Chapter 4

Counterparty Credit Risk and Collateral Risk

Counterparty credit risk and collateral risk are other forms of credit risk, where the underlying credit risk is not directly generated by the economic objective of the financial transaction. Therefore, it can reduce the P&L of the portfolio and create a loss even if the business objective is reached. A typical example is the purchase transaction of a credit default swap. In this case, we have previously seen that the protection buyer is hedged against the credit risk if the reference entity defaults. This is partially true, because the protection buyer faces the risk that the protection seller also defaults. In this example, we see that the total P&L of the financial transaction is the direct P&L of the economic objective minus the potential loss due to the transaction settlement. Another example concerns the collateral risk, since the P&L of the financial transaction is directly affected by the mark-to-market of the collateral portfolio.

In this chapter, we study the counterparty credit risk (CCR) and show its computation. We also focus on the regulatory framework that has evolved considerably since the collapse of the LTCM hedge fund in 1997, which has shocked the entire financial system, not because of the investor losses, but because of the indirect losses generated by the counterparty credit risk[1]. The second section is dedicated to the credit valuation adjustment (CVA), which can be considered as the '*little brother*' of the CCR. This risk has been mainly identified with the bankruptcy of Lehman Brothers, which has highlighted the market risk of CCR. Finally, Section three reviews different topics associated to the collateral risk management, particularly in the repo markets.

4.1 Counterparty credit risk

We generally make the distinction between credit risk (CR) and counterparty credit risk (CCR). The counterparty credit risk on market transactions is the risk that the counterparty could default before the final settlement of the transaction's cash flows. For instance, if the bank buys a CDS protection on a firm and the seller of the CDS protection defaults before the maturity of the contract, the bank could not be hedged against the default of the firm. Another example of CCR is the delivery/settlement risk. Indeed, few financial transactions are settled on the same-day basis and the difference between the payment date and the delivery date is generally between one and five business days. There is then a counterparty credit risk if one counterparty defaults when the payment date is not synchronized with the delivery date. This settlement risk is low when it is expressed as a percent of the notional amount because the maturity mismatch is short, but it concerns large amounts from an aggregate point of view. In a similar way, when an OTC contract has a positive mark-to-

[1] Chapter 8 on page 453 describes the impact of the LTCM bankruptcy on systemic risk.

market, the bank suffers a loss if the counterparty defaults. To reduce this risk, the bank can put in place bilateral netting agreements. We note that this risk disappears (or more precisely decreases) when the bank uses an exchange, because the counterparty credit risk is transferred to the central counterparty clearing house, which guarantees the expected cash flows.

4.1.1 Definition

BCBS (2004a) measures the counterparty credit risk by the replacement cost of the OTC derivative. Let us consider two banks A and B that have entered into an OTC contract $\mathfrak{C}$. We assume that the bank B defaults before the maturity of the contract. According to Pykhtin and Zhu (2006), Bank A can then face two situations:

- The current value of the contract $\mathfrak{C}$ is negative. In this case, Bank A closes out the position and pays the market value of the contract to Bank B. To replace the contract $\mathfrak{C}$, Bank A can enter with another counterparty C into a similar contract $\mathfrak{C}'$. For that, Bank A receives the market value of the contract $\mathfrak{C}'$ and the loss of the bank is equal to zero.
- The current value of the contract $\mathfrak{C}$ is positive. In this case, Bank A close out the position, but receives nothing from Bank B. To replace the contract, Bank A can enter with another counterparty C into a similar contract $\mathfrak{C}'$. For that, Bank A pays the market value of the contract $\mathfrak{C}'$ to C. In this case, the loss of the bank is exactly equal to the market value.

We note that the counterparty exposure is then the maximum of the market value and zero. Moreover, the counterparty credit risk differs from the credit risk by two main aspects (Canabarro and Duffie, 2003):

1. The counterparty credit risk is bilateral, meaning that both counterparties may face losses. In the previous example, Bank B is also exposed to the risk that Bank A defaults.
2. The exposure at default is uncertain, because we don't know what will be the replacement cost of the contract when the counterparty defaults.

Using the notations introduced in the previous chapter, we deduce that the credit loss of an OTC portfolio is:

$$L = \sum_{i=1}^{n} \mathrm{EAD}_i \left(\boldsymbol{\tau}_i\right) \cdot \mathrm{LGD}_i \cdot \mathbb{1}\left\{\boldsymbol{\tau}_i \leq T_i\right\}$$

This is the formula of a credit portfolio loss, except that the exposure at default is random and depends on different factors: the default time of the counterparty, the evolution of market risk factors and the correlation between the market value of the OTC contract and the default of the counterparty.

Let $\mathrm{MtM}(t)$ be the mark-to-market value of the OTC contract at time t. The exposure at default is defined as:

$$\mathrm{EAD} = \max\left(\mathrm{MtM}\left(\boldsymbol{\tau}\right), 0\right)$$

If we consider a portfolio of OTC derivatives with the same counterparty entity, the exposure at default is the sum of positive market values:

$$\mathrm{EAD} = \sum_{i=1}^{n} \max\left(\mathrm{MtM}_i\left(\boldsymbol{\tau}\right), 0\right)$$

This is why the bank may be interested in putting in place a global netting agreement:

$$\begin{aligned}\mathrm{EAD} &= \max\left(\sum_{i=1}^{n}\mathrm{MtM}_i\left(\boldsymbol{\tau}\right), 0\right) \\ &\leq \sum_{i=1}^{n}\max\left(\mathrm{MtM}_i\left(\boldsymbol{\tau}\right), 0\right)\end{aligned}$$

In practice, it is extremely complicated and rare that two counterparties succeed in signing such agreement. Most of the time, there are several netting agreements on different trading perimeters (equities, bonds, interest rate swaps, etc.). In this case, the exposure at default is:

$$\mathrm{EAD} = \sum_{k}\max\left(\sum_{i\in\mathcal{N}_k}\mathrm{MtM}_i\left(\boldsymbol{\tau}\right), 0\right) + \sum_{i\notin\cup\mathcal{N}_k}\max\left(\mathrm{MtM}_i\left(\boldsymbol{\tau}\right), 0\right)$$

where $\mathcal{N}_k$ corresponds to the k^{th} netting arrangement and defines a netting set. Since the default of Lehman Brothers, we observe a strong development of (global and partial) netting agreements in order to reduce potential losses, but also the capital charge induced by counterparty credit risk.

Example 43 *Banks A and B have traded five OTC products, whose mark-to-market values*[2] *are given in the table below:*

t	1	2	3	4	5	6	7	8
$\mathfrak{C}_1$	5	5	3	0	−4	0	5	8
$\mathfrak{C}_2$	−5	10	5	−3	−2	−8	−7	−10
$\mathfrak{C}_3$	0	2	−3	−4	−6	−3	0	5
$\mathfrak{C}_4$	2	−5	−5	−5	2	3	5	7
$\mathfrak{C}_5$	−1	−3	−4	−5	−7	−6	−7	−6

If we suppose that there is no netting agreement, the counterparty exposure of Bank A corresponds to the second row in Table 4.1. We notice that the exposure changes over time. If there is a netting agreement, we obtain lower exposures. We now consider a more complicated situation. We assume that Banks A and B have two netting agreements: one on equity OTC contracts ($\mathfrak{C}_1$ and $\mathfrak{C}_2$) and one on fixed income OTC contracts ($\mathfrak{C}_3$ and $\mathfrak{C}_4$). In this case, we obtain results given in the last row in Table 4.1. For instance, the exposure at default for $t = 8$ is calculated as follows:

$$\mathrm{EAD} = \max\left(8 - 10, 0\right) + \max\left(5 + 7, 0\right) + \max\left(-6, 0\right) = 12$$

TABLE 4.1: Counterparty exposure of Bank A

t	1	2	3	4	5	6	7	8
No netting	7	17	8	0	2	3	10	20
Global netting	1	9	0	0	0	0	0	4
Partial netting	2	15	8	0	0	0	5	12

If we consider Bank B, the counterparty exposure is given in Table 4.2. This illustrates the bilateral nature of the counterparty credit risk. Indeed, except if there is a global netting arrangement, both banks have a positive counterparty exposure.

[2] They are calculated from the viewpoint of Bank A.

TABLE 4.2: Counterparty exposure of Bank B

t	1	2	3	4	5	6	7	8
No netting	6	8	12	17	19	17	14	16
Global netting	0	0	4	17	17	14	4	0
Partial netting	1	6	12	17	17	14	9	8

Remark 49 *In the previous example, we have assumed that the mark-to-market value of the OTC contract for one bank is exactly the opposite of the mark-to-market value for the other bank. In practice, banks calculate mark-to-model prices, implying that they can differ from one bank to another one.*

4.1.2 Modeling the exposure at default

In order to understand the counterparty credit risk, we begin by an example and illustrate the time-varying property of the exposure at default. Then, we introduce the different statistical measures that are useful for characterizing the EAD and show how to calculate them.

4.1.2.1 An illustrative example

Example 44 *We consider a bank that buys* 1 000 *ATM call options, whose maturity is one-year. The current value of the underlying asset is equal to* \$100*. We assume that the interest rate r and the cost-of-carry parameter b are equal to* 5%*. Moreover, the implied volatility of the option is considered as a constant and is equal to* 20%*.*

By considering the previous parameters, the value $\mathcal{C}_0$ of the call option[3] is equal to \$10.45. At time t, the mark-to-market of this derivative exposure is defined by:

$$\operatorname{MtM}(t) = n_C \cdot (\mathcal{C}(t) - \mathcal{C}_0)$$

where n_C and $\mathcal{C}(t)$ are the number and the value of call options. Let $e(t)$ be the exposure at default. We have:

$$e(t) = \max(\operatorname{MtM}(t), 0)$$

At the initial date of the trade, the mark-to-market value and the counterparty exposure are zero. When $t > 0$, the mark-to-market value is not equal to zero, implying that the counterparty exposure $e(t)$ may be positive. In Table 4.3, we have reported the values taken by $\mathcal{C}(t)$, $\operatorname{MtM}(t)$ and $e(t)$ for two scenarios of the underlying price $S(t)$. If we consider the first scenario, the counterparty exposure is equal to zero during the first three months, because the mark-to-market value is negative. The counterparty exposure is then positive for the next four months. For instance, it is equal to \$2 519 at the end of the fourth month[4]. In the case of the second scenario, the counterparty exposure is always equal to zero except for two months. Therefore, we notice that the counterparty exposure is time-varying and depends of the trajectory of the underlying price. This implies that the counterparty exposure cannot be calculated once and for all at the initial date of the trade. Indeed, the counterparty exposure changes with time. Moreover, we don't known what the future price of the underlying asset will be. That's why we are going to simulate it.

[3] We use the Black-Scholes formula given by Equation (2.10) on page 94 to price the option.

[4] We have:

$$\operatorname{MtM}(t) = 1\,000 \times (12.969 - 10.450) = \$2\,519$$

TABLE 4.3: Mark-to-market and counterparty exposure of the call option

t	Scenario #1				Scenario #2			
	$S(t)$	$\mathcal{C}(t)$	$\text{MtM}(t)$	$e(t)$	$S(t)$	$\mathcal{C}(t)$	$\text{MtM}(t)$	$e(t)$
1M	97.58	8.44	−2 013	0	91.63	5.36	−5 092	0
2M	98.19	8.25	−2 199	0	89.17	3.89	−6 564	0
3M	95.59	6.26	−4 188	0	97.60	7.35	−3 099	0
4M	106.97	12.97	2 519	2 519	97.59	6.77	−3 683	0
5M	104.95	10.83	382	382	96.29	5.48	−4 970	0
6M	110.73	14.68	4 232	4 232	97.14	5.29	−5 157	0
7M	113.20	16.15	5 700	5 700	107.71	11.55	1 098	1 098
8M	102.04	6.69	−3 761	0	105.71	9.27	−1 182	0
9M	115.76	17.25	6 802	6 802	107.87	10.18	−272	0
10M	103.58	5.96	−4 487	0	108.40	9.82	−630	0
11M	104.28	5.41	−5 043	0	104.68	5.73	−4 720	0
1Y	104.80	4.80	−5 646	0	115.46	15.46	5 013	5 013

We note $\text{MtM}(t_1; t_2)$ the mark-to-market value between dates t_1 and t_2. By construction, we have:

$$\text{MtM}(0; t) = \text{MtM}(0; t_0) + \text{MtM}(t_0; t)$$

where 0 is the initial date of the trade, t_0 is the current date and t is the future date. This implies that the mark-to-market value at time t has two components:

1. the current mark-to-market value $\text{MtM}(0; t_0)$ that depends on the past trajectory of the underlying price;

2. and the future mark-to-market value $\text{MtM}(t_0; t)$ that depends on the future trajectory of the underlying price.

In order to evaluate the second component, we need to define the probability distribution of $S(t)$. In our example, we can assume that the underlying price follows a geometric Brownian motion:

$$\mathrm{d}S(t) = \mu S(t)\,\mathrm{d}t + \sigma S(t)\,\mathrm{d}W(t)$$

We face here an issue because we have to define the parameters μ and σ. There are two approaches:

1. the first method uses the historical probability measure $\mathbb{P}$, meaning that the parameters μ and σ are estimated using historical data;

2. the second method considers the risk-neutral probability measure $\mathbb{Q}$, which is used to price the OTC derivative.

While the first approach is more relevant to calculate the counterparty exposure, the second approach is more frequent because it is easier for a bank to implement it. Indeed, $\mathbb{Q}$ is already available because of the hedging portfolio, which is not the case of $\mathbb{P}$. In our example, this is equivalent to set μ and σ to their historical estimates $\hat{\mu}$ and $\hat{\sigma}$ if we consider the historical probability measure $\mathbb{P}$, while they are equal to the interest rate r and the implied volatility Σ if we consider the risk-neural probability measure $\mathbb{Q}$.

In Figure 4.1, we report an illustration of scenario generation when the current date t_0 is 6 months. This means that the trajectory of the asset price $S(t)$ is given when $t \leq t_0$ whereas it is simulated when $t > t_0$. At time $t_0 = 0.5$, the asset price is equal to \$114.77. We deduce

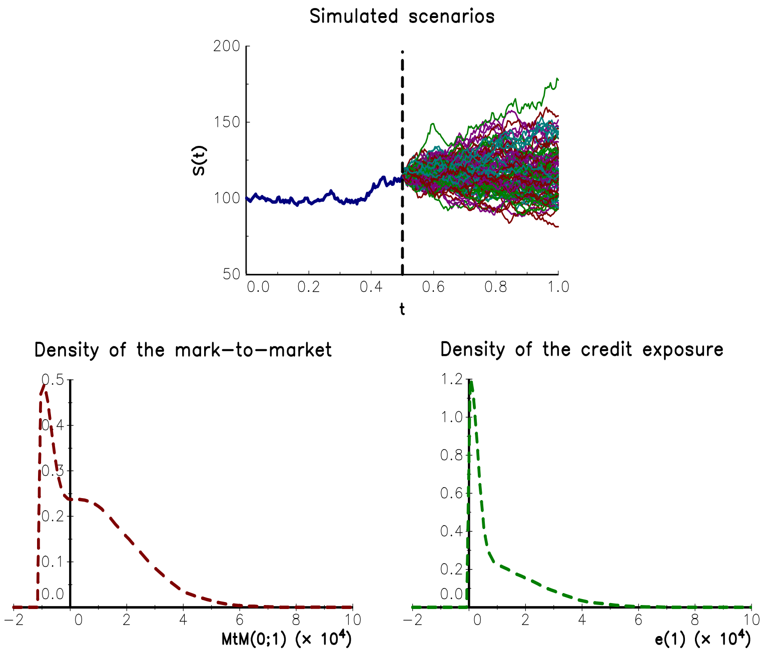

FIGURE 4.1: Probability density function of the counterparty exposure after six months

that the option price $\mathcal{C}(t_0)$ is equal to \$18.17. The mark-to-market value is then positive and equal to \$7 716. Using 10 000 simulated scenarios, we estimate the probability density function of the mark-to-market value $\mathrm{MtM}(0;1)$ at the maturity date (bottom/left panel in Figure 4.1) and deduce the probability density function of the counterparty exposure $e(1)$ (bottom/right panel in Figure 4.1). We notice that the probability to obtain a negative mark-to-market at the maturity date is significant. Indeed, it is equal to 36% because it remains 6 months and the asset price may sufficiently decrease. Of course, this probability depends on the parameters used for simulating the trajectories, especially the trend μ. Using a risk-neutral approach has the advantage to limit the impact of this coefficient.

Remark 50 *The mark-to-market value presents a very high skew, because it is bounded. Indeed, the worst-case scenario is reached when the asset price $S(1)$ is lower than the strike $K = 100$. In this case, we obtain:*

$$\begin{aligned}\mathrm{MtM}(0;1) &= 1\,000 \times (0 - 10.45) \\ &= -\$10\,450\end{aligned}$$

We suppose now that the current date is nine months. During the last three months, the asset price has changed and it is now equal to \$129.49. The current counterparty exposure has then increased and is equal to[5] \$20 294. In Figure 4.2, we observe that the shape of the probability density function has changed. Indeed, the skew has been highly reduced, because it only remains three months before the maturity date. The price is then sufficiently high that the probability to obtain a positive mark-to-market at the settlement date is almost equal to 100%. This is why the two probability density functions are very similar.

We can use the previous approach of scenario generation in order to represent the evolution of counterparty exposure. In Figure 4.3, we consider two observed trajectories of the

[5] Using the previous parameters, the BS price of the call option is now equal to \$30.74.

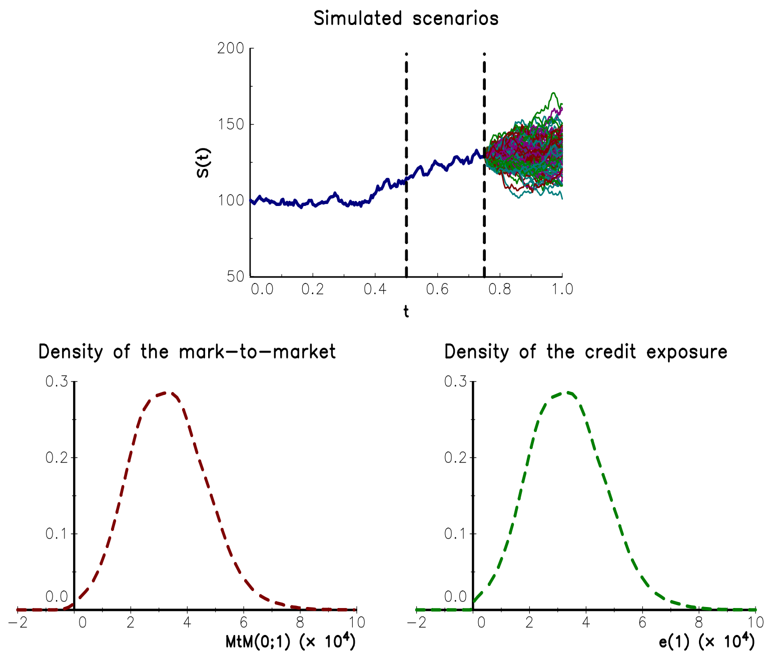

FIGURE 4.2: Probability density function of the counterparty exposure after nine months

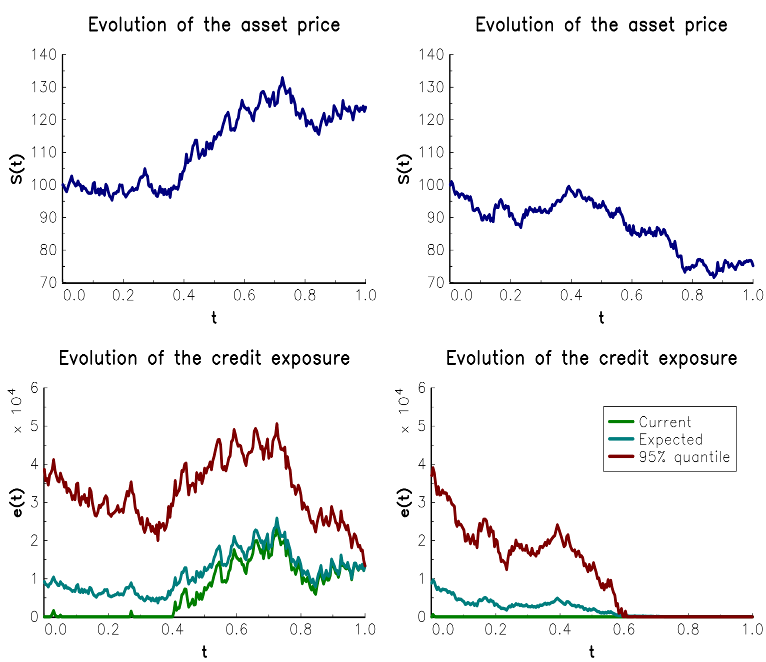

FIGURE 4.3: Evolution of the counterparty exposure

asset price. For each trajectory, we report the current exposure, the expected exposure and the 95% quantile of the counterparty exposure at the maturity date. All these counterparty measures converge at the maturity date, but differ before because of the uncertainty between the current date and the maturity date.

4.1.2.2 Measuring the counterparty exposure

We define the counterparty exposure at time t as the random credit exposure[6]:

$$e(t) = \max(\text{MtM}(0;t), 0) \tag{4.1}$$

This counterparty exposure is also known as the potential future exposure (PFE). When the current date t_0 is not equal to the initial date 0, the counterparty exposure can be decomposed in two parts:

$$\begin{aligned} e(t) &= \max(\text{MtM}(0;t_0) + \text{MtM}(t_0;t), 0) \\ &= \max(\text{MtM}(0;t_0), 0) + \\ &\quad (\max(\text{MtM}(0;t_0) + \text{MtM}(t_0;t), 0) - \max(\text{MtM}(0;t_0), 0)) \end{aligned}$$

The first component is the current exposure, which is always positive:

$$\text{CE}(t_0) = \max(\text{MtM}(0;t_0), 0)$$

The second component is the credit variation between t_0 and t. While the current mark-to-market value is negative, the second component can only be a positive value. However, the credit variation may also be negative if the future mark-to-market value is negative. Let us denote by $\mathbf{F}_{[0,t]}$ the cumulative distribution function of the potential future exposure $e(t)$. The peak exposure (PE) is the quantile of the counterparty exposure at the confidence level α:

$$\begin{aligned} \text{PE}_\alpha(t) &= \mathbf{F}_{[0,t]}^{-1}(\alpha) \\ &= \{\inf x : \Pr\{e(t) \leq x\} \geq \alpha\} \end{aligned} \tag{4.2}$$

The maximum value of the peak exposure is referred as the maximum peak exposure[7] (MPE):

$$\text{MPE}_\alpha(0;t) = \sup_s \text{PE}_\alpha(0;s) \tag{4.3}$$

We now introduce the traditional counterparty credit risk measures:

- The expected exposure (EE) is the average of the distribution of the counterparty exposure at the future date t:

$$\begin{aligned} \text{EE}(t) &= \mathbb{E}[e(t)] \\ &= \int_0^\infty x \, \mathrm{d}\mathbf{F}_{[0,t]}(x) \end{aligned} \tag{4.4}$$

- The expected positive exposure (EPE) is the weighted average over time $[0,t]$ of the expected exposure:

$$\begin{aligned} \text{EPE}(0;t) &= \mathbb{E}\left[\frac{1}{t}\int_0^t e(s)\,\mathrm{d}s\right] \\ &= \frac{1}{t}\int_0^t \text{EE}(s)\,\mathrm{d}s \end{aligned} \tag{4.5}$$

[6] The definitions introduced in this paragraph come from Canabarro and Duffie (2003) and the Basel II framework.

[7] It is also known as the maximum potential future exposure (MPFE).

- The effective expected exposure (EEE) is the maximum expected exposure that occurs at the future date t or any prior date:

$$\begin{aligned} \mathrm{EEE}(t) &= \sup_{s \leq t} \mathrm{EE}(s) \\ &= \max\left(\mathrm{EEE}\left(t^{-}\right), \mathrm{EE}(t)\right) \end{aligned} \tag{4.6}$$

- Finally, the effective expected positive exposure (EEPE) is the weighted average over time $[0, t]$ of the effective expected exposure:

$$\mathrm{EEPE}(0; t) = \frac{1}{t} \int_0^t \mathrm{EEE}(s)\, \mathrm{d}s \tag{4.7}$$

We can make several observations concerning the previous measures. Some of them are defined with respect to a future date t. This is the case of $\mathrm{PE}_\alpha(t)$, $\mathrm{EE}(t)$ and $\mathrm{EEE}(t)$. The others depend on the time period $[0; t]$, typically a one-year time horizon. Previously, we have considered the counterparty measure $e(t)$, which defines the potential future exposure between the initial date 0 and the future date t. We can also use other credit measures like the potential future exposure between the current date t_0 and the future date t:

$$e(t) = \max\left(\mathrm{MtM}(t_0; t), 0\right)$$

The counterparty exposure $e(t)$ can be defined with respect to one contract or to a basket of contracts. In this last case, we have to take into account netting arrangements.

4.1.2.3 Practical implementation for calculating counterparty exposure

We consider again Example 44 and assume that the current date t_0 is the initial date $t = 0$. Using 50 000 simulations, we have calculated the different credit measures with respect to the time t and reported them in Figure 4.4. For that, we have used the risk-neutral distribution probability $\mathbb{Q}$ in order to simulate the trajectory of the asset price $S(t)$. Let $\{t_0, t_1, \ldots, t_n\}$ be the set of discrete times. We note n_S the number of simulations and $S_j(t_i)$ the value of the asset price at time t_i for the j^{th} simulation. For each simulated trajectory, we then calculate the option price $\boldsymbol{\mathcal{C}}_j(t_i)$ and the mark-to-market value:

$$\mathrm{MtM}_j(t_i) = n_C \cdot \left(\boldsymbol{\mathcal{C}}_j(t_i) - \boldsymbol{\mathcal{C}}_0\right)$$

Therefore, we deduce the potential future exposure:

$$e_j(t_i) = \max\left(\mathrm{MtM}_j(t_i), 0\right)$$

The peak exposure at time t_i is estimated using the order statistics:

$$\mathrm{PE}_\alpha(t_i) = e_{\alpha n_S : n_S}(t_i) \tag{4.8}$$

We use the empirical mean to calculate the expected exposure:

$$\mathrm{EE}(t_i) = \frac{1}{n_S} \sum_{j=1}^{n_S} e_j(t_i) \tag{4.9}$$

For the expected positive exposure, we approximate the integral by the following sum:

$$\mathrm{EPE}(0; t_i) = \frac{1}{t_i} \sum_{k=1}^{i} \mathrm{EE}(t_k)\, \Delta t_k \tag{4.10}$$

If we consider a fixed-interval scheme with $\Delta t_k = \Delta t$, we obtain:

$$\begin{aligned} \mathrm{EPE}(0;t_i) &= \frac{\Delta t}{t_i}\sum_{k=1}^{i}\mathrm{EE}(t_k) \\ &= \frac{1}{i}\sum_{k=1}^{i}\mathrm{EE}(t_k) \end{aligned} \tag{4.11}$$

By definition, the effective expected exposure is given by the following recursive formula:

$$\mathrm{EEE}(t_i) = \max\left(\mathrm{EEE}(t_{i-1}), \mathrm{EE}(t_i)\right) \tag{4.12}$$

where $\mathrm{EEE}(0)$ is initialized with the value $\mathrm{EE}(0)$. Finally, the effective expected positive exposure is given by:

$$\mathrm{EEPE}(0;t_i) = \frac{1}{t_i}\sum_{k=1}^{i}\mathrm{EEE}(t_k)\,\Delta t_k \tag{4.13}$$

In the case of a fixed-interval scheme, this formula becomes:

$$\mathrm{EEPE}(0;t_i) = \frac{1}{i}\sum_{k=1}^{i}\mathrm{EEE}(t_k) \tag{4.14}$$

If we consider Figure 4.4, we observe that the counterparty exposure is increasing with respect to the time horizon[8]. This property is due to the fact that the credit risk evolves according to a square-root-of-time rule $\sqrt{t}$. In the case of an interest rate swap, the counterparty exposure takes the form of a bell-shaped curve. In fact, there are two opposite effects that determine the counterparty exposure (Pykhtin and Zhu, 2007):

- the diffusion effect of risk factors increases the counterparty exposure over time, because the uncertainty is greater in the future and may produce very large potential future exposures compared to the current exposure;
- the amortization effect decreases the counterparty exposure over time, because it reduces the remaining cash flows that are exposed to default.

In Figure 4.5, we have reported counterparty exposure in the case of an interest swap with a continuous amortization. The peak exposure initially increases because of the diffusion effect and generally reaches its maximum at one-third of the remaining maturity. It then decreases because of the amortization effect. This is why it is equal to zero at the maturity date when the swap is fully amortized.

4.1.3 Regulatory capital

The Basel II Accord includes three approaches to calculate the capital requirement for the counterparty credit risk: current exposure method (CEM), standardized method (SM) and internal model method (IMM). In March 2014, the Basel Committee decided to replace non-internal model approaches (CEM and SM) by a more sensitive approach called standardized approach (or SA-CCR), which is has been implemented since January 2017.

Each approach defines how the exposure at default EAD is calculated. The bank uses this estimate with the appropriated credit approach (SA or IRB) in order to measure the capital requirement. In the SA approach, the capital charge is equal to:

$$\mathcal{K} = 8\% \cdot \mathrm{EAD} \cdot \mathrm{RW}$$

[8]This implies that $\mathrm{MPE}_\alpha(0;t) = \mathrm{PE}_\alpha(t)$ and $\mathrm{EEE}(t) = \mathrm{EE}(t)$.

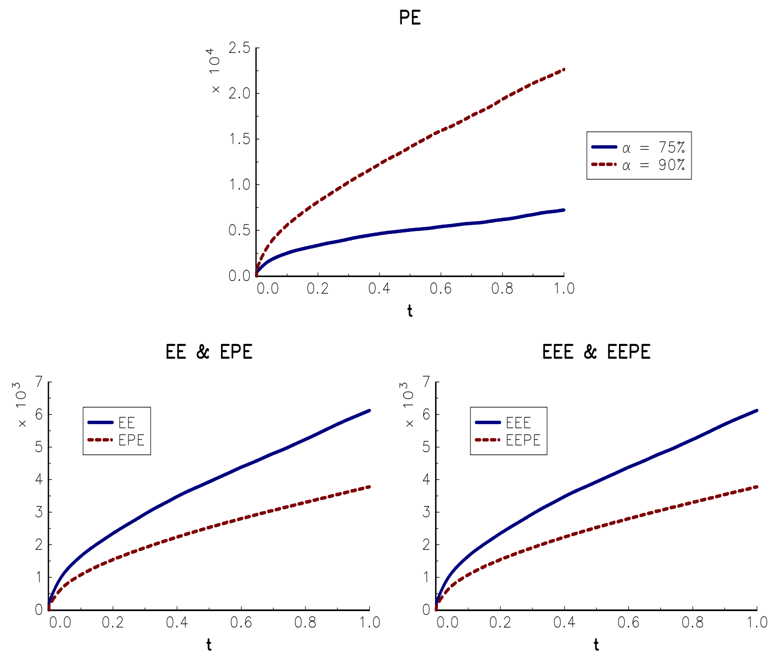

FIGURE 4.4: Counterparty exposure profile of options

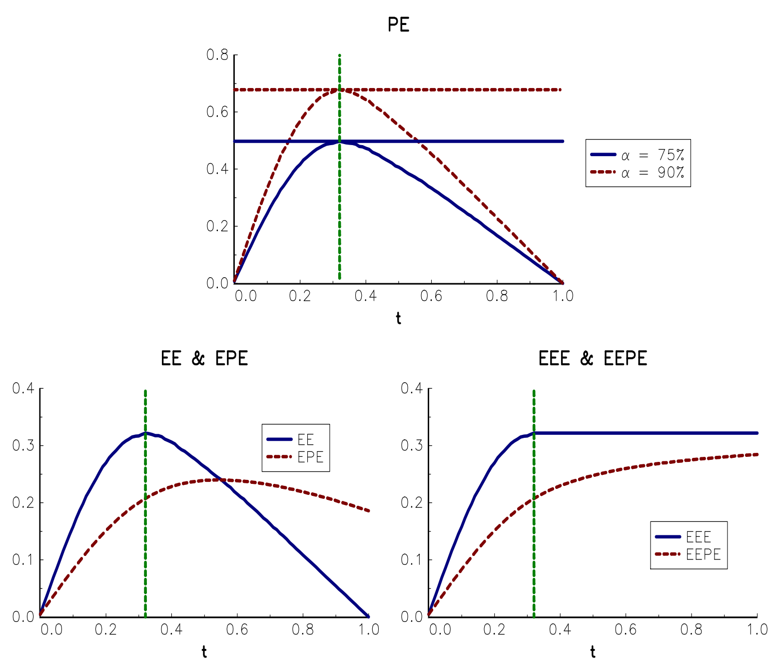

FIGURE 4.5: Counterparty exposure profile of interest rate swaps

where RW is the risk weight of the counterparty. In the IRB approach, we recall that:

$$\mathcal{K} = \mathrm{EAD} \cdot \mathrm{LGD} \cdot \left(\Phi \left(\frac{\Phi^{-1}(\mathrm{PD}) + \sqrt{\rho(\mathrm{PD})}\Phi^{-1}(0.999)}{\sqrt{1-\rho(\mathrm{PD})}} \right) - \mathrm{PD} \right) \cdot \varphi(\mathrm{M})$$

where LGD and PD are the loss given default and the probability of default, which apply to the counterparty. The correlation $\rho(\mathrm{PD})$ is calculated using the standard formula (3.35) given on page 184.

4.1.3.1 Internal model method

In the internal model method, the exposure at default is calculated as the product of a scalar α and the one-year effective expected positive exposure[9]:

$$\mathrm{EAD} = \alpha \cdot \mathrm{EEPE}(0;1)$$

The Basel Committee has set the value α at 1.4. The maturity M used in the IRB formula is equal to one year if the remaining maturity is less or equal than one year. Otherwise, it is calculated as follows[10]:

$$\mathrm{M} = \min\left(1 + \frac{\sum_{k=1} \mathbb{1}\{t_k > 1\} \cdot \mathrm{EE}(t_k) \cdot \Delta t_k \cdot B_0(t_k)}{\sum_{k=1} \mathbb{1}\{t_k \leq 1\} \cdot \mathrm{EEE}(t_k) \cdot \Delta t_k \cdot B_0(t_k)}, 5\right)$$

Under some conditions, the bank may uses its own estimates for α. Let LEE be the loan equivalent exposure such that:

$$\mathcal{K}(\mathrm{LEE} \cdot \mathrm{LGD} \cdot \mathbb{1}\{\boldsymbol{\tau} \leq T\}) = \mathcal{K}(\mathrm{EAD}(\boldsymbol{\tau}) \cdot \mathrm{LGD} \cdot \mathbb{1}\{\boldsymbol{\tau} \leq T\}) \tag{4.15}$$

The loan equivalent exposure is then the deterministic exposure at default, which gives the same capital than the random exposure at default $\mathrm{EAD}(\boldsymbol{\tau})$. Using a one-factor credit risk model, Canabarro *et al.* (2003) showed that:

$$\alpha = \frac{\mathrm{LEE}}{\mathrm{EPE}}$$

This is the formula that banks must use in order to estimate α, subject to a floor of 1.2.

Example 45 *We assume that the one-year effective expected positive exposure with respect to a given counterparty is equal to* \$50.2 *mn.*

In Table 4.4, we have reported the required capital $\mathcal{K}$ for different values of PD under the foundation IRB approach. The maturity M is equal to one year and we consider the 45% supervisory factor for the loss given default. The exposure at default is calculated with $\alpha = 1.4$. We show the impact of the Basel III multiplier applied to the correlation. In this example, if the default probability of the counterparty is equal to 1%, this induces an additional required capital of 27.77%.

[9]If the remaining maturity τ of the product is less than one year, the exposure at default becomes:

$$\mathrm{EAD} = \alpha \cdot \mathrm{EEPE}(0;\tau)$$

[10]The maturity has then a cap of five years.

TABLE 4.4: Capital charge of counterparty credit risk under the FIRB approach

	PD	1%	2%	3%	4%	5%
Basel II	ρ (PD) (in %)	19.28	16.41	14.68	13.62	12.99
	$\mathcal{K}$ (in \$ mn)	4.12	5.38	6.18	6.82	7.42
Basel III	ρ (PD) (in %)	24.10	20.52	18.35	17.03	16.23
	$\mathcal{K}$ (in \$ mn)	5.26	6.69	7.55	8.25	8.89
	$\Delta\mathcal{K}$ (in %)	27.77	24.29	22.26	20.89	19.88

4.1.3.2 Non-internal model methods (Basel II)

Under the current exposure method (CEM), we have:

$$\text{EAD} = \text{CE}(0) + A$$

where CE (0) is the current exposure and A is the add-on value. In the views of the Basel Committee, CE (0) represents the replacement cost, whereas the add-on reflects the potential future exposure of the contract. For a single OTC transaction, A is the product of the notional and the add-on factor, which is given in Table 4.5. For a portfolio of OTC transactions with netting agreements, the exposure at default is the sum of the current net exposure plus a net add-one value A_N, which is defined as follows:

$$A_N = (0.4 + 0.6 \cdot \text{NGR}) \cdot A_G$$

where $A_G = \sum_i A_i$ is the gross add-on, A_i is the add-on of the i^{th} transaction and NGR is the ratio between the current net and gross exposures.

TABLE 4.5: Regulatory add-on factors for the current exposure method

Residual Maturity	Fixed Income	FX and Gold	Equity	Precious Metals	Other Commodities
0−1Y	0.0%	1.0%	8.0%	7.0%	10.0%
1Y−5Y	0.5%	5.0%	8.0%	7.0%	12.0%
5Y+	1.5%	7.5%	10.0%	8.0%	15.0%

Example 46 *We consider a portfolio of four OTC derivatives, which are traded with the same counterparty:*

Contract	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$
Asset class	*Fixed income*	*Fixed income*	*Equity*	*Equity*
Notional (in \$ mn)	100	40	20	10
Maturity	2Y	6Y	6M	18M
Mark-to-market (in \$ mn)	3.0	−2.0	2.0	−1.0

We assume that there are two netting arrangements: one concerning fixed income derivatives and another one for equity derivatives.

In the case where there is no netting agreement, we obtain these results:

Contract	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	Sum
CE (0) (in \$ mn)	3.0	0.0	2.0	0.0	5.0
Add-on (in %)	0.5	1.5	8.0	8.0	
A (in \$ mn)	0.5	0.6	1.6	0.8	3.5

The exposure at default is then equal to \$8.5 mn. If we take into account the two netting agreements, the current net exposure becomes:

$$\mathrm{CE}(0) = \max(3-2,0) + \max(2-1,0) = \$2 \text{ mn}$$

We deduce that NGR is equal to 2/5 or 40%. It follows that:

$$A_N = (0.4 + 0.6 \times 0.4) \times 3.5 = \$2.24 \text{ mn}$$

Finally, the exposure at default is equal to \$4.24 mn.

The standardized method was designed for banks that do not have the approval to apply the internal model method, but would like to have a more sensitive approach that the current exposure method. In this framework, the exposure at default is equal to:

$$\mathrm{EAD} = \beta \cdot \max\left(\sum_i \mathrm{CMV}_i, \sum_j \mathrm{CCF}_j \cdot \left|\sum_{i \in j} \mathrm{RPT}_i\right|\right)$$

where CMV_i is the current market value of transaction i, CCF_j is the supervisory credit conversion factor with respect to the hedging set j and RPT_i is the risk position from transaction i. The supervisory scaling factor β is set to 1.4. In this approach, the risk positions have to be grouped into hedging sets, which are defined by similar instruments (e.g. same commodity, same issuer, same currency, etc.). The risk position $\sum_{i \in j} \mathrm{RPT}_i$ is the sum of notional values of linear instruments and delta-equivalent notional values of non-linear instruments, which belong to the hedging set j. The credit conversion factors ranges from 0.3% to 10%. The initial goal of the Basel Committee was to provide an approach which mimics the internal model method[11]. However, the SM approach was never really used by banks. Indeed, it didn't interest advanced banks that preferred to implement the IMM, and it was too complicated for the other banks that have used the CEM.

4.1.3.3 SA-CCR method (Basel III)

The SA-CCR has been adopted by the Basel Committee in March 2014 in order to replace non-internal models approaches since January 2017. The main motivation the Basel Committee was to propose a more-sensitive approach, which can easily be implemented:

> "*Although being more risk-sensitive than the CEM, the SM was also criticized for several weaknesses. Like the CEM, it did not differentiate between margined and unmargined transactions or sufficiently capture the level of volatilities observed over stress periods in the last five years. In addition, the definition of hedging set led to operational complexity resulting in an inability to implement the SM, or implementing it in inconsistent ways*" (BCBS, 2014b, page 1).

The exposure at default under the SA-CCR is defined as follows:

$$\mathrm{EAD} = \alpha \cdot (\mathrm{RC} + \mathrm{PFE})$$

where RC is the replacement cost (or the current exposure), PFE is the potential future exposure and α is equal to 1.4. We can view this formula as an approximation of the IMM calculation, meaning that RC + PFE represents a stylized EEPE value. The PFE add-on is given by:

$$\mathrm{PFE} = \gamma \cdot \sum_{q=1}^{5} A^{(\mathcal{C}_q)}$$

[11] Indeed, the β multiplier coefficient is the equivalent of the α multiplier coefficient, whereas the rest of the expression can be interpreted as an estimate of the effective expected positive exposure.

where γ is the multiplier and $A^{(\mathcal{C}_q)}$ is the add-on of the asset class $\mathcal{C}_q$ (interest rate, foreign exchange, credit, equity and commodity). We have:

$$\gamma = \min\left(1, 0.05 + 0.95 \cdot \exp\left(\frac{\mathrm{MtM}}{1.90 \cdot \sum_{q=1}^{5} A^{(\mathcal{C}_q)}}\right)\right)$$

where MtM is the mark-to-market value of the derivative transactions minus the haircut value of net collateral held. We notice that γ is equal to 1 when the mark-to-market is positive and $\gamma \in [5\%, 1]$ when the net mark-to-market is negative. Figure 4.6 shows the relationship between the ratio $\mathrm{MtM} \Big/ \sum_{q=1}^{5} A^{(\mathcal{C}_q)}$ and the multiplier γ. The role of γ is then to reduce the potential future exposure in the case of negative mark-to-market.

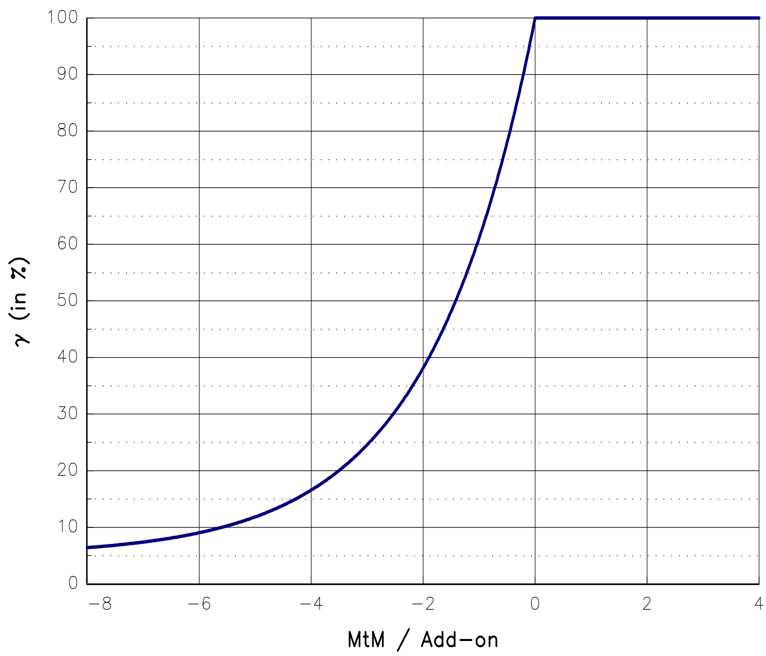

FIGURE 4.6: Impact of negative mark-to-market on the PFE multiplier

The general steps for calculating the add-on are the following. First, we have to determine the primary risk factors of each transaction in order to classify the transaction into one or more asset classes. Second, we calculate an adjusted notional amount d_i at the transaction level[12] and a maturity factor $\mathcal{MF}_i$, which reflects the time horizon appropriate for this type of transactions. For unmargined transactions, we have:

$$\mathcal{MF}_i = \sqrt{\min\left(\mathrm{M}_i, 1\right)}$$

[12]The trade-level adjusted notional d_i is defined as the product of current price of one unit and the number of units for equity and commodity derivatives, the notional of the foreign currency leg converted to domestic currency for foreign exchange derivatives and the product of the trade notional amount and the supervisory duration $\mathcal{SD}_i$ for interest rate and credit derivatives. The supervisory duration $\mathcal{SD}_i$ is defined as follows:

$$\mathcal{SD}_i = 20 \cdot \left(e^{-0.05 \cdot S_i} - e^{-0.05 \cdot E_i}\right)$$

where S_i and E_i are the start and end dates of the time period referenced by the derivative instrument.

where M_i is the remaining maturity of the transaction and is floored by 10 days. For margined transactions, we have:

$$\mathcal{MF}_i = \frac{3}{2}\sqrt{\mathrm{M}_i^\star}$$

where $\mathrm{M}_i^\star$ is the appropriate margin period of risk (MPOR). Then, we apply a supervisory delta adjustment $\boldsymbol{\Delta}_i$ to each transaction[13] and a supervisory factor $\mathcal{SF}_j$ to each hedging set j in order to take the volatility into account. The add-on of one transaction i has then the following expression:

$$A_i = \mathcal{SF}_j \cdot (\boldsymbol{\Delta}_i \cdot d_i \cdot \mathcal{MF}_i)$$

Finally, we apply an aggregation method to calculate the add-on $A^{(\mathcal{C}_q)}$ of the asset class $\mathcal{C}_q$ by considering correlations between hedging sets. Here are the formulas that determine the add-on values:

- The add-on for interest rate derivatives is equal to:

$$A^{(\mathrm{ir})} = \sum_j \mathcal{SF}_j \cdot \sqrt{\sum_{k=1}^{3}\sum_{k'=1}^{3} \rho_{k,k'} \cdot D_{j,k} \cdot D_{j,k'}}$$

  where notations j and k refer to currency j and maturity bucket[14] k and the effective notional $D_{j,k}$ is calculated according to:

$$D_{j,k} = \sum_{i \in (j,k)} \boldsymbol{\Delta}_i \cdot d_i \cdot \mathcal{MF}_i$$

- For foreign exchange derivatives, we obtain:

$$A^{(\mathrm{fx})} = \sum_j \mathcal{SF}_j \cdot \left| \sum_{i \in j} \boldsymbol{\Delta}_i \cdot d_i \cdot \mathcal{MF}_i \right|$$

  where the hedging set j refers to currency pair j.

- The add-on for credit and equity derivatives use the same formula:

$$A^{(\mathrm{credit/equity})} = \sqrt{\left(\sum_k \rho_k \cdot A_k\right)^2 + \sum_k (1 - \rho_k^2) \cdot A_k^2}$$

  where k represents entity k and:

$$A_k = \mathcal{SF}_k \cdot \sum_{i \in k} \boldsymbol{\Delta}_i \cdot d_i \cdot \mathcal{MF}_i$$

- In the case of commodity derivatives, we have:

$$A^{(\mathrm{commodity})} = \sum_j \sqrt{\left(\rho_j \cdot \sum_k A_{j,k}\right)^2 + \left(1 - \rho_j^2\right) \cdot \sum_k A_{j,k}^2}$$

  where j indicates the hedging set, k corresponds to the commodity type and:

$$A_{j,k} = \mathcal{SF}_{j,k} \cdot \sum_{i \in (j,k)} \boldsymbol{\Delta}_i \cdot d_i \cdot \mathcal{MF}_i$$

[13] For instance $\boldsymbol{\Delta}_i$ is equal to -1 for a short position, $+1$ for a long position, the Black-Scholes delta for an option position, etc.

[14] The three maturity buckets k are (1) less than one year, (2) between one and five years and (3) more than five years.

TABLE 4.6: Supervisory parameters for the SA-CCR approach

Asset class		$\mathcal{SF}_j$	ρ_k			Σ_i
Interest rate	0−1Y	0.50%	100%			50%
	1Y−5Y	0.50%	70%	100%		50%
	5Y+	0.50%	30%	70%	100%	50%
Foreign exchange		4.00%				15%
Credit	AAA	0.38%		50%		100%
	AA	0.38%		50%		100%
	A	0.42%		50%		100%
	BBB	0.54%		50%		100%
	BB	1.06%		50%		100%
	B	1.60%		50%		100%
	CCC	6.00%		50%		100%
	IG index	0.38%		80%		80%
	SG index	1.06%		80%		80%
Equity	Single name	32.00%		50%		120%
	Index	20.00%		80%		75%
Commodity	Electricity	40.00%		40%		150%
	Oil & gas	18.00%		40%		70%
	Metals	18.00%		40%		70%
	Agricultural	18.00%		40%		70%
	Other	18.00%		40%		70%

Source: BCBS (2014b).

For interest rate derivatives, hedging sets correspond to all derivatives in the same currency (e.g. USD, EUR, JPY). For currency, they consists of all currency pairs (e.g. USD/EUR, USD/JPY, EUR/JPY). For credit and equity, there is a single hedging set, which contains all the entities (both single names and indices). Finally, there are four hedging sets for commodity derivatives: energy (electricity, oil & gas), metals, agricultural and other. In Table 4.6, we give the supervisory parameters[15] for the factor $\mathcal{SF}_j$, the correlation[16] ρ_k and the implied volatility Σ_i in order to calculate Black-Scholes delta exposures. We notice that the value of the supervisory factor can differ within one hedging set. For instance, it is equal to 0.38% for investment grade (IG) indices, while it takes the value 1.06% for speculative grade (SG) indices.

Example 47 *The netting set consists of four interest rate derivatives[17]:*

Trade	*Instrument*	*Currency*	*Maturity*	*Swap*	*Notional*	MtM
1	*IRS*	*USD*	*9M*	*Payer*	4	0.10
2	*IRS*	*USD*	*4Y*	*Receiver*	20	−0.20
3	*IRS*	*USD*	*10Y*	*Payer*	20	0.70
4	*Swaption 10Y*	*USD*	*1Y*	*Receiver*	5	0.50

This netting set consists of only one hedging set, because the underlying assets of all these derivative instruments are USD interest rates. We report the different calculations in

[15] Source: BCBS (2014b).
[16] We notice that we consider cross-correlations between the three time buckets for interest rate derivatives.
[17] For the swaption, the forward rate swap and the strike value are equal to 6% and 5%.

the following table:

i	k	S_i	E_i	$\mathcal{SD}_i$	Δ_i	d_i	$\mathcal{MF}_i$	D_i
1	1	0.00	0.75	0.74	1.00	2.94	0.87	2.55
2	2	0.00	4.00	3.63	−1.00	72.51	1.00	−72.51
3	3	0.00	10.00	7.87	1.00	157.39	1.00	157.39
4	3	1.00	11.00	7.49	−0.27	37.43	1.00	−10.08

where k indicates the time bucket, S_i is the start date, E_i is the end date, $\mathcal{SD}_i$ is the supervisory duration, $\boldsymbol{\Delta}_i$ is the delta, d_i is the adjusted notional, $\mathcal{MF}_i$ is the maturity factor and D_i is the effective notional. For instance, we obtain the following results for the swaption transaction:

$$\begin{aligned}
\mathcal{SD}_i &= 20 \times \left(e^{-0.05\times 1} - e^{-0.05\times 10}\right) = 7.49 \\
\boldsymbol{\Delta}_i &= -\Phi\left(-\frac{\ln(6\%/5\%)}{0.5\times\sqrt{1}} + \frac{1}{2}\times 0.5\times\sqrt{1}\right) = -0.27 \\
d_i &= 7.49 \times 5 = 37.43 \\
\mathcal{MF}_i &= \sqrt{1} = 1 \\
D_i &= -0.27 \times 37.43 \times 1 = -10.08
\end{aligned}$$

We deduce that the effective notional of time buckets is respectively equal to $D_1 = 2.55$, $D_2 = -72.51$ and $D_3 + D_4 = 147.30$. It follows that:

$$\begin{aligned}
\sum_{k=1}^{3}\sum_{k'=1}^{3}\rho_{k,k'}D_{j,k}D_{j,k'} &= 2.55^2 - 2\times 70\% \times 2.55 \times 72.51 + \\
&\quad 72.51^2 - 2\times 70\% \times 72.51 \times 147.30 + \\
&\quad 147.30^2 + 2 \times 30\% \times 2.55 \times 147.30 \\
&= 11\,976.1
\end{aligned}$$

While the supervisory factor is 0.50%, the add-on value $A^{(\mathrm{ir})}$ is then equal to 0.55. The replacement cost is:

$$\mathrm{RC} = \max(0.1 - 0.2 + 0.7 + 0.5, 0) = 1.1$$

Because the mark-to-market of the netting set is positive, the PFE multiplier is equal to 1. We finally deduce that:

$$\mathrm{EAD} = 1.4 \times (1.1 + 1 \times 0.55) = 2.31$$

Remark 51 *Annex 4 of BCBS (2014b) contains four examples of SA-CCR calculations and presents also several applications including different hedging sets, netting sets and asset classes.*

Even if SA-CCR is a better approach for measuring the counterparty credit risk than CEM and SM, its conservative calibration has been strongly criticized, in particular the value of α. For instance, the International Swaps and Derivatives Association reports many examples, where the EAD calculated with SA-CCR is a multiple of the EAD calculated with CEM and IMM[18]. This is particularly true when the mark-to-market is negative and the hedging set is unmargined. In fact, the industry considers that $\alpha \approx 1$ is more appropriate than $\alpha = 1.4$.

[18] www.isda.org/a/qTiDE/isda-letter-to-the-bcbs-on-sa-ccr-march-2017.pdf

4.1.4 Impact of wrong way risk

According to ISDA (2014b), the wrong way risk (WWR) is defined as the risk that "*occurs when exposure to a counterparty or collateral associated with a transaction is adversely correlated with the credit quality of that counterparty*". This means that the exposure at default of the OTC contract and the default risk of the counterparty are not independent, but positively correlated. Generally, we distinguish two types of wrong way risk:

1. general (or conjectural) wrong way risk occurs when the credit quality of the counterparty is correlated with macroeconomic factors, which also impact the value of the transaction;

2. specific wrong way risk occurs when the correlation between the exposure at default and the probability of default is mainly explained by some idiosyncratic factors.

For instance, general WWR arises when the level of interest rates both impacts the mark-to-market of the transaction and the creditworthiness of the counterparty. An example of specific WWR is when Bank A buys a CDS protection on Bank B from Bank C, and the default probabilities of B and C are highly correlated. In this case, if the credit quality of B deteriorates, both the mark-to-market of the transaction and the default risk of C increase.

Remark 52 *Right way risk (RWR) corresponds to the situation where the counterparty exposure and the default risk are negatively correlated. In this case, the mark-to-market of the transaction decreases as the counterparty approaches the default. By definition, RWR is less a concern from a regulation point of view.*

4.1.4.1 An example

Let us assume that the mark-to-market of the OTC contract is given by a Brownian motion:

$$\mathrm{MtM}(t) = \mu + \sigma W(t)$$

If we note $e(t) = \max(\mathrm{MtM}(t), 0)$, we have:

$$\begin{aligned}
\mathbb{E}[e(t)] &= \int_{-\infty}^{\infty} \max\left(\mu + \sigma\sqrt{t}x, 0\right)\phi(x)\,\mathrm{d}x \\
&= \mu \int_{-\mu/(\sigma\sqrt{t})}^{\infty} \phi(x)\,\mathrm{d}x + \sigma\sqrt{t}\int_{-\mu/(\sigma\sqrt{t})}^{\infty} x\phi(x)\,\mathrm{d}x \\
&= \mu\left(1 - \Phi\left(-\frac{\mu}{\sigma\sqrt{t}}\right)\right) + \sigma\sqrt{t}\left[-\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}\right]_{-\mu/(\sigma\sqrt{t})}^{\infty} \\
&= \mu\Phi\left(\frac{\mu}{\sigma\sqrt{t}}\right) + \sigma\sqrt{t}\phi\left(\frac{\mu}{\sigma\sqrt{t}}\right)
\end{aligned}$$

We consider the Merton approach for modeling the default time $\boldsymbol{\tau}$ of the counterparty. Let $B(t) = \Phi^{-1}(1 - \mathbf{S}(t))$ be the default barrier, where $\mathbf{S}(t)$ is the survival function of the counterparty. We assume that the dependence between the mark-to-market $\mathrm{MtM}(t)$ and the survival time is equal to the Normal copula $\mathbf{C}(u_1, u_2; \rho)$ with parameter ρ. Redon (2006) shows that[19]:

$$\begin{aligned}\mathbb{E}\left[e\left(t\right) \mid \boldsymbol{\tau}=t\right] &= \mathbb{E}\left[e\left(t\right) \mid B\left(t\right)=B\right] \\ &= \mu_B \Phi\left(\frac{\mu_B}{\sigma_B}\right)+\sigma_B \phi\left(\frac{\mu_B}{\sigma_B}\right)\end{aligned}$$

where $\mu_B=\mu+\rho\sigma\sqrt{t}B$ and $\sigma_B=\sqrt{1-\rho^2}\sigma\sqrt{t}$. With the exception of $\rho=0$, we have:

$$\mathbb{E}\left[e\left(t\right)\right] \neq \mathbb{E}\left[e\left(t\right) \mid \boldsymbol{\tau}=t\right]$$

In Figure 4.7, we report the conditional distribution of the mark-to-market given that the default occurs at time $t=1$. The parameters are $\mu=0$, $\sigma=1$ and $\boldsymbol{\tau}\sim\mathcal{E}\left(\lambda\right)$ where λ is calibrated to fit the one-year probability of default PD[20]. We notice that the exposure at default decreases with the correlation ρ when PD is equal to 1% (top/left panel), whereas it increases with the correlation ρ when PD is equal to 99% (top/right panel). We verify the stochastic dominance of the mark-to-market with respect to the default probability. Figure 4.8 shows the relationship between the conditional expectation $\mathbb{E}\left[e\left(t\right) \mid \boldsymbol{\tau}=t\right]$ and the different parameters[21]. As expected, the exposure at default is an increasing function of μ, σ, ρ and PD.

4.1.4.2 Calibration of the α factor

In the internal model method, the exposure at default is computed by scaling the effective expected positive exposure:

$$\text{EAD}=\alpha\cdot\text{EEPE}\left(0;1\right)$$

where α is the scaling factor. In this framework, we assume that the mark-to-market of the OTC transaction and the default risk of the counterparty are not correlated. Therefore, the Basel Committee requires that the calibration of the scaling factor α incorporates the general wrong way risk. According to BCBS (2006), we have[22]:

$$\alpha=\frac{\boldsymbol{\mathcal{K}}\left(\text{EAD}\left(\tau\right)\cdot\text{LGD}\cdot\mathbb{1}\left\{\tau\leq T\right\}\right)}{\boldsymbol{\mathcal{K}}\left(\text{EPE}\cdot\text{LGD}\cdot\mathbb{1}\left\{\tau\leq T\right\}\right)}$$

[19] Since we have $1-\mathbf{S}\left(t\right)\sim\mathcal{U}_{[0,1]}$, it follows that $B\left(t\right)\sim\mathcal{N}\left(0,1\right)$. We deduce that the random vector $\left(\text{MtM}\left(t\right),B\left(t\right)\right)$ is normally distributed:

$$\left(\begin{array}{c}\text{MtM}\left(t\right)\\ B\left(t\right)\end{array}\right)\sim\mathcal{N}\left(\left(\begin{array}{c}\mu\\ 0\end{array}\right),\left(\begin{array}{cc}\sigma^2 t & \rho\sigma\sqrt{t}\\ \rho\sigma\sqrt{t} & 1\end{array}\right)\right)$$

because the correlation $\rho\left(\text{MtM}\left(t\right),B\left(t\right)\right)$ is equal to the Normal copula parameter ρ. Using the conditional expectation formula given on page 1062, it follows that:

$$\text{MtM}\left(t\right) \mid B\left(t\right)=B\sim\mathcal{N}\left(\mu_B,\sigma_B^2\right)$$

where:

$$\mu_B=\mu+\rho\sigma\sqrt{t}\left(B-0\right)$$

and:

$$\sigma_B^2=\sigma^2 t-\rho^2\sigma^2 t=\left(1-\rho^2\right)\sigma^2 t$$

[20] We have $1-e^{-\lambda}=\text{PD}$.

[21] The default values are $\mu=0$, $\sigma=1$, $\text{PD}=90\%$ and $\rho=50\%$.

[22] Using standard assumptions (single factor model, fined-grained portfolio, etc.), the first-order approximation is:

$$\alpha\approx\frac{\text{LEE}}{\text{EPE}}$$

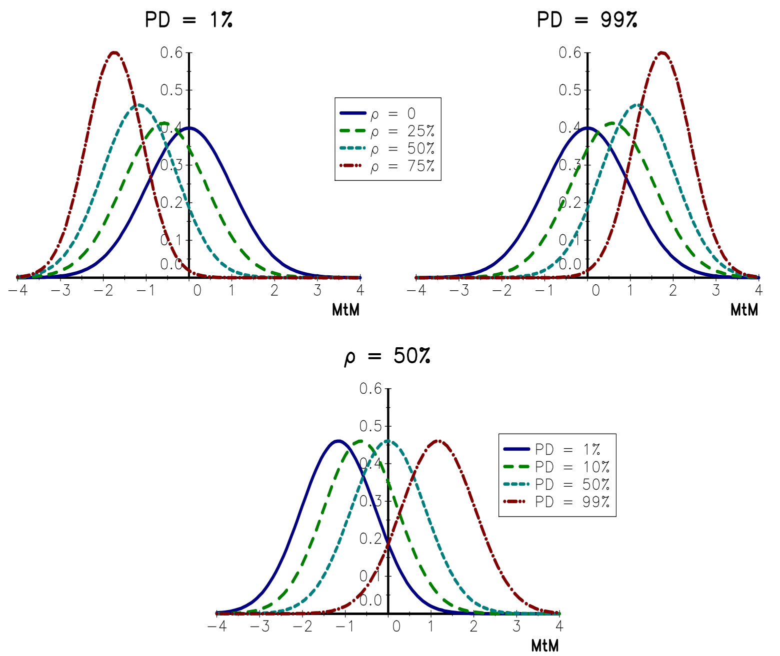

FIGURE 4.7: Conditional distribution of the mark-to-market

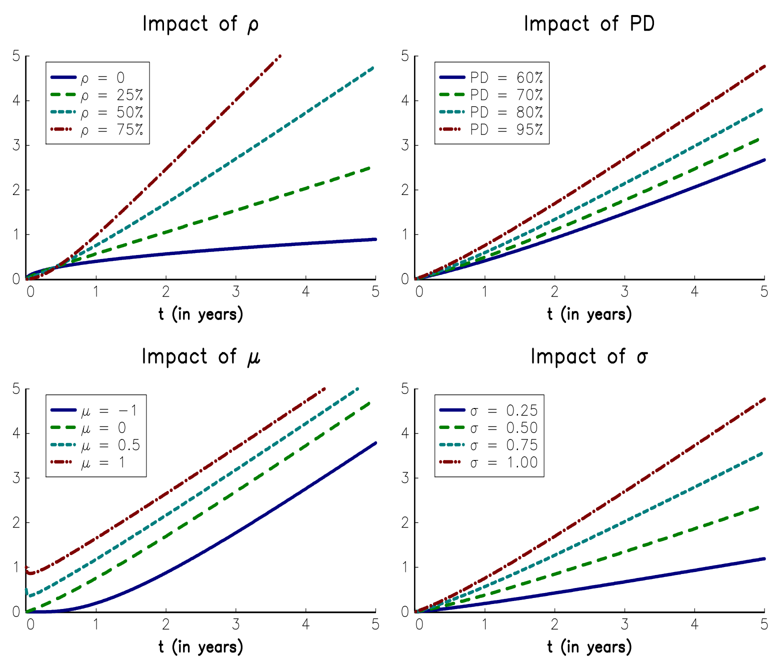

FIGURE 4.8: Conditional expectation of the exposure at default

Again, the Basel Committee considers a conservative approach, since they use EPE instead of EEPE for defining the denominator of α.

The calibration of α for a bank portfolio is a difficult task, because it is not easy to consider a joint modeling of market and credit risk factors. Let us write the portfolio loss as follows:

$$L = \sum_{i=1}^{n} \mathrm{EAD}\left(\boldsymbol{\tau}_i, \mathcal{F}_1, \ldots, \mathcal{F}_m\right) \cdot \mathrm{LGD}_i \cdot \mathbb{1}\left\{\boldsymbol{\tau}_i \leq T_i\right\}$$

where $\mathcal{F} = (\mathcal{F}_1, \ldots, \mathcal{F}_m)$ are the market risk factors and $\boldsymbol{\tau} = (\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_n)$ are the default times. Wrong way risk implies to correlate the random vectors $\mathcal{F}$ and $\boldsymbol{\tau}$. Given a small portfolio with a low number of transactions and counterparty entities, we can simulate the portfolio loss and calculate the corresponding α, but this Monte Carlo exercise is unrealistic for a comprehensive bank portfolio. Nevertheless, we can estimate α for more or less canonical portfolios. For instance, according to Cespedes *et al.* (2010), the scaling factor α may range from 0.7 to 1.4. When market and credit risks are uncorrelated, α is close to one. α is less than one for general right way risks, while it is larger than one for general wrong way risks. However, for realistic market-credit correlations, α is below 1.2.

Remark 53 *The treatment of specific wrong way risk is different. First, the bank must identify all the counterparty entities where specific WWR is significant, and monitor these operations. Second, the bank must calculate a conservative* EAD *figure.*

Remark 54 *The modeling of wrong way risk implies to correlate market and credit risk factors. The main approach is to specify a copula model. As the dimension of the problem is high (m risk factors and n counterparties), Cespedes* et al. *(2010) propose to consider a resampling approach. Another way is to relate the hazard rate of survival functions with the value of the contract (Hull and White, 2012). These two approaches will be discussed in the next section.*

4.2 Credit valuation adjustment

CVA is the adjustment to the risk-free (or fair) value of derivative instruments to account for counterparty credit risk. Thus, CVA is commonly viewed as the market price of CCR. The concept of CVA was popularized after the 2008 Global Financial Crisis, even if investments bank started to use CVA in the early 1990s (Litzenberger, 1992; Duffie and Huang, 1996). Indeed, during the global financial crisis, banks suffered significant counterparty credit risk losses on their OTC derivatives portfolios. However, according to BCBS (2010), roughly two-thirds of these losses came from CVA markdowns on derivatives and only one-third were due to counterparty defaults. In a similar way, the Financial Service Authority concluded that CVA losses were five times larger than CCR losses for UK banks during the period 2007-2009. In this context, BCBS (2010) included CVA capital charge in the Basel III framework, whereas credit-related adjustments were introduced in the accounting standard IFRS 13 also called *Fair Value Measurement*[23]. Nevertheless, the complexity of CVA raises several issues (EBA, 2015a). This is why questions around the CVA are not stabilized and new standards are emerging, but they only provide partial answers.

[23] IFRS 13 was originally issued in May 2011 and became effective after January 2013.

4.2.1 Definition

4.2.1.1 Difference between CCR and CVA

In order to understand the credit valuation adjustment, it is important to make the distinction between CCR and CVA. CCR is the credit risk of OTC derivatives associated to the default of the counterparty, whereas CVA is the market risk of OTC derivatives associated to the credit migration of the two counterparties. This means that CCR occurs at the default time. On the contrary, CVA impacts the market value of OTC derivatives before the default time.

Let us consider an example with two banks A and B and an OTC contract $\mathfrak{C}$. The P&L $\Pi_{A|B}$ of Bank A is equal to:

$$\Pi_{A|B} = \text{MtM} - \text{CVA}_B$$

where MtM is the risk-free mark-to-market value of $\mathfrak{C}$ and CVA_B is the CVA with respect to Bank B. We assume that Bank A has traded the same contract with Bank C. It follows that:

$$\Pi_{A|C} = \text{MtM} - \text{CVA}_C$$

In a world where there is no counterparty credit risk, we have:

$$\Pi_{A|B} = \Pi_{A|C} = \text{MtM}$$

If we take into account the counterparty credit risk, the two P&Ls of the same contract are different because Bank A does not face the same risk:

$$\Pi_{A|B} \neq \Pi_{A|C}$$

In particular, if Bank A wants to close the two exposures, it is obvious that the contact $\mathfrak{C}$ with the counterparty B has more value than the contact $\mathfrak{C}$ with the counterparty C if the credit risk of B is lower than the credit risk of C. In this context, the notion of mark-to-market is complex, because it depends on the credit risk of the counterparties.

Remark 55 *If the bank does not take into account CVA to price its OTC derivatives, it does not face CVA risk. This situation is now marginal because of the accounting standards IFRS 13.*

4.2.1.2 CVA, DVA and bilateral CVA

Previously, we have defined the CVA as the market risk related to the credit risk of the counterparty. According to EBA (2015a), it should reflect today's best estimate of the potential loss on the OTC derivative due to the default of the counterparty. In a similar way, we can define the debit value adjustment (DVA) as the credit-related adjustment capturing the entity's own credit risk. In this case, DVA should reflect the potential gain on the OTC derivative due to the entity's own default. If we consider our previous example, the expression of the P&L becomes:

$$\Pi_{A|B} = \text{MtM} + \underbrace{\text{DVA}_A - \text{CVA}_B}_{\text{Bilateral CVA}}$$

The combination of the two credit-related adjustments is called the bivariate CVA. We then obtain the following cases:

1. if the credit risk of Bank A is lower than the credit risk of Bank B ($\text{DVA}_A < \text{CVA}_B$), the bilateral CVA of Bank A is negative and reduces the value of the OTC portfolio from the perspective of Bank A;

2. if the credit risk of Bank A is higher than the credit risk of Bank B ($\mathrm{DVA}_A > \mathrm{CVA}_B$), the bilateral CVA of Bank A is positive and increases the value of the OTC portfolio from the perspective of Bank A;

3. if the credit risk of Bank A is equivalent to the credit risk of Bank B, the bilateral CVA is equal to zero.

We notice that the DVA of Bank A is the CVA of Bank A from the perspective of Bank B:

$$\mathrm{CVA}_A = \mathrm{DVA}_A$$

We also have $\mathrm{DVA}_B = \mathrm{CVA}_B$, which implies that the P&L of Bank B is equal to:

$$\begin{aligned}\Pi_{B|A} &= -\mathrm{MtM} + \mathrm{DVA}_B - \mathrm{CVA}_A \\ &= -\mathrm{MtM} + \mathrm{CVA}_B - \mathrm{DVA}_A \\ &= -\Pi_{A|B}\end{aligned}$$

We deduce that the P&Ls of Banks A and B are coherent in the bilateral CVA framework as in the risk-free MtM framework. This is not true if we only consider the (unilateral or one-sided) CVA or DVA adjustment.

In order to define more precisely CVA and DVA, we introduce the following notations:

- The positive exposure $e^+(t)$ is the maximum between 0 and the risk-free mark-to-market:
$$e^+(t) = \max(\mathrm{MtM}(t), 0)$$
This quantity was previously denoted by $e(t)$ and corresponds to the potential future exposure in the CCR framework.

- The negative exposure $e^-(t)$ is the difference between the risk-free mark-to-market and the positive exposure:
$$e^-(t) = \mathrm{MtM}(t) - e^+(t)$$
We also have:
$$\begin{aligned}e^-(t) &= -\min(\mathrm{MtM}(t), 0) \\ &= \max(-\mathrm{MtM}(t), 0)\end{aligned}$$
The negative exposure is then the equivalent of the positive exposure from the perspective of the counterparty.

The credit value adjustment is the risk-neutral discounted expected value of the potential loss:

$$\mathrm{CVA} = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}\{\boldsymbol{\tau}_B \le T\} \cdot e^{-\int_0^{\boldsymbol{\tau}_B} r_t \,\mathrm{d}t} \cdot L\right]$$

where T is the maturity of the OTC derivative, $\boldsymbol{\tau}_B$ is the default time of Bank B and L is the counterparty loss:

$$L = (1 - \boldsymbol{\mathcal{R}}_B) \cdot e^+(\boldsymbol{\tau}_B)$$

Using usual assumptions[24], we obtain:

$$\mathrm{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T B_0(t)\,\mathrm{EpE}(t)\,\mathrm{d}\mathbf{F}_B(t) \tag{4.16}$$

[24] The default time and the discount factor are independent and the recovery rate is constant.

where $\operatorname{EpE}(t)$ is the risk-neutral discounted expected positive exposure:

$$\operatorname{EpE}(t) = \mathbb{E}^{\mathbb{Q}}\left[e^{+}(t)\right]$$

and $\mathbf{F}_B$ is the cumulative distribution function of $\boldsymbol{\tau}_B$. Knowing that the survival function $\mathbf{S}_B(t)$ is equal to $1 - \mathbf{F}_B(t)$, we deduce that:

$$\text{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t) \operatorname{EpE}(t) \, \mathrm{d}\mathbf{S}_B(t) \tag{4.17}$$

In a similar way, the debit value adjustment is defined as the risk-neutral discounted expected value of the potential gain:

$$\text{DVA} = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}\{\boldsymbol{\tau}_A \le T\} \cdot e^{-\int_0^{\boldsymbol{\tau}_A} r_t \, \mathrm{d}t} \cdot G\right]$$

where $\boldsymbol{\tau}_A$ is the default time of Bank A and:

$$G = (1 - \boldsymbol{\mathcal{R}}_A) \cdot e^{-}(\boldsymbol{\tau}_A)$$

Using the same assumptions than previously, it follows that:

$$\text{DVA} = (1 - \boldsymbol{\mathcal{R}}_A) \cdot \int_0^T -B_0(t) \operatorname{EnE}(t) \, \mathrm{d}\mathbf{S}_A(t) \tag{4.18}$$

where $\operatorname{EnE}(t)$ is the risk-neutral discounted expected negative exposure:

$$\operatorname{EnE}(t) = \mathbb{E}^{\mathbb{Q}}\left[e^{-}(t)\right]$$

We deduce that the bilateral CVA is:

$$\begin{aligned} \text{BCVA} &= \text{DVA} - \text{CVA} \\ &= (1 - \boldsymbol{\mathcal{R}}_A) \cdot \int_0^T -B_0(t) \operatorname{EnE}(t) \, \mathrm{d}\mathbf{S}_A(t) - \\ &\quad (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t) \operatorname{EpE}(t) \, \mathrm{d}\mathbf{S}_B(t) \end{aligned} \tag{4.19}$$

When we calculate the bilateral CVA as the difference between the DVA and the CVA, we consider that the DVA does not depend on $\boldsymbol{\tau}_B$ and the CVA does not depend on $\boldsymbol{\tau}_A$. In the more general case, we have:

$$\text{BCVA} = \mathbb{E}^{\mathbb{Q}}\left[\begin{array}{c} \mathbb{1}\{\boldsymbol{\tau}_A \le \min(T, \boldsymbol{\tau}_B)\} \cdot e^{-\int_0^{\boldsymbol{\tau}_A} r_t \, \mathrm{d}t} \cdot G - \\ \mathbb{1}\{\boldsymbol{\tau}_B \le \min(T, \boldsymbol{\tau}_A)\} \cdot e^{-\int_0^{\boldsymbol{\tau}_B} r_t \, \mathrm{d}t} \cdot L \end{array}\right] \tag{4.20}$$

In this case, the calculation of the bilateral CVA requires considering the joint survival function of $(\boldsymbol{\tau}_A, \boldsymbol{\tau}_B)$.

Remark 56 *If we assume that the yield curve is flat and $\mathbf{S}_B(t) = e^{-\lambda_B t}$, we have $\mathrm{d}\mathbf{S}_B(t) = -\lambda_B e^{-\lambda_B t} \, \mathrm{d}t$ and:*

$$\begin{aligned} \text{CVA} &= (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T e^{-rt} \operatorname{EpE}(t) \lambda_B e^{-\lambda_B t} \, \mathrm{d}t \\ &= s_B \cdot \int_0^T e^{-(r + \lambda_B)t} \operatorname{EpE}(t) \, \mathrm{d}t \end{aligned}$$

We notice that the CVA is the product of the CDS spread and the discounted value of the expected positive exposure.

Example 48 *Let us assume that the mark-to-market value is given by:*

$$\mathrm{MtM}(t) = N \int_t^T f(t,T) B_t(s)\, \mathrm{d}s - N \int_t^T f(0,T) B_t(s)\, \mathrm{d}s$$

where N and T are the notional and the maturity of the swap, and $f(t,T)$ is the instantaneous forward rate which follows a geometric Brownian motion:

$$\mathrm{d}f(t,T) = \mu f(t,T)\, \mathrm{d}t + \sigma f(t,T)\, \mathrm{d}W(t)$$

We also assume that the yield curve is flat – $B_t(s) = e^{-r(s-t)}$ – and the risk-neutral survival function is $\mathbf{S}(t) = e^{-\lambda t}$.

Syrkin and Shirazi (2015) show that[25]:

$$\mathrm{EpE}(t) = N f(0,T) \varphi(t,T) \left(e^{\mu t} \Phi \left(\left(\frac{\mu}{\sigma} + \frac{1}{2}\sigma \right) \sqrt{t} \right) - \Phi \left(\left(\frac{\mu}{\sigma} - \frac{1}{2}\sigma \right) \sqrt{t} \right) \right)$$

where:

$$\varphi(t,T) = \frac{1 - e^{-r(T-t)}}{r}$$

It follows that the CVA at time t is equal to:

$$\mathrm{CVA}(t) = \mathcal{s}_B \cdot \int_t^T e^{-(r+\lambda)(u-t)}\, \mathrm{EpE}(u)\, \mathrm{d}u$$

We consider the following numerical values: $N = 1000$, $f(0,T) = 5\%$, $\mu = 2\%$, $\sigma = 25\%$, $T = 10$ years and $\boldsymbol{\mathcal{R}}_B = 50\%$. In Figure 4.9, we have reported the value of $\mathrm{CVA}(t)$ when λ is respectively equal to 20 and 100 bps. By construction, the CVA is maximum at the starting date.

4.2.1.3 Practical implementation for calculating CVA

In practice, we calculate CVA and DVA by approximating the integral by a sum:

$$\mathrm{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \sum_{t_i \leq T} B_0(t_i) \cdot \mathrm{EpE}(t_i) \cdot (\mathbf{S}_B(t_{i-1}) - \mathbf{S}_B(t_i))$$

and:

$$\mathrm{DVA} = (1 - \boldsymbol{\mathcal{R}}_A) \cdot \sum_{t_i \leq T} B_0(t_i) \cdot \mathrm{EnE}(t_i) \cdot (\mathbf{S}_A(t_{i-1}) - \mathbf{S}_A(t_i))$$

where $\{t_i\}$ is a partition of $[0,T]$. For the bilateral CVA, the expression (4.20) can be evaluated using Monte Carlo methods.

We notice that the approximation of $\mathrm{d}\mathbf{S}_B(t)$ is equal to the default probability of Bank B between two consecutive trading dates:

$$\begin{aligned} \mathbf{S}_B(t_{i-1}) - \mathbf{S}_B(t_i) &= \Pr\{t_{i-1} < \boldsymbol{\tau}_B \leq t_i\} \\ &= \mathrm{PD}_B(t_{i-1}, t_i) \end{aligned}$$

and we may wonder what is the best approach for estimating $\mathrm{PD}_B(t_{i-1}, t_i)$. A straightforward solution is to use the default probabilities computed by the internal credit system.

[25] See Exercise 4.4.5 on page 303.

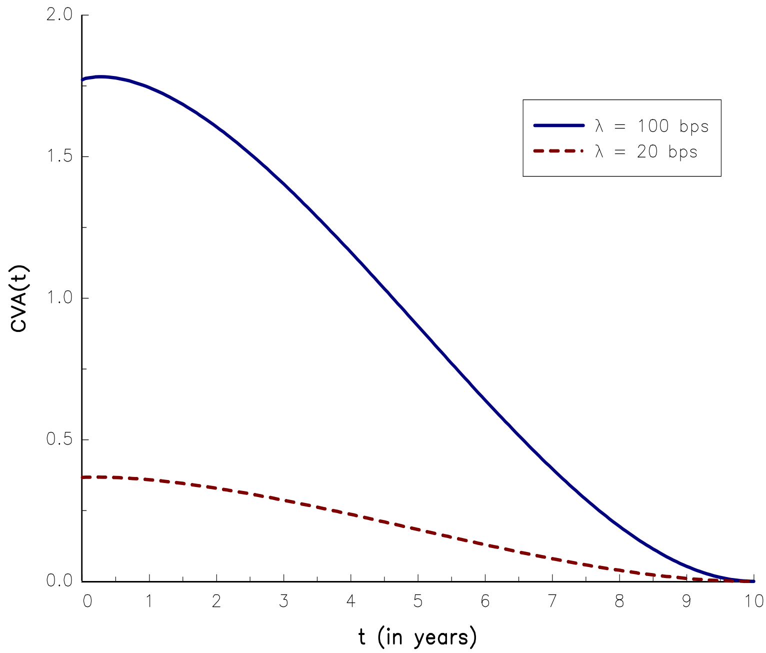

FIGURE 4.9: CVA of fixed-float swaps

However, there is a fundamental difference between CCR and CVA. Indeed, CCR is a default risk and must then be calculated using the historical probability measure $\mathbb{P}$. On the contrary, CVA is a market price, implying that it is valued under the risk-neutral probability measure $\mathbb{Q}$. Therefore, $\mathrm{PD}_B(t_{i-1}, t_i)$ is a risk-neutral probability. Using the credit triangle relationship, we know that the CDS spread $\mathcal{s}$ is related to the intensity λ:

$$\mathcal{s}_B(t) = (1 - \mathcal{R}_B) \cdot \lambda_B(t)$$

We deduce that:

$$\begin{aligned} \mathbf{S}_B(t) &= \exp\left(-\lambda_B(t) \cdot t\right) \\ &= \exp\left(-\frac{\mathcal{s}_B(t) \cdot t}{1 - \mathcal{R}_B}\right) \end{aligned}$$

It follows that the risk-neutral probability of default $\mathrm{PD}_B(t_{i-1}, t_i)$ is equal to:

$$\mathrm{PD}_B(t_{i-1}, t_i) = \exp\left(-\frac{\mathcal{s}_B(t_{i-1}) \cdot t_{i-1}}{1 - \mathcal{R}_B}\right) - \exp\left(-\frac{\mathcal{s}_B(t_i) \cdot t_i}{1 - \mathcal{R}_B}\right)$$

4.2.2 Regulatory capital

The capital charge for the CVA risk has been introduced by the Basel Committee in December 2010 after the Global Financial Crisis. At that moment, banks had the choice between two approaches: the advanced method (AM-CVA) and the standardized method (SM-CVA). However, the Basel Committee completely changed the CVA framework in December 2017 with two new approaches (BA-CVA and SA-CVA) that will replace the previous approaches (AM-CVA and SM-CVA) with effect from January 2022. It is the first time that the Basel Committee completely flip-flopped within the same accord, since these different approaches are all part of the Basel III Accord.

4.2.2.1 The 2010 version of Basel III

Advanced method The advanced method (or AM-CVA) can be considered by banks that use IMM and VaR models. In this approach, we approximate the integral by the middle Riemann sum:

$$\mathrm{CVA} = \mathrm{LGD}_B \cdot \sum_{t_i \leq T} \left(\frac{\mathrm{EpE}\,(t_{i-1})\, B_0\,(t_{i-1}) + B_0\,(t_i)\,\mathrm{EpE}\,(t_i)}{2} \right) \cdot \mathrm{PD}_B\,(t_{i-1}, t_i)$$

where $\mathrm{LGD} = 1 - \boldsymbol{\mathcal{R}}_B$ is the risk-neutral loss given default of the counterparty B and $\mathrm{PD}_B\,(t_{i-1}, t_i)$ is the risk neutral probability of default between t_{i-1} and t_i:

$$\mathrm{PD}_B\,(t_{i-1}, t_i) = \max \left(\exp \left(-\frac{\boldsymbol{s}\,(t_{i-1})}{\mathrm{LGD}_B} \cdot t_{i-1} \right) - \exp \left(-\frac{\boldsymbol{s}\,(t_i)}{\mathrm{LGD}_B} \cdot t_i \right), 0 \right)$$

We notice that a zero floor is added in order to verify that $\mathrm{PD}_B\,(t_{i-1}, t_i) \geq 0$. The capital charge is then equal to:

$$\boldsymbol{\mathcal{K}} = 3 \cdot (\mathrm{CVA} + \mathrm{SCVA})$$

where CVA is calculated using the last one-year period and SCVA is the stressed CVA based on a one-year stressed period of credit spreads.

Standardized method In the standardized method (or SM-CVA), the capital charge is equal to:

$$\boldsymbol{\mathcal{K}} = 2.33 \cdot \sqrt{h} \cdot \sqrt{\left(\frac{1}{2} \sum_i w_i \cdot \Omega_i - w^\star_{\mathrm{index}} \cdot \Omega^\star_{\mathrm{index}} \right)^2 + \frac{3}{4} \sum_i w_i^2 \cdot \Omega_i^2} \tag{4.21}$$

where:

$$\Omega_i = \mathrm{M}_i \cdot \mathrm{EAD}_i \cdot \frac{1 - e^{-0.05 \cdot \mathrm{M}_i}}{0.05 \cdot \mathrm{M}_i} - \mathrm{M}_i^\star \cdot H_i^\star \cdot \frac{1 - e^{-0.05 \cdot \mathrm{M}_i^\star}}{0.05 \cdot \mathrm{M}_i^\star}$$

$$\Omega^\star_{\mathrm{index}} = \mathrm{M}^\star_{\mathrm{index}} \cdot H^\star_{\mathrm{index}} \cdot \frac{1 - e^{-0.05 \cdot \mathrm{M}^\star_{\mathrm{index}}}}{0.05 \cdot \mathrm{M}^\star_{\mathrm{index}}}$$

In this formula, h is the time horizon (one year), w_i is the weight of the i^{th} counterparty based on its rating, M_i is the effective maturity of the i^{th} netting set, EAD_i is the exposure at default of the i^{th} netting set, $\mathrm{M}_i^\star$ is the maturity adjustment factor for the single name hedge, $H_i^\star$ is the hedging notional of the single name hedge, $w^\star_{\mathrm{index}}$ is the weight of the index hedge, $\mathrm{M}^\star_{\mathrm{index}}$ is the maturity adjustment factor for the index hedge and $H^\star_{\mathrm{index}}$ is the hedging notional of the index hedge. In this formula, EAD_i corresponds to the CCR exposure at default calculated with the CEM or IMM approaches.

Remark 57 *We notice that the Basel Committee recognizes credit hedges (single-name CDS, contingent CDS and CDS indices) for reducing CVA volatility. If there is no hedge, we obtain:*

$$\boldsymbol{\mathcal{K}} = 2.33 \cdot \sqrt{h} \cdot \sqrt{\frac{1}{4} \left(\sum_i w_i \cdot \mathrm{M}_i \cdot \mathrm{EAD}_i \right)^2 + \frac{3}{4} \sum_i w_i^2 \cdot \mathrm{M}_i^2 \cdot \mathrm{EAD}_i^2}$$

The derivation of Equation (4.21) is explained in Pykhtin (2012). We consider a Gaussian random vector $X = (X_1, \ldots, X_n)$ with $X_i \sim \mathcal{N}\left(0, \sigma_i^2\right)$. We assume that the random variables $X_1, \ldots, X_n$ follow a single risk factor model such that the correlation $\rho\,(X_i, X_j)$

is constant and equal to ρ. We consider another random variable $X_{n+1} \sim \mathcal{N}\left(0, \sigma_{n+1}^2\right)$ such that $\rho\left(X_i, X_{n+1}\right)$ is also constant and equal to ρ_{n+1}. Let Y be the random variable defined as the sum of X_i's minus X_{n+1}:

$$Y = \sum_{i=1}^{n} X_i - X_{n+1}$$

It follows that $Y \sim \mathcal{N}\left(0, \sigma_Y^2\right)$ where:

$$\sigma_Y^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\rho \sum_{i=1}^{n} \sum_{j=1}^{i} \sigma_i \sigma_j - 2\rho_{n+1}\sigma_{n+1} \sum_{i=1}^{n} \sigma_i + \sigma_{n+1}^2$$

We finally deduce that:

$$\mathbf{F}_Y^{-1}(\alpha) = \Phi^{-1}(\alpha) \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2\rho \sum_{i=1}^{n} \sum_{j=1}^{i} \sigma_i \sigma_j - 2\rho_{n+1}\sigma_{n+1} \sum_{i=1}^{n} \sigma_i + \sigma_{n+1}^2}$$

Equation (4.21) is obtained by setting $\sigma_i = w_i \Omega_i$, $\sigma_{n+1} = w_{\text{index}}^{\star} \Omega_{\text{index}}^{\star}$, $\rho = 25\%$, $\rho_{n+1} = 50\%$ and $\alpha = 99\%$. This means that X_i is the CVA net exposure of the i^{th} netting set (including individual hedges) and X_{n+1} is the macro hedge of the CVA based on credit indices.

4.2.2.2 The 2017 version of Basel III

There are now two approaches available for calculating CVA risk: the basic approach (BA-CVA) and the standardized approach (SA-CVA). However, if the bank has a few exposure on counterparty credit risk[26], it may choose to set its CVA capital requirement equal to its CCR capital requirement.

Basic approach Under the basic approach, the capital requirement is equal to:

$$\mathcal{K} = \beta \cdot \mathcal{K}^{\text{Reduced}} + (1 - \beta) \cdot \mathcal{K}^{\text{Hedged}}$$

where $\mathcal{K}^{\text{Reduced}}$ and $\mathcal{K}^{\text{Hedged}}$ are the capital requirements without and with hedging recognition. The reduced version of the BA-CVA is obtained by setting β to 100%. A bank that actively hedges CVA risks may choose the full version of the BA-CVA. In this case, β is set to 25%.

For the reduced version, we have:

$$\mathcal{K}^{\text{Reduced}} = \sqrt{\left(\rho \cdot \sum_j \text{SCVA}_j\right)^2 + (1 - \rho^2) \cdot \sum_j \text{SCVA}_j^2}$$

where $\rho = 50\%$ and SCVA_j is the CVA capital requirement for the j^{th} counterparty:

$$\text{SCVA}_j = \frac{1}{\alpha} \cdot \text{RW}_j \cdot \sum_k \text{DF}_k \cdot \text{EAD}_k \cdot \text{M}_k$$

In this formula, α is set to 1.4, RW_j is the risk weight for counterparty j, k is the netting set, DF_k is the discount factor, EAD_k is the CCR exposure at default and M_k is the effective

[26] The materiality threshold is €100 bn for the notional amount of non-centrally cleared derivatives.

maturity. These last three quantities are calculated at the netting set level. If the bank use the IMM to calculate the exposure at default, DF_k is equal to one, otherwise we have:

$$\mathrm{DF}_k = \frac{1 - e^{-0.05 \cdot \mathrm{M}_k}}{0.05 \cdot \mathrm{M}_k}$$

RW_j depends on the credit quality of the counterparty (IG/HY) and its sector and is given in Table 4.7.

TABLE 4.7: Supervisory risk weights (BA-CVA)

Sector	Credit quality	
	IG	HY/NR
Sovereign	0.5%	3.0%
Local government	1.0%	4.0%
Financial	5.0%	12.0%
Basic material, energy, industrial, agriculture, manufacturing, mining and quarrying	3.0%	7.0%
Consumer goods and services, transportation and storage, administrative and support service activities	3.0%	8.5%
Technology, telecommunication	2.0%	5.5%
Health care, utilities, professional and technical activities	1.5%	5.0%
Other sector	5.0%	12.0%

Source: BCBS (2017c).

The full version of the BA-CVA recognizes eligible hedging transactions that are used for mitigating the credit spread component of the CVA risk. They correspond to single-name CDS and index CDS transactions. $\mathcal{K}^{\mathrm{Hedged}}$ depends on three components:

$$\mathcal{K}^{\mathrm{Hedged}} = \sqrt{K_1 + K_2 + K_3}$$

According to BCBS (2017c), the first term aggregates the systematic components of the CVA risk:

$$K_1 = \left(\rho \cdot \sum_j \left(\mathrm{SCVA}_j - \mathrm{SNH}_j\right) - \mathrm{IH}\right)^2$$

where SNH_j is the CVA reduction for counterparty j due to single-name hedging and IH is the global CVA reduction due to index hedging. The second term aggregates the idiosyncratic components of the CVA risk:

$$K_2 = \left(1 - \rho^2\right) \cdot \sum_j \left(\mathrm{SCVA}_j - \mathrm{SNH}_j\right)^2$$

Finally, the third term corresponds to the hedging misalignment risk because of the mismatch between indirect hedges and single-name hedges:

$$K_3 = \sum_j \mathrm{HMA}_j$$

The single-name hedge SNH_j is calculated as follows:

$$\mathrm{SNH}_j = \sum_{h \in j} \varrho_{h,j} \cdot (\mathrm{RW}_h \cdot \mathrm{DF}_h \cdot N_h \cdot \mathrm{M}_h)$$

where h represents the single-name CDS transaction, $\varrho_{h,j}$ is the supervisory correlation, DF_h is the discount factor[27], N_h is the notional and M_h is the remaining maturity. These quantities are calculated at the single-name CDS level. The correlation $\varrho_{h,j}$ between the credit spread of the counterparty and the credit spread of the CDS can take three values: 100% if CDS h directly refers to counterparty j, 80% if CDS h has a legal relation with counterparty j, and 50% if CDS h and counterparty j are of the same sector and region. For the index hedge IH, we have a similar formula:

$$\mathrm{IH} = \sum_{h'} \mathrm{RW}_{h'} \cdot \mathrm{DF}_{h'} \cdot N_{h'} \cdot \mathrm{M}_{h'}$$

where h' represents the index CDS transaction. The other quantities $\mathrm{RW}_{h'}$, $\mathrm{DF}_{h'}$, $N_{h'}$ and $\mathrm{M}_{h'}$ are defined exactly as previously except that they are applied at the index CDS level. For the risk weight, its value is the weighted average of risk weights of RW_j:

$$\mathrm{RW}_{h'} = 0.7 \cdot \sum_{j \in h'} w_j \cdot \mathrm{RW}_j$$

where w_j is the weight of the counterparty/sector j in the index CDS h'. We notice that this formula reduces to $\mathrm{RW}_{h'} = 0.7 \cdot \mathrm{RW}_j$ when we consider a sector-specific index. Finally, we have

$$\mathrm{HMA}_j = \sum_{h \in j} \left(1 - \varrho_{h,j}^2\right) \cdot \left(\mathrm{RW}_h \cdot \mathrm{DF}_h \cdot N_h \cdot \mathrm{M}_h\right)^2$$

Remark 58 *In the case where there is no hedge, we have* $\mathrm{SNH}_j = 0$, $\mathrm{HMA}_j = 0$, $\mathrm{IH} = 0$, *and* $\mathcal{K} = \mathcal{K}^{\mathrm{Reduced}}$. *If there is no hedging misalignment risk and no index CDS hedging, we have:*

$$\mathcal{K} = \sqrt{\left(\rho \cdot \sum_j \mathcal{K}_j\right)^2 + (1 - \rho^2) \cdot \sum_j \mathcal{K}_j^2}$$

where $\mathcal{K}_j = \mathrm{SCVA}_j - \mathrm{SNH}_j$ *is the single-name capital requirement for counterparty* j.

Example 49 *We assume that the bank has three financial counterparties A, B and C, that are respectively rated IG, IG and HY. There are 4 OTC transactions, whose characteristics are the following:*

Transaction k	1	2	3	4
Counterparty	*A*	*A*	*B*	*C*
EAD_k	100	50	70	20
M_k	1	1	0.5	0.5

In order to reduce the counterparty credit risk, the bank has purchased a CDS protection on A for an amount of \$75 *mn, a CDS protection on B for an amount of* \$10 *mn and a HY Financial CDX for an amount of* \$10 *mn. The maturity of hedges exactly matches the maturity of transactions. However, the CDS protection on B is indirect, because the underlying name is not B, but B′ which is the parent company of B.*

[27] We have:

$$\mathrm{DF}_h = \frac{1 - e^{-0.05 \cdot \mathrm{M}_h}}{0.05 \cdot \mathrm{M}_h}$$

where M_h is the remaining maturity.

We first begin to calculate the discount factors DF_k for the four transactions. We obtain $\mathrm{DF}_1 = \mathrm{DF}_2 = 0.9754$ and $\mathrm{DF}_3 = \mathrm{DF}_4 = 0.9876$. Then we calculate the single-name capital for each counterparty. For example, we have:

$$\begin{aligned}
\mathrm{SCVA}_A &= \frac{1}{\alpha} \times \mathrm{RW}_A \times (\mathrm{DF}_1 \times \mathrm{EAD}_1 \times \mathrm{M}_1 + \mathrm{DF}_2 \times \mathrm{EAD}_2 \times \mathrm{M}_2) \\
&= \frac{1}{1.4} \times 5\% \times (0.9754 \times 100 \times 1 + 0.9754 \times 50 \times 1) \\
&= 5.225
\end{aligned}$$

We also find that $\mathrm{SCVA}_B = 1.235$ and $\mathrm{SCVA}_C = 0.847$. It follows that $\sum_j \mathrm{SCVA}_j = 7.306$ and $\sum_j \mathrm{SCVA}_j^2 = 29.546$. The capital requirement without hedging is equal to:

$$\mathcal{K}^{\text{Reduced}} = \sqrt{(0.5 \times 7.306)^2 + (1 - 0.5^2) \times 29.546} = 5.959$$

We notice that it is lower than the sum of individual capital charges. In order to take into account the hedging effect, we calculate the single-name hedge parameters:

$$\mathrm{SNH}_A = 5\% \times 100\% \times 0.9754 \times 75 \times 1 = 3.658$$

and:

$$\mathrm{SNH}_B = 5\% \times 80\% \times 0.9876 \times 10 \times 0.5 = 0.198$$

Since the CDS protection is on B' and not B, there is a hedging misalignment risk:

$$\mathrm{HMA}_B = 0.05^2 \times \left(1 - 0.80^2\right) \times (0.9876 \times 10 \times 0.5)^2 = 0.022$$

For the CDX protection, we have:

$$\mathrm{IH} = (0.7 \times 12\%) \times 0.9876 \times 10 \times 0.5 = 0.415$$

Then, we obtain $K_1 = 1.718$, $K_2 = 3.187$, $K_3 = 0.022$ and $\mathcal{K}^{\text{Hedged}} = 2.220$. Finally, the capital requirement is equal to \$3.154 mn:

$$\mathcal{K} = 0.25 \times 5.959 + 0.75 \times 2.220 = 3.154$$

Standardized approach The standardized approach for CVA follows the same principles than the standardized approach SA-TB for the market risk of the trading book. The main difference is that SA-CVA is only based on delta and vega risks, and does not include curvature, jump-to-default and residual risks:

$$\mathcal{K} = \mathcal{K}^{\text{Delta}} + \mathcal{K}^{\text{Vega}}$$

For computing the capital charge, we first consider two portfolios: the CVA portfolio and the hedging portfolio. For each risk (delta and vega), we calculate the weighted CVA sensitivity of each risk factor $\mathcal{F}_j$:

$$\mathrm{WS}_j^{\text{CVA}} = S_j^{\text{CVA}} \cdot \mathrm{RW}_j$$

and:

$$\mathrm{WS}_j^{\text{Hedge}} = S_j^{\text{Hedge}} \cdot \mathrm{RW}_j$$

where S_j and RW_j are the net sensitivity of the CVA or hedging portfolio with respect to the risk factor and the risk weight of $\mathcal{F}_j$. Then, we aggregate the weighted sensitivity in order to obtain a net figure:

$$\mathrm{WS}_j = \mathrm{WS}_j^{\text{CVA}} + \mathrm{WS}_j^{\text{Hedge}}$$

Second, we calculate the capital requirement for the risk bucket $\mathcal{B}_k$:

$$\mathcal{K}_{\mathcal{B}_k} = \sqrt{\sum_j \mathrm{WS}_j^2 + \sum_{j' \neq j} \rho_{j,j'} \cdot \mathrm{WS}_j \cdot \mathrm{WS}_{j'} + 1\% \cdot \sum_j \left(\mathrm{WS}_j^{\mathrm{Hedge}}\right)^2}$$

where $\mathcal{F}_j \in \mathcal{B}_k$. Finally, we aggregate the different buckets for a given risk class:

$$\mathcal{K}^{\mathrm{Delta/Vega}} = m_{\mathrm{CVA}} \cdot \sqrt{\sum_k \mathcal{K}_{\mathcal{B}_k}^2 + \sum_{k' \neq k} \gamma_{k,k'} \cdot \mathcal{K}_{\mathcal{B}_k} \cdot \mathcal{K}_{\mathcal{B}_{k'}}}$$

where $m_{\mathrm{CVA}} = 1.25$ is the multiplier factor. As in the case of SA-TB, SA-CVA is then based on the following set of parameters: the sensitivities S_j of the risk factors that are calculated by the bank; the risk weights RW_j of the risk factors; the correlation $\rho_{j,j'}$ between risk factors within a bucket; the correlation $\gamma_{k,k'}$ between the risk buckets. The values of these parameters are not necessarily equal to those of SA-TB[28]. For instance, the correlations $\rho_{j,j'}$ and $\gamma_{k,k'}$ are generally lower. The reason is that these correlations reflect the dependence between credit risk factors and not market risk factors.

Remark 59 *Contrary to the SA-TB, the bank must have the approval of the supervisory authority to use the SA-CVA. Otherwise, it must use the BA-CVA framework.*

4.2.3 CVA and wrong/right way risk

The wrong way or right way risk is certainly the big challenge when modeling CVA. We have already illustrated this point in the case of the CCR capital requirement, but this is even more relevant when computing the CVA capital requirement. The reason is that the bank generally manages the CVA risk because it represents a huge cost in terms of regulatory capital and it impacts on a daily basis the P&L of the trading book. For that, the bank generally puts in place a CVA trading desk, whose objective is to mitigate CVA risks. Therefore, the CVA desk must develop a fine modeling of WWR/RWR risks in order to be efficient and to be sure that the hedging portfolio does not create itself another source of hidden wrong way risk. This is why the CVA modeling is relatively complex, because we cannot assume in practice that market and credit risks are not correlated.

We reiterate that the definition of the CVA is[29]:

$$\mathrm{CVA} = \mathbb{E}\left[\mathbb{1}\left\{\boldsymbol{\tau} \leq T\right\} \cdot e^{-\int_0^{\boldsymbol{\tau}} r_t \,\mathrm{d}t} \cdot \left(1 - \boldsymbol{\mathcal{R}}\right) \cdot e^+\left(\boldsymbol{\tau}\right)\right]$$

where $e^+(t) = \max(\omega, 0)$ and ω is the random variable that represents the mark-to-market[30]. If we assume that the recovery rate is constant and interest rates are deterministic, we obtain:

$$\begin{aligned}
\mathrm{CVA} &= \left(1 - \boldsymbol{\mathcal{R}}\right) \cdot \int_0^T \int_{-\infty}^{+\infty} B_0(t) \max(\omega, 0) \,\mathrm{d}\mathbf{F}(\omega, t) \\
&= \left(1 - \boldsymbol{\mathcal{R}}\right) \cdot \int_0^T \int_{-\infty}^{+\infty} B_0(t) \max(\omega, 0) \,\mathrm{d}\mathbf{C}\left(\mathbf{F}_\omega(\omega), \mathbf{F}_\tau(t)\right)
\end{aligned}$$

[28] See BCBS (2017c) on pages 119-127.

[29] In order to obtain more concise formulas, we delete the reference to the counterparty B and we write $\boldsymbol{\mathcal{R}}$ instead of $\boldsymbol{\mathcal{R}}_B$.

[30] We implicitly assume that the mark-to-market is a stationary process. In fact, this assumption is not verified. However, we use this simplification to illustrate how the dependence between the counterparty exposure and the default times changes the CVA figure.

where $\mathbf{F}(\omega, t)$ is the joint distribution of the mark-to-market and the default time and $\mathbf{C}$ is the copula between ω and $\boldsymbol{\mathcal{R}}$. If we assume that $\mathbf{C} = \mathbf{C}^{\perp}$, we retrieve the traditional CVA formula[31]:

$$\begin{aligned} \text{CVA} &= (1 - \boldsymbol{\mathcal{R}}) \cdot \int_0^T \int_{-\infty}^{+\infty} B_0(t) \max(\omega, 0) \; \mathrm{d}\mathbf{F}_\omega(\omega) \; \mathrm{d}\mathbf{F}_\tau(t) \\ &= (1 - \boldsymbol{\mathcal{R}}) \cdot \int_0^T B_0(t) \, \text{EpE}(t) \; \mathrm{d}\mathbf{F}_\tau(t) \end{aligned}$$

where $\text{EpE}(t)$ is the expected positive exposure:

$$\text{EpE}(t) = \int_{-\infty}^{+\infty} \max(\omega, 0) \; \mathrm{d}\mathbf{F}_\omega(\omega) = \mathbb{E}\left[e^+(t)\right]$$

Otherwise, we have to model the dependence between the mark-to-market and the default time. In what follows, we consider two approaches: the copula model introduced by Cespedes *et al.* (2010) and the hazard rate model of Hull and White (2012).

The copula approach The Monte Carlo CVA is calculated as following:

$$\text{CVA} = (1 - \boldsymbol{\mathcal{R}}) \cdot \sum_{t_i \le T} B_0(t_i) \left(\frac{1}{n_S} \sum_{s=1}^{n_S} e_s^+(t_i; \omega_s) \right) (\mathbf{F}_{\boldsymbol{\tau}}(t_i) - \mathbf{F}_{\boldsymbol{\tau}}(t_{i-1}))$$

where $e^+(t_i; \omega_s)$ is the counterparty exposure of the s^{th} simulated scenario ω_s and n_S is the number of simulations. If market and credit risk factors are correlated, the Monte Carlo CVA becomes:

$$\text{CVA} = (1 - \boldsymbol{\mathcal{R}}) \cdot \sum_{t_i \le T} \sum_{s=1}^{n_S} B_0(t_i) \, e_s^+(t_i; \omega_s) \, \pi_{s,i} \tag{4.22}$$

where[32]:

$$\pi_{s,i} = \Pr\{\omega = \omega_s, t_i < \boldsymbol{\tau} \le t_i\}$$

The objective is then to calculate the joint probability by assuming a copula function $\mathbf{C}$ between ω and $\boldsymbol{\tau}$. For that, we assume that the scenarios ω_s are ordered. Let $U = \mathbf{F}_\omega(\omega)$ and $V = \mathbf{F}_{\boldsymbol{\tau}}(\boldsymbol{\tau})$ be the integral transform of ω and $\boldsymbol{\tau}$. Since U and V are uniform random variables, we obtain:

$$\begin{aligned} \pi_{s,i} &= \Pr\{\omega_{s-1} < \omega \le \omega_s, t_i < \boldsymbol{\tau} \le t_i\} \\ &= \Pr\{u_{s-1} < U \le u_s, v_{i-1} < V \le v_i\} \\ &= \mathbf{C}(u_s, v_i) - \mathbf{C}(u_{s-1}, v_i) - \mathbf{C}(u_s, v_{i-1}) + \mathbf{C}(u_{s-1}, v_{i-1}) \end{aligned} \tag{4.23}$$

Generally, we don't know the analytical expression of $\mathbf{F}_\omega$. This is why we replace it by the empirical distribution $\hat{\mathbf{F}}_\omega$ where the probability of each scenario is equal to $1/n_S$.

In order to define the copula function $\mathbf{C}$, Rosen and Saunders (2012) consider a market-credit version of the Basel model. Let $Z_m = \Phi^{-1}(\mathbf{F}_\omega(\omega))$ and $Z_c = \Phi^{-1}(\mathbf{F}_{\boldsymbol{\tau}}(\boldsymbol{\tau}))$ be the

[31] See Equation (4.16) on page 280.

[32] In the case where ω and $\boldsymbol{\tau}$ are independent, we retrieve the previous formula because we have:

$$\begin{aligned} \pi_{s,i} &= \Pr\{\omega = \omega_s\} \cdot \Pr\{t_i \le \boldsymbol{\tau} \le t_i\} \\ &= \frac{\mathbf{F}_{\boldsymbol{\tau}}(t_i) - \mathbf{F}_{\boldsymbol{\tau}}(t_{i-1})}{n_S} \end{aligned}$$

normalized latent random variables for market and credit risks. Rosen and Saunders use the one-factor model specification:

$$\left\{ \begin{array}{l} Z_m = \rho_m X + \sqrt{1-\rho_m^2}\,\varepsilon_m \\ Z_c = \rho_c X + \sqrt{1-\rho_c^2}\,\varepsilon_c \end{array} \right.$$

where X is the systematic risk factor that impacts both market and credit risks, ε_m and ε_c are the idiosyncratic market and credit risk factors, and ρ_m and ρ_c are the market and credit correlations with the common risk factor. It follows that the market-credit correlation is equal to:

$$\rho_{m,c} = \mathbb{E}\left[Z_m Z_c\right] = \rho_m \rho_c$$

We deduce that the dependence between Z_m and Z_c is a Normal copula with parameter $\rho_{m,c} = \rho_m \rho_c$, and we can write:

$$Z_m = \rho_{m,c} Z_c + \sqrt{1-\rho_{m,c}^2}\,\varepsilon_{m,c}$$

where $\varepsilon_{m,c} \sim \mathcal{N}(0,1)$ is an independent specific risk factor. Since the expression of the Normal copula is $\mathbf{C}(u,v;\rho_{m,c}) = \Phi_2\left(\Phi^{-1}(u), \Phi^{-1}(\mathbf{v}); \rho_{m,c}\right)$, Equation (4.23) becomes[33]:

$$\begin{array}{rcl} \pi_{s,i} & = & \Phi_2\left(\Phi^{-1}\left(\dfrac{s}{n_S}\right), \Phi^{-1}\left(\mathbf{F}_{\boldsymbol{\tau}}(t_i)\right); \rho_{m,c}\right) - \\ & & \Phi_2\left(\Phi^{-1}\left(\dfrac{s-1}{n_S}\right), \Phi^{-1}\left(\mathbf{F}_{\boldsymbol{\tau}}(t_i)\right); \rho_{m,c}\right) - \\ & & \Phi_2\left(\Phi^{-1}\left(\dfrac{s}{n_S}\right), \Phi^{-1}\left(\mathbf{F}_{\boldsymbol{\tau}}(t_{i-1})\right); \rho_{m,c}\right) + \\ & & \Phi_2\left(\Phi^{-1}\left(\dfrac{s-1}{n_S}\right), \Phi^{-1}\left(\mathbf{F}_{\boldsymbol{\tau}}(t_{i-1})\right); \rho_{m,c}\right) \end{array}$$

This approach is called the ordered-scenario copula model (OSC), because it is based on the ordering trick of the scenarios ω_s. Rosen and Saunders (2012) also propose different versions of the CVA discretization leading to different expressions of Equation (4.22). For instance, if we assume that the default occurs exactly at time t_i and not in the interval $[t_{i-1}, t_i]$, we have:

$$\pi_{s,i} \approx \pi_{s|i} \cdot \Pr\left\{t_i < \boldsymbol{\tau} \leq t_i\right\}$$

and:

$$\begin{array}{rcl} \pi_{s|i} & = & \Pr\left\{\omega = \omega_s \mid \boldsymbol{\tau} = t_i\right\} \\ & = & \Pr\left\{\omega_{s-1} < \omega \leq \omega_s \mid \boldsymbol{\tau} = t_i\right\} \\ & = & \Pr\left\{u_{s-1} < U \leq u_s \mid V = v_i\right\} \\ & = & \partial_2 \mathbf{C}(u_s, v_i) - \partial_2 \mathbf{C}(u_{s-1}, v_i) \\ & = & \partial_2 \mathbf{C}\left(\dfrac{s}{n_S}, \mathbf{F}_{\boldsymbol{\tau}}(t_i)\right) - \partial_2 \mathbf{C}\left(\dfrac{s-1}{n_S}, \mathbf{F}_{\boldsymbol{\tau}}(t_i)\right) \end{array}$$

In the case of the Rosen-Saunders model, we use the expression of the conditional Normal copula given on page 737:

$$\partial_2 \mathbf{C}(u,v;\rho_{m,c}) = \Phi\left(\frac{\Phi^{-1}(u) - \rho_{m,c}\Phi^{-1}(v)}{\sqrt{1-\rho_{m,c}^2}}\right)$$

[33] By definition, we have $\mathbf{F}_{\omega}^{-1}(\varpi_s) = s/n_S$ because the scenarios are ordered.

The hazard rate approach In Basel II, wrong way risk is addressed by introducing the multiplier $\alpha = 1.4$, which is equivalent to change the values of the mark-to-market. In the Rosen-Saunders model, wrong way risk is modeled by changing the joint probability of the mark-to-market and the default times. Hull and White (2012) propose a third approach, which consists in changing the values of the default probabilities. They consider that the hazard rate is a deterministic function of the mark-to-market: $\lambda(t) = \lambda(t, \mathrm{MtM}(t))$. For instance, they use two models:

$$\lambda(t, \mathrm{MtM}(t)) = e^{a(t)+b\cdot\mathrm{MtM}(t)} \tag{4.24}$$

and:

$$\lambda(t, \mathrm{MtM}(t)) = \ln\left(1 + e^{a(t)+b\cdot\mathrm{MtM}(t)}\right) \tag{4.25}$$

The case $b < 0$ corresponds to the right way risk, whereas $b > 0$ corresponds to the wrong way risk. When $b = 0$, the counterparty exposure is independent from the credit risk of the counterparty.

Hull and White (2012) propose a two-step procedure to calibrate $a(t)$ and b. First, they assume that the term structure of the hazard rate is flat. Given two pairs $(\mathrm{MtM}_1, \mathcal{s}_1)$ and $(\mathrm{MtM}_2, \mathcal{s}_2)$, $a(0)$ and b satisfy the following system of equations:

$$\left\{\begin{array}{l}(1-\boldsymbol{\mathcal{R}})\cdot\lambda(0,\mathrm{MtM}_1) = \mathcal{s}_1\\(1-\boldsymbol{\mathcal{R}})\cdot\lambda(0,\mathrm{MtM}_2) = \mathcal{s}_2\end{array}\right.$$

The solution is:

$$\left\{\begin{array}{l}b = \dfrac{\ln\lambda_2 - \ln\lambda_1}{\mathrm{MtM}_2 - \mathrm{MtM}_1}\\a(0) = \ln\lambda_1 - b\cdot\mathrm{MtM}_1\end{array}\right.$$

where $\lambda_i = \mathcal{s}_i/(1-\boldsymbol{\mathcal{R}})$ for Model (4.24) and $\lambda_i = \exp(\mathcal{s}_i/(1-\boldsymbol{\mathcal{R}})) - 1$ for Model (4.25). Hull and White (2012) consider the following example. They assume that the 5Y CDS spread of the counterparty is 300 bps when the mark-to-market is \$3 mn, and 600 bps when the mark-to-market is \$20 mn. If the recovery rate is set to 40%, the calibrated parameters are $a(0) = -3.1181$ and $b = 0.0408$ for Model (4.24) and $a(0) = -3.0974$ and $b = 0.0423$ for Model (4.25). The second step of the procedure consists in calibrating the function $a(t)$ given the value of b estimated at the first step. Since we have:

$$\mathbf{S}(t) = e^{-\int_0^t \lambda(s,\mathrm{MtM}(s))\,\mathrm{d}s}$$

and:

$$\mathbf{S}(t) = \exp\left(-\frac{\mathcal{s}(t)\cdot t}{1-\boldsymbol{\mathcal{R}}}\right)$$

the function $a(t)$ must verify that the survival probability calculated with the model is equal to the survival probability calculated with the credit spread:

$$e^{-\sum_{k=0}^{i}\lambda(t_k,\mathrm{MtM}(t_k))\cdot(t_k-t_{k-1})} = \exp\left(-\frac{\mathcal{s}(t_i)\cdot t_i}{1-\boldsymbol{\mathcal{R}}}\right)$$

In the case where the CVA is calculated with the Monte Carlo method, we have:

$$\frac{1}{n_S}\sum_{s=1}^{n_S}\prod_{k=0}^{i} e^{-\lambda(t_k,\omega_s(t_k))\cdot(t_k-t_{k-1})} = \exp\left(-\frac{\mathcal{s}(t_i)\cdot t_i}{1-\boldsymbol{\mathcal{R}}}\right)$$

where $\omega_s(t_k)$ is the s^{th} simulated value of $\text{MtM}(t_k)$. Therefore, $a(t)$ is specified as a piecewise linear function and we use the bootstrap method[34] for calibrating $a(t)$ given the available market CDS spreads[35].

4.3 Collateral risk

4.3.1 Definition

When there is a margin agreement, the counterparty needs to post collateral and the exposure at default becomes:

$$e^+(t) = \max(\text{MtM}(t) - C(t), 0) \tag{4.26}$$

where $C(t)$ is the collateral value at time t. Generally, the collateral transfer occurs when the mark-to-market exceeds a threshold H:

$$C(t) = \max(\text{MtM}(t - \delta_C) - H, 0) \tag{4.27}$$

H is the minimum collateral transfer amount whereas $\delta_C \geq 0$ is the margin period of risk (MPOR). According to the Financial Conduct Authority (FCA), the margin period of risk "*stands for the time period from the most recent exchange of collateral covering a netting set of financial instruments with a defaulting counterparty until the financial instruments are closed out and the resulting market risk is re-hedged*". It can be seen as the necessary time period for posting the collateral. In many models, δ_C is set to zero in order to obtain analytical formulas. However, this is not realistic from a practical point of view. From a regulatory point of view, δ_C is generally set to five or ten days (Cont, 2018).

If we combine Equations (4.26) and (4.27), it follows that:

$$\begin{aligned} e^+(t) &= \max(\text{MtM}(t) - \max(\text{MtM}(t-\delta_C) - H, 0), 0) \\ &= \text{MtM}(t) \cdot \mathbb{1}\{0 \leq \text{MtM}(t), \text{MtM}(t-\delta_C) < H\} + \\ &\quad (\text{MtM}(t) - \text{MtM}(t-\delta_C) + H) \cdot \\ &\quad \mathbb{1}\{H \leq \text{MtM}(t-\delta_C) \leq \text{MtM}(t) + H\} \end{aligned}$$

We obtain some special cases:

- When $H = +\infty$, $C(t)$ is equal to zero and we obtain:

$$e^+(t) = \max(\text{MtM}(t), 0)$$

- When $H = 0$, the collateral $C(t)$ is equal to $\text{MtM}(t-\delta_C)$ and the counterparty exposure becomes:

$$\begin{aligned} e^+(t) &= \max(\text{MtM}(t) - \text{MtM}(t-\delta_C), 0) \\ &= \max(\text{MtM}(t-\delta_C, t), 0) \end{aligned}$$

  The counterparty credit risk corresponds to the variation of the mark-to-market $\text{MtM}(t-\delta_C, t)$ during the liquidation period $[t-\delta_C, t]$.

[34] This method is presented on page 204.

[35] Generally, they correspond to the following maturities: 1Y, 3Y, 5Y, 7Y and 10Y.

- When δ_C is set to zero, we deduce that:

$$\begin{aligned} e^+(t) &= \max(\mathrm{MtM}(t) - \max(\mathrm{MtM}(t) - H, 0), 0) \\ &= \mathrm{MtM}(t) \cdot \mathbb{1}\{0 \leq \mathrm{MtM}(t) < H\} + H \cdot \mathbb{1}\{H \leq \mathrm{MtM}(t)\} \end{aligned}$$

- When δ_C is set to zero and there is no minimum collateral transfer amount, the counterparty credit risk vanishes:

$$e^+(t) = 0$$

This last case is interesting, because it gives an indication how to reduce the counterparty risk:

$$H \searrow 0 \text{ or } \delta_C \searrow 0 \Rightarrow e^+(t) \searrow 0$$

In the first panel in Figure 4.10, we have simulated the mark-to-market of a portfolio for a two-year period. In the second panel, we have reported the counterparty exposure when there is no collateral. The other panels show the collateral $C(t)$ and the counterparty exposure $e^+(t)$ for different values of δ_C and H. When there is no margin period of risk, we verify that the exposure is capped at the collateral threshold H in the fourth panel. When the threshold is equal to zero, the counterparty exposure corresponds to the lag effect due to the margin period of risk as illustrated in the sixth panel. The riskier situation corresponds to the combination of the threshold risk and the margin period of risk (eighth panel).

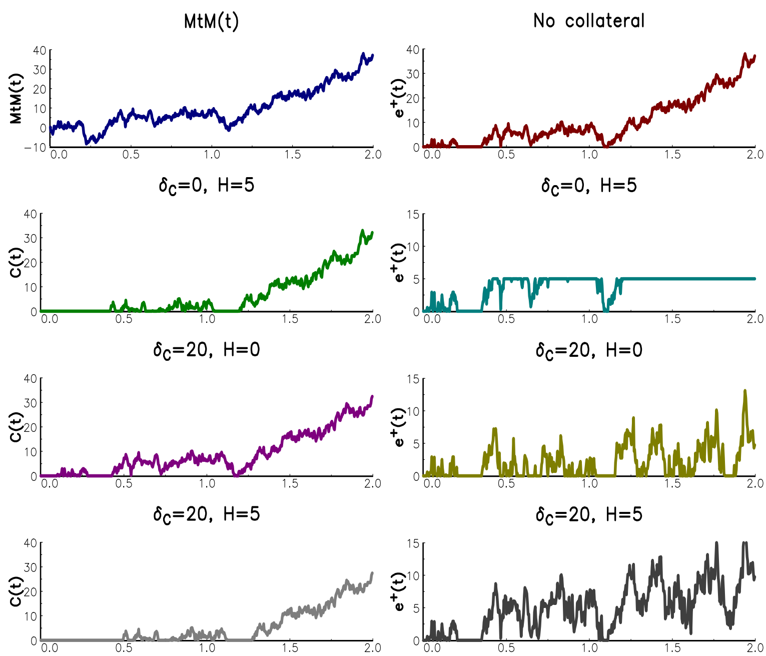

FIGURE 4.10: Impact of collateral on the counterparty exposure

4.3.2 Capital allocation

Taking into account collateral in the CVA computation is relatively straightforward when we use Monte Carlo simulations. In fact, the CVA formula remains the same, only the computation of the expected positive exposure $\mathrm{EpE}(t)$ is changed. However, as mentioned by Pykhtin and Rosen (2010), the big issue is the allocation of the capital. In Section 2.3 on page 104, we have seen that the capital allocation is given by the Euler allocation principle. Let $\mathcal{R}(w)$ be the risk measure of Portfolio $w = (w_1, \ldots, w_n)$. Under some assumptions, we reiterate that:

$$\mathcal{R}(w) = \sum_{i=1}^{n} \mathcal{RC}_i$$

where $\mathcal{RC}_i$ is the risk contribution of the i^{th} component:

$$\mathcal{RC}_i = w_i \cdot \frac{\partial\,\mathcal{R}(w)}{\partial\, w_i}$$

The components can be assets, credits, trading desks, etc. For instance, in the case of credit risk, the IRB formula gives the risk contribution of a loan within a portfolio. In the case of a CVA portfolio, we have:

$$\mathrm{CVA}(w) = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}(t; w)\,\mathrm{d}\mathbf{S}_B(t)$$

where $\mathrm{EpE}(t; w)$ is the expected positive exposure with respect to the portfolio w. The Euler allocation principle becomes:

$$\mathrm{CVA}(w) = \sum_{i=1}^{n} \mathrm{CVA}_i(w)$$

where $\mathrm{CVA}_i(w)$ is the CVA risk contribution of the i^{th} component:

$$\mathrm{CVA}_i(w) = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}_i(t; w)\,\mathrm{d}\mathbf{S}_B(t)$$

and $\mathrm{EpE}_i(t; w)$ is the EpE risk contribution of the i^{th} component:

$$\mathrm{EpE}_i(t; w) = w_i \cdot \frac{\partial\,\mathrm{EpE}(t; w)}{\partial\, w_i}$$

Therefore, the difficulty for computing the CVA risk contribution is to compute the EpE risk contribution.

We consider the portfolio $w = (w_1, \ldots, w_n)$, which is composed of n OTC contracts. The mark-to-market of the portfolio is equal:

$$\mathrm{MtM}(t) = \sum_{i=1}^{n} w_i \cdot \mathrm{MtM}_i(t)$$

where $\mathrm{MtM}_i(t)$ is the mark-to-market for the contract $\mathfrak{C}_i$. In the general case, the counterparty exposure is given by:

$$e^+(t) = \mathrm{MtM}(t) \cdot \mathbb{1}\{0 \le \mathrm{MtM}(t) < H\} + H \cdot \mathbb{1}\{\mathrm{MtM}(t) \ge H\}$$

If there is no collateral, we have:

$$\begin{aligned} e^+(t) &= \mathrm{MtM}(t) \cdot \mathbb{1}\{\mathrm{MtM}(t) \ge 0\} \\ &= \sum_{i=1}^{n} w_i \cdot \mathrm{MtM}_i(t) \cdot \mathbb{1}\{\mathrm{MtM}(t) \ge 0\} \end{aligned}$$

We deduce that:

$$\frac{\partial \mathbb{E}\left[e^{+}(t)\right]}{\partial w_i} = \mathbb{E}\left[\mathrm{MtM}_i(t) \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq 0\right\}\right]$$

and:

$$\mathrm{EpE}_i(t; w) = \mathbb{E}\left[w_i \cdot \mathrm{MtM}_i(t) \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq 0\right\}\right]$$

Computing the EpE (or CVA) risk contribution is then straightforward in this case. In the general case, Pykhtin and Rosen (2010) notice that $\mathrm{EpE}(t; w)$ is not a homogeneous function of degree one because of the second term $\mathbb{E}\left[H \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right]$. The idea of these authors is then to allocate the threshold risk to the individual contracts:

$$\mathbb{E}\left[H \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right] = H \cdot \sum_{i=1}^{n} \mathbb{E}\left[\omega_i \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right]$$

by choosing an appropriate value of ω_i such that $\sum_{i=1}^{n} \omega_i = 1$. They consider two propositions. Type A Euler allocation is given by:

$$\begin{aligned}\mathrm{EpE}_i(t; w) &= \mathbb{E}\left[w_i \cdot \mathrm{MtM}_i(t) \cdot \mathbb{1}\left\{0 \leq \mathrm{MtM}(t) < H\right\}\right] + \\ & H \cdot \frac{\mathbb{E}\left[\mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right] \cdot \mathbb{E}\left[w_i \cdot \mathrm{MtM}_i(t) \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right]}{\mathbb{E}\left[\mathrm{MtM}(t) \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right]}\end{aligned}$$

whereas type B Euler allocation is given by:

$$\begin{aligned}\mathcal{RC}_i &= \mathbb{E}\left[w_i \cdot \mathrm{MtM}_i(t) \cdot \mathbb{1}\left\{0 \leq \mathrm{MtM}(t) < H\right\}\right] + \\ & H \cdot \mathbb{E}\left[\frac{w_i \cdot \mathrm{MtM}_i(t)}{\mathrm{MtM}(t)} \cdot \mathbb{1}\left\{\mathrm{MtM}(t) \geq H\right\}\right]\end{aligned}$$

Pykhtin and Rosen (2010) consider the Gaussian case when the mark-to-market for the contract $\mathfrak{C}_i$ is given by:

$$\mathrm{MtM}_i(t) = \mu_i(t) + \sigma_i(t) X_i$$

where $(X_1, \ldots, X_n) \sim \mathcal{N}(0_n, \rho)$ and $\rho = (\rho_{i,j})$ is the correlation matrix. Let $\mu_w(t)$ and $\sigma_w(t)$ be the expected value and volatility of the portfolio mark-to-market $\mathrm{MtM}(t)$. The authors show that[36] the expected positive exposure is the sum of three components:

$$\mathrm{EpE}(t; w) = \mathrm{EpE}_{\mu}(t; w) + \mathrm{EpE}_{\sigma}(t; w) + \mathrm{EpE}_{H}(t; w)$$

where $\mathrm{EpE}_{\mu}(t; w)$ is the mean component:

$$\mathrm{EpE}_{\mu}(t; w) = \mu_w(t) \cdot \left(\Phi\left(\frac{\mu_w(t)}{\sigma_w(t)}\right) - \Phi\left(\frac{\mu_w(t) - H}{\sigma_w(t)}\right)\right)$$

$\mathrm{EpE}_{\sigma}(t; w)$ is the volatility component:

$$\mathrm{EpE}_{\sigma}(t; w) = \sigma_w(t) \cdot \left(\phi\left(\frac{\mu_w(t)}{\sigma_w(t)}\right) - \phi\left(\frac{\mu_w(t) - H}{\sigma_w(t)}\right)\right)$$

and $\mathrm{EpE}_{H}(t; w)$ is the collateral threshold component:

$$\mathrm{EpE}_{H}(t; w) = H \cdot \Phi\left(\frac{\mu_w(t) - H}{\sigma_w(t)}\right)$$

[36] See Exercise 4.4.6 on page 303.

We notice that $\mathrm{EpE}_{\mu}(t;w)$ and $\mathrm{EpE}_{H}(t;w)$ are always positive, while $\mathrm{EpE}_{\sigma}(t;w)$ may be positive or negative. When there is no collateral agreement, $\mathrm{EpE}_{H}(t;w)$ is equal to zero and $\mathrm{EpE}(t;w)$ depends on the ratio $\mu_w(t)/\sigma_w(t)$. Concerning the risk contributions, Pykhtin and Rosen (2010) obtain a similar decomposition:

$$\mathrm{EpE}_i(t;w) = \mathrm{EpE}_{\mu,i}(t;w) + \mathrm{EpE}_{\sigma,i}(t;w) + \mathrm{EpE}_{H,i}(t;w)$$

where:

$$\begin{aligned}
\mathrm{EpE}_{\mu,i}(t;w) &= w_i \cdot \mu_i(t) \cdot \left(\Phi\left(\frac{\mu_w(t)}{\sigma_w(t)}\right) - \Phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right)\right) \\
\mathrm{EpE}_{\sigma,i}(t;w) &= w_i \cdot \gamma_i(t) \cdot \sigma_i(t) \cdot \left(\phi\left(\frac{\mu_w(t)}{\sigma_w(t)}\right) - \phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right)\right) \\
\mathrm{EpE}_{H,i}(t;w) &= H \cdot \Phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right) \cdot \frac{\psi_i}{\psi_w}
\end{aligned}$$

$\gamma_i(t) = \sigma(t)^{-1}\sum_{j=1}^{n} w_j \cdot \rho_{i,j} \cdot \sigma_j(t)$ and:

$$\frac{\psi_i}{\psi_w} = \frac{w_i \cdot \mu_i \cdot (t)\,\Phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right) + w_i \cdot \gamma_i(t) \cdot \sigma_i(t) \cdot \phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right)}{\mu_w(t) \cdot \Phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right) + \sigma_w(t) \cdot \phi\left(\frac{\mu_w(t)-H}{\sigma_w(t)}\right)}$$

Example 50 *We consider a portfolio of two contracts $\mathfrak{C}_1$ and $\mathfrak{C}_2$ with the following characteristics: $\mu_1(t) = \$1$ mn, $\sigma_1(t) = \$1$ mn, $\mu_2(t) = \$1$ mn, $\sigma_2(t) = \$1$ mn and $\rho_{1,2} = 0\%$.*

We first calculate the expected positive exposure $\mathrm{EpE}(t;w)$ when we change the value of $\mu_2(t)$ and there is no collateral agreement. Results are given in Figure 4.11. In the first panel, we observe that $\mathrm{EpE}(t;w)$ increases with respect to $\mu_2(t)$. We notice that the mean component is the most important contributor when the expected value of the portfolio mark-to-market is high and positive[37]:

$$\frac{\mu_w(t)}{\sigma_w(t)} \to \infty \Rightarrow \left\{\begin{array}{l} \mathrm{EpE}_{\mu}(t;w) \to \mathrm{EpE}(t;w) \\ \mathrm{EpE}_{\sigma}(t;w) \to 0 \end{array}\right.$$

The risk contribution $\mathrm{EpE}_1(t;w)$ and $\mathrm{EpE}_2(t;w)$ are given in the second panel in Figure 4.11. The risk contribution of the second contract is negative when $\mu_2(t)$ is less than -1. This illustrate the diversification effect, implying that some trades can negatively contributes to the CVA risk. This is why the concept of netting sets is important when computing the CVA capital charge. In Figure 4.12, we have done the same exercise when we consider different values of the correlation $\rho_{1,2}$. We observe that the impact of this parameter is not very important except when the correlation is negative. The reason is that the correlation matrix has an impact on the volatility $\sigma_w(t)$ of the portfolio mark-to-market, but not on the expected value $\mu_w(t)$. We now consider that $\mu_1(t) = \mu_2(t) = 1$, $\sigma_1(t) = \sigma_2(t) = 1$ and $\rho_{1,2} = 0$. In Figure 4.13, we analyze the impact of the collateral threshold H. We notice that having a tighter collateral agreement (or a lower threshold H) allows to reduce the counterparty exposure. However, this reduction is not monotonous. It is very important when H is close to zero, but there is no impact when H is large.

[37] In this limit case, we obtain:

$$\mathrm{EpE}(t;w) = \mu_w(t) = \sum_{i=1}^{n} w_i \mu_i(t)$$

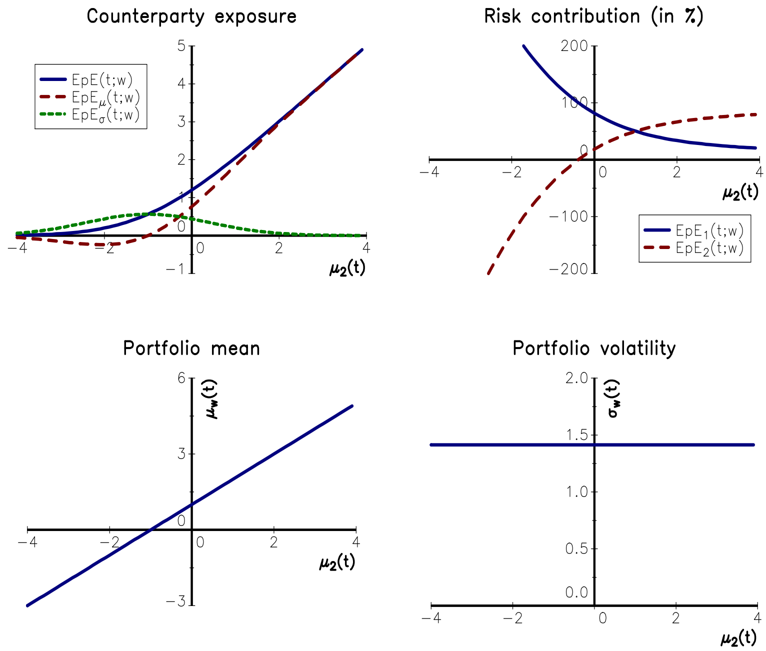

FIGURE 4.11: Impact of $\mu_i(t)/\sigma_i(t)$ on the counterparty exposure

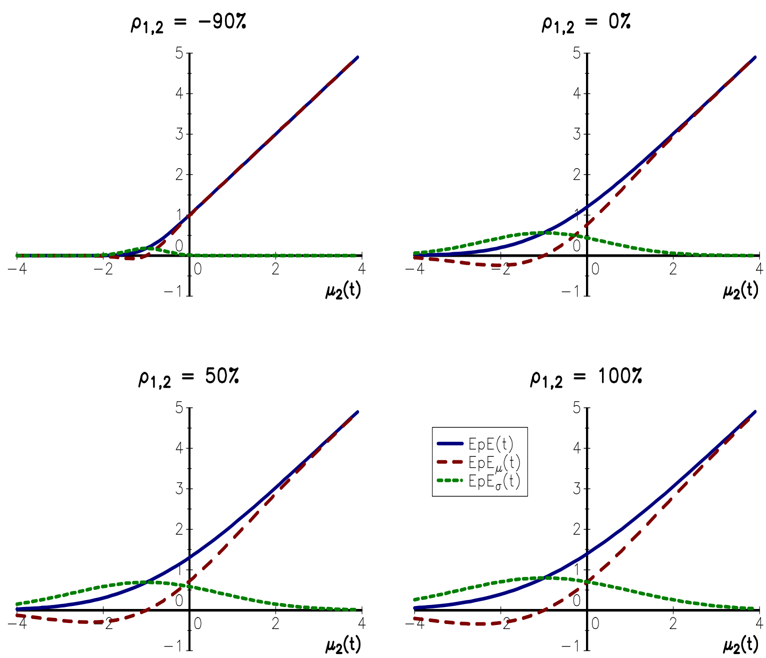

FIGURE 4.12: Impact of the correlation on the counterparty exposure

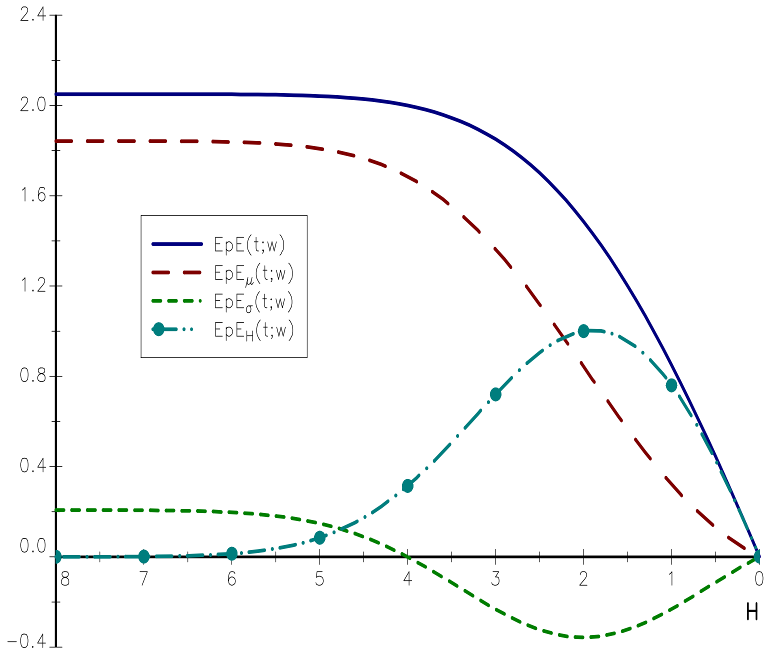

FIGURE 4.13: Decomposition of the counterparty exposure when there is a collateral agreement

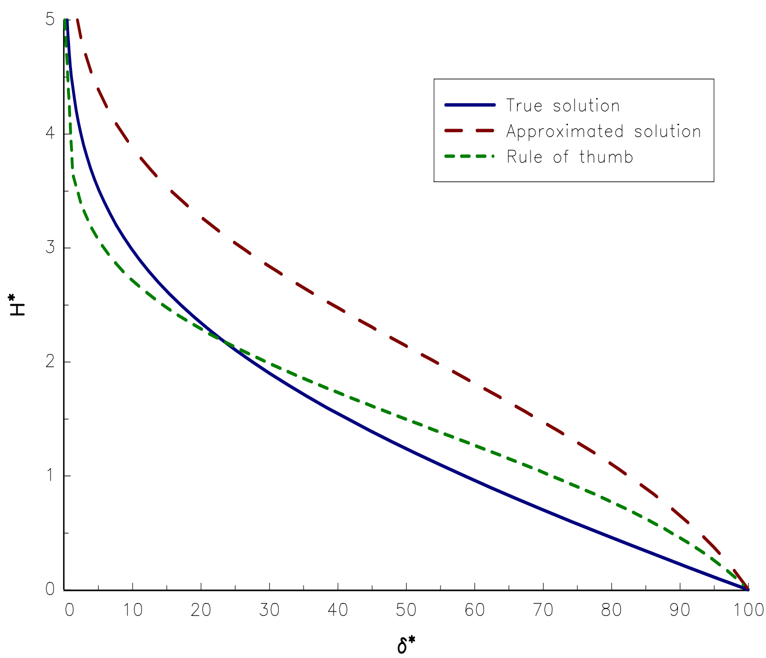

FIGURE 4.14: Optimal collateral threshold

The impact of the threshold can be measured by the ratio:

$$\delta(H) = \frac{\mathrm{EpE}(t; w, \infty) - \mathrm{EpE}(t; w, H)}{\mathrm{EpE}(t; w, \infty)}$$

where $\mathrm{EpE}(t; w, H)$ is the expected positive exposure for a given threshold H. If we would like to reduce the counterparty exposure by $\delta^\star$, we have to solve the non-linear equation $\delta(H) = \delta^\star$ in order to find the optimal value $H^\star$. We can also approximate $\mathrm{EpE}(t; w, H)$ by its mean contribution:

$$\begin{aligned} \delta(H) &\approx \delta_\mu(H) \\ &= \frac{\mu_w(t) \cdot \Phi(\zeta_H)}{\mathrm{EpE}_\mu(t; w, \infty)} \end{aligned}$$

In this case, the solution of the non-linear equation $\delta_\mu(H) = \delta^\star$ is equal to[38]:

$$H^\star = \mu_w(t) - \sigma_w(t) \cdot \Phi^{-1}\left(\frac{\mathrm{EpE}_\mu(t; w, \infty)}{\mu_w(t)} \cdot \delta^\star\right)$$

The computation of $H^\star$ is then straightforward since we have only to calculate $\mu_w(t)$, $\sigma_w(t)$ and the mean contribution $\mathrm{EpE}_\mu(t; w, \infty)$ when there is no collateral agreement. However, the value of $H^\star$ is overestimated because $\mathrm{EpE}_\mu(t; w, H)$ is lower than $\mathrm{EpE}(t; w, H)$. A rule of thumb is then to adjust the solution $H^\star$ by a factor[39], which is generally equal to 0.75. In Figure 4.14, we have represented the optimal collateral threshold $H^\star$ for the previous example.

4.4 Exercises

4.4.1 Impact of netting agreements in counterparty credit risk

The table below gives the current mark-to-market of 7 OTC contracts between Bank A and Bank B:

	Equity			Fixed income		FX	
	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
A	+10	−5	+6	+17	−5	−5	+1
B	−11	+6	−3	−12	+9	+5	+1

The table should be read as follows: Bank A has a mark-to-market equal to +10 for the contract $\mathfrak{C}_1$ whereas Bank B has a mark-to-market equal to −11 for the same contract, Bank A has a mark-to-market equal to −5 for the contract $\mathfrak{C}_2$ whereas Bank B has a mark-to-market equal to +6 for the same contract, etc.

1. (a) Explain why there are differences between the MtM values of a same OTC contract.
   (b) Calculate the exposure at default of Bank A.
   (c) Same question if there is a global netting agreement.
   (d) Same question if the netting agreement only concerns equity products.

[38] The solution $H^\star$ can be viewed as a quantile of the probability distribution of the portfolio mark-to-market: $\mathrm{MtM}(t) \sim \mathcal{N}\left(\mu_w(t), \sigma_w^2(t)\right)$.

[39] The underlying idea is that $\mathrm{EpE}_\mu(t; w, H) \approx 75\% \cdot \mathrm{EpE}(t; w, H)$.

2. In the following, we measure the impact of netting agreements on the exposure at default.

   (a) We consider an OTC contract $\mathfrak{C}$ between Bank A and Bank B. The mark-to-market $\mathrm{MtM}_1(t)$ of Bank A for the contract $\mathfrak{C}$ is defined as follows:

   $$\mathrm{MtM}_1(t) = x_1 + \sigma_1 W_1(t)$$

   where $W_1(t)$ is a Brownian motion. Calculate the potential future exposure of Bank A.

   (b) We consider a second OTC contract between Bank A and Bank B. The mark-to-market is also given by the following expression:

   $$\mathrm{MtM}_2(t) = x_2 + \sigma_2 W_2(t)$$

   where $W_2(t)$ is a second Brownian motion that is correlated with $W_1(t)$. Let ρ be this correlation such that $\mathbb{E}[W_1(t) W_2(t)] = \rho t$. Calculate the expected exposure of bank A if there is no netting agreement.

   (c) Same question when there is a global netting agreement between Bank A and Bank B.

   (d) Comment on these results.

4.4.2 Calculation of the effective expected positive exposure

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$.

1. Define the concepts of peak exposure $\mathrm{PE}_\alpha(t)$, maximum peak exposure $\mathrm{MPE}_\alpha(0;t)$, expected exposure $\mathrm{EE}(t)$, expected positive exposure $\mathrm{EPE}(0;t)$, effective expected exposure $\mathrm{EEE}(t)$ and effective expected positive exposure $\mathrm{EEPE}(0;t)$.

2. Calculate these different quantities when the potential future exposure is $e(t) = \sigma \cdot \sqrt{t} \cdot X$ where $X \sim \mathcal{U}_{[0,1]}$.

3. Same question when $e(t) = \exp\left(\sigma \cdot \sqrt{t} \cdot X\right)$ where $X \sim \mathcal{N}(0,1)$.

4. Same question when $e(t) = \sigma \cdot \left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right) \cdot X$ where $X \sim \mathcal{U}_{[0,1]}$.

5. Same question when $e(t) = \sigma \cdot \sqrt{t} \cdot X$ where X is a random variable defined on $[0,1]$ with the following probability density function[40]:

   $$f(x) = \frac{x^a}{a+1}$$

6. Comment on these results.

[40] We assume that $a > 0$.

4.4.3 Calculation of the capital charge for counterparty credit risk

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N and σ be the notional and the volatility of the underlying contract. We assume that $e(t) = N \cdot \sigma \cdot \sqrt{t} \cdot X$ where $0 \leq X \leq 1$, $\Pr\{X \leq x\} = x^{\gamma}$ and $\gamma > 0$.

1. Calculate the peak exposure $\mathrm{PE}_{\alpha}(t)$, the expected exposure $\mathrm{EE}(t)$ and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

2. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N is equal to \$3 mn, the maturity T is one year, the volatility σ is set to 20% and γ is estimated at 2.

   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.

   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[41].

4.4.4 Calculation of CVA and DVA measures

We consider an OTC contract with maturity T between Bank A and Bank B. We denote by $\mathrm{MtM}(t)$ the risk-free mark-to-market of Bank A. The current date is set to $t = 0$ and we assume that:

$$\mathrm{MtM}(t) = N \cdot \sigma \cdot \sqrt{t} \cdot X$$

where N is the notional of the OTC contract, σ is the volatility of the underlying asset and X is a random variable, which is defined on the support $[-1, 1]$ and whose density function is:

$$f(x) = \frac{1}{2}$$

1. Define the concept of positive exposure $e^{+}(t)$. Show that the cumulative distribution function $\mathbf{F}_{[0,t]}$ of $e^{+}(t)$ has the following expression:

   $$\mathbf{F}_{[0,t]}(x) = \mathbb{1}\left\{0 \leq x \leq \sigma\sqrt{t}\right\} \cdot \left(\frac{1}{2} + \frac{x}{2 \cdot N \cdot \sigma \cdot \sqrt{t}}\right)$$

   where $\mathbf{F}_{[0,t]}(x) = 0$ if $x \leq 0$ and $\mathbf{F}_{[0,t]}(x) = 1$ if $x \geq \sigma\sqrt{t}$.

2. Deduce the value of the expected positive exposure $\mathrm{EpE}(t)$.

3. We note $\boldsymbol{\mathcal{R}}_B$ the fixed and constant recovery rate of Bank B. Give the mathematical expression of the CVA.

4. By using the definition of the lower incomplete gamma function $\gamma(s, x)$, show that the CVA is equal to:

   $$\mathrm{CVA} = \frac{N \cdot (1 - \boldsymbol{\mathcal{R}}_B) \cdot \sigma \cdot \gamma\left(\frac{3}{2}, \lambda_B T\right)}{4\sqrt{\lambda_B}}$$

   when the default time of Bank B is exponential with parameter λ_B and interest rates are equal to zero.

[41] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

5. Comment on this result.

6. By assuming that the default time of Bank A is exponential with parameter λ_A, deduce the value of the DVA without additional computations.

4.4.5 Approximation of the CVA for an interest rate swap

This exercise is based on the results of Syrkin and Shirazi (2015).

1. Calculate $\mathrm{EpE}(t) = \mathbb{E}[\max(\mathrm{MtM}(t), 0)]$ when the mark-to-market is equal to $\mathrm{MtM}(t) = Ae^X - B$ and $X \sim \mathcal{N}\left(\mu_X, \sigma_X^2\right)$.

2. We define the mark-to-market of the interest rate swap as follows:

$$\mathrm{MtM}(t) = N \int_t^T f(t, T) B_t(s) \, \mathrm{d}s - N \int_t^T f(0, T) B_t(s) \, \mathrm{d}s$$

   where N and T are the notional and the maturity of the swap, and $f(t, T)$ is the instantaneous forward rate. Comment on this formulation. By assuming that $f(t, T)$ follows a geometric Brownian motion:

$$\mathrm{d}f(t, T) = \mu f(t, T) \, \mathrm{d}t + \sigma f(t, T) \, \mathrm{d}W(t)$$

   and the yield curve is flat – $B_t(s) = e^{-r(s-t)}$, calculate the value of the mark-to-market. Deduce the confidence interval of $\mathrm{MtM}(t)$ with a confidence level α.

3. Calculate the expected mark-to-market and the expected counterparty exposure.

4. Give the expression of the CVA at time t if we assume that the default time is exponentially distributed: $\boldsymbol{\tau} \sim \mathcal{E}(\lambda)$.

5. Retrieve the approximation of the CVA found by Syrkin and Shirazi (2015).

6. We consider the following numerical values: $N = 1000$, $f(0, T) = 5\%$, $\mu = 2\%$, $\sigma = 25\%$, $T = 10$ years, $\lambda = 1\%$ and $\boldsymbol{\mathcal{R}} = 50\%$.

   (a) Calculate the 90% confidence interval of $\mathrm{MtM}(t)$.

   (b) Compare the time profile of $\mathrm{EpE}(t)$ and $\mathbb{E}[\mathrm{MtM}(t)]$.

   (c) Compare the time profile of $\mathrm{CVA}(t)$ and its approximation.

   (d) What do you think about the numerical value of μ?

4.4.6 Risk contribution of CVA with collateral

This exercise is based on the results of Pykhtin and Rosen (2010).

1. We consider the portfolio $w = (w_1, \ldots, w_n)$, which is composed of n OTC contracts. We assume that the mark-to-market for the contract $\mathfrak{C}_i$ is given by:

$$\mathrm{MtM}_i(t) = \mu_i(t) + \sigma_i(t) X_i$$

   where $X_i \sim \mathcal{N}(0, 1)$. Determine the probability distribution of the portfolio mark-to-market:

$$\mathrm{MtM}(t) = \sum_{i=1}^{n} w_i \cdot \mathrm{MtM}_i(t)$$

   when $(X_1, \ldots, X_n) \sim \mathcal{N}(0_n, \rho)$ and $\rho = (\rho_{i,j})$ is the correlation matrix.

2. Calculate the correlation $\gamma_i(t)$ between $\mathrm{MtM}_i(t)$ and $\mathrm{MtM}(t)$.

3. Calculate the expected value of the counterparty exposure $e^+(t) = \max(\mathrm{MtM}(t) - C(t), 0)$ when the collateral value is given by $C(t) = \max(\mathrm{MtM}(t) - H, 0)$.

4. We consider the case where there is no collateral: $C(t) = 0$. What is the implicit value of H? Deduce the expression of $\mathrm{EpE}(t; w) = \mathbb{E}[e^+(t)]$. Calculate the risk contribution $\mathcal{RC}_i$ of the contract $\mathfrak{C}_i$. Show that $\mathrm{EpE}(t; w)$ satisfies the Euler allocation principle.

5. We consider the case where there is a collateral: $C(t) \neq 0$. Calculate the risk contribution $\mathcal{RC}_i$ of the contract $\mathfrak{C}_i$. Demonstrate that:

$$\sum_{i=1}^{n} \mathcal{RC}_i = \mathrm{EpE}(t; w) - H \cdot \Phi\left(\frac{\mu_w(t) - H}{\sigma_w(t)}\right)$$

   where $\mu_w(t)$ and $\sigma_w(t)$ are the expected value and volatility of $\mathrm{MtM}(t)$. Comment on this result.

6. Find the risk contribution $\mathcal{RC}_i$ of type A Euler allocation.

7. Find the risk contribution $\mathcal{RC}_i$ of type B Euler allocation.

8. We consider the Merton approach for modeling the default time $\boldsymbol{\tau}$ of the counterparty:

$$X_i = \varrho_i X_B + \sqrt{1 - \varrho_i^2}\,\eta_i$$

   where $X_B \sim \mathcal{N}(0,1)$ and the idiosyncratic risk $\eta_i \sim \mathcal{N}(0,1)$ are independent. Calculate the correlation $\varrho_w(t)$ between $\mathrm{MtM}(t)$ and X_B. Deduce the relationship between $\mathrm{MtM}(t)$ and X_B.

9. Let $B(t) = \Phi^{-1}(1 - \mathbf{S}(t))$ be the default barrier and $\mathbf{S}(t)$ the survival function of the counterparty. How to compute the conditional counterparty exposure $\mathbb{E}[e^+(t) \mid \boldsymbol{\tau} = t]$ and the corresponding risk contribution $\mathcal{RC}_i$? Give their expressions.