Hedge Fund Replication and Alternative Beta

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Outline

The Success of the Hedge Fund Industry

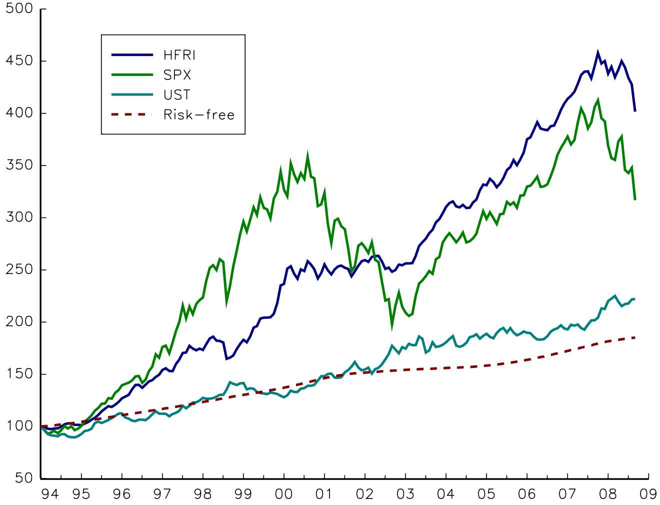
- Hedge-Funds (HF) deliver higher Sharpe ratios than Buy-and-Hold strategies on traditional asset classes[1]:

	HFRI	SPX	UST
Annualized Return	9.94%	8.18%	5.60%
1Y Volatility	7.06%	14.3%	6.95%
Sharpe	0.77	0.26	0.18

- The performance behaviour of the HF industry was very good during the equity bear market between 2000 and 2003.

[1] Jan. 1994 - Sep. 2008

Performance between January 1994 and September 2008



Motivation

- Appealing characteristics of Hedge-Fund returns
  - Superior risk-return profiles
  - Moderate or time-varying correlation with standard assets (approximate call option)
- However, investment in Hedge-Funds is limited for many investors
  - Regulatory or minimum size constraints (retail, institutional)
  - Main criticisms:
    - **Lack of transparency** (risk management – black box)
    - **Poor liquidity** (HF: monthly or quarterly, FoHF monthly, weekly or daily) → particularly relevant in period of stress (redemption problems)
    - **Fees** (problem of pricing rather than high fees)
- Emergence of "clones"
  - To alleviate HF limitations for investors seeking exposure to HF type returns: daily trading, no size limitations, etc.
  - Clones cannot compete with the best single hedge funds, but they give a significant part of the performance of the hedge fund industry.
  - Distinction HF Tracker / HF Risk Management.

Replication Methods

Replication of Strategies

Systematic (quantitative) replication of strategies known to be followed by some funds

- **Examples**
  FX carry trade, selling of volatility, momentum strategy
- **Strengths**
  Transparency, exposure to non-linear payoffs and earnings of associated premiums
- **Weaknesses**
  allocation methodologies, amount to creation of a new fund

Replication of Payoff Distribution (Kat Approach)

Strategy reproducing volatility, skewness, kurtosis and correlation of conventional asset classes to HF

- **Strengths**
  Weak dependency on current economic and financial conditions (according to Harry Kat)
- **Weaknesses**
  no replication of average returns, no time horizon, big entry ticket (amount to creation of tailor-made HF), exposed to breakdown in distribution and correlation

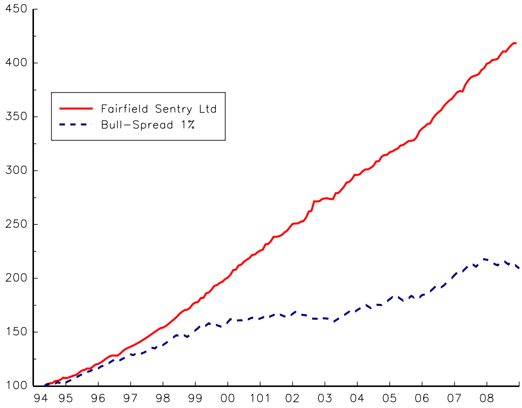
Factor Models

Reproduce the beta and risk/return profile of (the average of) hedge funds through investment in liquid instruments on standard asset classes

Our Focus !

Replication of Strategies

These tools are very useful, in particular since the Madoff Fraud [Clauss et al., 2009].



Replication of Payoff Distribution

- This approach has been proposed by Harry Kat.
- Mathematical framework by [Hocquard et al., 2008].
- Main idea :
  1. Because Payoff function $\Rightarrow$ return distribution, one may find the payoff function that implies the desired HF return distribution.
  2. One can then generate that distribution by buying this payoff or by replicating the payoff.

Main idea behind Factor Models

Assumption

The structure of all HF returns can be summarized by a set of risk factors $\{r^{(i)}, i = 1, \ldots, m\}$.

Comment

- Arbitrage Pricing Theory (APT).
- Extension of Sharpe's style analysis for performance assessment.

A typical replication procedure

- Step 1: Factor model for HF returns: $r_k^{\mathrm{HF}} = \sum_{i=1}^{m} \beta^{(i)} r_k^{(i)} + \varepsilon_k$
- Step 2: Identification of the replicating portfolio strategy

$$r_k^{\mathrm{Clone}} = \sum_{i=1}^{m} \hat{\beta}^{(i)} r_k^{(i)}$$

Previous Works
Overview

- Linear models with linear assets [Fung and Hsieh, 1997, Amenc et al., 2007, Hasanhodzic and Lo, 2007]
  - Different types of factors have been included depending on the type of strategies followed by hedge funds.
    e.g. Convertible and Fixed Income Arbitrage, Event Driven, Long/Short Equity, etc.
  - More factors to improve in-sample (and out-sample) fit?
    Difference between replication from an academic and a practitioner point of view (explaining vs. replicating as an investment)
- Option-based models
  Some authors have introduced options on an equity index as part of the factors [Diez de los Rios and Garcia, 2008]

$$r_k^{\mathrm{HF}} = \sum_{i=1}^{m} \beta^{(i)} r_k^{(i)} + \beta^{m+1} \max(r_k^{(1)} - s_k, 0) + \varepsilon_k$$

Previous Works
Estimation procedures

- Classically, estimation and calibration procedures (in chronological order):
  - Full factor model regressions,
  - stepwise regressions (versus economic selection of factors),
  - and rolling-windows OLS (to try to capture dynamic allocation).
- More recently, state-space modeling has been introduced to model and estimate HF returns:
  - Markov Regime-Switching Model: $r_k^{\mathrm{HF}} = \sum_{i=1}^{m} \beta^{(i)}(S_k) r_k^{(i)} + \varepsilon_k$ with $\varepsilon_k \sim \mathcal{N}\left(0, \sigma^2(S_k)\right)$ and S_k is a discrete variable representing the state of the nature [Amenc et al., 2008];
  - and Kalman Filter [Roncalli and Teiletche, 2008].

Previous Works
Results

- [Amenc et al., 2007] find that linear factor models fail the test of robustness, giving poor out-of-sample results.
- It seems that **economic selection** of factors provides a significant improvement over other methodologies on the tracking error in the out-of-sample robustness test.
- Capturing the *unobservable* dynamic allocation is very difficult, and estimates can vary greatly at balancing dates
- Non-linear models still represent a methodological challenge from a replication point-of-view

Tracking Problems

Definition

The following two equations define a tracking problem (TP) [Arulampalam et al., 2002]:

$$\left\{ \begin{array}{lcll} \mathbf{x}_k & = & f(t_k, \mathbf{x}_{k-1}, \nu_k) & \text{(Transition Equation)} \\ \mathbf{z}_k & = & h(t_k, \mathbf{x}_k, \eta_k) & \text{(Measurement Equation)} \end{array} \right.$$

where:

- $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the state vector, and $\mathbf{z}_k \in \mathbb{R}^{n_z}$ the measurement vector at step k.
- ν_k et η_k are mutually independent i.i.d noise processes.
- The functions f and h can be non-linear functions.

The goal in a tracking problem is to estimate $\mathbf{x}_k$, the current state at step k using all available measurements $\mathbf{z}_{1:k}$.

Global Tactical Asset Allocation as a Tracking Problem

- Assume a factor model with m asset classes acting as factors. Let r_k^{F} be the return of the GTAA fund and $r_k^{(i)}$ be the return of the i^{th} asset class ('the factors') at time index k. We assume that:

$$r_k^{\mathrm{F}} = \sum_{i=1}^{m} \beta_k^{(i)} r_k^{(i)} + \eta_k$$

and $\beta_{(t)}^{(i)} = \beta_k^{(i)}$ for $t \in \left]t_{k-1}, t_k\right[$ where $\beta_{(t)}^{(i)}$ is the weight of the i^{th} asset class at time t.

- We associate the following tracking problem:

$$\begin{cases} \beta_k & = & f\left(t_k, \beta_{k-1}, \nu_k\right) \\ r_k^{\mathrm{F}} & = & \mathbf{r}_k^\top \beta_k + \eta_k \end{cases}$$

where the vector of weights $\beta_k = \left(\beta_k^{(1)}, \ldots, \beta_k^{(m)}\right)^\top$ is the state vector, and r_k^{F} is the measurement.

Solving with Kalman Filter

Description

- f is linear:

$$\beta_k = \beta_{k-1} + \nu_k$$

  and ν_k and η_k are mutually independent i.i.d Gaussian noises.

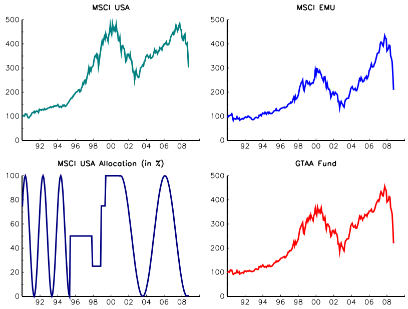
- The Kalman Filter is a recursive algorithm providing the optimal solution in the linear Gaussian case. At each time index k, among other things, it provides us with :
  - $\beta^{(i)}_{k|k-1}$ the prediction of the exposures;
  - $\beta^{(i)}_{k|k}$ the filtered estimate of the exposures;
  - $\beta^{(i)}_{k|n}$ the smoothed estimate of the exposures.

If $\nu_k = 0$, $\beta^{(i)}_{k|k}$ is the recursive OLS estimate whereas $\beta^{(i)}_{k|n}$ is the OLS estimate.

Solving with Kalman Filter

An Example

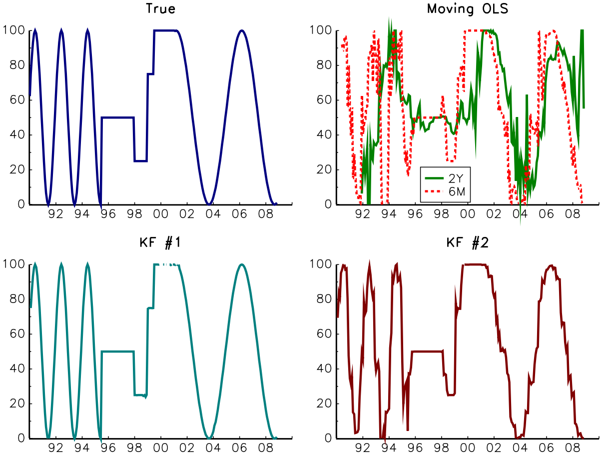
We constructed a GTAA fund allocating w_k^{US} in MSCI USA and $w_k^{\mathrm{EU}} = 1 - w_k^{\mathrm{US}}$ in MSCI EU.



- Moving OLS: choice of the lag window?
- Kalman Filter:
  - $\beta_0 \sim \mathcal{N}(b_0, \mathbf{0})$ with $b_0 = \left(w_0^{\mathrm{US}}, w_0^{\mathrm{EU}}\right)^\top$
  - $\eta_k = 0$
  - $v_k \sim \mathcal{N}(\mathbf{0}, Q)$
    - KF #1: Q is a full matrix.
    - KF #2: Q is a diagonal matrix (correlation between $\beta_k^{(1)}$ and $\beta_k^{(2)}$ is zero).

Solving with Kalman Filter

Results



Link between GTAA and HF replications

The return of a hedge fund may be decomposed into two components:

$$r_k^{\mathrm{HF}} = \underbrace{\sum_{i=1}^{m} \beta_k^{(i)} r_k^{(i)}}_{\text{GTAA factors}} + \underbrace{\sum_{i=m+1}^{p} \beta_k^{(i)} r_k^{(i)}}_{\text{HF factors}}$$

The idea of HF replication is to replicate the first part. Let's note $\eta_k = \sum_{i=m+1}^{p} w_k^{(i)} r_k^{(i)}$. The TP system becomes:

$$\begin{cases} \beta_k = \beta_{k-1} + \nu_k \\ r_k^{\mathrm{HF}} = \mathbf{r}_k^\top \beta_k + \eta_k \end{cases}$$

$\Rightarrow$ HF replication is done at the industry level because of the term η_k which captures the performance of stock picking strategies, illiquid assets, non-linear assets, high frequency strategies, etc.

$$\left.\begin{array}{l} \text{Aggregation} \\ \text{Efficient market hypothesis} \end{array}\right\} \overset{?}{\Longrightarrow} \begin{cases} \mathbb{E}\left[\nu_k\right] = 0 \\ \sigma\left[\nu_k\right] \ll \sigma\left[r_k^{\mathrm{HF}}\right] \end{cases}$$

$\Rightarrow$ Long/Short Equity $\neq$ Equity Market Neutral.

Replicating with Kalman Filter
Some remarks about model specification

- Initialization step: $\beta_0 \sim \mathcal{N}(b_0, V_0) \Rightarrow$ Diffuse prior distribution.
- Factors specification and system identification: importance of low correlation between factors
- Specification of the covariance matrix Q of the transition equation (diagonal or not ? Should it be constant ?)

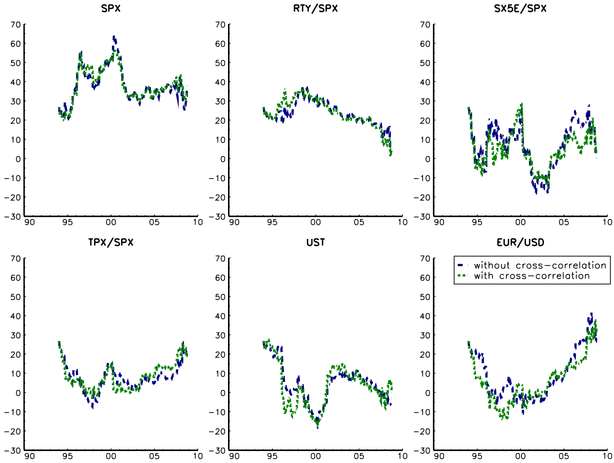
Replicating with Kalman Filter

An example

- We consider the replication of the Hedge Fund Research (HFR) Index using 6 asset classes:
  - an equity exposure in the S&P 500 index (SPX)
  - a L/S position between Russell 2000 and S&P 500 indexes (RTY/SPX)
  - a L/S position between DJ Eurostoxx 50 and S&P 500 indexes (SX5E/SPX)
  - a L/S position between Topix and S&P 500 indexes (TPX/SPX)
  - a bond position in the 10-years US Treasury (UST)
  - and a FX position in the EUR/USD
- For realistic results, exposures are assumed to be realised using futures (hedged in USD) with a monthly sampling period.
- Study period: January 1994 – September 2008.

Replicating with Kalman Filter

Results: Evolution of the weights $\hat{\beta}^{(i)}_{k|k-1}$



Replicating with Kalman Filter

Decomposing HF returns into alpha and beta

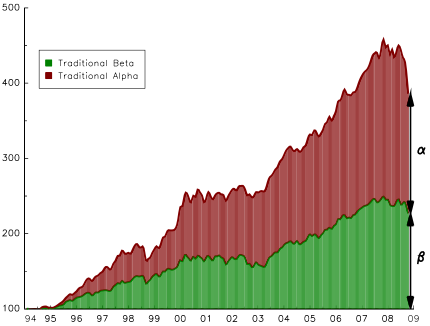
Following [Hasanhodzic and Lo, 2007, Roncalli and Teiletche, 2008], one can introduce the following concepts:

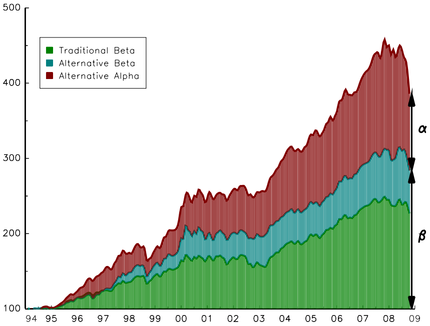
$$\begin{array}{rcll} r_k^{\mathrm{HF}} - r_k^{(0)} & = & \sum_{i=1}^m \bar{\beta}^{(i)} \left(r_k^{(i)} - r_k^{(0)} \right) & \text{(Traditional Beta)} \\ & + & \sum_{i=1}^m \left(\hat{\beta}_{k|k-1}^{(i)} - \bar{\beta}^{(i)} \right) \left(r_k^{(i)} - r_k^{(0)} \right) & \text{(Alternative Beta)} \\ & + & \left(r_k^{\mathrm{HF}} - \left(\left(1 - \sum_{i=1}^m \hat{\beta}_{k|k-1}^{(i)} \right) r_k^{(0)} + \sum_{i=1}^m \hat{\beta}_{k|k-1}^{(i)} r_k^{(i)} \right) \right) & \text{(Alternative Alpha)} \end{array}$$

where $r_k^{(0)}$ represents the risk-free return and $\bar{\beta}^{(i)} = \frac{1}{n} \sum_{k=1}^n \hat{\beta}_{k|k-1}^{(i)}$.

Replicating with Kalman Filter

Performance attribution between alpha and beta





Replicating with Kalman Filter

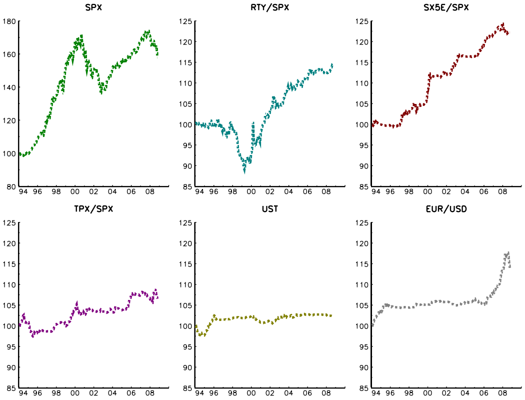
Decomposition of the yearly performance

Period	Traditional Alpha	Traditional Beta	Alternative Alpha	Alternative Beta	Total
1994	0.43	1.13	0.68	0.88	1.56
1995	6.99	13.56	7.00	13.55	21.50
1996	11.77	8.35	12.18	7.95	21.10
1997	7.21	8.94	-2.61	19.93	16.79
1998	-3.98	6.87	-4.44	7.39	2.62
1999	15.56	13.62	7.96	21.61	31.29
2000	3.03	1.90	3.63	1.31	4.98
2001	4.08	0.53	2.11	2.47	4.62
2002	4.39	-5.59	0.74	-2.18	-1.45
2003	2.99	16.08	3.96	15.00	19.55
2004	1.23	7.71	1.83	7.08	9.03
2005	2.30	6.84	1.44	7.74	9.30
2006	2.32	10.33	1.10	11.67	12.89
2007	5.30	4.43	3.35	6.39	9.96
2008	-5.96	-5.13	-4.90	-6.19	-10.78
1994-2008	3.80	5.92	2.22	7.55	9.94
1997-2008	3.14	5.46	1.14	7.55	8.77
2000-2008	2.20	4.02	1.48	4.75	6.30

Replicating with Kalman Filter

Performance attribution of the replicated strategy

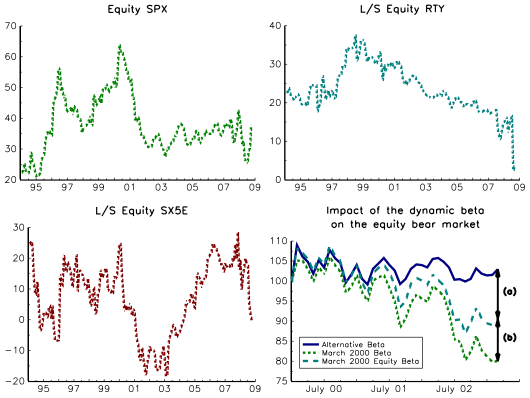
- The main contributor is the long equity exposure.
- Three other strategies have a good contribution:
  - the two L/S equity strategies on small caps and Eurozone;
  - and the FX position EUR/USD.
- TPX/SPX and US Treasury Bonds have little impact. Nonetheless, in the replication process, they can help track the volatility.



Replicating with Kalman Filter

The equity bear market between 2000 and 2003

- Highest exposure to equity was in March 2000 ($\approx 64\%$)
- After March 2000, it appears that the good performance of the HF industry may be explained by two components (cf. bottom right figure):
  - **(a)** a decrease of directional equity leverage;
  - **(b)** a good position on L/S equity on US small caps (RTY) and a good bet on L/S equity between DJ Eurostoxx 50 and SP500 (SX5E).



The Problem of Factors Selection

Two questions about adding or deleting a factor:

1. Improvement of the performance of the replication ?
2. Pertinence ? (risk management $\neq$ HF tracker)

		$\hat{\mu}_{1Y}$	π_{AB}	σ_{TE}	ρ	τ	ρ_S
	6F	7.55	75.93	3.52	87.35	67.10	84.96
+	CREDIT	7.35	73.91	3.51	87.46	67.30	85.11
+	GSCI	7.46	75.07	3.55	87.42	68.74	86.52
+	VIX	6.55	65.94	4.05	83.71	67.29	85.14
+	BUND	7.75	77.94	3.54	87.09	66.95	84.84
+	JPY/USD	7.37	74.18	3.56	87.02	66.42	84.23
+	USD/GBP	7.48	75.25	3.58	86.81	66.66	84.63
+	MXEF/SPX	7.56	76.06	3.03	90.68	72.92	89.94
−	SPX	6.42	64.56	6.31	47.51	32.19	45.82
−	RTY/SPX	7.08	71.20	4.66	75.92	54.02	73.55
−	SX5E/SPX	6.51	65.47	3.73	85.88	68.19	85.94
−	TPX/SPX	7.34	73.82	3.72	85.78	64.43	82.30
−	UST	7.86	79.13	3.50	87.47	66.92	84.79
−	EUR/USD	6.57	66.08	3.60	86.59	66.66	84.70
	7F	7.82	78.64	3.05	90.55	72.92	89.95

Which Strategy may be replicated?

Results with the 6F model

Name	π_{AB}	σ_{TE}	ρ	τ	ρ_S
HFRI Event - Driven (Total)	73.08	4.16	78.20	59.58	78.55
HFRI ED: Merger Arbitrage	62.72	2.93	65.23	43.75	60.87
HFRI ED: Private Issue/Registered Daily	34.11	6.73	31.15	24.90	36.17
HFRI ED:Distressed / Restructuring	79.77	4.70	58.36	41.50	57.14
HFRI Equity Hedge (Total)	53.97	4.50	87.35	68.65	87.12
HFRI EH: Energy / Basic Materials	41.41	17.15	46.20	30.39	42.98
HFRI EH: Equity Market Neutral	70.59	2.84	45.77	32.31	44.52
HFRI EH: Quant. Directional	69.59	5.23	92.48	75.32	91.63
HFRI EH: Short Bias	4.48	10.18	86.34	70.69	87.18
HFRI EH: Technology / Healthcare	56.81	10.69	82.66	61.39	78.99
HFRI Emerging Markets (Total)	71.99	10.37	69.99	47.27	64.78
HFRI Emerging Markets: Asia Excluding-Japan	82.47	9.99	63.84	47.23	64.56
HFRI Emerging Markets: Global	60.29	10.52	65.94	46.14	62.47
HFRI Emerging Markets: Russia/E Europe	35.21	25.73	43.47	30.52	43.60
HFRI Emerging Markets:Latin America	73.96	13.61	61.68	39.54	55.36
HFRI Macro (Total)	69.97	5.71	61.82	44.21	62.28
HFRI Macro:Syst. Diversified	49.26	6.06	65.87	57.99	76.70
HFRI Relative Value (Total)	67.94	3.07	56.46	40.48	55.74
HFRI RV: Yield Alternatives	81.76	5.98	52.23	32.30	45.27
HFRI RV:Fixed Income - Asset Backed	50.81	4.29	7.53	4.75	6.77
HFRI RV:Fixed Income - Conversion Arbeit	80.26	4.36	52.48	30.30	43.74
HFRI RV:Fixed Income - Corporate	105.67	4.23	51.41	35.73	49.96
HFRI RV:Multi - Strategy	90.95	3.06	56.41	41.59	58.42
HFRI FOF: Conservative	85.13	2.98	63.09	47.63	65.09
HFRI FOF: Diversified	91.91	4.64	70.65	50.62	68.71
HFRI FOF: Market Defensive	68.56	5.09	44.05	29.13	41.17
HFRI FOF: Strategic	98.25	5.62	79.14	58.81	77.02
HFRI Fund of Funds Composite	92.31	4.23	73.40	53.74	71.94
HFRI Fund Weighted Hedge Fund	75.93	3.52	87.35	67.10	84.96

Hedge Fund Replication: The Non-Linear Non-Gaussian Case

- HF Returns are not Gaussians
  - negative skewness and positive excess kurtosis.
- Non-Linearities in HF Returns
  - Non-linearities in hedge-fund returns have been documented from the very start of hedge-fund replication – see, e.g., [Fung and Hsieh, 1997].
  - It appears that non-linearities are important for some strategies but not for the entire industry [Diez de los Rios and Garcia, 2008].
  - Non-linearities may be due to positions in derivative instruments or uncaptured dynamic strategies – see, e.g., [Merton, 1981].
  - No successful hedge fund replication using non-linear models has ever been done

The Gaussian Distribution Assumption

Framework

We consider the following tracking problem:

$$\begin{cases} r_k^{\mathrm{HF}} = \mathbf{r}_k^\top \beta_k + \eta_k \\ \beta_k = \beta_{k-1} + \nu_k \\ \eta_k \sim \mathscr{H} \end{cases}$$

with $\mathscr{H}$ a non Gaussian distribution. TP may be solved using Particle Filters.

Assuming that $\mathscr{H}$ is a Skew t distribution $\mathscr{S}(\mu_\eta, \sigma_\eta, \alpha_\eta, \nu_\eta)$, we consider three estimation methods:

(PF #1) We estimate by ML the parameters of $\mathscr{H}$ using the KF tracking errors.

(PF #2) We estimate the $m+3$ parameters by GMM with the moment conditions (classical MM + two moments on the skewness and kurtosis).

(PF #3) The estimates are those of (PF #2) except for the parameter $\hat{\alpha}_\eta$ which is forced to -10.

The Gaussian Distribution Assumption

Results with SIR algorithm and 50000 particles

	$\hat{\mu}_{1Y}$	$\hat{\sigma}_{1Y}$	s	γ_1	γ_2
HF	9.94	7.06	0.77	-0.57	2.76
LKF[2]	7.55	6.91	0.45	-0.02	2.25
PF #1	7.76	7.44	0.45	-0.03	2.02
PF #2	7.57	7.28	0.43	-0.11	1.93
PF #3	6.90	7.99	0.31	-0.57	2.88
	π_{AB}	σ_{TE}	ρ	τ	ρ_S
LKF	75.93	3.52	87.35	67.10	84.96
PF #1	78.09	4.03	84.71	63.49	81.94
PF #2	76.13	4.25	82.51	61.60	80.20
PF #3	69.43	5.11	77.62	54.75	73.55

Conclusion

With linear assets, higher kurtosis and negative skewness come at the cost of a higher tracking error σ_{TE}.

$\Rightarrow$ It is not the right way to do it.

[2] LKF = 6F model + KF.

Taking into account Non-Linear Assets

Using option factors

Problem: Results are data dependent

We must be careful with backtests on options. Generally, what is realized is not exactly what has been predicted because of liquidity, bid/ask spread, size amount (e.g., backtest with VIX).

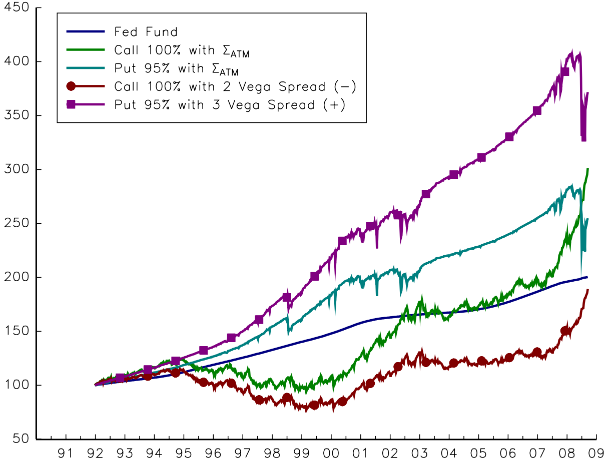
Example

We consider a systematic strategy of selling 1M put (respectively call) options with strike 95% (respectively 100%) at the end of the month. Results in a daily basis are reported in the next Figure.

Backtests clearly depend on the rebalancing dates (e.g., the end of the month is certainly a most favorable time for selling put options). Results are dependent on the implied volatility data and on skew's and bid/ask spread's assumptions.

Taking into account Non-Linear Assets

Using option factors



Taking into account Non-Linear Assets

Using option factors with exogenous strikes

The TP system becomes

$$\left\{ \begin{array}{l} r_k^{\mathrm{HF}} = \sum_{i=1}^m \beta_k^{(i)} r_k^{(i)} + \beta_k^{(m+1)} r_k^{(m+1)}(s_k) + \eta_k \\ \beta_k = \beta_{k-1} + \nu_k \end{array} \right.$$

where $r_k^{(m+1)}(s_k)$ is the return of a systematic one-month option selling strategy on S&P 500 and s_k is the (exogenous) strike of the option at time index k.

$\Rightarrow$ The TP remains linear with respect to the state variables and may be solved using Kalman Filter.

Taking into account Non-Linear Assets

Using option factors with exogenous strikes

Table: Results of replicating the HFRI index

	s_k	$\hat{\mu}_{1Y}$	$\hat{\sigma}_{1Y}$	s	γ_1	γ_2
HF		9.94	7.06	0.77	-0.57	2.76
LKF		7.55	6.91	0.45	-0.02	2.25
	95%	7.61	6.93	0.46	-0.21	2.91
Call	100%	7.92	6.94	0.50	-0.22	2.92
	105%	8.14	6.88	0.54	-0.06	2.40
	95%	7.77	6.98	0.48	-0.22	3.35
Put	100%	8.15	6.97	0.53	-0.20	3.29
	105%	8.27	6.92	0.55	-0.04	2.60
		π_{AB}	σ_{TE}	ρ	τ	ρ_S
LKF		75.93	3.52	87.35	67.10	84.96
	95%	76.62	3.55	87.12	65.48	83.56
Call	100%	79.74	3.58	86.95	65.53	83.61
	105%	81.90	3.55	87.07	66.81	84.57
	95%	78.20	3.49	87.68	66.61	84.33
Put	100%	81.98	3.53	87.40	66.60	84.22
	105%	83.21	3.48	87.62	67.68	85.21

Taking into account Non-Linear Assets

Using option factors with exogenous strikes

Table: Results of replicating the HFRI Relative Value index

	s_k	$\hat{\mu}_{1Y}$	$\hat{\sigma}_{1Y}$	s	γ_1	γ_2
HF		8.50	3.62	1.11	-2.76	15.22
LKF		5.77	2.75	0.52	0.16	1.18
	95%	6.44	2.97	0.69	-0.48	3.29
Call	100%	6.71	3.02	0.77	-0.46	3.69
	105%	7.13	2.97	0.92	0.12	1.97
	95%	6.75	4.28	0.55	-6.49	66.57
Put	100%	6.97	3.07	0.84	-0.84	6.06
	105%	6.81	2.85	0.85	0.24	1.80
		π_{AB}	σ_{TE}	ρ	τ	ρ_S
LKF		67.94	3.07	56.46	40.48	55.74
	95%	75.76	3.01	59.68	38.38	53.10
Call	100%	78.99	3.05	58.74	37.10	51.05
	105%	83.88	3.20	54.06	38.58	53.30
	95%	79.49	3.38	64.42	38.94	54.77
Put	100%	82.00	3.03	59.74	37.60	52.40
	105%	80.13	3.15	54.63	39.26	54.44

Taking into account Non-Linear Assets

Using option factors with endogenous strikes

The TP system becomes

$$\left\{ \begin{array}{l} r_k^{\mathrm{HF}} = \sum_{i=1}^{m} \beta_k^{(i)} r_k^{(i)} + \beta_k^{(m+1)} r_k^{(m+1)} (s_k) + \eta_k \\ \begin{pmatrix} \beta_k \\ s_k \end{pmatrix} = \begin{pmatrix} \beta_{k-1} \\ s_{k-1} \end{pmatrix} + \begin{pmatrix} \nu_k \\ \varepsilon_k \end{pmatrix} \\ \begin{pmatrix} \nu_k \\ \varepsilon_k \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mathbf{0} \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & \sigma_s^2 \end{pmatrix} \right) \end{array} \right.$$

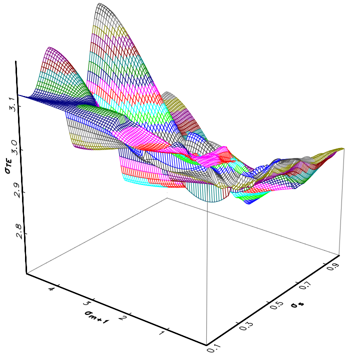
$\Rightarrow$ The TP is not linear with respect to the state variables and may be solved using Particles Filters.

Taking into account Non-Linear Assets

Using option factors with endogenous strikes

Problems with maximum likelihood method: computational time and convergence. ⇒ We prefer to use a grid approach.

Figure: Grid approach applied to the HFRI RV index



Taking into account Non-Linear Assets

Using option factors with endogenous strikes

Figure: Exposures of the linear assets for the HFRI RV index

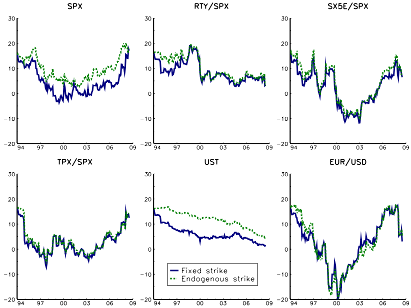
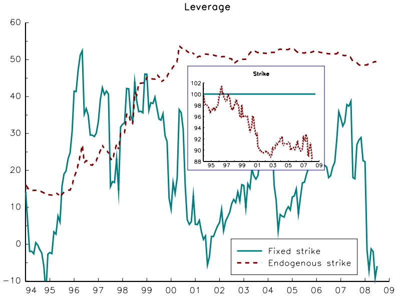


Figure: Option exposures and strikes for the HFRI RV index



What is Alpha?

- Notice the alpha is formulated as the unexplained residual of the replication strategy against the benchmarked HF. Thus, the alpha aggregates the performance of all uncaptured effects.
- Considering the performance of the state-space modeling and the monthly frequency of our replication, we consider that the uncaptured strategies remaining in the alpha could be generated either by dynamic trading in derivatives instruments or trading at ultra high frequencies or in illiquid assets (real estate, private equity, distress securities).
- A Core-Satellite approach to replication is more appropriated than trying to perfectly replicate HF.

Breakdown on the HF performance

- 75% corresponds to alternative beta which may be reproduced by the tracker;
- 25% is the alternative alpha of which:
  - 10% corresponds in fact to alternative beta which may not be implemented and are lost due to the dynamic allocation;
  - 15% makes up a component that we call the pure alternative alpha: optional, high-frequency and/or illiquid strategies

d	$\hat{\mu}_{1Y}^{\mathrm{HF}}$	$\hat{\mu}_{1Y}^{\mathrm{Clone}}$	π_{AB}	σ_{TE}	ρ	τ	ρ_S
-1	9.94	7.55	75.93	3.52	87.35	67.10	84.96
0	9.94	8.39	84.45	1.94	96.17	80.18	94.55
1	9.94	8.42	84.77	2.05	95.71	80.09	94.42
2	9.94	8.26	83.11	2.22	94.96	78.42	93.59

Table: Impact of time lags implementation

The Core/Satellite Approach

Problem

Improving the quality of replication requires investing in illiquid or optional strategies. $\Rightarrow$ **This is in contradiction with the investment philosophy of Hedge Fund Replication.**

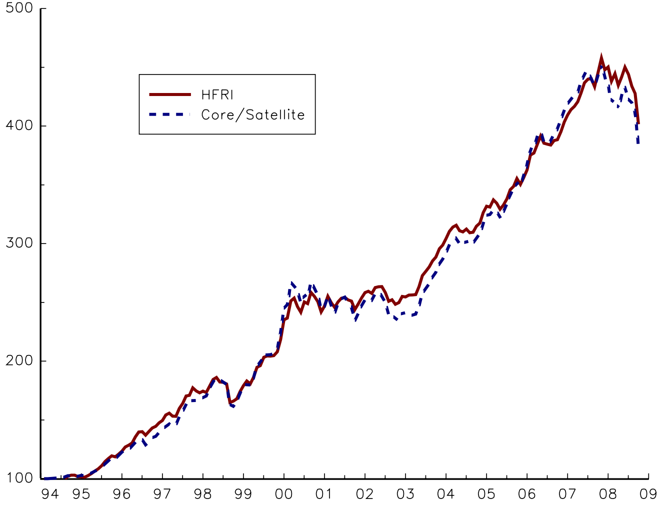
Solution

- Core: alternative beta
- Satellite: illiquid or optional strategies

Example

- 70% of alternative beta
- 10% of opt/quant strategies (SGI Vol premium & JPM Carry Max)
- 10% of real estate (UK IPD & NCREIF property)
- 10% of private equity (LPX buyout & LPX venture)

The Core/Satellite Approach



HF Replication in Practice

Asset managers and commercial investment banks have launched hedge fund replication products since 2006:

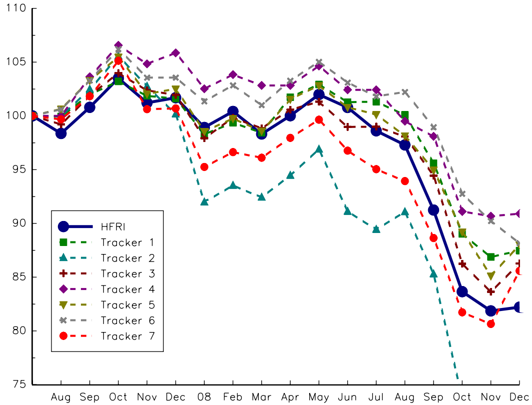
- BNP Paribas (Innocap Salto), Deutsche Bank (ARB), Goldman Sachs (ART), JPMorgan (HF AltBeta), Merrill Lynch (Torrus Factor Index), Société Générale (SGI Alternative Beta), etc.
- BlueWhite AI (ABF), Partners Group (ABS), Sgam AI (T-REX), SSgA (Premia), etc.

Products exist in several forms: Fund, Index, Short/Reverse, Leveraged, ETF, etc.

The three main applications are: investing, managing liquidity and hedging (especially since the HF liquidity crisis in 2008).

HF Replication in Practice

Figure: Comparison[3] HFRI / Trackers



[3] Hedged in Euro.

Main Ideas

- We must distinguish Hedge Fund Replication (HFR) for Risk Management purposes and Investment purposes.
- Does HFR work?
  - No for single hedge funds, specific strategies.
  - Yes if you consider the HF industry as a whole and for some specific strategies (long/short equity for example).
- From an investment point of view, building a HF tracker does not mean building a new hedge fund and it must have the following characteristics:
  - Liquidity (using only liquid futures).
  - Transparency (for example by giving the portfolio composition).
- What is the best replication method?
  - Certainly the Kalman filter.
  - Rolling-window OLS suffers of lack of reactivity.
  - Particle filters are not necessary (if you don't need options).
- Do we have to take into account non-linearities in HFR?
  - No if you want to build an investment vehicle.
  - Perhaps yes for risk management purposes (but more work is required on the subject).

Statistics Description

- $\hat{\mu}_{1Y}$ is the annualized performance;
- π_{AB} the proportion of the HFRI index performance explained by the clone;
- σ_{TE} is the yearly tracking error;
- ρ, τ and ρ_S are respectively the linear correlation, the Kendall tau and the Spearman rho between the monthly returns of the clone and the HFRI index;
- s is the sharpe ratio;
- γ_1 is the skewness;
- γ_2 is the excess kurtosis.

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Kalman Filter

If one assumes the tracking problem to be linear and Gaussian, we have

$$\left\{ \begin{array}{l} \mathbf{x}_k = \mathbf{c}_k + F_k \mathbf{x}_{k-1} + \nu_k \\ \mathbf{z}_k = \mathbf{d}_k + H_k \mathbf{x}_k + \eta_k \end{array} \right.$$

with $\nu_k \sim \mathcal{N}(0, Q_k)$ and $\eta_k \sim \mathcal{N}(0, S_k)$. Let $p(\mathbf{x}_0) = \phi\left(\mathbf{x}_0, \hat{\mathbf{x}}_0, \hat{\mathbf{P}}_0\right)$ be the initial distribution of the state vector. We have

$$\begin{array}{rcl} p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}) & = & \phi\left(\mathbf{x}_{k-1}, \hat{\mathbf{x}}_{k|k-1}, \hat{\mathbf{P}}_{k|k-1}\right) \\ p(\mathbf{x}_k \mid \mathbf{z}_{1:k}) & = & \phi\left(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k}, \hat{\mathbf{P}}_{k|k}\right) \end{array}$$

with

$$\left\{ \begin{array}{l} \hat{\mathbf{x}}_{k|k-1} = \mathbf{c}_k + F_k \hat{\mathbf{x}}_{k-1|k-1} \\ \hat{\mathbf{P}}_{k|k-1} = F_k \hat{\mathbf{P}}_{k-1|k-1} F_k^\top + Q_k \\ \hat{\mathbf{z}}_{k|k-1} = \mathbf{d}_k + H_k \hat{\mathbf{x}}_{k|k-1} \\ \hat{\mathbf{e}}_k = \mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1} \\ \hat{V}_k = H_k \hat{\mathbf{P}}_{k|k-1} H_k^\top + S_k \\ \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \hat{\mathbf{P}}_{k|k-1} H_k^\top \hat{V}_k^{-1} \hat{\mathbf{e}}_k \\ \hat{\mathbf{P}}_{k|k} = \hat{\mathbf{P}}_{k|k-1} - \hat{\mathbf{P}}_{k|k-1} H_k^\top \hat{V}_k^{-1} H_k \hat{\mathbf{P}}_{k|k-1} \end{array} \right.$$

Particle Filters

Let $\left\{\mathbf{x}_k^i, w_k^i\right\}_{i=1}^{N_s}$ denotes a set of support points $\left\{\mathbf{x}_k^i, i = 1, \ldots, N_s\right\}$ and their associated weights $\left\{w_k^i, i = 1, \ldots, N_s\right\}$ characterizing the posterior density $p\left(\mathbf{x}_k \mid \mathbf{z}_{0:k}\right)$. The posterior density at time k can then be approximated as

$$p\left(\mathbf{x}_k \mid \mathbf{z}_k\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(\mathbf{x}_k - \mathbf{x}_k^i\right)$$

If $p\left(x\right) \propto \pi\left(x\right)$ and let $x^s \sim q\left(x\right)$ be samples from the importance density $q\left(\cdot\right)$, we have by Bayes rule

$$w_k^i \propto w_{k-1}^i \frac{p\left(\mathbf{z}_k \mid \mathbf{x}_k^i\right) \times p\left(\mathbf{x}_k^i \mid \mathbf{x}_{k-1}^i\right)}{q\left(\mathbf{x}_k^i \mid \mathbf{x}_{k-1}^i, \mathbf{z}_k\right)}$$

This equation is the core of Particle filters. Considering different assumptions leads to different numerical algorithms (SIS, GPF, SIR, RPF, etc.).

Particle Filters

The GTAA example