

Introduction to Risk Parity and Budgeting

Chapter 4 – Application to Bond Portfolios

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[†] Evry University & Lyxor Asset Management, France

Instructors may find the description of the book at the following addresses:

<http://www.crcpress.com/product/isbn/9781482207156>

<http://www.thierry-roncalli.com/RiskParityBook.html>

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Figure: Term structure of spot and forward interest rates (in %)

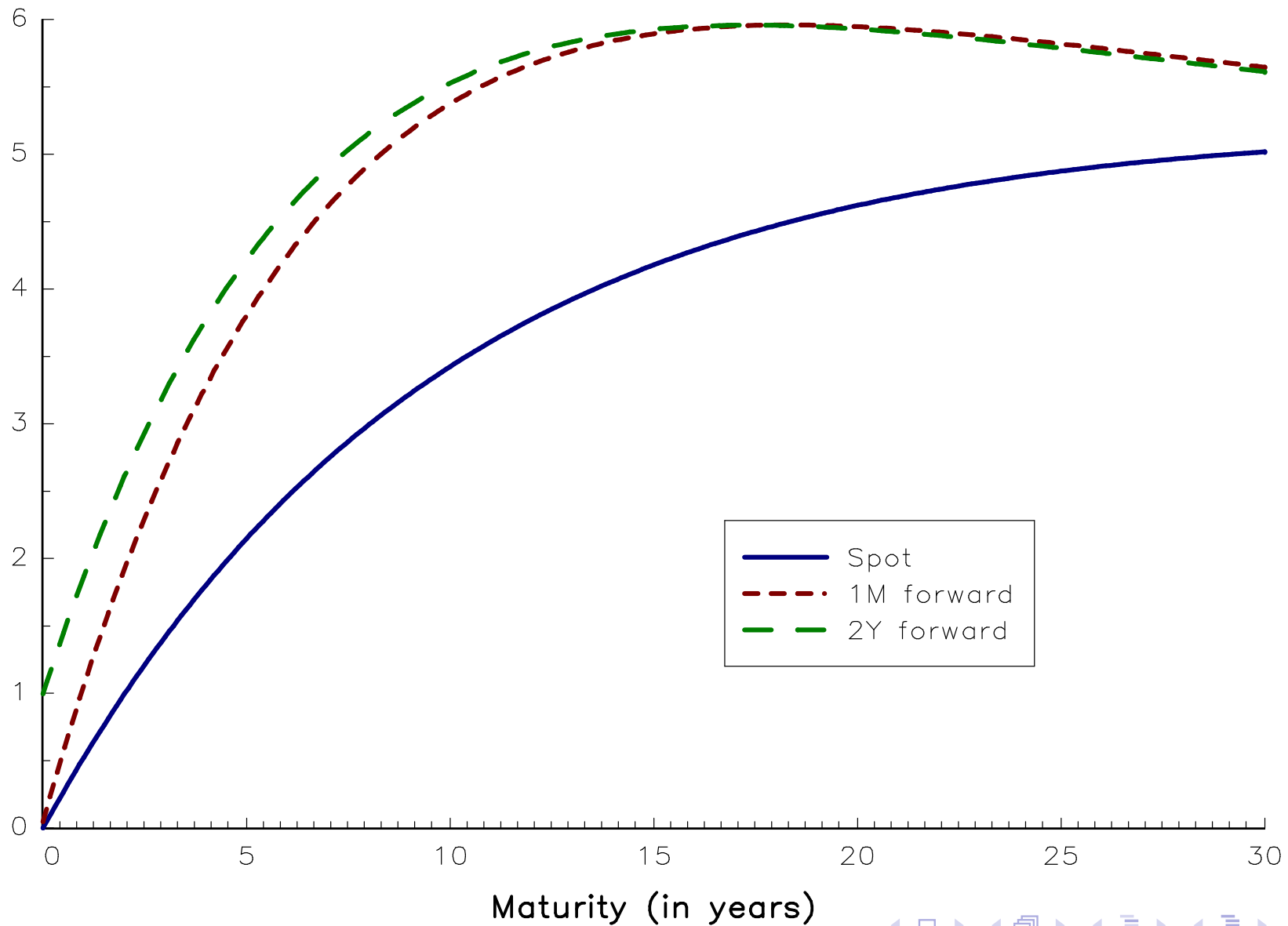


Figure: PCA factors of the US yield curve (Jan. 2003 – Jun. 2012)

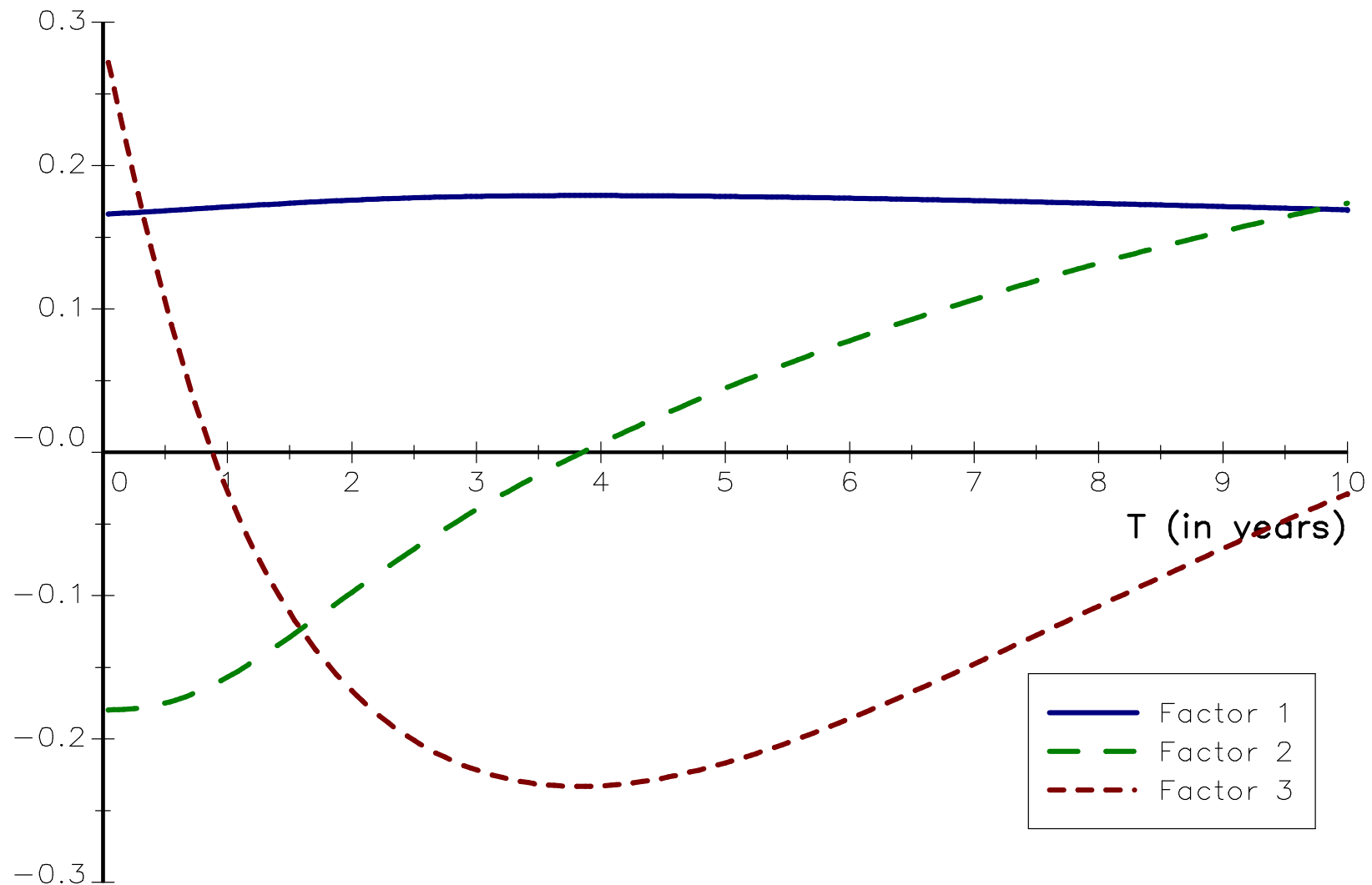


Figure: Cash flows of a bond with a fixed coupon rate

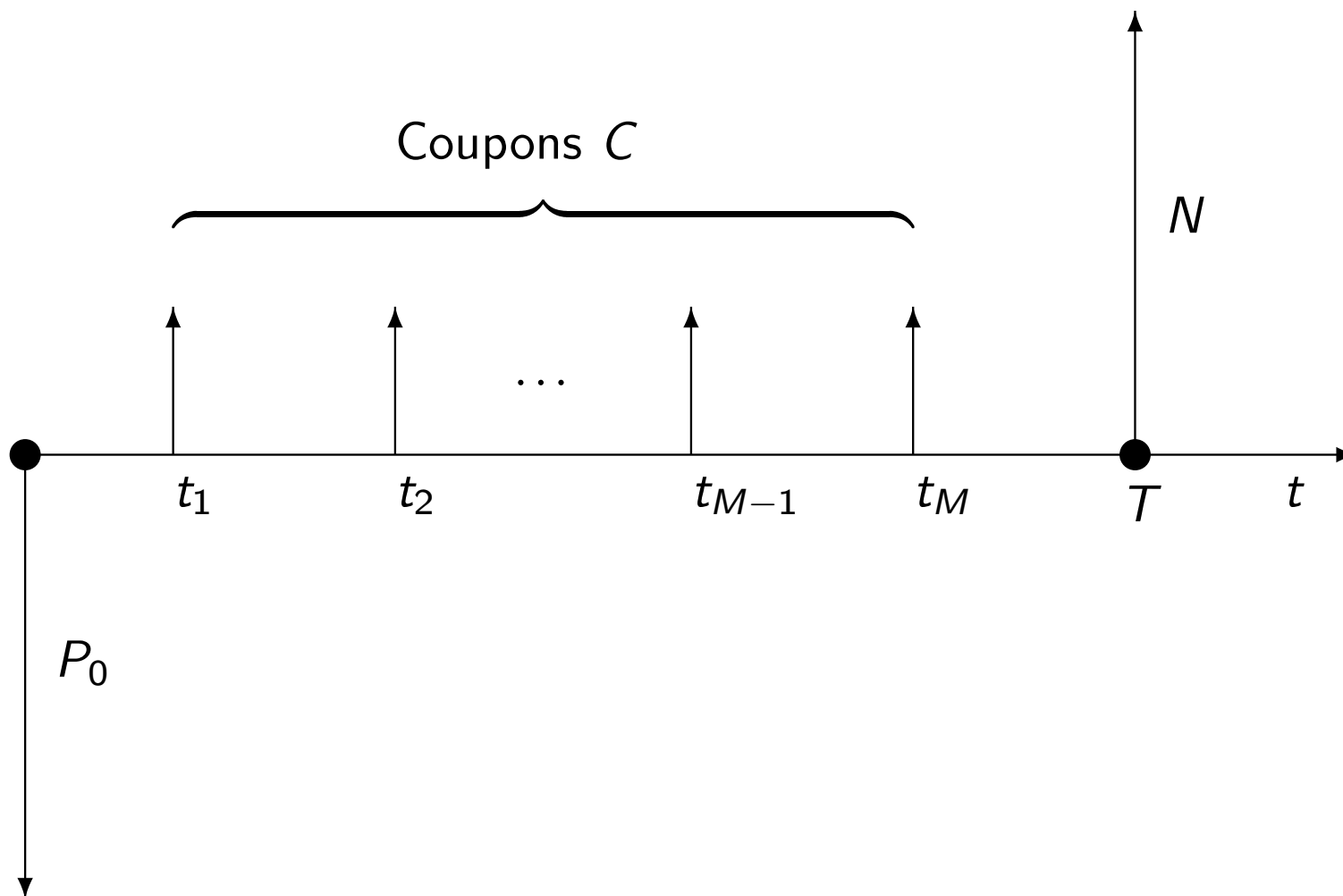
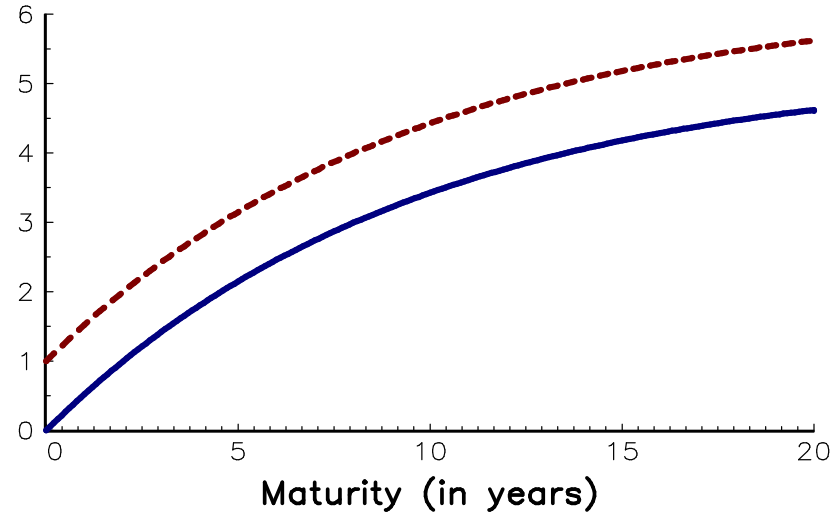
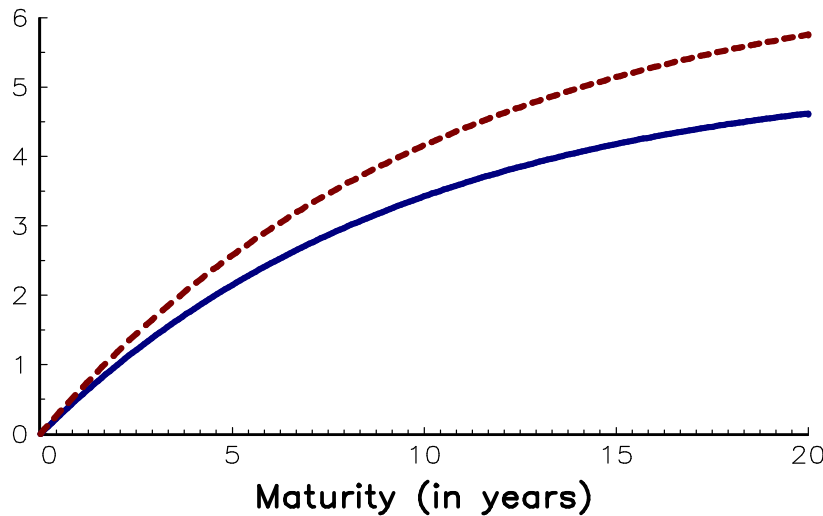


Figure: Movements of the yield curve

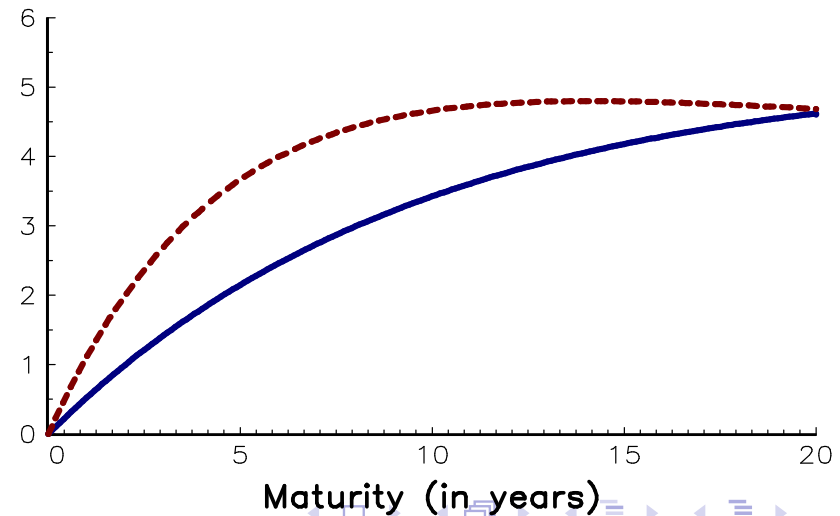
Parallel shift



Steepness of the slope



Curvature movement



Tables 4.1 & 4.2, Pages 199 & 200

Table: Price, yield to maturity and sensitivity of bonds

T	$R_0(T)$	$B_0(T)$	P_0	r^*	S
1	0.52	99.48	104.45	0.52	-104.45
2	0.99	98.03	107.91	0.98	-210.86
3	1.42	95.83	110.50	1.39	-316.77
4	1.80	93.04	112.36	1.76	-420.32
5	2.15	89.82	113.63	2.08	-520.16

Table: Impact of a parallel shift of the yield curve on the bond with five-year maturity

Δr^* (in bps)	\check{P}_0	ΔP	\hat{P}_0	ΔP	$S \cdot \Delta r^*$
-50	116.26	2.63	116.26	2.63	2.60
-30	115.20	1.57	115.20	1.57	1.56
-10	114.15	0.52	114.15	0.52	0.52
0	113.63	0.00	113.63	0.00	0.00
10	113.11	-0.52	113.11	-0.52	-0.52
30	112.08	-1.55	112.08	-1.55	-1.56
50	111.06	-2.57	111.06	-2.57	-2.60

Figure: Cash flows of a bond with default risk

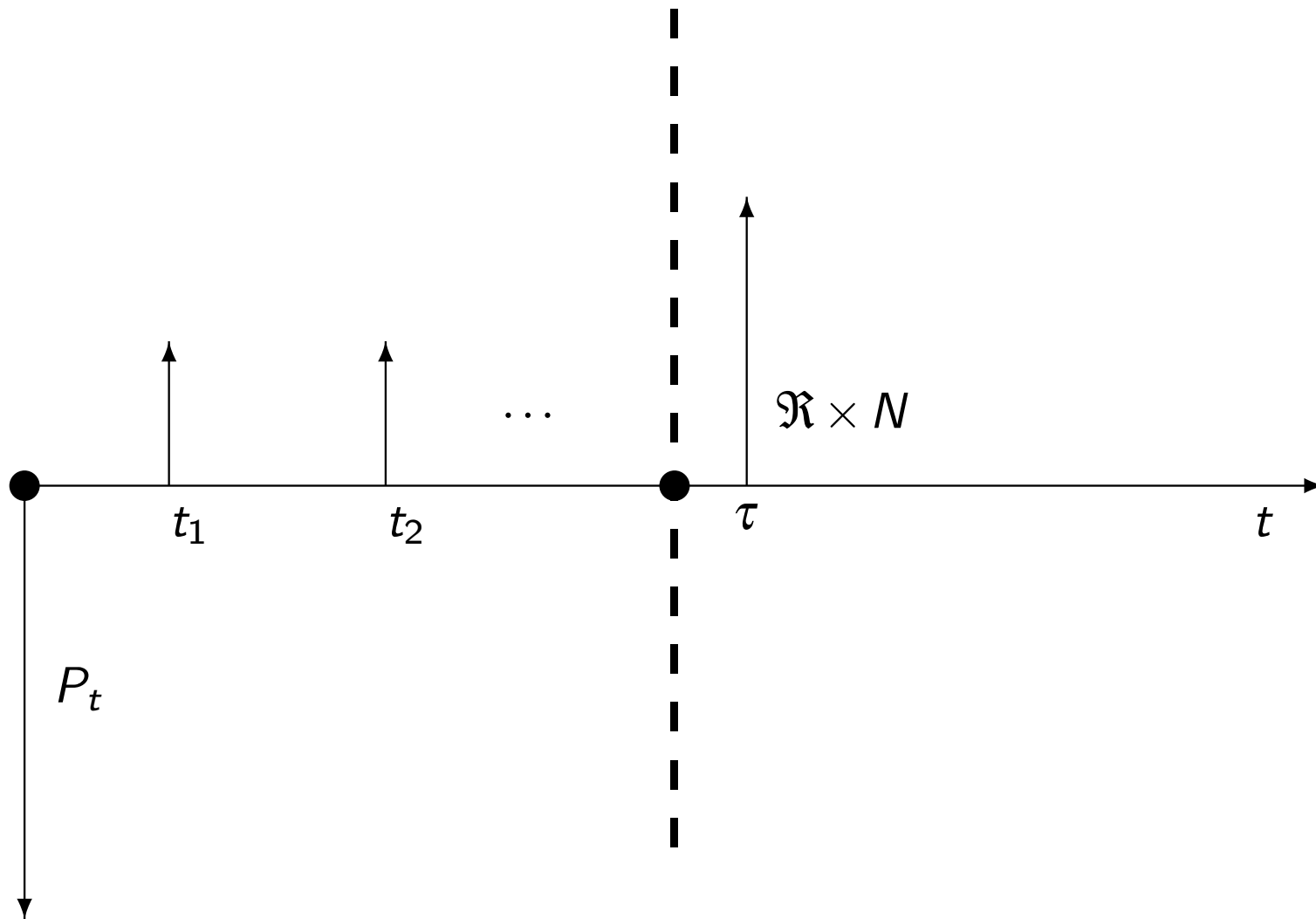


Table: Computation of the credit spread s

\mathfrak{R} (in %)	λ (in bps)	PD (in bps)	P_t (in \$)	\tilde{r}^* (in %)	s (in bps)
0	0	0.0	110.1	3.24	0.0
	10	10.0	109.2	3.34	9.9
	200	198.0	93.5	5.22	198.1
	1000	951.6	50.4	13.13	988.9
40	0	0.0	110.1	3.24	0.0
	10	10.0	109.6	3.30	6.0
	200	198.0	99.9	4.41	117.1
	1000	951.6	73.3	8.23	498.8
80	0	0.0	110.1	3.24	0.0
	10	10.0	109.9	3.26	2.2
	200	198.0	106.4	3.66	41.7
	1000	951.6	96.3	4.85	161.4

Figure: Evolution of the zero-coupon interest rates and the intensity (June 2010 – June 2012)

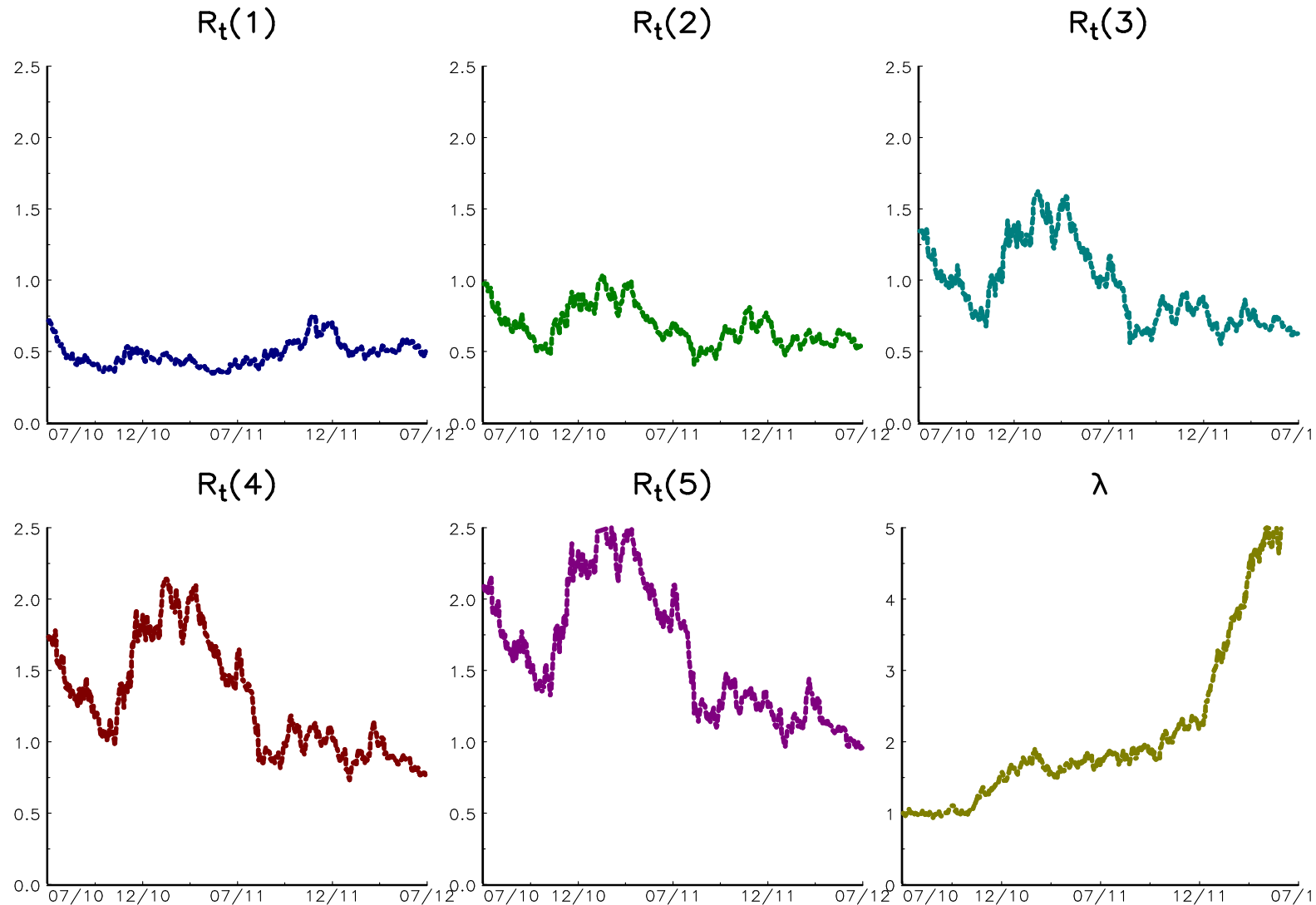


Table: Pricing of the bond

t_m	$C(t_m)$	$R_t(t_m)$	$B_t(t_m)$	$\delta(t_m)$
1	5	0.493	0.995	-4.975
2	5	0.544	0.989	-9.892
3	5	0.626	0.981	-14.721
4	5	0.773	0.970	-19.391
5	105	0.960	0.953	-500.400

Table: Risk measure and decomposition of the bond exposure

t_m	G1	G2	H1	H2	H3	ES
1	6.3	6.5	-1.6	7.2	6.4	15.1
2	41.6	42.3	29.4	50.1	46.7	81.7
3	100.7	102.4	102.4	121.7	114.2	172.6
4	169.8	172.8	195.1	206.6	193.0	271.0
5	4941.7	5032.2	5658.7	5928.3	5623.6	7399.3
\mathcal{R}	5260.2	5356.3	5984.0	6313.9	5984.0	7939.6

Table: Risk allocation of the bond portfolio

	G1	G2	H1	H2	H3	ES
R	4244.6	4321.0	4749.2	5114.1	4749.2	6552.3
	Risk contribution with respect to $R_t(t_m)$					
0.5	0.2	0.2	-0.3	0.1	0.1	0.3
1.0	3.8	3.9	-0.9	3.8	3.8	8.1
1.5	3.2	3.2	1.0	3.5	3.4	6.4
2.0	221.3	224.8	150.1	255.6	244.6	417.0
2.5	10.9	11.1	9.5	12.7	12.2	19.3
3.0	41.1	41.8	41.0	48.7	45.9	69.0
3.5	20.9	21.2	22.8	25.0	23.3	34.0
4.0	68.4	69.6	78.0	82.6	76.5	108.4
4.5	1904.4	1938.5	2184.5	2310.6	2132.3	2930.2
5.0	1970.5	2006.7	2263.5	2371.3	2207.2	2959.7
sum	4244.6	4321.0	4749.2	5114.1	4749.2	6552.3
	Risk contribution with respect to $\bar{\omega}_i$					
#1	2100.2	2138.6	2393.6	2525.5	2351.6	3175.8
#2	1939.5	1974.2	2217.5	2351.9	2171.3	2990.1
#3	205.0	208.2	138.2	236.6	226.4	386.4
sum	4244.6	4321.0	4749.2	5114.1	4749.2	6552.3
	Risk contribution with respect to $\bar{\omega}_i$ (in %)					
#1	49.5	49.5	50.4	49.4	49.5	48.5
#2	45.7	45.7	46.7	46.0	45.7	45.6
#3	4.8	4.8	2.9	4.6	4.8	5.9

Table: Risk decomposition of the bond portfolio with respect to the PCA factors

Factor	G1	G1	H1	H2	H3	ES
1	4198.4	4274.7	4406.2	4974.6	4406.2	6718.0
2	39.0	39.0	475.3	120.6	475.3	-139.8
3	6.0	6.0	-66.1	25.2	-66.1	-42.8
4	1.1	1.1	-57.5	-11.3	-57.5	25.2
5	0.1	0.1	-0.4	2.9	-0.4	-4.3
6	0.0	0.0	0.1	-0.1	0.1	0.0
7	0.0	0.0	-5.8	-2.3	-5.8	-2.2
8	0.0	0.0	0.9	2.9	0.9	-0.6
9	0.0	0.0	0.2	0.2	0.2	0.2
10	0.0	0.0	-3.5	1.4	-3.5	-1.3
sum	4244.6	4321.0	4749.2	5114.1	4749.2	6552.3

Figure: Loss distribution of the bond portfolio with and without default risk

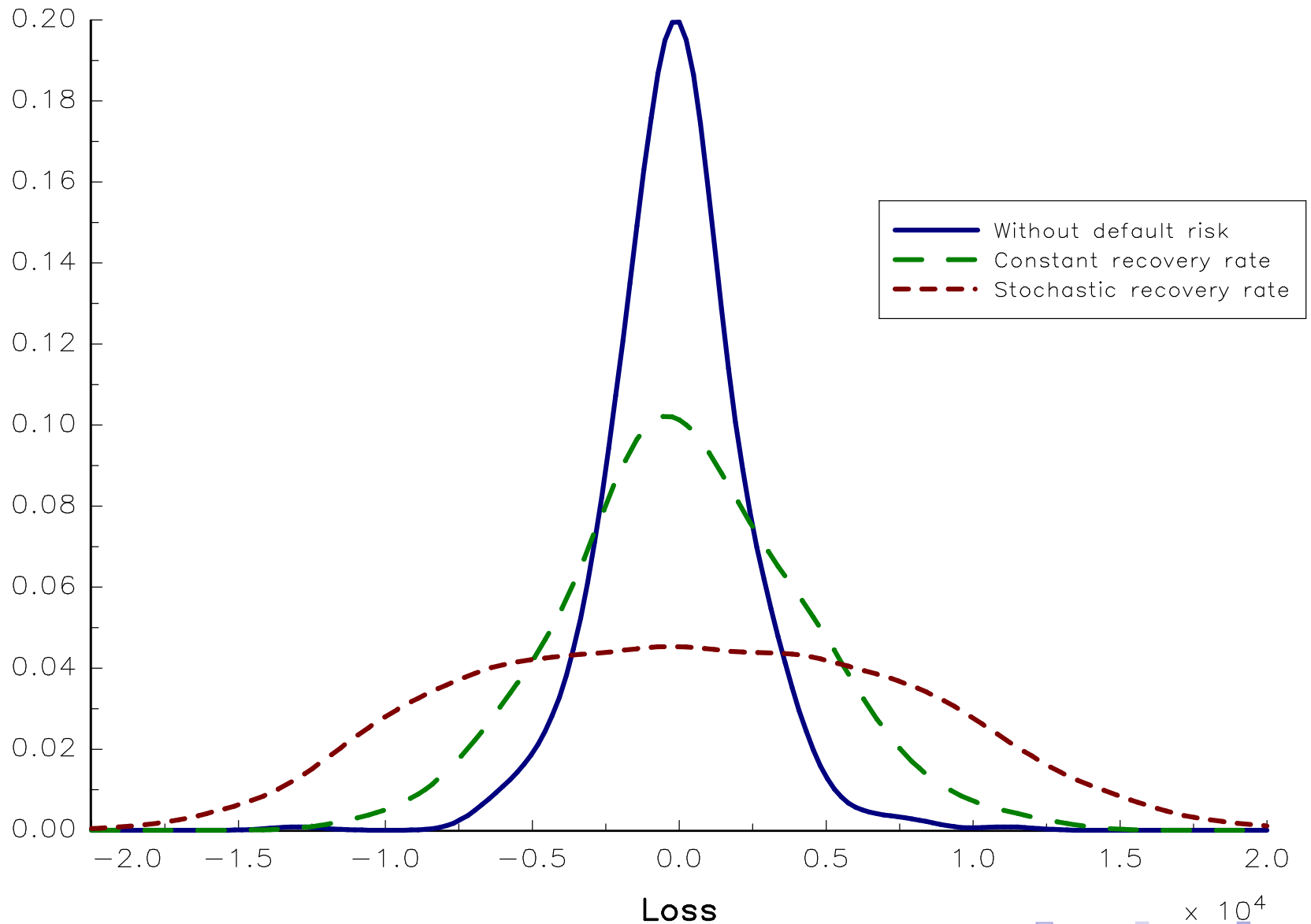


Table: Characteristics of the bond portfolio

Bond	x_i (in %)	D_i (in years)	λ_i (in bps)	p_i (in %)	$\mu(G_i)$ (in %)
1	20.00	5.00	100	4.88	70.00
2	30.00	6.00	120	6.95	70.00
3	10.00	8.00	85	6.57	50.00
4	15.00	7.00	115	7.73	50.00
5	25.00	5.00	250	11.75	50.00

Table: Normalized risk contributions \mathcal{RC}_i^* of the bond portfolio (in %)

Bond	(30%, 99%)		(50%, 99%)		(30%, 99.9%)	
	VaR	ES	VaR	ES	VaR	ES
1	18.2	19.0	19.1	20.2	19.8	20.3
2	33.9	34.2	34.4	34.7	34.5	34.6
3	7.8	7.9	8.0	8.1	8.0	8.1
4	12.9	12.9	12.9	12.8	12.8	12.8
5	27.1	26.0	25.6	24.2	24.9	24.1
$\mathcal{R}(x)$	24.9	30.1	36.2	42.8	36.6	40.4

Figure: Risk factor contributions of the EW Portfolio #1

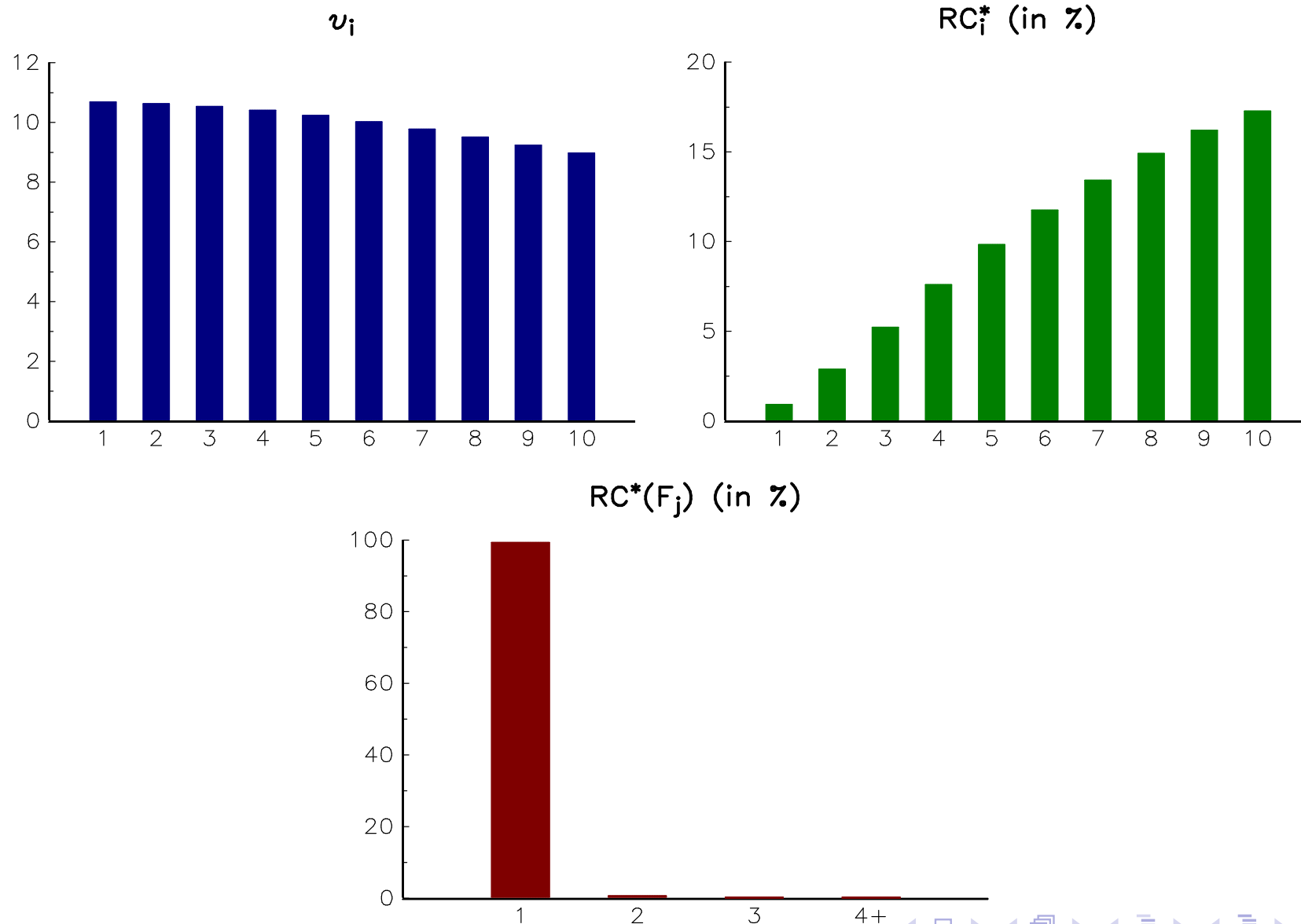


Figure: Risk factor contributions of the long-short Portfolio #2

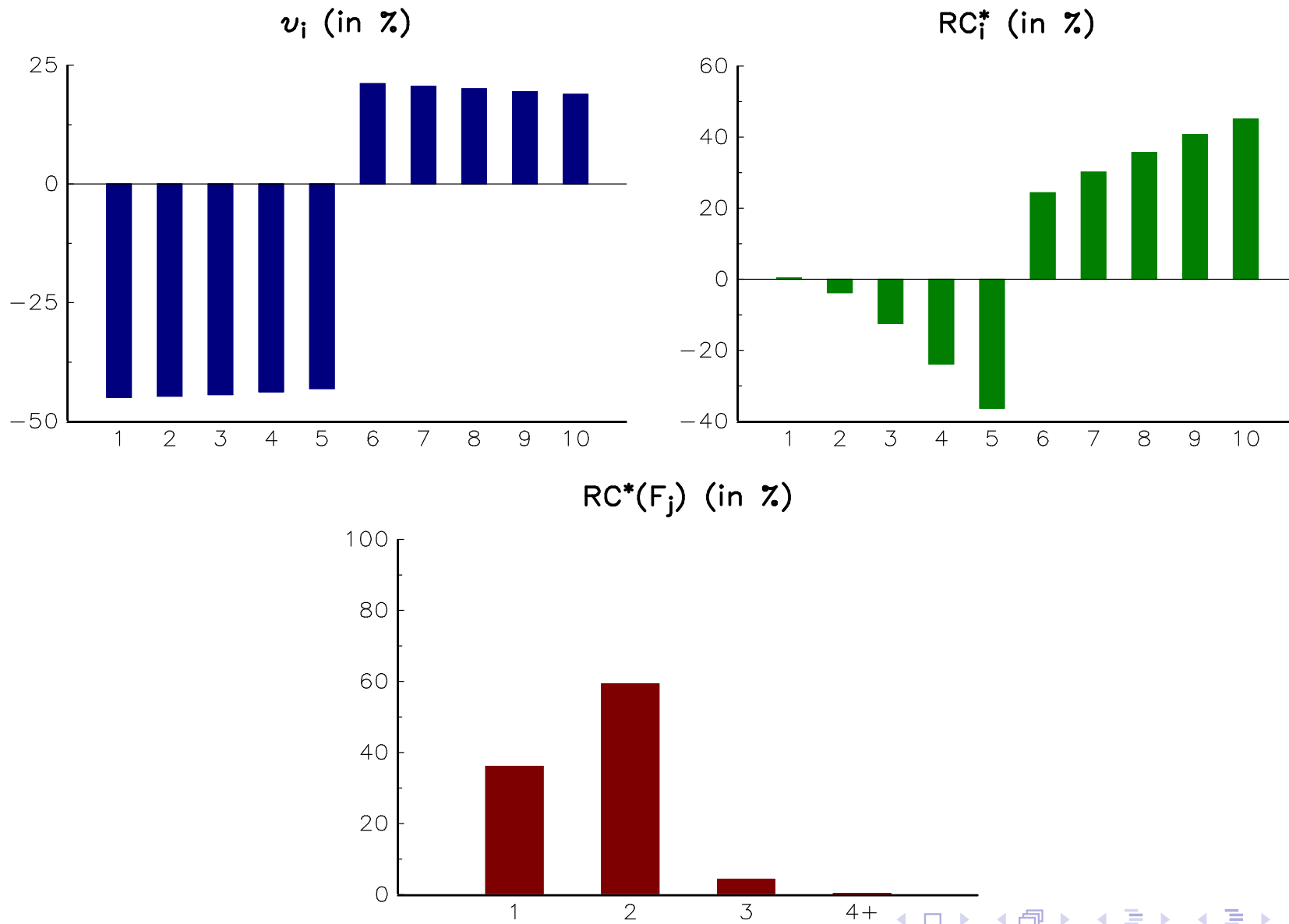


Figure: Risk factor contributions of the long-short Portfolio #3

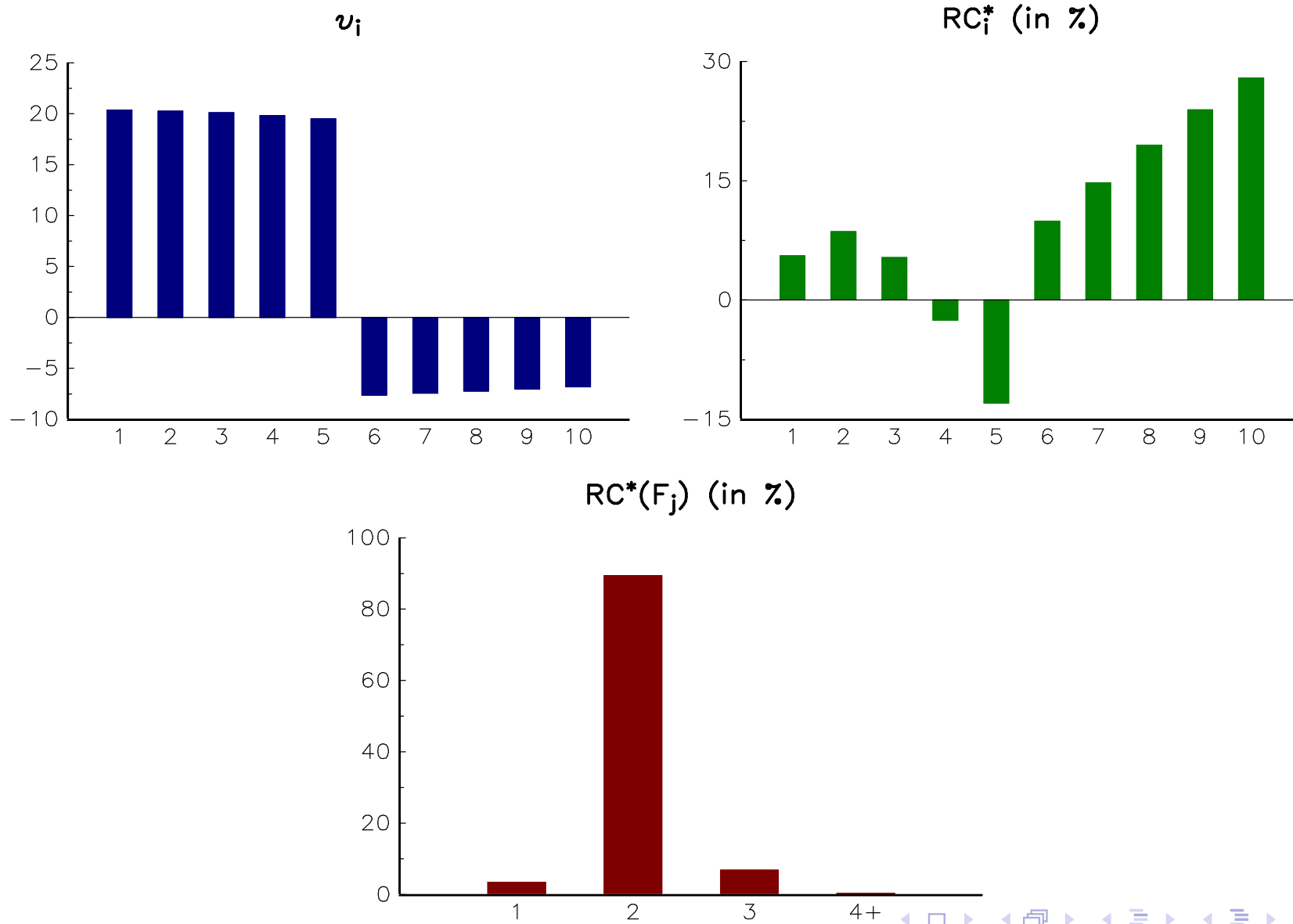


Figure: Risk factor contributions of the long-short Portfolio #4

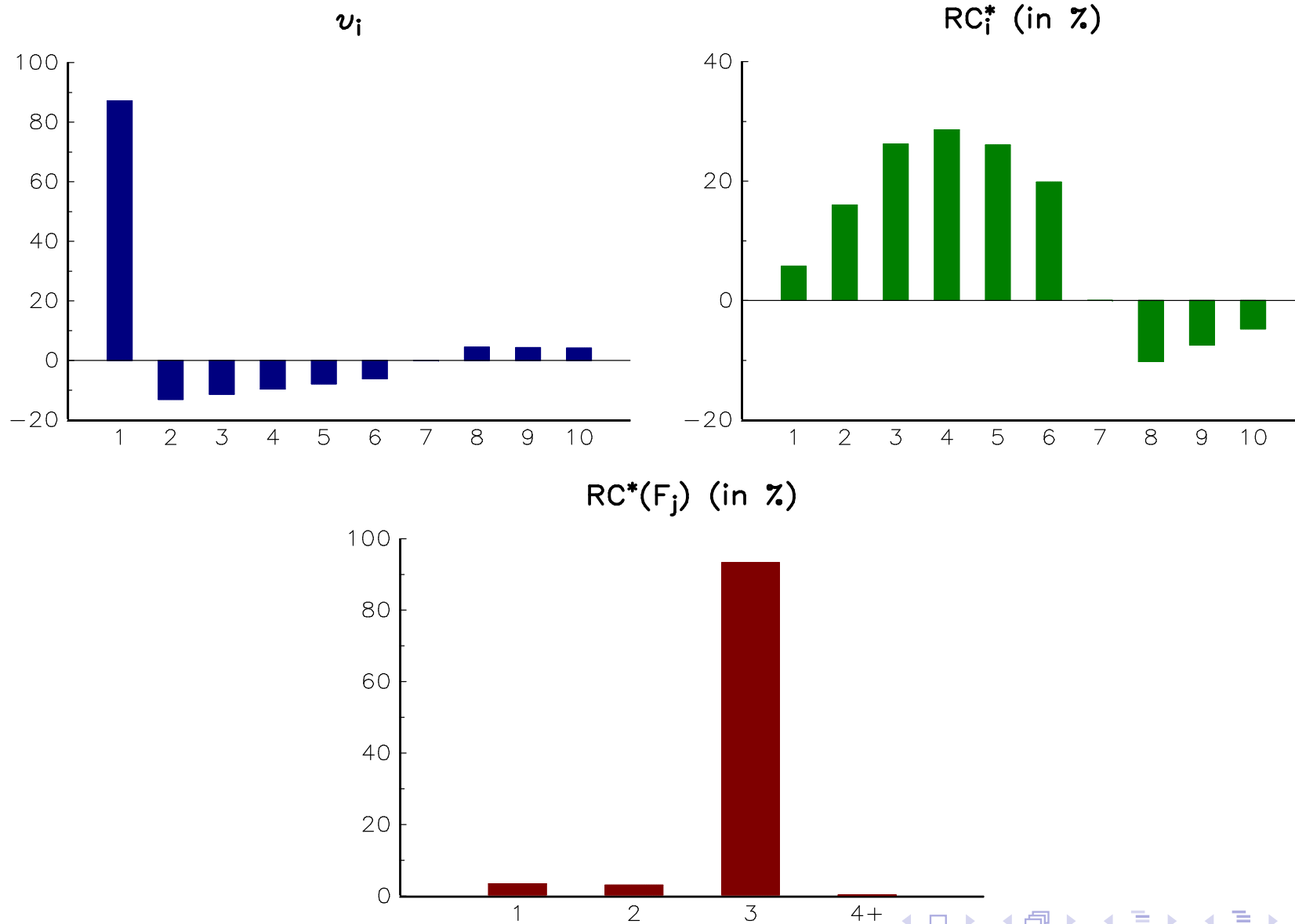


Figure: P&L of the barbell portfolios due to a YTM variation

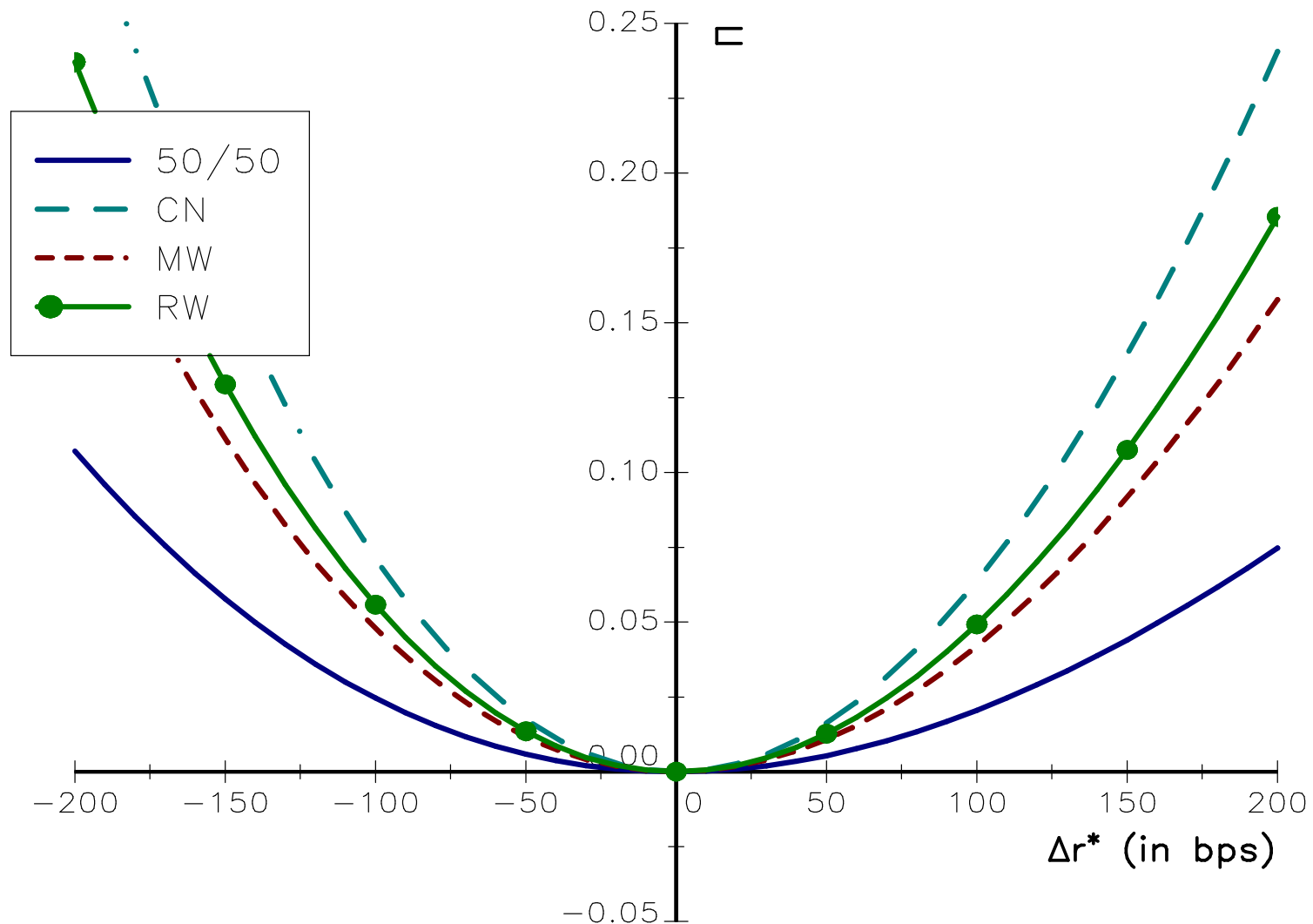


Figure: Risk factor contributions of the barbell portfolios

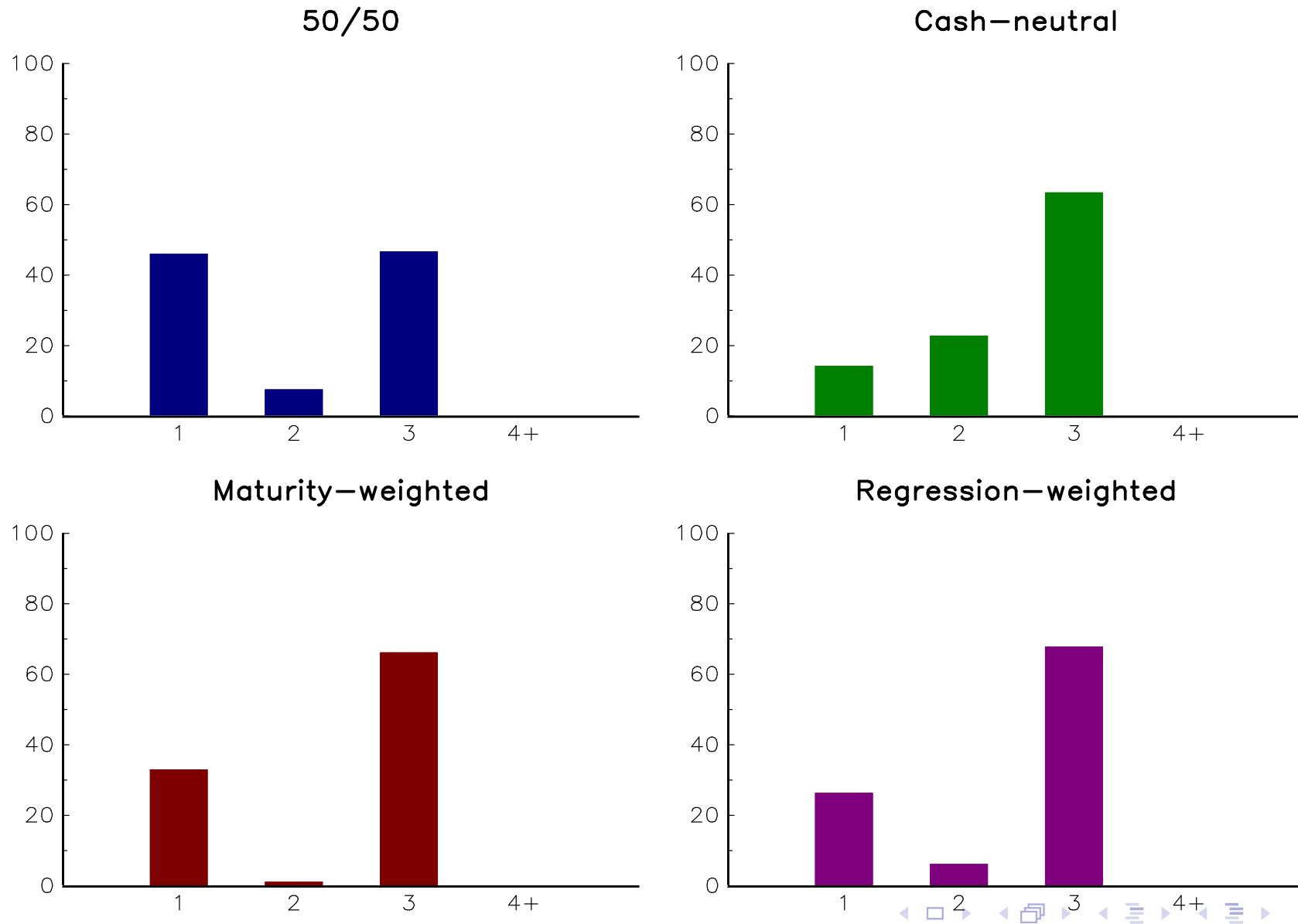


Table: Some measures of country risk (October 2011)

Country	Rating	ECR		CDS Spread	
		Score	Rank	01/09/11	04/10/11
Austria	AAA	84.01	13	123	186
Belgium	AA+	77.81	19	249	309
Finland	AAA	86.96	8	64	85
France	AAA	80.90	16	163	201
Germany	AAA	84.98	11	76	122
Greece	CCC	52.38	65	2291	5736
Ireland	BBB+	62.33	43	781	726
Italy	A	71.20	30	384	487
Netherlands	AAA	86.67	9	80	117
Portugal	BBB-	61.35	44	957	1167
Spain	AA	66.71	36	376	391
Norway	AAA	93.44	1	44	52
Switzerland	AAA	90.31	3	58	79
Denmark	AAA	89.21	4	100	153
Sweden	AAA	88.74	5	54	66
United Kingdom	AAA	80.22	17	76	102
United States	AA+	82.10	15	52	52
Japan	AA-	74.66	25	102	155

Table: ML estimate of the parameter β_i (Jan. 2008 – Jun. 2012)

Country	AT	BE	FI	FR	DE	GR
$\hat{\beta}_i$	0.953	0.969	0.780	0.806	0.853	1.219
$\hat{\sigma}(\beta_i)$ (in %)	1.015	1.410	1.415	1.085	1.638	0.462
Country	IE	IT	NL	PT	ES	AC
$\hat{\beta}_i$	0.786	1.033	0.790	0.911	1.042	0.922
$\hat{\sigma}(\beta_i)$ (in %)	0.633	1.697	0.870	0.950	1.776	1.178

Table: Spread $s_i(t)$ (in bps)

Country	Jan. 08	Jan. 09	Jan. 10	Jan. 11	Jan. 12
Austria	6	128	85	100	190
Belgium	12	83	54	218	316
Finland	6	57	27	33	78
France	7	54	32	101	222
Germany	6	45	27	58	104
Greece	22	228	282	1074	8786
Ireland	15	177	160	615	726
Italy	21	165	108	238	503
Netherlands	6	83	32	63	122
Portugal	18	92	91	500	1093
Spain	20	103	113	350	394
Median	12	92	85	218	316

Table: Estimated values of the volatility σ_i^s (in %)

Country	Jan. 08	Jan. 09	Jan. 10	Jan. 11	Jan. 12
Austria	56.6	96.0	72.4	76.3	69.7
Belgium	65.8	70.2	83.4	73.2	74.1
Finland	103.9	107.6	80.9	61.7	66.7
France	50.2	92.5	97.1	77.5	68.5
Germany	69.2	96.8	76.2	72.3	65.0
Greece	60.4	57.3	64.4	89.3	85.2
Ireland	76.5	97.4	63.2	78.1	52.1
Italy	48.8	65.5	62.8	90.6	74.3
Netherlands	81.7	108.6	78.3	61.2	66.5
Portugal	56.6	64.4	84.2	106.6	54.8
Spain	67.7	63.7	69.2	92.0	72.8
Mean	67.0	83.6	75.6	79.9	68.1

Figure: Average correlation of credit spreads (in %)

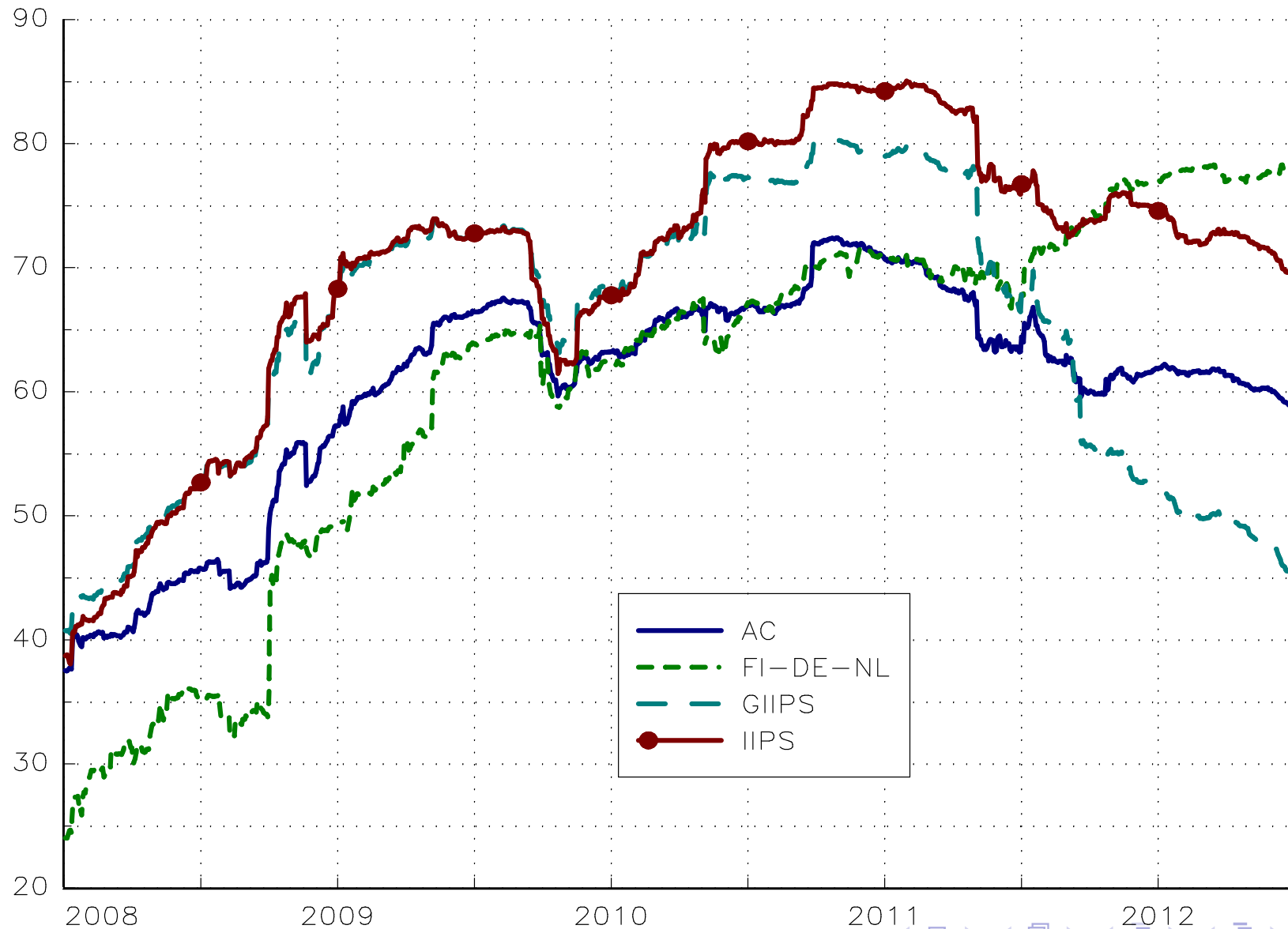


Table: Market-based parameters (March 1, 2012)

Country	$s_i(t)$	σ_i^5	$\rho_{i,j}$		
Germany	76 bps	66.0%	1.00		
France	166 bps	70.9%	0.86	1.00	
Italy	356 bps	74.2%	0.73	0.80	1.00

Table: Computing the credit risk measure σ_i^c for one bond

Bond	Country	B_i	D_i	σ_i^c
1	Germany	12	8.2	4.11%
2	France	15	7.1	8.36%
3	Italy	16	6.5	17.17%
4	Italy	8	5.9	15.58%

Tables 4.17 & 4.18 & 4.19, Pages 228 & 229

Table: Credit risk measure of the portfolio with three bonds

Bond	x_i	MR_i	RC_i	RC_i^*
1	27.91	3.48	0.97	9.85
2	34.88	7.65	2.67	27.12
3	37.21	16.67	6.20	63.03
Risk measure			9.84	

Table: Credit risk measure of the portfolio with four bonds

Bond	x_i	MR_i	RC_i	RC_i^*
1	23.53	3.39	0.80	7.45
2	29.41	7.47	2.20	20.55
3	31.37	16.87	5.29	49.52
4	15.69	15.31	2.40	22.47
Risk measure			10.69	

Table: Credit risk measure of the portfolio with the Italian meta-bond

Bond	x_i	MR_i	RC_i	RC_i^*
1	23.53	3.39	0.80	7.45
2	29.41	7.47	2.20	20.55
3'	47.06	16.35	7.70	71.99
Risk measure			10.69	

Table: Weights and risk contribution of the EGBI portfolio (in %)

Country	Jan. 08		Jan. 10		Jan. 11		Jan. 12	
	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*
Austria	3.9	1.7	3.8	4.5	4.0	1.9	4.2	2.8
Belgium	6.3	6.7	6.1	4.8	6.3	6.1	6.2	7.5
Finland	1.3	0.4	1.2	0.3	1.3	0.1	1.5	0.3
France	19.9	10.4	20.2	9.6	22.1	11.7	23.5	19.6
Germany	24.3	12.3	21.6	7.2	22.9	5.8	23.4	8.0
Greece	5.2	8.5	5.0	15.6	0.0	0.0	0.0	0.0
Ireland	1.0	1.0	1.9	3.0	2.1	6.2	1.7	2.2
Italy	22.6	42.1	23.1	35.2	23.4	38.3	20.8	40.3
Netherlands	5.5	1.8	5.3	2.1	6.1	1.4	6.5	2.6
Portugal	2.2	2.7	2.4	2.8	2.1	7.4	1.5	2.6
Spain	7.8	12.4	9.5	14.9	9.6	21.1	10.7	14.0
$\mathcal{R}(x)$	0.3		2.8		8.3		10.7	

Figure: Dynamics of the risk contributions (EGBI portfolio)

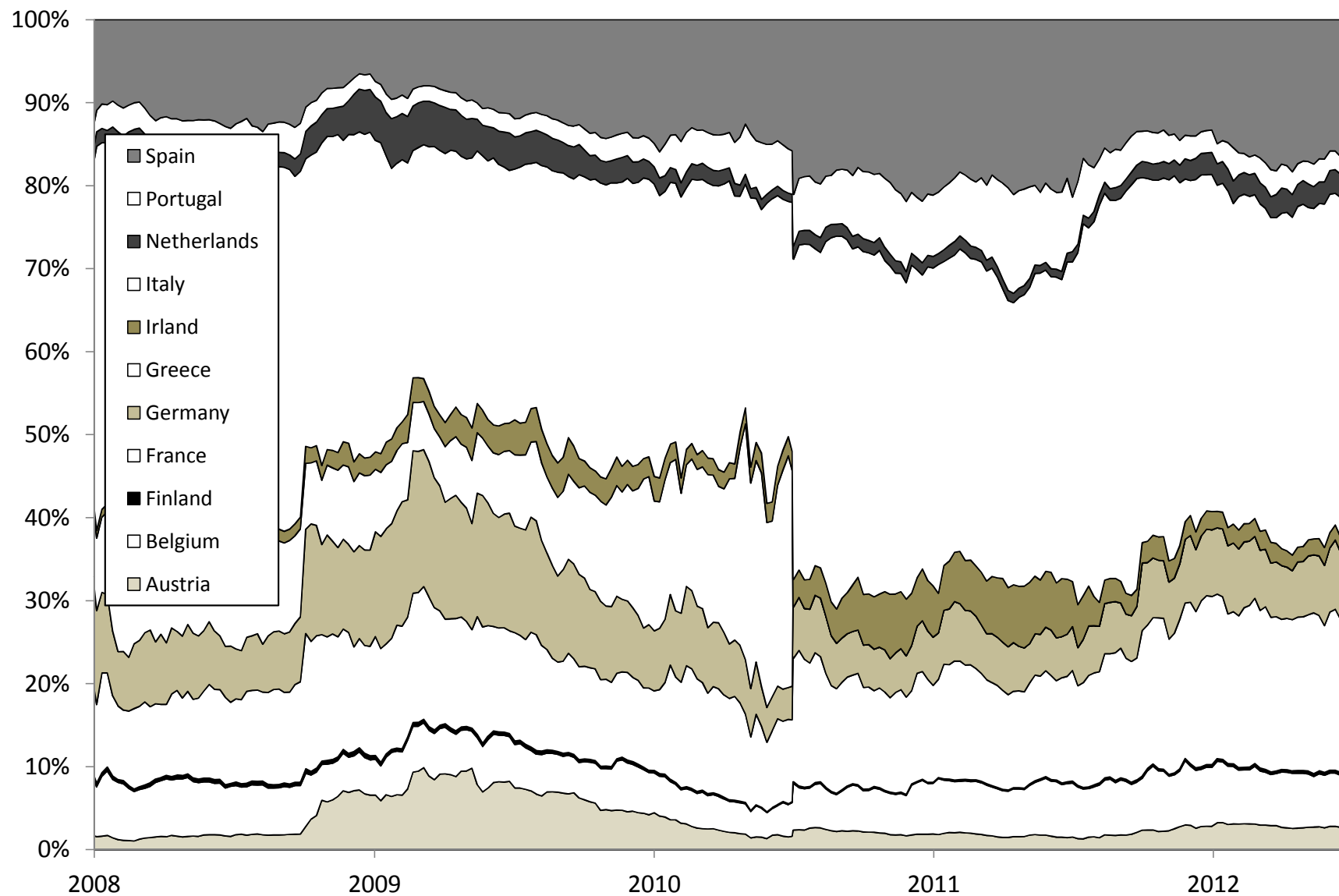


Table: Weights and risk contribution of the DEBT-WB indexation (in %)

Country	Jan. 08		Jan. 10		Jan. 11		Jan. 12	
	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*
Austria	3.9	1.7	3.8	4.5	3.9	1.6	4.2	2.6
Belgium	6.3	6.7	6.1	4.8	6.0	5.0	6.1	7.1
Finland	1.3	0.4	1.2	0.3	1.2	0.1	1.5	0.3
France	19.9	10.4	20.2	9.6	21.2	9.4	23.3	18.4
Germany	24.3	12.3	21.6	7.2	21.9	4.7	23.2	7.5
Greece	5.2	8.5	5.0	15.6	4.3	19.2	1.0	5.3
Ireland	1.0	1.0	1.9	3.0	2.0	5.1	1.7	2.1
Italy	22.6	42.1	23.1	35.2	22.4	30.6	20.6	38.3
Netherlands	5.5	1.8	5.3	2.1	5.9	1.1	6.5	2.5
Portugal	2.2	2.7	2.4	2.8	2.0	6.1	1.5	2.5
Spain	7.8	12.4	9.5	14.9	9.2	17.0	10.6	13.3
$\mathcal{R}(x)$	0.3		2.8		9.7		11.1	

Figure: Dynamics of the risk contributions (DEBT-WB indexation)

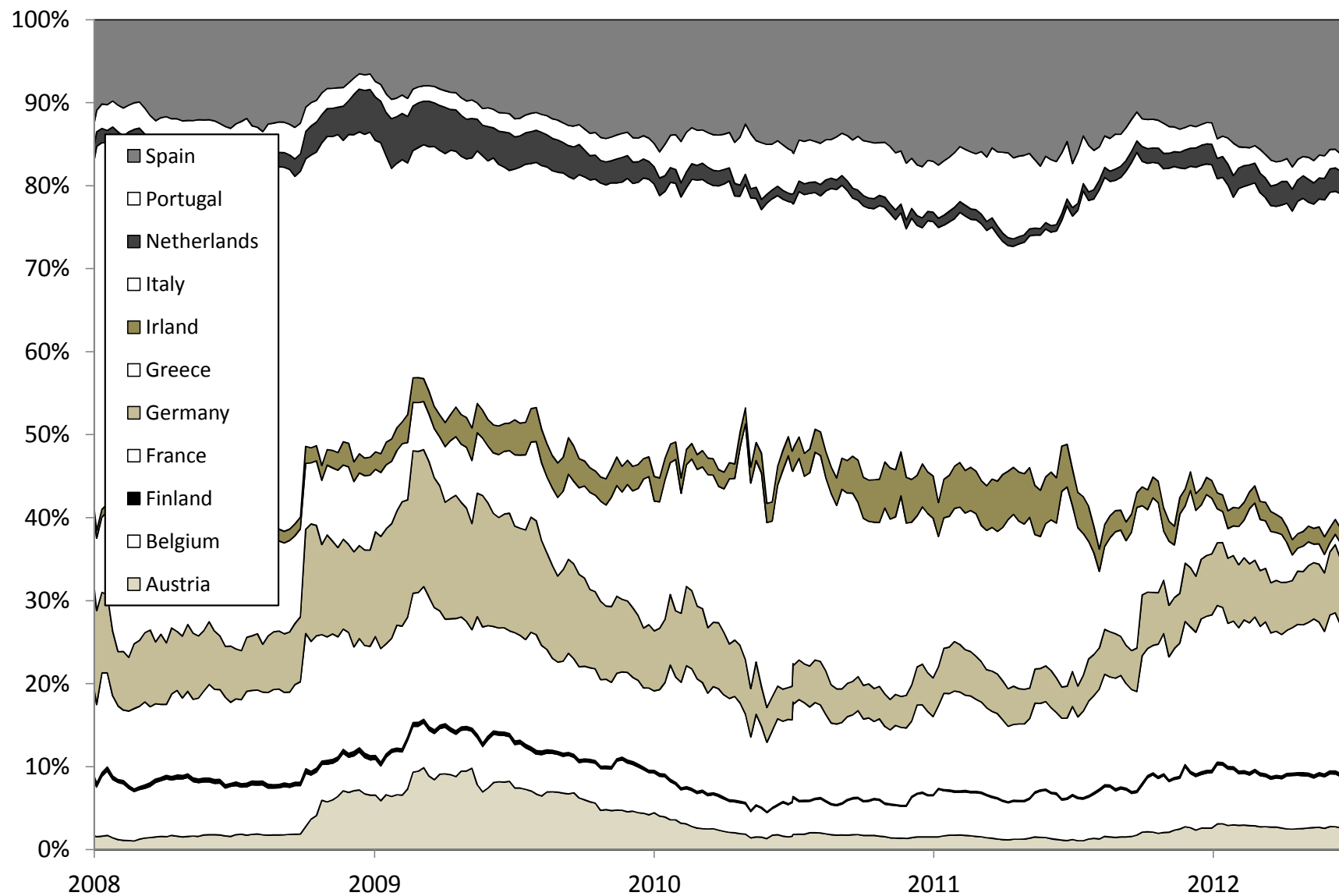


Table: Weights and risk contribution of the GDP-WB indexation (in %)

Country	Jan. 08		Jan. 10		Jan. 11		Jan. 12	
	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*	x_i	RC_i^*
Austria	3.1	1.4	3.1	4.2	3.2	1.5	3.3	1.9
Belgium	3.8	4.3	3.9	3.5	4.0	3.7	4.0	4.3
Finland	2.0	0.7	2.0	0.5	2.1	0.2	2.1	0.4
France	21.3	11.8	21.5	12.1	21.5	11.0	21.7	15.6
Germany	27.4	15.1	27.5	10.8	27.8	6.9	28.1	8.3
Greece	2.6	4.2	2.7	9.1	2.5	12.2	2.3	19.9
Ireland	2.1	2.3	1.8	3.2	1.6	4.7	1.7	2.1
Italy	17.4	32.5	17.2	29.5	17.1	26.7	17.0	29.0
Netherlands	6.5	2.4	6.5	2.9	6.6	1.5	6.5	2.3
Portugal	1.9	2.4	1.9	2.6	1.9	6.5	1.8	3.0
Spain	12.0	22.9	11.9	21.5	11.8	25.2	11.5	13.3
$\mathcal{R}(x)$	0.3		2.5		8.5		11.3	

Figure: Dynamics of the risk contributions (GDP-WB indexation)

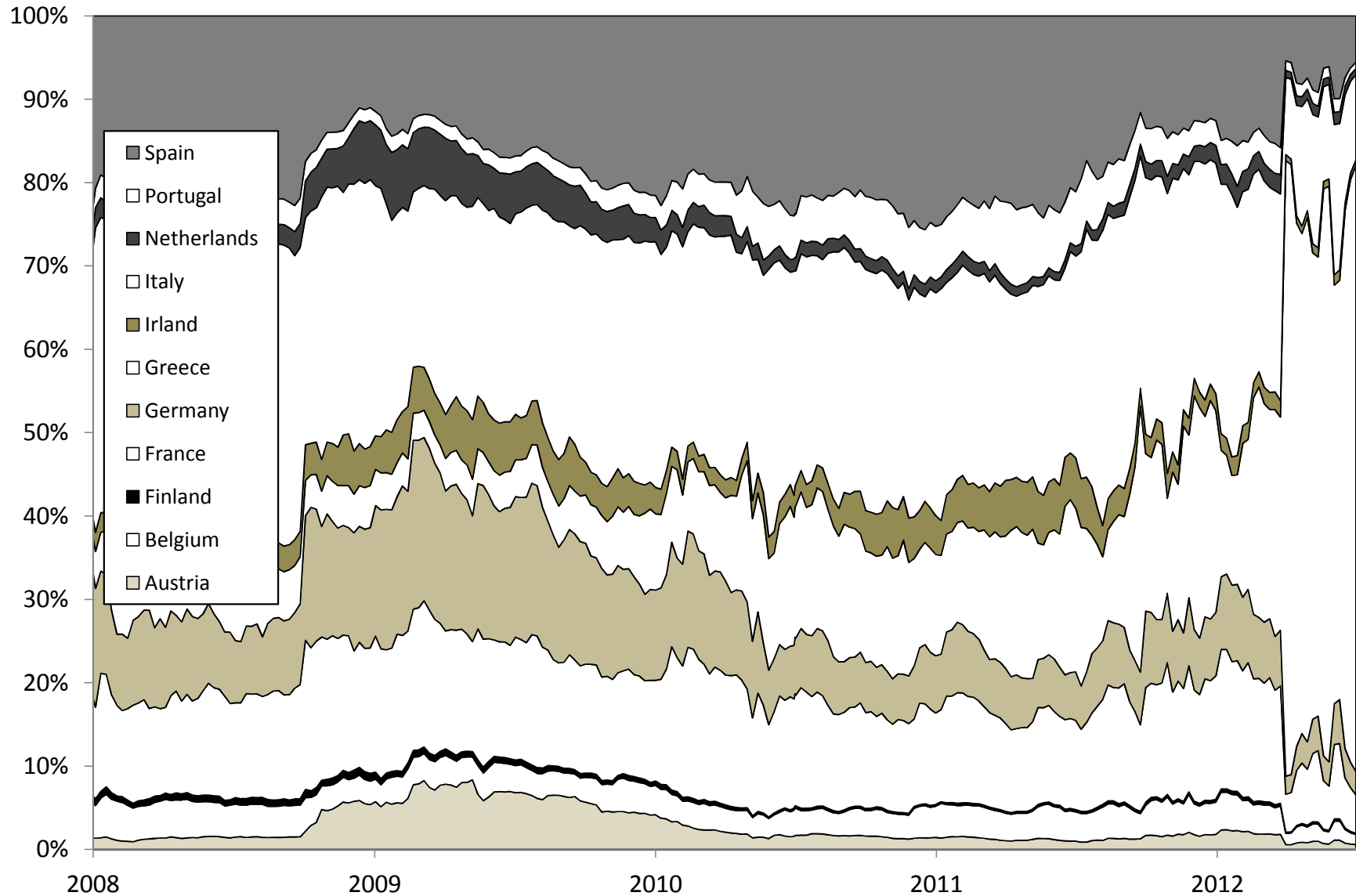


Table: Risk budgets and weights of the DEBT-RB indexation (in %)

Country	Jan. 08		Jan. 10		Jan. 11		Jan. 12	
	b_i	x_i	b_i	x_i	b_i	x_i	b_i	x_i
Austria	3.9	6.3	3.8	2.2	3.9	4.4	4.2	3.9
Belgium	6.3	4.2	6.1	5.1	6.0	3.3	6.1	3.6
Finland	1.3	2.6	1.2	3.1	1.2	5.5	1.5	5.3
France	19.9	26.1	20.2	24.5	21.2	19.8	23.3	19.3
Germany	24.3	31.6	21.6	38.5	21.9	43.4	23.2	42.7
Greece	5.2	2.5	5.0	1.1	4.3	0.5	1.0	0.2
Ireland	1.0	0.7	1.9	0.8	2.0	0.4	1.7	0.8
Italy	22.6	10.0	23.1	10.4	22.4	7.3	20.6	7.5
Netherlands	5.5	10.6	5.3	8.8	5.9	12.8	6.5	11.1
Portugal	2.2	1.4	2.4	1.3	2.0	0.3	1.5	0.5
Spain	7.8	3.9	9.5	4.1	9.2	2.4	10.6	5.1
$\mathcal{R}(x)$	0.2		1.8		4.4		7.3	

Figure: Evolution of the weights (DEBT-RB indexation)

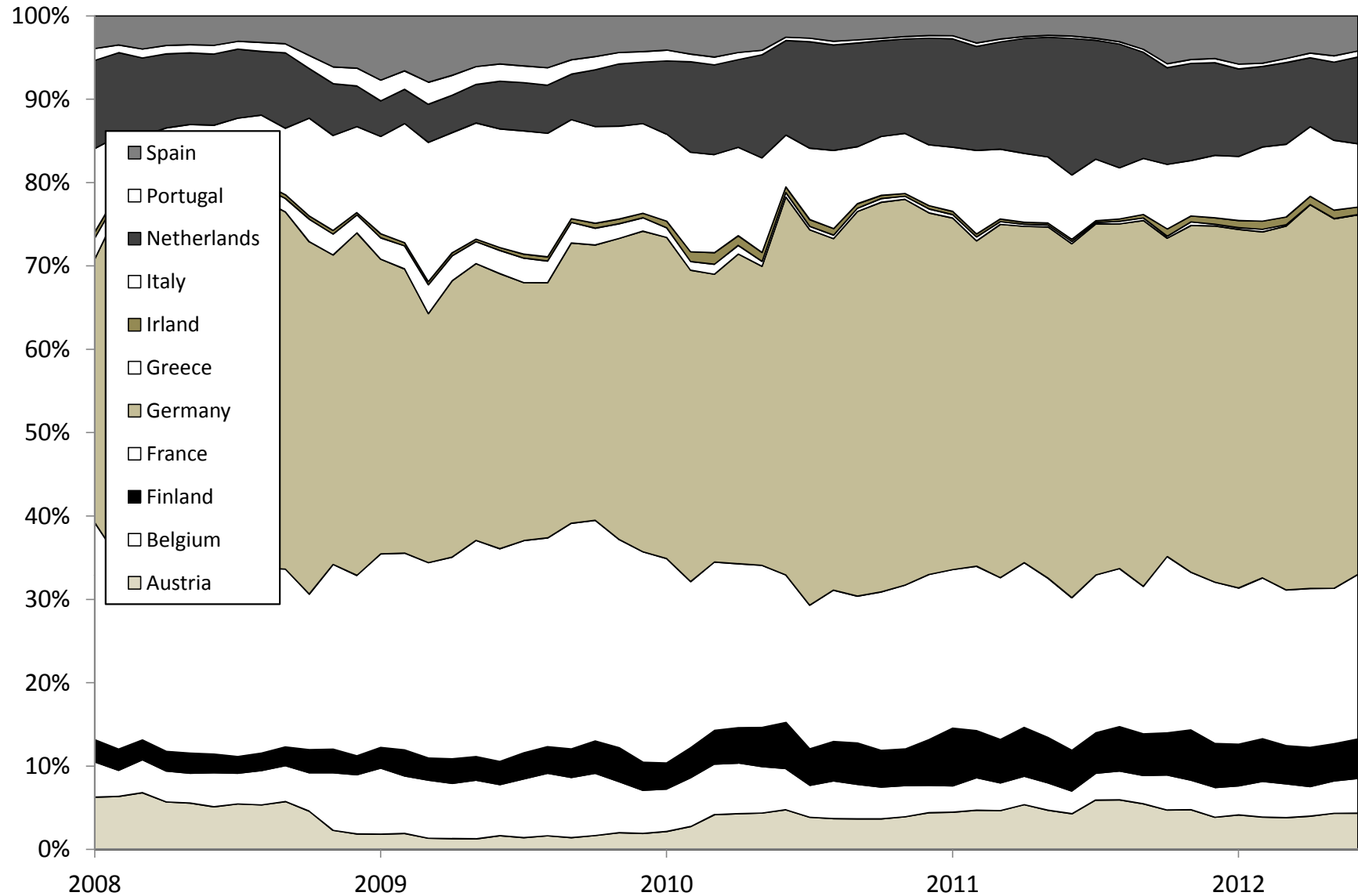


Table: Risk budgets and weights of the GDP-RB indexation (in %)

Country	Jan. 08		Jan. 10		Jan. 11		Jan. 12	
	b_i	x_i	b_i	x_i	b_i	x_i	b_i	x_i
Austria	3.1	4.8	3.1	1.7	3.2	3.2	3.3	2.9
Belgium	3.8	2.5	3.9	3.0	4.0	1.9	4.0	2.2
Finland	2.0	4.0	2.0	4.6	2.1	8.0	2.1	6.6
France	21.3	26.9	21.5	23.3	21.5	17.8	21.7	16.8
Germany	27.4	33.6	27.5	43.6	27.8	48.0	28.1	47.8
Greece	2.6	1.3	2.7	0.6	2.5	0.3	2.3	0.3
Ireland	2.1	1.4	1.8	0.7	1.6	0.3	1.7	0.8
Italy	17.4	7.7	17.2	7.3	17.1	5.1	17.0	5.9
Netherlands	6.5	11.2	6.5	9.7	6.6	12.6	6.5	10.5
Portugal	1.9	1.2	1.9	1.0	1.9	0.3	1.8	0.6
Spain	12.0	5.4	11.9	4.7	11.8	2.7	11.5	5.5
$\mathcal{R}(x)$	0.2		1.7		3.9		6.8	

Figure: Evolution of the weights (GDP-RB indexation)

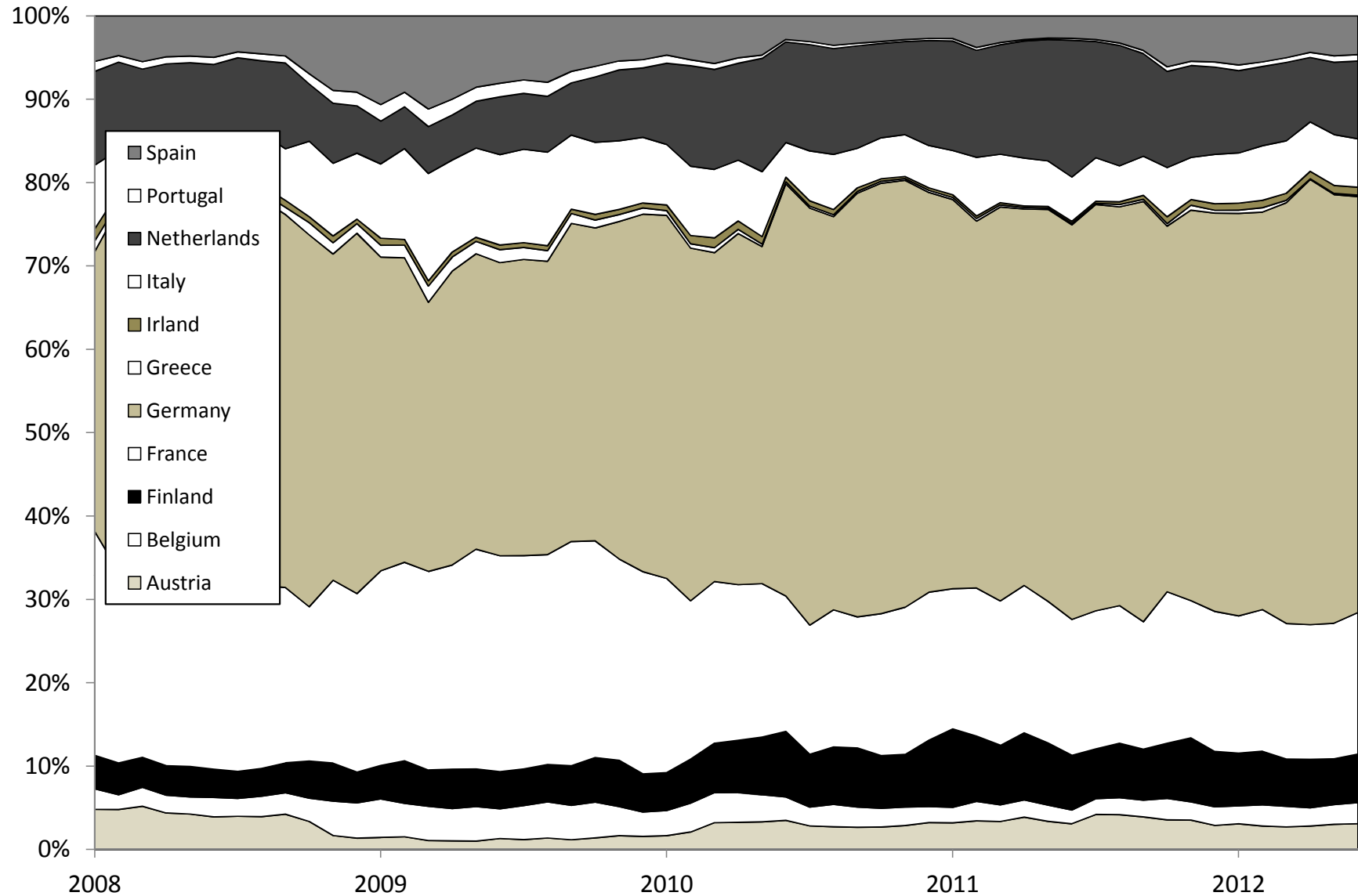


Table: Main statistics of bond indexations (Jan. 2008 – Jun. 2012)

Statistics	WB			RB	
	EGBI	DEBT	GDP	DEBT	GDP
$\mu(x)$	4.81	4.52	4.75	6.26	6.43
$\sigma(x)$	4.60	4.63	4.49	4.44	4.49
$SR(x r)$	0.70	0.63	0.70	1.05	1.07
$\mathcal{MDD}(x)$	-6.80	-7.94	-6.90	-6.29	-6.34
$\sigma(x b)$	0.00	0.35	0.67	2.18	2.48
$IR(x b)$		-0.79	-0.08	0.63	0.62
$\rho(x b)$		99.71	98.93	88.47	85.10
$\beta(x b)$		1.00	0.96	0.85	0.83

Figure: Dynamics of the credit risk measure (in %)

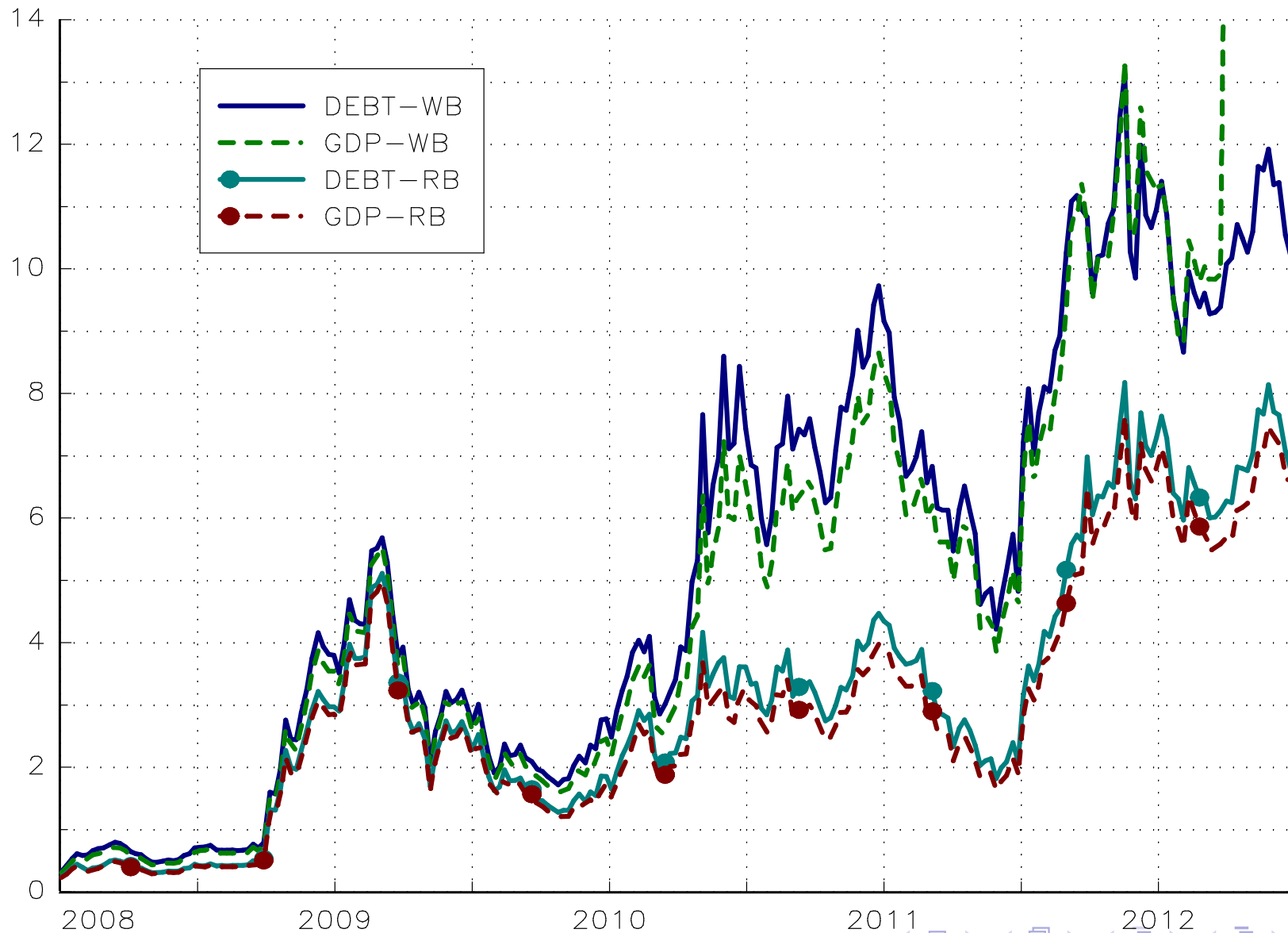


Figure: Evolution of the GIIPS risk contribution (in %)

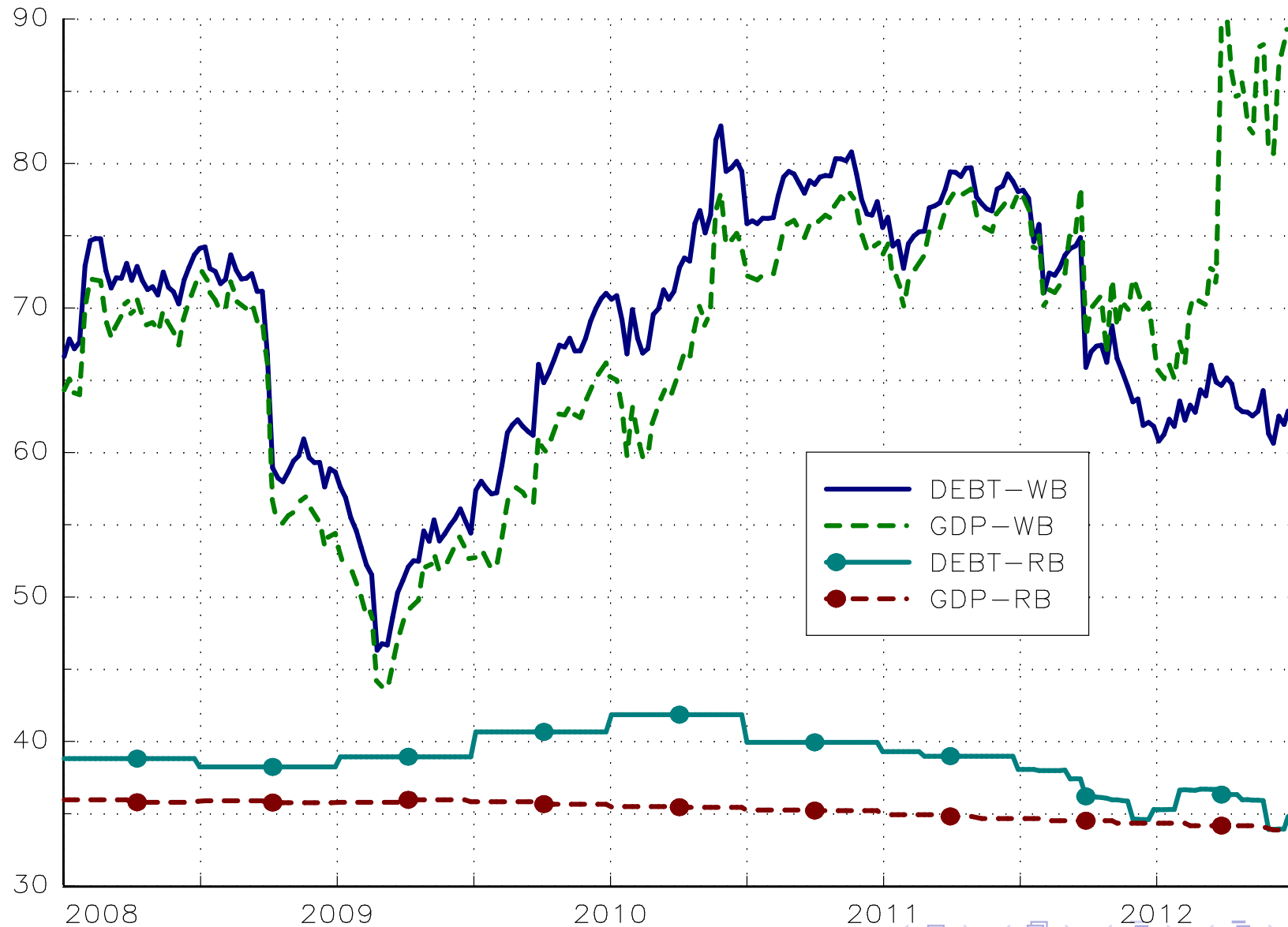


Figure: Simulated performance of the bond indexations

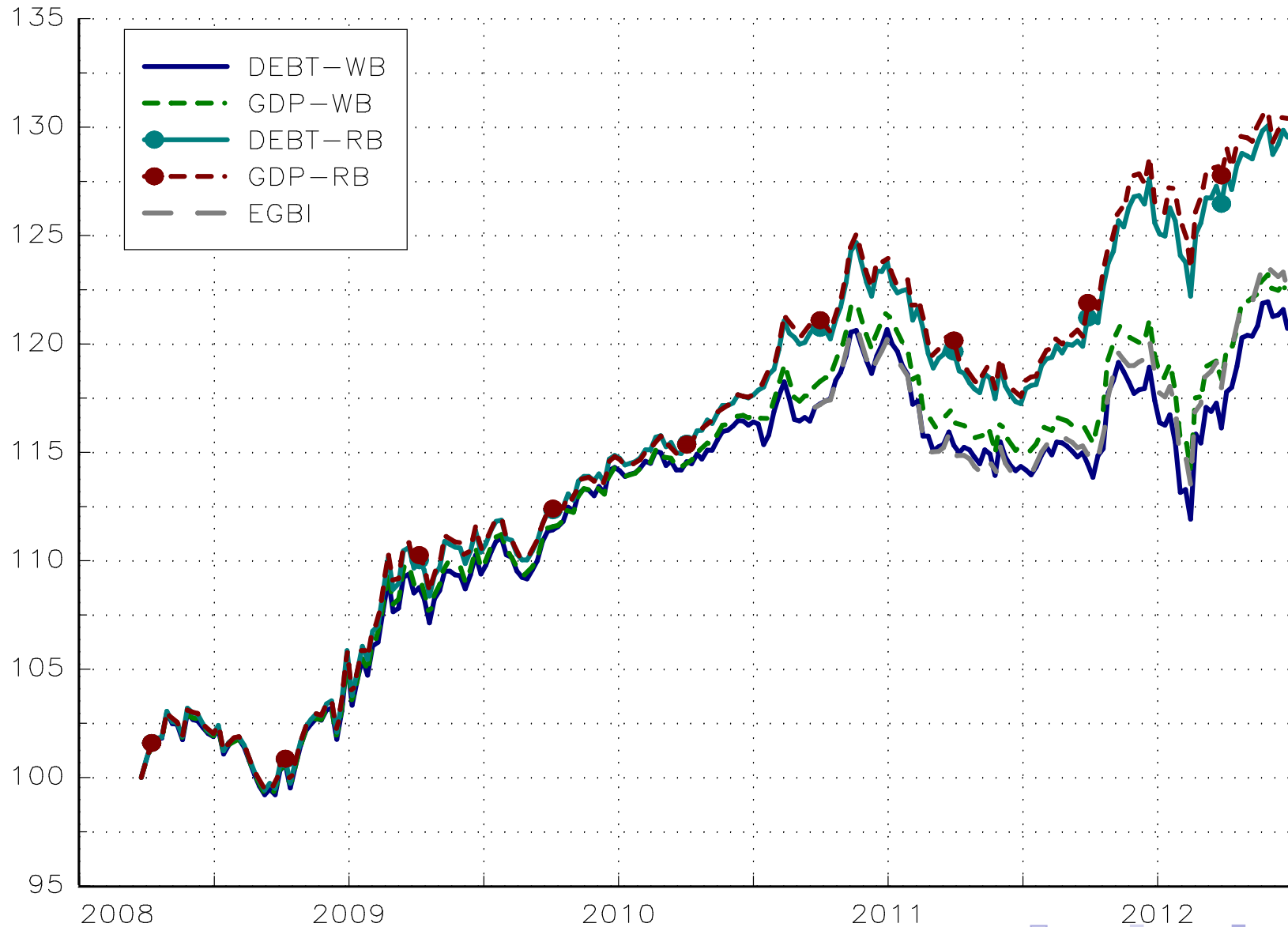


Figure: Comparing the dynamic allocation for four countries

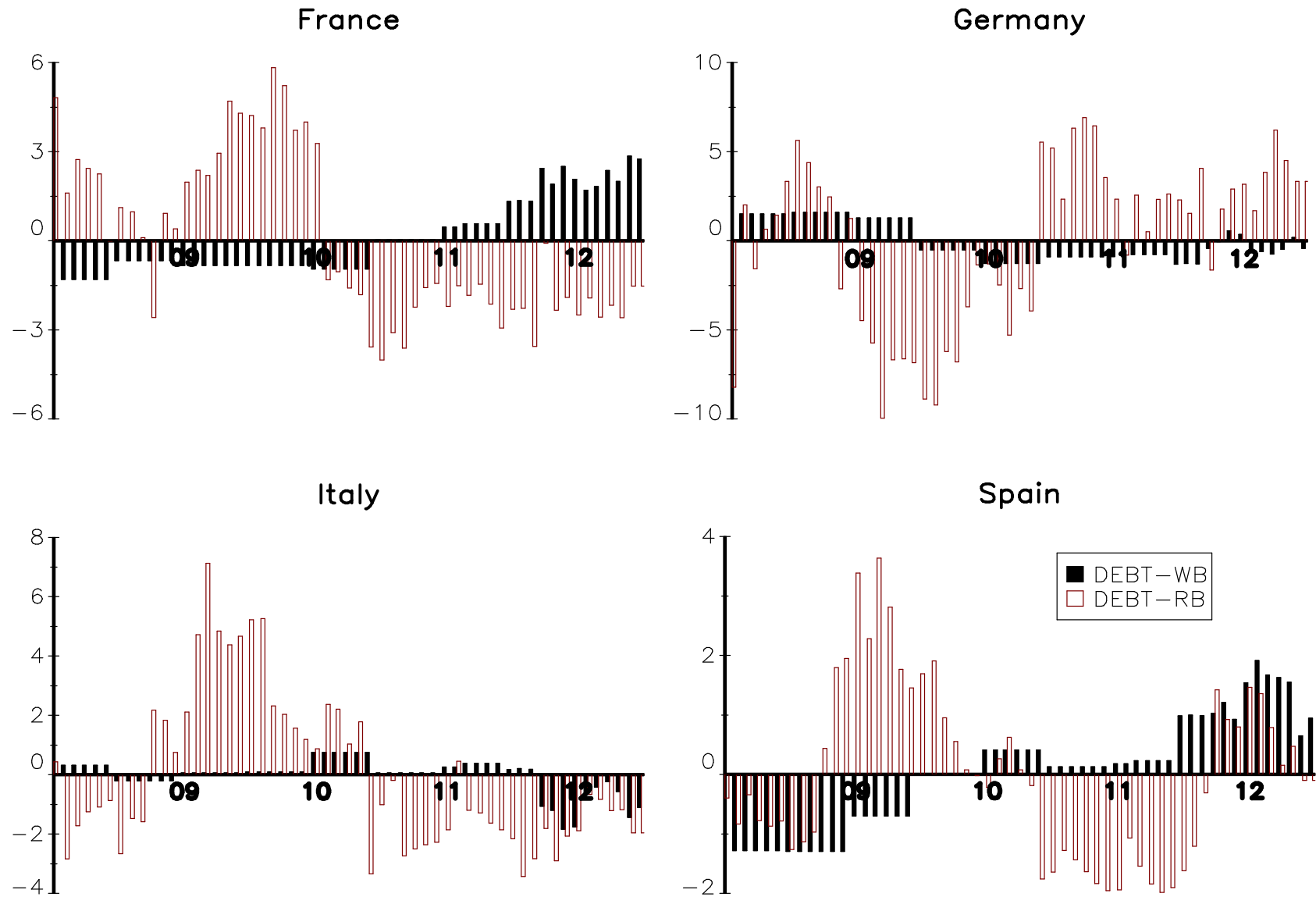


Figure: Comparison with active management

