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# The Market Measure of Carbon Risk and its Impact on the Minimum Variance Portfolio\*

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## Abstract

Like ESG investing, climate change is an important concern for asset managers and owners, and a new challenge for portfolio construction. Until now, investors have mainly measured carbon risk using fundamental approaches, such as with carbon intensity metrics. Nevertheless, it has not been proven that asset prices are directly impacted by these fundamental-based measures. In this paper, we focus on another approach, which consists in measuring the sensitivity of stock prices with respect to a carbon risk factor. In our opinion, carbon betas are market-based measures that are complementary to carbon intensities or fundamental-based measures when managing investment portfolios, because carbon betas may be viewed as an extension or forward-looking measure of the current carbon footprint. In particular, we show how this new metric can be used to build minimum variance strategies and how they impact their portfolio construction.

**Keywords:** Carbon, climate change, risk factor, carbon beta, carbon intensity, minimum variance portfolio.

**JEL classification:** C61, G11.

## Key findings:

1. Measuring carbon risk is different if we consider a fundamental-based approach by using carbon intensity metrics or a market-based approach by using carbon betas.
2. Managing relative carbon risk implies to overweight green firms, whereas managing absolute carbon risk implies having zero exposure to the carbon risk factor. The first approach is an active management bet, while the second case is an immunization investment strategy.
3. Both specific and systematic carbon risks are important when building a minimum variance portfolio and justify combining fundamental and market approaches of carbon risk.

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# 1 INTRODUCTION

According to Mark Carney (2019), climate change is one of the big current challenges faced by the financial sector with the goal to accelerate the transition to a low carbon economy. It concerns all the financial institutions: central banks, commercial banks, insurance companies, asset managers, asset owners, etc. Among the several underlying topics, the risk management of climate change will be one of the pillars of the future regulation in order to ensure financial sector resilience to a tail risk. Since risk management must concern both physical and transition risks (Carney, 2015), incorporating climate change when managing banks' credit portfolios is not obvious. The question is how climate change impacts the default probability of issuers. The same issue occurs when we consider stock and bond portfolios of asset managers and owners. Indeed, we have to understand how asset prices react to climate change. This is why we have to develop new risk metrics in order to assess the relationship between climate change and asset returns. However, we face data collection issues when we consider this broad subject. Therefore, we focus here on carbon risk since it is the main contributor to climate change<sup>1</sup> and we have more comprehensive and robust data on carbon metrics at the issuer level.

The general approach to managing an investment portfolio's carbon risk is to reduce or control the portfolio's carbon footprint, for instance by considering CO<sub>2</sub> or CO<sub>2e</sub> emissions. This approach assumes that the carbon risk will materialize and that having a portfolio with a lower exposure to CO<sub>2</sub> emissions will help to avoid some future losses. The main assumption of this approach is then to postulate that firms that currently have high carbon footprints will be penalized in the future in comparison with firms that currently have low carbon footprints. In this paper, we use an alternative approach. We define carbon risk from a financial point of view, and we consider that the carbon risk of equities corresponds to the market risk priced in by the stock market. This carbon financial risk can be decomposed into a common (or systematic) risk factor and a specific (or idiosyncratic) risk factor. Since identifying the specific risk is impossible, we focus on the common risk factor that drives the carbon risk. The objective is then to build a market-based risk measure to manage the carbon risk in investment portfolios. This is exactly the framework proposed by Gorgen et al. (2019) in their seminal paper.

In this framework, the carbon financial risk of a stock corresponds to its price sensitivity to the carbon risk factor. This carbon beta is a market-based relative risk and may be viewed as an extension or forward-looking measure of the carbon footprint, where the objective is to be more exposed to green firms than to brown ones. In this case, this is equivalent to promoting stocks with a negative carbon beta over stocks with a positive carbon beta. This approach of relative carbon risk differs from the approach of absolute carbon risk, which is measured at the stock level by the absolute value of the carbon beta, because absolute carbon risk considers that both large positive and negative carbon beta values incur a financial risk that must be reduced. This is an agnostic or neutral method, contrary to the first method which is more related to investors' moral values or convictions.

Since the 2008 Global Financial Crisis, institutional investors have widely used minimum variance (MV) strategies to reduce their equity investments' market risk. While the original idea of these strategies was to reduce the portfolio's volatility, today the goal of minimum variance strategies is to manage the largest financial unrewarded risks and not just volatility risk. This is why sophisticated MV programs also include idiosyncratic valuation risk, reputational risk, etc. In this context, incorporating climate risk into minimum variance portfolios is natural. Therefore, we propose a two-factor model that is particularly adapted to this investment strategy and show that the solution depends on whether we would like to manage relative or absolute carbon risk.

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<sup>1</sup>This implies that we consider transition risks, not physical risks.

## 2 THE MARKET MEASURE OF CARBON RISK

To manage a portfolio’s carbon risk, carbon risk needs to be measured at the company level. There are different ways to measure this risk, including the fundamental and market approaches. In this paper, we favor the second approach because it provides a better assessment of the impact of climate-related transition risks on each company’s stock price. Moreover, the market-based approach allows us to mitigate the issue of a lack of climate change-relevant information. In what follows, we present this latest approach by using the mimicking portfolio for carbon risk developed by Gorgen et al. (2019). We compare this seminal approach with a simplified approach, which consists in using direct metrics such as carbon intensity. Once carbon betas are computed, we can analyze the carbon risk of each company priced in by the stock market and compare it with the carbon intensity, which is the most used fundamental-based measure of carbon risk. We also discuss the difference between relative and absolute carbon risk.

### 2.1 Measuring carbon risk

Measuring a company’s carbon risk using the carbon beta of its stock price was first proposed by Gorgen et al. (2019). In what follows, we summarize their approach and test alternative approaches. Moreover, we suggest using the Kalman filter in order to estimate the dynamic carbon beta of stock prices.

**The Carima approach** The goal of the carbon risk management (Carima) project, developed by Gorgen et al. (2019), is to develop “a *quantitative tool in order to assess the opportunities of profits and the risks of losses that occur from the transition process*”. The Carima approach combines a market-based approach and a fundamental approach. Indeed, the carbon risk of a firm or a portfolio is measured by considering the dynamics of stock prices which are partly determined by climate policies and transition processes towards a green economy. Nevertheless, a prior fundamental approach is important to quantify carbon risk. In practical terms, the fundamental approach consists in defining a carbon risk score for each stock in an investment universe using a set of objective measures, whereas the market approach consists in building a brown minus green or BMG carbon risk factor, and computing the risk sensitivity of stock prices with respect to this BMG factor. Therefore, the carbon factor is derived from climate change-relevant information from numerous firms.

In the Carima approach, the BMG factor is developed using a large amount of climate-relevant information provided by different databases. In the following, we detail the methodology used by the Carima project to construct the BMG factor. Two steps are required to develop this new common risk factor: (1) the development of a scoring system to determine if a firm is green, neutral or brown and (2) the construction of a mimicking factor portfolio for carbon risk which has a long exposure to brown firms and a short exposure to green firms. The first step consists in defining a brown green score (BGS) using a fundamental approach to assess the carbon risk of different firms. This scoring system uses four ESG databases over the period from 2010 to 2016: Thomson Reuters ESG, MSCI ESG Ratings, Sustainalytics ESG ratings and the Carbon Disclosure Project (CDP) climate change questionnaire. Overall, 55 carbon risk proxy variables are retained. Each variable is transformed into a dummy derived with respect to the median, meaning that 1 corresponds to a brown value and 0 corresponds to a green value. Then, Gorgen et al. (2019) classified the variables into three different dimensions that may affect the stock value of a firm in the event of unexpected shifts towards a low carbon economy: (1) value chain, (2) public perception and (3) adaptability. The value chain dimension mainly deals with current emissions while the adaptability dimension reflects potential future emissions determined in particular by emission reduction targets and environmental R&D spending. Then, three scores are created

and correspond to the average of all variables contained in each dimension: the *value chain* VC, the *public perception* PP and the *non-adaptability* NA. It follows that each score has a range between 0 and 1. Gorgen et al. (2019) proposed defining the brown green score (BGS) using the following equation:

$$\text{BGS}_i(t) = \frac{2}{3} (0.7 \cdot \text{VC}_i(t) + 0.3 \cdot \text{PP}_i(t)) + \frac{\text{NA}_i(t)}{3} (0.7 \cdot \text{VC}_i(t) + 0.3 \cdot \text{PP}_i(t)) \quad (1)$$

The higher the BGS value, the browner the firm. The second step consists in constructing a brown minus green (BMG) risk factor. Here the Carima project considers an average BGS for each stock that corresponds to the mean value of the BGS over the period in question, from 2010 to 2016. The construction of the BMG factor follows the methodology of Fama and French (1992), which consists in splitting the stocks into six portfolios: small green (SG), small neutral (SN), small brown (SB), big green (BG), big neutral (BN) and big brown (BB). Then, the return of the BMG factor is defined as follows:

$$R_{\text{bmg}}(t) = \frac{1}{2} (R_{\text{SB}}(t) + R_{\text{BB}}(t)) - \frac{1}{2} (R_{\text{SG}}(t) + R_{\text{BG}}(t)) \quad (2)$$

where the returns of each portfolio are value-weighted.

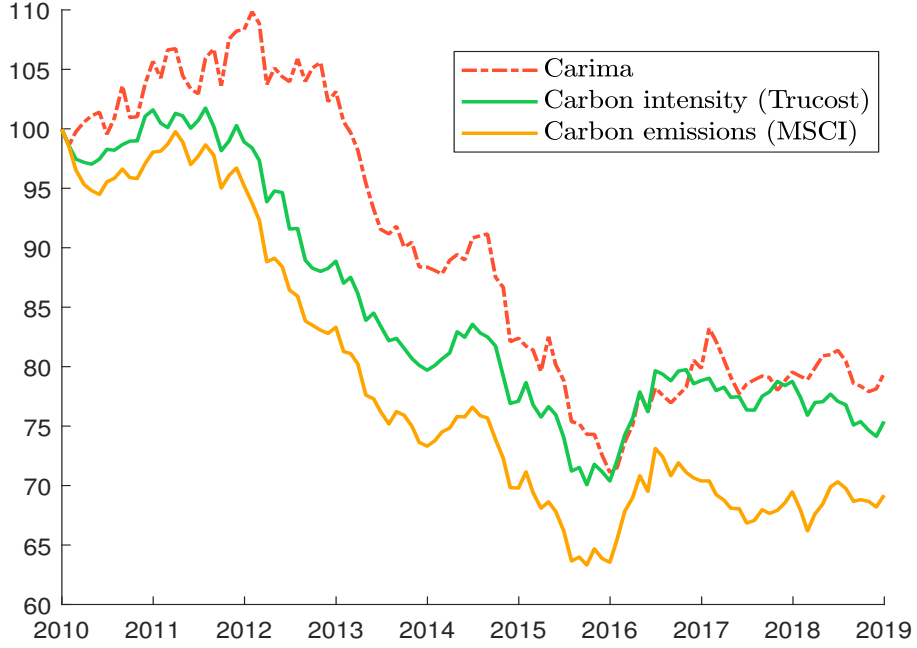
**Alternative approaches** Since the Carima approach is based on 55 variables from 4 ESG databases, it may be complicated for investors and academics to reproduce the BMG factor of Gorgen et al. (2019). This is why Roncalli et al. (2020) proposed several proxies that may be easily computed. They used the same approach to build the BMG factor, but replaced the brown green score by simple scoring systems using a single variable. Among the different tested factors<sup>2</sup>, Roncalli et al. (2020) showed that the Carima BMG factor is highly correlated to two BMG factors based on (1) the carbon intensity derived on the three scopes (Trucost dataset) and (2) the MSCI carbon emissions exposure score (MSCI, 2020). In Exhibit 1, we report the cumulative performance of these two factors and the Carima factor. We observe that the three factors are very similar and highly correlated. On average, we observe that brown firms slightly outperformed green firms from 2010 to 2012. Then, the cumulative return fell by almost 35% because of the unexpected path in the transition process towards a low carbon economy. From 2016 to the end of the study period, brown firms created a slight excess performance. Overall, the best-in-class green stocks outperform the worst-in-class green stocks over the study period with an annual return of 2.52% for the Carima factor, 3.09% for the carbon intensity factor and 4.01% for the factor built with the carbon emissions exposure score.

**Estimation of the carbon beta** Gorgen et al. (2019) and Roncalli et al. (2020) tested several models to estimate the carbon beta by considering different sets of risk factors that include market, size, value and momentum risk factors. While the first authors used a static approach by assuming that the carbon beta is constant over the period, the second authors proposed a dynamic approach by assuming that the betas are time-varying. This is more realistic since carbon betas may evolve with the introduction of a climate-related policy, a firm’s environmental controversies, a change in the firm’s environmental strategy, an increased incorporation of carbon risk into portfolio strategies, etc. In what follows, we consider the dynamic approach with a two-factor model. Let  $R_i(t)$  be the monthly return of stock  $i$  at time  $t$ . We assume that:

$$R_i(t) = \alpha_i(t) + \beta_{\text{mkt},i}(t) R_{\text{mkt}}(t) + \beta_{\text{bmg},i}(t) R_{\text{bmg}}(t) + \varepsilon_i(t) \quad (3)$$

<sup>2</sup>To build these factors, they have considered the stocks that were present in the MSCI World index during the 2010-2018 period.

Exhibit 1: Cumulative performance of the BMG factors



where  $R_{\text{mkt}}(t)$  is the return of the market risk factor,  $R_{\text{bmg}}(t)$  is the return of the BMG factor and  $\varepsilon_i(t) \sim \mathcal{N}(0, \tilde{\sigma}_i^2)$  is a white noise. The alpha component and the beta sensitivities follow a random walk:

$$\begin{cases} \alpha_i(t) = \alpha_i(t-1) + \eta_{\text{alpha},i}(t) \\ \beta_{\text{mkt},i}(t) = \beta_{\text{mkt},i}(t-1) + \eta_{\text{mkt},i}(t) \\ \beta_{\text{bmg},i}(t) = \beta_{\text{bmg},i}(t-1) + \eta_{\text{bmg},i}(t) \end{cases} \quad (4)$$

where  $\eta_{\text{alpha},i}(t) \sim \mathcal{N}(0, \sigma_{\text{alpha},i}^2)$ ,  $\eta_{\text{mkt},i}(t) \sim \mathcal{N}(0, \sigma_{\text{mkt},i}^2)$  and  $\eta_{\text{bmg},i}(t) \sim \mathcal{N}(0, \sigma_{\text{bmg},i}^2)$  are three independent white noise processes.

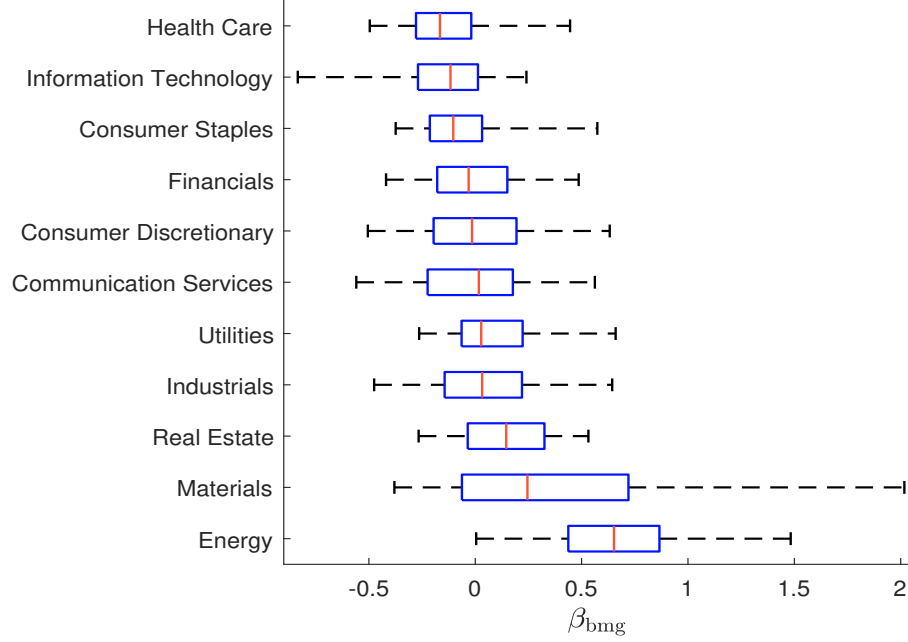
In the sequel of the paper, we use the Carima factor to estimate the carbon beta. For the market factor, we use the time series provided by Kenneth French on his website. We estimate  $\alpha_i(t)$ ,  $\beta_{\text{mkt},i}(t)$  and  $\beta_{\text{bmg},i}(t)$  for the stocks that belong to the MSCI World index between January 2010 and December 2018<sup>3</sup> using the Kalman filter (Fabozzi and Francis, 1978). Moreover, we scale the Carima risk factor so that it has the same volatility as the market risk factor over the entire period, implying that the magnitude of the carbon beta  $\beta_{\text{bmg},i}(t)$  may be understandable and comparable to the magnitude of the market beta  $\beta_{\text{mkt},i}(t)$ .

The average carbon beta of a stock is equal to 0.05, which is close to zero, whereas the monthly variation of the carbon beta has a standard deviation of 6.24%. If we consider the market beta, the figures become respectively 1.02 and 5.45%. Therefore, the time volatility of the carbon beta is larger than the time volatility of the market beta. In Exhibit 2, we report the GICS sector analysis of the carbon sensitivities at the end of December 2018. The box plots provide the median, the quartiles and the 5% and 95% quantiles of the carbon beta. We notice that on average, the energy, materials and real estate sector have a positive

<sup>3</sup>More precisely, we only consider the stocks that were in the MSCI World index for at least three years during the 2010-2018 period and we take into account only the returns for the period during which the stock is in the MSCI World index.

carbon beta<sup>4</sup> whereas the other sectors have a neutral or negative carbon beta. The results differ slightly from those obtained by Gorgen et al. (2019) and Roncalli et al. (2020), who provided a sector analysis by considering a constant carbon beta over the period 2010–2018.

Exhibit 2: Box plots of the dynamic carbon betas at the end of 2018



The average carbon beta  $\beta_{\text{bmg},\mathcal{R}}(t)$  for the region  $\mathcal{R}$  at time  $t$  is calculated as follows:

$$\beta_{\text{bmg},\mathcal{R}}(t) = \frac{\sum_{i \in \mathcal{R}} \beta_{\text{bmg},i}(t)}{\text{card } \mathcal{R}}$$

In Exhibit 3, we report  $\beta_{\text{bmg},\mathcal{R}}(t)$  for several MSCI universes at the end of each year: World (WD), North America (NA), EMU, Europe-ex-EMU (EU) and Japan (JP). Whatever the study period, the carbon beta  $\beta_{\text{bmg},\mathcal{R}}(t)$  is positive in North America, which implies that American stocks are negatively influenced by an acceleration in the transition process towards a green economy. The average carbon beta is always negative in the Eurozone. Overall, the Eurozone has always a lower average carbon beta than the world as a whole, whereas the opposite is true for North America. Nevertheless, the negative sensitivity of European equity returns has dramatically decreased since 2010 and the BMG betas are getting closer for North America and the Eurozone.

## 2.2 Absolute versus relative carbon risk

In the previous paragraph, the relative carbon risk of a stock  $i$  at time  $t$  is measured by its carbon beta value:

$$\mathcal{RCR}_i(t) = \beta_{\text{bmg},i}(t)$$

A majority of investors will prefer stocks with a negative carbon beta over stocks with a positive carbon beta. However, an investment portfolio with a negative carbon beta is

<sup>4</sup>This is in line with Bouchet and Le Guenedal (2020) who demonstrated that credit risks are more material in the energy and materials sectors. Therefore, the market perceives these sectors as the entry points for systemic financial carbon risks.

Exhibit 3: Relative carbon risk by region (end of year)

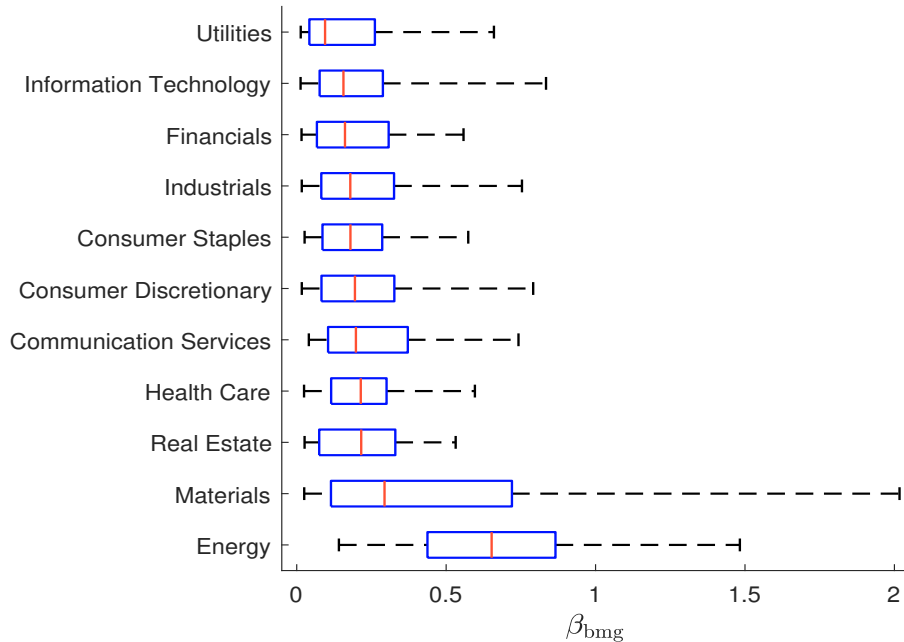
Year	WD	NA	EMU	EU	JP
2010	-0.02	0.13	-0.47	-0.16	0.02
2011	-0.04	0.11	-0.48	-0.14	-0.07
2012	-0.04	0.05	-0.32	-0.04	-0.13
2013	-0.02	0.07	-0.21	0.05	-0.22
2014	-0.04	0.01	-0.21	-0.02	-0.14
2015	0.00	0.04	-0.20	0.05	-0.08
2016	0.02	0.10	-0.21	0.01	-0.09
2017	0.03	0.12	-0.23	-0.03	-0.04
2018	0.06	0.10	-0.08	0.07	-0.02

exposed to the risk that brown firms outperform green firms. In this case, reducing the portfolio's carbon risk means having a carbon beta as close as possible to zero. This is why we introduce the concept of absolute carbon risk, which is equal to the absolute value of the carbon beta:

$$ACR_i(t) = |\beta_{bmg,i}(t)|$$

Exhibit 4 presents the sector analysis of the absolute carbon risk at the end of December 2018. From this point of view, utilities is the least exposed sector to absolute carbon risk, whereas the energy and materials are the sectors that are the most exposed.

Exhibit 4: Box plots of the absolute carbon risk at the end of 2018



$ACR_i(t)$  is also a pricing magnitude measure of the carbon risk. Let us consider an investment universe with two stocks. We assume that  $\beta_{bmg,1}(1) = 0.5$  and  $\beta_{bmg,2}(1) = -0.5$ . On average, the relative carbon risk is equal to zero, whereas the absolute carbon risk is equal to 0.5. One year later, we obtain  $\beta_{bmg,1}(2) = 1$  and  $\beta_{bmg,2}(2) = -1$ . In this case, the relative carbon risk of the investment universe has not changed and is always equal to zero.



However, its absolute carbon risk has increased and is now equal to 1. It is obvious that the carbon risk is priced in more in the second period than in the first period.

We have reported the absolute carbon risk by region in Exhibit 5. We notice that the carbon risk was priced in more in 2011 and 2012, because of the pricing magnitude in the Eurozone. In this region, the absolute carbon risk has dramatically decreased from 50% in 2011 to 27% in 2018. More globally, we observe a convergence between the different developed regions. One exception is Japan, where the absolute carbon risk is 50% lower than in Europe and North America.

Exhibit 5: Absolute carbon risk by region (end of year)

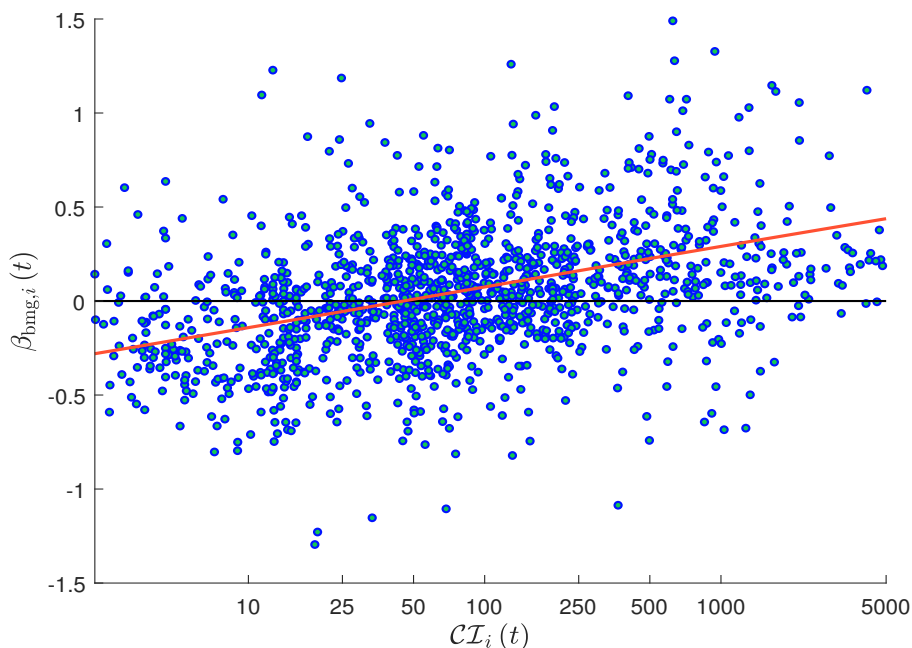
Year	WD	NA	EMU	EU	JP
2010	0.35	0.32	0.50	0.35	0.30
2011	0.34	0.32	0.51	0.32	0.31
2012	0.28	0.24	0.40	0.24	0.27
2013	0.28	0.26	0.31	0.24	0.30
2014	0.27	0.26	0.30	0.24	0.26
2015	0.27	0.29	0.27	0.27	0.20
2016	0.29	0.31	0.30	0.30	0.20
2017	0.27	0.29	0.30	0.28	0.20
2018	0.28	0.29	0.27	0.29	0.20

### 2.3 Comparison between market and fundamental measures of carbon risk

ESG rating agencies have developed many fundamental measures and scores to assess a firm’s carbon risk. For instance, the most well-known is the carbon intensity  $\mathcal{CI}_i(t)$ , which involves scopes 1, 2 and 3. In this paper, a firm’s carbon risk corresponds to the carbon beta priced in by the financial market. It is not obvious that there is a strong relationship between fundamental and market measures, because we may observe wide discrepancies between the market perception of the carbon risk and the carbon intensity of the firm. For instance, the linear correlation between  $\mathcal{CI}_i(t)$  and  $\beta_{\text{bmg},i}(t)$  is equal to 17.4% at the end of December 2018. If we consider the BMG factor built directly with the carbon intensity (Exhibit 1), the correlation increases but remains relatively low since it is equal to 21.4% at the end of December 2018. The relationship between  $\mathcal{CI}_i(t)$  and  $\beta_{\text{bmg},i}(t)$  is then more complex as seen in Exhibit 6.

This result is easily understandable because the stock market incorporates dimensions other than carbon intensity to price in the carbon risk. From a fundamental point of view, if the carbon intensity of two firms is equal to 100, they present the same carbon risk. Nevertheless, we know that their risks depend on other factors and parameters. For instance, it is difficult to compare two firms with the same carbon intensity if they belong to two different sectors or countries. The trajectory of the carbon intensity is also another important factor. For instance, the risk is not the same if one firm has dramatically decreased its carbon intensity in recent years. Moreover, the adaptability issue, the capacity to transform its business with investments in green R&D or its financial resources to absorb transition costs (Bouchet and Le Guenedal, 2020) are other important parameters that impact the market perception of the firm’s carbon risk. Therefore, carbon intensity is less appropriate to describe financial risks than carbon beta. In other words, the carbon beta is an integrated measure of the different fundamental factors affecting a firm’s carbon risk.

Exhibit 6: Scatter plot of  $\mathcal{CI}_i(t)$  and  $\beta_{\text{bm},i}(t)$  at the end of 2018



In Exhibit 7, we report the correlation between  $\mathcal{CI}_i(t)$  and  $\beta_{\text{bm},i}(t)$  at the end of December 2018. We notice that it is higher in the Eurozone than in other regions. In particular, the correlation in Japan is very low (less than 5%). Moreover, we observe that it also differs with respect to the sector. For instance, the financial sector presents the lowest correlation value, certainly because the carbon risk of financial institutions is less connected to their greenhouse gas emissions than their (green and brown) investments and financing programs.

Exhibit 7: Correlation in % between  $\mathcal{CI}_i(t)$  and  $\beta_{\text{bm},i}(t)$  at the end of 2018

Sector	WD	NA	EMU	EU	JP
Financials	18.2	20.1	29.2	20.9	-1.5
Energy	18.2	17.8	31.8	24.5	3.8
Materials	20.3	24.8	37.2	28.0	5.4
Information Technology	20.4	21.0	34.2	26.1	3.2
Health Care	20.9	21.3	34.5	26.3	4.2
Consumer Staples	21.5	22.3	35.1	26.4	4.3
Communication Services	21.6	22.2	32.2	24.7	6.6
Consumer Discretionary	22.3	23.1	37.8	25.8	2.6
Real Estate	22.4	22.5	34.3	26.1	6.1
Industrials	23.6	23.8	38.7	31.6	8.1
Utilities	26.6	29.8	26.5	26.1	8.4
All sectors	21.4	22.3	33.8	26.2	4.6

### 3 INCORPORATING CARBON RISK INTO MINIMUM VARIANCE PORTFOLIOS

There is an increasing appetite from fund managers of minimum variance portfolios to take into account carbon risk because of two main reasons. First, it is a financial and regulation risk that may negatively impact stock returns. The second reason is that it is highly sought after by institutional investors. In what follows, we show how to incorporate carbon risk into these strategies. In particular, we provide an analytical formula which is useful to understand the impact of carbon betas on the minimum variance portfolio and the covariance matrix of stock returns. We also discuss the different practical implementations of minimum variance portfolios when we consider market and fundamental measures of carbon risk.

#### 3.1 Analytical results

In this paragraph, we extend the famous formula of the minimum variance portfolio when we complement the market risk factor with the BMG factor. Then, we illustrate how the minimum variance portfolio selects stocks in the presence of carbon risk.

**Extension of the one-factor GMV formula** We consider the global minimum variance (GMV) portfolio, which corresponds to this optimization program:

$$\begin{aligned} x^* &= \arg \min \frac{1}{2} x^\top \Sigma x \\ \text{s.t. } & \mathbf{1}_n^\top x = 1 \end{aligned} \quad (5)$$

where  $x$  is the vector of portfolio weights and  $\Sigma$  is the covariance matrix of stock returns. In the capital asset pricing model, we recall that:

$$R_i(t) = \alpha_i + \beta_{\text{mkt},i} R_{\text{mkt}}(t) + \varepsilon_i(t) \quad (6)$$

where  $R_i(t)$  is the return of asset  $i$ ,  $R_{\text{mkt}}(t)$  is the return of the market factor,  $\varepsilon_i(t) \sim \mathcal{N}(0, \tilde{\sigma}_i^2)$  is the idiosyncratic risk and  $\tilde{\sigma}_i$  is the idiosyncratic volatility. Clarke et al. (2011) and Scherer (2011) showed that:

$$x_i^* = \frac{\sigma^2(x^*)}{\tilde{\sigma}_i^2} \left( 1 - \frac{\beta_{\text{mkt},i}}{\beta_{\text{mkt}}^*} \right) \quad (7)$$

where  $\beta_{\text{mkt}}^*$  is a threshold and  $\sigma^2(x^*)$  is the variance of the GMV portfolio. Therefore, we note that the minimum variance portfolio is exposed to stocks with low specific volatility  $\tilde{\sigma}_i$  and low beta  $\beta_{\text{mkt},i}$ . More precisely, if asset  $i$  has a market beta  $\beta_{\text{mkt},i}$  smaller than the threshold  $\beta_{\text{mkt}}^*$ , the weight of this asset is positive:  $x_i^* > 0$ . If  $\beta_{\text{mkt},i} > \beta_{\text{mkt}}^*$ , then  $x_i^* < 0$ . Clarke et al. (2011) extended Formula (7) to the long-only case, where  $\beta_{\text{mkt}}^*$  is another threshold. In this case, if  $\beta_{\text{mkt},i} < \beta_{\text{mkt}}^*$ ,  $x_i^* > 0$  and if  $\beta_{\text{mkt},i} \geq \beta_{\text{mkt}}^*$ ,  $x_i^* = 0$ .

We consider an extension of the CAPM by including the BMG risk factor:

$$R_i(t) = \alpha_i + \beta_{\text{mkt},i} R_{\text{mkt}}(t) + \beta_{\text{bmg},i} R_{\text{bmg}}(t) + \varepsilon_i(t) \quad (8)$$

where  $R_{\text{bmg}}(t)$  is the return of the BMG factor and  $\beta_{\text{bmg},i}$  is the BMG sensitivity (or the carbon beta) of stock  $i$ . Moreover, we assume that  $R_{\text{mkt}}(t)$  and  $R_{\text{bmg}}(t)$  are uncorrelated. Roncalli et al. (2020) showed that the GMV portfolio is defined as:

$$x_i^* = \frac{\sigma^2(x^*)}{\tilde{\sigma}_i^2} \left( 1 - \frac{\beta_{\text{mkt},i}}{\beta_{\text{mkt}}^*} - \frac{\beta_{\text{bmg},i}}{\beta_{\text{bmg}}^*} \right) \quad (9)$$

where  $\beta_{\text{mkt}}^*$  and  $\beta_{\text{bmg}}^*$  are two threshold values. In the case of long-only portfolios, we obtain a similar formula:

$$x_i^* = \frac{\sigma^2(x^*)}{\tilde{\sigma}_i^2} \max\left(1 - \frac{\beta_{\text{mkt},i}}{\beta_{\text{mkt}}^*} - \frac{\beta_{\text{bmg},i}}{\beta_{\text{bmg}}^*}; 0\right) \quad (10)$$

but with other values of the thresholds  $\beta_{\text{mkt}}^*$  and  $\beta_{\text{bmg}}^*$ .

**Interpretation of these results** Contrary to the single-factor model, the impact of sensitivities is more complex in the two-factor model. Indeed, we know that  $\bar{\beta}_{\text{mkt}} \approx 1$  and  $\bar{\beta}_{\text{bmg}} \approx 0$ . It follows that  $\beta_{\text{mkt}}^*$  is positive, but  $\beta_{\text{bmg}}^*$  may be positive or negative<sup>5</sup>. We deduce that the ratio  $\frac{\beta_{\text{mkt},i}}{\beta_{\text{mkt}}^*}$  is an increasing function of  $\beta_{\text{mkt},i}$ , but the ratio  $\frac{\beta_{\text{bmg},i}}{\beta_{\text{bmg}}^*}$  may be an increasing or a decreasing function of  $\beta_{\text{bmg},i}$ . The GMV portfolio will then always prefer stocks with low market betas, but not necessarily stocks with low carbon betas. For instance, it may prefer stocks with high carbon betas if  $\beta_{\text{bmg}}^*$  is negative.

In the long-only case, a stock is selected if it satisfies the following inequality:

$$\frac{\beta_{\text{mkt},i}}{\beta_{\text{mkt}}^*} + \frac{\beta_{\text{bmg},i}}{\beta_{\text{bmg}}^*} \leq 1$$

Therefore, we notice that there is a trade-off between  $\beta_{\text{mkt},i}$  and  $\beta_{\text{bmg},i}$ . Nevertheless, Roncalli et al. (2020) showed that the long-only MV portfolio tends to prefer stocks with low absolute carbon risk.

We recall that the volatility of stock  $i$  is equal to  $\sigma_i^2 = \beta_{\text{mkt},i}^2 \sigma_{\text{mkt}}^2 + \beta_{\text{bmg},i}^2 \sigma_{\text{bmg}}^2 + \tilde{\sigma}_i^2$ , whereas the covariance between stocks  $i$  and  $j$  is equal to  $\sigma_{i,j}^2 = \beta_{\text{mkt},i} \beta_{\text{mkt},j} \sigma_{\text{mkt}}^2 + \beta_{\text{bmg},i} \beta_{\text{bmg},j} \sigma_{\text{bmg}}^2$ . Therefore, choosing stocks with low volatilities implies considering stocks with low values of  $\beta_{\text{bmg},i}^2$ . In a similar way, removing stocks with high positive correlations implies removing stocks with high values of  $\beta_{\text{bmg},i} \beta_{\text{bmg},j}$ . This explains that the MV portfolio will prefer stocks with low values of  $|\beta_{\text{bmg},i}|$ .

## 3.2 Practical implementations

We now apply the previous framework to the MSCI World index at December 2018, and illustrate the difference between absolute and relative carbon risk when we consider the minimum variance portfolio. Moreover, we compare these market-based approaches with implementations of minimum variance portfolios that use fundamental carbon risk metrics.

**Impact of carbon risk** In Exhibit 8, we indicate the stocks that make up the MV portfolio with respect to their beta values  $\beta_{\text{mkt},i}$  and  $\beta_{\text{bmg},i}$ . We find that the most important axis is the market beta. Indeed, the market risk of a stock determines whether the stock is included in the MV portfolio or not whereas the carbon risk adjusts the weights of the asset. As we can see, the portfolio overweights assets whose market and carbon sensitivities are both close to zero. This solution is satisfactory if the original motivation is to reduce the portfolio's absolute carbon risk, but it is not satisfactory if the objective is to manage the portfolio's relative carbon risk.

**Considering relative carbon risk** In order to circumvent the previous drawback, we can directly add a constraint in the optimization program:

$$\beta_{\text{bmg}}(x) = \sum_{i=1}^n x_i \times \beta_{\text{bmg},i} \leq \beta_{\text{bmg}}^+ \quad (11)$$

<sup>5</sup>Moreover, it generally takes a high absolute value.

Exhibit 8: Weights of the long-only MV portfolio

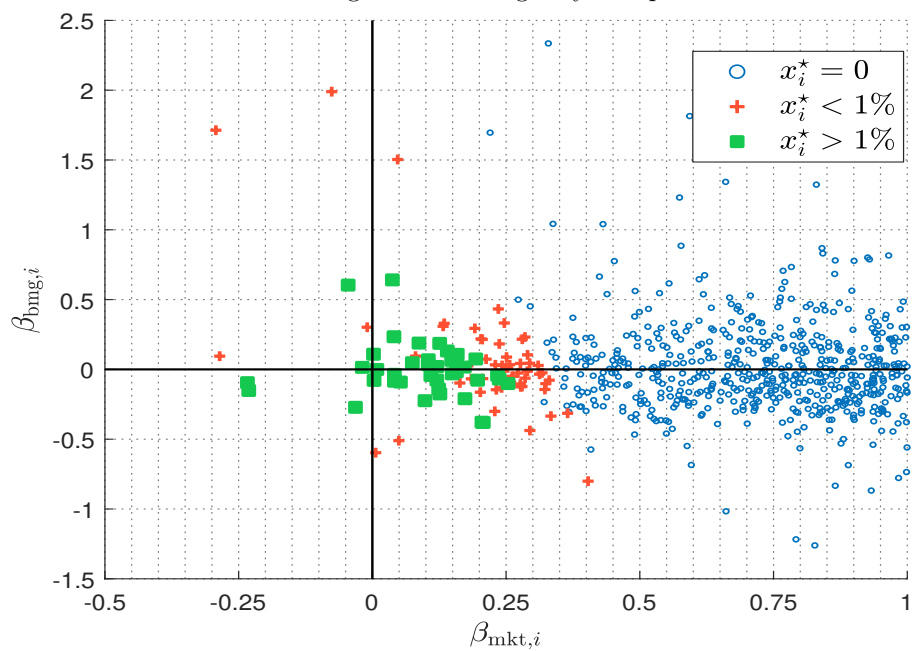
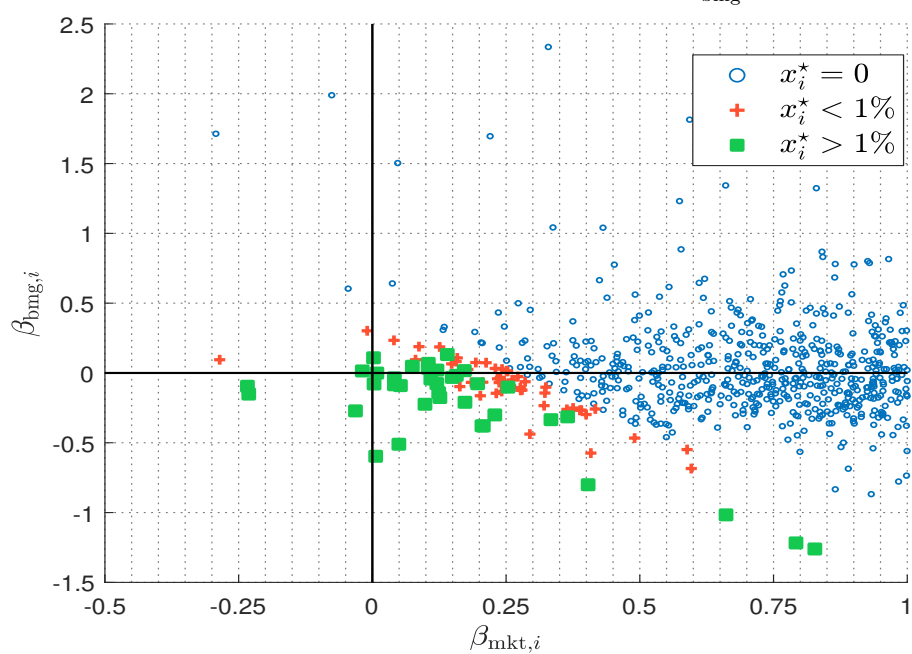


Exhibit 9: Weights of the constrained MV portfolio ( $\beta_{bmg}^+ = -0.25$ )



where  $\beta_{\text{bmg}}(x)$  is the carbon beta of portfolio  $x$  and  $\beta_{\text{bmg}}^+$  is the maximum tolerance of the investor with respect to the relative carbon risk. We consider the previous example. If we would like to impose a carbon sensitivity of lower than  $-0.25$ , we obtain results given in Exhibit 9. The comparison with Exhibit 8 shows that the MV portfolio tends to select stocks with both a low market sensitivity and a negative carbon beta. Moreover, large weights are associated with large negative values of  $\beta_{\text{bmg},i}$  on average.

**Managing both market and fundamental risk measures** The previous method is not the standard approach when managing carbon risk in investment portfolios. Indeed, the asset management industry generally considers constraints on carbon intensity measures. Following Andersson et al. (2016), we can impose individual constraints on the different stocks:

$$x_i = 0 \quad \text{if} \quad \mathcal{CI}_i \leq \mathcal{CI}^+ \quad (12)$$

or we can use a global constraint:

$$\mathcal{WACI}(x) = \sum_{i=1}^n x_i \times \mathcal{CI}_i \leq \mathcal{WACI}^+ \quad (13)$$

where  $\mathcal{CI}_i$  is the carbon intensity of stock  $i$  and  $\mathcal{WACI}(x)$  is the weighted average carbon intensity of portfolio  $x$ .  $\mathcal{CI}^+$  and  $\mathcal{WACI}^+$  are the individual and portfolio thresholds that are accepted by investors.

We may wonder whether managing the fundamental measure of carbon risk is equivalent to managing the market measure of carbon risk<sup>6</sup>. A preliminary answer has been provided previously since we have found that the correlation between  $\beta_{\text{bmg},i}$  and  $\mathcal{CI}_i$  is less than 30% on average. In Exhibit 10, we compute the minimum variance portfolio by considering several threshold values of  $\beta_{\text{bmg}}^+$ . We notice that using a lower value of  $\beta_{\text{bmg}}^+$  reduces the value of  $\mathcal{WACI}(x)$ , but  $\mathcal{WACI}(x)$  remains very high because some issuers have a low common carbon risk, but a high idiosyncratic carbon risk. We have also reported the number of stocks  $\mathcal{N}(x)$  in the MV portfolio. As expected, it decreases when we impose a stronger constraint. Exhibit 11 is a variant of Exhibit 10 by considering a constraint  $\mathcal{WACI}^+$  on the portfolio's carbon intensity instead of a constraint  $\beta_{\text{bmg}}^+$  on the portfolio's carbon beta. Here, the impact on the portfolio's carbon beta is low when we strengthen the constraint. Indeed, the portfolio's carbon beta  $\beta_{\text{bmg}}(x)$  is equal to 1.43% when we target a carbon intensity of 50, whereas it drops to 1.33% when the constraint on the carbon intensity is set to 50.

Exhibit 10: Minimum variance portfolios with a relative carbon beta constraint

$\beta_{\text{bmg}}^+$	$\beta_{\text{bmg}}(x)$	$\mathcal{WACI}(x)$	$\mathcal{N}(x)$
	1.43%	538	105
-10.00%	-10.00%	501	100
-20.00%	-20.00%	422	89
-40.00%	-40.00%	289	70

These results show that the two optimization problems give two different solutions in terms of carbon risk. Therefore, it makes sense to combine the approaches by imposing two constraints:

$$\begin{cases} \mathcal{WACI}(x) \leq \mathcal{WACI}^+ \\ \beta_{\text{bmg}}(x) \leq \beta_{\text{bmg}}^+ \end{cases} \quad (14)$$

<sup>6</sup>In what follows, we also impose that  $\mathcal{CI}^+ = 4000$ .

Exhibit 11: Minimum variance portfolios with a carbon intensity constraint

$\mathcal{WACI}^+$	$\mathcal{WACI}(x)$	$\beta_{\text{bmg}}(x)$	$\mathcal{N}(x)$
500	500	1.43%	105
250	250	1.37%	103
100	100	1.36%	98
50	50	1.33%	82

Exhibit 12: Minimum variance portfolios with carbon beta and intensity constraints

$\mathcal{WACI}^+$	$\mathcal{WACI}(x)$	$\beta_{\text{bmg}}(x)$	$\mathcal{N}(x)$	$\mathcal{WO}(x)$
500	430	-20.00%	111	74.65%
250	250	-20.00%	86	75.26%
100	100	-20.00%	79	74.87%
50	50	-20.00%	74	74.99%

Moreover, the threshold  $\beta_{\text{bmg}}^+$  allows us to reduce the common carbon risk, but not the idiosyncratic carbon risk. The WACI constraint circumvents this problem. Exhibit 12 presents the results for several values of  $\mathcal{WACI}^+$  when  $\beta_{\text{bmg}}^+$  is equal to  $-20\%$ . For instance, we notice that the WACI constraint is not reached when  $\mathcal{WACI}^+ = 500$  and  $\beta_{\text{bmg}}^+ = -20\%$ . The last column of Exhibit 12 corresponds to the portfolio’s weight overlap with respect to the optimized portfolio based on the WACI constraint, meaning that we compare the portfolio optimized with the BMG and WACI constraints to the portfolio optimized with the WACI constraint. In this example, we notice that the weight overlap  $\mathcal{WO}(x)$  is equal to 75% on average. This means that 25% of the minimum variance portfolio allocation is changed when we add the market carbon constraint  $\beta_{\text{bmg}}^+ = -20\%$ .

## 4 CONCLUSION

This paper considers the seminal approach of G3rgen et al. (2019) to measuring carbon risk. While many asset managers and owners use carbon intensity, we focus on the carbon beta which is priced in by the market. The carbon beta is estimated using a two-step approach. First, we build a brown-minus-green risk factor. Second, we perform a Kalman filtering in order to obtain the time-varying carbon beta. By considering this dynamic framework, we highlight several stylized facts. We show that this market measure is very different from a traditional fundamental measure of the carbon risk. The main reason is that carbon intensity is not the only dimension that is priced in by the market.

Another important result is the difference between relative and absolute carbon risk. Investors that are sensitive to relative carbon risk prefer stocks with a negative carbon beta over stocks with a positive carbon beta, whereas investors that are sensitive to absolute carbon risk prefer stocks with a carbon beta close to zero. Managing relative carbon risk implies having a negative exposure to the carbon risk factor, whereas managing absolute carbon risk implies having zero exposure to the carbon risk factor. The first case is an active management bet since the performance may be negative if brown stocks outperform green stocks. Nevertheless, this approach reduces exposure to firms that face a threat of environmental regulation (Maxwell et al., 2000). The second case is an immunization investment strategy against carbon risk. However, this hedging strategy is not widely implemented by institutional and passive investors because of their moral values and convictions, and they generally prefer to implement relative carbon risk strategies.

Introducing carbon risk into a minimum variance portfolio is a hot topic among as-

set managers and owners. Indeed, the goal of a minimum variance portfolio is to build a low volatility strategy on the equity market. This is achieved by considering a strong risk management approach on several dimensions. Originally, the strategy only focused on the portfolio's volatility. Since the 2008 Global Financial Crisis, it has included other risk dimensions that can burst the equity market such as credit risk and valuation risk. Climate risk has become another important dimension, especially because minimum variance strategies are massively implemented by ESG institutional investors. In this context, the question of carbon metrics is important. In this paper, we show that managing the carbon intensity of minimum variance portfolios has little impact on their carbon beta. The opposite is not true, but the effect of managing the carbon beta on carbon intensity is limited. This is why we propose combining the market and fundamental approaches to carbon risk. Another issue concerns the choice of the market carbon risk measure. We show that the optimization program of a minimum variance portfolio naturally considers absolute carbon risk. However, relative carbon risk can also be an option if the investor's goal is not to hedge the carbon risk, but to be a green investor.

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