Risk Management & Financial Regulation
Final Examination

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Please write entirely your answers. The correction of exercises will be available in the next release of the lecture notes.

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel II?
3. How is calculated the capital requirement with the internal model approach in Basel III (FRTB)?

3 Credit risk

1. What is the definition of the default in Basel II?
2. Describe the standard approach (SA) to compute the capital requirement.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?

4 Counterparty credit risk (CCR) and credit value adjustment (CVA)

1. Define the concept of counterparty credit risk. What differences do you make between the CCR capital charge in Basel II and the CVA capital charge in Basel III?
2. How is calculated the CCR capital requirement? Explain why the exposure-at-default (EAD) has to be estimated.
3. How is calculated the CVA capital requirement?

5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to calculate the capital charge.
3. Describe the loss distribution approach (LDA) to calculate the capital charge.

6 Market risk of a long/short portfolio

We consider an investment universe, which is composed of two stocks A and B. The current prices of the two stocks are respectively equal to \$50 and \$20. Their volatilities are equal to 25% and 20% whereas the cross-correlation is equal to +12.5%. The portfolio is long of 2 stocks A and short of 5 stocks B.

1. Gaussian risk measure

   (a) Calculate the Gaussian value-at-risk at the 99% confidence level for a ten-day time horizon.

   (b) Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.

2. Historical risk measure
   The ten worst scenarios of daily stock returns (expressed in %) among the last 250 historical scenarios are the following:

s	1	2	3	4	5	6	7	8	9	10
R_A	-0.6	-3.7	-5.8	-4.2	-3.7	0.0	-5.7	-4.3	-1.7	-4.1
R_B	5.7	2.3	-0.7	0.6	0.9	4.5	-1.4	0.0	2.3	-0.2
$R_A - R_B$	-6.3	-6.0	-5.1	-4.8	-4.6	-4.5	-4.3	-4.3	-4.0	-3.9

   (a) Calculate the historical value-at-risk at the 99% confidence level for a ten-day time horizon.

   (b) Calculate the historical expected shortfall at the 97.5% confidence level for a ten-day time horizon.

   (c) Give an approximation of the capital charge under Basel II, Basel II.5 and Basel III/FRTB Standards by considering the historical risk measure[1].

7 Credit spreads

We consider a CDS 3M with two-year maturity and \$1 mn notional principal. The recovery rate $\mathcal{R}$ is equal to 40% whereas the spread $\mathcal{s}$ is equal to 150 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection buyer B if the reference entity defaults in one year and two months?

2. What is the relationship between $\mathcal{s}$, $\mathcal{R}$ and λ? What is the implied one-year default probability at the inception date?

3. Seven months later, the CDS spread has increased and is equal to 450 bps. Estimate the new default probability. The protection buyer B decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 1.189.

8 Calculation of the capital charge for counterparty credit risk

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N and σ be the notional and the volatility of the underlying contract. We assume that $e(t) = N\sigma\sqrt{t}X$ with $0 \leq X \leq 1$, $\Pr\{X \leq x\} = x^{\gamma}$ and $\gamma > 0$.

1. Calculate the peak exposure $\mathrm{PE}_{\alpha}(t)$, the expected exposure $\mathrm{EE}(t)$ and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

[1] We assume that the multiplicative factor is equal to 3, and the 'stressed' risk measure is 2 times the 'normal' risk measure.

2. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N is equal to \$3 mn, the maturity T is one year, the volatility σ is set to 20% and γ is estimated at 2.

   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.

   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[2].

9 Parametric estimation of the loss severity distribution

We assume that the severity losses are log-logistic distributed $X_i \sim \mathcal{LL}(\alpha, \beta)$ with:

$$\mathbf{F}(x; \alpha, \beta) = \frac{(x/\alpha)^{\beta}}{1 + (x/\alpha)^{\beta}}$$

1. Calculate the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.

2. Show that the ML estimators satisfy the following first-order conditions:

$$\left\{ \begin{array}{l} \sum_{i=1}^{n} \mathbf{F}\left(x_i; \hat{\alpha}, \hat{\beta}\right) = n/2 \\ \sum_{i=1}^{n} \left(2\mathbf{F}\left(x_i; \hat{\alpha}, \hat{\beta}\right) - 1\right) \ln x_i = n/\hat{\beta} \end{array} \right.$$

3. What does the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$ become if we assume that the losses were collected beyond a threshold H?

10 The bivariate Pareto copula

We consider the bivariate Pareto distribution:

$$\begin{array}{rcl} \mathbf{F}(x_1, x_2) & = & 1 - \left(\dfrac{\theta_1 + x_1}{\theta_1}\right)^{-\alpha} - \left(\dfrac{\theta_2 + x_2}{\theta_2}\right)^{-\alpha} + \\ & & \left(\dfrac{\theta_1 + x_1}{\theta_1} + \dfrac{\theta_2 + x_2}{\theta_2} - 1\right)^{-\alpha} \end{array}$$

where $x_1 \geq 0$, $x_2 \geq 0$, $\theta_1 > 0$, $\theta_2 > 0$ and $\alpha > 0$.

1. Show that the marginal functions of $\mathbf{F}(x_1, x_2)$ correspond to univariate Pareto distributions[3].

2. Find the copula function associated to the bivariate Pareto distribution.

3. Deduce the copula density function.

4. Show that the bivariate Pareto copula function has no lower tail dependence ($\lambda^{-} = 0$), but an upper tail dependence ($\lambda^{+} = 2^{-\alpha}$).

5. Do you think that the bivariate Pareto copula family can reach the copula functions $\mathbf{C}^{-}$, $\mathbf{C}^{\perp}$ and $\mathbf{C}^{+}$? Justify your answer.

6. Let X_1 and X_2 be two Pareto-distributed random variables, whose parameters are (α_1, θ_1) and (α_2, θ_2).

[2] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We remind that $\Phi^{-1}(0.01) \approx -2.33$ and $\Phi^{-1}(0.999) \approx 3.09$. We can also use the approximation $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

[3] We remind that the univariate Pareto distribution $\mathcal{P}(\alpha, \theta)$ can be written as:

$$\mathbf{F}(x) = 1 - \left(\frac{\theta + x}{\theta}\right)^{-\alpha}$$

where $x \geq 0$, $\theta > 0$ and $\alpha > 0$.

(a) Show that the linear correlation between X_1 and X_2 is equal to 1 if and only if the parameters α_1 and α_2 are equal.

(b) Show that the linear correlation between X_1 and X_2 can never reached the lower bound -1.

(c) Build a new bivariate Pareto distribution by assuming that the marginal distributions are $\mathcal{P}(\alpha_1, \theta_1)$ and $\mathcal{P}(\alpha_2, \theta_2)$ and the dependence is a bivariate Pareto copula function with parameter α. What is the relevance of this approach for building bivariate Pareto distributions?

11 Extreme value theory in the bivariate case

1. What is an extreme value (EV) copula $\mathbf{C}$?

2. Show that $\mathbf{C}^{\perp}$ and $\mathbf{C}^{+}$ are EV copulas. Why $\mathbf{C}^{-}$ cannot be an EV copula?

3. We define the Gumbel-Hougaard copula as follows:

$$\mathbf{C}(u_1, u_2) = \exp\left(-\left[(-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}\right]^{1/\theta}\right)$$

   with $\theta \geq 1$. Verify that it is an EV copula.

4. What is the definition of the upper tail dependence λ? What is its usefulness in multivariate extreme value theory?

5. Let $f(x)$ and $g(x)$ be two functions such that $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = 0$. If $g'(x_0) \neq 0$, L'Hospital's rule states that:

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$

   Deduce that the upper tail dependence λ of the Gumbel-Hougaard copula is $2 - 2^{1/\theta}$. What is the correlation of two extremes when $\theta = 1$?