Course 2023-2024 in Financial Risk Management
Lecture 4. Counterparty Credit Risk and Collateral Risk

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[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

1. **Overview**
   The objective of this course is to understand the theoretical and practical aspects of risk management
2. **Prerequisites**
   M1 Finance or equivalent
3. **ECTS**
   4
4. **Keywords**
   Finance, Risk Management, Applied Mathematics, Statistics
5. **Hours**
   Lectures: 36h, Training sessions: 15h, HomeWork: 30h
6. **Evaluation**
   There will be a final three-hour exam, which is made up of questions and exercises
7. **Course website**
   `http://www.thierry-roncalli.com/RiskManagement.html`

Objective of the course

The objective of the course is twofold:

1. knowing and understanding the financial regulation (banking and others) and the international standards (especially the Basel Accords)
2. being proficient in risk measurement, including the mathematical tools and risk models

Class schedule

Course sessions

- September 15 (6 hours, AM+PM)
- September 22 (6 hours, AM+PM)
- September 19 (6 hours, AM+PM)
- October 6 (6 hours, AM+PM)
- October 13 (6 hours, AM+PM)
- October 27 (6 hours, AM+PM)

Tutorial sessions

- October 20 (3 hours, AM)
- October 20 (3 hours, PM)
- November 10 (3 hours, AM)
- November 10 (3 hours, PM)
- November 17 (3 hours, PM)

Class times: Fridays 9:00am-12:00pm, 1:00pm–4:00pm, University of Evry, Room 209 IDF

Agenda

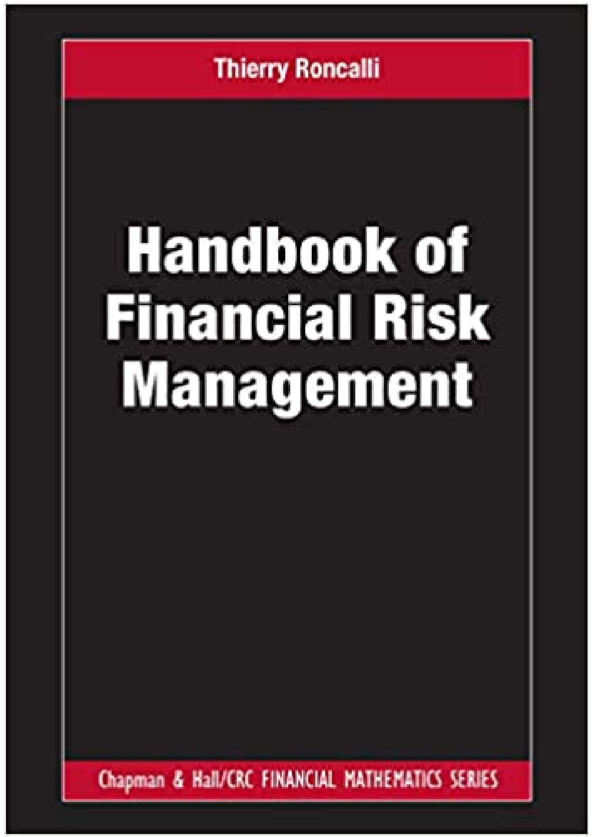
- Lecture 1: Introduction to Financial Risk Management
- Lecture 2: Market Risk
- Lecture 3: Credit Risk
- Lecture 4: Counterparty Credit Risk and Collateral Risk
- Lecture 5: Operational Risk
- Lecture 6: Liquidity Risk
- Lecture 7: Asset Liability Management Risk
- Lecture 8: Model Risk
- Lecture 9: Copulas and Extreme Value Theory
- Lecture 10: Monte Carlo Simulation Methods
- Lecture 11: Stress Testing and Scenario Analysis
- Lecture 12: Credit Scoring Models

Agenda

- Tutorial Session 1: Market Risk
- Tutorial Session 2: Credit Risk
- Tutorial Session 3: Counterparty Credit Risk and Collateral Risk
- Tutorial Session 4: Operational Risk & Asset Liability Management Risk
- Tutorial Session 5: Copulas, EVT & Stress Testing

Textbook

- Roncalli, T. (2020), *Handbook of Financial Risk Management*, Chapman & Hall/CRC Financial Mathematics Series.



Additional materials

- Slides, tutorial exercises and past exams can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagement.html`

- Solutions of exercises can be found in the companion book, which can be downloaded at the following address:

  `http://www.thierry-roncalli.com/RiskManagementBook.html`

Agenda

Counterparty credit risk and collateral risk are other forms of credit risk, where the underlying credit risk is not directly generated by the economic objective of the financial transaction

$\Rightarrow$ The portfolio can suffer a loss even if the business objective is reached

Some examples:

- 1997: LTCM (CCR)
- 2008: Lehman Brothers (CVA)
- 2011: ETF & Repo markets (Collateral risk)

Credit risk (CR) $\neq$ Counterparty credit risk (CCR)

CR:

- Loan $\Rightarrow$ credit risk (which is rewarded by a credit spread)
- CDS $\Rightarrow$ credit risk of the firm

CCR:

- Option $\Rightarrow$ counterparty credit risk (because the settlement is not guaranteed)
- CDS $\Rightarrow$ counterpart credit risk (if one counterparty defaults before the firm)

Definition

Definition

BCBS (2006) measures the counterparty credit risk by the replacement cost of the OTC derivative

Definition

Let us consider two banks A and B that have entered into an OTC contract $\mathfrak{C}$. We assume that the bank B defaults before the maturity of the contract. Bank A can then face two situations:

- The current value of the contract $\mathfrak{C}$ is negative $\Rightarrow$ Bank A closes out the position, pays the market value of the contract to Bank B, enters with another counterparty into a similar contract and receives the market value of the contract
- The current value of the contract $\mathfrak{C}$ is positive $\Rightarrow$ Bank A closes out the position, receives nothing from Bank B, enters with another counterparty into a similar contract and pays the market value of the contract

Loss = maximum between zero and the market value

This loss is not a market risk, a credit risk but a counterparty credit risk

CCR is more complex than CR

1. The counterparty credit risk is bilateral, meaning that both counterparties may face losses (Banks A and B)
2. The exposure at default is uncertain, because we don't know what will be the replacement cost of the contract when the counterparty defaults

The credit loss of an OTC portfolio is:

$$L = \sum_{i=1}^{n} \mathrm{EAD}_i \left(\boldsymbol{\tau}_i\right) \cdot \mathrm{LGD}_i \cdot \mathbb{1} \left\{\boldsymbol{\tau}_i \leq T_i\right\}$$

$\Rightarrow$ The exposure at default is random and depends on different factors:

- The default time of the counterparty
- The evolution of market risk factors
- The correlation between the market value of the OTC contract and the default of the counterparty

Exposure at default

Exposure at default

We have:

$$\mathrm{EAD} = \max \left(\mathrm{MtM} \left(\boldsymbol{\tau} \right), 0 \right)$$

Table: EAD of a portfolio

No netting	$\mathrm{EAD} =$	$\sum_{i=1}^{n} \max \left(\mathrm{MtM}_i \left(\boldsymbol{\tau} \right), 0 \right)$
Global netting	$\mathrm{EAD} =$	$\max \left(\sum_{i=1}^{n} \mathrm{MtM}_i \left(\boldsymbol{\tau} \right), 0 \right)$
Netting sets	$\mathrm{EAD} =$	$\sum_k \max \left(\sum_{i \in \mathcal{N}_k} \mathrm{MtM}_i \left(\boldsymbol{\tau} \right), 0 \right) +$ $\sum_{i \notin \cup \mathcal{N}_k} \max \left(\mathrm{MtM}_i \left(\boldsymbol{\tau} \right), 0 \right)$

Exposure at default

Example

Banks A and B have traded five OTC products, whose mark-to-market values[a] are given in the table below:

t	1	2	3	4	5	6	7	8
$\mathfrak{C}_1$	5	5	3	0	−4	0	5	8
$\mathfrak{C}_2$	−5	10	5	−3	−2	−8	−7	−10
$\mathfrak{C}_3$	0	2	−3	−4	−6	−3	0	5
$\mathfrak{C}_4$	2	−5	−5	−5	2	3	5	7
$\mathfrak{C}_5$	−1	−3	−4	−5	−7	−6	−7	−6

[a] They are calculated from the viewpoint of Bank A.

- No netting
- Global netting
- Partial netting = equity OTC contracts ($\mathfrak{C}_1$ and $\mathfrak{C}_2$) and fixed income OTC contracts ($\mathfrak{C}_3$ and $\mathfrak{C}_4$)

Exposure at default

Table: Counterparty exposure of Bank A

t	1	2	3	4	5	6	7	8
No netting	7	17	8	0	2	3	10	20
Global netting	1	9	0	0	0	0	0	4
Partial netting*	2	15	8	0	0	0	5	12

(*) Partial netting for $t = 8$: $\mathrm{EAD} = \max(8 - 10, 0) + \max(5 + 7, 0) + \max(-6, 0) = 12$

Table: Counterparty exposure of Bank B

t	1	2	3	4	5	6	7	8
No netting	6	8	12	17	19	17	14	16
Global netting	0	0	4	17	17	14	4	0
Partial netting	1	6	12	17	17	14	9	8

An illustrative example

Example

We consider a bank that buys 1 000 ATM call options, whose maturity is one-year. The current value of the underlying asset is equal to \$100. We assume that the interest rate r and the cost-of-carry parameter b are equal to 5%. Moreover, the implied volatility of the option is considered as a constant and is equal to 20%

We have:

$$\mathrm{MtM}\left(t\right) = n_C \cdot \left(\mathcal{C}\left(t\right) - \mathcal{C}_0\right)$$

where n_C and $\mathcal{C}\left(t\right)$ are the number and the market value of call options. The initial value of the call option is given by the Black-Scholes formula and we have $\mathcal{C}_0 = \$10.45$

The exposure at default $e\left(t\right)$ is equal to:

$$e\left(t\right) = \max\left(\mathrm{MtM}\left(t\right), 0\right)$$

An illustrative example

Table: Mark-to-market and counterparty exposure of the call option

t	Scenario #1				Scenario #2			
	$S(t)$	$\mathcal{C}(t)$	$\mathrm{MtM}(t)$	$e(t)$	$S(t)$	$\mathcal{C}(t)$	$\mathrm{MtM}(t)$	$e(t)$
1M	97.58	8.44	−2 013	0	91.63	5.36	−5 092	0
2M	98.19	8.25	−2 199	0	89.17	3.89	−6 564	0
3M	95.59	6.26	−4 188	0	97.60	7.35	−3 099	0
4M	106.97	12.97	2 519	2 519	97.59	6.77	−3 683	0
5M	104.95	10.83	382	382	96.29	5.48	−4 970	0
6M	110.73	14.68	4 232	4 232	97.14	5.29	−5 157	0
7M	113.20	16.15	5 700	5 700	107.71	11.55	1 098	1 098
8M	102.04	6.69	−3 761	0	105.71	9.27	−1 182	0
9M	115.76	17.25	6 802	6 802	107.87	10.18	−272	0
10M	103.58	5.96	−4 487	0	108.40	9.82	−630	0
11M	104.28	5.41	−5 043	0	104.68	5.73	−4 720	0
1Y	104.80	4.80	−5 646	0	115.46	15.46	5 013	5 013

An illustrative example

We have:

$$\mathrm{MtM}\left(0; t\right) = \mathrm{MtM}\left(0; t_0\right) + \mathrm{MtM}\left(t_0; t\right)$$

where 0 is the initial date of the trade, t_0 is the current date and t is the future date

$\Rightarrow$ This implies that the mark-to-market value at time t has two components:

1. The current mark-to-market value $\mathrm{MtM}\left(0; t_0\right)$ that depends on the past trajectory of the underlying price
2. The future mark-to-market value $\mathrm{MtM}\left(t_0; t\right)$ that depends on the future trajectory of the underlying price

How to calculate $\mathrm{MtM}\left(t_0; t\right)$?

- Historical probability measure $\mathbb{P}$
- Risk-neutral probability measure $\mathbb{Q}$

An illustrative example

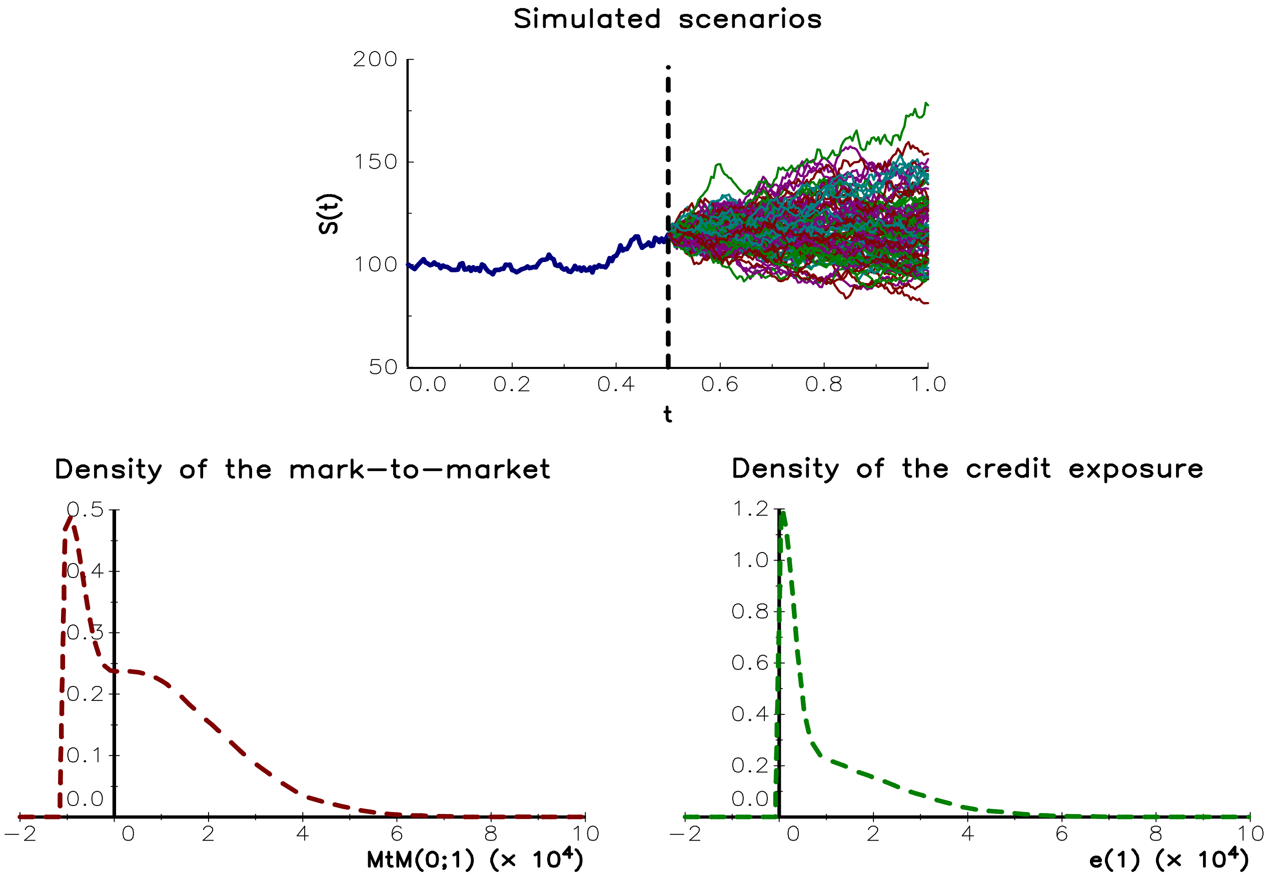


Figure: Probability density function of the counterparty exposure after six months

An illustrative example

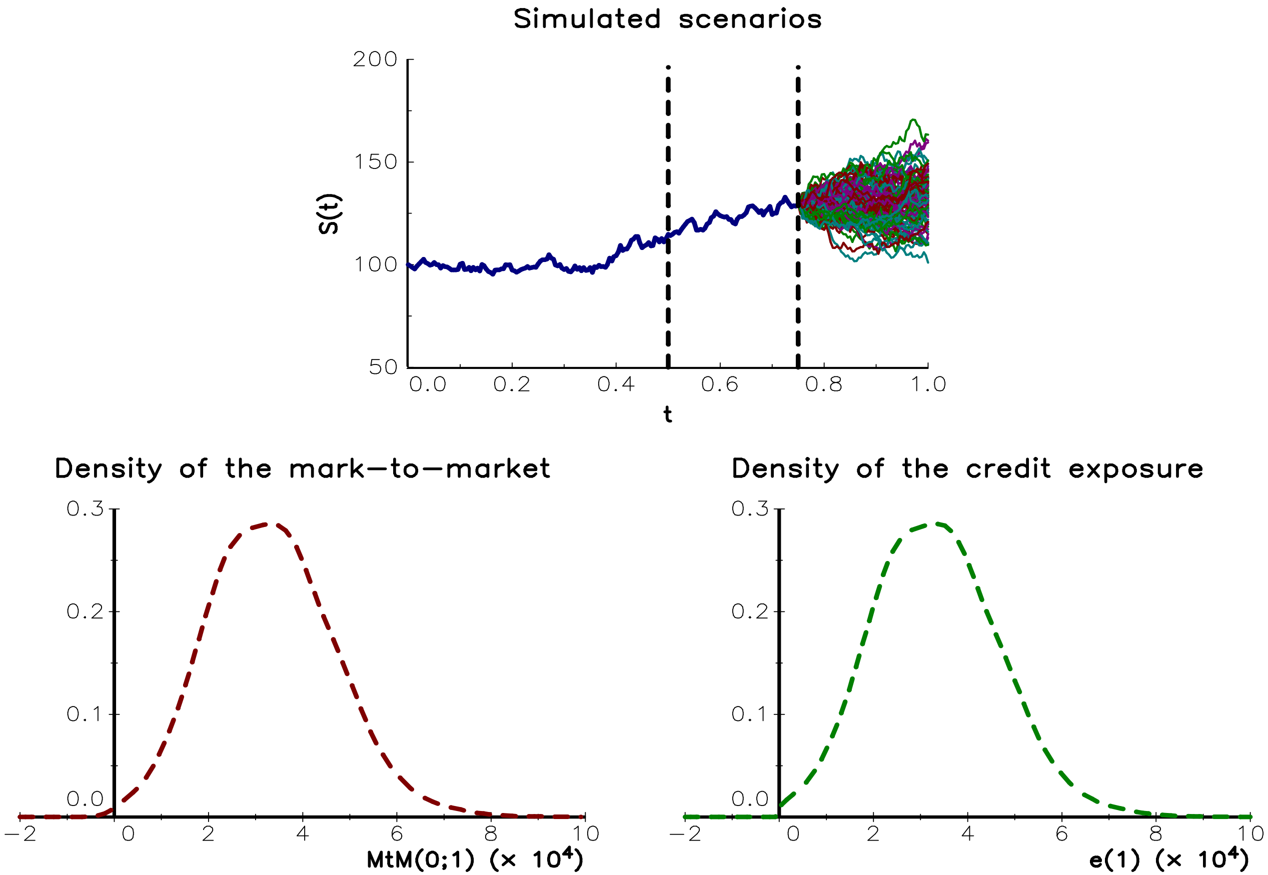


Figure: Probability density function of the counterparty exposure after nine months

An illustrative example

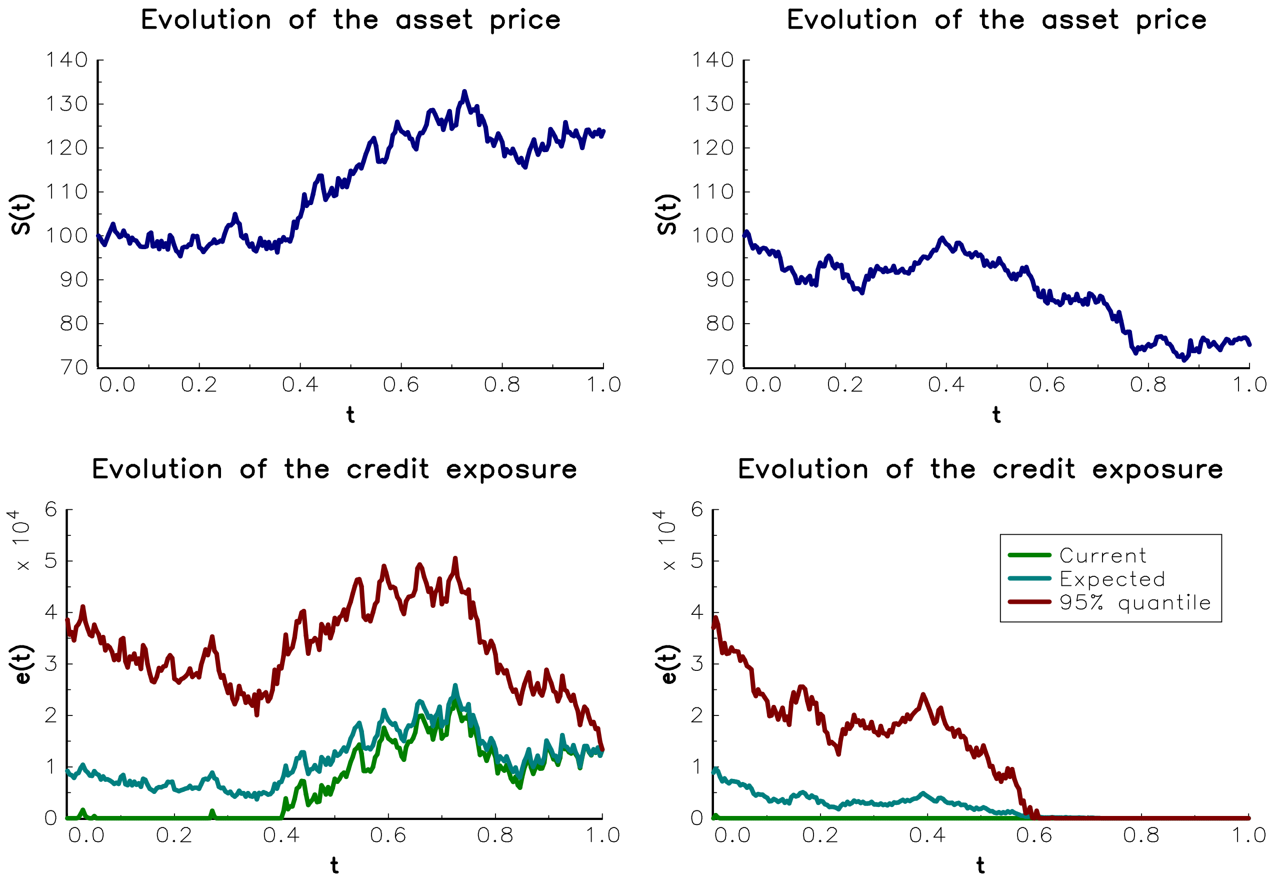


Figure: Evolution of the counterparty exposure

Measuring the counterparty exposure

- The counterparty exposure (or the potential future exposure – PFE) is equal to:

$$e(t) = \max(\mathrm{MtM}(0;t), 0)$$

- The current exposure is defined as:

$$\mathrm{CE}(t_0) = \max(\mathrm{MtM}(0;t_0), 0)$$

- $\mathbf{F}_{[0,t]}$ is the cumulative distribution function of the potential future exposure $e(t)$
- The peak exposure (PE) is the quantile of the counterparty exposure at the confidence level α:

$$\mathrm{PE}_\alpha(t) = \mathbf{F}_{[0,t]}^{-1}(\alpha) = \{\inf x : \Pr\{e(t) \leq x\} \geq \alpha\}$$

- The maximum peak exposure (MPE) is equal to:

$$\mathrm{MPE}_\alpha(0;t) = \sup_s \mathrm{PE}_\alpha(0;s)$$

Measuring the counterparty exposure

- The expected exposure (EE) is the average of the distribution of the counterparty exposure at the future date t:

$$\mathrm{EE}(t) = \mathbb{E}[e(t)] = \int_0^\infty x \,\mathrm{d}\mathbf{F}_{[0,t]}(x)$$

- The expected positive exposure (EPE) is the weighted average over time $[0, t]$ of the expected exposure:

$$\mathrm{EPE}(0; t) = \mathbb{E}\left[\frac{1}{t}\int_0^t e(s)\,\mathrm{d}s\right] = \frac{1}{t}\int_0^t \mathrm{EE}(s)\,\mathrm{d}s$$

- The effective expected exposure (EEE) is the maximum expected exposure that occurs at the future date t or any prior date:

$$\mathrm{EEE}(t) = \sup_{s\leq t} \mathrm{EE}(s) = \max\left(\mathrm{EEE}\left(t^-\right), \mathrm{EE}(t)\right)$$

- The effective expected positive exposure (EEPE) is the weighted average over time $[0, t]$ of the effective expected exposure:

$$\mathrm{EEPE}(0; t) = \frac{1}{t}\int_0^t \mathrm{EEE}(s)\,\mathrm{d}s$$

Exercise I

Exercise (HFRM, Exercise 4.4.2, Question 3, page 301)

We assume that:

$$e(t) = \exp\left(\sigma \cdot \sqrt{t} \cdot X\right)$$

where $X \sim \mathcal{N}(0,1)$

Solution of $\mathbf{F}_{[0,t]}$

- We have:

$$\begin{aligned}\mathbf{F}_{[0,t]}(x) &= \Pr\left\{e^{\sigma\sqrt{t}X} \leq x\right\} \\ &= \Pr\left\{\sigma\sqrt{t}X \leq \ln x\right\} \\ &= \Phi\left(\frac{\ln x}{\sigma\sqrt{t}}\right)\end{aligned}$$

with $x \in [0, \infty]$

- We deduce that the probability density function is equal to:

$$\begin{aligned}f_{[0,t]}(x) &= \frac{\partial\,\mathbf{F}_{[0,t]}(x)}{\partial\,x} \\ &= \frac{1}{x\sigma\sqrt{t}}\phi\left(\frac{\ln x}{\sigma\sqrt{t}}\right)\end{aligned}$$

We recognize the pdf of the log-normal distribution:

$$e(t) \sim \mathcal{LN}\left(0, \sigma^2 t\right)$$

Solution of PE

- We have:

$$\mathrm{PE}_{\alpha}\left(t\right)=\mathbf{F}_{[0,t]}^{-1}\left(\alpha\right)$$

It follows that:

$$\begin{aligned}\Phi\left(\frac{\ln x}{\sigma\sqrt{t}}\right)=\alpha & \Leftrightarrow \quad \frac{\ln x}{\sigma\sqrt{t}}=\Phi^{-1}\left(\alpha\right) \\ & \Leftrightarrow \quad x=\exp\left(\Phi^{-1}\left(\alpha\right)\sigma\sqrt{t}\right)\end{aligned}$$

We conclude that:

$$\mathrm{PE}_{\alpha}\left(t\right)=e^{\Phi^{-1}(\alpha)\sigma\sqrt{t}}$$

- It is obvious that $e^{\Phi^{-1}(\alpha)\sigma\sqrt{t}}$ is maximum when t is equal to the maturity T:

$$\mathrm{MPE}_{\alpha}\left(0;T\right)=\sup_{t}\mathrm{PE}_{\alpha}\left(t\right)=e^{\Phi^{-1}(\alpha)\sigma\sqrt{T}}$$

Solution of EE

- The expected exposure is the average of the potential future exposure:

$$\begin{aligned}\mathrm{EE}(t) &= \mathbb{E}[e(t)] \\ &= \int x \,\mathrm{d}\mathbf{F}_{[0,t]}(x) \\ &= \int x\, f_{[0,t]}(x)\,\mathrm{d}x\end{aligned}$$

We can compute the integral or we can use the property that $e(t) \sim \mathcal{LN}\left(0, \sigma^2 t\right)$. Since we know that:

$$\mathbb{E}\left[\mathcal{LN}\left(\mu, \sigma^2\right)\right] = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

we conclude that:

$$\mathrm{EE}(t) = \exp\left(\frac{1}{2}\sigma^2 t\right)$$

Solution of EPE

- We have:

$$\begin{aligned}
\mathrm{EPE}(0;t) &= \frac{1}{t}\int_0^t \mathrm{EE}(s)\,\mathrm{d}s \\
&= \frac{1}{t}\int_0^t e^{\frac{1}{2}\sigma^2 s}\,\mathrm{d}s \\
&= \frac{1}{t}\left[\frac{e^{\frac{1}{2}\sigma^2 s}}{\frac{1}{2}\sigma^2}\right]_0^t \\
&= \frac{2e^{\frac{1}{2}\sigma^2 t}-2}{\sigma^2 t}
\end{aligned}$$

Solution of EEE

- Since the function $e^{\frac{1}{2}\sigma^2 t}$ is increasing with respect to t, we deduce that the effective expected exposure is equal to the expected exposure:

$$\begin{aligned}\mathrm{EEE}(t) &= \sup_{s \leq t} \mathrm{EE}(s) \\ &= \exp\left(\frac{1}{2}\sigma^2 t\right)\end{aligned}$$

Solution of EEPE

- It follows that:

$$\begin{aligned}
\mathrm{EEPE}\left(0;t\right) &= \frac{1}{t}\int_0^t \mathrm{EEE}\left(s\right)\,\mathrm{d}s \\
&= \frac{1}{t}\int_0^t \mathrm{EE}\left(s\right)\,\mathrm{d}s \\
&= \mathrm{EPE}\left(0;t\right) \\
&= \frac{2e^{\frac{1}{2}\sigma^2 t}-2}{\sigma^2 t}
\end{aligned}$$

Solution

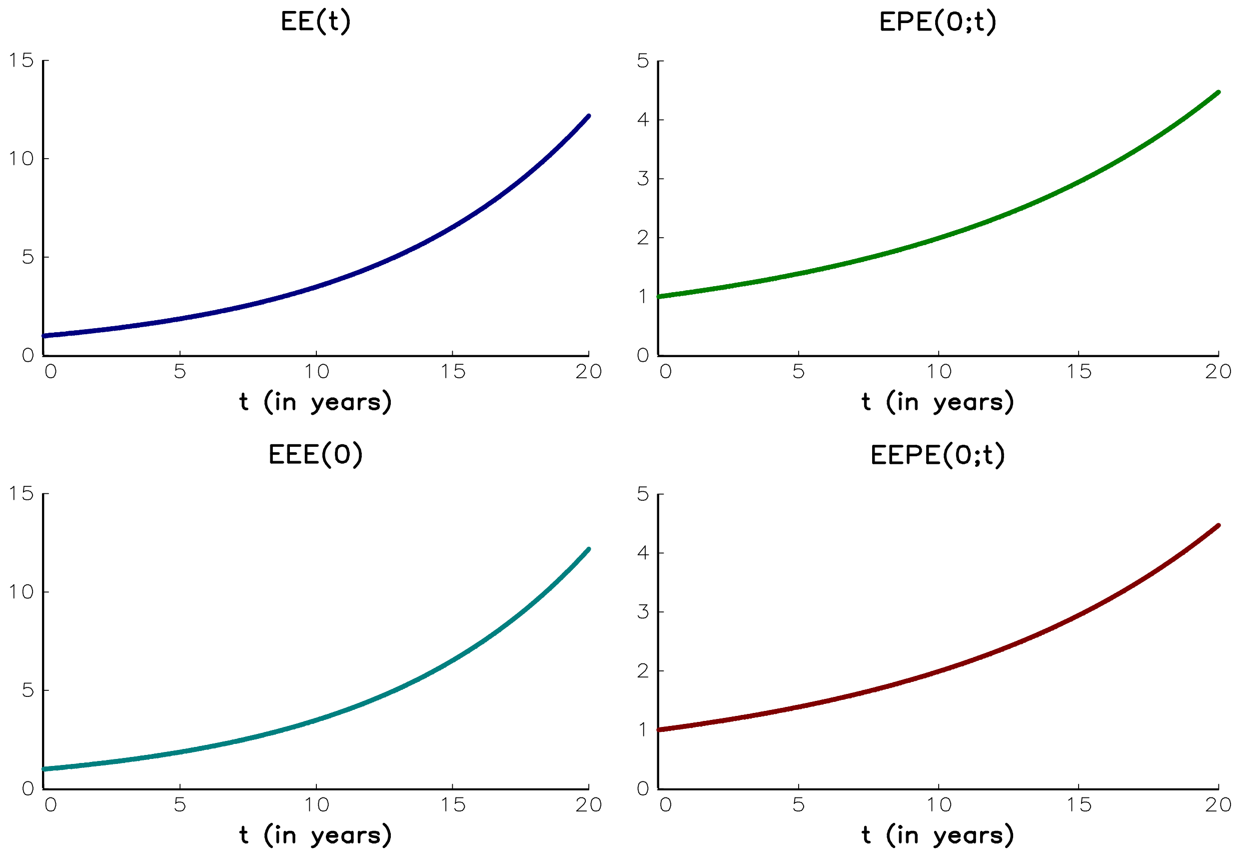


Figure: Credit exposure when $e(t) = \exp\left(\sigma\sqrt{t}\mathcal{N}(0,1)\right)$

Exercise II

Exercise (HFRM, Exercise 4.4.2, Question 4, page 301)

We assume that:

$$e(t) = \sigma \cdot \left(t^3 - \frac{7}{3} T t^2 + \frac{4}{3} T^2 t \right) \cdot X$$

where $X \sim \mathcal{U}_{[0,1]}$

Solution

Solution (HFRM-CB, pages 75-76)

$$\mathbf{F}_{[0,t]}(x) = \frac{x}{\sigma\left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)} \text{ with } x \in \left[0, \sigma\left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)\right]$$

$$\mathrm{PE}_\alpha(0) = \alpha\sigma\left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)$$

$$\mathrm{MPE}_\alpha(0;t) = \mathbb{1}\{t < t^\star\} \times \mathrm{PFE}_\alpha(0;t) + \mathbb{1}\{t \geq t^\star\} \times \mathrm{PFE}_\alpha(0;t^\star)$$

$$\mathrm{EE}(t) = \frac{1}{2}\sigma\left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)$$

$$\mathrm{EPE}(0;t) = \sigma\left(\frac{9t^3 - 28Tt^2 + 24T^2 t}{72}\right)$$

$$\mathrm{EEE}(t) = \mathbb{1}\{t < t^\star\} \times \mathrm{EE}(t) + \mathbb{1}\{t \geq t^\star\} \times \mathrm{EE}(t^\star)$$

$$t^\star = \left(\frac{7 - \sqrt{13}}{9}\right)T$$

Solution

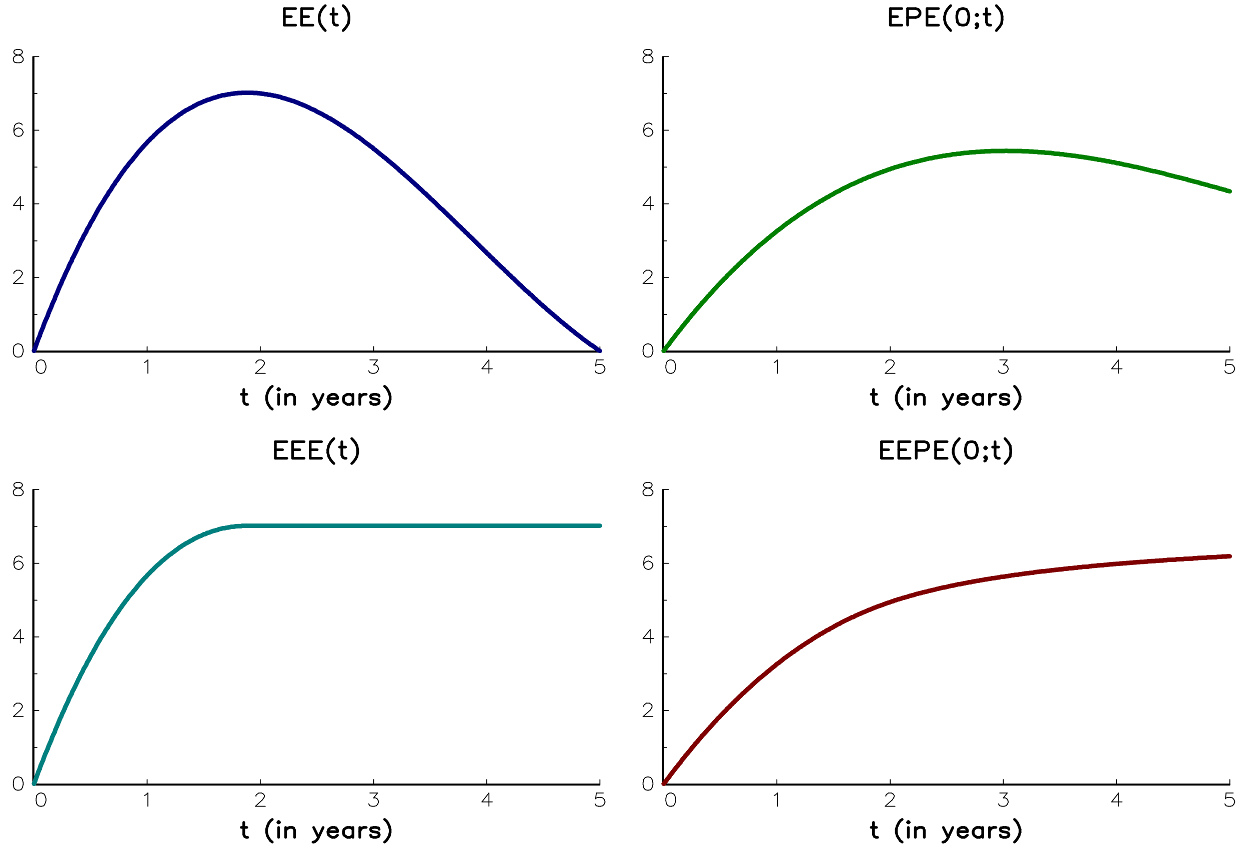


Figure: Credit exposure when $e(t) = \sigma \left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)\mathcal{U}_{[0,1]}$

Practical implementation for calculating counterparty exposures

- In practice, we use Monte Carlo simulations and the risk-neutral distribution probability $\mathbb{Q}$
- We consider a set of discrete times $\{t_0, t_1, \ldots, t_n\}$
- We note $\mathrm{MtM}_j(t_i)$ the simulated mark-to-market value for the j^{th} simulation at time at time t_i
- We note n_S the number of Monte Carlo simulations

Remark

If we consider the introductory example, we simulate $S_j(t_i)$ the value of the asset price at time t_i for the j^{th} simulation. For each simulated trajectory, we then calculate the option price $\mathcal{C}_j(t_i)$ and the mark-to-market value:

$$\mathrm{MtM}_j(t_i) = n_C \cdot (\mathcal{C}_j(t_i) - \mathcal{C}_0)$$

Practical implementation

Given a sample of n_S simulated exposures for $t \in \{t_0, t_1, \ldots, t_n\}$:

$$e_j(t_i) = \max(\mathrm{MtM}_j(t_i), 0)$$

we deduce the following estimators:

- The peak exposure at time t_i is estimated using the order statistics:

$$\mathrm{PE}_\alpha(t_i) = e_{\alpha n_S : n_S}(t_i)$$

- We use the empirical mean to calculate the expected exposure:

$$\mathrm{EE}(t_i) = \frac{1}{n_S}\sum_{j=1}^{n_S} e_j(t_i)$$

Practical implementation

- For the expected positive exposure, we approximate the integral by the following sum:

$$\mathrm{EPE}\left(0; t_i\right) = \frac{1}{t_i} \sum_{k=1}^{i} \mathrm{EE}\left(t_k\right) \Delta t_k$$

If we consider a fixed-interval scheme with $\Delta t_k = \Delta t$, we obtain:

$$\mathrm{EPE}\left(0; t_i\right) = \frac{\Delta t}{t_i} \sum_{k=1}^{i} \mathrm{EE}\left(t_k\right) = \frac{1}{i} \sum_{k=1}^{i} \mathrm{EE}\left(t_k\right)$$

Practical implementation

- By definition, the effective expected exposure is given by the following recursive formula:

$$\mathrm{EEE}(t_i) = \max\left(\mathrm{EEE}(t_{i-1}), \mathrm{EE}(t_i)\right)$$

  where $\mathrm{EEE}(0)$ is initialized with the value $\mathrm{EE}(0)$

- Finally, the effective expected positive exposure is given by:

$$\mathrm{EEPE}(0; t_i) = \frac{1}{t_i} \sum_{k=1}^{i} \mathrm{EEE}(t_k) \Delta t_k$$

  In the case of a fixed-interval scheme, this formula becomes:

$$\mathrm{EEPE}(0; t_i) = \frac{1}{i} \sum_{k=1}^{i} \mathrm{EEE}(t_k)$$

The square-root profile of CCR

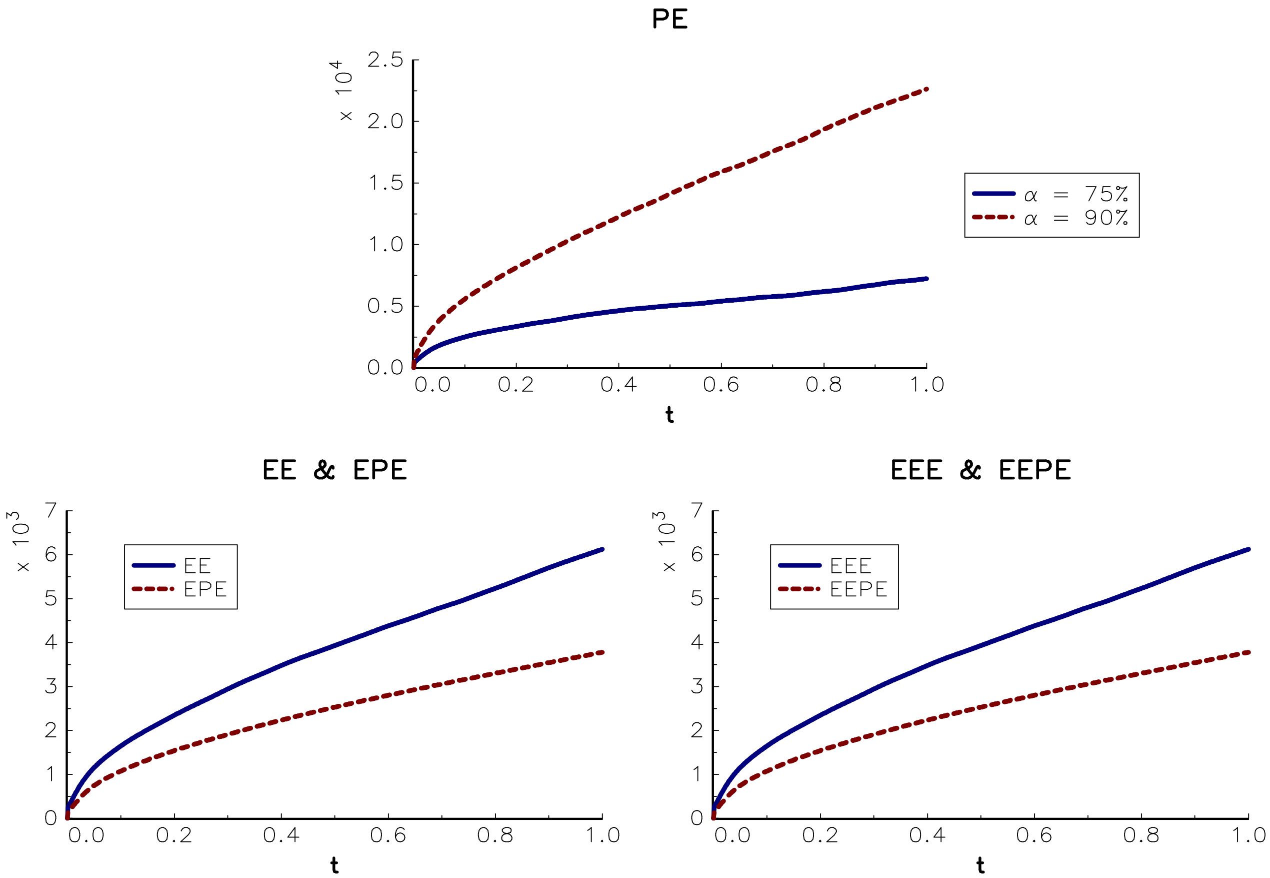


Figure: Counterparty exposure profile of options

The bell-shaped profile of CCR

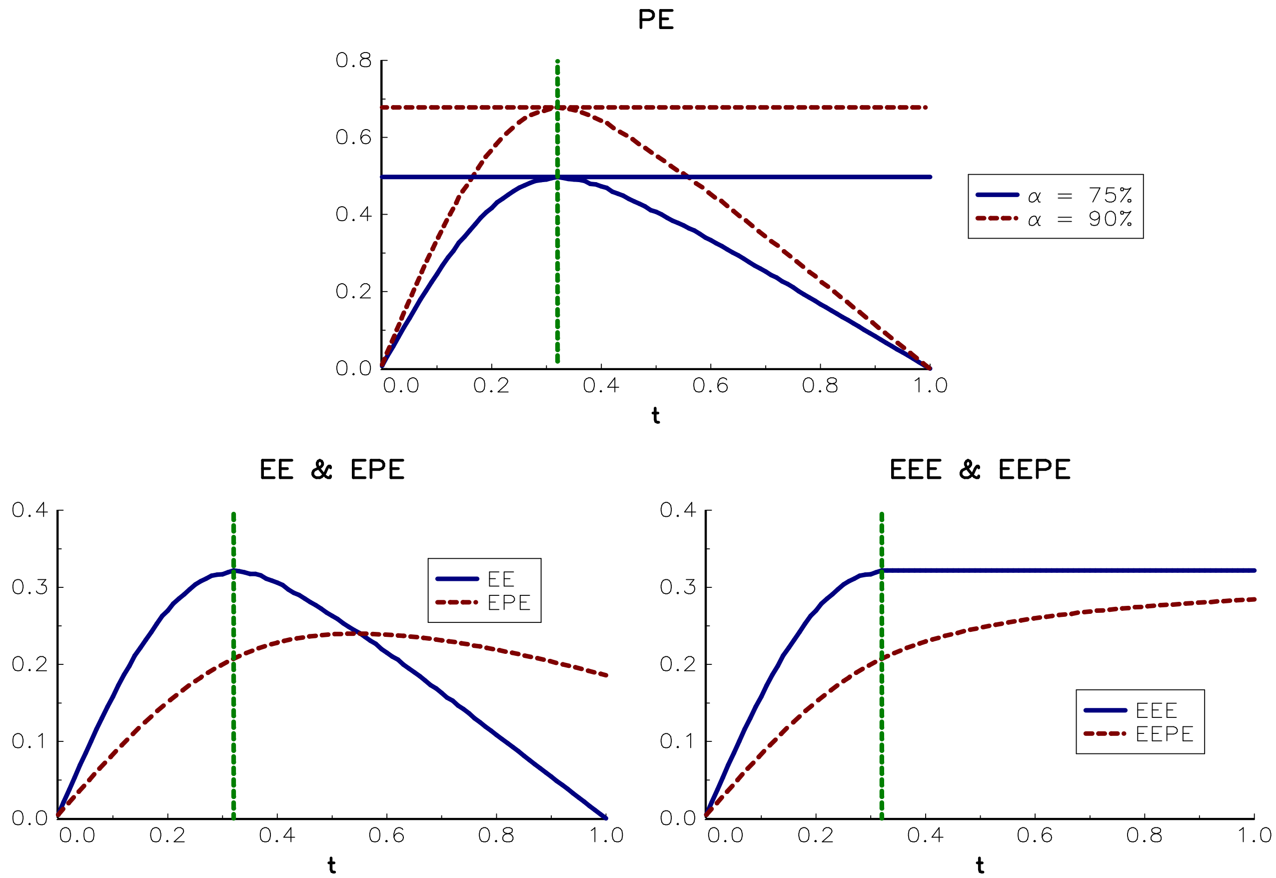


Figure: Counterparty exposure profile of interest rate swaps

Regulatory capital

Basel II

- Non-internal model methods
  1. Current exposure method (CEM)
  2. Standardized method (SM)
- Internal model method (IMM)

Basel III

- Standardized approach (SA-CCR)
- Internal model method (IMM)

Each approach defines how the exposure at default EAD is calculated. In the SA approach, the capital charge is equal to:

$$\mathcal{K} = 8\% \cdot \mathrm{EAD} \cdot \mathrm{RW}$$

In the IRB approach, we recall that:

$$\mathcal{K} = \mathrm{EAD} \cdot \mathrm{LGD} \cdot \left(\Phi \left(\frac{\Phi^{-1}(\mathrm{PD}) + \sqrt{\rho(\mathrm{PD})}\Phi^{-1}(0.999)}{\sqrt{1 - \rho(\mathrm{PD})}} \right) - \mathrm{PD} \right) \cdot \varphi(\mathrm{M})$$

Regulatory capital

Internal model method (Basel II and III)

We have:

$$\mathrm{EAD} = \alpha \cdot \mathrm{EEPE}\left(0; \min\left(T, 1\right)\right)$$

where α is equal to 1.4 and T is the maturity of the OTC contract

Remark

Under some conditions, the bank may uses its own estimates for α, but it must be larger than 1.2

Regulatory capital

Internal model method (Basel II and III)

Example

We assume that the one-year effective expected positive exposure with respect to a given counterparty is equal to \$50.2 mn. The LGD is equal to 45% and the maturity is set to one year.

Table: Capital charge of counterparty credit risk under the FIRB approach

	PD	1%	2%	3%	4%	5%
Basel II	ρ (PD) (in %)	19.28	16.41	14.68	13.62	12.99
	$\mathcal{K}$ (in \$ mn)	4.12	5.38	6.18	6.82	7.42
Basel III	ρ (PD) (in %)	24.10	20.52	18.35	17.03	16.23
	$\mathcal{K}$ (in \$ mn)	5.26	6.69	7.55	8.25	8.89
	$\Delta\mathcal{K}$ (in %)	27.77	24.29	22.26	20.89	19.88

Regulatory capital

SA-CCR method (Basel III)

The exposure at default under the SA-CCR is defined as follows:

$$\mathrm{EAD} = \alpha \cdot (\mathrm{RC} + \mathrm{PFE})$$

where RC is the replacement cost (or the current exposure), PFE is the potential future exposure and α is equal to 1.4

Remark

We can view this formula as an approximation of the IMM calculation, meaning that $\mathrm{RC} + \mathrm{PFE}$ *represents a stylized* EEPE *value*

$\Rightarrow$ SA-CCR is close to SA-TB (see HFRM on pages 270-274)

Impact of wrong way risk

Definition

The wrong way risk (WWR) is defined as the risk that "*occurs when exposure to a counterparty or collateral associated with a transaction is adversely correlated with the credit quality of that counterparty*". This means that the exposure at default of the OTC contract and the default risk of the counterparty are positively correlated

Two types of wrong way risk:

1. General (or conjectural) wrong way risk occurs when the credit quality of the counterparty is correlated with macroeconomic factors, which also impact the value of the transaction (e.g. level of interest rates)
2. Specific wrong way risk occurs when the correlation between the exposure at default and the probability of default is mainly explained by some idiosyncratic factors (e.g. Bank A buys a CDS protection on Bank B from Bank C)

Impact of wrong way risk

An example

We assume that:

$$\mathrm{MtM}(t) = \mu + \sigma W(t)$$

If we note $e(t) = \max(\mathrm{MtM}(t), 0)$, we have:

$$\begin{aligned}
\mathbb{E}[e(t)] &= \int_{-\infty}^{\infty} \max\left(\mu + \sigma\sqrt{t}x, 0\right)\phi(x)\,\mathrm{d}x \\
&= \mu \int_{-\mu/(\sigma\sqrt{t})}^{\infty} \phi(x)\,\mathrm{d}x + \sigma\sqrt{t}\int_{-\mu/(\sigma\sqrt{t})}^{\infty} x\phi(x)\,\mathrm{d}x \\
&= \mu\left(1 - \Phi\left(-\frac{\mu}{\sigma\sqrt{t}}\right)\right) + \sigma\sqrt{t}\left[-\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}\right]_{-\mu/(\sigma\sqrt{t})}^{\infty} \\
&= \mu\Phi\left(\frac{\mu}{\sigma\sqrt{t}}\right) + \sigma\sqrt{t}\phi\left(\frac{\mu}{\sigma\sqrt{t}}\right)
\end{aligned}$$

Impact of wrong way risk

An example

Two assumptions:

$\mathcal{H}_1$ Merton model with the default barrier $B(t) = \Phi^{-1}(1 - \mathbf{S}(t))$

$\mathcal{H}_2$ The dependence between the mark-to-market $\mathrm{MtM}(t)$ and the survival time is given by the Normal copula $\mathbf{C}(u_1, u_2; \rho)$ with parameter ρ

Impact of wrong way risk

An example

Since we have $1 - \mathbf{S}(t) \sim \mathcal{U}_{[0,1]}$, it follows that $B(t) \sim \mathcal{N}(0,1)$. We deduce that the random vector $(\mathrm{MtM}(t), B(t))$ is normally distributed:

$$\begin{pmatrix} \mathrm{MtM}(t) \\ B(t) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 t & \rho\sigma\sqrt{t} \\ \rho\sigma\sqrt{t} & 1 \end{pmatrix}\right)$$

because the correlation $\rho(\mathrm{MtM}(t), B(t))$ is equal to the Normal copula parameter ρ. Using the conditional expectation formula (Lecture 2, Slide 114), it follows that:

$$\mathrm{MtM}(t) \mid B(t) = B \sim \mathcal{N}\left(\mu_B, \sigma_B^2\right)$$

where:

$$\mu_B = \mu + \rho\sigma\sqrt{t}\,(B - 0)$$

and:

$$\sigma_B^2 = \sigma^2 t - \rho^2\sigma^2 t = \left(1 - \rho^2\right)\sigma^2 t$$

Impact of wrong way risk

An example

We deduce that:

$$\mathbb{E}\left[e(t) \mid \boldsymbol{\tau} = t\right] = \mathbb{E}\left[e(t) \mid B(t) = B\right] = \mu_B \Phi\left(\frac{\mu_B}{\sigma_B}\right) + \sigma_B \phi\left(\frac{\mu_B}{\sigma_B}\right)$$

where:

$$\mu_B = \mu + \rho\sigma\sqrt{t}B$$

and:

$$\sigma_B = \sqrt{1-\rho^2}\sigma\sqrt{t}$$

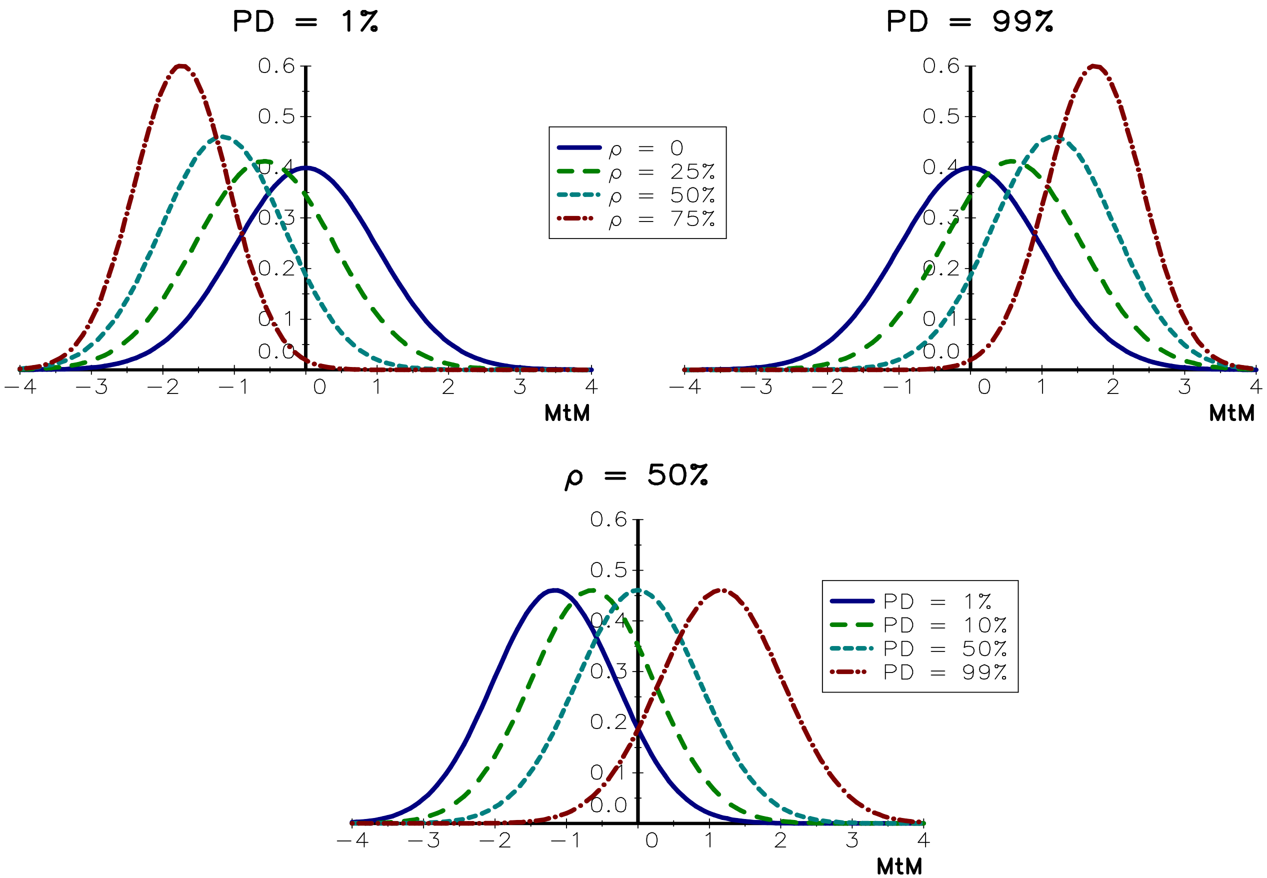
With the exception of $\rho = 0$, we have:

$$\mathbb{E}\left[e(t)\right] \neq \mathbb{E}\left[e(t) \mid \boldsymbol{\tau} = t\right]$$

Impact of wrong way risk

An example

Figure: Conditional distribution of the mark-to-market

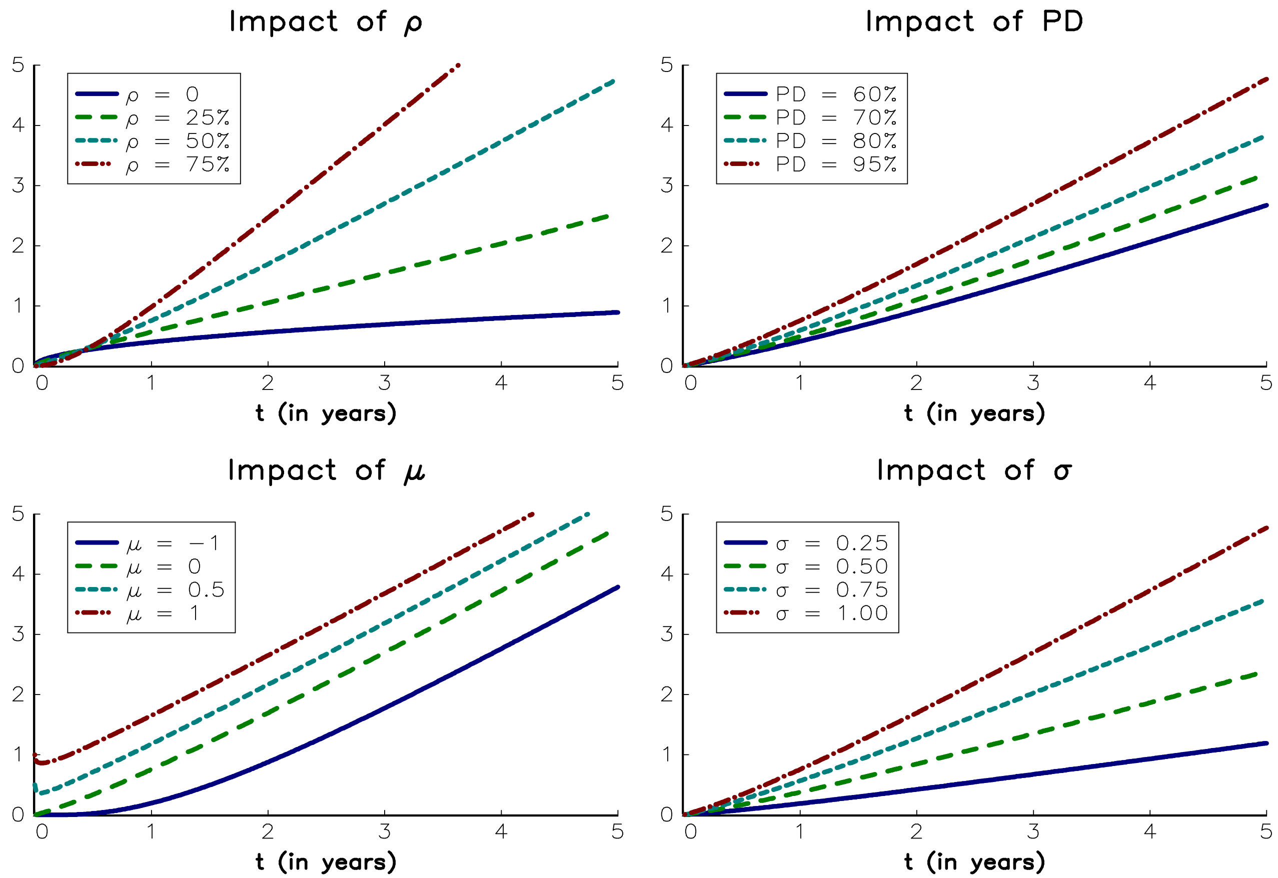


(*) The default occurs at time $t = 1$, and the parameters are $\mu = 0$, $\sigma = 1$ and $\boldsymbol{\tau} \sim \mathcal{E}(\lambda)$

Impact of wrong way risk

An example

Figure: Conditional expectation of the exposure at default



(*) The default values are $\mu = 0$, $\sigma = 1$, $\mathrm{PD} = 90\%$ and $\rho = 50\%$

Impact of wrong way risk

Calibration of the α factor

$\Rightarrow$ A difficult task:

$$L = \sum_{i=1}^{n} \mathrm{EAD}\left(\boldsymbol{\tau}_i, \mathcal{F}_1, \ldots, \mathcal{F}_m\right) \cdot \mathrm{LGD}_i \cdot \mathbb{1}\left\{\boldsymbol{\tau}_i \leq T_i\right\}$$

where $\mathcal{F} = (\mathcal{F}_1, \ldots, \mathcal{F}_m)$ are the market risk factors and $\boldsymbol{\tau} = (\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_n)$ are the default times

WWR implies to correlate the random vectors $\mathcal{F}$ and $\boldsymbol{\tau}$

CVA versus CCR

Definition

CVA is the adjustment to the risk-free (or fair) value of derivative instruments to account for counterparty credit risk. Thus, CVA is commonly viewed as the market price of CCR

- CCR concerns the default risk of the counterparty $\Rightarrow$ credit risk

  CCR may induce a loss

- CVA concerns the credit risk of the counterparty before the default $\Rightarrow$ market risk

  CVA impacts the mark-to-market of the OTC contract

2008 GFC & Lehman Brothers bankruptcy

Banks suffered significant CCR losses on their OTC derivatives portfolios:

- $^2/_3$ of these losses came from CVA markdowns on derivatives
- $^1/_3$ were due to counterparty defaults

Fair valuation

We consider two banks A and B and an OTC contract $\mathfrak{C}$. The P&L $\Pi_{A|B}$ of Bank A is equal to:

$$\Pi_{A|B} = \mathrm{MtM} - \mathrm{CVA}_B$$

where MtM is the risk-free mark-to-market value of $\mathfrak{C}$ and CVA_B is the CVA with respect to Bank B. We assume that Bank A has traded the same contract with Bank C. It follows that:

$$\Pi_{A|C} = \mathrm{MtM} - \mathrm{CVA}_C$$

In a world where there is no counterparty credit risk, we have:

$$\Pi_{A|B} = \Pi_{A|C} = \mathrm{MtM}$$

Fair valuation

If we take into account the counterparty credit risk, the two P&Ls of the same contract are different because Bank A does not face the same risk:

$$\Pi_{A|B} \neq \Pi_{A|C}$$

In particular, if Bank A wants to close the two exposures, it is obvious that the contact $\mathfrak{C}$ with the counterparty B has more value than the contact $\mathfrak{C}$ with the counterparty C if the credit risk of B is lower than the credit risk of C

CVA, DVA and bilateral CVA

- CVA is the market risk related to the credit risk of the counterparty
- DVA (debit value adjustment) is the credit-related adjustment capturing the entity's own credit risk
- BCVA (bilateral CVA) is the combination of the two credit-related adjustments:

$$\Pi_{A|B} = \mathrm{MtM} + \underbrace{\mathrm{DVA}_A - \mathrm{CVA}_B}_{\text{Bilateral CVA}}$$

- If the credit risk of Bank A is lower than the credit risk of Bank B, the bilateral CVA of Bank A is negative and reduces the value of the OTC portfolio from the perspective of Bank A
- If the credit risk of Bank A is higher than the credit risk of Bank B, the bilateral CVA of Bank A is positive and increases the value of the OTC portfolio from the perspective of Bank A
- If the credit risk of Banks A and B is the same, the bilateral CVA is equal to zero

BCVA and the coherency property

The DVA of Bank A is the CVA of Bank A from the perspective of Bank B:

$$\mathrm{CVA}_A = \mathrm{DVA}_A$$

We also have $\mathrm{DVA}_B = \mathrm{CVA}_B$, which implies that the P&L of Bank B is equal to:

$$\begin{aligned} \Pi_{B|A} &= -\mathrm{MtM} + \mathrm{DVA}_B - \mathrm{CVA}_A \\ &= -\mathrm{MtM} + \mathrm{CVA}_B - \mathrm{DVA}_A \\ &= -\Pi_{A|B} \end{aligned}$$

Remark

We deduce that the P&Ls of Banks A and B are coherent in the bilateral CVA framework as in the risk-free MtM framework

Notations

- The positive exposure $e^{+}(t)$ is the maximum between 0 and the risk-free mark-to-market:

$$e^{+}(t) = \max(\mathrm{MtM}(t), 0)$$

This quantity was previously denoted by $e(t)$ and corresponds to the potential future exposure in the CCR framework

- The negative exposure $e^{-}(t)$ is the difference between the risk-free mark-to-market and the positive exposure:

$$e^{-}(t) = \mathrm{MtM}(t) - e^{+}(t) = \max(-\mathrm{MtM}(t), 0)$$

The negative exposure is then the equivalent of the positive exposure from the perspective of the counterparty

The CVA formula

CVA is the risk-neutral discounted expected value of the potential loss:

$$\mathrm{CVA} = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}\left\{\boldsymbol{\tau}_B \leq T\right\} \cdot e^{-\int_0^{\boldsymbol{\tau}_B} r_t \, \mathrm{d}t} \cdot L\right]$$

where:

- T is the maturity of the OTC derivative
- $\boldsymbol{\tau}_B$ is the default time of Bank B
- L is the counterparty loss:

$$L = (1 - \boldsymbol{\mathcal{R}}_B) \cdot e^{+}(\boldsymbol{\tau}_B)$$

The CVA formula

Using usual assumptions, we obtain:

$$\mathrm{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T B_0(t)\,\mathrm{EpE}(t)\,\mathrm{d}\mathbf{F}_B(t)$$

where:

- $\mathrm{EpE}(t)$ is the risk-neutral expected positive exposure:

$$\mathrm{EpE}(t) = \mathbb{E}^{\mathbb{Q}}\left[e^+(t)\right]$$

- $\mathbf{F}_B$ is the cumulative distribution function of $\boldsymbol{\tau}_B$

The CVA formula

Since $\mathbf{S}_B(t) = 1 - \mathbf{F}_B(t)$, we obtain:

$$\mathrm{CVA} = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}(t)\,\mathrm{d}\mathbf{S}_B(t)$$

The DVA formula

The debit value adjustment is defined as the risk-neutral discounted expected value of the potential gain:

$$\mathrm{DVA} = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}\left\{\tau_A \leq T\right\} \cdot e^{-\int_0^{\tau_A} r_t \,\mathrm{d}t} \cdot G\right]$$

where:

- τ_A is the default time of Bank A
- G is the counterparty gain:

$$G = \left(1 - \mathcal{R}_A\right) \cdot e^{-}\left(\tau_A\right)$$

The DVA formula

$$\mathrm{DVA} = \left(1 - \mathcal{R}_A\right) \cdot \int_0^T -B_0\left(t\right) \mathrm{EnE}\left(t\right) \,\mathrm{d}\mathbf{S}_A\left(t\right)$$

where $\mathrm{EnE}\left(t\right)$ is the risk-neutral expected negative exposure:

$$\mathrm{EnE}\left(t\right) = \mathbb{E}^{\mathbb{Q}}\left[e^{-}\left(t\right)\right]$$

The two BCVA formulas

Independent case ($\boldsymbol{\tau}_B \perp \boldsymbol{\tau}_A$)

$$\begin{aligned}
\mathrm{BCVA} &= \mathrm{DVA} - \mathrm{CVA} \\
&= (1 - \boldsymbol{\mathcal{R}}_A) \cdot \int_0^T -B_0(t)\,\mathrm{EnE}(t)\;\mathrm{d}\mathbf{S}_A(t) - \\
&\quad (1 - \boldsymbol{\mathcal{R}}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}(t)\;\mathrm{d}\mathbf{S}_B(t)
\end{aligned}$$

General case

We must consider the joint survival function of $(\boldsymbol{\tau}_A, \boldsymbol{\tau}_B)$:

$$\mathrm{BCVA} = \mathbb{E}^{\mathbb{Q}}\left[\begin{array}{l} \mathbb{1}\left\{\boldsymbol{\tau}_A \le \min\left(T, \boldsymbol{\tau}_B\right)\right\} \cdot e^{-\int_0^{\boldsymbol{\tau}_A} r_t\,\mathrm{d}t} \cdot G - \\ \mathbb{1}\left\{\boldsymbol{\tau}_B \le \min\left(T, \boldsymbol{\tau}_A\right)\right\} \cdot e^{-\int_0^{\boldsymbol{\tau}_B} r_t\,\mathrm{d}t} \cdot L \end{array}\right]$$

Interpretation of the CVA measure

If we assume that the yield curve is flat and $\mathbf{S}_B(t) = e^{-\lambda_B t}$, we have:

$$\mathrm{d}\mathbf{S}_B(t) = -\lambda_B e^{-\lambda_B t}\,\mathrm{d}t$$

and:

$$\begin{aligned} \mathrm{CVA} &= (1 - \mathcal{R}_B) \cdot \int_0^T e^{-rt}\,\mathrm{EpE}(t)\,\lambda_B e^{-\lambda_B t}\,\mathrm{d}t \\ &= \mathcal{s}_B \cdot \int_0^T e^{-(r+\lambda_B)t}\,\mathrm{EpE}(t)\,\mathrm{d}t \end{aligned}$$

$\Rightarrow$ CVA is the product of the CDS spread and the discounted value of the expected positive exposure

Exercise III

Exercise (HFRM, Exercise 4.4.5, page 303)

We assume that the mark-to-market value is given by:

$$\mathrm{MtM}(t) = N \int_t^T f(t,T) B_t(s) \, \mathrm{d}s - N \int_t^T f(0,T) B_t(s) \, \mathrm{d}s$$

where N and T are the notional and the maturity of the swap, and $f(t,T)$ is the instantaneous forward rate which follows a geometric Brownian motion:

$$\mathrm{d}f(t,T) = \mu f(t,T) \, \mathrm{d}t + \sigma f(t,T) \, \mathrm{d}W(t)$$

We also assume that the yield curve is flat – $B_t(s) = e^{-r(s-t)}$ – and the risk-neutral survival function is $\mathbf{S}(t) = e^{-\lambda t}$

Solution

Solution (Syrkin and Shirazi, 2015; HFRM-CB, Section 4.4.5, pages 82-85)

We have:

$$\mathrm{CVA}(t) = s_B \cdot \int_t^T e^{-(r+\lambda)(u-t)}\, \mathrm{EpE}(u)\, \mathrm{d}u$$

where:

$$\mathrm{EpE}(t) = Nf(0,T)\,\varphi(t,T)\left(e^{\mu t}\Phi\left(\left(\frac{\mu}{\sigma}+\frac{1}{2}\sigma\right)\sqrt{t}\right) - \Phi\left(\left(\frac{\mu}{\sigma}-\frac{1}{2}\sigma\right)\sqrt{t}\right)\right)$$

and:

$$\varphi(t,T) = \frac{1-e^{-r(T-t)}}{r}$$

Numerical example: $N = 1000$, $f(0,T) = 5\%$, $\mu = 2\%$, $\sigma = 25\%$, $T = 10$ years and $\mathcal{R}_B = 50\%$

Solution

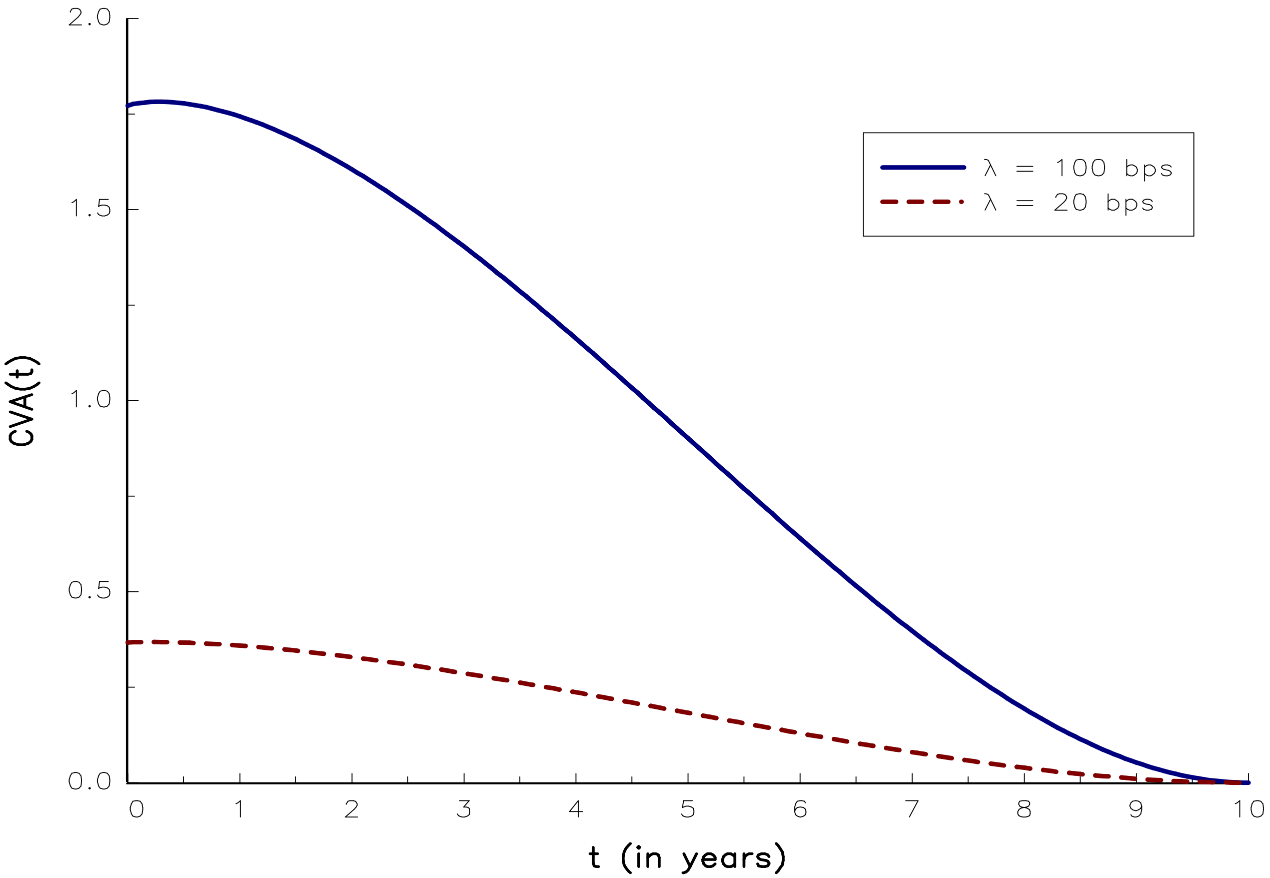


Figure: CVA of fixed-float swaps

Practical implementation

We approximate the integral by a sum:

$$\mathrm{CVA} = (1 - \mathcal{R}_B) \cdot \sum_{t_i \leq T} B_0(t_i) \cdot \mathrm{EpE}(t_i) \cdot (\mathbf{S}_B(t_{i-1}) - \mathbf{S}_B(t_i))$$

and:

$$\mathrm{DVA} = (1 - \mathcal{R}_A) \cdot \sum_{t_i \leq T} B_0(t_i) \cdot \mathrm{EnE}(t_i) \cdot (\mathbf{S}_A(t_{i-1}) - \mathbf{S}_A(t_i))$$

where $\{t_i\}$ is a partition of $[0, T]$

Practical implementation

We have:

$$\mathbf{S}_B(t_{i-1}) - \mathbf{S}_B(t_i) = \Pr\{t_{i-1} < \boldsymbol{\tau}_B \leq t_i\} = \mathrm{PD}_B(t_{i-1}, t_i)$$

$\mathrm{PD}_B(t_{i-1}, t_i)$ is a risk-neutral probability

The credit triangle relationship is:

$$\mathcal{s}_B(t) = (1 - \boldsymbol{\mathcal{R}}_B) \cdot \lambda_B(t)$$

We deduce that:

$$\mathbf{S}_B(t) = \exp(-\lambda_B(t) \cdot t) = \exp\left(-\frac{\mathcal{s}_B(t) \cdot t}{1 - \boldsymbol{\mathcal{R}}_B}\right)$$

and:

$$\mathrm{PD}_B(t_{i-1}, t_i) = \exp\left(-\frac{\mathcal{s}_B(t_{i-1}) \cdot t_{i-1}}{1 - \boldsymbol{\mathcal{R}}_B}\right) - \exp\left(-\frac{\mathcal{s}_B(t_i) \cdot t_i}{1 - \boldsymbol{\mathcal{R}}_B}\right)$$

Comparison with AM-CVA (2010 version of Basel III)

BCBS approximates the integral by the middle Riemann sum:

$$\mathrm{CVA} = \mathrm{LGD}_B \cdot \sum_{t_i \leq T} \left(\frac{\mathrm{EpE}\left(t_{i-1}\right) B_0\left(t_{i-1}\right) + B_0\left(t_i\right) \mathrm{EpE}\left(t_i\right)}{2} \right) \cdot \mathrm{PD}_B\left(t_{i-1}, t_i\right)$$

where:

- $\mathrm{LGD} = 1 - \mathcal{R}_B$ is the risk-neutral loss given default of the counterparty B
- $\mathrm{PD}_B\left(t_{i-1}, t_i\right)$ is the risk neutral probability of default between t_{i-1} and t_i:

$$\mathrm{PD}_B\left(t_{i-1}, t_i\right) = \max\left(\exp\left(-\frac{s\left(t_{i-1}\right)}{\mathrm{LGD}_B} \cdot t_{i-1}\right) - \exp\left(-\frac{s\left(t_i\right)}{\mathrm{LGD}_B} \cdot t_i\right), 0\right)$$

Basel III

2010 version of Basel III

- Standardized method (SM-CVA)
- Advanced method (AM-CVA)

2017 version of Basel III

- Basic approach (BA-CVA)
- Standardized approach (SA-CVA)

$\Rightarrow$ The Basel Committee completely flip-flopped within the same accord, since the 2017 version will replace the 2010 version in January 2022

Basic approach (BA-CVA)

The capital requirement is equal to:

$$\mathcal{K} = \beta \cdot \mathcal{K}^{\text{Reduced}} + (1 - \beta) \cdot \mathcal{K}^{\text{Hedged}}$$

where $\mathcal{K}^{\text{Reduced}}$ and $\mathcal{K}^{\text{Hedged}}$ are the capital requirements without and with hedging recognition

- The reduced version of the BA-CVA is obtained by setting β to 100%
- A bank that actively hedges CVA risks may choose the full version of the BA-CVA and $\beta = 25\%$

Reduced version

We have:

$$\mathcal{K}^{\mathrm{Reduced}} = \sqrt{\left(\rho \cdot \sum_j \mathrm{SCVA}_j\right)^2 + (1-\rho^2) \cdot \sum_j \mathrm{SCVA}_j^2}$$

where:

- $\rho = 50\%$
- SCVA_j is the CVA capital requirement for the j^{th} counterparty:

$$\mathrm{SCVA}_j = \frac{1}{\alpha} \cdot \mathrm{RW}_j \cdot \sum_k \mathrm{DF}_k \cdot \mathrm{EAD}_k \cdot \mathrm{M}_k$$

- $\alpha = 1.4$
- RW_j is the risk weight for counterparty j
- k is the netting set, DF_k is the discount factor, EAD_k is the CCR exposure at default, M_k is the effective maturity

Reduced version

RW_j depends on the credit quality of the counterparty (IG/HY) and its sector:

Table: Supervisory risk weights (BA-CVA)

Sector	Credit quality IG	Credit quality HY/NR
Sovereign	0.5%	3.0%
Local government	1.0%	4.0%
Financial	5.0%	12.0%
Basic material, energy, industrial, agriculture, manufacturing, mining and quarrying	3.0%	7.0%
Consumer goods and services, transportation and storage, administrative and support service activities	3.0%	8.5%
Technology, telecommunication	2.0%	5.5%
Health care, utilities, professional and technical activities	1.5%	5.0%
Other sector	5.0%	12.0%

Hedged version

The full version of the BA-CVA recognizes hedging instruments (single-name CDS and index CDS):

$$\mathcal{K}^{\text{Hedged}} = \sqrt{K_1 + K_2 + K_3}$$

where:

1. K_1 aggregates the systematic risk components of the CVA risk:

$$K_1 = \left(\rho \cdot \sum_j \left(\mathrm{SCVA}_j - \mathrm{SNH}_j \right) - \mathrm{IH} \right)^2$$

2. K_2 aggregates the idiosyncratic risk components of the CVA risk:

$$K_2 = \left(1 - \rho^2\right) \cdot \sum_j \left(\mathrm{SCVA}_j - \mathrm{SNH}_j \right)^2$$

3. K_3 corresponds to the hedging misalignment risk because of the mismatch between indirect and single-name hedges:

$$K_3 = \sum_j \mathrm{HMA}_j$$

Hedged version

Single-name hedging

SNH_j is the CVA reduction for counterparty j due to single-name hedging

$$\mathrm{SNH}_j = \sum_{h \in j} \varrho_{h,j} \cdot (\mathrm{RW}_h \cdot \mathrm{DF}_h \cdot N_h \cdot \mathrm{M}_h)$$

where:

- h represents the single-name CDS transaction, $\varrho_{h,j}$ is the supervisory correlation, DF_h is the discount factor, N_h is the notional and M_h is the remaining maturity
- These quantities are calculated at the single-name CDS level
- The correlation $\varrho_{h,j}$ between the credit spread of the counterparty and the credit spread of the CDS can take three values:
  1. 100% if CDS h directly refers to counterparty j
  2. 80% if CDS h has a legal relation with counterparty j
  3. 50% if CDS h and counterparty j are of the same sector and region

Hedged version

Index hedging

IH is the global CVA reduction due to index hedging:

$$\mathrm{IH} = \sum_{h'} \mathrm{RW}_{h'} \cdot \mathrm{DF}_{h'} \cdot N_{h'} \cdot \mathrm{M}_{h'}$$

where:

- h' represents the index CDS transaction
- The risk weight is the weighted average of risk weights of RW_j:

$$\mathrm{RW}_{h'} = 0.7 \cdot \sum_{j \in h'} w_j \cdot \mathrm{RW}_j$$

  where w_j is the weight of the counterparty/sector j in the index CDS h'

Hedged version

Hedging mismatch

The hedging misalignment risk is equal to:

$$\mathrm{HMA}_j = \sum_{h \in j} \left(1 - \varrho_{h,j}^2\right) \cdot \left(\mathrm{RW}_h \cdot \mathrm{DF}_h \cdot N_h \cdot \mathrm{M}_h\right)^2$$

Basic approach (BA-CVA)

Special cases

If there is no hedge, we have $\mathrm{SNH}_j = 0$, $\mathrm{HMA}_j = 0$, $\mathrm{IH} = 0$, and

$$\mathcal{K} = \mathcal{K}^{\mathrm{Reduced}}$$

If there is no hedging misalignment risk and no index CDS hedging, we have:

$$\mathcal{K} = \sqrt{\left(\rho \cdot \sum_j \mathcal{K}_j\right)^2 + (1 - \rho^2) \cdot \sum_j \mathcal{K}_j^2}$$

where $\mathcal{K}_j = \mathrm{SCVA}_j - \mathrm{SNH}_j$ is the single-name capital requirement for counterparty j

Exercise IV

Exercise

We assume that the bank has three financial counterparties A, B and C, that are respectively rated IG, IG and HY. There are 4 OTC transactions, whose characteristics are the following:

Transaction k	1	2	3	4
Counterparty	A	A	B	C
EAD_k	100	50	70	20
M_k	1	1	0.5	0.5

In order to reduce the counterparty credit risk, the bank has purchased a CDS protection on A for an amount of \$75 mn, a CDS protection on B for an amount of \$10 mn and a HY Financial CDX for an amount of \$10 mn. The maturity of hedges exactly matches the maturity of transactions. However, the CDS protection on B is indirect, because the underlying name is not B, but B' which is the parent company of B

Solution ($\mathcal{K}^{\text{Reduced}}$)

- We calculate the discount factors DF_k for the four transactions: $\text{DF}_1 = \text{DF}_2 = 0.9754$ and $\text{DF}_3 = \text{DF}_4 = 0.9876$
- We calculate the single-name capital for each counterparty:

$$\begin{aligned}
\text{SCVA}_A &= \frac{1}{\alpha} \times \text{RW}_A \times (\text{DF}_1 \times \text{EAD}_1 \times \text{M}_1 + \text{DF}_2 \times \text{EAD}_2 \times \text{M}_2) \\
&= \frac{1}{1.4} \times 5\% \times (0.9754 \times 100 \times 1 + 0.9754 \times 50 \times 1) \\
&= 5.225
\end{aligned}$$

We also find that $\text{SCVA}_B = 1.235$ and $\text{SCVA}_C = 0.847$

- It follows that $\sum_j \text{SCVA}_j = 7.306$ and $\sum_j \text{SCVA}_j^2 = 29.546$
- The capital requirement without hedging is equal to:

$$\mathcal{K}^{\text{Reduced}} = \sqrt{(0.5 \times 7.306)^2 + (1 - 0.5^2) \times 29.546} = 5.959$$

Solution ($\mathcal{K}^{\mathrm{Hedged}}$)

- We calculate the single-name hedge parameters:

$$\mathrm{SNH}_A = 5\% \times 100\% \times 0.9754 \times 75 \times 1 = 3.658$$

and:

$$\mathrm{SNH}_B = 5\% \times 80\% \times 0.9876 \times 10 \times 0.5 = 0.198$$

- Since the CDS protection is on B' and not B, there is a hedging misalignment risk:

$$\mathrm{HMA}_B = 0.05^2 \times \left(1 - 0.80^2\right) \times (0.9876 \times 10 \times 0.5)^2 = 0.022$$

- For the CDX protection, we have:

$$\mathrm{IH} = (0.7 \times 12\%) \times 0.9876 \times 10 \times 0.5 = 0.415$$

- We obtain $K_1 = 1.718$, $K_2 = 3.187$, $K_3 = 0.022$ and

$$\mathcal{K}^{\mathrm{Hedged}} = \sqrt{1.718^2 + 3.187^2 + 0.022^2} = 2.220$$

Solution (regulatory capital)

The capital requirement is equal to \$3.154 mn:

$$\mathcal{K} = 0.25 \times 5.959 + 0.75 \times 2.220 = 3.154$$

Standardized approach (SA-CVA)

Remark

SA-CVA ≈ SA-TB

$$\mathcal{K} = \mathcal{K}^{\mathrm{Delta}} + \mathcal{K}^{\mathrm{Vega}}$$

- Two portfolios:
  1. The CVA portfolio
  2. The hedging portfolio
- For each risk (delta and vega), we calculate the weighted CVA sensitivity of each risk factor $\mathcal{F}_j$:

$$\mathrm{WS}_j^{\mathrm{CVA}} = S_j^{\mathrm{CVA}} \cdot \mathrm{RW}_j$$

and:

$$\mathrm{WS}_j^{\mathrm{Hedge}} = S_j^{\mathrm{Hedge}} \cdot \mathrm{RW}_j$$

where S_j and RW_j are the net sensitivity of the CVA or hedging portfolio with respect to the risk factor and the risk weight of $\mathcal{F}_j$

Standardized approach (SA-CVA)

- We aggregate the weighted sensitivity in order to obtain a net figure:

$$\mathrm{WS}_j = \mathrm{WS}_j^{\mathrm{CVA}} + \mathrm{WS}_j^{\mathrm{Hedge}}$$

- We calculate the capital requirement for the risk bucket $\mathcal{B}_k$:

$$\mathcal{K}_{\mathcal{B}_k} = \sqrt{\sum_j \mathrm{WS}_j^2 + \sum_{j' \neq j} \rho_{j,j'} \cdot \mathrm{WS}_j \cdot \mathrm{WS}_{j'} + 1\% \cdot \sum_j \left(\mathrm{WS}_j^{\mathrm{Hedge}}\right)^2}$$

  where $\mathcal{F}_j \in \mathcal{B}_k$

- We aggregate the different buckets for a given risk class:

$$\mathcal{K}^{\mathrm{Delta/Vega}} = m_{\mathrm{CVA}} \cdot \sqrt{\sum_k \mathcal{K}_{\mathcal{B}_k}^2 + \sum_{k' \neq k} \gamma_{k,k'} \cdot \mathcal{K}_{\mathcal{B}_k} \cdot \mathcal{K}_{\mathcal{B}_{k'}}}$$

  where $m_{\mathrm{CVA}} = 1.25$ is the multiplier factor

CVA and wrong/right way risk

- CVA trading desk
- How to be sure that the CVA hedging portfolio does not create itself another source of hidden wrong way risk?
- In practice, **market and credit risks are correlated!**
- Two approaches
  1. The copula model (Cespedes *et al.*, 2010)
  2. The hazard rate model (Hull and White, 2012)

Exposure at default

- In the case of a margin agreement, the counterparty needs to post collateral and the exposure at default becomes:

$$e^{+}(t) = \max\left(\mathrm{MtM}(t) - C(t), 0\right)$$

  where $C(t)$ is the collateral value at time t

- The collateral transfer occurs when the mark-to-market exceeds a threshold H:

$$C(t) = \max\left(\mathrm{MtM}(t - \delta_C) - H, 0\right)$$

  where:
  - H is the minimum collateral transfer amount
  - $\delta_C \geq 0$ is the margin period of risk (MPOR)

- We obtain:

$$\begin{aligned} e^{+}(t) &= \mathrm{MtM}(t) \cdot \mathbb{1}\left\{0 \leq \mathrm{MtM}(t), \mathrm{MtM}(t - \delta_C) < H\right\} + \\ & \quad \left(\mathrm{MtM}(t) - \mathrm{MtM}(t - \delta_C) + H\right) \cdot \\ & \quad \mathbb{1}\left\{H \leq \mathrm{MtM}(t - \delta_C) \leq \mathrm{MtM}(t) + H\right\} \end{aligned}$$

Special cases

- When $H = +\infty$, $C(t)$ is equal to zero and we obtain:

$$e^{+}(t) = \max\left(\mathrm{MtM}(t), 0\right)$$

- When $H = 0$, the collateral $C(t)$ is equal to $\mathrm{MtM}(t - \delta_C)$ and the counterparty exposure becomes:

$$e^{+}(t) = \max\left(\mathrm{MtM}(t) - \mathrm{MtM}(t - \delta_C), 0\right) = \max\left(\mathrm{MtM}(t - \delta_C, t), 0\right)$$

  The CCR corresponds to the variation of the mark-to-market $\mathrm{MtM}(t - \delta_C, t)$ during the liquidation period $[t - \delta_C, t]$

- When δ_C is set to zero, we deduce that:

$$e^{+}(t) = \mathrm{MtM}(t) \cdot \mathbb{1}\left\{0 \leq \mathrm{MtM}(t) < H\right\} + H \cdot \mathbb{1}\left\{H \leq \mathrm{MtM}(t)\right\}$$

- When δ_C is set to zero and there is no minimum collateral transfer amount, the counterparty credit risk vanishes:

$$e^{+}(t) = 0$$

Illustration

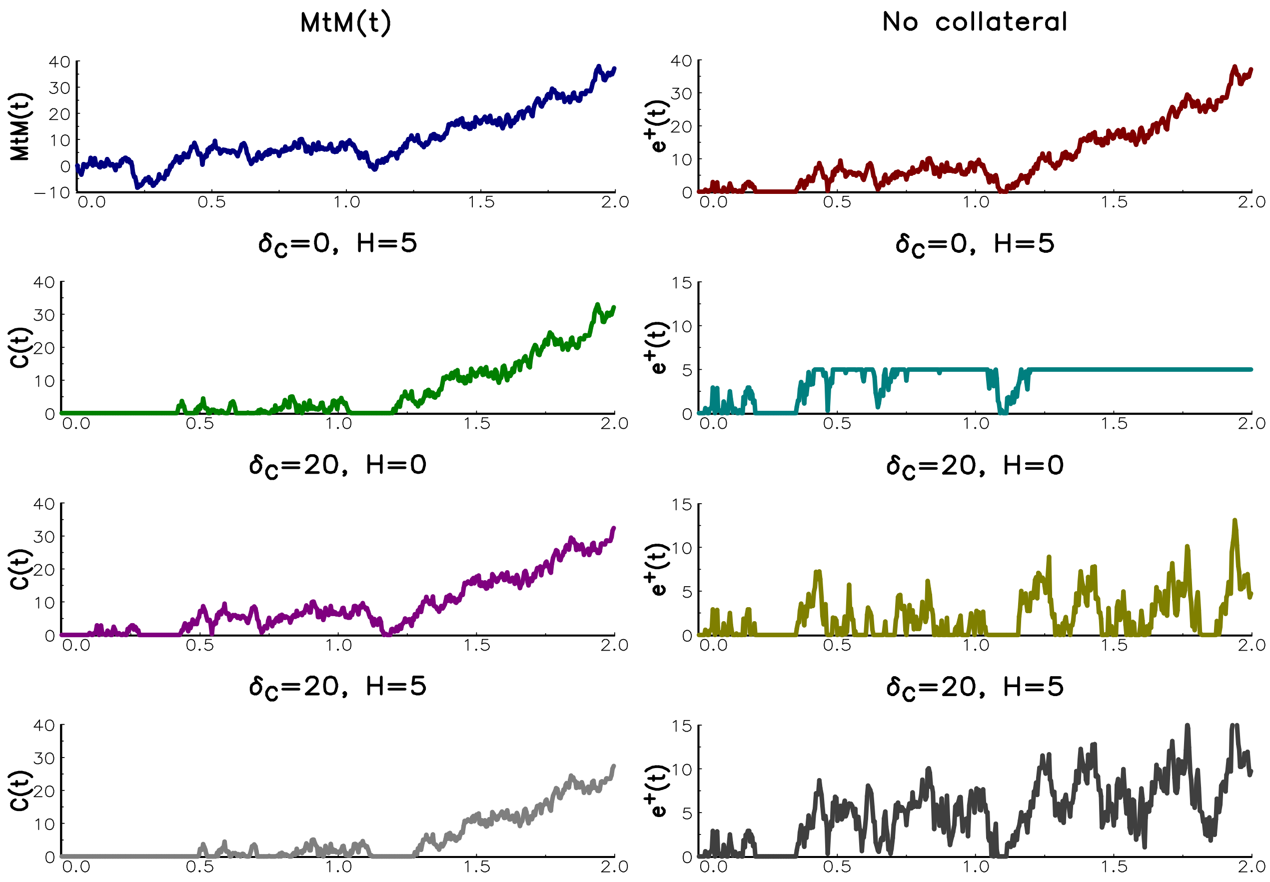


Figure: Impact of collateral on the counterparty exposure

Collateral risk management

Two ways to reduce the counterparty risk:

1. Reducing the haircut ($H \searrow 0$)
2. Reducing the margin period of risk ($\delta_C \searrow 0$)

Trade-off between risk and operational cost & process

Risk allocation

We recall the Euler allocation principle:

$$\mathcal{R}(w) = \sum_{i=1}^{n} \mathcal{RC}_i = \sum_{i=1}^{n} w_i \cdot \frac{\partial \, \mathcal{R}(w)}{\partial \, w_i}$$

Risk allocation

Application to a CVA portfolio

$$\mathrm{CVA}(w) = (1 - \mathcal{R}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}(t; w)\,\mathrm{d}\mathbf{S}_B(t)$$

where $\mathrm{EpE}(t; w)$ is the expected positive exposure with respect to the portfolio w. The Euler allocation principle becomes:

$$\mathrm{CVA}(w) = \sum_{i=1}^{n} \mathrm{CVA}_i(w)$$

where $\mathrm{CVA}_i(w)$ is the CVA risk contribution of the i^{th} component:

$$\mathrm{CVA}_i(w) = (1 - \mathcal{R}_B) \cdot \int_0^T -B_0(t)\,\mathrm{EpE}_i(t; w)\,\mathrm{d}\mathbf{S}_B(t)$$

and $\mathrm{EpE}_i(t; w)$ is the EpE risk contribution of the i^{th} component

Risk allocation

What is the challenge?

Computing the EpE risk contribution:

$$\mathrm{EpE}_i\left(t;w\right) = w_i \cdot \frac{\partial\ \mathrm{EpE}\left(t;w\right)}{\partial\ w_i}$$

Very difficult and almost impossible $\Rightarrow$ needs simplification

Exercises

- Counterparty credit risk (CCR)
  - Exercise 4.4.1 – Impact of netting agreements in counterparty credit risk
  - Exercise 4.4.2 – Calculation of the effective expected positive exposure
  - Exercise 4.4.3 – Calculation of the capital charge for counterparty credit risk
- Credit valuation adjustment (CVA)
  - Exercise 4.4.4 – Calculation of CVA and DVA measures
  - Exercise 4.4.5 – Approximation of the CVA for an interest rate swap

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