Managing Sovereign Credit Risk in Bond Portfolios using the Risk Budgeting Approach[1]

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[1] The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.

Outline

Some issues on the asset management industry
(after the 2008 financial crisis)

Transition in the investment industry

- Concentration of assets under management
- Risk aversion of large institutional investors becomes higher
- Funding ratios are smaller (weakness of retirement systems)
- Pressure for more transparency and robustness
- Risk management as important as performance management

Transition in the passive indexation

- Robustness of market-cap indexation?
- Equity indexes = trend-following strategy
- Lack of portfolio construction rules $\Rightarrow$ Risk concentration
- Alternative-weighted indexation = passive indexes where the weights are not based on market capitalization

Some issues on the asset management industry
(after the 2008 financial crisis)

Emergence of heuristic solutions

- Strong criticism of Markowitz optimization
- 2011: success of minimum-variance, erc, mdp/msr and risk parity strategies $\Rightarrow$ These portfolio constructions only depend on risks, not on expected returns
- Special cases of the risk budgeting approach

$\Rightarrow$ Risk budgeting allocation = widely used by market practitioners (multi-asset classes, strategic asset allocation, equity portfolios)

Objective of this talk

- What could we say about risk budgeting allocation from a theoretical point of view?
- How to adapt risk budgeting techniques to manage bond portfolios?

Weight budgeting versus risk budgeting

Let $x = (x_1, \ldots, x_n)$ be the weights of n assets in the portfolio. Let $\mathscr{R}(x_1, \ldots, x_n)$ be a coherent and convex risk measure. We have:

$$\begin{aligned} \mathscr{R}(x_1, \ldots, x_n) &= \sum_{i=1}^{n} x_i \cdot \frac{\partial \mathscr{R}(x_1, \ldots, x_n)}{\partial x_i} \\ &= \sum_{i=1}^{n} \mathrm{RC}_i(x_1, \ldots, x_n) \end{aligned}$$

Let $b = (b_1, \ldots, b_n)$ be a vector of budgets such that $b_i \geq 0$ and $\sum_{i=1}^{n} b_i = 1$. We consider two allocation schemes:

1. Weight budgeting (WB)

$$x_i = b_i$$

2. Risk budgeting[2] (RB)

$$\mathrm{RC}_i = b_i \cdot \mathscr{R}(x_1, \ldots, x_n)$$

[2] The ERC portfolio is a special case when $b_i = 1/n$.

Application to the volatility risk measure

Let Σ be the covariance matrix of the assets returns. We assume that the risk measure $\mathscr{R}(x)$ is the volatility of the portfolio $\sigma(x) = \sqrt{x^\top \Sigma x}$. We have:

$$
\begin{aligned}
\frac{\partial \mathscr{R}(x)}{\partial x} &= \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} \\
\mathrm{RC}_i(x_1, \ldots, x_n) &= x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \\
\sum_{i=1}^n \mathrm{RC}_i(x_1, \ldots, x_n) &= \sum_{i=1}^n x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} = x^\top \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} = \sigma(x)
\end{aligned}
$$

The risk budgeting portfolio is defined by this system of equations:

$$
\left\{
\begin{array}{l}
x_i \cdot (\Sigma x)_i = b_i \cdot \left(x^\top \Sigma x\right) \\
x_i \geq 0 \\
\sum_{i=1}^n x_i = 1
\end{array}
\right.
$$

An example

Illustration

- 3 assets
- Volatilities are respectively 20%, 30% and 15%
- Correlations are set to 60% between the 1st asset and the 2nd asset and 10% between the first two assets and the 3rd asset

The marginal risk for the first asset is:

$$\lim_{\varepsilon \to 0} \frac{\sigma(x_1+\varepsilon, x_2, x_3) - \sigma(x_1, x_2, x_3)}{(x_1+\varepsilon) - x_1}$$

If $\varepsilon = 1\%$, we have:

$$\frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq \frac{16.72\% - 16.54\%}{1\%} = 18.01\%$$

Weight budgeting (or traditional) approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	**50.00%**	17.99%	9.00%	54.40%
2	**25.00%**	25.17%	6.29%	38.06%
2	**25.00%**	4.99%	1.25%	7.54%
Volatility			16.54%	

Risk budgeting approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	41.62%	16.84%	7.01%	**50.00%**
2	15.79%	22.19%	3.51%	**25.00%**
2	42.58%	8.23%	3.51%	**25.00%**
Volatility			14.02%	

ERC approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	30.41%	15.15%	4.61%	**33.33%**
2	20.28%	22.73%	4.61%	**33.33%**
2	49.31%	9.35%	4.61%	**33.33%**
Volatility			13.82%	

Some analytical solutions

- The case of uniform correlation[3] $\rho_{i,j} = \rho$
  - ERC portfolio ($b_i = 1/n$)

$$x_i(\rho) = \frac{\sigma_i^{-1}}{\sum_{j=1}^{n} \sigma_j^{-1}}$$

  - RB portfolio

$$x_i\left(-\frac{1}{n-1}\right) = \frac{\sigma_i^{-1}}{\sum_{j=1}^{n} \sigma_j^{-1}}, \; x_i(0) = \frac{\sqrt{b_i}\sigma_i^{-1}}{\sum_{j=1}^{n} \sqrt{b_j}\sigma_j^{-1}}, \; x_i(1) = \frac{b_i\sigma_i^{-1}}{\sum_{j=1}^{n} b_j\sigma_j^{-1}}$$

- The general case

$$x_i = \frac{b_i\beta_i^{-1}}{\sum_{j=1}^{n} b_j\beta_j^{-1}}$$

where β_i is the beta of the asset i with respect to the RB portfolio.

[3] The solution is noted $x_i(\rho)$.

The RB portfolio is a minimum variance (MV) portfolio subject to a constraint of weight diversification

Let us consider the following minimum variance optimization problem:

$$x^{\star}(c) = \arg\min \sqrt{x^{\top}\Sigma x} \quad \text{u.c.} \quad \begin{cases} \sum_{i=1}^{n} b_i \ln x_i \geq c \\ \mathbf{1}^{\top} x = 1 \\ x \geq \mathbf{0} \end{cases}$$

- if $c = c^{-} = -\infty$, $x^{\star}(c^{-}) = x_{\mathrm{MV}}$ (no weight diversification)
- if $c = c^{+} = \sum_{i=1}^{n} b_i \ln b_i$, $x^{\star}(c^{+}) = x_{\mathrm{WB}}$ (no variance minimization)
- $\exists\, c^0 : x^{\star}(c^0) = x_{\mathrm{RB}}$ (variance minimization and weight diversification)

$\Longrightarrow$ if $b_i = 1/n$, $x_{\mathrm{RB}} = x_{\mathrm{ERC}}$ (variance minimization, weight diversification and perfect risk diversification[4])

[4] The Gini coefficient of the risk measure is then equal to 0.

Other properties

- Existence and uniqueness
  - If $b_i > 0$, the solution exists and is unique.
  - If $b_i \geq 0$, there may be several solutions[5].
- The RB portfolio is a portfolio located between the minimum variance and the weight budgeting portfolios:

$$\sigma_{\mathrm{MV}} \leq \sigma_{\mathrm{RB}} \leq \sigma_{\mathrm{WB}}$$

- If the RB portfolio is optimal (in the Markowitz sense), the ex-ante performance contributions are equal to the risk budgets.
- The EW, MV, MDP and ERC portfolios could be interpreted as RB portfolios[6].

[5] see Bruder and Roncalli (2012) for a full characterization of the solutions.

[6] For example, the equally-weighted (EW) portfolio could be viewed as a risk budgeting portfolio when the risk budget is proportional to the beta of the asset.

Time to rethink the bond portfolios management

Two main problems:

1. Benchmarks = debt-weighted indexation (the weights are based on the notional amount of the debt)
2. Fund management driven by the search of yield with little consideration for risk (carry position $\neq$ arbitrage position)

$\Rightarrow$ Time to rethink bond indexes? (Toloui, 2010)

For the application, we consider the euro government bond portfolios. The benchmark is the Citigroup EGBI index[7].

[7] This index is very close to the EuroMTS index.

Bond indexation schemes

Debt weighting

It is defined by[a]:

$$w_i = \frac{\mathrm{DEBT}_i}{\sum_{i=1}^{n} \mathrm{DEBT}_i}$$

[a]Two forms of debt-weighting are considered : DEBT (with the 11 countries) and DEBT* (without Greece after July 2010). This last one corresponds to the weighting scheme of the EGBI index.

Alternative weighting

1. Fundamental indexation
   The GDP-weighting is defined by:
   $$w_i = \frac{\mathrm{GDP}_i}{\sum_{i=1}^{n} \mathrm{GDP}_i}$$
2. Risk-based indexation
   The DEBT-RB and GDP-RB weightings are defined by:
   $$\begin{aligned} b_i &= \frac{\mathrm{DEBT}_i}{\sum_{i=1}^{n} \mathrm{DEBT}_i} \\ b_i &= \frac{\mathrm{GDP}_i}{\sum_{i=1}^{n} \mathrm{GDP}_i} \end{aligned}$$

Choosing the right measure of credit risk

- Volatility of price returns $\neq$ a good measure of credit risk
- Correlation of price returns $\neq$ a good measure of contagion
- A better measure is the yield spread, but its computation can be difficult as it needs to first define a reference risk-free rate.

$\Rightarrow$ One of the best measure is the CDS spread (it does not depend on the currency, the yield curve or the duration).

The SABR CDS model

Let $S_i(t)$ be the spread of the i^{th} issuer. We have:

$$\mathrm{d}S_i(t) = \sigma_i^S \cdot S_i(t)^{\beta_i} \cdot \mathrm{d}W_i(t)$$

Moreover, we assume that the correlation between the brownian motions $W_i(t)$ and $W_j(t)$ is $\Gamma_{i,j}$.

Calibration of the β_i parameter

We assume that we observe spreads at some given known dates $t_0, \ldots, t_n$. Let $S_{i,j}$ be the observed spread for the i^{th} country at date t_j. The log-likelihood function for the i^{th} country is:

$$\ell = -\frac{n}{2}\ln 2\pi - n \ln \sigma_i^S - \frac{1}{2}\sum_{j=1}^{n} \ln\left(t_j - t_{j-1}\right) - \beta_i \sum_{j=1}^{n} \ln S_{i,j-1} - \frac{1}{2}\sum_{j=1}^{n} \frac{\left(S_{i,j} - S_{i,j-1}\right)^2}{\left(\sigma_i^S S_{i,j-1}^{\beta_i}\right)^2 \left(t_j - t_{j-1}\right)}$$

Figure: Results for the period January 2008-August 2011

Country	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES	Average
estimate	0.996	1.017	0.816	0.786	0.899	1.070	0.836	1.157	0.793	1.013	1.148	0.957
std-dev.	1.10%	2.00%	1.60%	1.60%	2.00%	1.10%	0.70%	1.70%	0.90%	1.10%	2.10%	1.45%

$\Rightarrow$ We assume that $\beta_i = 1$ (ML estimation is then easy to compute).

Computing the credit risk measure of a bond portfolio

Let $w = (w_1, \ldots, w_n)$ be the weights of bonds in the portfolio. The risk measure is[8]:

$$\mathscr{R}(x) = \sqrt{w^\top \Sigma w} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j \Sigma_{i,j}}$$

with $\Sigma_{i,j} = \Gamma_{i,j} \cdot \sigma_i^B \cdot \sigma_j^B$ and $\sigma_i^B = D_i \cdot \sigma_i^S \cdot S_i(t)$, where D_i is the duration of the bond i, σ_i^S is the CDS volatility of the corresponding issuer, $S_i(t)$ is the CDS level and $\Gamma_{i,j}$ is the correlation between the CDS relative variations of issuers corresponding to the bonds i and j.

$\mathscr{R}(w)$ is the volatility of the CDS basket which would perfectly hedge the credit risk of the bond portfolio.

$\mathscr{R}(w)$ depends on 3 "CDS" parameters $S_i(t)$, σ_i^S and $\Gamma_{i,j}$ and two "portfolio" parameters w_i and D_i.

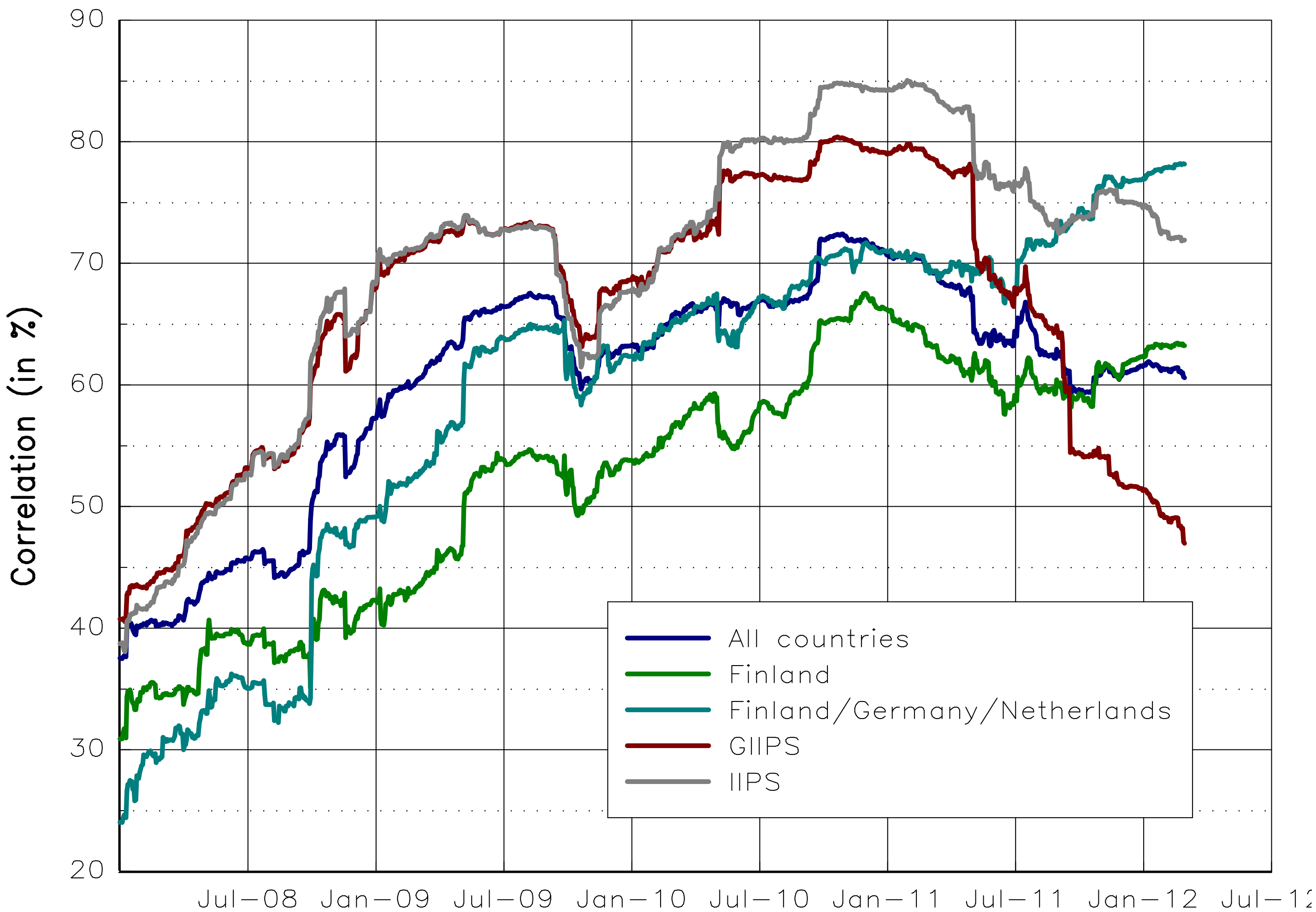
[8] We have $\mathrm{d}\ln B_t(D_i) = -D_i \cdot \mathrm{d}R(t) - D_i \cdot \mathrm{d}S_i(t)$ with $B_t(D_i)$ the zero-coupon of maturity D_i and $R(t)$ the "risk-free" interest rate. It comes that $\sigma_i^B = D_i \cdot \sigma_i^S \cdot S_i(t)$.

Statistics as of March 1st, 2012

Table: CDS levels $S_i(t)$, volatilities σ_i^S and correlations $\Gamma_{i,j}$

Country	Spread	Volatility	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	158	73.5%	100%										
Belgium	223	73.1%	80%	100%									
Finland	64	68.8%	75%	75%	100%								
France	166	70.9%	87%	85%	78%	100%							
Germany	76	66.0%	82%	78%	73%	86%	100%						
Greece	8,871	163.4%	9%	12%	9%	6%	6%	100%					
Ireland	581	51.9%	62%	72%	57%	67%	66%	16%	100%				
Italy	356	74.2%	74%	86%	72%	80%	73%	11%	71%	100%			
Netherlands	94	67.7%	79%	79%	78%	85%	83%	6%	64%	74%	100%		
Portugal	1,175	56.1%	55%	66%	50%	60%	57%	15%	79%	67%	54%	100%	
Spain	356	72.5%	74%	80%	66%	75%	69%	9%	69%	81%	66%	64%	100%

Evolution of the correlation matrix



Defining the risk contribution

Our credit risk measure $\mathscr{R}(w) = \sqrt{w^\top \Sigma w}$ is a convex risk measure. It means that:

$$\begin{aligned} \mathscr{R}(w_1, \ldots, w_n) &= \sum_{i=1}^{n} w_i \cdot \frac{\partial\, \mathscr{R}(w_1, \ldots, w_m)}{\partial\, w_i} \\ &= \sum_{i=1}^{n} RC_i \end{aligned}$$

We can then break the risk measure down into n individual sources of risk.

The risk contribution RC_i is an increasing function of the parameters D_i, $S_i(t)$ and σ_i^S.

Some results for the EGBI index

Figure: EGBI weights and risk contributions

Country	July-08		July-09		July-10		July-11		March-12	
	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC
Austria	4.1%	1.7%	3.6%	7.7%	4.1%	2.3%	4.3%	1.5%	4.2%	3.0%
Belgium	6.2%	6.1%	6.5%	5.1%	6.3%	5.7%	6.4%	6.5%	6.3%	6.6%
Finland	1.2%	0.4%	1.3%	0.5%	1.3%	0.2%	1.6%	0.2%	1.6%	0.3%
France	20.5%	9.8%	20.4%	13.2%	22.2%	15.1%	23.1%	13.3%	23.2%	19.0%
Germany	24.4%	6.1%	22.3%	13.0%	22.9%	6.0%	22.1%	5.3%	22.4%	7.3%
Greece	4.9%	11.4%	5.4%	8.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ireland	1.0%	1.3%	1.5%	4.3%	2.1%	3.3%	1.4%	5.4%	1.7%	2.3%
Italy	22.1%	45.2%	22.4%	29.5%	23.4%	38.7%	23.1%	38.5%	22.1%	39.7%
Netherlands	5.3%	1.7%	5.3%	4.1%	6.1%	1.6%	6.2%	1.2%	6.2%	2.6%
Portugal	2.4%	3.9%	2.3%	2.3%	2.1%	6.3%	1.6%	6.6%	1.4%	3.0%
Spain	7.8%	12.4%	9.1%	11.8%	9.6%	20.9%	10.3%	21.3%	10.8%	16.2%
Sovereign Risk Measure	0.70%		2.59%		6.12%		4.02%		8.62%	

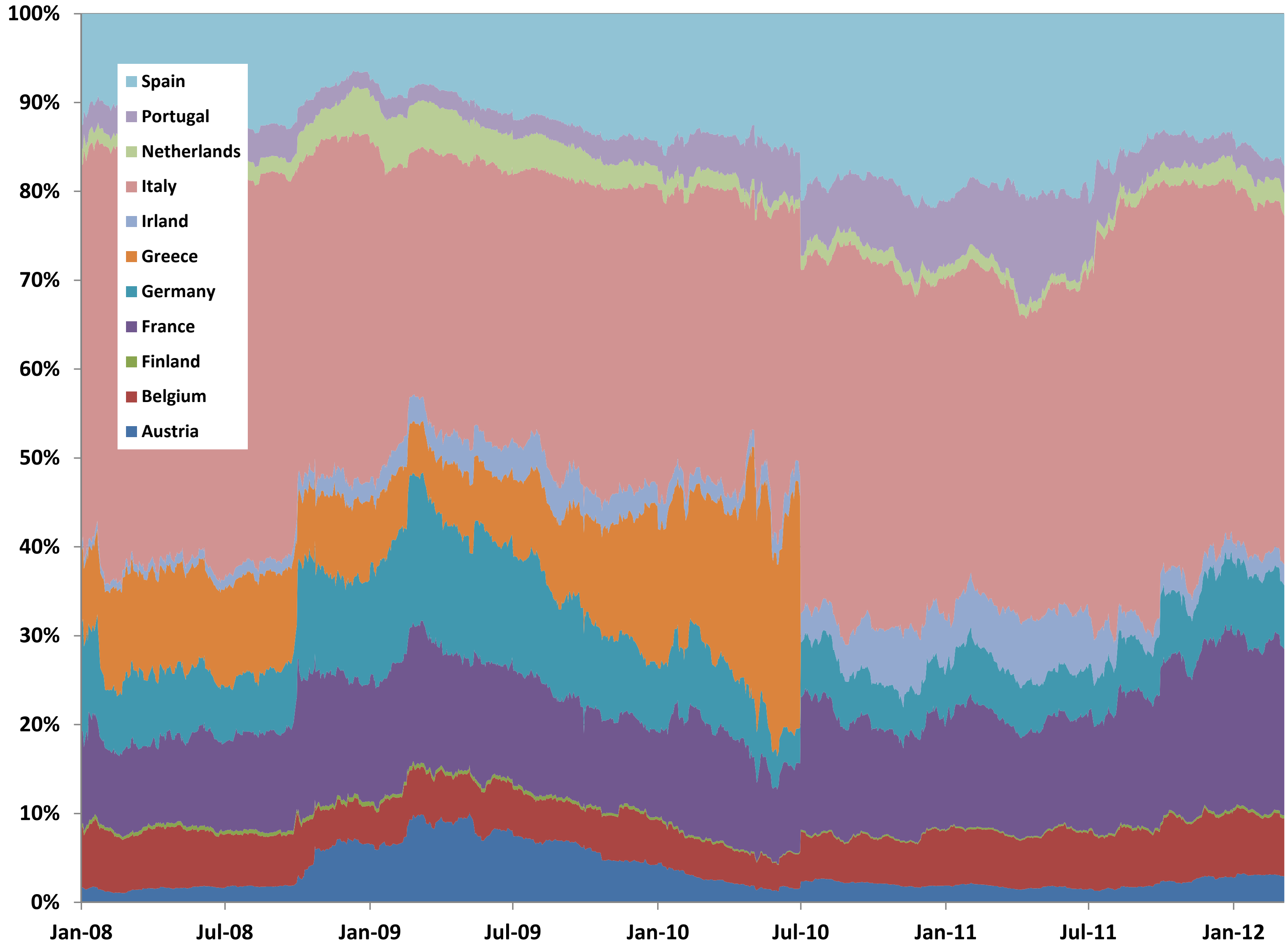
⇒ Small changes in weights but large changes in risk contributions.

⇒ The sovereign credit risk measure has highly increased (the largest value 12.5% is obtained in November 25$^{\text{th}}$ 2011).

⇒ If we think that the EGBI portfolio is optimal, we expect that 60% of the performance will come from Italy and France.

Some results for the EGBI index

Evolution of the risk contributions



GDP indexation

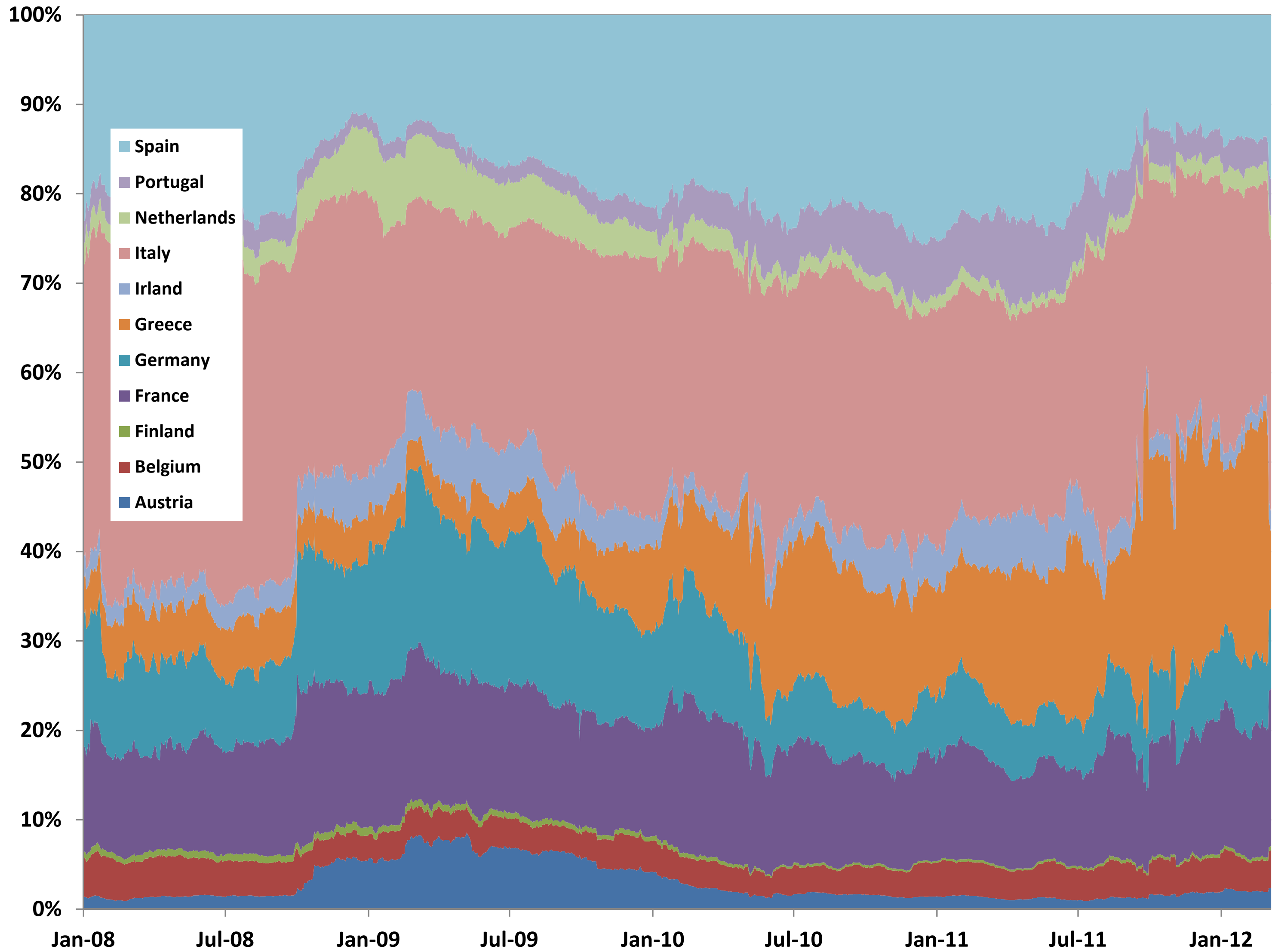
Figure: Weights and risk contributions of the GDP indexation

Country	July-08		July-09		July-10		July-11		March-12	
	GDP	RC	GDP	RC	GDP	RC	GDP	RC	GDP	RC
Austria	3.1%	1.4%	3.1%	7.0%	3.1%	1.7%	3.2%	1.0%	3.4%	2.0%
Belgium	3.8%	4.0%	3.8%	3.2%	3.9%	3.3%	4.0%	3.5%	4.0%	3.4%
Finland	2.0%	0.8%	1.9%	0.7%	2.0%	0.3%	2.1%	0.3%	2.1%	0.4%
France	21.2%	11.2%	21.5%	14.9%	21.4%	13.4%	21.5%	10.6%	21.7%	14.0%
Germany	27.4%	7.6%	27.2%	17.0%	27.7%	6.7%	27.9%	5.8%	27.8%	7.2%
Greece	2.6%	6.2%	2.7%	4.4%	2.6%	15.7%	2.4%	19.8%	2.4%	26.9%
Ireland	2.0%	3.0%	1.9%	5.6%	1.8%	2.6%	1.7%	5.9%	1.7%	1.9%
Italy	17.4%	37.5%	17.3%	23.5%	17.2%	25.8%	17.0%	23.9%	17.1%	24.6%
Netherlands	6.5%	2.5%	6.5%	5.3%	6.5%	1.6%	6.6%	1.2%	6.5%	2.1%
Portugal	1.9%	3.3%	1.9%	2.0%	1.9%	5.3%	1.9%	6.7%	1.9%	3.4%
Spain	12.0%	22.6%	12.0%	16.5%	11.8%	23.7%	11.8%	21.4%	11.6%	14.1%
Sovereign Risk Measure	0.64%		2.47%		6.59%		4.56%		9.41%	

$\Rightarrow$ RC of Debt and GDP indexations are different, but sovereign credit risk measures are similar.

GDP indexation

Evolution of the risk contributions



GDP-RB indexation

Figure: Weights and risk contributions of the GDP-RB indexation

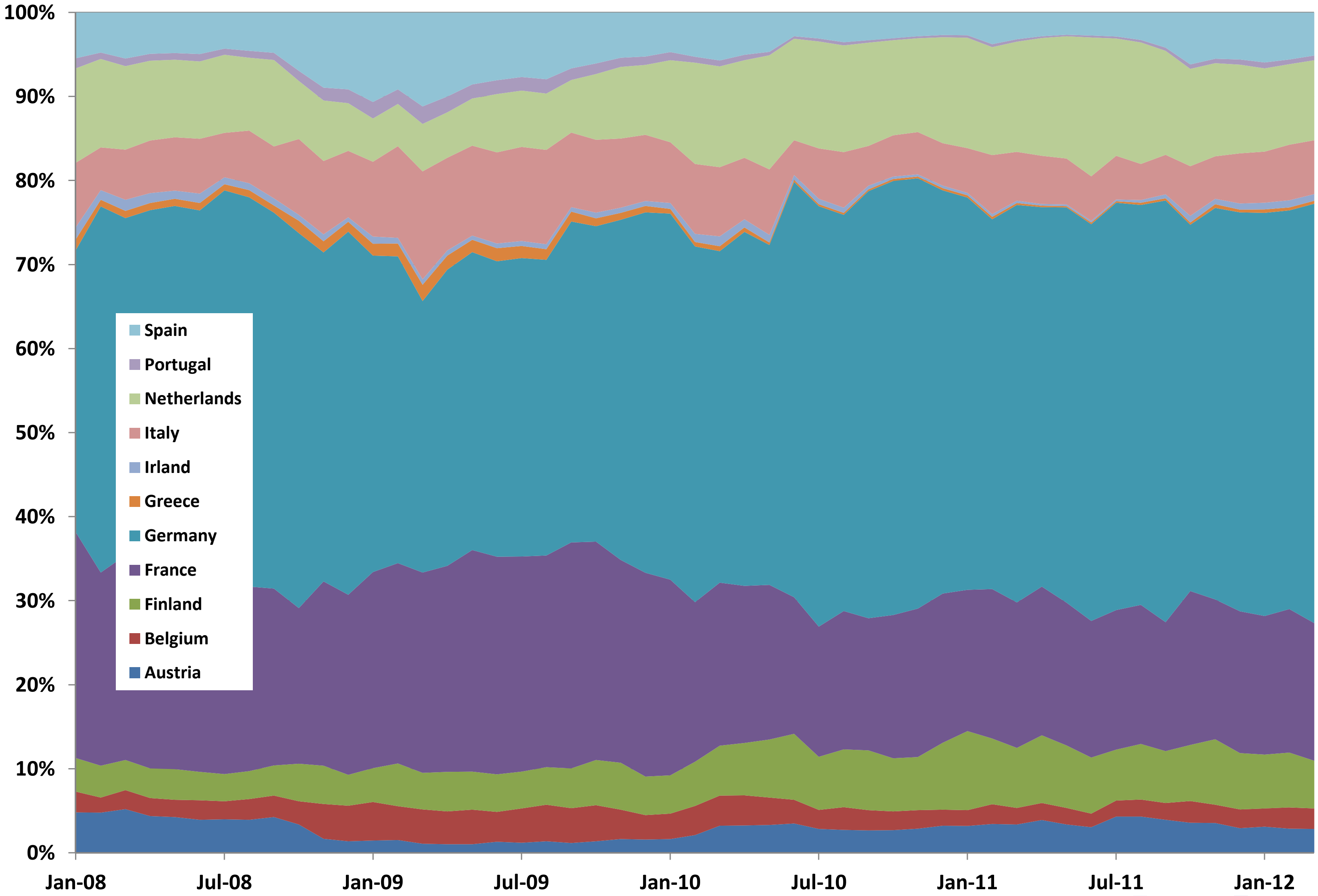
Country	July-08		July-09		July-10		July-11		March-12	
	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights
Austria	3.1%	3.9%	3.1%	1.2%	3.1%	2.9%	3.2%	4.2%	3.4%	2.8%
Belgium	3.8%	2.1%	3.8%	4.1%	3.9%	2.2%	4.0%	1.9%	4.0%	2.4%
Finland	2.0%	3.2%	1.9%	4.4%	2.0%	6.3%	2.1%	6.0%	2.1%	5.7%
France	21.2%	22.0%	21.5%	25.6%	21.4%	15.5%	21.5%	16.5%	21.7%	16.4%
Germany	27.4%	47.8%	27.2%	35.5%	27.7%	50.0%	27.9%	48.7%	27.8%	49.9%
Greece	2.6%	0.7%	2.7%	1.4%	2.6%	0.2%	2.4%	0.2%	2.4%	0.3%
Ireland	2.0%	0.8%	1.9%	0.6%	1.8%	0.6%	1.7%	0.2%	1.7%	0.8%
Italy	17.4%	5.3%	17.3%	11.2%	17.2%	6.0%	17.0%	5.2%	17.1%	6.4%
Netherlands	6.5%	9.2%	6.5%	6.7%	6.5%	12.8%	6.6%	14.0%	6.5%	9.5%
Portugal	1.9%	0.7%	1.9%	1.6%	1.9%	0.4%	1.9%	0.2%	1.9%	0.6%
Spain	12.0%	4.2%	12.0%	7.7%	11.8%	3.1%	11.8%	2.9%	11.6%	5.1%
Sovereign Risk Measure	0.39%		2.10%		3.25%		1.91%		5.43%	

$\Rightarrow$ RB indexation is very different from WB indexation, in terms of weights, RC and credit risk measures.

$\Rightarrow$ The dynamics of the GDP-RB is relatively smooth (monthly turnover $\simeq 7\%$, $\max = 20\%$, $\min = 1.8\%$).

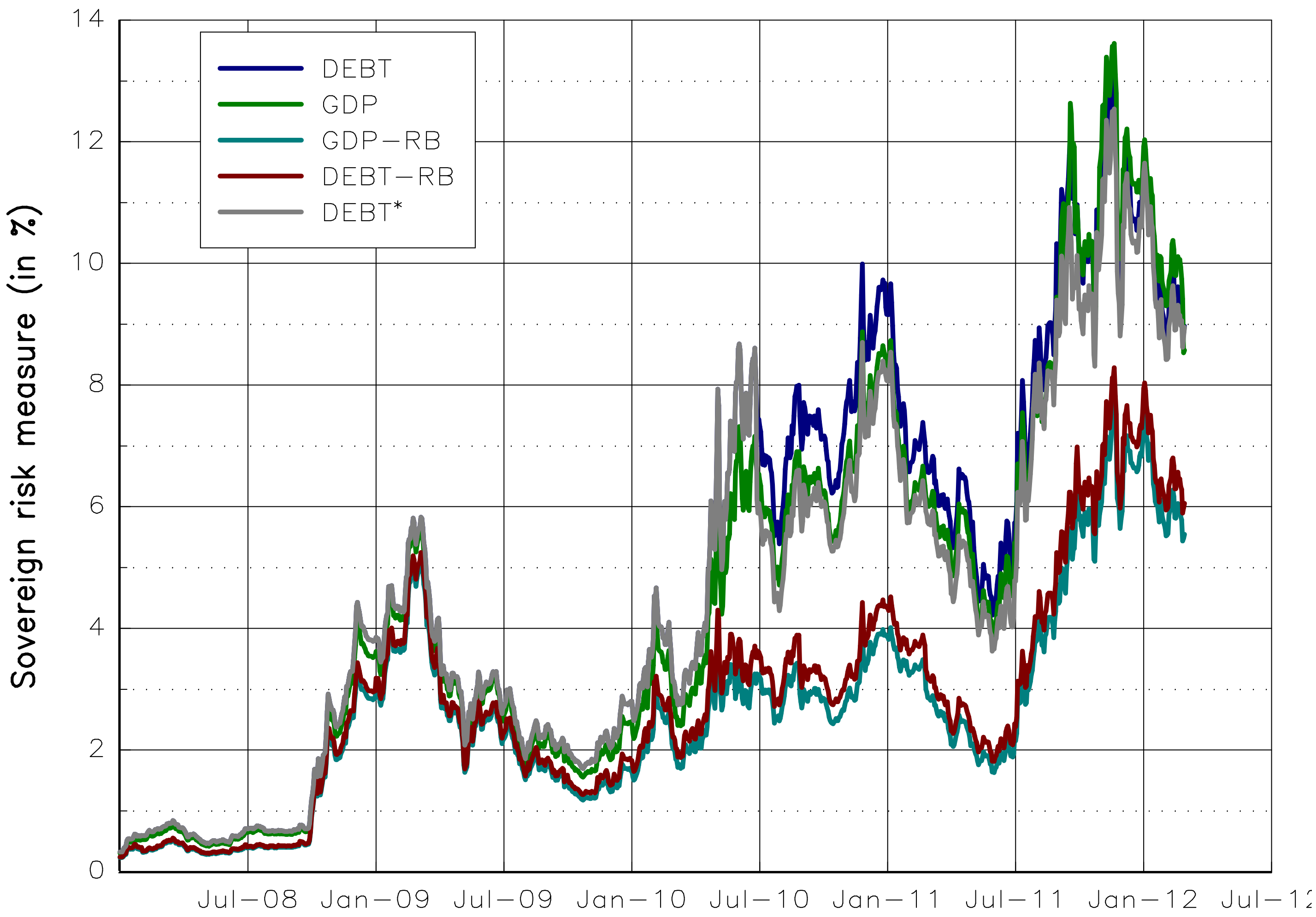
GDP-RB indexation

Evolution of weights



Comparison of the indexing schemes

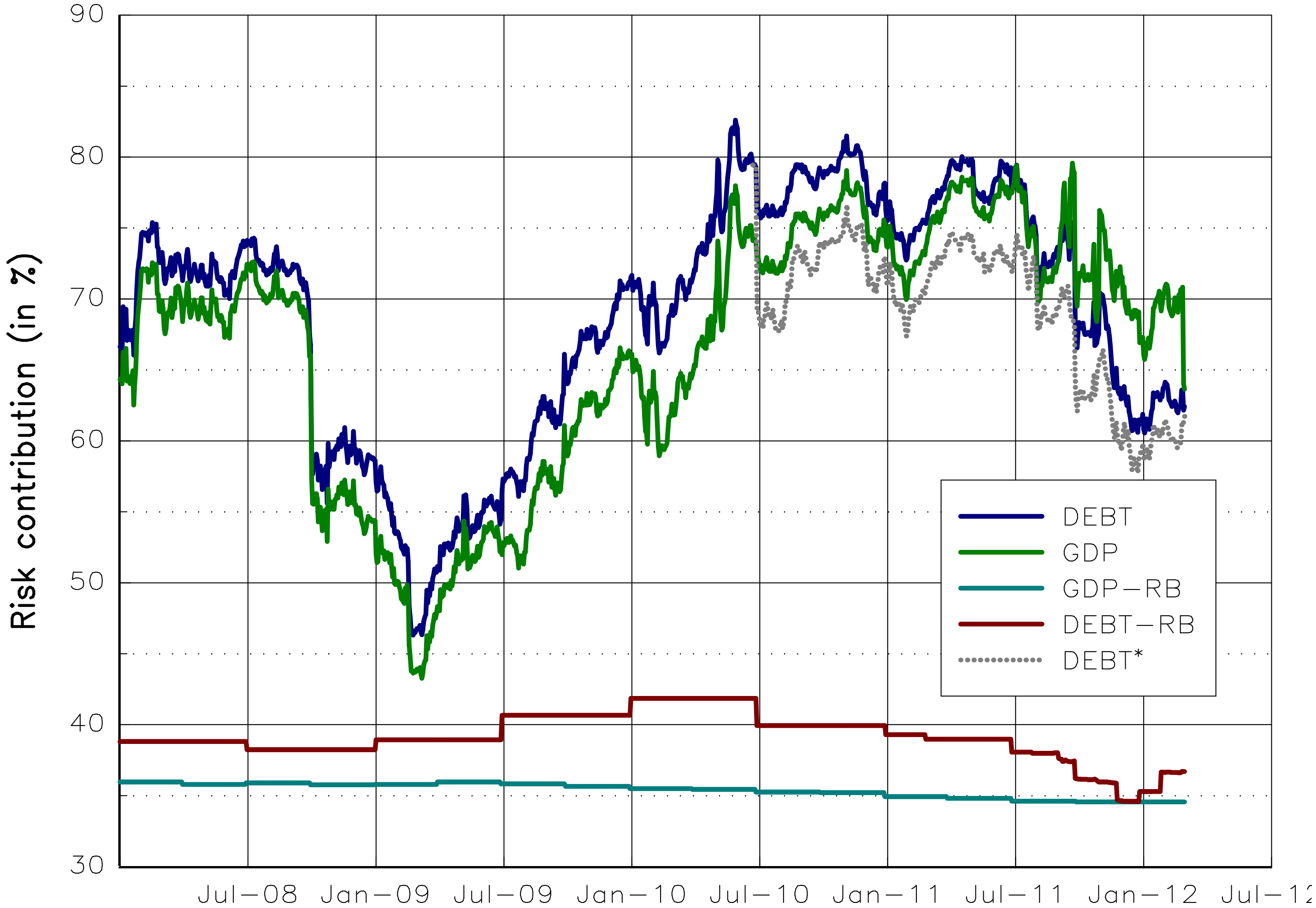
Evolution of the risk measure



⇒ We verify that the risk measure of the RB indexation is smaller than the one of the WB indexation.

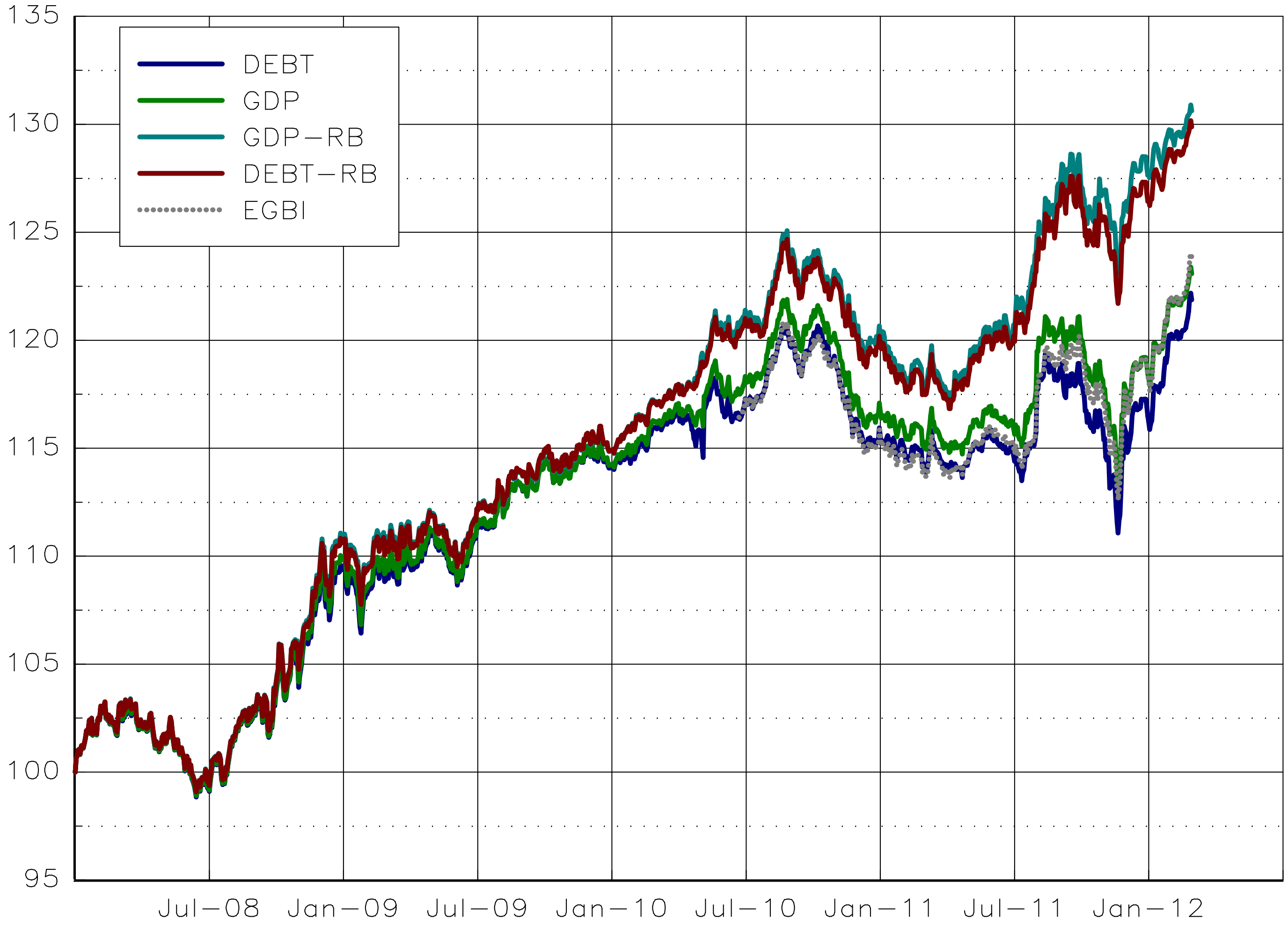
Comparison of the indexing schemes

Evolution of the GIIPS risk contribution



Comparison of the indexing schemes

Performance simulations



⇒ RB indexation / WB indexation = better performance, same volatility and smaller drawdowns.

Comparison with active management

- Database: Morningstar
- Category: Bond EURO Government
- 216 funds[9]

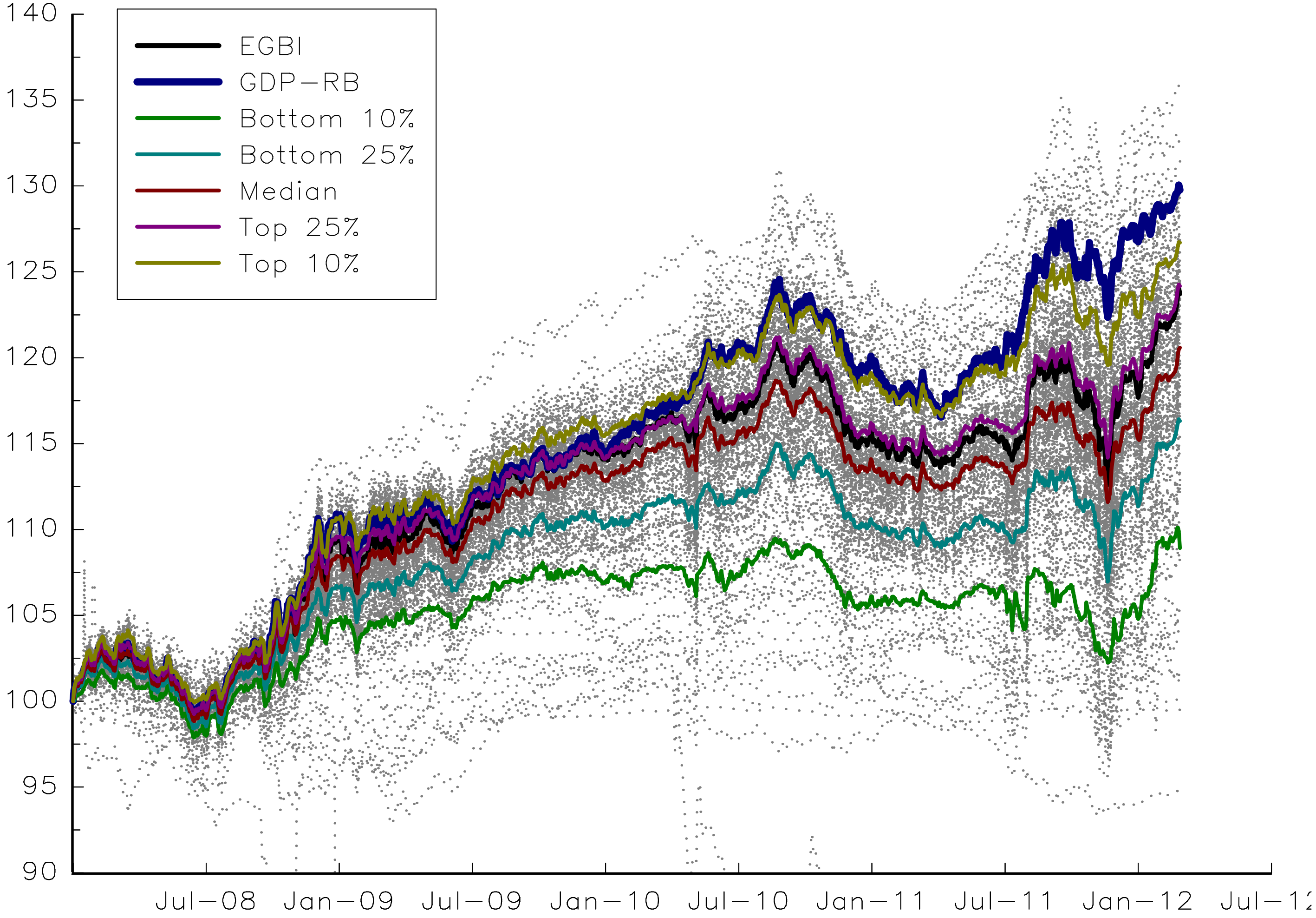
The Academic Rule[10]:

$$\text{Average Performance of Active Management} = \text{Performance of the Index} - \text{Management Fees}$$

⇒ Implied fees for Bond EURO Government: 61 bps / year[11]

[9] We don't take into account the survivorship bias.
[10] There is a large literature on this subject, see e.g. Blake *et al.* (1993).
[11] This figure was only 36 bps / year for the period 01/2008 - 08/2011.

Comparison with active management



#(funds > GDP-RB) = 6

Perf. of GDP-RB Index†
=
Perf. of Top 10%
+ 58 bps / year

† Transaction costs = 15 bps / year

Conclusion

- Credit risk must be managed in bond portfolios.
- Debt-weighted indexation has some limits.

- Fundamental indexation is not enough and must be completed with risk-based methods.
- Risk-budgeting approach is a good compromise between managing the performance and managing the risk.
- Some business related issues:
  1. Asset classes (sovereign bonds, corporate bonds, high yield, global aggregate, etc.) ?
  2. Risk measures (CDS spreads $\Rightarrow$ AS spreads) ?
  3. Clusters (country, sector, etc.) ?

- Risk-budgeting approach = new theoretical results (Bruder and Roncalli, 2012).
- Risk-budgeting approach = good model for Solvency II.

For Further Reading I

B. Bruder, T. Roncalli.
Managing Risk Exposures using the Risk Budgeting Approach.
SSRN, www.ssrn.com/abstract=2009778, January 2012.

B. Bruder, P. Hereil, T. Roncalli.
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