Retirement Solutions
Accumulation, Decumulation & Longevity Risk

Thierry Roncalli*, Benjamin Bruder, Camille Schittly & Jiali Xu

*Amundi Investment Institute, Amundi Asset Management[1], France

Amundi Investment Solutions

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[1]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

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https://research-center.amundi.com

Retirement journey

At what age did you first estimate your retirement pension?

- 45
- 50
- 55
- 60
- Just before retiring
- Never

Retirement journey

Can we reduce retirement issues to these two questions:

1. **Gestion pilotée ou gestion libre?**
2. **Sortie en capital ou en rente?**

Retirement journey

Hot topic in Europe

- Savings and investments union (SIU) strategy
- Ensuring adequate retirement income for EU citizens (November 2025) $\Rightarrow$ The package of measures includes a Commission Recommendation, two legislative proposals, and also clarifies the *prudent person principle*
- New version of the Directive on Institutions for Occupational Retirement Provision (IORP) II
- Review of the Pan-European Personal Pension Product (PEPP) Regulation
- The **Spanish pension reform** came into effect in April 2025
- The Danish parliament raised the retirement age in Denmark to **70 years in 2040** (74 years in 2060)
- **German Pension Reform Package 2025**
- Italy launches **new auto-enrolment regime** that will come into force on 1 July 2026

Retirement journey

Pension markets

Table: Assets earmarked for retirement in the OECD (in USD trillion)

	2000	2005	2010	2015	2020	2024
Pension providers	15.3	22.3	29.2	36.9	54.8	63.1
Public pension reserve funds	1.5	2.9	4.9	5.4	6.9	6.7
Total	16.8	25.2	34.1	42.3	61.7	69.8

Source: OECD (2025), Pension Markets in Focus 2025, 50 pages.

- Growth: 11.6% in 2023, 7.1% in 2024
- Growth projection: **7% per year over the next 5 years**
- DB growth = 4% vs DC growth: 11.4% (**DB share = 32% in 2024 vs. 40% in 2014**)
- Pension assets ≈ 75% of GDP

Pension markets

Figure: 2024 pension assets by country (in USD billion)

Market	Private Assets	Pillar II	Market	Private Assets	Pillar II
US	37992	35017	Brazil	232	
Japan	3300	1266	Hong Kong	224	
Canada	3267	3126	Chile	198	175
UK	**3139**	**2562**	**France**	**166**	**306**
Australia	2639	2089	Ireland	149	144
Netherlands	**1747**	**1541**	Spain	142	166
Switzerland	**1433**	**1273**	Denmark		781
South Korea	1098	547	Sweden		561
Germany	**556**	**268**	Israel		307
China	480		Belgium		224
Mexico	342	301	New Zealand		78
Malaysia	326		**Norway**		**44**
Finland	299	169	**Greece**		**2**
India	270		Total	58511	51177
Italy	258	230	Total ex US	20522	16160
South Africa	257		Source	TAI/GPAS	OliverWyman

Source: Thinking Ahead Institute (2025), Global Pension Assets Study, 44 pages & Oliver Wyman and Morgan Stanley (2025), Longevity Unlocked: Retiring in the Age of Aging, 53 pages.

Longevity risk

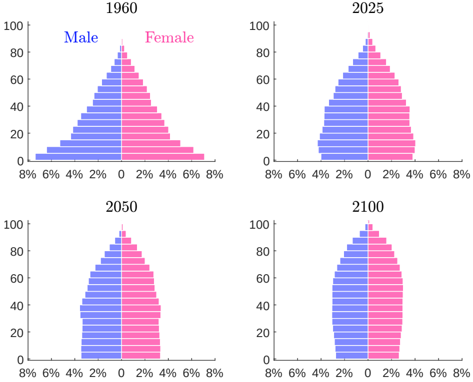
Longevity risk and retirement solutions

Longevity risk is a systemic risk for countries and DB pension funds

Longevity risk is an idiosyncratic/specific risk for individuals

Demographic transformation and longevity risk

Figure: Population pyramid (world, 1960–2100)



Source: United Nations (2024) & Authors' calculations.

	1950	2025	2050	2100
Life expectancy (yr)	46.4	**73.5**	77.0	**81.7**
Median age (yr)	22.2	**30.9**	36.1	**42.1**
60+ (%)	7.9	**14.9**	21.8	**29.7**
Dependency ratio*	5.6	**11.2**	19.0	**31.3**

*The old-age dependency ratio measures the size of the population aged 65 and over relative to the working-age population (25-64). It is expressed as number of dependents per 100 persons of working age.

Longevity risk is a systemic risk for developed countries

Figure: Western Europe

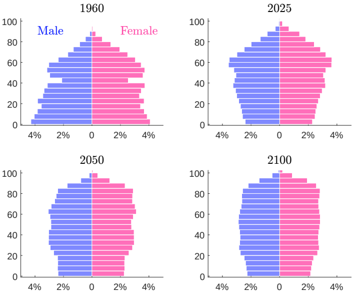


Figure: US

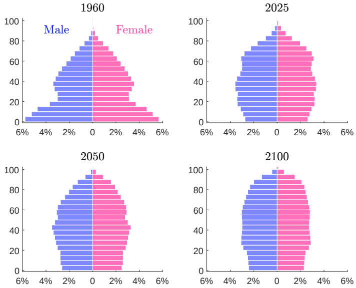
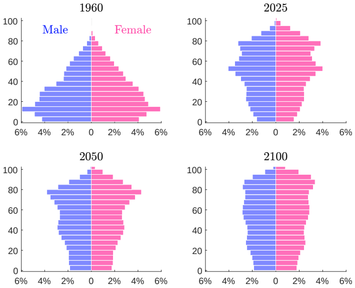


Figure: Japan



Source: United Nations (2024) & Authors' calculations.

Longevity risk is a systemic risk for developing countries

Figure: China

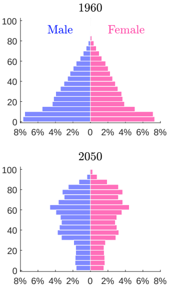


Figure: India

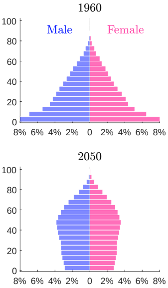
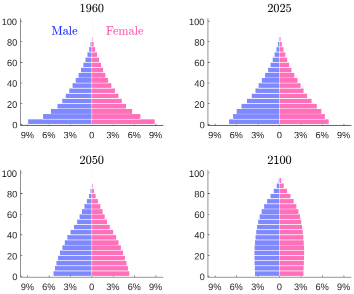


Figure: Africa



Source: United Nations (2024) & Authors' calculations.

Demographic transformation and longevity risk

Table: Old-age dependency ratio

	1950	2025	2050	2100
Brazil	3.0	11.0	24.8	48.6
Canada	8.0	21.6	32.1	43.3
China	5.3	**14.5**	37.3	**80.8**
France	11.5	**27.3**	38.8	**46.4**
Germany	8.3	26.5	41.1	46.3
Hong Kong	2.6	**22.7**	67.2	**144.8**
India	3.7	7.1	14.7	40.6
Italy	8.2	28.7	55.3	59.4
Japan	5.5	40.1	57.5	60.9
Saudi Arabia	3.7	2.5	7.4	20.2
Singapore	2.8	12.7	30.1	64.0
Taiwan	2.4	**18.4**	51.0	**71.4**
USA	7.8	19.8	28.9	39.8
Middle Africa	4.7	**3.9**	4.4	**11.9**

Retirement solutions and public finance

Table: Public expenditure on old-age and survivor benefits in % of government spending and GDP

Country	Gov't spending		% of GDP		Country	Gov't spending		% of GDP	
	2000	2019	2000	2019		2000	2019	2000	2019
Australia	12.8	10.3	4.7	4.3	Austria	23.9	26.8	12.2	13.0
Belgium	17.8	20.6	8.8	10.7	Canada	10.1	11.3	4.2	5.0
Denmark	12.0	16.4	6.3	8.1	Finland	15.5	22.4	7.4	11.9
France	22.2	**24.3**	11.5	**13.4**	Germany	22.8	23.1	10.9	10.4
Greece	21.9	**32.7**	10.2	**15.7**	Iceland	4.6	**6.6**	2.1	**2.9**
Ireland	10.3	13.7	3.1	**3.3**	Italy	29.0	**32.8**	13.5	**15.9**
Netherlands	10.9	11.8	4.6	5.0	Norway	11.1	13.8	4.7	7.1
Poland	24.3	26.2	10.5	10.9	Portugal	18.3	29.3	7.8	12.4
Spain	21.5	26.7	8.4	11.3	Sweden	12.8	14.2	6.8	7.0
Switzerland	17.8	19.6	5.9	6.4	United Kingdom	13.4	11.5	4.8	4.9
United States	16.4	18.6	5.7	7.1	OECD	16.2	**18.1**	6.5	**7.7**

Source: OECD (2023, Table 8.2, page 211).

Retirement solutions and public finance

Table: Projections of public expenditure on pensions, 2023–60, percentage of GDP

Country	2025	2030	2035	2040	2045	2050	2055	2060	2060/2025
Belgium	13.1	13.6	14.1	14.4	14.6	14.8	15.1	15.4	2.3%
Canada	7.2	7.8	8.0	8.1	8.1	8.1	8.1	8.3	1.1%
Czechia	7.9	8.0	8.4	9.1	10.0	10.6	11.0	11.0	**3.1%**
France	14.2	14.3	14.3	14.1	13.9	13.7	13.6	13.5	**−0.7%**
Germany	10.5	10.8	11.2	11.1	11.0	11.0	11.1	11.2	0.7%
Greece	13.2	12.7	13.4	13.7	14.0	14.0	13.3	12.7	**−0.5%**
Hungary	7.8	7.7	8.1	9.0	10.2	10.7	11.0	11.5	**3.7%**
Ireland	3.7	4.2	4.7	5.0	5.5	6.0	6.2	6.5	2.8%
Italy	16.1	16.6	17.2	17.1	16.5	15.5	14.4	13.7	**−2.4%**
Luxembourg	9.3	9.7	10.6	11.2	11.8	12.5	13.6	15.0	**5.7%**
Netherlands	6.8	7.3	7.7	8.0	7.9	7.9	7.9	8.0	1.2%
Spain	13.7	14.3	15.4	16.2	16.9	17.3	17.2	16.9	**3.2%**
United Kingdom	8.2	7.9	8.2	8.3	8.3	8.5	8.9	9.5	1.3%
Brazil	8.5	8.8	9.4	10.2	11.3	12.3	13.2	13.9	**5.4%**
Japan	9.1	8.8	8.7	8.8	9.1	9.3	9.5	9.7	0.6%
Korea	2.0	2.7	3.4	4.4	5.4	6.3	7.0	7.7	**5.7%**
United States	5.3	5.6	5.8	5.9	5.9	5.9	6.0	6.1	0.8%
OECD32	9.0	9.3	9.5	9.7	9.9	10.0	10.1	10.3	1.3%

Source: OECD (2025, Table 8.4, page 227).

Pension system typology

Financing

Benefit Promise	Pay-As-You-Go	Fully Funded
Defined Benefit	**PAYG-DB** • U.S. Social Security • France Social Security • AGIRC–ARRCO • . . .	**Funded DB** • U.S. corporate DB plans • Canadian funded DB (e.g., HOOPP, OMERS) • UK legacy DB • . . .
Defined Contribution	**PAYG-DC (Notional DC)** • Sweden NDC • Italy NDC • Latvia NDC • Poland NDC • . . .	**Funded DC** • U.S. 401(k), 403(b), IRA • UK master trusts (e.g., The People's Pension, NEST) • France PER • . . .

Pension replacement rates

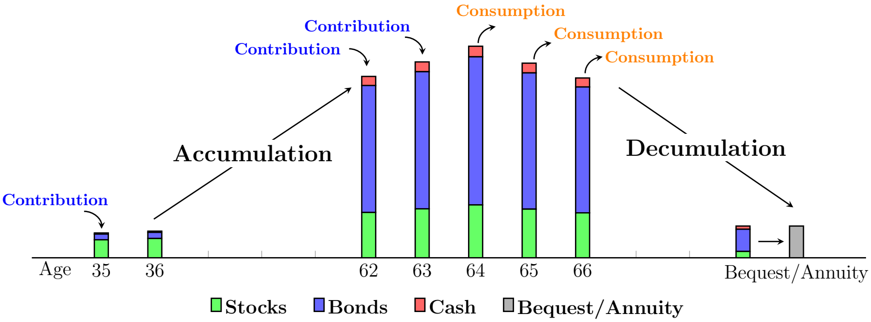
Table: Pension replacement rates from mandatory (public/private) & voluntary pension schemes (%)

Country	Mandatory		Voluntary		Country	Mandatory		Voluntary	
	Gross	Net	Gross	Net		Gross	Net	Gross	Net
Australia	**26.0**	33.7			Austria	74.1	87.4		
Belgium	43.5	60.9	52.4	73.8	Canada	36.8	44.2	57.0	66.0
Denmark	73.1	77.3			Estonia	28.1	34.4	47.4	54.7
Finland	58.4	65.1			France	**57.6**	**71.9**		
Germany	43.9	55.3	54.7	69.5	Greece	**80.8**	**90.0**		
Iceland	43.1	52.1			Ireland	26.2	36.1	55.7	**74.3**
Israel	38.0	47.3	51.7	63.2	Italy	76.1	82.6		
Japan	32.4	38.8			Lithuania	**18.2**	28.9	30.1	47.9
Mexico	55.5	62.4	64.7	72.7	Netherlands	74.7	93.2		
New Zealand	39.7	43.5	54.9	61.9	Norway	44.5	54.8		
Poland	**29.3**	40.3			Portugal	73.9	**98.8**		
Spain	**80.4**	86.5			Sweden	62.3	65.3		
Switzerland	39.9	45.3			United Kingdom	41.9	54.4		
United States	39.1	50.5	73.2	**87.7**	OECD	50.7	61.4	55.2	66.9

Source: OECD (2023, Table 4.5, page 159).

Accumulation and decumulation in funded DC plans

- Accumulation: build assets through contributions and returns, gradually de-risking.
- Decumulation: turn assets into sustainable income via withdrawals/annuitization.



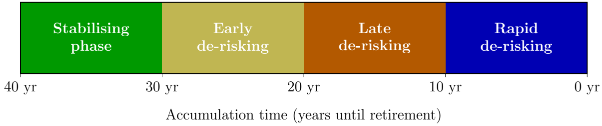
Accumulation strategies

Lifecycle vs. lifestyle

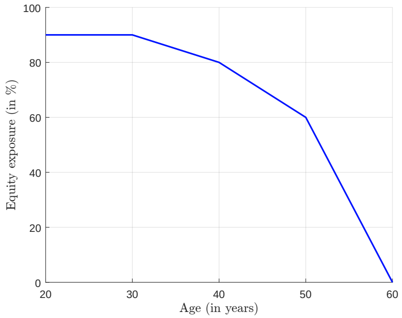
Bogle rule

"Investors should hold a percentage of bonds equal to their age, with the remainder in equities." (John C. Bogle, founder of Vanguard).

FTSE Lifecycle Screened Select Index



Deterministic glide path



1. *Stabilising phase (Years 31-40)*
   Maintain approximately 90% equities
2. *Early gradual de-risking (Years 21-30)*
   Reduce equity allocation by 1% per year
3. *Mid-term gradual de-risking (Years 11-20)*
   Reduce equity allocation by 2% per year
4. *Late rapid de-risking (Years 1-10)*
   Equity allocation declines linearly to 0%

Merton framework

- Let α_t be the allocation in the risky asset S_t
- We denote by μ_t and σ_t the expected return and the volatility of the asset S_t
- r_t is the return of the **risk-free asset** (a mix of cash and zero-coupon maturing at date T)
- T is the retirement date
- c_t is the contribution at time t
- CRRA utility function $\mathcal{U}(x) = \frac{x^\gamma}{\gamma}$ with $\gamma \leq 1$
- Without contribution, the solution is the constant-mix portfolio:

$$\bar{\alpha}_t = \frac{\mu_t - r_t}{(1-\gamma)\,\sigma_t^2}$$

- With contribution, the solution is dynamic:

$$\alpha_t^\star = \bar{\alpha}_t \left(1 + \frac{H_t}{X_t}\right)$$

where X_t is the current wealth and $H_t = \int_t^T e^{-\int_t^s r_u \,\mathrm{d}u} \boldsymbol{c}_s \,\mathrm{d}s$ is the present value of future contributions

Rationale of the glide path

- **Future contribution matters!**
- **Human-to-financial capital ratio**

$$\frac{H_t}{X_t} = \frac{\text{Forward wealth}}{\text{Current wealth}} = \frac{\text{Human capital}}{\text{Financial capital}} := \mathbf{HFCR}$$

- Solution (without constraints)

$$\text{Dynamic allocation} = \text{Constant-mix allocation} \times (1 + \mathbf{HFCR})$$

- This solution is valid for three types of models
  1. Tangency portfolio dynamic optimization (HJB equation solved with finite-difference algorithm)
  2. Dynamic multi-asset portfolio optimization (HJB equation solved with Howard policy-iteration algorithm)
  3. Macro-finance model with inflation risk (HJB equation solved with Howard policy-iteration algorithm)

What is the human capital?

Figure: Spot contribution function c_t

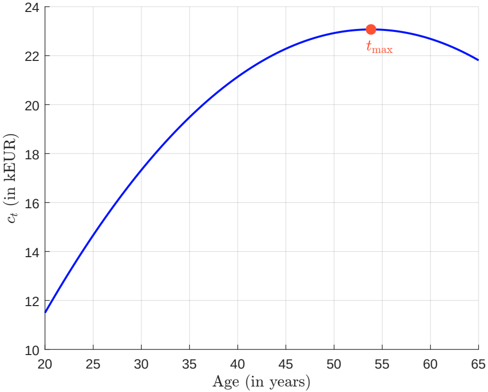
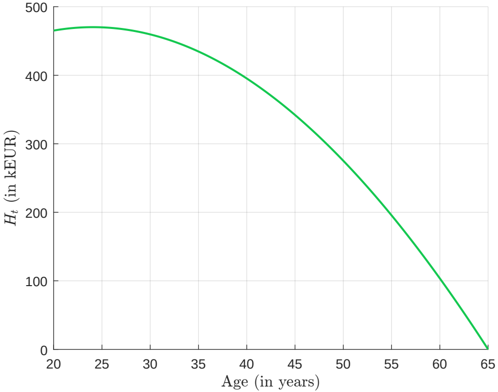


Figure: Forward contribution H_t (human capital)



Source: INSEE & Author's calculations.

What is a glide path?

Figure: Optimal allocation

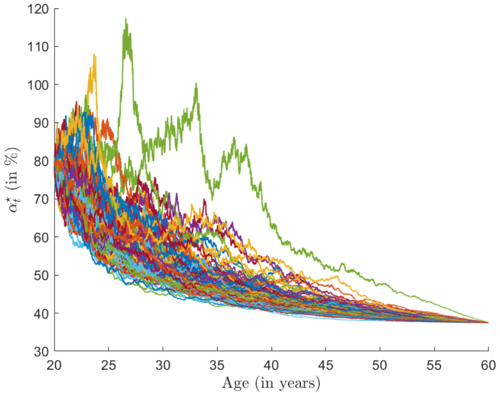
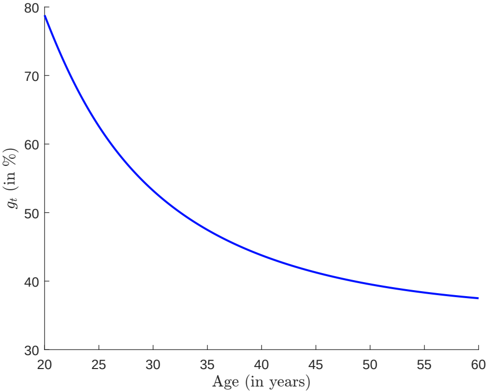


Figure: Dynamic glide path



Definition of the glide path

Definition

The glide path is the mathematical expectation of the optimal risky exposure at the inception date:

$$g_t = \mathbb{E}\left[\alpha_t^\star \mid \mathcal{F}_{t_0}\right] = \frac{\mu_t - r_t}{(1-\gamma)\,\sigma_t^2}\left(1 + H_t \mathbb{E}\left[\frac{1}{X_t} \middle| \mathcal{F}_{t_0}\right]\right)$$

$\Rightarrow$ Practical considerations for modeling target date funds:

The shape of the glide path is known at the inception date t_0
while the shape of the optimal risk exposure is known at the current date t

No leverage $\Rightarrow$ The constrained glide path is defined by the conditional expectation:

$$g_t^c = \mathbb{E}\left[\alpha_t^\star \mid \alpha_t^\star \in [0,1]\right]$$

Personalization of retirement solutions

Table: Main principles of retirement accumulation strategies

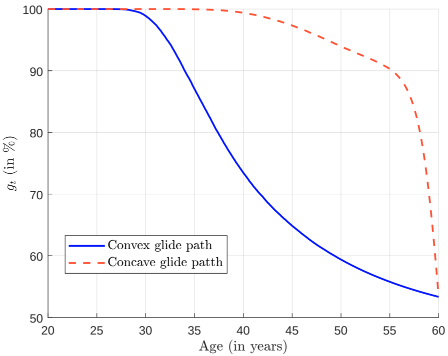
Rule	More risky assets		Fewer risky assets
#1	Young	$>$	Old
#2	Risk taker	$>$	Risk averse
#3	Low current wealth	$>$	High current wealth
#4	High expected future income	$>$	Low expected future income
#4'	High human capital	$>$	Low human capital
#5	High risk premium	$>$	Low risk premium
#6	Low risky-asset volatility	$>$	High risky-asset volatility
#7	Positive stock/bond correlation	$>$	Negative stock/bond correlation
#8	Certain income	$>$	Uncertain income
#9	Income uncorrelated with equities	$>$	Income correlated with equities

What is the shape of a glide path?

Why does theory predict convex glide paths?

Why do practitioners implement concave glide paths?

Figure: Convex vs. concave glide path



What is the shape of a glide path?

Figure: Constrained vs. unconstrained glide path

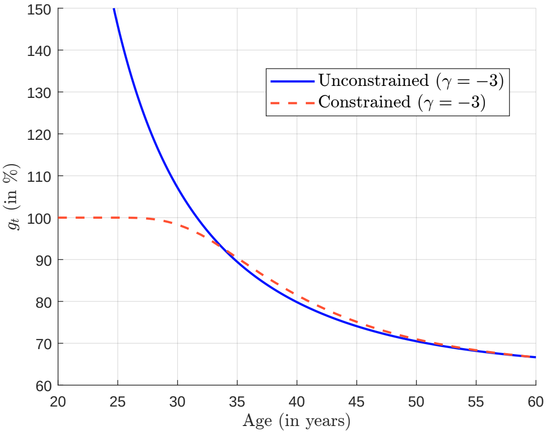
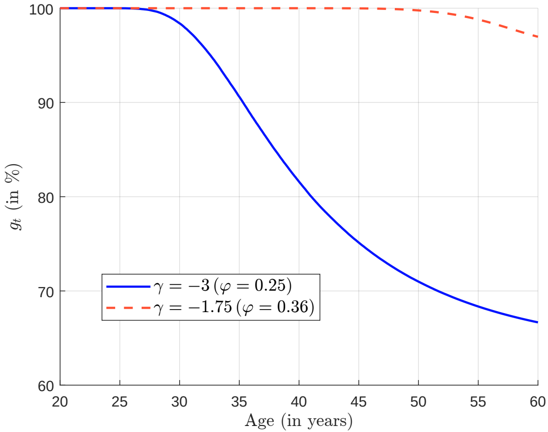


Figure: Impact of the risk aversion



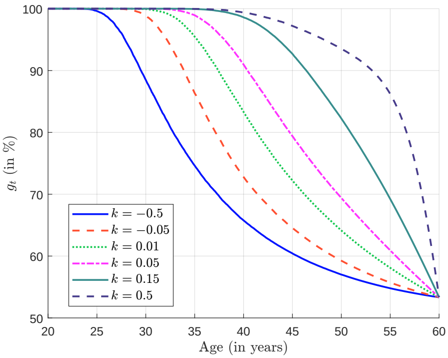
What is the shape of a glide path?

We solve the **convex vs. concave** glide path puzzle

- Leverage ratio: Individuals cannot borrow or use leverage, therefore they need to keep a risky exposure longer
- Time-varying risk aversion: Young investors typically exhibit lower risk aversion than old investors
- Concave contribution function: Contribution rates typically increase with age

The concave vs. convex puzzle

Figure: Effect of k on the convexity of g_t

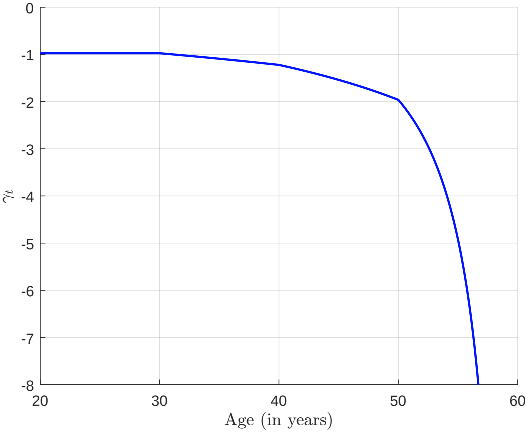


$$\gamma_t = \gamma_0 + (\gamma_T - \gamma_0) \frac{e^{k(t-t_0)} - 1}{e^{k(T-t_0)} - 1}, \quad \gamma_0 > \gamma_T$$

- Shape of γ_t: $k > 0 \Rightarrow \gamma_t$ concave, $k < 0 \Rightarrow \gamma_t$ convex
- Baseline (constant risk aversion): g_t tends to be convex
- Leverage constraint ($\alpha_t^\star \in [0,1]$): no leverage early $\Rightarrow$ reduced curvature $\Rightarrow g_t$ more concave
- Time-varying risk aversion: $(1-\gamma_t)^{-1}$ increases faster near retirement $\Rightarrow$ accelerated de-risking $\Rightarrow g_t$ concave
- Increasing contributions: H_t (PV of future contributions) concave $\Rightarrow$ the curvature of H_t/m_t induces a concave profile of g_t

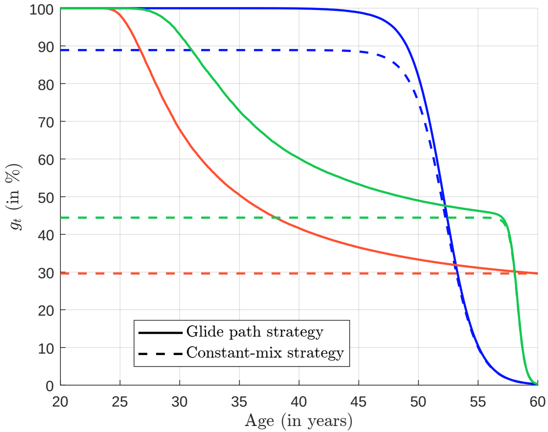
The concave vs. convex puzzle

Figure: Implied risk aversion of the FTSE Lifecycle Screened Select Index



The lifecycle vs. lifestyle puzzle

Figure: Glide path vs. constant-mix strategies



- Risk aversion can be calibrated from the constant-mix strategy (reverse engineering)
  1. Conservative (20/80)
  2. Balanced (50/50)
  3. Agressive (80/20)
- Puzzle: At the retirement date, the solution is the CM strategy, not a zero-equity exposure
- To solve this puzzle, risk aversion must be time-varying!

Private assets and retirement solutions

- 26 June: BlackRock published a press release on retirement planning & private market benefits: "*BlackRock's Custom Glidepath to Power Great Gray Trust Company's New Target Date Retirement Solution with Access to Private Markets*"

  "***New BlackRock research shows how purpose-built private markets exposures can deliver 50 bps of additional performance for target date funds***"

- 14 July: AllianceBernstein published the white paper "*Clearing Hurdles: Lending Private Assets in DC Plans*"
- 16 July: Goldman Sachs Asset Management published the research paper "*Private Markets: A New Frontier for Retirement Savings*"
- 24 August: Schroders published its annual survey "*Schroders 2025 US Retirement Survey*"

  "***Schroders study finds nearly half of retirement plan participants would invest in private markets***"

Private assets and retirement solutions

KKR's Freise argued that the pension industry needs to be transformed to enable savers to invest in private markets:

> "***In an aging society, people need to be having comfort and trust and confidence in their retirement and retirement savings. Our pension system needs to reform itself in terms of giving access to investments that actually produce the returns that can allow you to retire with that kind of saving pot.***" *(Bloomberg, September 25, 2025)*

Mansion House Compact II (May 2025)

- Allocate at least 5% of DC default assets to unlisted equity by 2030
- 9 organisations (Aegon, Aviva, Legal & General, M&G, Mercer, Nest, Phoenix, Scottish Widows and Smart Pension) initially signed the pledge, with Aon and Natwest Cushon also joining the initiative later

Rationale

- **Horizon Match**

  TDFs have multi-decade horizons and can hold long-term, illiquid assets (PE, private credit, infrastructure) to capture the illiquidity premium

- **Diversification**

  Low correlation to public stocks/bonds; improves risk-adjusted returns for the same risk budget

- **Inflation Resilience**

  Infrastructure and real assets often have inflation-linked cash flows/contract escalators, adding an inflation hedge

Are DC investors in fact the real long-term investors?

Real assets and retirement solutions

Main finding

Our simulations shows that including real assets could deliver **20 and 40 bps of additional annualized net performance** in the global and Eurozone universes, respectively.

Impact of inflation on glide paths

Inclusion of the inflation risk

- The solution becomes:

$$\text{Dynamic Allocation} = \textbf{Growth Portfolio} \times \textbf{(1+HFCR)} + \textbf{Hedging Demand} \times \textbf{LHP}$$

- LHP = Liability-hedging portfolio (LDI)
- Convergence between DB and DC approaches

- The LHP depends on covariance risk between inflation and asset returns
- The weights increase with the inflation volatility
- Generally, a positive (resp. negative) correlation between inflation and asset returns implies a positive (negative) weight

Impact of inflation on glide paths

The hedging demand is more complex

It depends critically on two factors:

- The relationship between expected inflation levels and asset risk premia (= **opportunity-set component**)
- The investor's objective function, particularly whether they prioritize terminal nominal wealth or real purchasing power (= **real-discounting component**)

This highlights that the concept of inflation hedging is not well-defined because inflation risk recovers several aspects (e.g., expected inflation vs. unexpected inflation)

$\Rightarrow$ We can obtain a partial hedge

Understanding the hedging demand

Figure: Heatmap of the hedging demand $\mathcal{H}_t^{(\pi)}$ for $\varrho = 0$

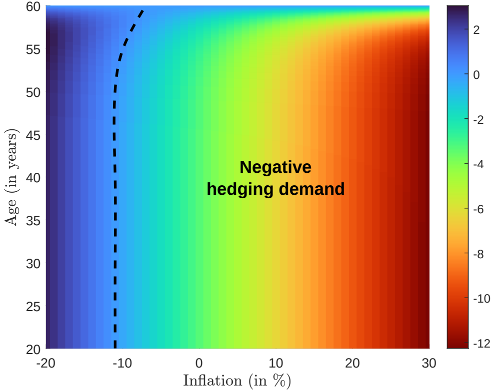
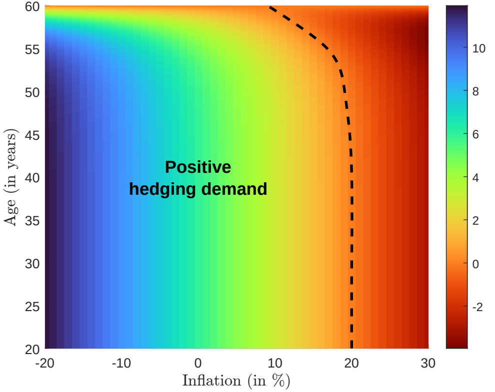


Figure: Heatmap of the hedging demand $\mathcal{H}_t^{(\pi)}$ for $\varrho = 1$



Decumulation strategies

Main sources of risk

- **Longevity risk:** How many years of income needed?
  - 20 years on average (median case)
  - 32 years for 5% of people in the population
  - **Managing the ruin probability**
- **Drawdown rule:** How much to spend each year?
  - Stable income (4% safe withdrawal rate of Bengen (1997))
  - Retirement cycle (U-shaped: leisure/travel, followed by low spending and then increased medical costs)
  - Surplus management
  - Bequest issue
- **Money management:** How to invest the portfolio?
  - Balanced strategy (50/50 portfolio)
  - Decumulation/withdrawal strategy (fixed amount/percentage, inflation-adjusted, dynamic rule)
  - Annuities (immediate, deferred, variable)
  - Tontines (longevity pooling)

What is the optimal policy?

Solution #1

Withdrawal strategy

Solution #2

Constant-mix strategy

Solution #2

Glide path with a drawdown rule + annuity purchase after n years

Solution #3

Forward annuity purchase + Glide path with a withdrawal rule

$\Rightarrow$ There is no market of forward annuities

Conclusion

Conclusion

- Longevity risk $\Rightarrow$ both a systemic and idiosyncratic risk $\Rightarrow$ DC plans $\nearrow$
- Two phases: Accumulation & Decumulation
- Accumulation challenges
  - Too late to begin risk & exposure to market risk
  - Standardized investment strategies through glide path approaches
  - Heterogenous implementation across asset manager (glide paths differ significantly from one provider to another)
  - Lifecycle complexity vs. lifestyle simplicity
- Decumulation challenges (more acute)
  - Exposure to longevity risk
  - Multiple competing goals (income & bequest) & irreversibility
  - Embedded option, & lottery-like features
  - Underdeveloped market characterized by limited supply $\Rightarrow$ Efficiency?
- Retirement solutions $\Rightarrow$ Hybrid products (insurance **and** asset management features)

Conclusion

Regulation challenge

How to improve the efficiency of the retirement solutions market and the financial literacy of the advisors/clients?

- Accumulation products (PER, PERCOL, PERO):

  Gestion pilotée ou gestion libre?

- Decumulation products:

  Sortie en capital ou en rente?

Merton framework for designing glide paths

Merton Framework

1. **Define the stochastic control and market setup**: exposure $\alpha_t^{(i)}$ for each risky asset i
2. **Specify the investor's wealth dynamics**:

$$\mathrm{d}X_t = ??\,\mathrm{d}t + ??\,\mathrm{d}W_t + \ldots$$

3. **Solve the Hamilton-Jacobi-Bellman (HJB) equation**: let $\mathcal{J}(t,x)$ be the value function associated with the investor's problem:

$$\mathcal{J}(t,x) = \sup_{\alpha_t} \mathbb{E}_t \left[\mathcal{U}(X_T) \mid X_t = x \right]$$

the function $\mathcal{J}(t,x)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\underbrace{\frac{\partial \mathcal{J}(t,x,\ldots)}{\partial t} + \cdots}_{\text{Pure derivative part}} + \underbrace{\max_{\alpha_t} \mathcal{H}(t,x,\ldots,\alpha_t)}_{\text{Optimization part}} = 0$$

Howard policy-iteration algorithm for solving the HJB equation

Howard policy-iteration uses a *greed* switching rule to update from any non-optimal policy to a dominating one, iterating until an optimal policy is found:

Policy Evaluation

Given α_t, solve $\frac{\partial \mathcal{J}(t,x,\ldots)}{\partial t} + \cdots + \mathcal{H}\left(t,x,\ldots,\alpha_t\right) = 0$

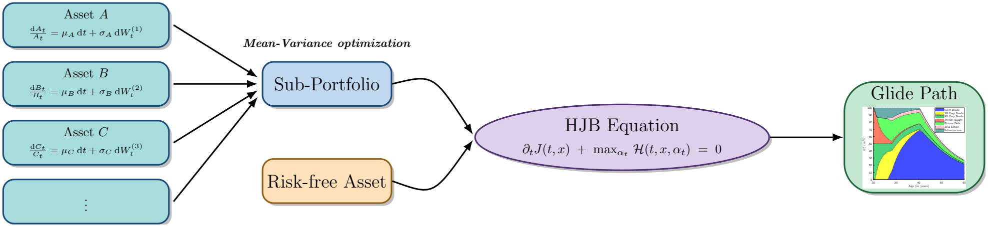
$$\underbrace{\frac{\partial \mathcal{J}\left(t,x,\ldots\right)}{\partial t}}_{\textbf{Pure derivative part}} + \cdots + \underbrace{\max_{\alpha_t} \mathcal{H}\left(t,x,\ldots,\alpha_t\right)}_{\textbf{Optimization part}} = 0$$

Policy Improvement

$$\alpha_t^\star = \arg\max_{\alpha_t} \mathcal{H}(t,x,\alpha_t)$$

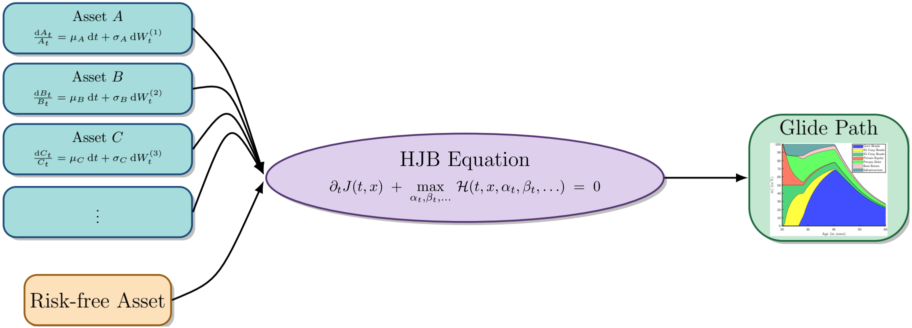
Basic modeling of glide path strategy

Two-stage approach



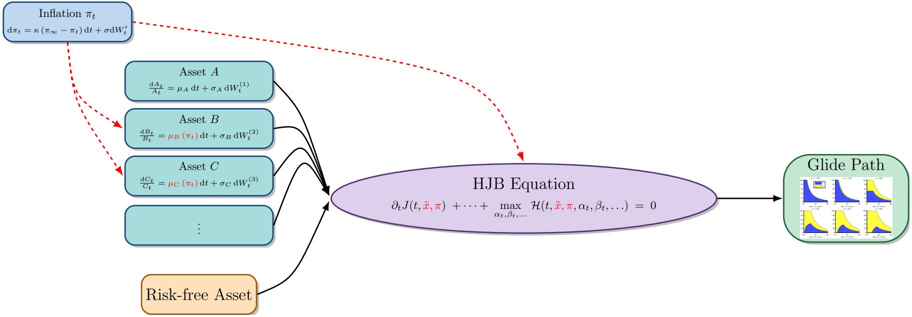
Extension to multi-asset classes

One-stage approach



Incorporating inflation risk

One-stage approach incorporating inflation risk



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