Liquidity Stress Testing in Asset Management Part 4. A Step-by-step Practical Guide*

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Abstract

This article is part of a comprehensive research project on liquidity risk in asset management, which can be divided into three dimensions. The first dimension covers liability liquidity risk (or funding liquidity) modeling, the second dimension focuses on asset liquidity risk (or market liquidity) modeling, and the third dimension considers the asset-liability management of the liquidity gap risk (or asset-liability matching). The purpose of this research is to propose a methodological and practical framework in order to perform liquidity stress testing programs, which comply with regulatory guidelines (ESMA, 2019a, 2020a) and are useful for fund managers. The review of the academic literature and professional research studies shows that there is a lack of standardized and analytical models. The aim of this research project is then to fill the gap with the goal of developing mathematical and statistical approaches, and providing appropriate answers.

The three dimensions have been developed in the published working papers: (1) modeling the liability liquidity risk (Roncalli *et al.*, 2021a), (2) modeling the asset liquidity risk (Roncalli *et al.*, 2021b) and (3) managing the asset-liability liquidity risk (Roncalli, 2021c). This fourth working paper provides three examples and the comprehensive details to compute the redemption coverage ratio, implement reverse stress testing and estimate the liquidation cost of the redemption portfolio. The portfolios have been chosen in order to cover the main asset classes: large-cap stocks, small-cap stocks, sovereign bonds and corporate bonds. Since we provide the data in the appendix, these basic examples are easily reproducible and may help quantitative analysts to understand the different steps to implement liquidity stress testing in asset management.

Keywords: liquidity risk, stress testing, asset-liability management, redemption coverage ratio, reverse stress testing, transaction cost, reproducible research, knowledge transfer.

JEL classification: C02, G32.

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1 Introduction

Since September 2020, the European Securities and Markets Authority (ESMA) has required asset managers to adopt a liquidity stress testing (LST) policy for their investment funds (ESMA, 2020a). More precisely, each asset manager must assess the liquidity risk factors across their funds in order to ensure that stress testing is tailored to the liquidity risk profile of each fund. The guidelines are described in two ESMA publications (ESMA, 2019a, 2020a). However, contrary to the banking regulation on liquidity risk, those regulatory texts do not contain any methodological aspects[1]. Even though they are complemented by two other ESMA publications (ESMA, 2019b, 2020b) and some IMF FSAP analysis (IMF, 2017, 2020), the absence of standardized models and parameter values can be a major hurdle for implementing LST policies, especially for small asset managers[2]. Certainly, this situation can be explained by the lack of maturity of this topic in the asset management industry.

In April 2020, we launched an ambitious research project in order to develop quantitative models and provide practical solutions for implementing LST programs. This research project has been built around three dimensions: liability liquidity risk, asset liquidity risk and asset-liability risk management. It resulted in three publications, each one considering a specific dimension: (1) modeling the liability liquidity risk (Roncalli *et al.*, 2021a), (2) modeling the asset liquidity risk (Roncalli *et al.*, 2021b) and (3) managing the asset-liability liquidity risk (Roncalli, 2021c). The discussions we had with the asset management industry show that these working papers may be viewed as too elaborate. Therefore, we have decided to complement them with a fourth working paper, which is a step-by-step practical guide. This working paper is equivalent to the publication of Bouveret (2017), but our research is reproducible since all the data are provided and described in the appendix.

In order to be concise and simple, we have only focused on three LST measures or tools. The first one is the redemption coverage ratio, which can be computed using a time to liquidation (TTL) approach or a high-quality liquid assets (HQLA) approach. The first approach requires us to define a redemption portfolio and a liquidation policy, whereas the second one is based on the concept of cash conversion factor. For the latter approach, we can use the figures provided by the Basel Committee or postulate a parametric function. The second tool or the reverse stress testing (RST) defines two measures: the liability RST scenario and the asset RST scenario. Finally, the third measure is the liquidity cost associated with the redemption scenario. For that, we need to specify the unit transaction cost function and combine it with the liquidation policy. Moreover, analyzing the redemption cost helps to determine the liquidity risk contribution of each asset.

This paper is organized as follows. Section Two deals with equity portfolios. Using a €1 bn investment in large-cap stocks, we show how to compute the redemption coverage ratio and implement reverse stress testing. Then, we conduct a transaction cost analysis in order to calculate the liquidation cost of the redemption portfolio. Using a second portfolio, we show how the transaction cost formulas are impacted by small-cap stocks. We also illustrate how several statistics change when we consider an asset-liability stress test scenario instead of a normal scenario. In Section Three, we do the same analysis with a bond portfolio. In particular, we highlight the differences between stock and bond portfolios. Finally, Section Four offers some concluding remarks.

[1]For instance, the redemption coverage ratio (RCR) is the main tool of LST programs. However, it is referred to only twice. First, ESMA defines it as: "*a measurement of the ability of a fund's assets to meet funding obligations arising from the liabilities side of the balance sheet, such as a redemption shock*" (ESMA-2020a, page 7). Second, ESMA states that "*an outcome of combined asset and liability LST may be a comparable metric or score, for example based on the RCR*" (ESMA-2020a, page 20).

[2]However, the research of Bouveret (2017) contains the basics of liquidity stress testing for investment funds and can be used as a beginner's guide.

2 The case of equity portfolios

We consider the equity portfolio described in Table 13 on page 26. This portfolio corresponds to a €1 bn investment[3] in the Eurostoxx 50 index at the end of October 2021. For each stock i, we have the number of shares ω_i held by the portfolio, the price P_i, the bid and ask quotes P_i^{bid} and P_i^{ask}, the annualized volatility σ_i and the current daily volume v_i.

2.1 Redemption coverage ratio

2.1.1 Liquidation ratio

Let q_i^+ be the maximum number of shares that can be sold during a trading day for the asset i. We note $q = (q_1, \ldots, q_n)$ the redemption portfolio and $q_i(h)$ the number of shares liquidated after h trading days. Following Roncalli *et al.* (2021b, Equation (22), page 14), we have:

$$q_i(h) = \min\left(\left(q_i - \sum_{k=0}^{h-1} q_i(k)\right)^+, q_i^+\right) \tag{1}$$

where $q_i(0) = 0$. The liquidation ratio $\mathcal{LR}(q; h)$ is then the proportion of the redemption scenario q that is liquidated after h trading days (Roncalli *et al.*, 2021b, Equation (23), page 14):

$$\mathcal{LR}(q; h) = \frac{\sum_{i=1}^{n} \sum_{k=1}^{h} q_i(k) \cdot P_i}{\sum_{i=1}^{n} q_i \cdot P_i} \tag{2}$$

where P_i is the price of the asset i.

We consider the vertical slicing approach (or pro-rata liquidation). We deduce that the redemption portfolio is defined as (Roncalli, 2021c, page 9):

$$q_i = \mathcal{R} \cdot \omega_i \tag{3}$$

The liquidation policy is given by (Roncalli *et al.*, 2021b, Equation (9), page 4):

$$q_i^+ = x_i^+ \cdot v_i \tag{4}$$

where x_i^+ is the trading limit expressed in %. In the sequel, we assume that $x_i^+ = 10\%$, which is a standard figure. In Table 1, we report the values of q_i, v_i, q_i^+ and $q_i(h)$. We observe that we need three trading days to liquidate the redemption portfolio when the redemption shock $\mathcal{R}$ is set to 80%. In fact, most exposures are liquidated in two days, but two stocks require three trading days: Flutter Entertainment ($i = 24$) and Linde ($i = 35$). We verify that the mark-to-market value of the liquidated portfolio is equal to the nominal redemption shock $\mathbb{R} = \mathcal{R} \cdot \mathrm{TNA}$:

$$\mathbb{V}(q) = \sum_{h=1}^{h^+} \sum_{i=1}^{n} q_i(h) \cdot P_i = \mathbb{R} \tag{5}$$

where h^+ is the liquidation period. In our case, we have $\mathbb{R} =$ €799 999 999.60.

[3] The exact value of the total net assets is equal to:

$$\mathrm{TNA} = \mathbb{V}(\omega) = \sum_{i=1}^{n} \omega_i \cdot P_i = 999\,999\,999.50\,€$$

Table 1: Liquidation of the redemption portfolio (TNA = €1 bn, $\mathcal{R}$ = 80%, vertical slicing)

i	ω_i	q_i	v_i	q_i^+	$q_i(1)$	$q_i(2)$	$q_i(3)$	$\sum_{h=1}^{3} q_i(h)$
1	59 106	47 284.8	514 842	51 484.2	47 284.8	0.0	0.0	47 284.8
2	8 883	7 106.4	56 255	5 625.5	5 625.5	1 480.9	0.0	7 106.4
3	150 027	120 021.6	629 509	62 950.9	62 950.9	57 070.7	0.0	120 021.6
4	184 310	147 448.0	1 316 600	131 660.0	131 660.0	15 788.0	0.0	147 448.0
5	130 520	104 416.0	750 684	75 068.4	75 068.4	29 347.6	0.0	104 416.0
6	268 123	214 498.4	1 736 372	173 637.2	173 637.2	40 861.2	0.0	214 498.4
7	131 520	105 216.0	754 901	75 490.1	75 490.1	29 725.9	0.0	105 216.0
8	651 421	521 136.8	4 358 304	435 830.4	435 830.4	85 306.4	0.0	521 136.8
9	3 192 430	2 553 944.0	54 130 721	5 413 072.1	2 553 944.0	0.0	0.0	2 553 944.0
10	5 544 072	4 435 257.6	72 371 040	7 237 104.0	4 435 257.6	0.0	0.0	4 435 257.6
11	242 317	193 853.6	2 473 040	247 304.0	193 853.6	0.0	0.0	193 853.6
12	181 800	145 440.0	2 444 130	244 413.0	145 440.0	0.0	0.0	145 440.0
13	136 334	109 067.2	1 183 053	118 305.3	109 067.2	0.0	0.0	109 067.2
14	365 067	292 053.6	2 390 614	239 061.4	239 061.4	52 992.2	0.0	292 053.6
15	251 719	201 375.2	1 434 050	143 405.0	143 405.0	57 970.2	0.0	201 375.2
16	265 762	212 609.6	2 672 846	267 284.6	212 609.6	0.0	0.0	212 609.6
17	206 041	164 832.8	1 579 517	157 951.7	157 951.7	6 881.1	0.0	164 832.8
18	60 148	48 118.4	339 768	33 976.8	33 976.8	14 141.6	0.0	48 118.4
19	311 879	249 503.2	2 536 147	253 614.7	249 503.2	0.0	0.0	249 503.2
20	1 026 503	821 202.4	9 225 311	922 531.1	821 202.4	0.0	0.0	821 202.4
21	2 459 244	1 967 395.2	30 518 046	3 051 804.6	1 967 395.2	0.0	0.0	1 967 395.2
22	795 234	636 187.2	19 419 467	1 941 946.7	636 187.2	0.0	0.0	636 187.2
23	95 262	76 209.6	491 647	49 164.7	49 164.7	27 044.9	0.0	76 209.6
24	55 520	44 416.0	212 501	21 250.1	21 250.1	21 250.1	1 915.8	44 416.0
25	1 840 196	1 472 156.8	14 316 692	1 431 669.2	1 431 669.2	40 487.6	0.0	1 472 156.8
26	351 837	281 469.6	7 543 014	754 301.4	281 469.6	0.0	0.0	281 469.6
27	413 417	330 733.6	3 643 730	364 373.0	330 733.6	0.0	0.0	330 733.6
28	1 235 905	988 724.0	15 954 487	1 595 448.7	988 724.0	0.0	0.0	988 724.0
29	5 774 696	4 619 756.8	106 942 206	10 694 220.6	4 619 756.8	0.0	0.0	4 619 756.8
30	23 113	18 490.4	204 628	20 462.8	18 490.4	0.0	0.0	18 490.4
31	161 807	129 445.6	786 412	78 641.2	78 641.2	50 804.4	0.0	129 445.6
32	261 645	209 316.0	2 285 287	228 528.7	209 316.0	0.0	0.0	209 316.0
33	290 422	232 337.6	2 489 971	248 997.1	232 337.6	0.0	0.0	232 337.6
34	76 637	61 309.6	372 415	37 241.5	37 241.5	24 068.1	0.0	61 309.6
35	162 956	130 364.8	578 973	57 897.3	57 897.3	57 897.3	14 570.2	130 364.8
36	83 427	66 741.6	364 566	36 456.6	36 456.6	30 285.0	0.0	66 741.6
37	44 351	35 480.8	256 421	25 642.1	25 642.1	9 838.7	0.0	35 480.8
38	64 954	51 963.2	363 103	36 310.3	36 310.3	15 652.9	0.0	51 963.2
39	282 801	226 240.8	2 135 693	213 569.3	213 569.3	12 671.5	0.0	226 240.8
40	120 062	96 049.6	794 444	79 444.4	79 444.4	16 605.2	0.0	96 049.6
41	362 506	290 004.8	1 551 395	155 139.5	155 139.5	34 865.3	0.0	290 004.8
42	345 779	276 623.2	1 859 400	185 940.0	185 940.0	90 683.2	0.0	276 623.2
43	180 140	144 112.0	818 822	81 882.2	81 882.2	62 229.8	0.0	144 112.0
44	237 952	190 361.6	1 136 151	113 615.1	113 615.1	76 746.5	0.0	190 361.6
45	660 350	528 280.0	10 497 975	1 049 797.5	528 280.0	0.0	0.0	528 280.0
46	835 885	668 708.0	6 596 020	659 602.0	659 602.0	9 106.0	0.0	668 708.0
47	247 964	198 371.2	1 943 066	194 306.6	194 306.6	4 064.6	0.0	198 371.2
48	189 196	151 356.8	912 539	91 253.9	91 253.9	60 102.9	0.0	151 356.8
49	57 954	46 363.2	1 071 749	107 174.9	46 363.2	0.0	0.0	46 363.2
50	163 680	130 944.0	976 446	97 644.6	97 644.6	33 299.4	0.0	130 944.0

The liquidation contribution of the trading day h is the proportion of the redemption portfolio liquidated on day h:

$$\mathcal{LC}(q;h) = \frac{\sum_{i=1}^{n} q_i(h) \cdot P_i}{\sum_{i=1}^{n} q_i \cdot P_i} \tag{6}$$

By construction, we verify that the sum of liquidation contributions is equal to the liquidation ratio:

$$\mathcal{LR}(q;h) = \sum_{k=1}^{h} \mathcal{LC}(q;k) \tag{7}$$

In Table 2, we report the liquidation contribution $\mathcal{LC}(q;h)$ and the liquidation ratio $\mathcal{LR}(q;h)$ for different values of the redemption rate. When $\mathscr{R}$ is equal to 90%, we liquidate 72.41% the first day, 26.06% the second day and 1.53% the third day.

Table 2: Liquidation ratio in % (TNA = €1 bn, vertical slicing)

$\mathscr{R}$	5%	10%	25%	50%	75%	90%
$\mathcal{LC}(q;1)$	100.00	100.00	100.00	96.43	81.31	72.41
$\mathcal{LC}(q;2)$	0.00	0.00	0.00	3.57	18.46	26.06
$\mathcal{LC}(q;3)$	0.00	0.00	0.00	0.00	0.23	1.53
$\mathcal{LR}(q;1)$	100.00	100.00	100.00	96.43	81.31	72.41
$\mathcal{LR}(q;2)$	100.00	100.00	100.00	100.00	99.77	98.47
$\mathcal{LR}(q;3)$	100.00	100.00	100.00	100.00	100.00	100.00

The liquidity risk profile depends on the size of the redemption portfolio. For instance, we can consider individual investment funds, whose total net assets are lower than \$1 bn. On the contrary, if we consider the largest asset managers, their aggregate exposure to the stocks of the Eurostoxx 50 index is greater than \$1 bn. In Figure 13 on page 38, we report the liquidation ratio[4] when the total net assets are respectively equal to 1, 5, 10 and 20 bn. If we consider a redemption rate of 10%, we obtain the results given in Figure 1 for different time horizons (one day, two days and one week). For instance, we obtain $\mathcal{LR}(q;1) = 37.43\%$, $\mathcal{LR}(q;2) = 66.91\%$ and $\mathcal{LR}(q;1) = 99.47\%$ when the total net assets are equal to €20 bn.

From the liquidation ratio, we can compute the liquidation time (Roncalli *et al.*, 2021b, page 18):

$$\begin{aligned} \mathcal{LT}(q;p) &= \mathcal{LR}^{-1}(q;p) \\ &= \{\inf h : \mathcal{LR}(q;h) \geq p\} \end{aligned} \tag{8}$$

The liquidation period h^+ is equal to $\mathcal{LT}(q;1)$. It measures the number of days required to liquidate 100% of the redemption portfolio. In practice, the redemption portfolio can have some small illiquid exposures on small-cap stocks. Therefore, it is better to define h^+ as the 99% quantile of the liquidation ratio:

$$h^+ = \mathcal{LR}^{-1}(q;99\%) \tag{9}$$

In Figure 2, we report the computation of the liquidation ratio for the previous example.

[4] For that, we scale the original portfolio $(\omega_1, \ldots, \omega_n)$ by a factor of m where m is respectively equal to 1, 5, 10 and 20.

Figure 1: Liquidation ratio $\mathcal{LR}(q;h)$ in % when the redemption rate is equal to 10%

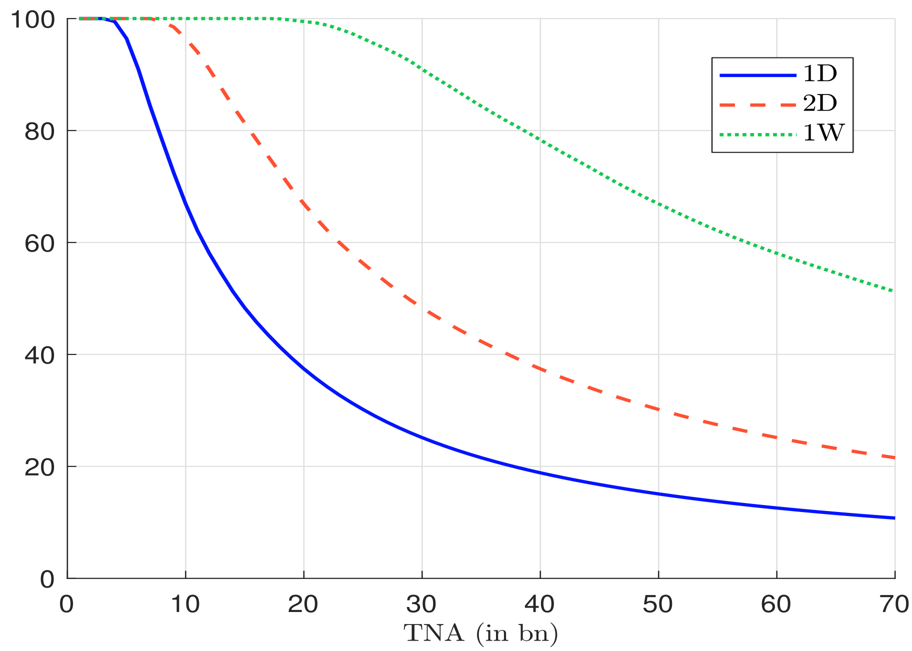
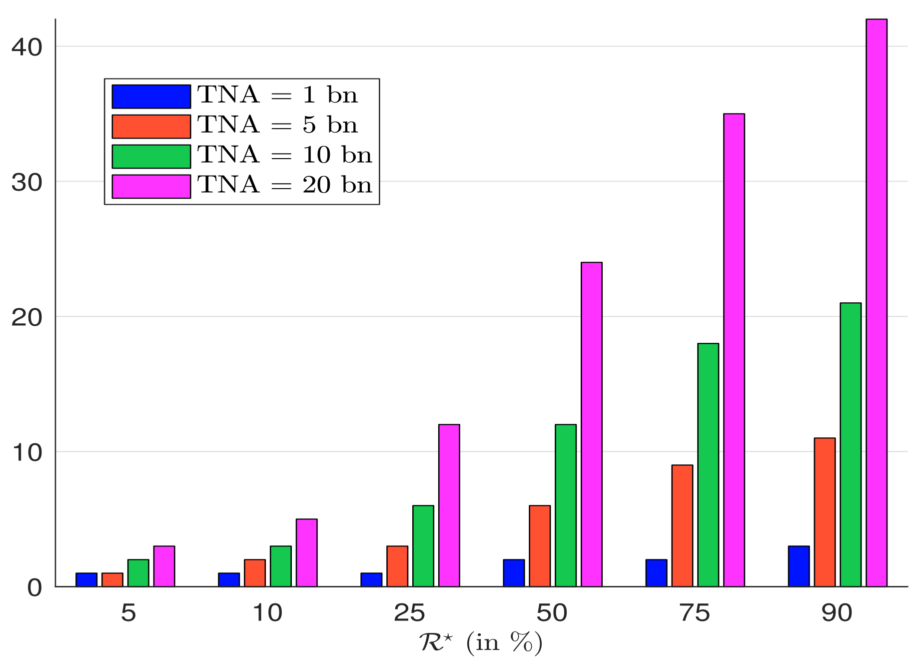


Figure 2: Liquidation time $h^{+} = \mathcal{LR}^{-1}(q;99\%)$ in number of trading days



2.1.2 Time to liquidation approach

We now turn to the computation of the redemption coverage ratio. Following Roncalli (2021c, Equation (12), page 6), we have:

$$\mathrm{RCR}(h) = \frac{\mathcal{LR}(q; h) \cdot \mathbb{V}(q)}{\mathcal{R} \cdot \mathbb{V}(\omega)} \tag{10}$$

where $\mathbb{V}(q) = \sum_{i=1}^{n} q_i \cdot P_i$ and $\mathbb{V}(\omega) = \sum_{i=1}^{n} \omega_i \cdot P_i = \mathrm{TNA}$. The liquidity shortfall is the amount of additional assets to be sold to satisfy the redemption. Its relative value (with respect to the total net assets) is equal to (Roncalli, 2021c, Equation (9), page 6):

$$\mathrm{LS}(h) = \mathcal{R} \cdot \max(0, 1 - \mathrm{RCR}(h)) \tag{11}$$

As noticed by Roncalli (2021c), the redemption coverage ratio is exactly equal to the liquidation ratio when the redemption shock $\mathbb{R} = \mathcal{R} \cdot \mathrm{TNA}$ is equal to the mark-to-market $\mathbb{V}(q)$ of the redemption portfolio. The reason is that the redemption portfolio is defined using the vertical slicing and its value is exactly equal to the redemption portfolio (Roncalli, 2021c, page 9). Therefore, in the case of the naive vertical slicing approach, we always have:

$$q = \mathcal{R} \cdot \omega \Longrightarrow \mathrm{RCR}(h) \leq 1 \tag{12}$$

It is obvious that the RCR cannot be computed with the naive vertical slicing approach. It is better to consider the waterfall approach: $q = \omega$. In this case, we use the following formula (Roncalli, 2021c, Equations (7) and (8), page 6):

$$\mathrm{RCR}(h) = \sum_{k=1}^{h} \frac{\sum_{i=1}^{n} \omega_i(k) \cdot P_i}{\mathcal{R} \cdot \mathrm{TNA}} = \frac{1}{\mathcal{R}} \sum_{k=1}^{h} \frac{\sum_{i=1}^{n} \omega_i(k) \cdot P_i}{\sum_{i=1}^{n} \omega_i \cdot P_i} \tag{13}$$

In Table 3, we report the redemption coverage ratio for different redemption rate values when the total net assets are equal to €1 bn. When the liquidation approach corresponds to the naive vertical slicing, we obtain the same figures as the liquidation ratio (see Table 2 on page 5). When we use the waterfall approach, the RCR is higher. Figure 3 shows the evolution of the RCR with respect to the TNA. In the case where the redemption rate is equal to 10%, the RCR is below one when the TNA is larger than 7.6 bn for the one-day time horizon, 15.1 bn for the two-day time horizon and 37.7 bn for the one-week time horizon.

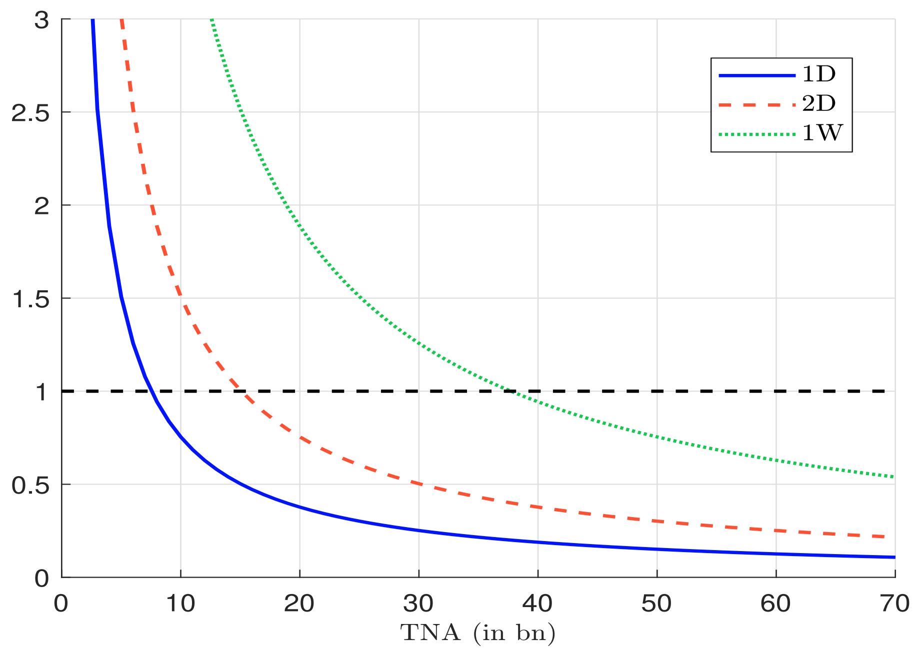
Table 3: Redemption coverage ratio (TNA = €1 bn)

Redemption rate $\mathcal{R}$		5%	10%	25%	50%	75%	90%
Vertical slicing	RCR (1)	1.00	1.00	1.00	0.96	0.81	0.72
	RCR (2)	1.00	1.00	1.00	1.00	1.00	0.98
	RCR (3)	1.00	1.00	1.00	1.00	1.00	1.00
Waterfall liquidation	RCR (1)	13.38	6.69	2.68	1.34	0.89	0.74
	RCR (2)	19.29	9.64	3.86	1.93	1.29	1.07
	RCR (3)	20.00	10.00	4.00	2.00	1.33	1.11

2.1.3 Impact of the stress test scenario

To compute the redemption coverage ratio in a stress period, we can shock the redemption rate $\mathcal{R}$ and the daily trading volume v_i. For the first parameter, we use a larger value

Figure 3: Redemption coverage ratio when the redemption rate is equal to 10% (waterfall liquidation)



$\mathcal{R}^{\text{stress}}$ for the redemption shock. For the second parameter, we recall that the liquidation policy is defined as $q_i^+ = x_i^+ \cdot v_i$. Following Roncalli *et al.* (2021b, page 56), we introduce a multiplicative parameter $m_v \leq 1$ so that the liquidation policy in a stress period becomes:

$$q_i^+ = m_v \cdot x_i^+ \cdot v_i \tag{14}$$

We consider the previous equity portfolio. We assume that the redemption rate $\mathcal{R}$ is equal to 5% in a normal period[5]. Results are given in Table 4. For the stress testing exercise, we consider a higher redemption rate value ($\mathcal{R}^{\text{stress}} = 20\%$) and different asset liquidity scenarios. Results are given in Table 5. For instance, if we assume that the asset liquidity is reduced by a factor of 2 ($m_v = 50\%$), the value of $\text{RCR}^{\text{stress}}(1)$ is equal to 0.38 for a one-day time horizon and a TNA of €5 bn, while it was equal to 3.02 in a normal period.

Table 4: Redemption coverage ratio in a normal period ($\mathcal{R} = 5\%$, waterfall liquidation)

TNA	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	13.38	3.02	1.51	0.75
$h = 2$	19.29	6.04	3.02	1.51
$h = 5$	20.00	13.38	7.49	3.77

Remark 1 *We can break down the impact of the stress test scenario by distinguishing the effect of the redemption shock and the impact of the asset liquidity.*

[5]This figure is far overestimated when it concerns normal market periods.

Table 5: Stress testing of the redemption coverage ratio ($\mathcal{R}^{\text{stress}} = 20\%$, waterfall liquidation)

	$m_v = 1.00$				$m_v = 0.75$			
TNA	€1 bn	€5 bn	€10 bn	€20 bn	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	3.35	0.75	0.38	0.19	2.67	0.57	0.28	0.14
$h = 2$	4.82	1.51	0.75	0.38	4.33	1.13	0.57	0.28
$h = 5$	5.00	3.35	1.87	0.94	5.00	2.67	1.41	0.71
	$m_v = 0.50$				$m_v = 0.10$			
TNA	€1 bn	€5 bn	€10 bn	€20 bn	€1 bn	€5 bn	€10 bn	€20 bn
$h = 1$	1.87	0.38	0.19	0.09	0.38	0.08	0.04	0.02
$h = 2$	3.35	0.75	0.38	0.19	0.75	0.15	0.08	0.04
$h = 5$	4.97	1.87	0.94	0.47	1.87	0.38	0.19	0.09

2.1.4 The case of small-cap portfolios

The previous example may be misleading because we obtain high redemption coverage ratio figures as we are considering large-cap liquid stocks. In fact, liquidity stress testing makes more sense when the portfolio contains small- and mid-cap stocks. In Table 14 on page 27, we consider a second portfolio, which is equally weighted on 20 stocks. Since some stocks present a low free-float market capitalization, liquidating this portfolio is more challenging. For instance, 144 trading days are required to liquidate 99% of a €1 bn exposure on this portfolio[6]. Therefore, it is unsurprising that we obtained the redemption coverage ratio values given in Tables 6 and 7. Using the waterfall approach, the redemption coverage ratio $\text{RCR}^{\text{stress}}(1)$ is below one for a €1 bn exposure even though the multiplicative factor m_v is equal to 1.

Table 6: Redemption coverage ratio in a normal period (small-cap portfolio, $\mathcal{R} = 5\%$, waterfall liquidation)

TNA	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	1.28	0.64	0.43	0.32
$h = 2$	2.56	1.28	0.85	0.64
$h = 5$	5.89	3.20	2.13	1.60

Table 7: Stress testing of the redemption coverage ratio (small-cap portfolio, $\mathcal{R}^{\text{stress}} = 20\%$, waterfall liquidation)

	$m_v = 1.00$				$m_v = 0.75$			
TNA	€1 bn	€2 bn	€3 bn	€4 bn	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	0.32	0.06	0.03	0.02	0.24	0.05	0.02	0.01
$h = 2$	0.64	0.13	0.06	0.03	0.48	0.10	0.05	0.02
$h = 5$	1.47	0.32	0.16	0.08	1.17	0.24	0.12	0.06
	$m_v = 0.50$				$m_v = 0.10$			
TNA	€1 bn	€2 bn	€3 bn	€4 bn	€1 bn	€2 bn	€3 bn	€4 bn
$h = 1$	0.16	0.03	0.02	0.01	0.03	0.01	0.00	0.00
$h = 2$	0.32	0.06	0.03	0.02	0.06	0.01	0.01	0.00
$h = 5$	0.80	0.16	0.08	0.04	0.16	0.03	0.02	0.01

[6] Figures 14 and 15 on page 38 show other liquidation statistics.

2.1.5 HQLA approach

The high-quality liquid assets (HQLA) method is explained in detail in Roncalli (2021c, Section 2.1.2, pages 13-18). Let CCF_k be the cash conversion factor (CCF) of the k^{th} HQLA class. The redemption coverage ratio is defined as:

$$\text{RCR}(h) = \frac{\sum_{k=1}^{m} w_k \cdot \text{CCF}_k(h)}{\mathcal{R}} \tag{15}$$

where w_k is the weight of the k^{th} HQLA class. In the case of the Basel III framework, $\text{CCF}_k(h)$ is fixed and does not depend on the time horizon h. In the case of the risk sensitive framework, Roncalli (2021c) proposed the following formula:

$$\text{CCF}_{k,j}(h) = \text{LF}_k(h) \cdot \left(1 - \text{DF}_k\left(\frac{h}{2}\right)\right) \cdot \left(1 - \text{SF}_k\left(\text{TNA}_j, \mathcal{H}_j\right)\right) \tag{16}$$

where $\text{LF}_k(h) \in [0,1]$ is the liquidity factor, $\text{DF}_k(\tau_h) \in [0, \text{MDD}_k]$ is the drawdown factor and $\text{SF}_k \in [0,1]$ is the specific risk factor associated to the fund j. Following Roncalli (2021c), we specify these functions as follows:

$$\begin{cases} \text{LF}_k(h) = \min\left(1.0, \lambda_k \cdot h\right) \\ \text{DF}_k(h) = \min\left(\text{MDD}_k, \eta_k \cdot \sqrt{h}\right) \\ \text{SF}_k\left(\text{TNA}_j, \mathcal{H}_j\right) = \min\left(\xi_k^{\text{size}}\left(\frac{\text{TNA}_j}{\text{TNA}^\star} - 1\right)^+ + \xi_k^{\text{concentration}}\left(\sqrt{\frac{\mathcal{H}_j}{\mathcal{H}^\star}} - 1\right)^+, \text{SF}^+\right) \end{cases} \tag{17}$$

where λ_k is the selling intensity, MDD_k is the maximum drawdown, η_k is the loss intensity, TNA_j is the total net assets and $\mathcal{H}_j$ is the Herfindahl index of the fund.

In the Basel framework, the value of $\text{CCF}_k(h)$ is set to 50%. Concerning the risk sensitive framework, we assume that $\lambda_k = 2\%$, $\eta_k = 5\%$, $\text{MDD}_k = 50\%$, $\xi_k^{\text{size}} = 10\%$, $\xi_k^{\text{concentration}} = 25\%$, $\text{TNA}^\star = 1$ bn, $\mathcal{H}^\star = 2\%$ and $\text{SF}^+ = 0.80$. Results are given[7] in Figures 4 and 5 when the stressed redemption rate $\mathcal{R}^{\text{stress}}$ is equal to 20%. We also compare the HQLA approach with the time-to-liquidation (TTL) approach[8].

2.2 Reverse stress testing

According to Roncalli (2021c), reverse stress testing consists in finding the liquidity scenario such that $\text{RCR}(h) = \text{RCR}^-$ where RCR^- is the minimum acceptable level of the redemption coverage ratio[9].

2.2.1 Liability RST scenario

From a liability perspective, reverse stress testing consists in finding the redemption shock above which the redemption coverage ratio is lower than the minimum acceptable level:

$$\text{RCR}(h) \leq \text{RCR}^- \Leftrightarrow \mathcal{R} \geq \mathcal{R}^{\text{RST}} \tag{18}$$

$\mathcal{R}^{\text{RST}}$ is computed by solving the non-linear equation:

$$\mathcal{R}^{\text{RST}} = \left\{\mathcal{R} \in [0,1] : \text{RCR}(h) = \text{RCR}^-\right\} \tag{19}$$

[7] For the small-cap portfolio, we assume that the selling intensity λ_k is reduced by 25%, the loss intensity η_k is increased by 20% and the threshold $\text{TNA}^\star$ is divided by a factor of two.

[8] The stress multiplier m_v is set to 50%.

[9] A standard value of RCR^- is 50%.

Figure 4: Redemption coverage ratio of the large-cap portfolio (TNA = €1 bn, $\mathcal{R}^{\text{stress}} = 20\%$ and $m_v = 50\%$)

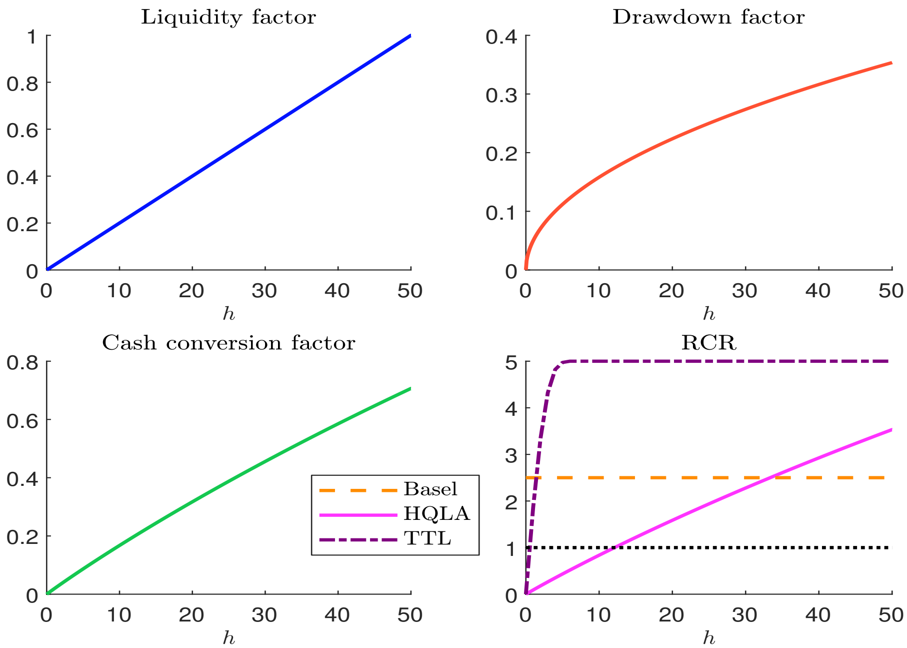
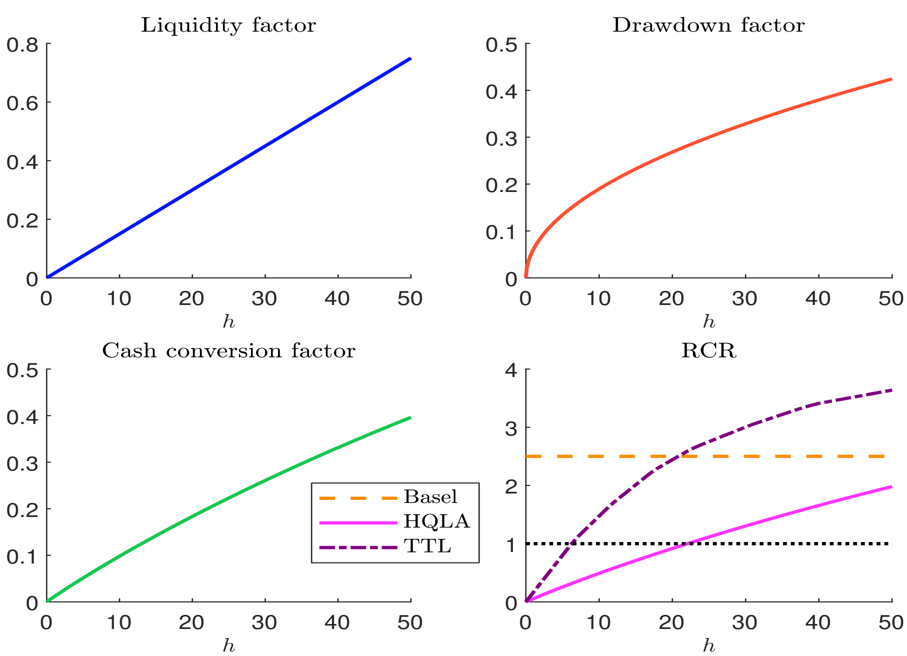


Figure 5: Redemption coverage ratio of the small-cap portfolio (TNA = €1 bn, $\mathcal{R}^{\text{stress}} = 20\%$ and $m_v = 50\%$)



In the case where the liquidation portfolio q does not depend on the redemption rate, we obtain an analytical expression:

$$\frac{\sum_{k=1}^{h}\sum_{i=1}^{n} q_i(k)\cdot P_i}{\mathcal{R}^{\mathrm{RST}}\cdot \mathrm{TNA}} = \mathrm{RCR}^{-} \Leftrightarrow \mathcal{R}^{\mathrm{RST}} = \frac{\sum_{k=1}^{h}\sum_{i=1}^{n} q_i(k)\cdot P_i}{\mathrm{RCR}^{-}\cdot \mathrm{TNA}} \tag{20}$$

For instance, this solution is valid for the waterfall approach since we have $q = \omega$. However, the analytical solution cannot be used for the pro-rata liquidation: $q = \mathcal{R}\cdot\omega$. In this case, we must solve the non-linear equation (19).

Table 8 gives the liability RST scenario $\mathcal{R}^{\mathrm{RST}}$ when the minimum acceptable redemption coverage ratio is set to 50% and we consider the pro-rata liquidation policy. We notice that $\mathcal{R}^{\mathrm{RST}}$ may be greater than one, implying that the size of the fund is too small to experience a liquidity stress test scenario. In this case, we report the total net assets $\mathrm{TNA}^{\mathrm{RST}} = \mathcal{R}^{\mathrm{RST}}\cdot\mathrm{TNA}$ such that the non-linear equation $RCR(h) = \mathrm{RCR}^{-}$ is satisfied for a 100% redemption rate. If we consider the large-cap portfolio with $m_v = 0.50$, the reverse stress testing scenario for the one-day time horizon is $\mathcal{R}^{\mathrm{RST}} = 72.1\%$. If $m_v = 1.00$, the value of $\mathcal{R}$ is always greater than one. Therefore, the reverse stress testing scenario for the one-day time horizon is given by the variable $\mathrm{TNA}^{\mathrm{RST}}$ instead of the metric $\mathcal{R}^{\mathrm{RST}}$. In our case, we obtain $\mathrm{TNA}^{\mathrm{RST}} = €1.441$ bn. Therefore, by assuming a 100% redemption shock, the size of the fund must be greater than €1.441 bn to breach the stress test scenario. On the contrary, if we consider the small-cap portfolio, reverse stress testing implies low values of the redemption rate $\mathcal{R}^{\mathrm{RST}}$. For example, if we observe a 50% reduction in the asset liquidity, a redemption scenario of 4.9% is sufficient to observe a redemption coverage ratio lower than 50% when the time horizon h is set to one trading day.

Table 8: Liability reverse stress testing scenario $\mathcal{R}^{\mathrm{RST}}$ in % (TNA = €1 bn, pro-rata liquidation, $\mathrm{RCR}^{-} = 50\%$)

	Large-cap portfolio				Small-cap portfolio			
m_v	1.00	0.75	0.50	0.10	1.00	0.75	0.50	0.10
$h = 1$	144.1	108.1	72.1	14.5	9.7	7.3	4.9	1.0
$h = 2$	288.2	216.2	144.1	28.9	19.3	14.5	9.7	2.0
$h = 3$	432.3	324.2	216.2	43.3	28.9	21.7	14.5	2.9
$h = 4$	576.3	432.3	288.2	57.7	38.5	28.9	19.3	3.9
$h = 5$	720.4	540.3	360.2	72.1	48.1	36.1	24.1	4.9

2.2.2 Asset RST scenario

From an asset perspective, reverse stress testing consists in finding the asset liquidity shock m_v below which the redemption coverage ratio is lower than the minimum acceptable level:

$$\mathrm{RCR}(h) \leq \mathrm{RCR}^{-} \Leftrightarrow m_v \leq m_v^{\mathrm{RST}} \tag{21}$$

Since the liquidation policy $q_i^{+} = m_v \cdot x_i^{+}\cdot v_i$ depends on the value m_v, there is no analytical solution. The numerical solution corresponds then to the root of the non-linear equation $\left\{m_v \in [0,1] : \mathrm{RCR}(h) = \mathrm{RCR}^{-}\right\}$.

For the large-cap portfolio, the values taken by m_v^{RST} are very small. This indicates that the asset liquidity must be dramatically reduced to observe a stress test scenario, which is not realistic. For instance, in the case where the redemption shock is equal to 10%, the current liquidity must be reduced by a factor greater than 10 (m_v^{RST} is equal to 0.07 for $h = 1$). In the case of the small-cap portfolio, the current liquidity is not enough to liquidate 10% of the total net assets in one day since we have $m_v^{\mathrm{RST}} = 1.04$.

Table 9: Asset reverse stress testing scenario m_v^{RST} (TNA = €1 bn, pro-rata liquidation, $\mathrm{RCR}^- = 50\%$)

	Large-cap portfolio				Small-cap portfolio			
$\mathcal{R}$	5%	10%	20%	50%	5%	10%	20%	50%
$h = 1$	0.04	0.07	0.14	0.35	0.52	1.04	2.08	5.20
$h = 2$	0.02	0.04	0.07	0.17	0.26	0.52	1.04	2.60
$h = 3$	0.01	0.02	0.05	0.12	0.17	0.35	0.69	1.73
$h = 4$	0.01	0.02	0.04	0.09	0.13	0.26	0.52	1.30
$h = 5$	0.01	0.01	0.03	0.07	0.11	0.21	0.42	1.04

2.3 Transaction cost analysis

2.3.1 Analytics of the transaction cost function

The cost function corresponds to the square-root-linear model described in Roncalli *et al.* (2021b, Section 2.4.2, page 11). Let $x_i(h)$ be the participation rate at the trading day h. It is equal to:

$$x_i(h) = \frac{q_i(h)}{v_i} \tag{22}$$

where v_i is the daily volume. The unit transaction cost function is equal to:

$$\boldsymbol{c}_i(x_i(h)) = \begin{cases} 1.25\, s_i + 0.40\, \sigma_i \sqrt{x_i(h)} & \text{if } x_i(h) \leq \tilde{x}_i \\ 1.25\, s_i + \dfrac{0.40}{\sqrt{\tilde{x}_i}} \sigma_i x_i & \text{if } \tilde{x}_i \leq x_i(h) \leq x_i^+ \\ +\infty & \text{if } x_i(h) > x_i^+ \end{cases} \tag{23}$$

where s_i is the bid-ask spread, σ_i is the daily volatility, $x_i^+ = 10\%$ and $\tilde{x}_i = \dfrac{2}{3} x_i^+$. We can break down this unit transaction cost as follows:

$$\boldsymbol{c}_i(x_i(h)) = \boldsymbol{c}_i^s + \boldsymbol{c}_i^{\boldsymbol{\pi}}(x_i(h)) \tag{24}$$

where $\boldsymbol{c}_i^s$ is the spread component and $\boldsymbol{c}_i^{\boldsymbol{\pi}}(x_i(h))$ is the price impact component. Let $Q_i = q_i \cdot P_i$ and $Q_i(h) = q_i(h) \cdot P_i$ be the nominal value of q_i and $q_i(h)$. The transaction cost of the trade associated to $q_i(h)$ is therefore equal to:

$$\mathcal{TC}_i(q_i(h)) = Q_i(h) \cdot \boldsymbol{c}_i(x_i(h)) \tag{25}$$

Again, we have the following decomposition:

$$\begin{cases} \mathcal{TC}_i(q_i(h)) = \mathcal{TC}_i^s(q_i(h)) + \mathcal{TC}_i^{\boldsymbol{\pi}}(q_i(h)) \\ \mathcal{TC}_i^s(q_i(h)) = Q_i(h) \cdot \boldsymbol{c}_i^s \\ \mathcal{TC}_i^{\boldsymbol{\pi}}(q_i(h)) = \mathcal{TC}_i(q_i(h)) - \mathcal{TC}_i^s \end{cases} \tag{26}$$

We can now compute the total and unit costs of the redemption portfolio:

$$\mathcal{TC}(q) = \sum_{h=1}^{h^+} \sum_{i=1}^{n} \mathcal{TC}_i(q_i(h)) \tag{27}$$

and:

$$\boldsymbol{c}(q) = \frac{\mathcal{TC}(q)}{\sum_{i=1}^{n} q_i \cdot P_i} \tag{28}$$

If we are interested in the contribution of the stock i or the trading day h, we have[10]:

$$\mathcal{TC}(q;h) = \sum_{i=1}^{n} \mathcal{TC}_i(q_i(h)) \tag{31}$$

and[11]:

$$\mathcal{TC}_i(q_i) = \sum_{h=1}^{h^+} \mathcal{TC}_i(q_i(h)) \tag{34}$$

We consider the redemption portfolio described in Table 1 on page 4. Using the daily volume, the annualized volatility and the bid-ask quotes given in Table 13 on page 26, we compute the daily participation rate[12] $x_i(h)$, the daily volatility[13] σ_i and the (half) bid-ask spread[14] s_i. Then, we define the spread and price impact component $\boldsymbol{c}_i^s$ and $\boldsymbol{c}_i^{\boldsymbol{\pi}}(x_i(h))$ and we deduce the unit transaction cost $\boldsymbol{c}_i(x_i(h))$. Results are given in Table 16 on page 30. For example, if we consider the first stock ($i=1$), we have $x_1(1) = 9.18\%$, $\sigma_1 = 1.59\%$, $s_1 = 0.89$ bps, $\boldsymbol{c}_1^s = 1.11$ bps, $\boldsymbol{c}_1^{\boldsymbol{\pi}}(x_1(1)) = 22.67$ bps and $\boldsymbol{c}_1(x_1(1)) = 23.78$ bps. When h is equal to 2 or 3, we have $\boldsymbol{c}_1(x_1(h)) = 0$ bps because $x_1(h) = 0$. Then, we compute $Q_i(h) = q_i(h) \cdot P_i$ and deduce $\mathcal{TC}_i(q_i(h))$ and $\mathcal{TC}_i(q_i)$ using Equations (25) and (34). Results are reported in Table 17 on page 31. The transaction cost to liquidate q_1 is equal to €31 936. Finally, liquidating the redemption portfolio implies a total cost $\mathcal{TC}(q)$ of €1 738 156. This represents a unit transaction cost $\boldsymbol{c}(q)$ of 21.73 bps. Table 18 on page 32 also provides the break-down between the spread component $\mathcal{TC}_i^s(q_i)$ and the price impact component $\mathcal{TC}_i^{\boldsymbol{\pi}}(q_i)$.

Remark 2 *From a trading viewpoint, computing the unit transaction cost $\boldsymbol{c}(q)$ makes a lot of sense because we compare the total transaction cost to the value of the redemption portfolio. From a liquidity stress testing viewpoint, it is better to normalize the total transaction cost by the total net assets:*

$$\tilde{\boldsymbol{c}}(q) = \frac{\mathcal{TC}(q)}{\text{TNA}} \tag{37}$$

[10] We have the following decomposition:

$$\mathcal{TC}^s(q;h) = \sum_{i=1}^{n} Q_i(h) \cdot \boldsymbol{c}_i^s \tag{29}$$

and:

$$\mathcal{TC}^{\boldsymbol{\pi}}(q;h) = \mathcal{TC}(q;h) - \mathcal{TC}^s(q;h) \tag{30}$$

[11] We have the following decomposition:

$$\mathcal{TC}_i^s(q_i) = Q_i \cdot \boldsymbol{c}_i^s \tag{32}$$

and:

$$\mathcal{TC}_i^{\boldsymbol{\pi}}(q_i) = \mathcal{TC}_i(q_i) - \mathcal{TC}_i^s(q_i) \tag{33}$$

[12] We use Equation (22).

[13] We have:

$$\sigma_i = \frac{\sigma_i^{\text{yearly}}}{\sqrt{260}} \tag{35}$$

[14] We have:

$$s_i = \frac{P_i^{\text{ask}} - P_i^{\text{bid}}}{P_i^{\text{ask}} + P_i^{\text{bid}}} \tag{36}$$

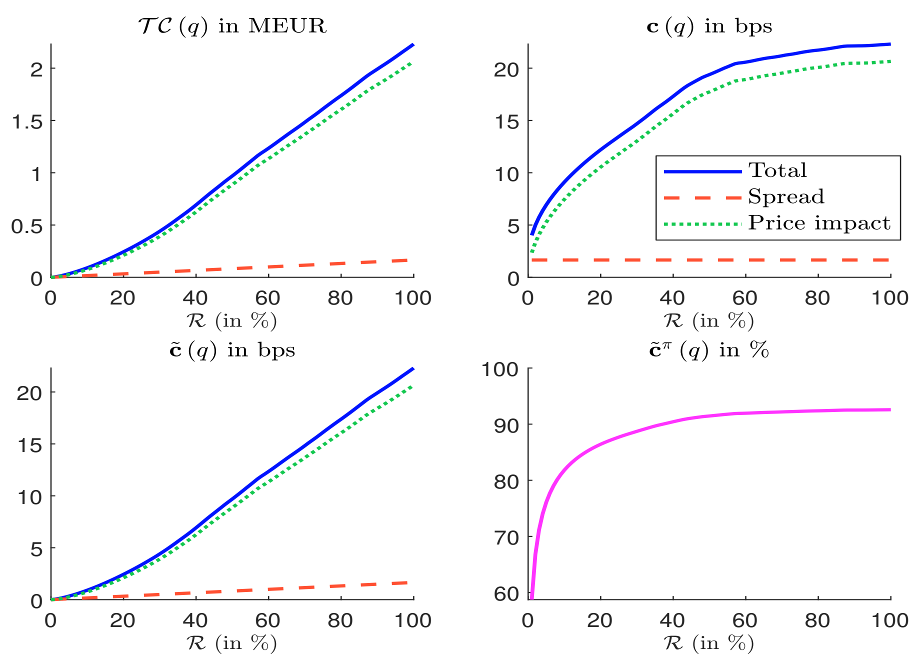
Indeed, the computation of $\boldsymbol{c}(q)$ *measures the real impact of the redemption portfolio by combining the redemption shock and the liquidation cost:*

$$\begin{aligned}\tilde{\boldsymbol{c}}(q) &= \frac{\mathcal{TC}(q)}{\sum_{i=1}^{n} \omega_i \cdot P_i} \\ &= \mathcal{R} \cdot \boldsymbol{c}(q) \end{aligned} \tag{38}$$

Moreover, without the implementation of swing pricing, this is the cost borne by the final investor. In the example above, we obtain $\tilde{\boldsymbol{c}}(q) = 17.38$ *bps and* $\boldsymbol{c}(q) = 21.73$ *bps.*

In Figure 6, we report the transaction cost functions $\mathcal{TC}(q)$, $\boldsymbol{c}(q)$ and $\tilde{\boldsymbol{c}}(q)$. For each cost function, we indicate the total value and also the breakdown between the spread and the price impact. In the last panel, we show the proportion of $\boldsymbol{c}(q)$ due to the price impact. In Figure 7, we illustrate the breakdown per asset when the redemption rate is equal to 5%.

Figure 6: Transaction cost of the large-cap portfolio (TNA = €1 bn)



2.3.2 The case of small-cap stocks

For small-cap stocks, the only difference concerns the sensitivity coefficients of the unit transaction cost function. Following Roncalli *et al.* (2021b, Equation (44), page 40), we have:

$$\boldsymbol{c}_i(x_i(h)) = \begin{cases} 1.40\, s_i + 0.50\, \sigma_i \sqrt{x_i(h)} & \text{if } x_i(h) \leq \tilde{x}_i \\ 1.40\, s_i + \dfrac{0.50}{\sqrt{\tilde{x}_i}} \sigma_i x_i & \text{if } \tilde{x}_i \leq x_i(h) \leq x_i^{+} \\ +\infty & \text{if } x_i(h) > x_i^{+} \end{cases} \tag{39}$$

We consider the small-cap portfolio. Figure 8 shows the breakdown per trading day when we assume that the redemption rate is equal to 5%. Finally, we obtain $\mathcal{TC}(q) = €147\,560$.

Figure 7: Breakdown per asset in € (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 5\%$)

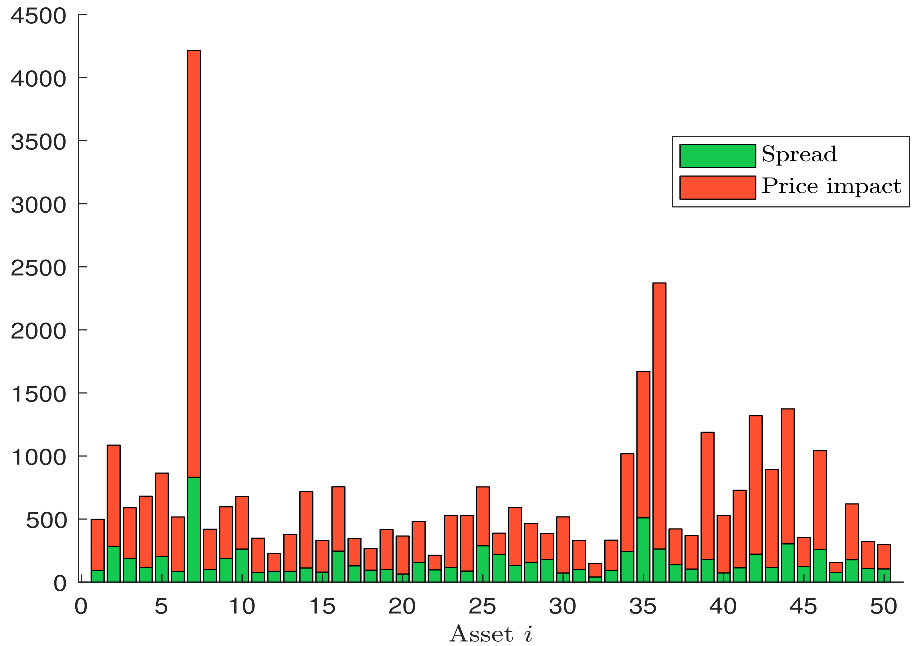
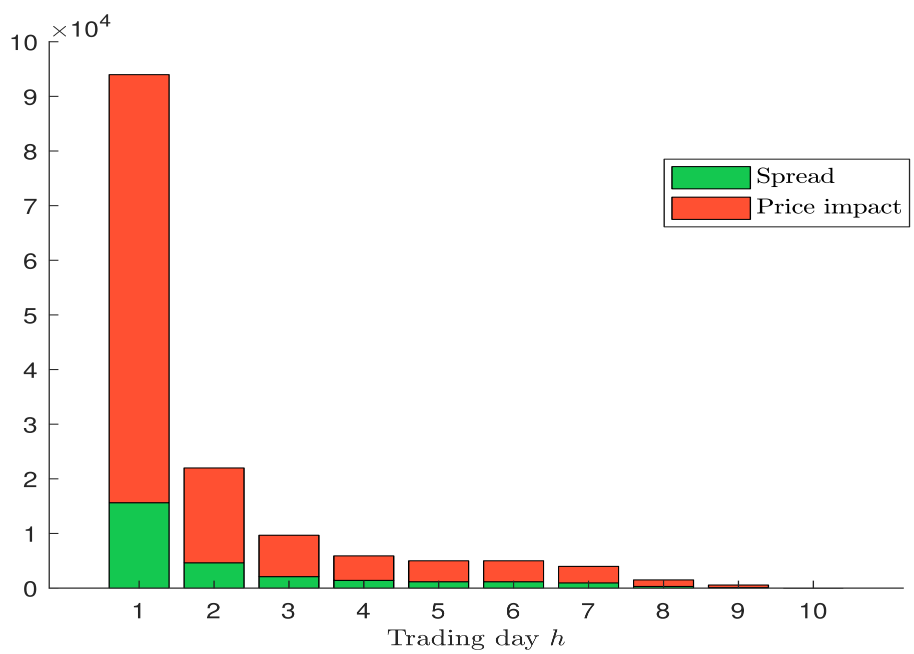


Figure 8: Breakdown per trading day in € (small-cap portfolio, TNA = €1 bn, $\mathcal{R} = 5\%$)



2.3.3 Stress testing

The transaction cost function remains the same for the stress period, but the three parameters s_i, σ_i and v_i change because we apply shocks to them. Since the stress test scenario is defined by the triplet $(\Delta_s, \Delta_\sigma, m_v)$, it follows that Equation (23) becomes:

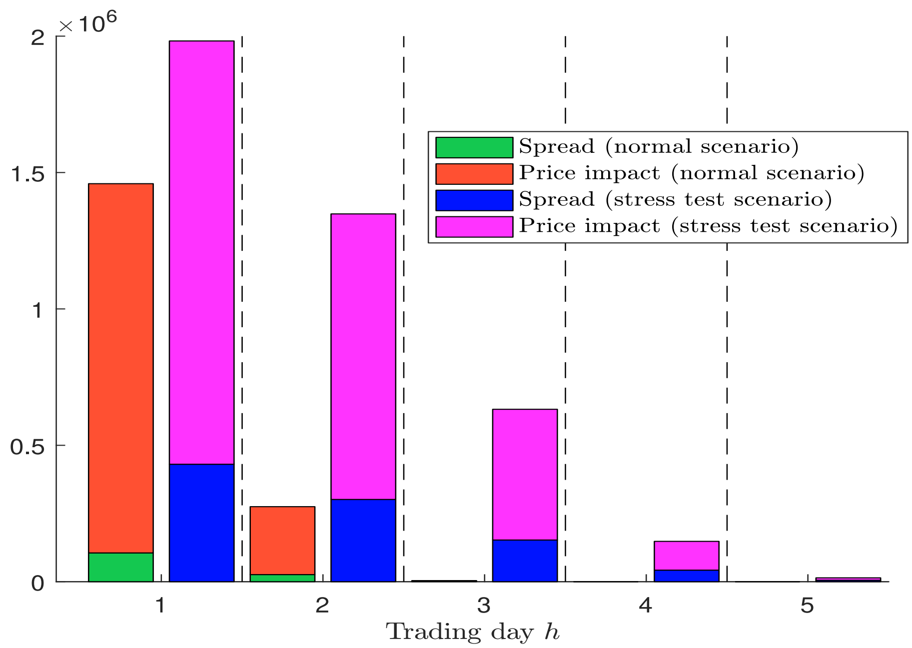
$$\boldsymbol{c}_i\left(x_i\left(h\right)\right)=\begin{cases}1.25\,\left(s_i+\Delta_s\right)+0.40\left(\sigma_i+\dfrac{\Delta_\sigma}{\sqrt{260}}\right)\sqrt{x_i} & \text{if } x_i\left(h\right)\leq\tilde{x}_i\\ 1.25\,\left(s_i+\Delta_s\right)+\dfrac{0.40}{\sqrt{\tilde{x}_i}}\left(\sigma_i+\dfrac{\Delta_\sigma}{\sqrt{260}}\right)x_i & \text{if } \tilde{x}_i\leq x_i\left(h\right)\leq x_i^{+}\\ +\infty & \text{if } x_i\left(h\right)>x_i^{+}\end{cases} \tag{40}$$

where:

$$x_i\left(h\right)=\frac{q_i\left(h\right)}{m_v\cdot v_i} \tag{41}$$

Therefore, a stress test scenario has three impacts: (1) for a given value of $q_i(h)$, the participation rate $x_i(h)$ is larger; (2) the transaction cost $\boldsymbol{c}_i(x_i(h))$ is higher because the participation rate, the bid-ask spread and the volatility are higher; (3) moreover, it takes more time to liquidate the redemption portfolio since we have $q_i^+ = m_v \cdot x_i^+ \cdot v_i$, meaning that the daily trading limits expressed in number of shares are reduced.

Figure 9: Breakdown per trading day in € (large-cap portfolio, TNA = €1 bn, $\mathcal{R}$ = 80%)



We consider the example described in Section 2.3.1 on page 13. Previously, we found that the total transaction cost $\mathcal{TC}(q)$ was equal to 1 738 156 euros. representing a redemption transaction cost $\boldsymbol{c}(q)$ of 21.73 bps and a portfolio transaction cost $\tilde{\boldsymbol{c}}(q)$ of 17.38 bps. Let us assume the following stress test scenario: $\Delta_s = 8$ bps, $\Delta_\sigma = 20\%$ and $m_v = 0.50$. We now obtain[15] $\mathcal{TC}^{\text{stress}}(q) = 4\,124\,811$ euros, $\boldsymbol{c}(q) = 51.56$ bps and $\tilde{\boldsymbol{c}}(q) = 41.25$ bps. In Figure 9, we compare the breakdown per trading day for the normal and stress cases.

[15] See Tables 19, 20 and 21 on page 33 for computational details.

3 The case of bond portfolios

The case of bond portfolios is similar to the case of equity portfolios. This is why we only focus on the most important differences when doing a liquidity stress testing exercise with bond portfolios.

3.1 Computing the liquidation portfolio

We define the redemption portfolio by the vector of liquidated bonds $q = (q_1, \ldots, q_n)$. Nevertheless, in the fixed-income universe, we may prefer to consider nominal values $Q_i = q_i \cdot P_i$ instead of real values q_i. Therefore, the redemption portfolio is also defined by $Q = (Q_1, \ldots, Q_n)$. Let $Q_i(h)$ be the exposure in the i^{th} bond liquidated after h trading days. We have:

$$Q_i(h) = \min\left(\left(Q_i - \sum_{k=0}^{h-1} Q_i(k)\right)^+, Q_i^+\right) \tag{42}$$

where $Q_i(0) = 0$ and Q_i^+ is the maximum trading limit that can be sold during a trading day for the bond i. Equation (42) is equivalent to Equation (1) on page 3 because we have:

$$q_i(h) = \frac{Q_i(h)}{P_i} \tag{43}$$

The formulas for $\mathcal{LR}(q; h)$, $\text{RCR}(h)$ and $\text{LS}(h)$ remain the same as previously.

We consider the portfolio described on page 28. It is made up of 11 sovereign bonds and 36 corporate bonds. In Table 15, we report the values of Q_i^+. We assume that Q_i^+ is equal to \$50 mn for US sovereign bonds, \$6 mn for senior corporate bonds and \$3 mn for non-senior corporate bonds (e.g. subordinated debt). If we consider a redemption rate equal to 30%, we obtain the liquidation values $Q_i(h)$ given in Table 10. We have $\mathcal{LR}(q; 1) = 0.9566$, $\mathcal{LR}(q; 2) = 0.9958$ and $\mathcal{LR}(q; 3) = 1$. The redemption portfolio can almost be liquidated during the first day, since only the exposures on the four subordinated bonds require more than one day to be sold.

3.2 Computing the redemption coverage ratio

We scale up the portfolio in order to obtain TNA = \$10 bn or TNA = \$20 bn. Results are given in Table 11. When the total net assets are equal to \$10 bn and the redemption rate is set to 30%, the redemption coverage ratio $\text{RCR}(1)$ is equal to 0.251 if we consider a pro-rata liquidation or a waterfall liquidation. Therefore, we notice that the liquidation policy may have no impact on the RCR. The reason lies in the fact that $Q_i(h)$ is equal to Q_i^+ when the time horizon h is low. This means that the trading limits are reached for all the bonds. In this case, we cannot use a bond i to liquidate the exposure on the bond j.

Stress testing is implemented by considering a shock on the trading limits:

$$Q_i(h) = \min\left(\left(Q_i - \sum_{k=0}^{h-1} Q_i(k)\right)^+, m_{Q^+} \cdot Q_i^+\right) \tag{44}$$

where $m_{Q^+} \leq 1$ is the stress parameter of the trading limit Q_i^+. For instance, if m_v is equal to 50%, we obtain the right panel in Table 11. We notice the high impact of the stress parameter m_{Q^+} on the redemption coverage ratio[16].

[16]See Figures 17 and 18 on page 40 for a more visual illustration.

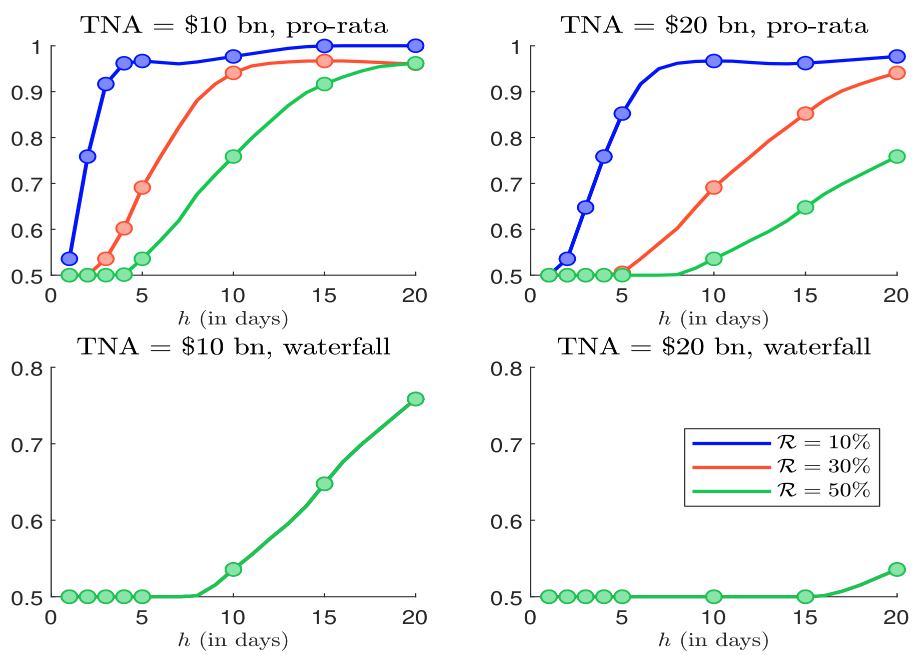
Table 10: Liquidation of the redemption portfolio (bond portfolio, TNA = \$1 bn, $\mathcal{R} = 30\%$, vertical slicing)

i	Q_i	Q_i^+	$Q_i(1)$	$Q_i(2)$	$Q_i(3)$	$\sum_{h=1}^{3} Q_i(h)$
1	16 255 353	50 000 000	16 255 353	0	0	16 255 353
2	12 718 447	50 000 000	12 718 447	0	0	12 718 447
3	12 017 318	50 000 000	12 017 318	0	0	12 017 318
4	13 233 346	50 000 000	13 233 346	0	0	13 233 346
5	12 063 483	50 000 000	12 063 483	0	0	12 063 483
6	11 544 025	50 000 000	11 544 025	0	0	11 544 025
7	21 235 655	50 000 000	21 235 655	0	0	21 235 655
8	20 407 521	50 000 000	20 407 521	0	0	20 407 521
9	20 072 491	50 000 000	20 072 491	0	0	20 072 491
10	13 683 284	50 000 000	13 683 284	0	0	13 683 284
11	26 768 829	50 000 000	26 768 829	0	0	26 768 829
12	3 802 896	6 000 000	3 802 896	0	0	3 802 896
13	3 282 954	6 000 000	3 282 954	0	0	3 282 954
14	1 476 301	6 000 000	1 476 301	0	0	1 476 301
15	2 064 731	6 000 000	2 064 731	0	0	2 064 731
16	2 954 414	6 000 000	2 954 414	0	0	2 954 414
17	2 359 638	6 000 000	2 359 638	0	0	2 359 638
18	1 887 457	6 000 000	1 887 457	0	0	1 887 457
19	3 538 056	6 000 000	3 538 056	0	0	3 538 056
20	6 906 942	3 000 000	3 000 000	3 000 000	906 942	6 906 942
21	6 153 350	3 000 000	3 000 000	3 000 000	153 350	6 153 350
22	2 639 184	6 000 000	2 639 184	0	0	2 639 184
23	4 773 841	6 000 000	4 773 841	0	0	4 773 841
24	3 007 583	6 000 000	3 007 583	0	0	3 007 583
25	5 735 256	3 000 000	3 000 000	2 735 256	0	5 735 256
26	6 210 205	3 000 000	3 000 000	3 000 000	210 205	6 210 205
27	3 561 300	6 000 000	3 561 300	0	0	3 561 300
28	2 978 950	6 000 000	2 978 950	0	0	2 978 950
29	4 166 994	6 000 000	4 166 994	0	0	4 166 994
30	2 979 133	6 000 000	2 979 133	0	0	2 979 133
31	2 920 499	6 000 000	2 920 499	0	0	2 920 499
32	2 358 223	6 000 000	2 358 223	0	0	2 358 223
33	2 652 842	6 000 000	2 652 842	0	0	2 652 842
34	4 958 507	6 000 000	4 958 507	0	0	4 958 507
35	3 535 819	6 000 000	3 535 819	0	0	3 535 819
36	2 650 657	6 000 000	2 650 657	0	0	2 650 657
37	2 662 310	6 000 000	2 662 310	0	0	2 662 310
38	3 140 845	6 000 000	3 140 845	0	0	3 140 845
39	4 155 263	6 000 000	4 155 263	0	0	4 155 263
40	2 803 320	6 000 000	2 803 320	0	0	2 803 320
41	2 293 116	6 000 000	2 293 116	0	0	2 293 116
42	1 946 863	6 000 000	1 946 863	0	0	1 946 863
43	2 656 800	6 000 000	2 656 800	0	0	2 656 800
44	2 295 612	6 000 000	2 295 612	0	0	2 295 612
45	3 267 302	6 000 000	3 267 302	0	0	3 267 302
46	2 270 545	6 000 000	2 270 545	0	0	2 270 545
47	2 952 538	6 000 000	2 952 538	0	0	2 952 538

Table 11: Redemption coverage ratio RCR (h) (bond portfolio, $\mathcal{R} = 30\%$)

		Normal period				Stress period			
		Pro-rata		Waterfall		Pro-rata		Waterfall	
	TNA	10	20	10	20	10	20	10	20
h	1	0.251	0.126	0.251	0.126	0.126	0.063	0.126	0.063
	2	0.503	0.251	0.503	0.251	0.251	0.126	0.251	0.126
	3	0.704	0.377	0.754	0.377	0.377	0.188	0.377	0.188
	4	0.835	0.503	1.005	0.503	0.503	0.251	0.503	0.251
	5	0.900	0.622	1.257	0.628	0.622	0.314	0.628	0.314
	6	0.928	0.704	1.508	0.754	0.704	0.377	0.754	0.377
	7	0.940	0.773	1.759	0.880	0.773	0.440	0.880	0.440
	8	0.948	0.835	2.006	1.005	0.835	0.503	1.005	0.503
	9	0.953	0.873	2.195	1.131	0.873	0.565	1.131	0.565
	10	0.957	0.900	2.346	1.257	0.900	0.622	1.257	0.628

Figure 10: Multiplier stress factor $m_{\mathrm{RCR}}(h)$ ($m_{Q^+} = 50\%$)



Remark 3 *The stress parameter m_{Q^+} is exactly equal to the previous stress parameter m_v. Indeed, we have $q_i^+ = x_i^+ \cdot v_i$ and $Q_i^+ = q_i^+ \cdot P_i$. The stressed values are defined as $q_i^{+,\text{stress}} = m_v \cdot q_i^+$ and $Q_i^{+,\text{stress}} = m_{Q^+} \cdot Q_i^+$. We deduce that $Q_i^{+,\text{stress}} = q_i^+ \cdot P_i = m_v \cdot q_i^+ \cdot P_i = m_v \cdot Q_i^+$ and $m_{Q^+} = m_v$.*

We note $\text{RCR}^{\text{normal}}(h)$ the redemption coverage ratio when m_{Q^+} (or m_v) is equal to one. The stressed value $\text{RCR}^{\text{stress}}(h)$ is computed with $m_{Q^+} < 1$. We define the multiplier stress factor of the redemption coverage ratio as:

$$m_{\text{RCR}}(h) = \frac{\text{RCR}^{\text{stress}}(h)}{\text{RCR}^{\text{normal}}(h)} \tag{45}$$

By construction, we have $\text{RCR}^{\text{stress}}(\infty) = \text{RCR}^{\text{normal}}(\infty)$, implying that:

$$m_{Q^+} \leq m_{\text{RCR}}(h) \leq 1 \tag{46}$$

In Figure 10, we report $m_{\text{RCR}}(h)$ when we consider the pro-rata liquidation and the waterfall liquidation. We notice that the stress test scenario has a bigger impact on the pro-rata liquidation than on the waterfall liquidation. In particular, in this example, $m_{\text{RCR}}(h)$ does not depend on the redemption shock when we use the waterfall liquidation.

3.3 Computing the transaction cost

In the case of bonds, the participation rate is defined with respect to the outstanding amount $\mathcal{M}_i$ and not the daily volume v_i:

$$y_i(h) = \frac{Q_i(h)}{\mathcal{M}_i} \tag{47}$$

Following Roncalli *et al.* (2021b), we use the following unit transaction cost for the sovereign bonds:

$$\boldsymbol{c}_i(y_i(h)) = 1.25\, s_i + 3.00\, \sigma_i y_i(h)^{0.25} \qquad \text{if } y_i(h) \leq y_i^+ \tag{48}$$

where:

$$y_i^+ = \frac{Q_i^+}{\mathcal{M}_i} \tag{49}$$

In the case of corporate bonds, the formula becomes:

$$\boldsymbol{c}_i(y_i(h)) = 1.50\, s_i + 0.125\ \text{DTS}_i\, y_i(h)^{0.25} \qquad \text{if } y_i(h) \leq y_i^+ \tag{50}$$

where DTS_i is the duration-times-spread measure of the bond i. Again, we can introduce a second transaction cost regime, where the price impact is linear. In this case, we have:

$$\boldsymbol{c}_i(y_i(h)) = \begin{cases} 1.25\, s_i + 3.00\, \sigma_i y_i(h)^{0.25} & \text{if } y_i(h) \leq \tilde{y}_i \\ 1.25\, s_i + \dfrac{3.00}{\tilde{y}_i^{0.75}}\, \sigma_i y_i(h) & \text{if } \tilde{y}_i \leq y_i(h) \leq y_i^+ \end{cases} \tag{51}$$

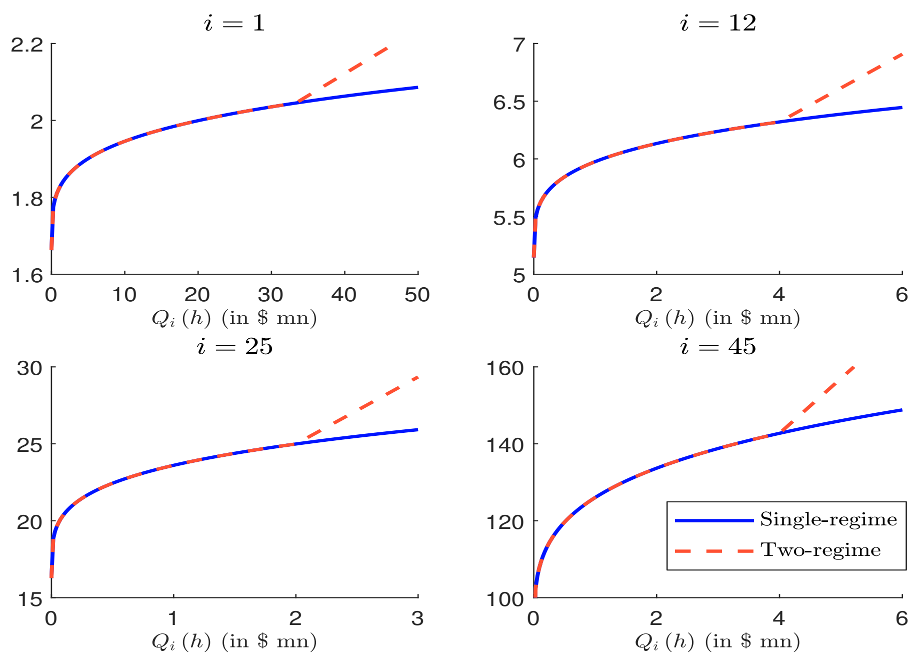
and:

$$\boldsymbol{c}_i(y_i(h)) = \begin{cases} 1.50\, s_i + 0.125\ \text{DTS}_i\, y_i(h)^{0.25} & \text{if } y_i(h) \leq \tilde{y}_i \\ 1.50\, s_i + \dfrac{0.125}{\tilde{y}_i^{0.75}}\ \text{DTS}_i\, y_i(h) & \text{if } \tilde{y}_i \leq y_i(h) \leq y_i^+ \end{cases} \tag{52}$$

where $\tilde{y}_i = \dfrac{2}{3} y_i^+$.

Using the data given in Table 15 on page 28, we compute the unit transaction cost with respect to $Q_i(h)$ in Figure 11. The first panel uses the formula of sovereign bonds while the three other panels use the formula of corporate bonds. $i = 1$ corresponds to the UST bond. The spread s_i is equal to 1.33 bps and the yearly volatility is equal to 16 bps. This explains the low transaction cost. $i = 45$ corresponds to the Petronas bond. The transaction cost is very high because the bid-ask spread s_i is equal to 57.15 bps and the DTS is equal to 2635 bps.

Figure 11: Unit transaction cost function



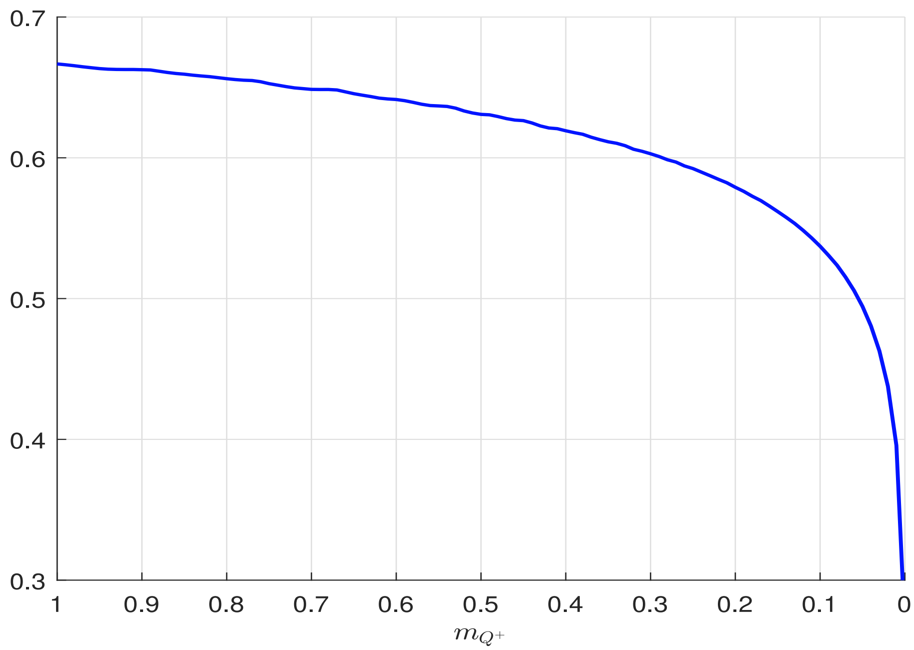
We consider the bond portfolio, whose total net assets are equal to \$10 bn. By assuming a 30% redemption rate and a vertical slicing approach, we obtain $\mathcal{TC}(q) = 10.68$ million dollars. This implies a liquidation cost $\boldsymbol{c}(q)$ of 35.60 bps and an investment cost $\tilde{\boldsymbol{c}}(q)$ of 10.68 bps. The price impact cost represents 68.90% of the total cost[17].

Let us consider the following stress test scenario: $\Delta_s = 3$ bps, $\Delta_{\sigma^{\text{yearly}}} = 2\%$, $\Delta_{\text{DTS}} = 100$ bps and $m_{Q^+} = 0.50$. We now obtain $\mathcal{TC}^{\text{stress}}(q) = 12.29$ million dollars. In Table 12, we compare the breakdown between the spread and price impact components. We notice that the ratio of the transaction cost that is explained by the price impact is greater for the normal scenario than for the stress test scenario. At first sight, this result may be curious, because we expect that price impacts increase when we stress the liquidity condition. The issue comes from the definition of the participation rate. In the case of stocks, we have $x_i(h) = \dfrac{q_i(h)}{v_i}$. When we stress the volume v_i by applying the stress factor m_v, we have two effects: an impact on $q_i(h)$ and an impact on v_i. Finally, the stressed value of $x_i(h)$ is greater or equal to the normal value of $x_i(h)$. This is not the case for bonds. Indeed, we have:

$$y_i(h) = \frac{Q_i(h)}{\mathcal{M}_i} \tag{53}$$

[17] More figures are given in Tables 22 and 23 on page 36.

Figure 12: Ratio of transaction cost explained by the price impact (bond portfolio, TNA = \$10 bn, $\mathcal{R} = 30\%$, vertical slicing)



When we stress the liquidity policy by applying the multiplicative factor m_{Q^+}, we have an impact on the liquidation shares $Q_i(h)$, but no effect on the outstanding amount $\mathcal{M}_i$. Therefore, the stressed value of $y_i(h)$ is less than the normal value of $y_i(h)$. Finally, the part of the transaction cost due to the price impact is reduced when we decrease m_{Q^+} (see Figure 12). In order to be consistent with the equity framework, we propose to scale the participation rate:

$$y_i^{\text{stress}^*}(h) = \frac{1}{m_{Q^+}} \left(\frac{Q_i(h)}{\mathcal{M}_i} \right) \tag{54}$$

This approach is called "*stress**" in order to distinguish it from the previous one. In Table 12 we report the transaction cost of the redemption portfolio when we assume that $\mathcal{R} = 30\%$ and the fund size is equal to \$10 bn. In the normal case, the investment cost is equal to 10.68 bps. However, this figure is not representative of a normal cost. Indeed, we use a too high redemption value ($\mathcal{R} = 30\%$). If we assume that $\mathcal{R} = 30\%$, we obtain $\tilde{\boldsymbol{c}}(q) = 1.53$ bps. This investment cost becomes 12.29 bps when we consider the stress test scenario. If we implement the correction given by Equation (54), we finally obtain $\tilde{\boldsymbol{c}}(q) = 13.75$ bps.

Table 12: Spread and price impact components in bps (bond portfolio, TNA = \$10 bn, $\mathcal{R} = 30\%$, vertical slicing)

Scenario	$\boldsymbol{c}^s(q)$	$\boldsymbol{c}^{\boldsymbol{\pi}}(q)$	$\boldsymbol{c}(q)$	$\tilde{\boldsymbol{c}}^s(q)$	$\tilde{\boldsymbol{c}}^{\boldsymbol{\pi}}(q)$	$\tilde{\boldsymbol{c}}(q)$
Normal ($\mathcal{R} = 5\%$)	11.07	19.51	30.58	0.55	0.98	1.53
Normal	11.07	24.53	35.60	3.32	7.36	10.68
Stress	15.12	25.84	40.96	4.54	7.75	12.29
Stress*	15.12	30.73	45.85	4.54	9.22	13.75

4 Conclusion

This working paper complements the publication of the three previous research papers dedicated to the liquidity stress testing in asset management (Roncalli *et al.*, 2021a,b; Roncalli, 2021c). Using three portfolios, we have explained how to compute the redemption coverage ratio, implement reverse stress testing and estimate the liquidation cost of the redemption portfolio. The first portfolio is an index portfolio corresponding to the Eurostoxx 50 index. The second portfolio is an active equity portfolio, which is made up of 20 small- and mid-cap stocks. Finally, the third portfolio corresponds to 47 sovereign and corporate bonds. These examples illustrate the main asset classes that are concerned by liquidity stress testing: large-cap stocks, small-cap stocks, sovereign bonds, corporate bonds. We have excluded derivatives from this research project because of the lack of research on this topic. Nevertheless, these examples must enable asset managers to understand the common basics of liquidity stress testing in asset management.

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Appendix

A Data

We use three portfolios[18]:

1. Table 13 presents a first equity portfolio based on the Eurostoxx 50 index. It is made up of 50 stocks with the following correspondence: (1) Adidas, (2) Adyen, (3) Air Liquide, (4) Airbus, (5) Allianz, (6) Anheuser-Busch, (7) ASML, (8) AXA, (9) Banco Santander, (10) BASF, (11) Bayer, (12) BMW, (13) BBVA, (14) BNP Paribas, (15) CRH, (16) Daimler, (17) Danone, (18) Deutsche Boerse, (19) Deutsche Post, (20) Deutsche Telekom, (21) Enel, (22) Eni, (23) EssilorLuxottica, (24) Flutter Entertainment, (25) Iberdrola, (26) Inditex, (27) Infineon Technolog, (28) ING, (29) Intesa Sanpaolo, (30) Kering, (31) Kon Ahold Delhaize, (32) Kone, (33) Koninklijke Philips, (34) L'Oreal, (35) Linde, (36) LVMH, (37) Muenchener Rueckve, (38) Pernod Ricard, (39) Prosus, (40) Safran, (41) Sanofi, (42) SAP, (43) Schneider Electric, (44) Siemens, (45) Stellantis, (46) TotalEnergies, (47) Universal Music, (48) Vinci, (49) Volkswagen, (50) Vonovia.

2. In Table 14, we consider a second equity portfolio with more small and mid-cap stocks. We have the following correspondence: (1) Heineken, (2) Zalando, (3) ArcelorMittal, (4) Kerry Group, (5) Porsche, (6) Puma, (7) Hannover Rueck, (8) Solvay, (9) Euronext, (10) Sodexo, (11) Gecina, (12) Ubisoft Entertainment, (13) Rational, (14) Deutsche Lufthansa, (15) Ackermans & van Haaren, (16) Aeroports de Paris, (17) Telecom Italia, (18) Christian Dior, (19) Sopra Steria Group, (20) Dassault Aviation.

3. The last portfolio is made up of 11 US bonds and 36 USD-denominated corporate bonds. Table 15 gives the different characteristics for the different bonds: (1) UST (T 1 5/8 11/15/22), (2) UST (T 0 1/4 05/15/24), (3) UST (T 0 3/8 08/15/24), (4) UST (T 1 5/8 08/15/29), (5) UST (T 2 3/8 05/15/29), (6) UST (T 3 1/8 11/15/28), (7) UST (T 1 5/8 05/15/31), (8) UST (T 1 1/4 08/15/31), (9) UST (T 1 1/8 02/15/31), (10) UST (T 2 3/8 05/15/51), (11) UST (T 2 08/15/51), (12) Apple (AAPL 3 02/09/24), (13) DNB Bank (DNBNO 0.856 09/30/25), (14) Equinor (EQNR 2 7/8 04/06/25), (15) Bank of America (BAC 3.093 10/01/25), (16) Comcast (CMCSA 3.7 04/15/24), (17) Cnooc Finance (CNOOC 3 05/09/23), (18) EMD Finance (MRKGR 3 1/4 03/19/25), (19) Boeing (BA 4.508 05/01/23), (20) Bank of America (BCHINA 5 11/13/24), (21) BP Capital Markets (BPLN 4 3/8 PERP), (22) Apple (AAPL 3.35 02/09/27), (23) Nestle (NESNVX 3 5/8 09/24/28), (24) Alibaba (BABA 3.4 12/06/27), (25) Bank of America (BAC 4.183 11/25/27), (26) Dai-Ichi Life Insurance (DAIL 4 PERP), (27) NTT Finance (NTT 1.162 04/03/26), (28) Anheuser-Busch (ABIBB 4 04/13/28), (29) Bat Capital (BATSLN 3.557 08/15/27), (30) Bayer (BAYNGR 4 1/4 12/15/25), (31) Nissan (NSANY 4.345 09/17/27), (32) Amazon (AMZN 1 1/2 06/03/30), (33) Alphabet (GOOGL 1.1 08/15/30), (34) Shell (RDSALN 2 3/4 04/06/30), (35) Bank of America (BAC 2.592 04/29/31), (36) BNP Paribas (BNP 2.871 04/19/32), (37) Petronas (PETMK 3 1/2 04/21/30), (38) British Telecom (BRITEL 9 5/8 12/15/30), (39) Deutsche Telekom (DT 8 1/4 06/15/30), (40) Orange (ORAFP 8 1/2 03/01/31), (41) Shell (RDSALN 3 1/4 04/06/50), (42) Abbott Laboratories (ABT 4 3/4 11/30/36), (43) Saudi Arabian Oil (ARAMCO 3 1/4 11/24/50), (44) Bank of America (BAC 4.443 01/20/48), (45) Petronas (PETMK 4.55 04/21/50), (46) Credit Suisse (CS 4 7/8 05/15/45) and (47) Telefonica (TELEFO 5.213 03/08/47).

[18] The raw data are available at https://www.researchgate.net/publication/356849853.

Table 13: Large-cap equity portfolio (Eurostoxx 50 index), TNA = €1 bn)

i	ω_i	P_i	P_i^{bid}	P_i^{ask}	σ_i^{yearly}	v_i
1	59 106	284.050	281.750	281.800	25.69	514 842
2	8 883	2630.500	2567.500	2568.500	31.14	56 255
3	150 027	143.600	142.880	142.920	13.75	629 509
4	184 310	112.000	111.640	111.660	26.42	1 316 600
5	130 520	200.950	200.150	200.200	21.75	750 684
6	268 123	54.460	53.280	53.290	27.08	1 736 372
7	131 520	698.400	690.100	690.300	31.82	754 901
8	651 421	24.415	24.760	24.765	18.72	4 358 304
9	3 192 430	5.641	5.988	5.990	33.74	54 130 721
10	5 544 072	3.2805	3.2465	3.2480	29.85	72 371 040
11	242 317	62.550	62.190	62.200	20.69	2 473 040
12	181 800	48.735	48.805	48.820	21.39	2 444 130
13	136 334	87.340	86.610	86.630	26.13	1 183 053
14	365 067	57.460	58.250	58.260	26.61	2 390 614
15	251 719	41.360	41.060	41.070	20.83	1 434 050
16	265 762	83.850	84.870	84.900	26.15	2 672 846
17	206 041	56.040	55.810	55.830	18.72	1 579 517
18	60 148	144.200	142.700	142.750	17.01	339 768
19	311 879	54.040	53.160	53.170	19.38	2 536 147
20	1 026 503	16.032	15.942	15.944	19.77	9 225 311
21	2 459 244	7.270	7.213	7.215	23.17	30 518 046
22	795 234	12.164	12.408	12.412	21.32	19 419 467
23	95 262	172.860	177.120	177.160	20.43	491 647
24	55 520	165.600	163.600	163.650	33.71	212 501
25	1 840 196	10.250	10.225	10.230	24.90	14 316 692
26	351 837	31.050	30.910	30.930	25.62	7 543 014
27	413 417	40.100	39.700	39.710	29.69	3 643 730
28	1 235 905	13.082	13.122	13.126	25.02	15 954 487
29	5 774 696	2.438	2.441	2.442	22.67	106 942 206
30	23 113	650.900	646.100	646.200	31.69	204 628
31	161 807	57.460	57.960	57.980	19.61	786 412
32	261 645	28.085	27.965	27.970	15.39	2 285 287
33	290 422	40.665	40.250	40.260	21.56	2 489 971
34	76 637	393.450	388.850	388.950	20.44	372 415
35	162 956	271.800	271.300	271.400	17.81	578 973
36	83 427	671.700	666.100	666.200	28.37	364 566
37	44 351	254.400	255.300	255.400	21.80	256 421
38	64 954	201.100	197.000	197.050	17.37	363 103
39	282 801	76.530	75.000	75.020	46.16	2 135 693
40	120 062	114.080	116.920	116.940	30.87	794 444
41	362 506	85.990	85.770	85.780	14.72	1 551 395
42	345 779	126.300	123.060	123.080	21.01	1 859 400
43	180 140	148.900	145.600	145.620	22.27	818 822
44	237 952	139.840	136.940	136.980	25.33	1 136 151
45	660 350	17.286	17.078	17.084	28.79	10 497 975
46	835 885	43.295	43.610	43.620	21.90	6 596 020
47	247 964	25.120	24.970	24.980	12.73	1 943 066
48	189 196	91.670	91.590	91.620	20.21	912 539
49	57 954	194.780	193.200	193.260	29.39	1 071 749
50	163 680	54.060	52.680	52.700	19.20	976 446

The portfolio holding ω_i and the daily volume v_i are measured in number of shares, the yearly volatility σ_i^{yearly} is expressed in %, whereas the prices (P_i, P_i^{bid} and P_i^{ask}) are in euros.

Table 14: Small-cap equity portfolio (Eurostoxx index), TNA = €1 bn)

i	ω_i	P_i	P_i^{bid}	P_i^{ask}	σ_i^{yearly}	v_i
1	505 766	98.860	98.840	98.860	17.68	432 593
2	645 495	77.460	77.460	77.480	34.36	725 019
3	1 830 496	27.315	27.310	27.320	41.55	5 513 446
4	436 110	114.650	114.650	114.750	16.88	221 468
5	592 839	84.340	84.300	84.320	27.48	535 423
6	447 628	111.700	111.650	111.750	21.57	291 008
7	309 311	161.650	161.600	161.700	20.60	92 108
8	473 261	105.650	105.600	105.700	17.23	134 879
9	531 915	94.000	93.950	94.050	21.40	201 095
10	597 944	83.620	83.600	83.660	26.02	298 374
11	407 997	122.550	122.500	122.550	17.73	90 460
12	1 066 100	46.900	46.890	46.910	30.24	566 323
13	56 883	879.000	878.400	879.400	34.17	8 328
14	7 423 908	6.735	6.734	6.738	38.20	12 831 900
15	324 464	154.100	154.000	154.200	14.60	22 415
16	413 908	120.800	120.600	120.750	30.83	90 763
17	153 940 887	0.325	0.324	0.325	25.92	179 923 300
18	70 521	709.000	709.000	709.500	28.73	4 061
19	289 687	172.600	172.500	172.700	26.86	19 850
20	537 923	92.950	92.850	93.000	24.35	41 414

The portfolio holding ω_i and the daily volume v_i are measured in number of shares, the yearly volatility σ_i^{yearly} is expressed in %, whereas the prices (P_i, P_i^{bid} and P_i^{ask}) are in euros.

Table 15: Bond portfolio (USD-denominated, TNA = $1 bn)

i	Isin	ω_i	P_i	s_i	σ_i^{yearly}	DTS_i	$\mathcal{M}_i$	Q_i^+
1	US912828TY62	529 725	102.288	1.33	0.16		121 993	50
2	US91282CCC38	427 182	99.243	1.56	0.99		88 769	50
3	US91282CCT62	403 263	99.334	1.21	1.46		83 876	50
4	US912828YB05	432 386	102.018	2.31	4.65		92 619	50
5	US9128286T26	372 123	108.060	1.46	4.31		84 427	50
6	US9128285M81	339 801	113.243	2.10	3.91		80 506	50
7	US91282CCB54	695 025	101.846	1.55	5.90		148 501	50
8	US91282CCS89	695 930	97.747	1.60	6.25		142 197	50
9	US91282CBL46	689 215	97.079	1.61	5.88		140 063	50
10	US912810SX72	412 836	110.482	2.86	15.15		95 481	50
11	US912810SZ21	880 939	101.289	3.10	17.88		91 407	50
12	US037833CG39	120 000	105.636	3.43	0.92	43	1 750	6
13	US25601B2A27	110 140	99.357	6.52	2.44	78	1 250	6
14	US29446MAD48	46 500	105.828	9.09	2.38	101	1 250	6
15	US06051GGT04	65 100	105.721	9.87	1.19	137	1 750	6
16	US20030NCR08	92 000	107.044	4.07	1.06	53	2 500	6
17	US12625GAC87	75 300	104.455	6.65	0.88	119	2 000	6
18	US26867LAL45	59 000	106.636	6.17	1.57	124	1 600	6
19	US097023CS21	112 300	105.018	2.19	1.30	101	3 000	6
20	US061202AA55	205 000	112.308	9.37	2.08	265	3 000	3
21	US05565QDU94	192 000	106.829	3.56	4.29	559	2 500	3
22	US037833CJ77	80 000	109.966	7.56	2.44	137	2 250	6
23	US641062AF17	142 000	112.062	13.08	3.94	243	1 250	6
24	US01609WAT99	92 800	108.031	10.18	3.74	518	2 550	6
25	US06051GGC78	170 000	112.456	10.85	2.74	392	2 000	3
26	US23380YAD94	190 500	108.665	26.74	2.10	555	2 500	3
27	US62954WAC91	120 000	98.925	7.94	2.20	168	3 000	6
28	US035240AL43	88 000	112.839	13.64	3.54	300	2 500	6
29	US05526DBB01	130 000	106.846	15.23	2.79	566	3 500	6
30	US07274NAJ28	89 000	111.578	11.56	2.04	257	2 500	6
31	US654744AC50	89 000	109.382	7.22	3.08	753	2 500	6
32	US023135BS49	80 500	97.649	14.43	4.97	296	2 000	6
33	US02079KAD90	93 800	94.273	12.76	5.05	425	2 250	6
34	US822582CG52	156 000	105.951	11.78	5.83	413	1 750	6
35	US06051GJB68	116 000	101.604	13.98	5.32	729	3 000	6
36	US09659W2P81	86 400	102.263	15.38	5.57	926	2 250	6
37	US716743AP46	82 000	108.224	23.37	10.20	763	2 250	6
38	US111021AE12	68 000	153.963	16.15	4.43	1193	2 670	6
39	US25156PAC77	92 000	150.553	14.01	3.75	839	3 500	6
40	US35177PAL13	60 000	155.740	17.46	4.65	763	2 460	6
41	US822582CH36	70 000	109.196	26.31	15.97	1619	2 000	6
42	US002824BG43	49 900	130.051	37.90	7.36	817	1 650	6
43	US80414L2L80	90 000	98.400	59.02	11.07	2872	2 250	6
44	US06051GGG82	60 000	127.534	56.44	12.08	1560	2 000	6
45	US716743AR02	87 000	125.184	57.14	12.43	2336	2 750	6
46	US225433AF86	58 600	129.155	29.29	10.24	1962	1 925	6
47	US87938WAU71	77 700	126.664	27.90	10.78	2635	2 500	6

The portfolio holding ω_i is measured in number of shares, the price P_i is in US dollars, the yearly volatility σ_i^{yearly} is expressed in %, the (half) bid-ask spread s_i and the duration-times-spread DTS_i are in bps, whereas the outstanding amount $\mathcal{M}_i$ and the daily trading limit Q_i^+ are expressed in $ mn.

B Additional results

This appendix contains additional tables and figures.

Table 16: Computation of the unit transaction cost $\boldsymbol{c}_i(x_i(h))$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	s_i	$\boldsymbol{c}_i^s$	σ_i	$x_i(h)$			$\boldsymbol{c}_i^{\boldsymbol{\pi}}(x_i(h))$			$\boldsymbol{c}_i(x_i(h))$		
1	0.89	1.11	1.59	9.18	0.00	0.00	22.67	0.00	0.00	23.78	0.00	0.00
2	1.95	2.43	1.93	10.00	2.63	0.00	29.92	12.53	0.00	32.35	14.97	0.00
3	1.40	1.75	0.85	10.00	9.07	0.00	13.21	11.98	0.00	14.96	13.73	0.00
4	0.90	1.12	1.64	10.00	1.20	0.00	25.38	7.18	0.00	26.50	8.30	0.00
5	1.25	1.56	1.35	10.00	3.91	0.00	20.90	10.67	0.00	22.46	12.23	0.00
6	0.94	1.17	1.68	10.00	2.35	0.00	26.02	10.31	0.00	27.19	11.48	0.00
7	1.45	1.81	1.97	10.00	3.94	0.00	30.57	15.66	0.00	32.38	17.47	0.00
8	1.01	1.26	1.16	10.00	1.96	0.00	17.99	6.50	0.00	19.25	7.76	0.00
9	1.67	2.09	2.09	4.72	0.00	0.00	18.18	0.00	0.00	20.27	0.00	0.00
10	2.31	2.89	1.85	6.13	0.00	0.00	18.33	0.00	0.00	21.22	0.00	0.00
11	0.80	1.00	1.28	7.84	0.00	0.00	15.58	0.00	0.00	16.59	0.00	0.00
12	1.54	1.92	1.33	5.95	0.00	0.00	12.94	0.00	0.00	14.86	0.00	0.00
13	1.15	1.44	1.62	9.22	0.00	0.00	23.14	0.00	0.00	24.59	0.00	0.00
14	0.86	1.07	1.65	10.00	2.22	0.00	25.57	9.83	0.00	26.64	10.90	0.00
15	1.22	1.52	1.29	10.00	4.04	0.00	20.01	10.39	0.00	21.53	11.91	0.00
16	1.77	2.21	1.62	7.95	0.00	0.00	19.98	0.00	0.00	22.19	0.00	0.00
17	1.79	2.24	1.16	10.00	0.44	0.00	17.99	3.07	0.00	20.22	5.30	0.00
18	1.75	2.19	1.05	10.00	4.16	0.00	16.34	8.61	0.00	18.53	10.80	0.00
19	0.94	1.18	1.20	9.84	0.00	0.00	18.32	0.00	0.00	19.49	0.00	0.00
20	0.63	0.78	1.23	8.90	0.00	0.00	16.91	0.00	0.00	17.69	0.00	0.00
21	1.39	1.73	1.44	6.45	0.00	0.00	14.59	0.00	0.00	16.33	0.00	0.00
22	1.61	2.01	1.32	3.28	0.00	0.00	9.57	0.00	0.00	11.59	0.00	0.00
23	1.13	1.41	1.27	10.00	5.50	0.00	19.63	11.89	0.00	21.04	13.30	0.00
24	1.53	1.91	2.09	10.00	10.00	0.90	32.39	32.39	7.94	34.30	34.30	9.85
25	2.44	3.06	1.54	10.00	0.28	0.00	23.92	3.28	0.00	26.98	6.34	0.00
26	3.23	4.04	1.59	3.73	0.00	0.00	12.28	0.00	0.00	16.32	0.00	0.00
27	1.26	1.57	1.84	9.08	0.00	0.00	25.89	0.00	0.00	27.47	0.00	0.00
28	1.52	1.90	1.55	6.20	0.00	0.00	15.45	0.00	0.00	17.36	0.00	0.00
29	2.05	2.56	1.41	4.32	0.00	0.00	11.69	0.00	0.00	14.25	0.00	0.00
30	0.77	0.97	1.97	9.04	0.00	0.00	27.51	0.00	0.00	28.48	0.00	0.00
31	1.73	2.16	1.22	10.00	6.46	0.00	18.84	12.36	0.00	21.00	14.52	0.00
32	0.89	1.12	0.95	9.16	0.00	0.00	13.54	0.00	0.00	14.66	0.00	0.00
33	1.24	1.55	1.34	9.33	0.00	0.00	19.33	0.00	0.00	20.88	0.00	0.00
34	1.29	1.61	1.27	10.00	6.46	0.00	19.64	12.89	0.00	21.25	14.50	0.00
35	1.84	2.30	1.10	10.00	10.00	2.52	17.11	17.11	7.01	19.41	19.41	9.31
36	0.75	0.94	1.76	10.00	8.31	0.00	27.26	22.64	0.00	28.20	23.58	0.00
37	1.96	2.45	1.35	10.00	3.84	0.00	20.94	10.59	0.00	23.39	13.04	0.00
38	1.27	1.59	1.08	10.00	4.31	0.00	16.69	8.95	0.00	18.27	10.53	0.00
39	1.33	1.67	2.86	10.00	0.59	0.00	44.35	8.82	0.00	46.02	10.49	0.00
40	0.86	1.07	1.91	10.00	2.09	0.00	29.66	11.07	0.00	30.73	12.14	0.00
41	0.58	0.73	0.91	10.00	8.69	0.00	14.14	12.29	0.00	14.87	13.02	0.00
42	0.81	1.02	1.30	10.00	4.88	0.00	20.19	11.51	0.00	21.20	12.53	0.00
43	0.69	0.86	1.38	10.00	7.60	0.00	21.40	16.26	0.00	22.25	17.12	0.00
44	1.46	1.83	1.57	10.00	6.75	0.00	24.34	16.44	0.00	26.16	18.26	0.00
45	1.76	2.20	1.79	5.03	0.00	0.00	16.02	0.00	0.00	18.22	0.00	0.00
46	1.15	1.43	1.36	10.00	0.14	0.00	21.04	2.02	0.00	22.47	3.45	0.00
47	2.00	2.50	0.79	10.00	0.21	0.00	12.23	1.44	0.00	14.73	3.95	0.00
48	1.64	2.05	1.25	10.00	6.59	0.00	19.42	12.87	0.00	21.46	14.91	0.00
49	1.55	1.94	1.82	4.33	0.00	0.00	15.16	0.00	0.00	17.10	0.00	0.00
50	1.90	2.37	1.19	10.00	3.41	0.00	18.45	8.80	0.00	20.82	11.17	0.00

The daily volatility σ_i and the daily participation rate $x_i(h)$ are expressed in %, whereas the bid-ask spread s_i and the unit transaction costs $\boldsymbol{c}_i^s$, $\boldsymbol{c}_i^{\boldsymbol{\pi}}(x_i(h))$ and $\boldsymbol{c}_i(x_i(h))$ are measured in bps.

Table 17: Computation of the total transaction cost $\mathcal{TC}_i\left(q_i\left(h\right)\right)$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	$Q_i\left(h\right)$			$\mathcal{TC}_i\left(q_i\left(h\right)\right)$			$\mathcal{TC}_i\left(q_i\right)$
1	13 431 247.44	0.00	0.00	31 936.75	0.00	0.00	31 936.75
2	14 797 877.75	3 895 507.45	0.00	47 874.31	5 830.54	0.00	53 704.85
3	9 039 749.24	8 195 352.52	0.00	13 523.51	11 248.99	0.00	24 772.50
4	14 745 920.00	1 768 256.00	0.00	39 081.24	1 467.05	0.00	40 548.28
5	15 084 994.98	5 897 400.22	0.00	33 877.67	7 212.12	0.00	41 089.78
6	9 456 281.91	2 225 300.95	0.00	25 712.16	2 554.23	0.00	28 266.39
7	52 722 285.84	20 760 568.56	0.00	170 729.19	36 278.74	0.00	207 007.93
8	10 640 799.22	2 082 755.76	0.00	20 480.97	1 616.00	0.00	22 096.97
9	14 406 798.10	0.00	0.00	29 198.99	0.00	0.00	29 198.99
10	14 549 862.56	0.00	0.00	30 872.51	0.00	0.00	30 872.51
11	12 125 542.68	0.00	0.00	20 112.48	0.00	0.00	20 112.48
12	7 088 018.40	0.00	0.00	10 535.97	0.00	0.00	10 535.97
13	9 525 929.25	0.00	0.00	23 421.99	0.00	0.00	23 421.99
14	13 736 468.04	3 044 931.81	0.00	36 592.48	3 319.27	0.00	39 911.75
15	5 931 230.80	2 397 647.47	0.00	12 772.78	2 855.88	0.00	15 628.67
16	17 827 314.96	0.00	0.00	39 565.34	0.00	0.00	39 565.34
17	8 851 613.27	385 616.84	0.00	17 902.33	204.55	0.00	18 106.88
18	4 899 454.56	2 039 218.72	0.00	9 079.77	2 201.99	0.00	11 281.76
19	13 483 152.93	0.00	0.00	26 283.29	0.00	0.00	26 283.29
20	13 165 516.88	0.00	0.00	23 292.62	0.00	0.00	23 292.62
21	14 302 963.10	0.00	0.00	23 351.72	0.00	0.00	23 351.72
22	7 738 581.10	0.00	0.00	8 966.85	0.00	0.00	8 966.85
23	8 498 610.04	4 674 981.41	0.00	17 880.92	6 216.75	0.00	24 097.67
24	3 519 016.56	3 519 016.56	317 256.48	12 069.30	12 069.30	312.50	24 451.10
25	14 674 609.30	414 997.90	0.00	39 590.09	263.12	0.00	39 853.21
26	8 739 631.08	0.00	0.00	14 262.89	0.00	0.00	14 262.89
27	13 262 417.36	0.00	0.00	36 426.36	0.00	0.00	36 426.36
28	12 934 487.37	0.00	0.00	22 448.98	0.00	0.00	22 448.98
29	11 262 967.08	0.00	0.00	16 047.96	0.00	0.00	16 047.96
30	12 035 401.36	0.00	0.00	34 275.96	0.00	0.00	34 275.96
31	4 518 723.35	2 919 220.82	0.00	9 487.95	4 238.94	0.00	13 726.89
32	5 878 639.86	0.00	0.00	8 618.38	0.00	0.00	8 618.38
33	9 448 008.50	0.00	0.00	19 728.26	0.00	0.00	19 728.26
34	14 652 668.17	9 469 593.95	0.00	31 129.91	13 728.40	0.00	44 858.31
35	15 736 486.14	15 736 486.14	3 960 180.36	30 551.75	30 551.75	3 687.74	64 791.24
36	24 487 898.22	20 342 434.50	0.00	69 044.22	47 969.50	0.00	117 013.72
37	6 523 350.24	2 502 965.28	0.00	15 259.67	3 264.04	0.00	18 523.71
38	7 302 001.33	3 147 798.19	0.00	13 344.15	3 315.46	0.00	16 659.62
39	16 344 458.53	969 749.90	0.00	75 209.91	1 016.95	0.00	76 226.86
40	9 063 017.15	1 894 321.22	0.00	27 848.80	2 299.77	0.00	30 148.57
41	13 340 445.60	11 597 067.15	0.00	19 838.80	15 102.83	0.00	34 941.63
42	23 484 222.00	11 453 288.16	0.00	49 789.93	14 346.06	0.00	64 135.98
43	12 192 259.58	9 266 017.22	0.00	27 133.61	15 862.95	0.00	42 996.55
44	15 887 935.58	10 732 230.56	0.00	41 565.42	19 601.77	0.00	61 167.20
45	9 131 848.08	0.00	0.00	16 635.12	0.00	0.00	16 635.12
46	28 557 468.59	394 244.27	0.00	64 179.58	136.08	0.00	64 315.65
47	4 880 981.79	102 102.75	0.00	7 191.19	40.30	0.00	7 231.49
48	8 365 245.01	5 509 632.84	0.00	17 955.14	8 216.72	0.00	26 171.86
49	9 030 624.10	0.00	0.00	15 446.58	0.00	0.00	15 446.58
50	5 278 667.08	1 800 165.56	0.00	10 989.73	2 010.43	0.00	13 000.16
Total	626 583 692.07	169 138 870.69	4 277 436.84	1 459 115.46	275 040.48	4 000.24	1 738 156.17

All the metrics are expressed in €.

Table 18: Break-down of the total transaction cost $\mathcal{TC}(q)$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	$\mathcal{TC}_i(q_i)$	$\mathcal{TC}_i^s(q_i)$	$\mathcal{TC}_i^{\pi}(q_i)$
1	31 936.75	1 489.58	30 447.17
2	53 704.85	4 549.60	49 155.26
3	24 772.50	3 015.24	21 757.26
4	40 548.28	1 848.88	38 699.40
5	41 089.78	3 275.63	37 814.15
6	28 266.39	1 370.18	26 896.21
7	207 007.93	13 308.25	193 699.67
8	22 096.97	1 605.70	20 491.27
9	29 198.99	3 006.93	26 192.06
10	30 872.51	4 200.63	26 671.88
11	20 112.48	1 218.50	18 893.98
12	10 535.97	1 361.34	9 174.63
13	23 421.99	1 374.67	22 047.31
14	39 911.75	1 800.42	38 111.33
15	15 628.67	1 267.64	14 361.03
16	39 565.34	3 937.82	35 627.51
17	18 106.88	2 068.53	16 038.35
18	11 281.76	1 519.24	9 762.52
19	26 283.29	1 585.06	24 698.23
20	23 292.62	1 032.23	22 260.39
21	23 351.72	2 478.33	20 873.38
22	8 966.85	1 558.94	7 407.91
23	24 097.67	1 859.21	22 238.47
24	24 451.10	1 404.75	23 046.35
25	39 853.21	4 610.61	35 242.60
26	14 262.89	3 533.16	10 729.73
27	36 426.36	2 087.65	34 338.71
28	22 448.98	2 463.90	19 985.08
29	16 047.96	2 883.21	13 164.75
30	34 275.96	1 164.15	33 111.82
31	13 726.89	1 603.83	12 123.05
32	8 618.38	656.86	7 961.52
33	19 728.26	1 466.90	18 261.36
34	44 858.31	3 876.68	40 981.63
35	64 791.24	8 161.31	56 629.92
36	117 013.72	4 206.10	112 807.62
37	18 523.71	2 209.30	16 314.41
38	16 659.62	1 657.44	15 002.18
39	76 226.86	2 885.32	73 341.54
40	30 148.57	1 171.36	28 977.21
41	34 941.63	1 817.07	33 124.56
42	64 135.98	3 548.54	60 587.44
43	42 996.55	1 842.10	41 154.45
44	61 167.20	4 859.11	56 308.09
45	16 635.12	2 004.83	14 630.29
46	64 315.65	4 148.76	60 166.89
47	7 231.49	1 247.02	5 984.47
48	26 171.86	2 839.95	23 331.90
49	15 446.58	1 752.57	13 694.02
50	13 000.16	1 679.36	11 320.80
Total	1 738 156.17	132 514.40	1 605 641.78

All the metrics are expressed in €.

Table 19: Stress testing — Liquidation portfolio $q_i(h)$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	q_i	q_i^+	$q_i(1)$	$q_i(2)$	$q_i(3)$	$q_i(4)$	$q_i(5)$
1	47 284.80	25 742.10	25 742.10	21 542.70	0.00	0.00	0.00
2	7 106.40	2 812.75	2 812.75	2 812.75	1 480.90	0.00	0.00
3	120 021.60	31 475.45	31 475.45	31 475.45	31 475.45	25 595.25	0.00
4	147 448.00	65 830.00	65 830.00	65 830.00	15 788.00	0.00	0.00
5	104 416.00	37 534.20	37 534.20	37 534.20	29 347.60	0.00	0.00
6	214 498.40	86 818.60	86 818.60	86 818.60	40 861.20	0.00	0.00
7	105 216.00	37 745.05	37 745.05	37 745.05	29 725.90	0.00	0.00
8	521 136.80	217 915.20	217 915.20	217 915.20	85 306.40	0.00	0.00
9	2 553 944.00	2 706 536.05	2 553 944.00	0.00	0.00	0.00	0.00
10	4 435 257.60	3 618 552.00	3 618 552.00	816 705.60	0.00	0.00	0.00
11	193 853.60	123 652.00	123 652.00	70 201.60	0.00	0.00	0.00
12	145 440.00	122 206.50	122 206.50	23 233.50	0.00	0.00	0.00
13	109 067.20	59 152.65	59 152.65	49 914.55	0.00	0.00	0.00
14	292 053.60	119 530.70	119 530.70	119 530.70	52 992.20	0.00	0.00
15	201 375.20	71 702.50	71 702.50	71 702.50	57 970.20	0.00	0.00
16	212 609.60	133 642.30	133 642.30	78 967.30	0.00	0.00	0.00
17	164 832.80	78 975.85	78 975.85	78 975.85	6 881.10	0.00	0.00
18	48 118.40	16 988.40	16 988.40	16 988.40	14 141.60	0.00	0.00
19	249 503.20	126 807.35	126 807.35	122 695.85	0.00	0.00	0.00
20	821 202.40	461 265.55	461 265.55	359 936.85	0.00	0.00	0.00
21	1 967 395.20	1 525 902.30	1 525 902.30	441 492.90	0.00	0.00	0.00
22	636 187.20	970 973.35	636 187.20	0.00	0.00	0.00	0.00
23	76 209.60	24 582.35	24 582.35	24 582.35	24 582.35	2 462.55	0.00
24	44 416.00	10 625.05	10 625.05	10 625.05	10 625.05	10 625.05	1 915.80
25	1 472 156.80	715 834.60	715 834.60	715 834.60	40 487.60	0.00	0.00
26	281 469.60	377 150.70	281 469.60	0.00	0.00	0.00	0.00
27	330 733.60	182 186.50	182 186.50	148 547.10	0.00	0.00	0.00
28	988 724.00	797 724.35	797 724.35	190 999.65	0.00	0.00	0.00
29	4 619 756.80	5 347 110.30	4 619 756.80	0.00	0.00	0.00	0.00
30	18 490.40	10 231.40	10 231.40	8 259.00	0.00	0.00	0.00
31	129 445.60	39 320.60	39 320.60	39 320.60	39 320.60	11 483.80	0.00
32	209 316.00	114 264.35	114 264.35	95 051.65	0.00	0.00	0.00
33	232 337.60	124 498.55	124 498.55	107 839.05	0.00	0.00	0.00
34	61 309.60	18 620.75	18 620.75	18 620.75	18 620.75	5 447.35	0.00
35	130 364.80	28 948.65	28 948.65	28 948.65	28 948.65	28 948.65	14 570.20
36	66 741.60	18 228.30	18 228.30	18 228.30	18 228.30	12 056.70	0.00
37	35 480.80	12 821.05	12 821.05	12 821.05	9 838.70	0.00	0.00
38	51 963.20	18 155.15	18 155.15	18 155.15	15 652.90	0.00	0.00
39	226 240.80	106 784.65	106 784.65	106 784.65	12 671.50	0.00	0.00
40	96 049.60	39 722.20	39 722.20	39 722.20	16 605.20	0.00	0.00
41	290 004.80	77 569.75	77 569.75	77 569.75	77 569.75	57 295.55	0.00
42	276 623.20	92 970.00	92 970.00	92 970.00	90 683.20	0.00	0.00
43	144 112.00	40 941.10	40 941.10	40 941.10	40 941.10	21 288.70	0.00
44	190 361.60	56 807.55	56 807.55	56 807.55	56 807.55	19 938.95	0.00
45	528 280.00	524 898.75	524 898.75	3 381.25	0.00	0.00	0.00
46	668 708.00	329 801.00	329 801.00	329 801.00	9 106.00	0.00	0.00
47	198 371.20	97 153.30	97 153.30	97 153.30	4 064.60	0.00	0.00
48	151 356.80	45 626.95	45 626.95	45 626.95	45 626.95	14 475.95	0.00
49	46 363.20	53 587.45	46 363.20	0.00	0.00	0.00	0.00
50	130 944.00	48 822.30	48 822.30	48 822.30	33 299.40	0.00	0.00

Table 20: Stress testing — Participation rate $x_i(h)$ and unit transaction cost $\boldsymbol{c}_i(x_i(h))$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	s_i	σ_i	$x_i(h)$					$\boldsymbol{c}_i(x_i(h))$				
1	8.89	2.83	10.00	8.37	0.00	0.00	0.00	55.01	47.85	0.00	0.00	0.00
2	9.95	3.17	10.00	10.00	5.26	0.00	0.00	61.57	61.57	41.54	0.00	0.00
3	9.40	2.09	10.00	10.00	10.00	8.13	0.00	44.18	44.18	44.18	38.12	0.00
4	8.90	2.88	10.00	10.00	2.40	0.00	0.00	55.72	55.72	28.95	0.00	0.00
5	9.25	2.59	10.00	10.00	7.82	0.00	0.00	51.67	51.67	42.92	0.00	0.00
6	8.94	2.92	10.00	10.00	4.71	0.00	0.00	56.41	56.41	36.51	0.00	0.00
7	9.45	3.21	10.00	10.00	7.88	0.00	0.00	61.60	61.60	51.02	0.00	0.00
8	9.01	2.40	10.00	10.00	3.91	0.00	0.00	48.46	48.46	30.27	0.00	0.00
9	9.67	3.33	9.44	0.00	0.00	0.00	0.00	60.81	0.00	0.00	0.00	0.00
10	10.31	3.09	10.00	2.26	0.00	0.00	0.00	60.78	31.47	0.00	0.00	0.00
11	8.80	2.52	10.00	5.68	0.00	0.00	0.00	50.10	35.06	0.00	0.00	0.00
12	9.54	2.57	10.00	1.90	0.00	0.00	0.00	51.69	26.08	0.00	0.00	0.00
13	9.15	2.86	10.00	8.44	0.00	0.00	0.00	55.76	48.84	0.00	0.00	0.00
14	8.86	2.89	10.00	10.00	4.43	0.00	0.00	55.85	55.85	35.42	0.00	0.00
15	9.22	2.53	10.00	10.00	8.08	0.00	0.00	50.75	50.75	43.24	0.00	0.00
16	9.77	2.86	10.00	5.91	0.00	0.00	0.00	56.55	40.04	0.00	0.00	0.00
17	9.79	2.40	10.00	10.00	0.87	0.00	0.00	49.44	49.44	21.21	0.00	0.00
18	9.75	2.30	10.00	10.00	8.32	0.00	0.00	47.75	47.75	41.79	0.00	0.00
19	8.94	2.44	10.00	9.68	0.00	0.00	0.00	49.01	47.78	0.00	0.00	0.00
20	8.63	2.47	10.00	7.80	0.00	0.00	0.00	48.99	40.60	0.00	0.00	0.00
21	9.39	2.68	10.00	2.89	0.00	0.00	0.00	53.21	29.95	0.00	0.00	0.00
22	9.61	2.56	6.55	0.00	0.00	0.00	0.00	38.25	0.00	0.00	0.00	0.00
23	9.13	2.51	10.00	10.00	10.00	1.00	0.00	50.26	50.26	50.26	21.45	0.00
24	9.53	3.33	10.00	10.00	10.00	10.00	1.80	63.51	63.51	63.51	63.51	29.80
25	10.44	2.78	10.00	10.00	0.57	0.00	0.00	56.19	56.19	21.43	0.00	0.00
26	11.23	2.83	7.46	0.00	0.00	0.00	0.00	46.75	0.00	0.00	0.00	0.00
27	9.26	3.08	10.00	8.15	0.00	0.00	0.00	59.31	50.50	0.00	0.00	0.00
28	9.52	2.79	10.00	2.39	0.00	0.00	0.00	55.16	29.19	0.00	0.00	0.00
29	10.05	2.65	8.64	0.00	0.00	0.00	0.00	47.98	0.00	0.00	0.00	0.00
30	8.77	3.21	10.00	8.07	0.00	0.00	0.00	60.63	51.06	0.00	0.00	0.00
31	9.73	2.46	10.00	10.00	10.00	2.92	0.00	50.21	50.21	50.21	28.95	0.00
32	8.89	2.19	10.00	8.32	0.00	0.00	0.00	45.12	39.40	0.00	0.00	0.00
33	9.24	2.58	10.00	8.66	0.00	0.00	0.00	51.48	46.14	0.00	0.00	0.00
34	9.29	2.51	10.00	10.00	10.00	2.93	0.00	50.46	50.46	50.46	28.77	0.00
35	9.84	2.34	10.00	10.00	10.00	10.00	5.03	48.63	48.63	48.63	48.63	33.35
36	8.75	3.00	10.00	10.00	10.00	6.61	0.00	57.41	57.41	57.41	41.80	0.00
37	9.96	2.59	10.00	10.00	7.67	0.00	0.00	52.61	52.61	43.27	0.00	0.00
38	9.27	2.32	10.00	10.00	8.62	0.00	0.00	47.49	47.49	42.54	0.00	0.00
39	9.33	4.10	10.00	10.00	1.19	0.00	0.00	75.23	75.23	29.54	0.00	0.00
40	8.86	3.15	10.00	10.00	4.18	0.00	0.00	59.94	59.94	36.87	0.00	0.00
41	8.58	2.15	10.00	10.00	10.00	7.39	0.00	44.09	44.09	44.09	35.37	0.00
42	8.81	2.54	10.00	10.00	9.75	0.00	0.00	50.42	50.42	49.45	0.00	0.00
43	8.69	2.62	10.00	10.00	10.00	5.20	0.00	51.47	51.47	51.47	34.77	0.00
44	9.46	2.81	10.00	10.00	10.00	3.51	0.00	55.38	55.38	55.38	32.89	0.00
45	9.76	3.03	10.00	0.06	0.00	0.00	0.00	59.07	15.27	0.00	0.00	0.00
46	9.15	2.60	10.00	10.00	0.28	0.00	0.00	51.69	51.69	16.89	0.00	0.00
47	10.00	2.03	10.00	10.00	0.42	0.00	0.00	43.95	43.95	17.75	0.00	0.00
48	9.64	2.49	10.00	10.00	10.00	3.17	0.00	50.68	50.68	50.68	29.81	0.00
49	9.55	3.06	8.65	0.00	0.00	0.00	0.00	53.00	0.00	0.00	0.00	0.00
50	9.90	2.43	10.00	10.00	6.82	0.00	0.00	50.03	50.03	38.06	0.00	0.00

The daily volatility σ_i and the daily participation rate $x_i(h)$ are expressed in %, whereas the bid-ask spread s_i and the unit transaction costs $\boldsymbol{c}_i(x_i(h))$ are measured in bps.

Table 21: Stress testing — Break-down of the total transaction cost $\mathcal{TC}(q)$ (large-cap portfolio, TNA = €1 bn, $\mathcal{R} = 80\%$, vertical slicing)

i	$\mathcal{TC}_i(q_i)$	$\mathcal{TC}_i^s(q_i)$	$\mathcal{TC}_i^{\pi}(q_i)$
1	69 498.63	14 920.83	54 577.80
2	107 290.00	23 242.98	84 047.02
3	73 910.28	20 250.34	53 659.94
4	87 281.60	18 363.05	68 918.54
5	103 263.27	24 258.03	79 005.24
6	61 463.65	13 051.76	48 411.89
7	430 680.96	86 791.11	343 889.85
8	57 872.23	14 329.25	43 542.97
9	87 604.76	17 413.73	70 191.03
10	80 581.70	18 750.49	61 831.21
11	54 141.87	13 344.04	40 797.83
12	33 736.06	8 449.35	25 286.71
13	50 102.25	10 900.60	39 201.65
14	87 508.74	18 581.82	68 926.92
15	40 467.87	9 596.51	30 871.36
16	89 878.23	21 765.14	68 113.09
17	44 580.36	11 305.76	33 274.60
18	31 915.40	8 457.91	23 457.49
19	65 268.45	15 068.21	50 200.23
20	59 659.28	14 197.75	45 461.53
21	68 639.06	16 781.30	51 857.76
22	29 601.62	9 297.52	20 304.10
23	64 977.96	15 032.80	49 945.16
24	45 645.94	8 760.04	36 885.90
25	83 351.95	19 700.22	63 651.73
26	40 860.80	12 272.79	28 588.01
27	73 414.82	15 350.07	58 064.75
28	64 855.26	15 398.39	49 456.88
29	54 038.96	14 146.18	39 892.79
30	67 823.25	13 199.55	54 623.71
31	35 944.56	9 041.78	26 902.78
32	24 997.62	6 535.50	18 462.12
33	46 297.30	10 914.91	35 382.39
34	117 072.56	27 998.94	89 073.61
35	166 258.54	43 594.46	122 664.07
36	244 729.74	49 036.44	195 693.30
37	45 147.21	11 235.62	33 911.59
38	48 068.44	12 107.24	35 961.21
39	125 825.97	20 199.53	105 626.44
40	61 311.15	12 128.69	49 182.46
41	105 645.33	26 754.59	78 890.75
42	175 033.80	38 486.05	136 547.75
43	105 152.21	23 300.38	81 851.83
44	141 145.26	31 479.28	109 665.98
45	53 687.02	11 136.67	42 550.35
46	148 277.36	33 100.47	115 176.89
47	21 632.44	6 230.10	15 402.34
48	67 548.14	16 714.83	50 833.30
49	47 858.61	10 783.19	37 075.42
50	33 262.97	8 758.19	24 504.78
Total	4 124 811.45	932 514.40	3 192 297.05

All the metrics are expressed in €.

Table 22: Break-down of the total transaction cost $\mathcal{TC}(q)$ (bond portfolio, TNA = \$10 bn, $\mathcal{R} = 30\%$, vertical slicing)

i	$\mathcal{TC}_i(q_i)$	$\mathcal{TC}_i^s(q_i)$	$\mathcal{TC}_i^\pi(q_i)$
1	36 012.29	27 024.52	8 987.77
2	69 885.29	24 800.97	45 084.32
3	82 529.58	18 176.19	64 353.39
4	255 188.46	38 211.29	216 977.17
5	212 256.83	22 015.86	190 240.97
6	199 177.04	30 303.07	168 873.97
7	457 182.39	41 144.08	416 038.31
8	475 933.42	40 815.04	435 118.38
9	448 409.64	40 395.89	408 013.75
10	783 664.66	48 917.74	734 746.92
11	1 897 014.61	103 729.21	1 793 285.40
12	26 113.54	19 565.90	6 547.64
13	43 144.34	32 107.29	11 037.05
14	26 290.67	20 129.36	6 161.31
15	41 572.40	30 568.34	11 004.05
16	23 824.88	18 036.70	5 788.18
17	34 493.76	23 537.39	10 956.36
18	27 033.64	17 468.42	9 565.22
19	24 224.80	11 622.52	12 602.29
20	152 183.10	97 077.07	55 106.03
21	140 308.97	32 858.89	107 450.08
22	43 327.04	29 928.35	13 398.69
23	145 124.09	93 662.76	51 461.33
24	103 949.26	45 925.79	58 023.47
25	168 041.37	93 341.29	74 700.07
26	356 675.94	249 091.31	107 584.63
27	63 633.66	42 415.08	21 218.57
28	94 233.98	60 949.31	33 284.67
29	175 895.36	95 194.98	80 700.38
30	80 175.43	51 658.16	28 517.28
31	112 149.16	31 629.01	80 520.15
32	78 265.60	51 043.74	27 221.86
33	92 505.62	50 775.40	41 730.22
34	170 309.73	87 616.82	82 692.91
35	165 002.62	74 146.13	90 856.49
36	152 015.61	61 150.66	90 864.95
37	168 452.93	93 327.29	75 125.64
38	211 293.24	76 086.97	135 206.27
39	206 312.96	87 322.85	118 990.11
40	150 154.94	73 418.95	76 735.99
41	232 066.72	90 497.82	141 568.90
42	174 454.08	110 679.19	63 774.90
43	517 530.40	235 206.50	282 323.90
44	331 010.74	194 346.51	136 664.23
45	550 434.34	280 040.49	270 393.85
46	270 117.83	99 756.39	170 361.44
47	410 992.48	123 563.71	287 428.78
Total	10 680 569.46	3 321 281.21	7 359 288.25

All the metrics are expressed in \$.

Table 23: Break-down of the total transaction cost $\mathcal{TC}(q)$ (bond portfolio, TNA = \$10 bn, $\mathcal{R} = 30\%$, vertical slicing)

h	$\mathcal{TC}_i(q;h)$	$\mathcal{TC}_i^s(q;h)$	$\mathcal{TC}_i^{\pi}(q;h)$
1	2 474 425.38	662 994.50	1 811 430.88
2	2 474 425.38	662 994.50	1 811 430.88
3	2 088 332.97	625 380.98	1 462 951.99
4	1 671 552.80	521 194.84	1 150 357.97
5	961 444.93	309 791.04	651 653.89
6	298 303.99	130 259.49	168 044.50
7	129 808.94	70 773.73	59 035.21
8	77 556.19	44 594.76	32 961.43
9	43 809.26	25 534.82	18 274.44
10	39 588.86	22 734.00	16 854.86
11	39 588.86	22 734.00	16 854.86
12	39 588.86	22 734.00	16 854.86
13	39 588.86	22 734.00	16 854.86
14	39 588.86	22 734.00	16 854.86
15	39 588.86	22 734.00	16 854.86
16	39 588.86	22 734.00	16 854.86
17	39 588.86	22 734.00	16 854.86
18	39 588.86	22 734.00	16 854.86
19	39 588.86	22 734.00	16 854.86
20	31 558.10	18 425.29	13 132.81
21	20 125.95	13 466.70	6 659.24
22	6 611.72	4 216.50	2 395.22
23	6 611.72	4 216.50	2 395.22
24	113.52	97.57	15.95
25	0.00	0.00	0.00
Total	10 680 569.46	3 321 281.21	7 359 288.25

All the metrics are expressed in \$.

Figure 13: Liquidation ratio $\mathcal{LR}(q;h)$ in % (equity portfolio, vertical slicing)

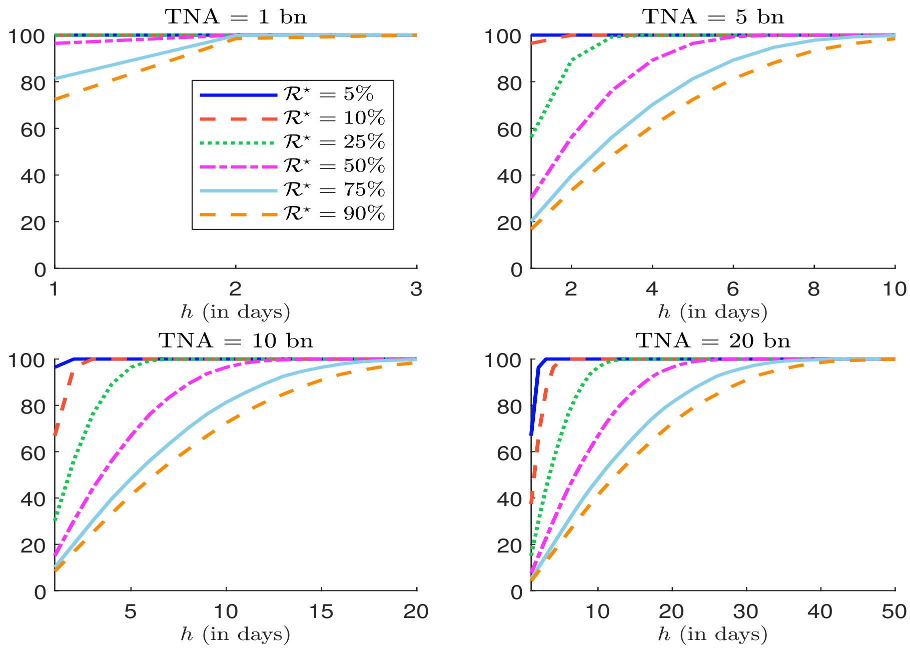

Figure 14: Liquidation ratio $\mathcal{LR}(q;h)$ in % (small-cap portfolio, vertical slicing)

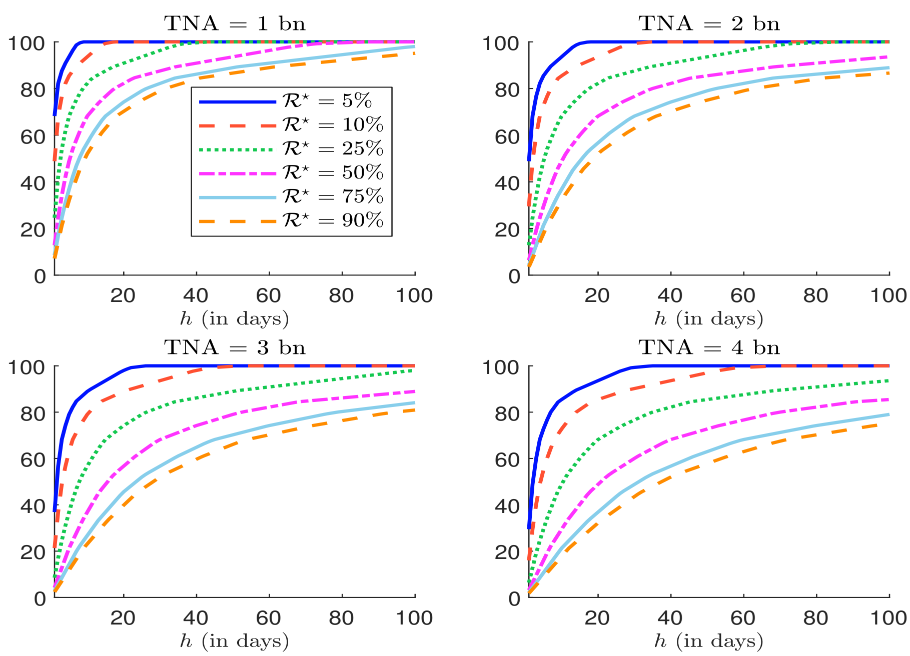

Figure 15: Liquidation time $h^{+} = \mathcal{LR}^{-1}(q; 99\%)$ in number of trading days (small-cap portfolio, vertical slicing)

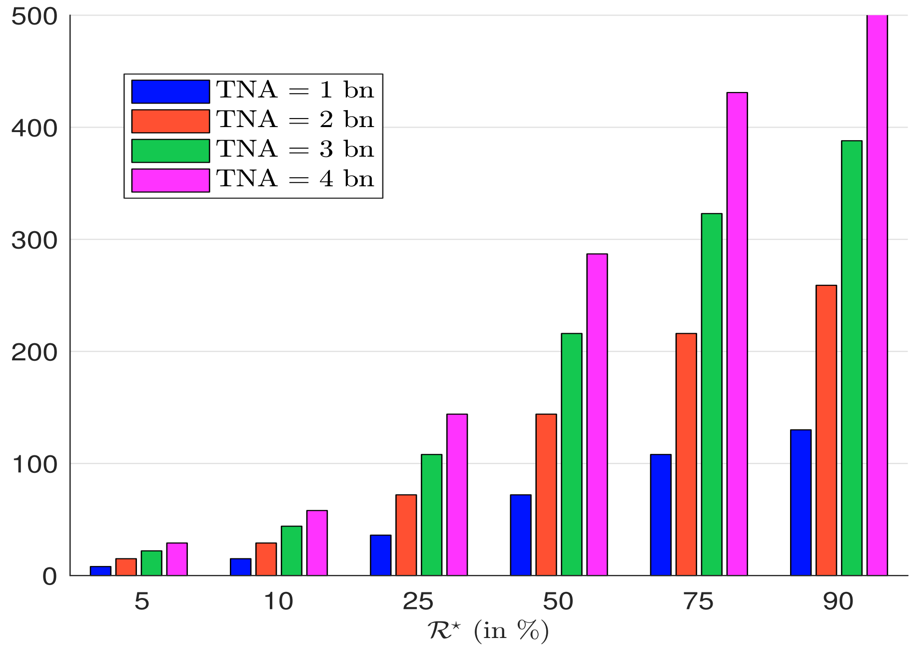

Figure 16: Transaction cost of the large-cap portfolio (TNA = €5 bn)

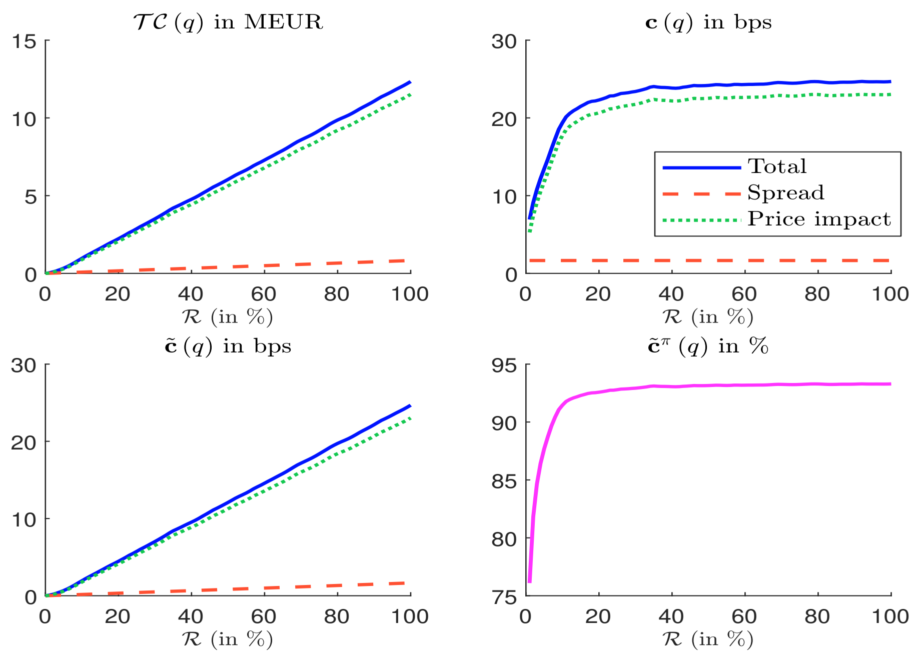

Figure 17: Normal redemption coverage ratio (bond portfolio, $\mathcal{R} = 30\%$, $m_{Q^+} = 1.0$)

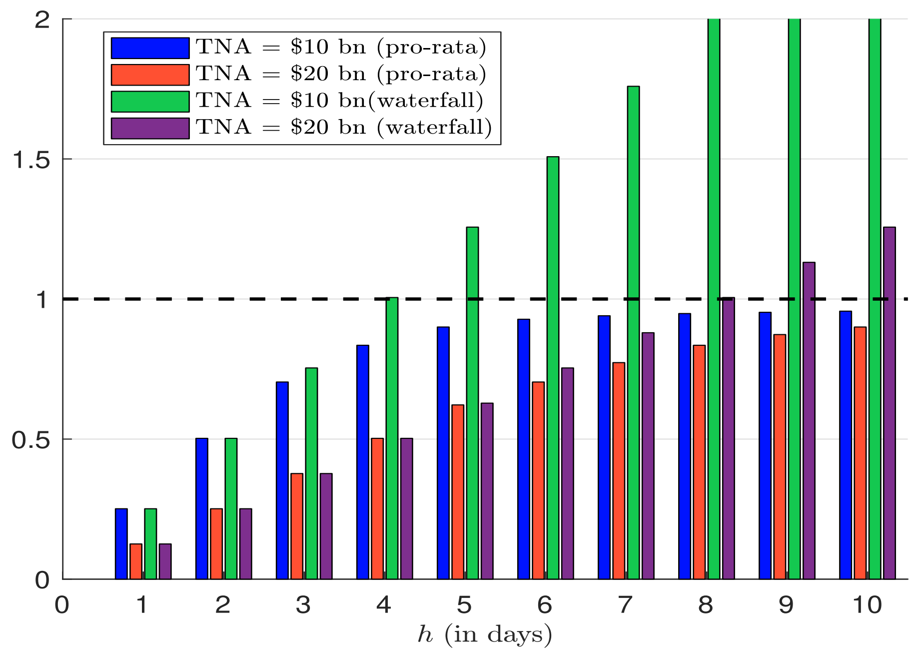

Figure 18: Stressed redemption coverage ratio (bond portfolio, $\mathcal{R} = 30\%$, $m_{Q^+} = 0.5$)