset and liability amortization

Table: Computation of the liquidity gap (mixed schedule, **first twelve months**)

t	Assets					Liabilities					$\mathcal{LG}_t$
	#1	#2	#3	#4	A_t	#1	#2	#3	#4	L_t	
1	99.4	49.9	39.6	110	298.8	119.2	78.7	70	30	297.9	−0.92
2	98.7	49.7	39.2	110	297.6	118.5	77.3	70	30	295.8	−1.83
3	98.1	49.6	38.8	110	296.4	117.7	76.0	70	30	293.7	−2.75
4	97.4	49.5	38.3	110	295.2	116.9	74.7	70	30	291.6	−3.66
5	96.8	49.3	37.9	110	294.0	116.1	73.3	70	30	289.4	−4.58
6	96.1	49.2	37.5	110	292.8	115.3	72.0	70	30	287.3	−5.49
7	95.4	49.1	37.1	110	291.6	114.5	70.7	70	30	285.2	−6.41
8	94.8	48.9	36.7	110	290.4	113.7	69.3	70	30	283.1	−7.32
9	94.1	48.8	36.3	110	289.2	112.9	68.0	70	30	280.9	−8.24
10	93.4	48.7	35.8	110	287.9	112.1	66.7	70	30	278.8	−9.15
11	92.8	48.5	35.4	110	286.7	111.3	65.3	70	30	276.7	−10.06
12	92.1	48.4	35.0	110	285.5	110.5	64.0	70	30	274.5	−10.97

Asset and liability amortization

Table: Computation of the liquidity gap (mixed schedule, **annual schedule**)

t	Assets					Liabilities					$\mathcal{LG}_t$
	#1	#2	#3	#4	A_t	#1	#2	#3	#4	L_t	
0	100.0	50.0	40.0	110	300.0	120.0	80.0	70	30	300.0	0.00
1	92.1	48.4	35.0	110	285.5	110.5	64.0	70	30	274.5	−10.97
2	83.8	46.7	30.0	110	270.4	100.5	48.0	70	30	248.5	−21.90
3	75.0	44.8	25.0	110	254.8	90.1	32.0	70	30	222.1	−32.76
4	65.9	42.7	20.0	110	238.6	79.0	16.0	70	30	195.0	−43.55
5	56.2	40.5	15.0	110	221.7	67.4		70	30	167.4	−54.27
6	46.1	38.1	10.0	110	204.2	55.3		70	30	155.3	−48.91
7	35.4	35.5	5.0		75.9	42.5		70	30	142.5	66.56
8	24.2	32.7			56.9	29.0		70	30	129.0	72.12
9	12.4	29.7			42.1	14.9		70	30	114.9	72.81
10		26.4			26.4				30	30.0	3.62
11		22.8			22.8				30	30.0	7.19
12		18.9			18.9				30	30.0	11.06
13		14.8			14.8				30	30.0	15.24
14		10.2			10.2				30	30.0	19.77
15		5.3			5.3				30	30.0	24.68
16					0.0				30	30.0	30.00

Impact of prepayment

We have:

$$N^c(t) = N(t) \cdot \mathbb{1}\{\boldsymbol{\tau} > t\}$$

where:

- $N^c(t)$ and $N(t)$ are the outstanding principal balances with and without prepayment
- $\boldsymbol{\tau}$ is the prepayment time of the debt instrument

We deduce that:

$$\mathbb{E}[N^c(t)] = \mathbf{S}(t) \cdot N(t)$$

where $\mathbf{S}(t) = \mathbb{E}[\mathbb{1}\{\boldsymbol{\tau} > t\}]$ is the survival function of $\boldsymbol{\tau}$

Remark

If $\boldsymbol{\tau} \sim \mathcal{E}(\lambda)$ where λ is the prepayment intensity, we obtain:

$$\mathbb{E}[N^c(t)] = e^{-\lambda t} \cdot N(t) \leq N(t)$$

Impact of prepayment

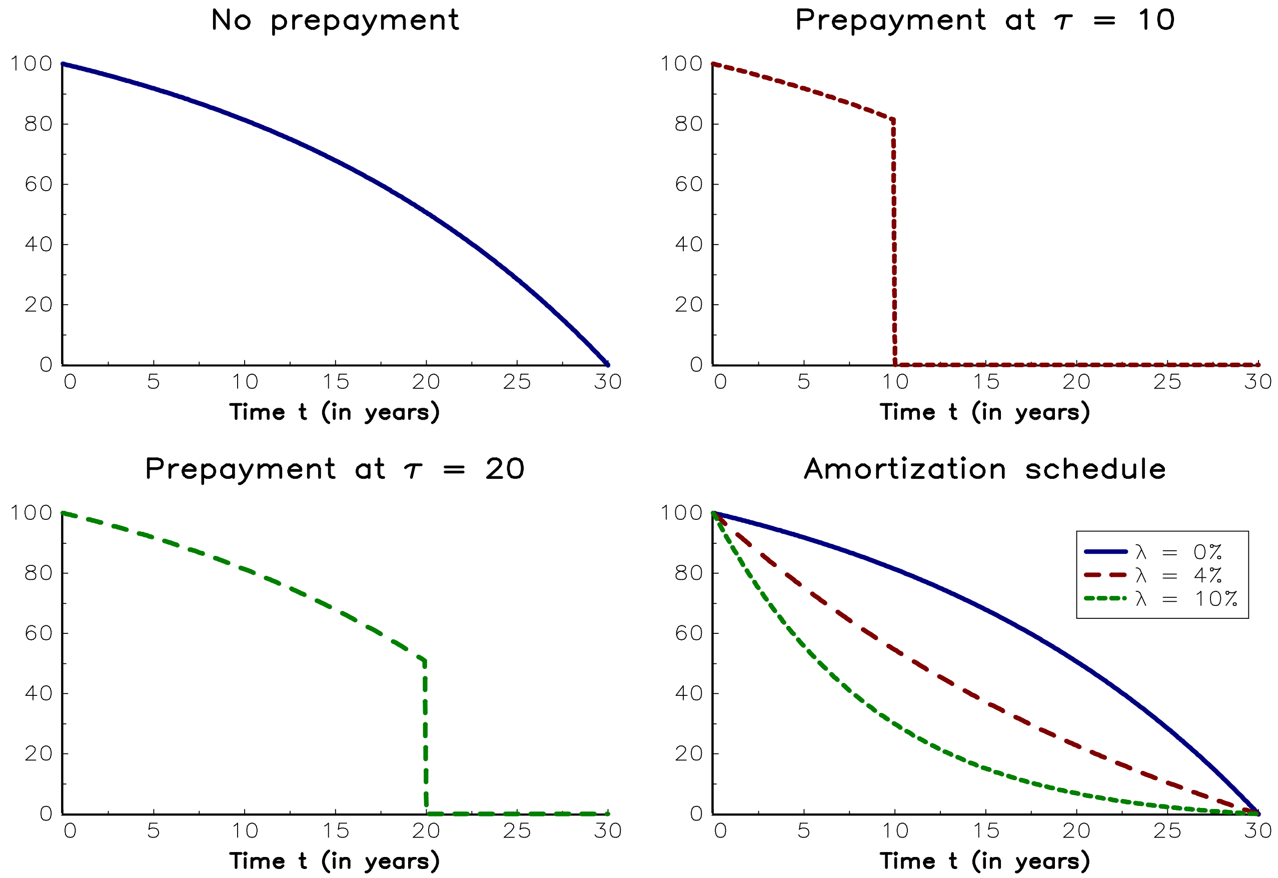


Figure: Conventional amortization schedule with prepayment risk

Impact of new production

Accounting identity

$$N(t) = N(t-1) - \mathrm{AM}(t) + \mathrm{NP}(t)$$

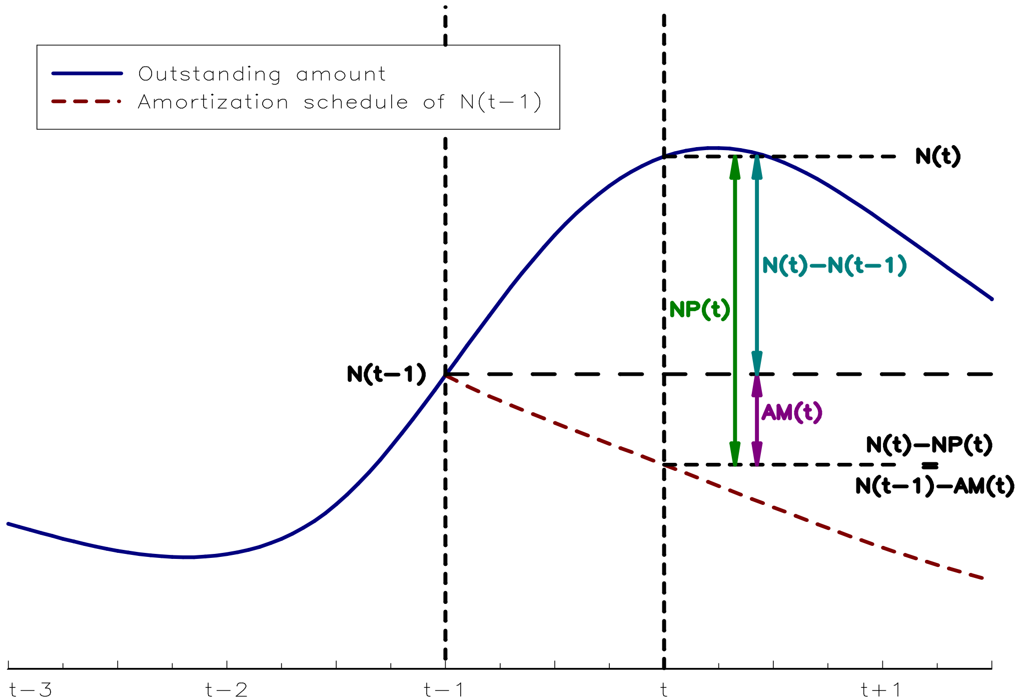


Figure: Impact of the new production on the outstanding amount

Dynamic analysis

- Run-off balance sheet
  A balance sheet where existing non-trading book positions amortize and are not replaced by any new business.
- Constant balance sheet
  A balance sheet in which the total size and composition are maintained by replacing maturing or repricing cash flows with new cash flows that have identical features.
- Dynamic balance sheet
  A balance sheet incorporating future business expectations, adjusted for the relevant scenario in a consistent manner.

Dynamic analysis
Notations

- $\text{NP}(t)$: New production at time t
- $\text{NP}(t,u)$: New production at time t that is present in the balance sheet at time $u \geq t$
- $\mathbf{S}(t,u)$: Survival function of the new production
- $f(t,u)$ is the density function associated to the survival function $\mathbf{S}(t,u)$
- $N(t,u)$: Non-amortized outstanding amount at time t that is present in the balance sheet at time $u \geq t$
- $\mathbf{S}^{\star}(t,u)$: Survival function of the outstanding amount

Dynamic analysis

Stock-flow analysis

- The amortization function $\mathbf{S}(t,u)$ is defined by:

$$\mathrm{NP}(t,u) = \mathrm{NP}(t) \cdot \mathbf{S}(t,u)$$

It measures the proportion of \$1 entering in the balance sheet at time t that remains present at time $u \geq t$:

$$N(t) = \int_{-\infty}^{t} \mathrm{NP}(s)\,\mathbf{S}(s,t)\,\mathrm{d}s$$

- The amortization function $\mathbf{S}^{\star}(t,u)$ is defined by:

$$N(t,u) = N(t) \cdot \mathbf{S}^{\star}(t,u)$$

It measures the proportion of \$1 of outstanding amount at time t that remains present at time $u \geq t$

$$\mathbf{S}^{\star}(t,u) = \frac{\int_{-\infty}^{t} \mathrm{NP}(s)\,\mathbf{S}(s,u)\,\mathrm{d}s}{\int_{-\infty}^{t} \mathrm{NP}(s)\,\mathbf{S}(s,t)\,\mathrm{d}s}$$

Dynamic analysis

Dynamics of the outstanding amount

We have:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -\int_{-\infty}^{t} \mathrm{NP}(s) f(s,t)\,\mathrm{d}s + \mathrm{NP}(t)$$

where $f(t,u) = -\partial_u \mathbf{S}(t,u)$ is the density function of the amortization

Dynamic analysis

Estimation of the dynamic liquidity gap

The dynamic liquidity gap at time t for a future date $u \geq t$ is given by:

$$\begin{aligned}\mathcal{LG}(t,u) &= \sum_{k \in \mathcal{L}iabilities} \left(N_k(t,u) + \int_t^u \mathrm{NP}_k(s)\, \mathbf{S}_k(s,u)\, \mathrm{d}s \right) - \\ & \quad \sum_{k \in \mathcal{A}ssets} \left(N_k(t,u) + \int_t^u \mathrm{NP}_k(s)\, \mathbf{S}_k(s,u)\, \mathrm{d}s \right)\end{aligned}$$

In the case of the run-off balance sheet, we obtain:

$$\mathcal{LG}(t,u) = \sum_{k \in \mathcal{L}iabilities} N_k(t,u) - \sum_{k \in \mathcal{A}ssets} N_k(t,u)$$

Dynamic analysis

Liquidity duration

The liquidity duration is the weighted average life (WAL) of the principal repayments:

$$\mathcal{D}(t) = \int_t^{\infty} (u - t) f(t, u) \, \mathrm{d}u$$

where $f(t, u)$ is the density function associated to the survival function $\mathbf{S}(t, u)$

Remark

All the previous formulas can be obtained in the discrete-time analysis (HFRM, Section 7.1.2.3, pages 385-392)

Dynamic analysis

Mathematical formulas

Table: Survival function and liquidity duration of some amortization schemes (HFRM, Exercise 7.4.3, page 450; HFRM-CB, Section 7.4.3, pages 126-128)

Amortization	$\mathbf{S}(t,u)$	$\mathcal{D}(t)$
Bullet	$\mathbb{1}\{t \leq u < t+m\}$	m
Constant	$\mathbb{1}\{t \leq u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)$	$\frac{m}{2}$
Exponential	$e^{-\lambda(u-t)}$	$\frac{1}{\lambda}$
Amortization	$\mathbf{S}^{\star}(t,u)$	$\mathcal{D}^{\star}(t)$
Bullet	$\mathbb{1}\{t \leq u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)$	$\frac{m}{2}$
Constant	$\mathbb{1}\{t \leq u < t+m\} \cdot \left(1 - \frac{u-t}{m}\right)^2$	$\frac{m}{3}$
Exponential	$e^{-\lambda(u-t)}$	$\frac{1}{\lambda}$
Amortization	$\mathrm{d}N(t)$	
Bullet	$\mathrm{d}N(t) = (\mathrm{NP}(t) - \mathrm{NP}(t-m))\,\mathrm{d}t$	
Constant	$\mathrm{d}N(t) = \left(\mathrm{NP}(t) - \frac{1}{m}\int_{t-m}^{t} \mathrm{NP}(s)\,\mathrm{d}s\right)\mathrm{d}t$	
Exponential	$\mathrm{d}N(t) = (\mathrm{NP}(t) - \lambda N(t))\,\mathrm{d}t$	

Dynamic analysis

Illustration

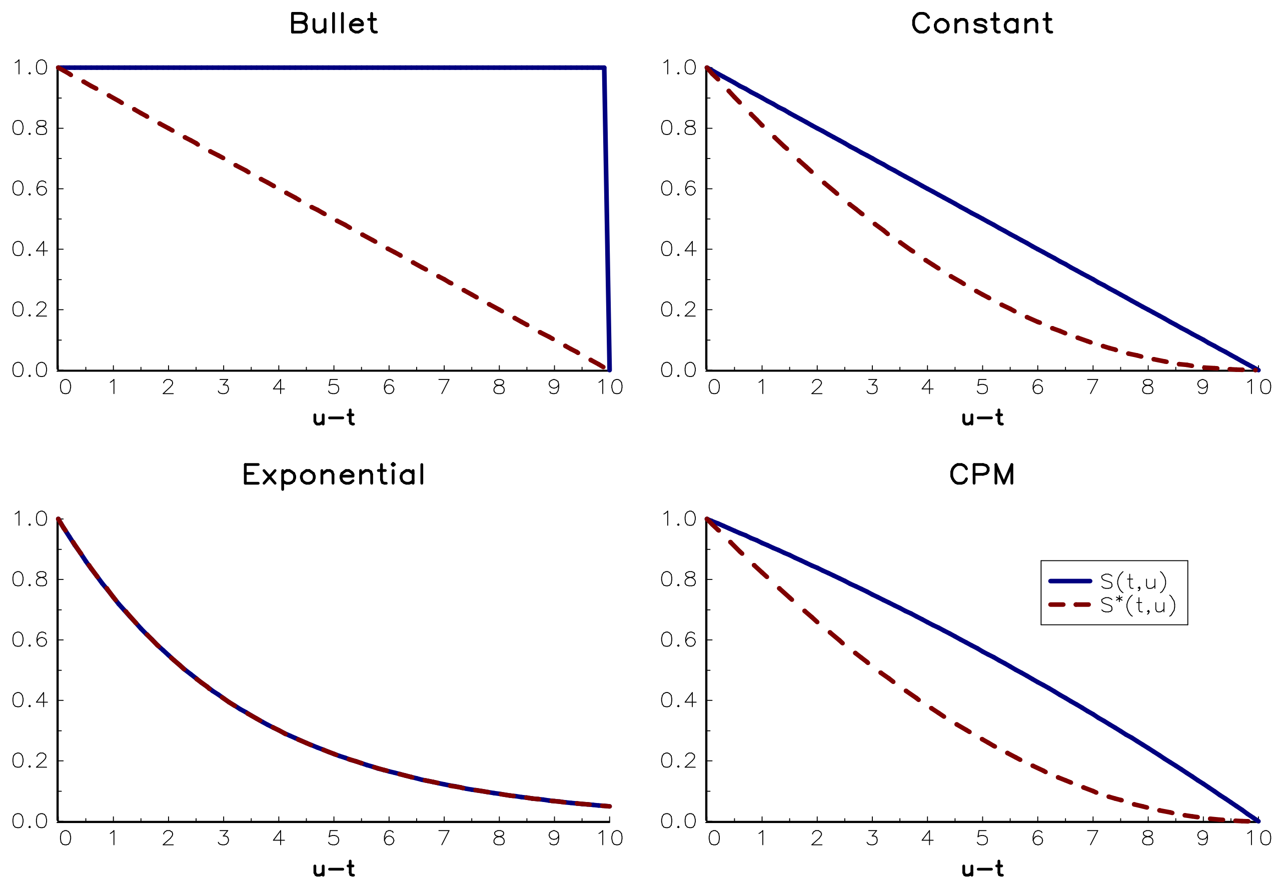


Figure: Amortization functions $\mathbf{S}(t,u)$ and $\mathbf{S}^{\star}(t,u)$

Definition

"IRRBB refers to the current or prospective risk to the bank' capital and earnings arising from adverse movements in interest rates that affect the bank's banking book positions. When interest rates change, the present value and timing of future cash flows change. This in turn changes the underlying value of a bank's assets, liabilities and off-balance sheet items and hence its economic value. Changes in interest rates also affect a bank's earnings by altering interest rate-sensitive income and expenses, affecting its net interest income" (BCBS, 2016)

1. Economic value (EV, EVE): changes in the net present value of the balance sheet
2. Earnings-based risk measures (EaR, NII): changes in the expected future profitability of the bank

Categories of IRR

3 main sources of interest rate risk

- Gap risk: mismatch risk arising from the term structure of banking book instruments
  - Repricing risk
  - Yield curve risk
- Basis risk: mismatch risk arising from different interest rate indices
  - Correlation risk of interest rate indices with the same maturity
- Option risk: option derivative positions and embedded options
  - Automatic option risk (caps, floors, swaptions and other interest rate derivatives)
  - Behavioral option risk
    - Prepayment risk
    - Early redemption risk (or withdrawal risk)
    - Non-maturity deposit (NMD)

Economic value (EV)

The economic value of a series of cash flows $\mathrm{CF} = \{\mathrm{CF}(t_k), t_k \geq t\}$ is the present value of these cash flows:

$$\mathrm{EV} = \mathbb{E}\left[\sum_{t_k \geq t} \mathrm{CF}(t_k) \cdot e^{-\int_t^{t_k} r(s)\,\mathrm{d}s}\right] = \sum_{t_k \geq t} \mathrm{CF}(t_k) \cdot B(t, t_k)$$

where $B(t, t_k)$ is the discount factor for the maturity date t_k

Application to the banking book

- We slot all notional repricing cash flows of assets and liabilities into a set of time buckets
- We calculate the net cash flows:

$$\mathrm{CF}(t_k) = \mathrm{CF}_A(t_k) - \mathrm{CF}_L(t_k)$$

  where $\mathrm{CF}_A(t_k)$ and $\mathrm{CF}_L(t_k)$ are the cash flows of assets and liabilities for the time bucket t_k
- The economic value is given by:

$$\begin{aligned}
\mathrm{EV} &= \sum_{t_k \geq t} \mathrm{CF}(t_k) \cdot B(t, t_k) \\
&= \sum_{t_k \geq t} \mathrm{CF}_A(t_k) \cdot B(t, t_k) - \sum_{t_k \geq t} \mathrm{CF}_L(t_k) \cdot B(t, t_k) \\
&= \mathrm{EV}_A - \mathrm{EV}_L
\end{aligned}$$

Stress testing of the economic value

- We note EV_s the economic value corresponding to the stress scenario s
- EV_0 is the base scenario and corresponds to the current term structure of interest rates
- We have:

$$\begin{aligned}\Delta \,\mathrm{EV}_s &= \mathrm{EV}_0 - \mathrm{EV}_s \\ &= \sum_{t_k \geq t} \mathrm{CF}_0 \left(t_k\right) \cdot B_0 \left(t, t_k\right) - \sum_{t_k \geq t} \mathrm{CF}_s \left(t_k\right) \cdot B_s \left(t, t_k\right)\end{aligned}$$

Economic value of equity (EVE)

The economic value of equity (EVE or EV_E) is a specific form of EV where equity is excluded from the cash flows

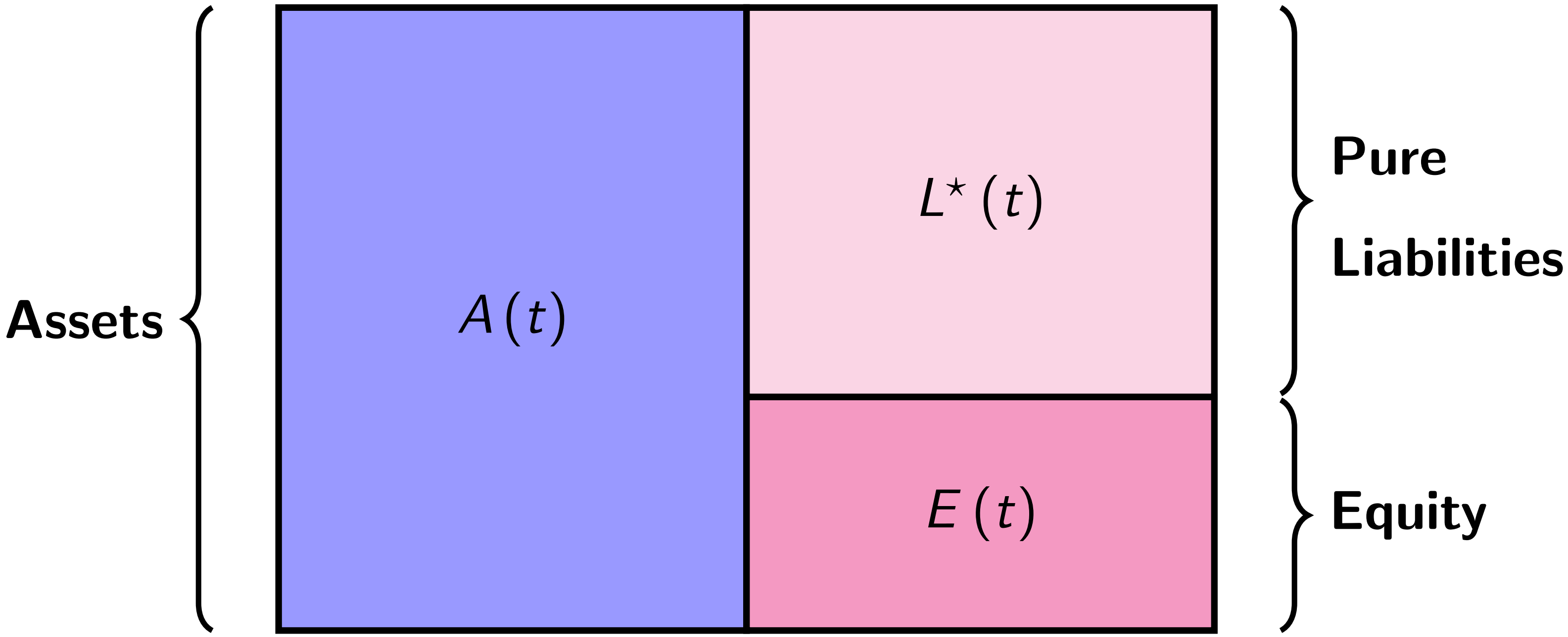


Figure: Relationship between $A(t)$, $L^{\star}(t)$ and $E(t)$

We have:

$$A(t) = L(t) = L^{\star}(t) + E(t)$$

Economic value of equity

Since the value of the capital is equal to $E(t) = A(t) - L^{\star}(t)$, we have:

$$\mathrm{EVE} = \mathrm{EV}_A - \mathrm{EV}_{L^{\star}}$$

and:

$$\Delta\,\mathrm{EVE}_s = \mathrm{EVE}_0 - \mathrm{EVE}_s$$

Remark

The economic value of equity is then equal to:

$$\mathrm{EVE} = \sum_{t_k \geq t} \mathrm{CF}_A(t_k) \cdot B(t, t_k) - \sum_{t_k \geq t} \mathrm{CF}_{L^{\star}}(t_k) \cdot B(t, t_k)$$

Net interest income (NII)

- The net interest income is the difference between the interest payments on assets and the interest payments of liabilities
- We have:
$$\Delta\,\mathrm{NII}_s = \mathrm{NII}_0 - \mathrm{NII}_s$$
- $\Delta\,\mathrm{NII}_s > 0$ indicates a loss if the stress scenario s occurs

Basel III

The risk measures are equal to the maximum of losses by considering the different scenarios:

$$\mathcal{R}(\mathrm{EVE}) = \max_s (\Delta \,\mathrm{EVE}_s, 0)$$

and:

$$\mathcal{R}(\mathrm{NII}) = \max_s (\Delta \,\mathrm{NII}_s, 0)$$

IRRBB

- No minimum capital requirements $\mathcal{K}$
- $\mathcal{R}(\mathrm{EVE}) \leq 15\% \times \mathrm{Tier}\,1$
- Pillar 2

Interest rate risk principles

- 9 IRR principles for banks (management, risk appetite, model governance process, capital adequacy policy, etc.)
- 3 IRR principles for supervisors (data collection, challenging the model assumptions, identification of outlier banks)

Some examples:

- To compute $\Delta\,\mathrm{EVE}$, banks must consider a run-off balance sheet assumption
- To compute $\Delta\,\mathrm{NII}$, banks must use a constant or dynamic balance sheet and a rolling 12-month period
- Banks must use:
  - Internal (historical and hypothetical) interest rate scenarios
  - 6 external interest rate scenarios

The standardized approach

5 steps for measuring the bank's IRRBB

1. The first step consists in allocating the interest rate sensitivities of the banking book to three categories
   1. amenable to standardization
   2. less amenable to standardization
   3. not amenable to standardization
2. Then, the bank must slot cash flows into 19 predefined time buckets: overnight (O/N), O/N–1M, ..., 10Y–15Y, 15Y–20Y, 20Y+
3. The bank determines $\Delta \mathrm{EVE}_{s,c}$ for each shock s and each currency c
4. The bank calculates the total measure for automatic interest rate option risk $\mathrm{KAO}_{s,c}$
5. The bank calculates the EVE for each shock s:

$$\mathcal{R}(\mathrm{EVE}_s) = \max\left(\sum_c \left(\Delta \mathrm{EVE}_{s,c} + \mathrm{KAO}_{s,c}\right)^+ ; 0\right)$$

The standardized EVE risk measure is equal to:

$$\mathcal{R}(\mathrm{EVE}) = \max_s \mathcal{R}(\mathrm{EVE}_s)$$

The standardized approach

The supervisory interest rate shock scenarios

Three shock sizes:

Shock size		USD/CAD/SEK	EUR/HKD	GBP	JPY	EM
$\mathbb{S}_0$	(parallel)	200	200	250	100	400
$\mathbb{S}_1$	(short)	300	250	300	100	500
$\mathbb{S}_2$	(long)	150	100	150	100	300

Given $\mathbb{S}_0$, $\mathbb{S}_1$ and $\mathbb{S}_2$, we calculate the following generic shocks for a given maturity t:

	Parallel shock $\Delta R^{(\text{parallel})}(t)$	Short rates shock $\Delta R^{(\text{short})}(t)$	Long rates shock $\Delta R^{(\text{long})}(t)$
Up	$+\mathbb{S}_0$	$+\mathbb{S}_1 \cdot e^{-t/\tau}$	$+\mathbb{S}_2 \cdot \left(1 - e^{-t/\tau}\right)$
Down	$-\mathbb{S}_0$	$-\mathbb{S}_1 \cdot e^{-t/\tau}$	$-\mathbb{S}_2 \cdot \left(1 - e^{-t/\tau}\right)$

where τ is equal to four years

The standardized approach

The supervisory interest rate shock scenarios

The six standardized interest rate shock scenarios are defined as follows:

1. Parallel shock up: $\Delta R^{(\text{parallel})}(t) = +\mathbb{S}_0$
2. Parallel shock down: $\Delta R^{(\text{parallel})}(t) = -\mathbb{S}_0$
3. Steepener shock (short rates down and long rates up):
$$\Delta R^{(\text{steepnener})}(t) = 0.90 \cdot \left|\Delta R^{(\text{long})}(t)\right| - 0.65 \cdot \left|\Delta R^{(\text{short})}(t)\right|$$
4. Flattener shock (short rates up and long rates down):
$$\Delta R^{(\text{flattener})}(t) = 0.80 \cdot \left|\Delta R^{(\text{short})}(t)\right| - 0.60 \cdot \left|\Delta R^{(\text{long})}(t)\right|$$
5. Short rates shock up:
$$\Delta R^{(\text{short})}(t) = +\mathbb{S}_1 \cdot e^{-t/\tau}$$
6. Short rates shock down:
$$\Delta R^{(\text{short})}(t) = -\mathbb{S}_1 \cdot e^{-t/\tau}$$

The standardized approach

The supervisory interest rate shock scenarios

Example

We assume that $\mathbb{S}_0 = 100$ bps, $\mathbb{S}_1 = 150$ bps and $\mathbb{S}_2 = 200$ bps. We would like to calculate the standardized shocks for the one-year maturity

- The parallel shock up is equal to $+100$ bps, while the parallel shock down is equal to -100 bps
- For the short rates shock, we obtain:

$$\Delta R^{(\mathrm{short})}(t) = 150 \times e^{-1/4} = 116.82 \text{ bps}$$

  for the up scenario and -116.82 bps for the down scenario
- Since we have $\left|\Delta R^{(\mathrm{short})}(t)\right| = 116.82$ and $\left|\Delta R^{(\mathrm{long})}(t)\right| = 44.24$, the steepener shock is equal to:

$$\Delta R^{(\mathrm{steepnener})}(t) = 0.90 \times 44.24 - 0.65 \times 116.82 = -36.12 \text{ bps}$$

  For the flattener shock, we have:

$$\Delta R^{(\mathrm{flattener})}(t) = 0.80 \times 116.82 - 0.60 \times 44.24 = 66.91 \text{ bps}$$

The standardized approach

The supervisory interest rate shock scenarios

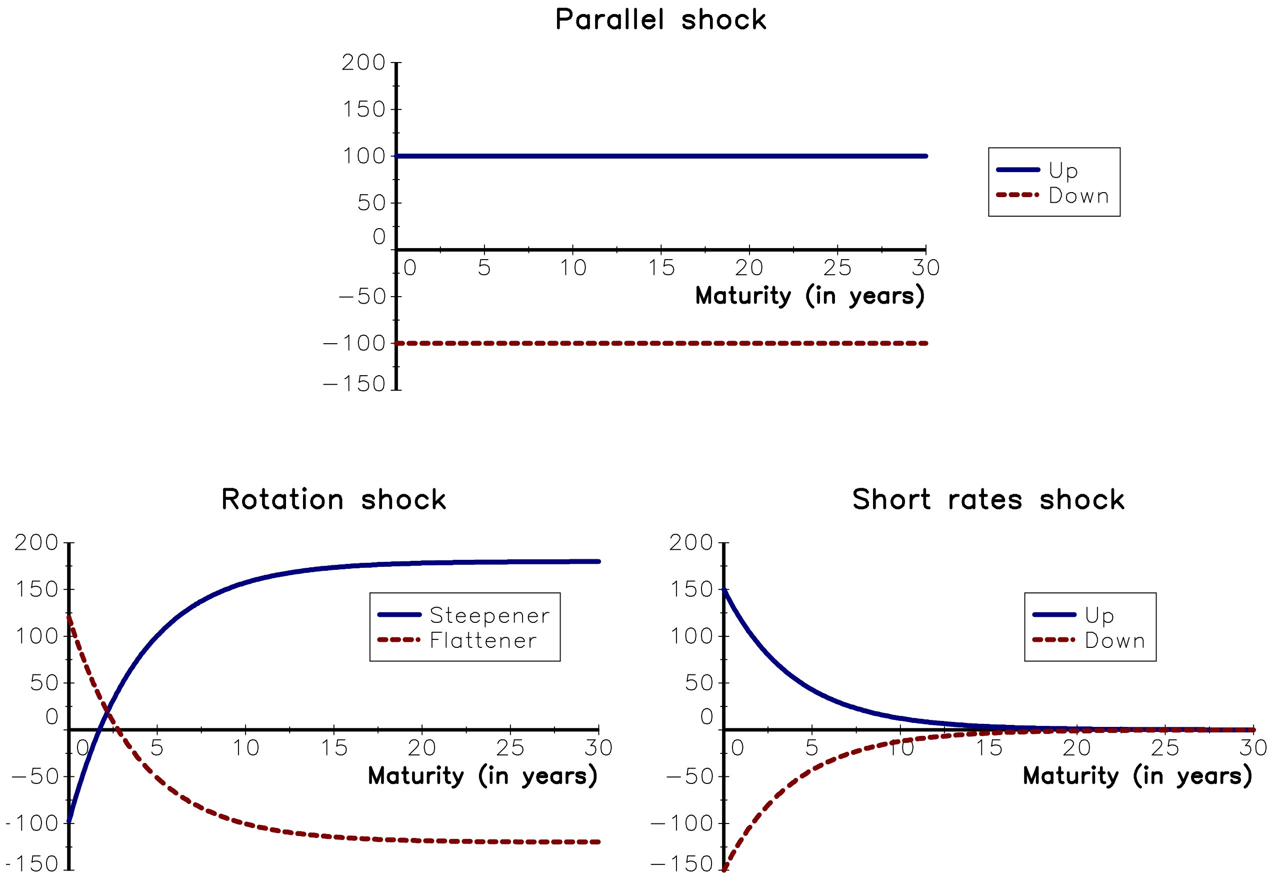


Figure: Interest rate shocks (in bps) with $(\mathbb{S}_0 = 100, \mathbb{S}_1 = 150, \mathbb{S}_2 = 200)$

The standardized approach

The supervisory interest rate shock scenarios

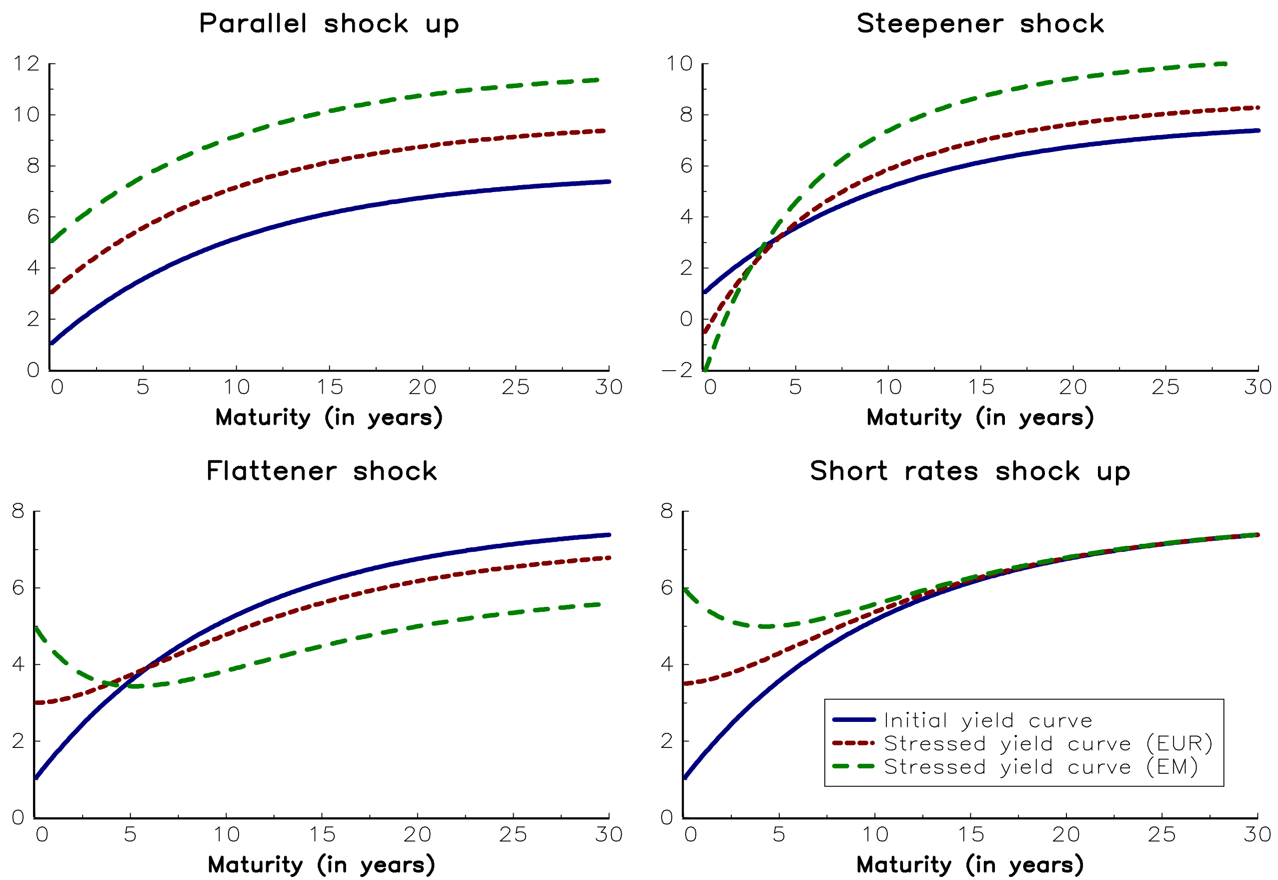


Figure: Stressed yield curve (in %)

The standardized approach

Treatment of NMDs

- Retail transactional (RT)
- Retail non-transactional (RNT)
- Wholesale (W)

- Difference between stable and non-stable part of each category
- The stable part of NMDs must be split between:
  - Core deposits
    - Maximum proportion: 90% for RT, 70% for RNT and 50% for W
    - Maximum maturity: 5Y for RT, 4.5Y for RNT and 4Y for W
  - Non-core deposits (overnight maturity)

The standardized approach

Prepayment risk

The bank estimates the baseline conditional prepayment rate (CPR) CPR_0 and calculates the stressed conditional prepayment rate as follows:

$$\mathrm{CPR}_s = \min\left(1, \gamma_s \cdot \mathrm{CPR}_0\right)$$

where γ_s is the multiplier for the scenario s and

- $\gamma_s = 0.8$ for the scenarios 1, 3 and 5 (parallel up, steepener and short rates up)
- $\gamma_s = 1.2$ for the scenarios 2, 4 and 6 (parallel down, flattener and short rates down)

The standardized approach

Early redemption

The term deposit redemption ratio (TDRR) is stressed as follows:

$$\mathrm{TDRR}_s = \min\left(1, \gamma_s \cdot \mathrm{TDRR}_0\right)$$

where γ_s is the multiplier for the scenario s and:

- $\gamma_s = 1.2$ for the scenarios 1, 4 and 5 (parallel up, flattener and short rates up)
- $\gamma_s = 0.8$ for the scenarios 2, 3 and 6 (parallel down, steepener and short rates down)

The standardized approach

Automatic interest rate options

The computation of the automatic interest rate option risk KAO_s is given by:

$$\mathrm{KAO}_s = \sum_{i \in \mathcal{S}} \Delta \, \mathrm{FVAO}_{s,i} - \sum_{i \in \mathcal{B}} \Delta \, \mathrm{FVAO}_{s,i}$$

where:

- $i \in \mathcal{S}$ (resp. $i \in \mathcal{B}$) denotes an automatic interest rate option which is sold (resp. bought) by the bank
- $\mathrm{FVAO}_{s,i}$ (resp. $\mathrm{FVAO}_{0,i}$) is the fair value of the automatic option i given the stressed (resp. current) yield curve and a relative increase in the implied volatility of 25% (resp. the current implied volatility)
- $\Delta \, \mathrm{FVAO}_{s,i}$ is the change in the fair value of the option:

$$\Delta \, \mathrm{FVAO}_{s,i} = \mathrm{FVAO}_{s,i} - \mathrm{FVAO}_{0,i}$$

The standardized approach

Example

We consider a USD-denominated balance sheet. The assets are composed of loans with the following cash flow slotting:

Instruments	Loans	Loans	Loans
Maturity	1Y	5Y	13Y
Cash flows	200	700	200

The liabilities are composed of retail deposit accounts, term deposits, debt and tier-one equity capital:

Instruments	Non-core deposits	Term deposits	Core deposits	Debt ST	Debt LT	Equity capital
Maturity	O/N	7M	3Y	4Y	8Y	
Cash flows	100	50	450	100	100	200

The standardized approach

Example

Table: Economic value of the assets

Bucket	t_k	$\mathrm{CF}_0(t_k)$	$R_0(t_k)$	$\mathrm{EV}_0(t_k)$
6	0.875	200	1.55%	197.31
11	4.50	700	3.37%	601.53
17	12.50	100	5.71%	48.98
EV_0				847.82

Table: Economic value of the pure liabilities

Bucket	t_k	$\mathrm{CF}_0(t_k)$	$R_0(t_k)$	$\mathrm{EV}_0(t_k)$
1	0.0028	100	1.00%	100.00
5	0.625	50	1.39%	49.57
9	2.50	450	2.44%	423.35
10	3.50	100	2.93%	90.26
14	7.50	100	4.46%	71.56
EV_0				734.73

The standardized approach

Example

Table: Stressed economic value of equity

Bucket	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$
			Assets			
$R_s(t_6)$	3.55%	-0.45%	0.24%	3.30%	3.96%	-0.87%
$R_s(t_{11})$	5.37%	1.37%	3.65%	3.54%	4.34%	2.40%
$R_s(t_{17})$	7.71%	3.71%	6.92%	4.96%	5.84%	5.58%
$\mathrm{EV}_s(t_6)$	193.89	200.80	199.57	194.31	193.20	201.52
$\mathrm{EV}_s(t_{11})$	549.76	658.18	594.03	596.91	575.74	628.48
$\mathrm{EV}_s(t_{17})$	38.15	62.89	42.13	53.83	48.18	49.79
EV_s	781.79	921.87	835.74	845.05	817.11	879.79
			Pure liabilities			
$R_s(t_1)$	3.00%	-1.00%	-0.95%	3.40%	4.00%	-2.00%
$R_s(t_5)$	3.39%	-0.61%	-0.08%	3.32%	3.96%	-1.17%
$R_s(t_9)$	4.44%	0.44%	2.03%	3.31%	4.05%	0.84%
$R_s(t_{10})$	4.93%	0.93%	2.90%	3.40%	4.18%	1.68%
$R_s(t_{14})$	6.46%	2.46%	5.31%	4.07%	4.92%	4.00%
$\mathrm{EV}_s(t_1)$	99.99	100.00	100.00	99.99	99.99	100.01
$\mathrm{EV}_s(t_5)$	48.95	50.19	50.02	48.97	48.78	50.37
$\mathrm{EV}_s(t_9)$	402.70	445.05	427.77	414.27	406.69	440.69
$\mathrm{EV}_s(t_{10})$	84.16	96.80	90.34	88.77	86.39	94.30
$\mathrm{EV}_s(t_{14})$	61.59	83.14	67.17	73.70	69.13	74.07
EV_s	697.39	775.18	735.31	725.71	710.98	759.43
			Equity			
EVE_s	84.41	146.68	100.43	119.34	106.13	120.37
$\Delta\,\mathrm{EVE}_s$	28.69	-33.58	12.67	-6.24	6.97	-7.27

The standardized approach

Example

- The current economic value of equity is equal to:

$$\mathrm{EVE}_0 = 847.82 - 734.73 = 113.09$$

- In the case of the first stress scenario, we have:

$$\mathrm{EVE}_1 = 781.79 - 697.39 = 84.41$$

and:

$$\Delta\,\mathrm{EVE}_1 = 113.10 - 84.41 = 28.69$$

- EVE decreases for scenarios 1, 3 and 5
- The EVE risk measure is equal to:

$$\mathcal{R}\left(\mathrm{EVE}\right) = \max_s \left(\Delta\,\mathrm{EVE}_s, 0\right) = 28.69$$

It represents 14.3% of the equity:

$$\frac{28.69}{200} = 14.3\%$$

The materiality test is not satisfied

Currency risk

- Currency hedging $\Rightarrow$ also the equity capital?
- Dollar funding
- Multi-currency balance sheet

Credit spread risk

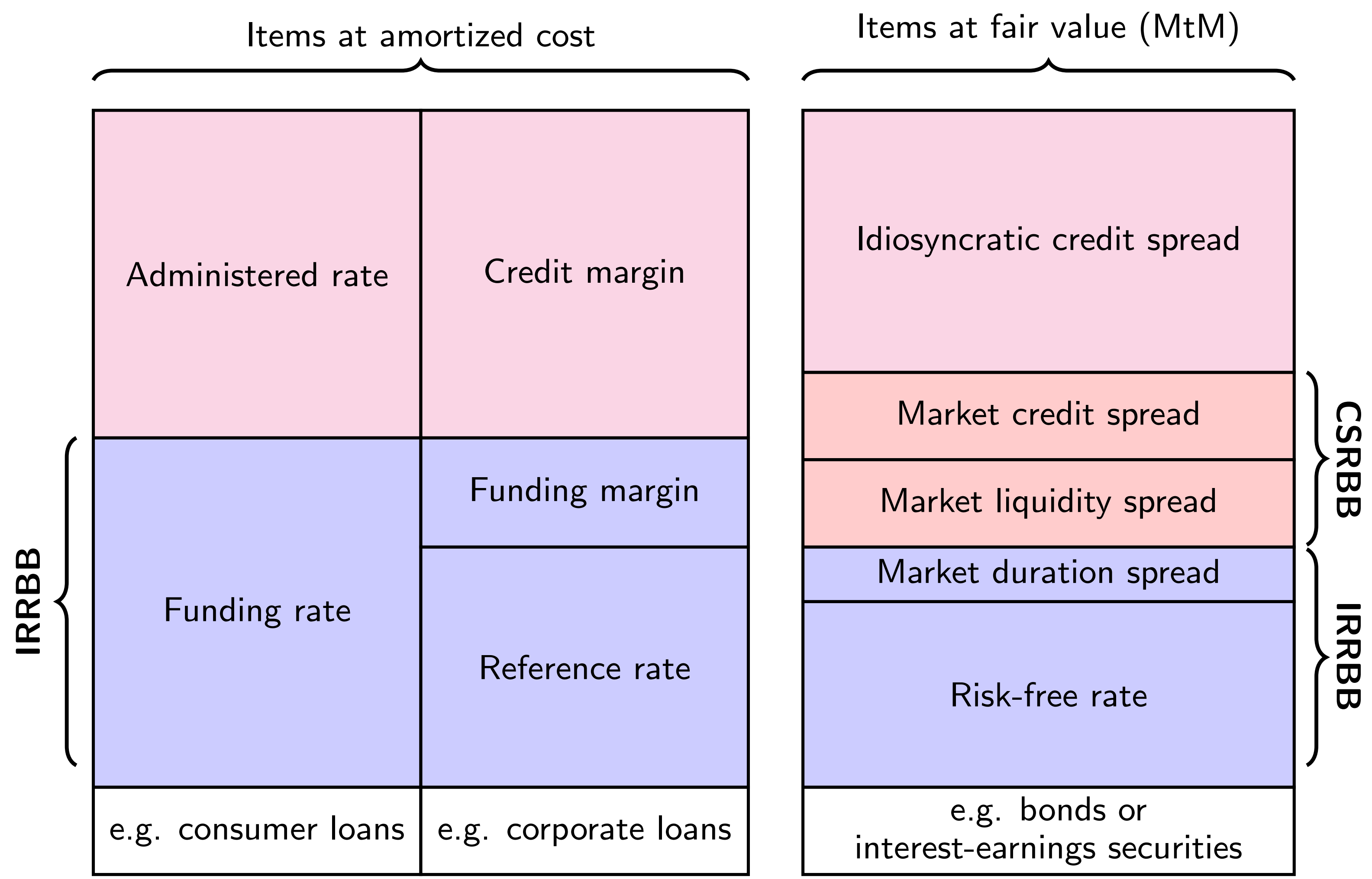


Figure: Components of interest rates

Macaulay duration

The Macaulay duration $\mathcal{D}$ is the weighted average of the cash flow maturities:

$$\mathcal{D} = \sum_{t_k \geq t} w(t, t_k) \cdot (t_k - t)$$

We have:

$$\frac{\partial V}{\partial y} = -\frac{\mathcal{D}}{1 + y} \cdot V = -\mathfrak{D} \cdot V$$

where $\mathfrak{D}$ is the modified duration

Application to a portfolio

The market value of the portfolio is composed of m cash flow streams: $V = \sum_{j=1}^{m} V_j$ while the duration of the portfolio is the average of individual durations: $\mathcal{D} = \sum_{j=1}^{m} w_j \cdot \mathcal{D}_j$ where $w_j = \frac{V_j}{V}$

Duration gap risk

Since $E(t) = A(t) - L^{\star}(t)$ and $\mathrm{EV}_E = \mathrm{EV}_A - \mathrm{EV}_{L^{\star}}$, the duration of equity is equal to:

$$\mathcal{D}_E = \frac{\mathrm{EV}_A}{\mathrm{EV}_A - \mathrm{EV}_{L^{\star}}} \cdot \mathcal{D}_A - \frac{\mathrm{EV}_{L^{\star}}}{\mathrm{EV}_A - \mathrm{EV}_{L^{\star}}} \cdot \mathcal{D}_{L^{\star}} = \frac{\mathrm{EV}_A}{\mathrm{EV}_A - \mathrm{EV}_{L^{\star}}} \cdot \mathcal{D}_{\mathcal{G}ap}$$

where the duration gap (also called DGAP) is equal to

$$\mathcal{D}_{\mathcal{G}ap} = \mathcal{D}_A - \frac{\mathrm{EV}_{L^{\star}}}{\mathrm{EV}_A} \cdot \mathcal{D}_{L^{\star}}$$

Duration gap risk

Another expression of the equity duration is:

$$\mathcal{D}_E = \frac{\mathrm{EV}_A}{\mathrm{EV}_E} \cdot \mathcal{D}_{\mathcal{G}ap} = \mathcal{L}_{A/E} \cdot \mathcal{D}_{\mathcal{G}ap}$$

where $\mathcal{L}_{A/E}$ is the leverage ratio

Relationship between EVE and duration gap

$$\begin{aligned}
\Delta \, \mathrm{EVE} &= \Delta \, \mathrm{EV}_E \\
&\approx -\mathcal{D}_E \cdot \mathrm{EV}_E \cdot \frac{\Delta y}{1+y} \\
&\approx -\mathcal{D}_{\mathcal{G}ap} \cdot \mathrm{EV}_A \cdot \frac{\Delta y}{1+y}
\end{aligned}$$

Illustration

We consider the following balance sheet:

Assets	V_j	$\mathcal{D}_j$	Liabilities	V_j	$\mathcal{D}_j$
Cash	5	0.0	Deposits	40	3.2
Loans	40	1.5	CDs	20	0.8
Mortgages	40	6.0	Debt	30	1.7
Securities	15	3.8	Equity capital	10	
Total	100		Total	100	

We have $\mathrm{EV}_A = 100$, $\mathrm{EV}_{L^\star} = 90$, $\mathrm{EV}_E = 10$ and:

$$\mathcal{L}_{A/E} = \frac{\mathrm{EV}_A}{\mathrm{EV}_E} = \frac{100}{10} = 10$$

The duration values are equal to:

$$\begin{aligned}
\mathcal{D}_A &= \frac{5}{100} \times 0 + \frac{40}{100} \times 1.5 + \frac{40}{100} \times 6.0 + \frac{15}{100} \times 3.8 = 3.57 \text{ years} \\
\mathcal{D}_{L^\star} &= \frac{40}{90} \times 3.2 + \frac{20}{90} \times 0.8 + \frac{30}{90} \times 1.7 = 2.17 \text{ years}
\end{aligned}$$

Illustration

We deduce that:

$$\mathcal{D}_{\mathcal{G}ap} = 3.57 - \frac{90}{100} \times 2.17 = 1.62 \text{ years}$$

If we assume that the current yield to maturity is equal to 3%, we obtain:

Δy	-2%	-1%	$+1\%$	$+2\%$
$\Delta\,\mathrm{EVE}$	3.15	1.57	−1.57	−3.15
$\dfrac{\Delta\,\mathrm{EVE}}{\mathrm{EVE}}$	31.46%	15.73%	−15.73%	−31.46%

Immunization of the balance sheet

We must have:

$$\Delta\,\mathrm{EVE} = 0 \Leftrightarrow \mathcal{D}_{\mathcal{G}ap} = 0 \Leftrightarrow \mathcal{D}_A - \frac{\mathrm{EV}_{L^\star}}{\mathrm{EV}_A} \cdot \mathcal{D}_{L^\star} = 0$$

Table: Bank balance sheet after immunization of the duration gap

Assets	V_j	$\mathcal{D}_j$	Liabilities	V_j	$\mathcal{D}_j$
Cash	5	0.0	Deposits	40	3.2
Loans	40	1.5	CDs	20	0.8
Mortgages	40	6.0	Debt	10.48	1.7
Securities	15	3.8	Zero-coupon bond	19.52	10.0
			Equity capital	10	0.0
Total	100		Total	100	

Income gap analysis

- If interest rates change, this induces a gap (or repricing) risk because the bank will have to reinvest assets and refinance liabilities at a different interest rate level in the future
- The gap is defined as the difference between rate sensitive assets (RSA) and rate sensitive liabilities (RSL):

$$\mathrm{GAP}\left(t,u\right)=\mathrm{RSA}\left(t,u\right)-\mathrm{RSL}\left(t,u\right)$$

  where t is the current date and u is the time horizon of the gap
- We can show that:

$$\Delta\,\mathrm{NII}\left(t,u\right)\approx\mathrm{GAP}\left(t,u\right)\cdot\Delta R$$

  where ΔR is the parallel shock of interest rates

Net interest income

The net interest income of the bank is the difference between interest rate revenues of its assets and interest rate expenses of its liabilities:

$$\mathrm{NII}(t,u) = \sum_{i \in \mathcal{A}ssets} N_i(t,u) \cdot R_i(t,u) - \sum_{j \in \mathcal{L}iabilities} N_j(t,u) \cdot R_j(t,u)$$

where $\mathrm{NII}(t,u)$ is the net interest income at time t for the maturity date u

$\Rightarrow$ Mathematical formulation (HFRM, pages 412-418)

Modeling customer rates

Client rates $\neq$ market rates

Several issues:

- Correlation
- Next repricing date (known or unknown?)
- Sensitivity of the customer rate with respect to the market rate

Hedging strategies

Using a forward rate agreement, we can show that:

$$\mathrm{NII}_{\mathcal{H}}(t,u) - \mathrm{NII}(t,u) = \mathrm{GAP}(t,u) \cdot \rho(t,u) \cdot (f(t,u) - r(u))$$

We can draw several conclusions:

- When the interest rate gap is closed, the bank does not need to hedge the net interest income
- When the correlation $\rho(t,u)$ between the customer rate and the market rate is equal to one, the notional of the hedge is exactly equal to the interest rate gap (it is lower in the general case)
- If the bank hedges the net interest income and if the gap is positive, a decrease of interest rates is not favorable

Hedging the interest rate gap depends on the expectations of the bank $\Longrightarrow$ partial hedging and macro hedging

Hedging instruments

- Interest rate swaps (IRS)
- Forward rate agreements (FRA)
- Swaptions

Funds transfer pricing

All liquidity and interest rate risks are transferred to the ALM unit:

- Business units can then lend or borrow funding at a given internal price
- This price is called the funds transfer price (FTP) or the internal transfer rate
- The FTP charges interests to the business unit for client loans, whereas the FTP compensates the business unit for raising deposits

Net interest margin

- The net interest margin (NIM) is equal to:

$$\begin{aligned}\mathrm{NIM}(t,u) &= \frac{\sum_{i\in\mathcal{A}ssets} N_i(t,u)\cdot R_i(t,u) - \sum_{j\in\mathcal{L}iabilities} N_j(t,u)\cdot R_j(t,u)}{\sum_{i\in\mathcal{A}ssets} N_i(t,u)} \\ &= \frac{\mathrm{RA}(t,u)\cdot R_{\mathrm{RA}}(t,u) - \mathrm{RL}(t,u)\cdot R_{\mathrm{RL}}(t,u)}{\mathrm{RA}(t,u)}\end{aligned}$$

  where R_{RA} and R_{RL} represent the weighted average interest rate of interest earning assets and interest bearing liabilities

- The net interest spread (NIS) is equal to:

$$\begin{aligned}\mathrm{NIS}(t,u) &= \frac{\sum_{i\in\mathcal{A}ssets} N_i(t,u)\cdot R_i(t,u)}{\sum_{i\in\mathcal{A}ssets} N_i(t,u)} - \frac{\sum_{j\in\mathcal{L}iabilities} N_j(t,u)\cdot R_j(t,u)}{\sum_{j\in\mathcal{L}iabilities} N_j(t,u)} \\ &= R_{\mathrm{RA}}(t,u) - R_{\mathrm{RL}}(t,u)\end{aligned}$$

- NIM is the profitability ratio of the assets whereas NIS is the interest rate spread captured by the bank

Net interest margin

Example

We consider the following interest earning and bearing items:

Assets	$N_i(t,u)$	$R_i(t,u)$	Liabilities	$N_j(t,u)$	$R_j(t,u)$
Loans	100	5%	Deposits	100	0.5%
Mortgages	100	4%	Debts	60	2.5%

Net interest margin

- The interest income is equal to $100 \times 5\% + 100 \times 4\% = 9$ whereas the interest expense is equal to $100 \times 0.5\% + 60 \times 2.5\% = 2$. We deduce that the net interest income is equal to:

$$\mathrm{NII}(t,u) = 9 - 2 = 7$$

- We have:

$$R_{\mathrm{RA}}(t,u) = \frac{100 \times 5\% + 100 \times 4\%}{100 + 100} = 4.5\%$$

and:

$$R_{\mathrm{RL}}(t,u) = \frac{100 \times 0.5\% + 60 \times 2.5\%}{100 + 60} = 1.25\%$$

Since $\mathrm{RA}(t,u) = 200$ and $\mathrm{RL}(t,u) = 160$, we deduce that:

$$\mathrm{NIM}(t,u) = \frac{200 \times 4.5\% - 160 \times 1.25\%}{200} = \frac{7}{200} = 3.5\%$$

and:

$$\mathrm{NIS}(t,u) = 4.5\% - 1.25\% = 3.25\%$$

Net interest margin

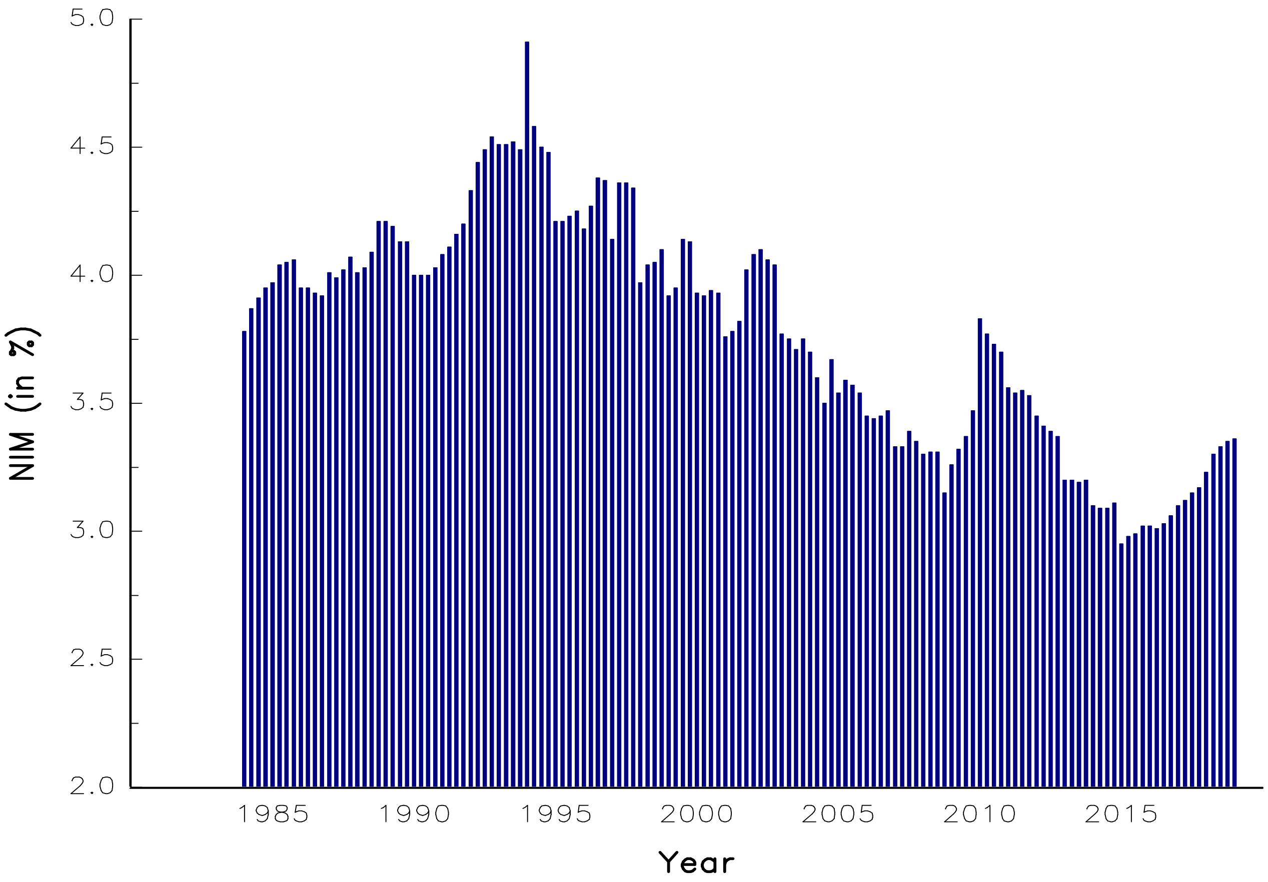


Figure: Evolution of the net interest margin in the US

Commercial margin

- The commercial margin rate is the spread between the client rate $R_i(t,u)$ of the asset i and the corresponding market rate $r(t,u)$:

$$m_i(t,u) = R_i(t,u) - r(t,u)$$

- For the liability j, we have:

$$m_j(t,u) = r(t,u) - R_j(t,u)$$

- In the case where we can perfectly match the asset i with the liability j, the commercial margin rate is the net interest spread:

$$m(t,u) = m_i(t,u) + m_j(t,u) = R_i(t,u) - R_j(t,u)$$

Commercial margin

Introducing a funds transfer pricing system is equivalent to interpose the ALM unit between the business unit and the market

- For assets, we have:

$$m_i(t,u) = \underbrace{\left(R_i(t,u) - \mathrm{FTP}_i(t,u)\right)}_{m_i^{(c)}(t,u)} + \underbrace{\left(\mathrm{FTP}_i(t,u) - r(t,u)\right)}_{m_i^{(t)}(t,u)}$$

  where:
  - $m_i^{(c)}(t,u)$ is the commercial margin rate of the business unit
  - $m_i^{(t)}(t,u)$ is the transformation margin rate of the ALM unit

- For liabilities, we have:

$$m_j(t,u) = \underbrace{\left(\mathrm{FTP}_j(t,u) - R_j(t,u)\right)}_{m_j^{(c)}(t,u)} + \underbrace{\left(r(t,u) - \mathrm{FTP}_j(t,u)\right)}_{m_j^{(t)}(t,u)}$$

Commercial margin

Commercial margins and funds transfer prices

The goal of FTP is to lock the commercial margin rates $m_i^{(c)}(t,u)$ and $m_j^{(c)}(t,u)$ over the lifetime of the product contract

Commercial margin

Example

We consider the following interest earning and bearing items:

Assets	$N_i(t,u)$	$R_i(t,u)$	Liabilities	$N_j(t,u)$	$R_j(t,u)$
Loans	100	5%	Deposits	100	0.5%
Mortgages	100	4%	Debts	60	2.5%

The FTP for the loans and the mortgages is equal to 3%, while the FTP for deposits is equal to 1.5% and the FTP for debts is equal to 2.5%. We assume that the market rate is equal to 2.5%

Solution

We obtain the following results:

Assets	$m_i^{(c)}(t,u)$	$m_i^{(t)}(t,u)$	Liabilities	$m_j^{(c)}(t,u)$	$m_j^{(t)}(t,u)$
Loans	2%	0.5%	Deposits	1.0%	1.0%
Mortgages	1%	0.5%	Debts	0.0%	0.0%

Commercial margin

The commercial margin of the bank is equal to:

$$M^{(c)} = 100 \times 2\% + 100 \times 1\% + 100 \times 1\% + 60 \times 0\% = 4$$

For the transformation margin, we have:

$$M^{(t)} = 100 \times 0.5\% + 100 \times 0.5\% + 100 \times 1.0\% + 60 \times 0\% = 2$$

We don't have $M^{(c)} + M^{(t)} = \mathrm{NII}$ because assets and liabilities are not compensated:

$$\mathrm{NII} - \left(M^{(c)} + M^{(t)} \right) = \left(\mathrm{RA}\left(t,u\right) - \mathrm{RL}\left(t,u\right)\right) \cdot r\left(t,u\right) = 40 \times 2.5\% = 1$$

The commercial margin of each product is:

- $M^{(c)}_{Loans} = 2$
- $M^{(c)}_{Mortgages} = 1$
- $M^{(c)}_{Deposits} = 1$

Computing the internal transfer rate

The reference rate

- Since we have $m_i^{(t)}(t,u) = \mathrm{FTP}_i(t,u) - r(t,u)$, internal prices are fair if the corresponding mark-to-market is equal to zero on average
- For a contract with a bullet maturity, this implies that:

$$\mathrm{FTP}_i(t,u) = \mathbb{E}[r(t,u)]$$

- The transformation margin can then be interpreted as an interest rate swap receiving a fixed leg $\mathrm{FTP}_i(t,u)$ and paying a floating leg $r(t,u)$

Remark

It follows that the funds transfer price is equal to the market swap rate at the initial date t with the same maturity than the asset item i

Computing the internal transfer rate

FTPs and the new production

If we assume that the commercial margin rate of the business unit is constant:

$$R(u) - \mathrm{FTP}(t,u) = m$$

we can show that:

$$\mathrm{FTP}(t,u) = R(u) + \frac{\mathbb{E}_t\left[\int_t^\infty B(t,u)\,\mathbf{S}(t,u)\,(r(u) - R(u))\,\mathrm{d}u\right]}{\mathbb{E}_t\left[\int_t^\infty B(t,u)\,\mathbf{S}(t,u)\,\mathrm{d}u\right]}$$

We deduce that:

- for a loan with a fixed rate, the funds transfer price is exactly the swap rate with the same maturity than the loan and the same amortization scheme than the new production
- if the client rate $R(u)$ is equal to the short-term market rate $r(u)$, the funds transfer price $\mathrm{FTP}(t,u)$ is also equal to $r(u)$

Non-maturity deposits

What is the maturity of NMDs?

- The deposit balance of the client A is equal to \$500 $\Rightarrow$ the duration of this deposit is equal to zero day
- We consider 1 000 clients $\Rightarrow$ the total amount that may be withdrawn today is then between \$0 and \$500 000
- We assume that the probability to withdraw \$500 at once is equal to 50% $\Rightarrow$ the probability that \$500 000 are withdrawn is less than $10^{-300}\%$!
- Since we have $\Pr\{S > 275000\} < 0.1\%$, we can decide that 55% of the deposit balance has a duration of zero day, 24.75% has a duration of one day, 11.14% has a duration of two days, etc.

The statistical duration of NMDs is long (and not short)

Non-maturity deposits

Dynamic modeling

In the case of non-maturity deposits, the hazard function rate $\lambda(t,u)$ of the amortization function $\mathbf{S}(t,u)$ does not depend on the entry date t:

$$\lambda(t,u) = \lambda(u)$$

We can show that (HFRM, pages 428-428):

$$\mathrm{d}N(t) = (\mathrm{NP}(t) - \lambda(t)N(t))\,\mathrm{d}t$$

If we assume that the new production and the hazard rate are constant – $\mathrm{NP}(t) = \mathrm{NP}$ and $\lambda(t) = \lambda$, we obtain:

$$\mathrm{d}N(t) = \lambda(N_{\infty} - N(t))\,\mathrm{d}t$$

Non-maturity deposits
Dynamic modeling

Two extensions:

- The Ornstein-Uhlenbeck model:

$$\mathrm{d}N(t) = \lambda \left(N_{\infty} - N(t) \right) \mathrm{d}t + \sigma \, \mathrm{d}W(t)$$

- The aggregate model:

$$D(t) = \underbrace{D_{\infty} e^{g(t-s)}}_{D_{\mathrm{long}}(s,t)} + \underbrace{\left(D_s - D_{\infty} \right) e^{(g-\lambda)(t-s)} + \varepsilon(t)}_{D_{\mathrm{short}}(s,t)}$$

where g is the growth rate of deposits

Non-maturity deposits

Stable vs non-stable deposits

Remark

We have:

$$D(t) = \underbrace{\varphi D_\infty e^{g(t-s)}}_{D_{\mathrm{stable}}(s,t)} + \underbrace{(D_s - D_\infty) e^{(g-\lambda)(t-s)} + \varepsilon(t) + (1-\varphi) D_\infty e^{g(t-s)}}_{D_{\mathrm{non-stable}}(s,t)}$$

Calibration of φ

We have:

$$\Pr\{D(t) \leq \varphi D_\infty\} = 1 - \alpha$$

IIf we consider the Ornstein-Uhlenbeck dynamics, we obtain:

$$\varphi = 1 - \frac{\sigma \Phi^{-1}(1-\alpha)}{D_\infty \sqrt{2\lambda}}$$

Non-maturity deposits

Stable vs non-stable deposits

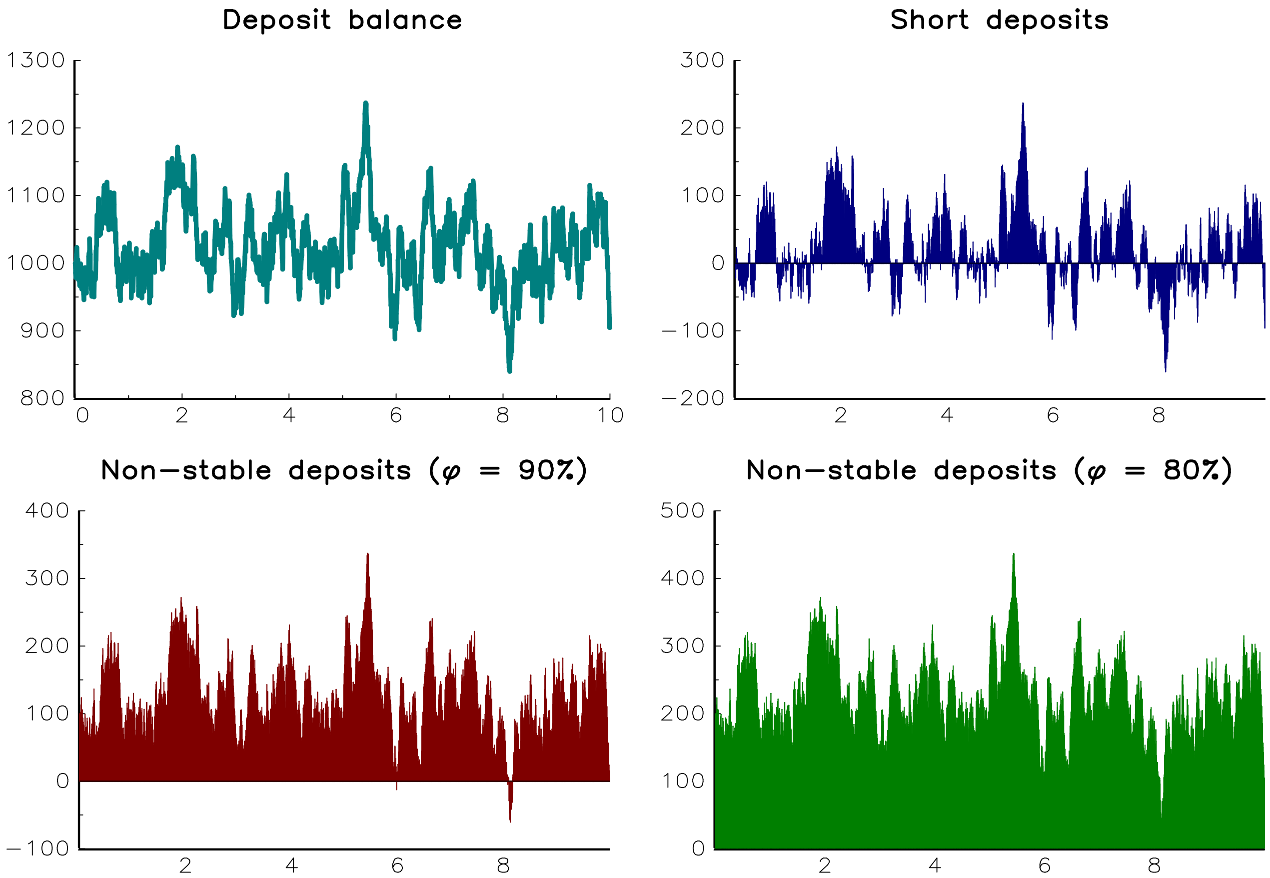


Figure: Stable and non-stable deposits

Non-maturity deposits

Behavioral modeling

The Hutchison-Pennacchi-Selvaggio framework

- The deposit rate $i(t)$ is exogenous and the bank account holder modifies his current deposit balance $D(t)$ to target a level $D^{\star}(t)$:

$$\ln D^{\star}(t) = \beta_0 + \beta_1 \ln i(t) + \beta_2 \ln Y(t)$$

  where $Y(t)$ is the income of the account holder

- The behavior of the bank account holder can be represented by a mean-reverting AR(1) process:

$$\ln D(t) - \ln D(t-1) = (1-\phi)\left(\ln D^{\star}(t) - \ln D(t-1)\right) + \varepsilon(t)$$

- The bank maximizes its profit $i^{\star}(t) = \arg\max \Pi(t)$ where the profit $\Pi(t)$ is equal to the revenue minus the cost:

$$\Pi(t) = r(t) \cdot D(t) - (i(t) + c(t)) \cdot D(t)$$

  $r(t)$ is the market interest rate and $c(t)$ is the cost of issuing deposits

- We can show that:

$$i^{\star}(t) = r(t) - s(t)$$

Non-maturity deposits

Behavioral modeling

The IRS framework (Jarrow and van Deventer, 1998)

- The current market value of deposits is the net present value of the cash flow stream $D(t)$:

$$V(0) = \mathbb{E}\left[\sum_{t=0}^{\infty} B(0, t+1)(r(t) - i(t)) D(t)\right]$$

  $V(0)$ as an exotic interest rate swap, where the bank receives the market rate and pays the deposit rate.

- We have:

$$\ln D(t) = \ln D(t-1) + \beta_0 + \beta_1 r(t) + \beta_2 (r(t) - r(t-1)) + \beta_3 t$$

  and:

$$i(t) = i(t) + \beta_0' + \beta_1' r(t) + \beta_2' (r(t) - r(t-1))$$

Non-maturity deposits

Behavioral modeling

Asymmetric adjustment models

- O'Brien model:

$$\Delta i(t) = \alpha(t) \cdot (\hat{\imath}(t) - i(t-1)) + \eta(t)$$

where $\hat{\imath}(t)$ is the conditional equilibrium deposit rate and:

$$\alpha(t) = \alpha^{+} \cdot \mathbb{1}\{\hat{\imath}(t) > i(t-1)\} + \alpha^{-} \cdot \mathbb{1}\{\hat{\imath}(t) < i(t-1)\}$$

- Frachot model:

$$\ln D(t) - \ln D(t-1) = (1-\phi)(\ln D^{\star}(t) - \ln D_{t-1}) + \delta_c(r(t), r^{\star})$$

where $\delta_c(r(t), r^{\star}) = \delta \cdot \mathbb{1}\{r(t) \leq r^{\star}\}$ and $r^{\star}$ is the interest rate floor

- OTS model:

$$d(t) = d(t-1) + \Delta \ln \left(\beta_0 + \beta_1 \arctan\left(\beta_2 + \beta_3 \frac{i(t)}{r(t)}\right) + \beta_4 i(t)\right) + \varepsilon(t)$$

Non-maturity deposits
Behavioral modeling

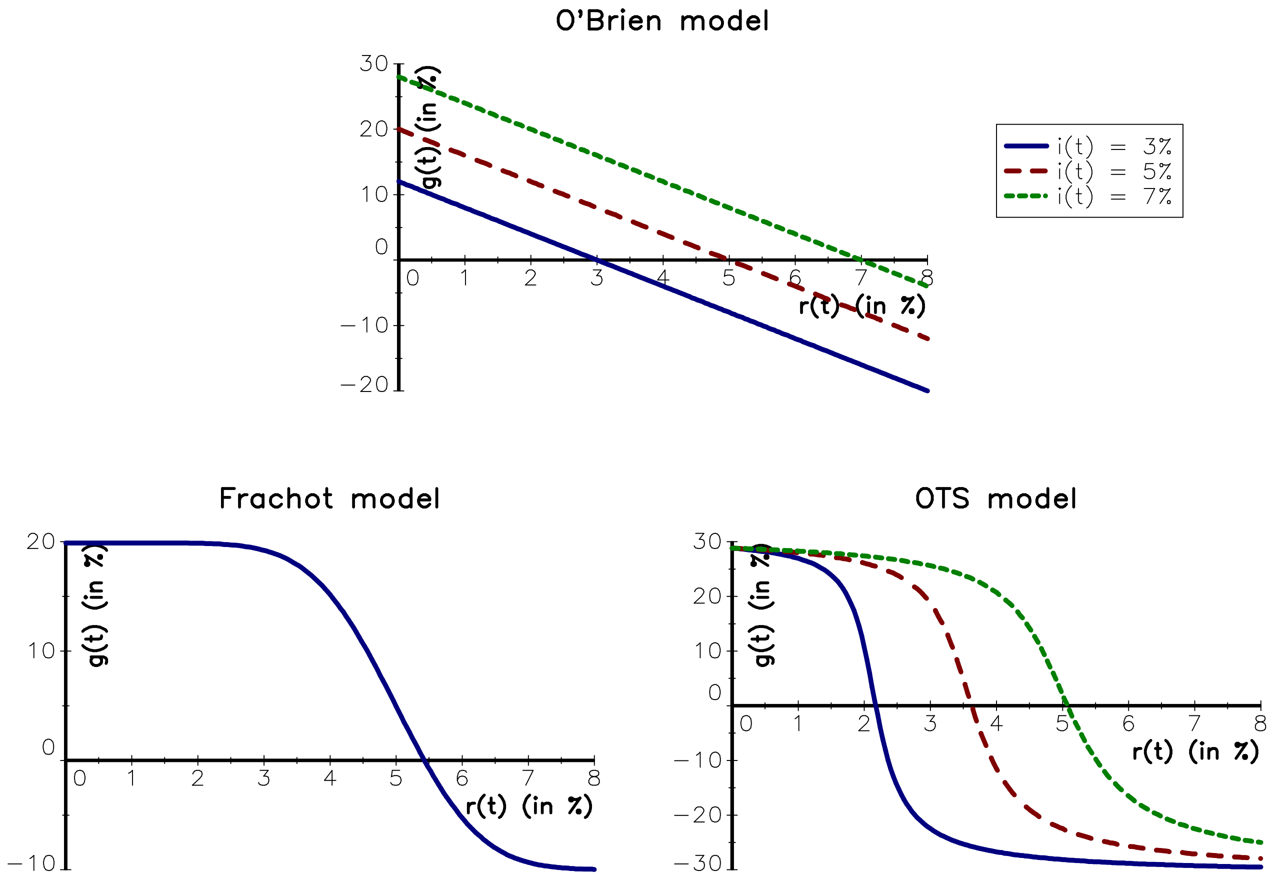


Figure: Impact of the market rate on the growth rate of deposits

Prepayment risk

Definition

A prepayment is the settlement of a debt or the partial repayment of its outstanding amount before its maturity date

Prepayment risk

Factors of prepayment

1. Refinancing:

$$\mathbb{P}(t) = \Pr\{\boldsymbol{\tau} \leq t\} = \vartheta(i_0 - i(t))$$

> *"A household with a 30-year fixed-rate mortgage of* \$200 000 *at an interest rate of* 6.0% *that refinances when rates fall to* 4.5% *(approximately the average rate decrease between 2008 and 2010 in the US) saves more than* \$60 000 *in interest payments over the life of the loan, even after accounting for refinance transaction costs. Further, when mortgage rates reached all-time lows in late 2012, with rates of roughly* 3.35% *prevailing for three straight months, this household with a contract rate of* 6.5% *would save roughly* \$130 000 *over the life of the loan by refinancing" (Keys, et al., 2016, pages 482-483).*

2. Housing turnover (marriage, divorce, death, children leaving home or changing jobs)

Prepayment risk

Structural models

The prepayment value is the premium of an American call option, meaning that we can derive the optimal option exercise. In this case, the prepayment strategy can be viewed as an arbitrage strategy between the market interest rate and the cost of refinancing

Prepayment risk
Reduced-form models

Rate, coupon or maturity incentive?

We assume that the mortgage rate drops from i_0 to $i(t)$

- The absolute difference of the annuity is equal to:

$$\mathfrak{D}_A(i_0, i(t)) = A(i_0, n) - A(i(t), n)$$

- The relative cumulative difference $\mathfrak{C}(i_0, i(t))$ is equal to:

$$\mathfrak{C}(i_0, i(t)) = \frac{\sum_{t=1}^{n} \mathfrak{D}_A(i_0, i(t))}{N_0}$$

- By assuming that the borrower continues to pay the same annuity, the maturity reduction is given by:

$$\mathfrak{N}(i_0, i(t)) = \{x \in \mathbb{N} : A(i(t), x) \geq A(i(t), n), A(i(t), x+1) < A(i(t), n)\}$$

Prepayment risk
Reduced-form models

Table: Impact of a new mortgage rate (100 KUSD, 5%, 10-year)

i (in %)	A (in \$)	$\mathfrak{D}_A$ (in \$) Monthly	Annually	$\mathfrak{D}_R$ (in %)	$\mathfrak{C}$ (in %)	$\mathfrak{M}$ (in years)
5.0	1 061					
4.5	1 036	24	291	2.3	2.9	9.67
4.0	1 012	48	578	4.5	5.8	9.42
3.5	989	72	862	6.8	8.6	9.17
3.0	966	95	1 141	9.0	11.4	8.92
2.5	943	118	1 415	11.1	14.2	8.75
2.0	920	141	1 686	13.2	16.9	8.50
1.5	898	163	1 953	15.3	19.5	8.33
1.0	876	185	2 215	17.4	22.2	8.17
0.5	855	206	2 474	19.4	24.7	8.00

Prepayment risk

Reduced-form models

Table: Impact of a new mortgage rate (100 KUSD, 5%, 20-year)

i (in %)	A (in \$)	$\mathfrak{D}_A$ (in \$) Monthly	Annually	$\mathfrak{D}_R$ (in %)	$\mathfrak{C}$ (in %)	$\mathfrak{N}$ (in years)
5.0	660					
4.5	633	27	328	4.1	6.6	18.67
4.0	606	54	648	8.2	13.0	17.58
3.5	580	80	960	12.1	19.2	16.67
3.0	555	105	1 264	16.0	25.3	15.83
2.5	530	130	1 561	19.7	31.2	15.17
2.0	506	154	1 849	23.3	37.0	14.50
1.5	483	177	2 129	26.9	42.6	14.00
1.0	460	200	2 401	30.3	48.0	13.50
0.5	438	222	2 664	33.6	53.3	13.00

Prepayment risk
Reduced-form models

Table: Impact of a new mortgage rate (100 KUSD, 10%, 10-year)

i (in %)	A (in \$)	$\mathfrak{D}_A$ (in \$) Monthly	Annually	$\mathfrak{D}_R$ (in %)	$\mathfrak{C}$ (in %)	$\mathfrak{N}$ (in years)
10.0	1 322					
9.0	1 267	55	657	4.1	6.6	9.33
8.0	1 213	108	1 299	8.2	13.0	8.75
7.0	1 161	160	1 925	12.1	19.3	8.33
6.0	1 110	211	2 536	16.0	25.4	7.92
5.0	1 061	261	3 130	19.7	31.3	7.58
4.0	1 012	309	3 709	23.3	37.1	7.25
3.0	966	356	4 271	26.9	42.7	6.92
2.0	920	401	4 816	30.4	48.2	6.67
1.0	876	445	5 346	33.7	53.5	6.50

Prepayment risk
Reduced-form models

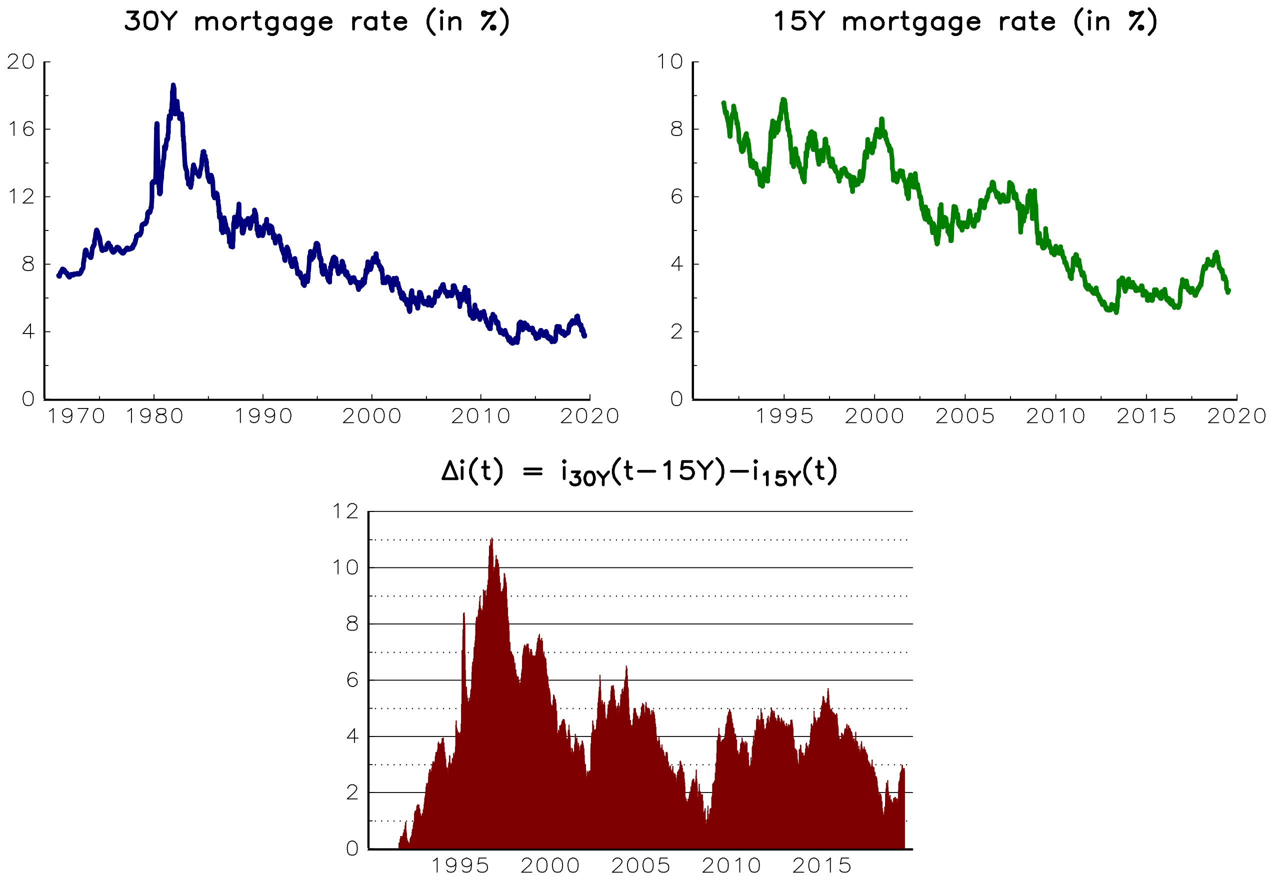


Figure: Evolution of 30-year and 15-year mortgage rates in the US

Prepayment risk

Survival function with prepayment risk

We have:

$$\mathbf{S}(t,u) = \mathbf{S}_c(t,u) \cdot \mathbf{S}_p(t,u)$$

where $\mathbf{S}_c(t,u)$ is the traditional amortization function (or the contract-based survival function) and $\mathbf{S}_p(t,u)$ is the prepayment-based survival function

Prepayment risk

Survival function with prepayment risk

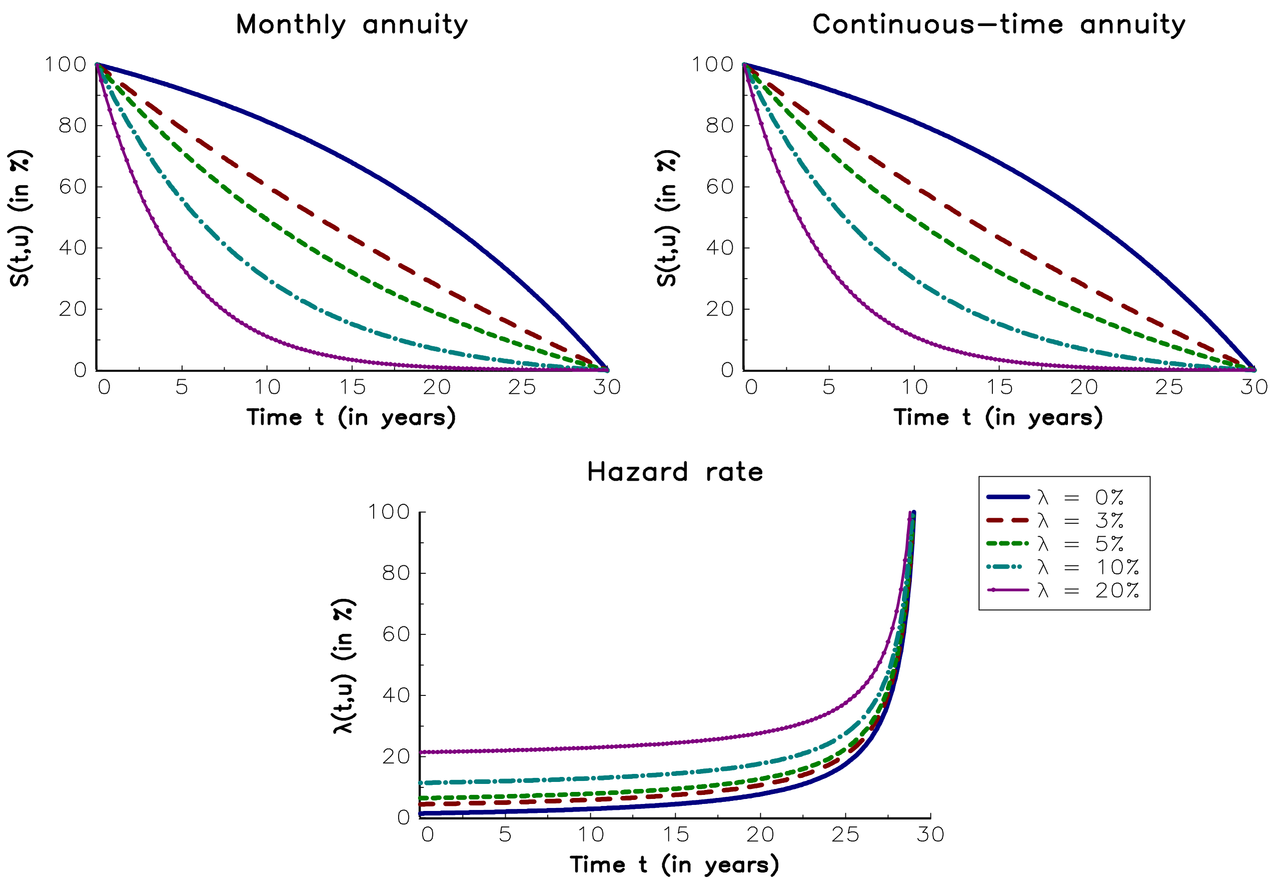


Figure: Survival function in the case of prepayment

Prepayment risk

Specification of the hazard function

- $\mathbf{S}_p(t,u)$ can be decomposed into the product of two survival functions:

$$\mathbf{S}_p(t,u) = \mathbf{S}_{\mathrm{refinancing}}(t,u) \cdot \mathbf{S}_{\mathrm{turnover}}(t,u)$$

- OTC model:

$$\lambda_p(t,u) = \lambda_{\mathrm{age}}(u-t) \cdot \lambda_{\mathrm{seasonality}}(u) \cdot \lambda_{\mathrm{rate}}(u)$$

where λ_{age} measures the impact of the loan age, $\lambda_{\mathrm{seasonality}}$ corresponds to the seasonality factor and λ_{rate} represents the influence of market rates

Prepayment risk

Specification of the hazard function

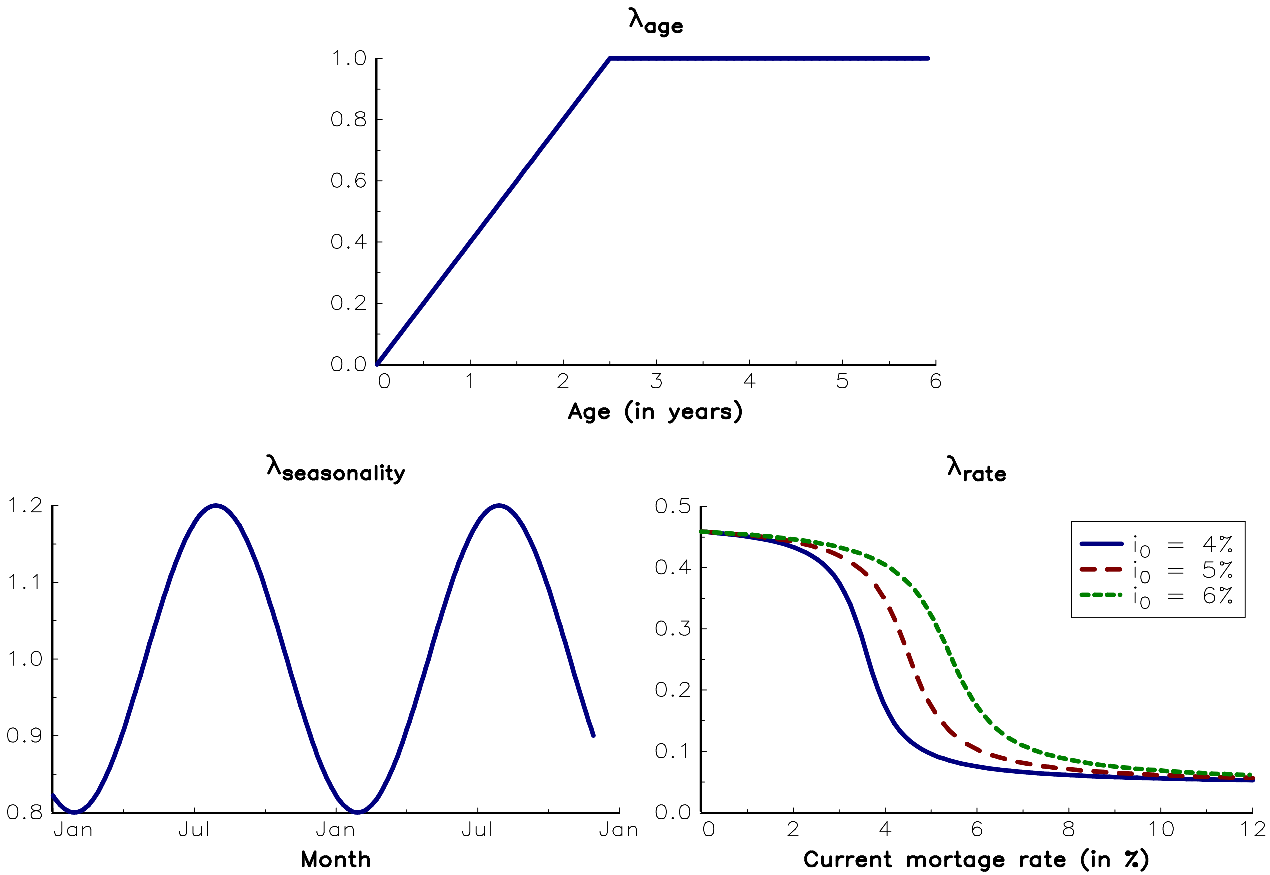


Figure: Components of the OTC model

Prepayment risk

Statistical measure of prepayment

- Single monthly mortality:

$$\mathrm{SMM} = \frac{\text{prepayments during the month}}{\text{outstanding amount at the beginning of the month}}$$

- The constant prepayment rate (CPR) and the SMM are related by the following equation:

$$\mathrm{CPR} = \left(1 - \left(1 - \mathrm{SMM}\right)\right)^{12}$$

- In IRRBB, the CPR is also known as the conditional prepayment rate:

$$\begin{aligned}
\mathrm{CPR}\left(u,t\right) &= \Pr\left\{u < \boldsymbol{\tau} \leq u+1 \mid \boldsymbol{\tau} \geq u\right\} \\
&= \frac{\mathbf{S}_p\left(t,u\right) - \mathbf{S}_p\left(t,u+1\right)}{\mathbf{S}_p\left(t,u\right)} \\
&= 1 - \exp\left(-\int_u^{u+1} \lambda_p\left(t,s\right)\,\mathrm{d}s\right)
\end{aligned}$$

Prepayment risk

Statistical measure of prepayment

Table: Conditional prepayment rates in June 2018 by coupon rate and issuance date

Year	2012	2013	2014	2015	2016	2017	2018
Coupon = 3%	9.6%	10.2%	10.9%	10.0%	8.7%	5.3%	3.1%
Coupon = 4.5%	16.1%	15.8%	16.6%	17.9%	17.4%	12.8%	5.3%
Difference	6.5%	5.6%	5.7%	8.0%	8.7%	7.6%	2.2%

Redemption risk

The funding risk of term deposits

- A term deposit, also known as time deposit or certificate of deposit (CD), is a fixed-term cash investment. The client deposits a minimum sum of money into a banking account in exchange for a fixed rate over a specified period
- When buying a term deposit, the investor can withdraw their funds only after the term ends
- Under some conditions, the investor may withdraw his term deposit before the maturity date if he pays early redemption costs and fees

Redemption risk

Early time deposit withdrawals may be motivated by two reasons:

1. Economic motivation: $i(t) \gg i_0$
2. Negative liquidity shocks of depositors

The redemption-based survival function of time deposits can be decomposed as:

$$\mathbf{S}_r(t,u) = \mathbf{S}_{\mathrm{economic}}(t,u) \cdot \mathbf{S}_{\mathrm{liquidity}}(t,u)$$

Redemption risk

Modeling the early withdrawal risk

Early withdrawals due to economic reasons

- We note t the current date, m the maturity of the time deposit and N_0 the initial investment at time 0
- The value of the time deposit at the maturity is equal to $V_0 = N_0 (1 + i_0)^m$
- The value of the investment for $\boldsymbol{\tau} = t$ becomes:

$$V_r(t) = N_0 \cdot (1 + (1 - \varphi(t))\, i_0)^t \cdot (1 + i(t))^{m-t} - C(t)$$

  where $\varphi(t)$ is the penalty parameter applied to interest paid and $C(t)$ is the break fee
- The rational investor redeems the term deposit if the refinancing incentive is positive:

$$\mathrm{RI}(t) = \frac{V_r(t) - V_0}{N_0} > 0$$

Redemption risk

Modeling the early withdrawal risk

Early withdrawals due to economic reasons

- We can assume that:

$$\lambda_{\mathrm{economic}}(t,u)=g\left(i(u)-i_0\right)$$

or:

$$\lambda_{\mathrm{economic}}(t,u)=g\left(r(u)-i_0\right)$$

Redemption risk

Modeling the early withdrawal risk

Early withdrawals due to negative liquidity shocks

We can decompose the hazard function into two effects:

$$\lambda_{\mathrm{liquidity}}(t,u) = \lambda_{\mathrm{structural}} + \lambda_{\mathrm{cyclical}}(u)$$

where $\lambda_{\mathrm{structural}}$ is the structural rate of redemption and $\lambda_{\mathrm{cyclical}}(u)$ is the liquidity component due to the economic cycle. A simple way to model $\lambda_{\mathrm{cyclical}}(u)$ is to consider a linear function of the GDP growth

Exercises

- Interest rate risk
  - Exercise 7.4.1 – Constant amortization of a loan
  - Exercise 7.4.2 – Computation of the amortization functions $\mathbf{S}(t, u)$ and $\mathbf{S}^{\star}(t, u)$
  - Exercise 7.4.3 – Continuous-time analysis of the constant amortization mortgage (CAM)
- Non-maturity deposits (NMD)
  - Exercise 7.4.4 – Valuation of non-maturity deposits
- Prepayment risk
  - Exercise 7.4.5 – Impact of prepayment on the amortization scheme of the CAM

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