Risk-Based Investing & Asset Management
Final Examination

Thierry Roncalli

January 29th, 2021

Deadline: March 28th, 2021

Remark 1 ***The final examination is composed of 2 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file or one program by exercise.***

- ***Concerning risk decomposition[1], present the results as follows:***

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^{\star}$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			✓	

- ***The report is a zipped file, whose filename is <u>yourname.zip</u> if you do the project alone or <u>yourname1-yourname2.zip</u> if you do the project in groups of two.***
- ***The zipped file is composed of three files:***
    1. ***the pdf document that contains the answers to the two exercises and a cover sheet with your names;***
    2. ***the program of each exercise with an explicit filename, e.g. exercise1.xls (if you use excel), exercise1.m (if you use matlab), exercise1.py (if you use python), exercise1.r (if you use R), etc.***

[1] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^{\star}$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

1 Mean-variance and risk-based portfolios

We consider an investment universe with 5 assets. We assume that their expected returns are equal to 5%, 7%, 6%, 10% and 8% whereas their volatilities are equal to 18%, 20%, 22%, 25% and 30%. The correlation matrix C is given by:

$$C = \begin{pmatrix} 100\% & & & & \\ 70\% & 100\% & & & \\ 20\% & 30\% & 100\% & & \\ -30\% & 20\% & 10\% & 100\% & \\ 0\% & 0\% & 0\% & 0\% & 100\% \end{pmatrix}$$

We assume that the risk measure is the portfolio volatility $\sigma(x)$.

1. Calculate the covariance matrix.
2. We consider long-short MVO portfolios such that:

$$\sum_{i=1}^{n} x_1 = 1$$

   (a) Using the γ-problem, find the optimal solution $x^\star(\gamma)$ when the coefficient γ is equal to 0, 0.10, 0.20, 0.50 and 1.00. For each optimized portfolio, computed its expected return $\mu(x^\star(\gamma))$ and its volatility $\sigma(x^\star(\gamma))$.

   (b) Draw the efficient frontier by considering granular values[2] of γ.

   (c) Using a bisection algorithm, calculate the MVO portfolio if we target an ex-ante volatility of 12% and 15%. Give the corresponding values of γ of the QP problem.

3. We consider long-short portfolios such that:

$$\sum_{i=1}^{n} x_1 = 1$$

   (a) Give the risk decomposition of the equally-weighted (EW) portfolio.

   (b) Give the risk decomposition of the minimum variance (MV) portfolio.

   (c) Give the risk decomposition of the equal risk contribution (ERC) portfolio.

   (d) Give the risk decomposition of the most diversified portfolio (MDP).

   (e) Place the four risk-based portfolios EW, GMV, ERC and MDP on the efficient frontier. Comment on these results.

   (f) For each asset i ($i = 1, \ldots, 5$), compute its correlation $\rho(e_i, x)$ with each risk-based portfolio x. Present the result in a 5×4 table. Comment on these results.

   (g) Compute the correlation matrix between the four risk-based portfolios. Comment on these results.

4. We consider long-only portfolios such that:

$$x_i \geq 0 \qquad \text{and} \qquad \sum_{i=1}^{n} x_1 = 1$$

   (a) Using a bisection algorithm, calculate the MVO portfolio if we target an ex-ante volatility of 12% and 15%. Give the corresponding values of γ of the QP problem.

[2] For instance, you can consider that $\gamma = -0.5, -0.4, \ldots, -0.1, 0, 0.05, 0.10, \ldots, 0.95, 1, 2, \ldots, 10$.

(b) Give the risk decomposition of the long-only minimum variance (MV) portfolio.

(c) Give the risk decomposition of the long-only most diversified portfolio (MDP).

(d) For each asset i ($i = 1, \ldots, 5$), compute its correlation $\rho(e_i, x)$ with each long-only risk-based portfolio x. Present the result in a 5×4 table. Comment on these results.

(e) Compute the correlation matrix between the four long-only risk-based portfolios. Comment on these results.

(f) What is the impact of the no short selling constraint?

2 Tilted portfolios with ESG and carbon intensity constraints

We consider the CAPM model:

$$R_i - r = \beta_i \cdot (R_m - r) + \varepsilon_i$$

where R_i is the return of Asset i, R_m is the return of the market portfolio, r is the risk free asset, β_i is the beta of Asset i with respect to the market portfolio and ε_i is the idiosyncratic risk. We assume that $R_m \perp \varepsilon_i$ and $\varepsilon_i \perp \varepsilon_j$. We note σ_m the volatility of the market portfolio and $\tilde{\sigma}_i$ the idiosyncratic volatility. We consider a universe of 5 assets with the following parameter values:

Asset i	1	2	3	4	5
β_i	0.30	0.50	0.90	1.30	2.00
$\tilde{\sigma}_i$	15%	16%	10%	11%	12%

and $\sigma_m = 20\%$. The risk free return is set to 1% and we assume that the expected return of the market portfolio is equal to $\mu_m = 6\%$.

1. We assume that the CAPM is valid.

   (a) Calculate the vector μ of expected returns.

   (b) Compute the covariance matrix Σ.

   (c) Deduce the volatility σ_i of the assets and find the correlation matrix $C = (\rho_{i,j})$ between asset returns.

   (d) Show that the tangency portfolio is a function of μ, Σ and r. Compute the tangency portfolio $x^\star$. Calculate $\mu(x^\star)$ and $\sigma(x^\star)$. Deduce the Sharpe ratio of the tangency portfolio.

   (e) Compute the beta coefficient $\beta_i(x^\star)$ of the five assets with respect to the tangency portfolio. Compute the implied expected return $\tilde{\mu}_i$:

   $$\tilde{\mu}_i = r + \beta_i(x^\star) \cdot (\mu(x^\star) - r)$$

   (f) Comment on these results.

2. We assume that:

Asset i	1	2	3	4	5
μ_i	3%	4%	5%	7%	10%
$\mathcal{S}_i^{\text{esg}}$	1.10	2.70	-0.90	-2.20	0.40
$\mathcal{CI}_i$	50	170	490	180	320
b_i	20%	20%	20%	20%	20%

   where μ_i, $\mathcal{S}_i^{\text{esg}}$, $\mathcal{CI}_i$ and b_i are respectively the expected return, the ESG score[3], the carbon intensity[4] and the benchmark weight of Asset i. The covariance matrix is given by the CAPM model and corresponds to the one calculated in Question 1.b. In what follows, we consider long-only portfolios.

   (a) Compute the ESG score $\mathcal{S}^{\text{esg}}(b)$ and the carbon intensity $\mathcal{CI}(b)$ of the benchmark b.

   (b) The current portfolio of the fund manager is equal to:

   $$x = \begin{pmatrix} 10\% \\ 10\% \\ 30\% \\ 30\% \\ 20\% \end{pmatrix}$$

[3] It corresponds to a z-score between -3 (worst score) and $+3$ (best score).

[4] It is measured in tCO_2e/\$ mn.

Compute the excess expected return $\mu(x \mid b)$, the tracking error volatility $\sigma(x \mid b)$, the ESG score $\mathcal{S}^{\text{esg}}(x)$ and the carbon intensity $\mathcal{CI}(x)$ of the portfolio x. Deduce its information ratio $\text{IR}(x \mid b)$. Comment on these results[5].

(c) We would like to tilt the benchmark b in order to improve its expected return. Formulate the γ-problem of portfolio optimization in the presence of a benchmark. Find the corresponding QP problem. We note $x^\star(\gamma)$ the optimized portfolio. Draw the efficient frontier betwen the tracking error volatility $\sigma(x^\star(\gamma) \mid b)$ and the excess expected return $\mu(x^\star(\gamma) \mid b)$.

(d) Draw the relationship between $\sigma(x^\star(\gamma) \mid b)$ and $\mathcal{S}^{\text{esg}}(x^\star(\gamma))$. Comment on these results.

(e) Draw the relationship between $\sigma(x^\star(\gamma) \mid b)$ and $\mathcal{CI}(x^\star(\gamma))$. Comment on these results.

(f) Find the optimal portfolio[6] $x^\star$ if we target an ex-ante tracking error volatility of 5%. Give the optimal value of γ and the expected excess return $\mu(x^\star \mid b)$. Compute also the ESG score $\mathcal{S}^{\text{esg}}(x^\star)$ and the carbon intensity $\mathcal{CI}(x^\star)$ of the optimal portfolio $x^\star$.

(g) We now assume that $\mu = \mathbf{0}_5$.

i. We would like to reduce the carbon intensity of the benchmark portfolio by 20%. Compute the optimal portfolio $x^\star$ such that it has the lowest tracking error volatility $\sigma(x \mid b)$. Give the values of $\sigma(x^\star \mid b)$, $\mathcal{S}^{\text{esg}}(x^\star)$ and $\mathcal{CI}(x^\star)$. Comment on these results.

ii. We would like to improve the ESG score of the benchmark portfolio by $+0.50$. Compute the optimal portfolio $x^\star$ such that it has the lowest tracking error volatility $\sigma(x \mid b)$. Give the values of $\sigma(x^\star \mid b)$, $\mathcal{S}^{\text{esg}}(x^\star)$ and $\mathcal{CI}(x^\star)$. Comment on these results.

iii. Same question if we mix the two constraints.

[5]In particular, do you think that Portfolio x is a good portfolio from the viewpoint of the financial analysis? Same question from the viewpoint of the extra-financial analysis

[6]It corresponds to the portfolio that maximizes the expected excess return given the tracking error volatility constraint.