Financial Risk Management Examination

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Please write entirely your answers.

1 The Basle II regulation

1. What are the main differences between the first Basle Accord and the second Basle Accord?
2. What are the three pillars of the Basle II Accord?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets that require capital.
2. How is computed the capital requirement with the internal model-based approach?
3. Define precisely the backtesting method.

3 Credit risk

1. What is the definition of the default in Basle II?
2. Describe the standard approach (SA) to compute the capital requirement.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?

4 Counterparty credit risk on OTC operations

1. Define the concept of counterparty credit risk. Give two examples.
2. Explain why the exposure at default (EaD) is considered as random.

5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to compute the capital charge.
3. Describe the loss distribution approach (LDA) to compute the capital charge.

6 Value at risk of a long/short portfolio

Let us consider a long/short portfolio composed of a long (buying) position in the asset A and a short (selling) position in the asset B. The current prices of the two stocks are equal to 100 euros.

1. Using the historical prices of the last 250 trading days of asset A and B, we estimate that the annual volatilities $\hat{\sigma}_A$ and $\hat{\sigma}_B$ are both equal to 20%, and that the correlation between asset returns is equal to 50%. Neglecting the mean effect, calculate the gaussian VaR of the long/short portfolio for a time horizon of 1 day and a confidence level of 99%.

2. How do we calculate the historical VaR? Using the historical returns of the last 250 trading days, the five worst scenarios of the 250 simulated daily PnLs of the portfolio are -3.37, -3.09, -2.72, -2.67 and -2.61 euros. Calculate the historical VaR for a time horizon of 1 day and a confidence level of 99%.

3. The long/short portfolio manager decides to sell a call option on the stock A. How do the previous calculations change if we consider the delta-normal method to measure the value at risk (we suppose that the delta of the option is equal to 50%)?

7 Parameter estimation for operational risk

1. Let us consider a sample of single losses $\{L_1, \ldots, L_n\}$. We assume that these losses are log-normal distributed $L_i \sim \mathcal{LN}(\mu, \sigma)$.

   (a) Show that the density function of the probability distribution $\mathcal{LN}(\mu, \sigma)$ is:

   $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)$$

   (b) Deduce the log-likelihood function of the sample $\{L_1, \ldots, L_n\}$.

   (c) Calculate the maximum likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$.

   (d) We assume that the losses $\{L_1, \ldots, L_n\}$ were collected beyond a threshold H. Calculate the log-likelihood function of the sample $\{L_1, \ldots, L_n\}$.

2. Let N_t be the number of losses for the period t. We note $\{N_1, \ldots, N_T\}$ a sample of N_t and we assume that N_t follows a Poisson distribution $\mathcal{P}(\lambda)$. We remind that:

   $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   (a) Compute the first moment $\mathbb{E}[N_t]$.

   (b) Show the following result:

   $$\mathbb{E}\left[\prod_{i=0}^{m} (N_t - i)\right] = \lambda^{m+1}$$

   Deduce the variance of N_t.

   (c) Propose two estimators based on the method of moments.

3. Let L be the random sum:

   $$L = \sum_{i=0}^{N_t} L_i$$

   where $L_i \sim \mathcal{LN}(\mu, \sigma)$, $L_i \perp L_j$ and $N_t \sim \mathcal{P}(\lambda)$.

(a) Calculate the mathematical expectation $\mathbb{E}[L]$.

(b) We remind that:

$$\left(\sum_{i=1}^{n} x_i\right)^2 = \sum_{i=1}^{n} x_i^2 + \sum_{i \neq j} x_i x_j$$

Show that:

$$\operatorname{var}(L) = \lambda e^{2\mu + 2\sigma^2}$$

(c) How can we estimate μ and σ if we have already calibrated λ?

8 Copula functions

1. Let $\tau = (\tau_1, \tau_2)$ be a random vector with distribution $\mathbf{F}$. We assume that τ_1 and τ_2 are two exponential default times with parameters λ_1 and λ_2. We also assume that the copula function $\mathbf{C}$ of the random vector τ is Normal with parameter ρ.

   (a) Let $U = (U_1, U_2)$ be a random vector with copula $\mathbf{C}$. We note Σ the matrix defined as follows:

   $$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

   Compute the Cholesky decomposition of Σ. Deduce an algorithm to simulate U.

   (b) We remind that the bivariate cumulative density function of the gaussian standardized vector $X = (X_1, X_2)$ with correlation ρ is:

   $$\Phi(x_1, x_2; \rho) = \int_{-\infty}^{x_1} \Phi\left(\frac{x_2 - \rho x}{\sqrt{1 - \rho^2}}\right) \phi(x) \, \mathrm{d}x$$

   Deduce the expression of the Normal copula by considering the change of variable $u = \Phi(x)$. Calculate then the conditional copula function $\mathbf{C}_{2|1}$.

   (c) Describe the simulation algorithm based on conditional distributions. Apply this algorithm to the Normal copula. Show that this algorithm is equivalent to the simulation algorithm based on the Cholesky decomposition.

   (d) How to simulate $\tau = (\tau_1, \tau_2)$ from the random variates (u_1, u_2) generated by one of the two previous algorithms.

2. We consider some special cases of ρ.

   (a) Show that if $\rho = 1$, we have the following relationship:

   $$\tau_1 = \frac{\lambda_2}{\lambda_1} \tau_2$$

   Deduce that the linear correlation between τ_1 and τ_2 is equal to $+1$:

   $$\rho \langle \tau_1, \tau_2 \rangle = +1$$

   (b) What becomes this relationship when $\rho = -1$? Deduce that the linear correlation between τ_1 and τ_2 is not equal to -1:

   $$\rho \langle \tau_1, \tau_2 \rangle > -1$$

9 Credit spreads

1. We model the default time τ as an exponential distribution of parameter λ. Write the associated distribution function $\mathbf{F}$ and survival function $\mathbf{S}$. How do we simulate the default time τ?

2. Let us consider a CDS 3M with a 1 year maturity. Give the flow chart assuming that the CDS protection leg is paid at default and the recovery rate is fixed and equal to R.

3. What is the spread s of the CDS? What is the relationship between s, R and λ?

4. We assume a recovery rate of 25%. What is the implied 1 year default probability of a counterpart whose CDS spread is equal to 200 bps?

10 Extreme value theory and stress-testing

1. Define the stress-testing. What is its usefulness in risk management? How is it used by the regulation? Give an example of stress-testing for credit risk.

2. Let X be the daily return of a portfolio. Using the standard assumptions, what is the probability distribution $\mathbf{G}_n$ of the maximum of daily returns for a period of n days if we suppose that $X \sim \mathcal{N}(\mu, \sigma)$:
$$\max(X_1, \ldots, X_n) \sim \mathbf{G}_n$$

3. Recall the Fisher-Tippet theorem characterizing the asymptotic distribution of $\max(X_1, \ldots, X_n)$ where X_i are *iid* random variables.

4. Extreme value theory in the bivariate case.

   (a) What is an extreme value (EV) copula $\mathbf{C}$?

   (b) Is the product copula $\mathbf{C}^{\perp}(u_1, u_2) = u_1 u_2$ an EV copula?

   (c) We define the Marshall-Olkin copula as follows:
   $$\mathbf{C}(u_1, u_2) = u_1^{1-\theta_1} u_2^{1-\theta_2} \min\left(u_1^{\theta_1}, u_2^{\theta_2}\right)$$
   with $\{\theta_1, \theta_2\} \in [0,1]^2$. Verify that it is an EV copula.

   (d) What is the definition of the upper tail dependence λ? What is its usefulness in multivariate extreme value theory?

   (e) Let $f(x)$ and $g(x)$ be two functions such that $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = 0$. If $g'(x_0) \neq 0$, L'Hospital's rule states that:
   $$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$
   Deduce that the upper tail dependence λ of the the Marshall-Olkin copula is $\min(\theta_1, \theta_2)$.

   (f) What is the correlation of two extremes when $\min(\theta_1, \theta_2) = 0$?