# Financial Risk Management Examination

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**Please write entirely your answers.**

## 1 The BCBS regulation

1. What are the main differences between the first Basle Accord and the second Basle Accord?
2. What are the three pillars of the Basle II Accord?
3. What are the new capital requirements imposed by Basle III?

## 2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets that require capital.
2. How is computed the capital requirement with the internal model-based approach?
3. Why do we need to compute two VaR measures for the internal model-based approach?

## 3 Credit risk

1. What is the definition of the default in Basle II?
2. Describe the standard approach (SA) to compute the capital requirement?
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?

## 4 Counterparty credit risk

1. Define the concept of counterparty credit risk. Give two examples.
2. How is computed the capital requirement for this type of risk?
3. Define the three methods to compute the exposure-at-default (EaD) parameter.

## 5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to compute the capital charge.
3. Describe the loss distribution approach (LDA) to compute the capital charge.

## 6 Value-at-risk of an equity portfolio

We consider an investment universe of two stocks $A$ and $B$ and an equity index $I$. The current prices of the two stocks are respectively equal to 150 and 200 euros. The other characteristics are the following:

| | Volatility | Correlation matrix | | |
|---|---|---|---|---|
| | | $A$ | $B$ | $I$ |
| $A$ | 20% | 100% | | |
| $B$ | 40% | 64% | 100% | |
| $I$ | 20% | 80% | 80% | 100% |

1. The portfolio $\mathcal{P}_0$ is composed of two stocks $A$ and one stock $B$. Compute the Gaussian VaR of $\mathcal{P}_0$ for a one-week time horizon and a 99% confidence level.

2. We would like to hedge the portfolio $\mathcal{P}_0$ by adding a **<u>short</u>** position $-W_I$ on the index $I$. We note $\mathcal{P}_1$ the hedged portfolio.

   (a) Compute the Gaussian VaR of $\mathcal{P}_1$ when $W_I = 500$?

   (b) Same question when $W_I = 560$.

   (c) What do you conclude?

3. We assume that the CAPM model is valid. It implies that we have the following relationships:

$$\left\{ \begin{array}{l} R_A = \beta_A R_I + \varepsilon_A \\ R_B = \beta_B R_I + \varepsilon_B \end{array} \right.$$

   where $R_A$ and $R_B$ are the returns of stocks $A$ and $B$, $\varepsilon_A$ and $\varepsilon_B$ are their idiosyncratic risks and $R_I$ is the return of the index $I$. We recall that $R_I$, $\varepsilon_A$ and $\varepsilon_B$ are independent. We set $\beta_A = 80\%$ and $\beta_B = 160\%$.

   (a) How do you explain the results obtained in Questions 2.(a) and 2.(b)?

   (b) Compute $\sigma(\varepsilon_A)$ and $\sigma(\varepsilon_B)$.

   (c) Retrieve the result obtained in Question 2.(b).

   (d) Show that the VaR of the hedged portfolio $\mathcal{P}_1$ is lower than the VaR of the original portfolio $\mathcal{P}_0$ if $W_I \leq 1120$.

## 7 Risk contribution in the Basle II model

Let us consider a portfolio of $I$ loans with maturity $M_i$. We denote $L$ the portfolio loss:

$$L = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathrm{LGD}_i \times 1\{\tau_i \leq M_i\}$$

We can show that, under some assumptions $(\mathcal{H})$, the expectation of the portfolio loss conditionally to the factors $X_1, \ldots, X_m$ is:

$$\mathbb{E}[L \mid X_1, \ldots, X_m] = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathbb{E}[\mathrm{LGD}_i] \times \mathrm{PD}_i(X_1, \ldots, X_m)$$

1. How do we obtain this expression? What are the necessary assumptions $(\mathcal{H})$? What do we call an infinitely granular portfolio?

2. Define the credit risk contribution.

3. Define the *expected loss* (EL) and the *unexpected loss* (UL). Show that the VaR measure is equal to the EL measure under the previous hypothesis $(\mathcal{H})$ if the default times are independent of the factors.

4. Write the expression of the loss quantile $\mathbf{F}^{-1}(\alpha)$ when we have a single factor $X \sim \mathbf{H}$. Why this expression is not relevant if at least one of the exposures $\mathrm{EAD}_i$ is negative? What do you conclude for the management of the credit portfolio?

5. In the Basle II model, we assume that the loan $i$ defaults before the maturity $M_i$ if a latent variable $Z_i$ goes below a barrier $B_i$:
$$\tau_i \leq M_i \Leftrightarrow Z_i \leq B_i$$
We consider that $Z_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$ where $Z_i$, $X$ and $\varepsilon_i$ are three independent Gaussian variables $\mathcal{N}(0,1)$. $X$ is the factor (or the systemic risk) and $\varepsilon_i$ is the idiosyncratic risk. Compute the conditional default probability.

6. Compute the quantile $\mathbf{F}^{-1}(\alpha)$.

7. Interpret the correlation $\rho$.

8. The previous risk contribution was obtained considering the assumptions $(\mathcal{H})$ and the framework of the default model defined in Question 5. What are the implications in terms of Pillar II?

# 8 Correlation and log-normal random variables

1. Let $X \sim \mathcal{N}(\mu, \sigma^2)$. We set $Y = e^X$ the log-normal random variable and we note $Y \sim \mathcal{LN}(\mu, \sigma^2)$.
   (a) Compute the density function of $Y$.
   (b) Deduce the expression of the first moment $\mathbb{E}[Y]$.
   (c) Let $m \geq 1$ be an integer. Show the following result:
   $$\mathbb{E}[Y^m] = e^{m\mu + \frac{1}{2}m^2\sigma^2}$$
   (d) Deduce the variance of $Y$.
   (e) Let $Y = (Y_1, \ldots, Y_n)$ be a sample of *iid* random variables. We assume that $Y_i \sim \mathcal{LN}(\mu, \sigma^2)$. Explain how to estimate the parameters $\mu$ and $\sigma$ by the generalized method of moments (GMM).

2. Let $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$. We also assume that $(X_1, X_2)$ is a Gaussian random vector and the correlation between $X_1$ and $X_2$ is equal to $\rho$.
   (a) Find the distribution of $X_1 + X_2$.
   (b) Deduce that the covariance between $Y_1 = e^{X_1}$ and $Y_2 = e^{X_2}$ is:
   $$\operatorname{cov}(Y_1, Y_2) = e^{\mu_1 + \frac{1}{2}\sigma_1^2} e^{\mu_2 + \frac{1}{2}\sigma_2^2} \left(e^{\rho\sigma_1\sigma_2} - 1\right)$$
   (c) Compute the correlation between $Y_1$ and $Y_2$. Then, find the cases when $\rho(Y_1, Y_2) = -1$ (resp. $\rho(Y_1, Y_2) = +1$).
   (d) By using the previous results, explain why the linear correlation can not be a concordance measure.

# 9 Credit spreads

1. We assume that the default time $\tau$ follows an exponential distribution $\mathcal{E}(\lambda)$ of parameter $\lambda$. Write the distribution function $\mathbf{F}$ and the survival function $\mathbf{S}$ of $\tau$. How do we simulate the default time $\tau$?

2. We consider a CDS 3M with a two-year maturity. Give the flow chart assuming that the protection leg is paid at default and the recovery rate is fixed and equal to $R$.

3. What is the spread $s$ of the CDS? What is the relationship between $s$, $R$ and $\lambda$?

4. We assume a recovery rate of 25%. What is the implied one-year default probability if the CDS spread is equal to 200 bps? What is the *relative value* strategy?

# 10 Extreme value theory and stress-testing

1. Define the stress-testing. What is its usefulness in risk management? How is it used by the regulation? Give an example of stress-testing for credit risk.

2. Let $X$ be the daily return of a portfolio. Using the standard assumptions, what is the probability distribution $\mathbf{G}_n$ of the maximum of daily returns for a period of $n$ days if we suppose that $X \sim \mathcal{N}(\mu, \sigma)$:
$$\max(X_1, \ldots, X_n) \sim \mathbf{G}_n$$

3. Recall the Fisher-Tippet theorem characterizing the asymptotic distribution $\mathbf{G}_\infty$ of $\max(X_1, \ldots, X_n)$ where $X_i$ are *iid* random variables.

4. Extreme value theory in the bivariate case.

   (a) What is an extreme value (EV) copula $\mathbf{C}$?

   (b) Is the product copula $\mathbf{C}^\perp(u_1, u_2) = u_1 u_2$ an EV copula?

   (c) We define the Gumbel-Houggaard copula as follows:
   $$\mathbf{C}(u_1, u_2) = \exp\left(-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta\right]^{1/\theta}\right)$$
   with $\theta \geq 1$. Verify that it is an EV copula.

   (d) What is the definition of the upper tail dependence $\lambda$? What is its usefulness in multivariate extreme value theory?

   (e) Let $f(x)$ and $g(x)$ be two functions such that $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = 0$. If $g'(x_0) \neq 0$, L'Hospital's rule states that:
   $$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$
   Deduce that the upper tail dependence $\lambda$ of the Gumbel-Houggaard copula is $2 - 2^{1/\theta}$.

   (f) What is the correlation of two extremes when $\theta = 1$?

5. Maximum domain of attraction in the bivariate case.

   (a) We note $a_n$ and $b_n$ the normalization parameters of the Fisher-Tippet theorem. We obtain the following results for three univariate distributions:

| Distribution | | $\mathbf{F}(x)$ | $a_n$ | $b_n$ | $\mathbf{G}_\infty(x)$ | |
|---|---|---|---|---|---|---|
| Exponential | $\mathcal{E}(\lambda)$ | $1 - e^{-\lambda x}$ | $\lambda^{-1}$ | $\lambda^{-1} \ln n$ | $\mathbf{\Lambda}(x)$ | $e^{-e^{-x}}$ |
| Uniform | $\mathcal{U}_{[0,1]}$ | $x$ | $n^{-1}$ | $1 - n^{-1}$ | $\mathbf{\Psi}_1(x - 1)$ | $e^{x-1}$ |
| Pareto | $\mathcal{P}(\theta, \alpha)$ | $1 - \left(\frac{\theta}{\theta + x}\right)^\alpha$ | $\theta \alpha^{-1} n^{1/\alpha}$ | $\theta n^{1/\alpha} - \theta$ | $\mathbf{\Phi}_\alpha\left(1 + \frac{x}{\alpha}\right)$ | $e^{-\left(1 + \frac{x}{\alpha}\right)^{-\alpha}}$ |

   We note $\mathbf{G}_\infty(x_1, x_2)$ the asymptotic distribution of the bivariate random vector[1] $(X_{1,n:n}, X_{2,n:n})$ where $X_{1,i}$ (resp. $X_{2,i}$) are *iid* random variables. What is the expression of $\mathbf{G}_\infty(x_1, x_2)$ when $X_{1,i}$ and $X_{2,i}$ are independent and:

   i. $X_{1,i} \sim \mathcal{E}(\lambda)$ and $X_{2,i} \sim \mathcal{U}_{[0,1]}$?

   ii. $X_{1,i} \sim \mathcal{E}(\lambda)$ and $X_{2,i} \sim \mathcal{P}(\theta, \alpha)$?

   (b) Same question if the dependence function between $X_{1,i}$ and $X_{2,i}$ is the Gaussian copula with parameter $\rho < 1$.

   (c) Same question if the dependence function between $X_{1,i}$ and $X_{2,i}$ is the Gaussian copula with parameter $\rho = 1$.

   (d) Same question if the dependence function between $X_{1,i}$ and $X_{2,i}$ is the Gumbel-Houggaard copula with parameter $\theta > 1$.

[1] We recall that $X_{1,n:n} = \max(X_{1,1}, \ldots, X_{1,n})$ and $X_{2,n:n} = \max(X_{2,1}, \ldots, X_{2,n})$.