Course 2023-2024 in Financial Risk Management Lecture 11. Stress Testing and Scenario Analysis

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¹The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

Overview

The objective of this course is to understand the theoretical and practical aspects of risk management

Prerequisites

M1 Finance or equivalent

ECTS

4

Get Keywords

Finance, Risk Management, Applied Mathematics, Statistics

6 Hours

Lectures: 36h, Training sessions: 15h, HomeWork: 30h

Evaluation

There will be a final three-hour exam, which is made up of questions and exercises

Course website

http://www.thierry-roncalli.com/RiskManagement.html

Objective of the course

The objective of the course is twofold:

- In the international standards (especially the Basel Accords)
- eing proficient in risk measurement, including the mathematical tools and risk models

Class schedule

Course sessions

- September 15 (6 hours, AM+PM)
- September 22 (6 hours, AM+PM)
- September 19 (6 hours, AM+PM)
- October 6 (6 hours, AM+PM)
- October 13 (6 hours, AM+PM)
- October 27 (6 hours, AM+PM)

Tutorial sessions

- October 20 (3 hours, AM)
- October 20 (3 hours, PM)
- November 10 (3 hours, AM)
- November 10 (3 hours, PM)
- November 17 (3 hours, PM)

Class times: Fridays 9:00am-12:00pm, 1:00pm-4:00pm, University of Evry, Room 209 IDF

Agenda

- Lecture 1: Introduction to Financial Risk Management
- Lecture 2: Market Risk
- Lecture 3: Credit Risk
- Lecture 4: Counterparty Credit Risk and Collateral Risk
- Lecture 5: Operational Risk
- Lecture 6: Liquidity Risk
- Lecture 7: Asset Liability Management Risk
- Lecture 8: Model Risk
- Lecture 9: Copulas and Extreme Value Theory
- Lecture 10: Monte Carlo Simulation Methods
- Lecture 11: Stress Testing and Scenario Analysis
- Lecture 12: Credit Scoring Models

Agenda

- Tutorial Session 1: Market Risk
- Tutorial Session 2: Credit Risk
- Tutorial Session 3: Counterparty Credit Risk and Collateral Risk
- Tutorial Session 4: Operational Risk & Asset Liability Management Risk
- Tutorial Session 5: Copulas, EVT & Stress Testing



Textbook

 Roncalli, T. (2020), Handbook of Financial Risk Management, Chapman & Hall/CRC Financial Mathematics Series.



Additional materials

 Slides, tutorial exercises and past exams can be downloaded at the following address:

http://www.thierry-roncalli.com/RiskManagement.html

 Solutions of exercises can be found in the companion book, which can be downloaded at the following address:

http://www.thierry-roncalli.com/RiskManagementBook.html

Agenda

- Lecture 1: Introduction to Financial Risk Management
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- Lecture 6: Liquidity Risk
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- Lecture 10: Monte Carlo Simulation Methods
- Lecture 11: Stress Testing and Scenario Analysis
- Lecture 12: Credit Scoring Models

"Stress testing is now a critical element of risk management for banks and a core tool for banking supervisors and macroprudential authorities" (BCBS, 2017, page 5).

Definition Methodologies

General objective

If we consider a trading book portfolio, we recall that:

$$L_{s}(w) = P_{t}(w) - g(\mathcal{F}_{1,s}, \ldots, \mathcal{F}_{m,s}; w)$$

In the case of a stress testing program, we have:

$$L_{\text{stress}}(w) = P_t(w) - g(\mathcal{F}_{1,\text{stress}},\ldots,\mathcal{F}_{m,\text{stress}};w)$$

where $(\mathcal{F}_{1,\text{stress}},\ldots,\mathcal{F}_{m,\text{stress}})$ is the stress scenario

Definition Methodologies

Scenario design and risk factors

2004 FSAP stress scenarios applied to the French banking system

- F_1 flattening of the yield curve due to an increase in interest rates: increase of 150 basis points (bp) in overnight rates, increase of 50 bp in 10-year rates, with interpolation for intermediate maturities
- F_5 share price decline of 30% in all stock markets
- F_9 flattening of the yield curve (increase of 150 basis points in overnight rates, increase of 50 bp in 10-year rates) together with a 30% drop in stock markets
- M_2 increase to USD 40 in the price per barrel of Brent crude for two years (an increase of 48% compared with USD 27 per barrel in the baseline case), without any reaction from the central bank; the increase in the price of oil leads to an increase in the general rate of inflation and a decline in economic activity in France together with a drop in global demand

Definition

Scenario design and risk factors

Classification

- historical scenario: "a stress test scenario that aims at replicating the changes in risk factor shocks that took place in an actual past episode"
- In the provide the second s hypothetical set of risk factor changes, which does not aim to replicate a historical episode of distress"

Macroeconomic scenario: "a stress test that implements a link between stressed macroeconomic factors [...] and the financial sustainability of either a single financial institution or the entire financial system"

a liquidity scenario: *"a liquidity stress test is the process of assessing"* the impact of an adverse scenario on institution's cash flows as well as on the availability of funding sources, and on market prices of liquid assets"

Definition Methodologies

Scenario design and risk factors



Figure: 2017 DFAST supervisory scenarios: Domestic variables

Definition Methodologies

Scenario design and risk factors



Figure: 2017 DFAST supervisory scenarios: International variables

Definition Methodologies

Firm-specific versus supervisory stress testing

Examples of hard trading limits:

- Unobservable parameters (e.g. correlations of basket options)
- Less liquid assets

Examples of supervisory stress testing:

- Financial sector assessment program (FSAP)
- Dodd-Frank Act stress test (DFAST)
- EU-wide stress testing

Historical approach

Table: Worst historical scenarios of the S&P 500 index

Sc.	1D		1W		1M	
1	1987-10-19	-20.47	1987-10-19	-27.33	2008-10-27	-30.02
2	2008-10-15	-9.03	2008-10-09	-18.34	1987-10-26	-28.89
3	2008-12-01	-8.93	2008-11-20	-17.43	2009-03-09	-22.11
4	2008-09-29	-8.79	2008-10-27	-13.85	2002-07-23	-19.65
5	1987-10-26	-8.28	2011-08-08	-13.01	2001-09-21	-16.89
Sc.	2M		3M		6M	
1	2008-11-20	-37.66	2008-11-20	-41.11	2009-03-09	-46.64
2	1987-10-26	-31.95	1987-11-30	-30.17	1974-09-13	-34.33
3	2002-07-23	-27.29	1974-09-13	-28.59	2002-10-09	-31.29
4	2009-03-06	-26.89	2002-07-23	-27.55	1962-06-27	-26.59
5	1962-06-22	-23.05	2009-03-09	-25.63	1970-05-26	-25.45

Definition Methodologies

Macro-economic approach



Figure: Macroeconomic approach of stress testing

Definition Methodologies

Macro-economic approach



Figure: Feedback effects in stress testing models

Definition Methodologies

Probabilistic approach

At first approximation, a stress scenario can be seen as an extreme quantile or value-at-risk \Rightarrow we can use EVT (extreme value theory)

Univariate stress scenarios Joint stress scenarios Conditional stress scenarios Reverse stress testing

Univariate stress scenarios

Let X be the random variable that produces the stress scenario S(X).
 If X ~ F and the relationship between L(w) and X is decreasing, we have:

$$\Pr\left\{X \leq \mathbb{S}\left(X\right)\right\} = \mathbf{F}\left(\mathbb{S}\left(X\right)\right)$$

• Given a stress scenario $\mathbb{S}(X)$, we deduce its severity:

$$\alpha = \mathbf{F}(\mathbb{S}(X))$$

• We can also compute the stressed value given the probability of occurrence α :

$$\mathbb{S}(X) = \mathbf{F}^{-1}(\alpha)$$

 $\alpha \approx 0 \ (\neq value-at-risk)$

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Univariate stress scenarios

Return time

- We have $\mathcal{T} = \alpha^{-1}$ and $\alpha = \mathcal{T}^{-1}$
- We reiterate that:

$$\mathcal{T} = \alpha^{-1} = n \cdot \left(1 - \alpha_{\rm GEV}\right)^{-1}$$

where n is the length of the block maxima

Table: Probability (in %) associated to the return period \mathcal{T} in years

Return period	1	5	10	20	30	50
Daily	0.3846	0.0769	0.0385	0.0192	0.0128	0.0077
Weekly	1.9231	0.3846	0.1923	0.0962	0.0641	0.0385
Monthly	8.3333	1.6667	0.8333	0.4167	0.2778	0.1667
$1 - lpha_{ m GEV}$	7.6923	1.5385	0.7692	0.3846	0.2564	0.1538

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Univariate stress scenarios

Table: GEV parameter estimates (in %) of MSCI USA and MSCI EMU indices

Daramatar	Long p	osition	Short position		
Farameter	MSCI USA	MSCI EMU	MSCI USA	MSCI EMU	
μ	1.242	1.572	1.317	1.599	
σ	0.720	0.844	0.577	0.730	
ξ	19.363	21.603	26.341	26.494	

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Univariate stress scenarios

Table: Stress scenarios (in %) of MSCI USA and MSCI EMU indices

Voor	Long p	osition	Short position		
rear	MSCI USA	MSCI EMU	MSCI USA	MSCI EMU	
5	-5.86	-7.27	5.69	7.16	
10	-7.06	-8.83	7.01	8.84	
25	-8.92	-11.29	9.17	11.60	
50	-10.56	-13.49	11.18	14.17	
75	-11.62	-14.94	12.54	15.91	
100	-12.43	-16.05	13.59	17.26	
Extreme statistic	-9.51	-10.94	11.04	10.87	
\mathcal{T}^{\star}	32.49	22.24	47.87	20.03	

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Univariate stress scenarios



Figure: Stress scenarios (in %) of MSCI USA and MSCI EMU indices

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Bivariate stress scenarios

- We note $p = \Pr \{X_{n:n,1} > \mathbb{S}(X_1), X_{n:n,2} > \mathbb{S}(X_2)\}$ the joint probability of stress scenarios $(\mathbb{S}(X_1), \mathbb{S}(X_2))$
- We have:

$$p = 1 - \mathbf{F}_1(\mathbb{S}(X_1)) - \mathbf{F}_2(\mathbb{S}(X_2)) + \mathbf{C}(\mathbf{F}_1(\mathbb{S}(X_1)), \mathbf{F}_2(\mathbb{S}(X_2)))$$

= $\mathbf{\overline{C}}(\mathbf{F}_1(\mathbb{S}(X_1)), \mathbf{F}_2(\mathbb{S}(X_2)))$

where $\mathbf{\bar{C}}(u_1, u_2) = 1 - u_1 - u_2 + \mathbf{C}(u_1, u_2)$

• We deduce that the failure area is represented by:

$$\left\{ \left(\mathbb{S}\left(X_{1}
ight),\mathbb{S}\left(X_{2}
ight)
ight)\in\mathbb{R}_{+}^{2}\midar{\mathbf{C}}\left(\mathbf{F}_{1}\left(\mathbb{S}\left(X_{1}
ight)
ight),\mathbf{F}_{2}\left(\mathbb{S}\left(X_{2}
ight)
ight)
ight)\leqrac{n}{\mathcal{T}}
ight\}$$

• We have:

$$\mathcal{T} = \frac{n}{\mathbf{\bar{C}}\left(\mathbf{F}_{1}\left(\mathbb{S}\left(X_{1}\right)\right), \mathbf{F}_{2}\left(\mathbb{S}\left(X_{2}\right)\right)\right)}$$

and:

$$\mathsf{max}\left(\mathcal{T}_1,\mathcal{T}_2\right) \leq \mathcal{T} \leq \mathit{n}\mathcal{T}_1\mathcal{T}_2$$

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Bivariate stress scenarios



Figure: Failure area of MSCI USA and MSCI EMU indices (blockwise dependence)

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Bivariate stress scenarios



Figure: Failure area of MSCI USA and MSCI EMU indices (daily dependence)

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Multivariate stress scenarios

$\Rightarrow \bar{\mathbf{C}}$ has a compliacted expression (see HFRM, Section 14.2.2.2, page 908)

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The conditional expectation solution

Given a joint stress scenario $\mathbb{S}(X) = (\mathbb{S}(X_1), \dots, \mathbb{S}(X_n))$, the conditional stress scenario of Y is:

$$\mathbb{S}(Y) = \mathbb{E}[Y_t \mid X_t = (\mathbb{S}(X_1), \dots, \mathbb{S}(X_n))]$$
$$= \beta_0 + \sum_{i=1}^n \beta_i \mathbb{S}(X_i)$$

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The conditional expectation solution

Logit transformation

• We use the following transformation:

$$Z_t = \ln\left(\frac{Y_t}{1 - Y_t}\right)$$

• We have:

$$Y_t = rac{\exp(Z_t)}{1 + \exp(Z_t)} = rac{1}{1 + \exp(-Z_t)} = h(Z_t)$$

where h(z) is the logit transformation

• We deduce that:

$$\mathbb{E}\left[Y_t \mid X_t = (x_1, \dots, x_n)\right] = \int_{-\infty}^{\infty} h\left(\beta_0 + \sum_{i=1}^n \beta_i X_{i,t} + \omega\right) \frac{1}{\sigma} \phi\left(\frac{\omega}{\sigma}\right) \, \mathrm{d}\omega$$

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The conditional expectation solution

Example

• We assume that the probability of default PD_t at time t is explained by the following linear regression model:

$$\ln\left(\frac{\mathrm{PD}_t}{1-\mathrm{PD}_t}\right) = -2.5 - 5g_t - 3\pi_t + 2u_t + \varepsilon_t$$

where $\varepsilon_t \sim \mathcal{N}(0, 0.25)$, g_t is the growth rate of the GDP, π_t is the inflation rate, and u_t is the unemployment rate

- The baseline scenario is defined by $g_t = 2\%$, $\pi_t = 2\%$ and $u_t = 5\%$
- The stress scenario is equal to $g_t = -8\%$, $\pi_t = 5\%$ and $u_t = 10\%$

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The conditional expectation solution



Figure: Probability density function of PD_t

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The conditional expectation solution

 \Rightarrow The conditional expectation is equal to 7.90% for the baseline scenario and 12.36% for the stress scenario

 \Rightarrow The figure of 7.90% can be interpreted as the long-run (or unconditional) probability of default that is used in the IRB formula (i.e. Pillar I)

 \Rightarrow The figure of 12.36% may be used in Pillar II

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The conditional expectation solution



Figure: Relationship between the macroeconomic variables and PD_t

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The conditional expectation solution

Table: Stress scenario of the probability of default

t	gt	π_t	Ut	$\mathbb{E}\left[\mathrm{PD}_t \mid \mathbb{S}\left(X\right)\right]$	$q_{90\%}\left(\mathbb{S}\left(X ight) ight)$
0	2.00	2.00	5.00	7.90	12.78
1	-6.00	2.00	6.00	11.45	18.26
2	-7.00	1.00	7.00	12.47	19.79
3	-9.00	1.00	9.00	14.03	22.14
4	-7.00	1.00	10.00	13.12	20.78
5	-7.00	2.00	11.00	13.01	20.59
6	-6.00	2.00	10.00	12.26	19.49
7	-4.00	4.00	9.00	10.49	16.80
8	-2.00	3.00	8.00	9.70	15.58
9	-1.00	3.00	7.00	9.11	14.68
10	2.00	3.00	6.00	7.82	12.68
11	4.00	3.00	6.00	7.14	11.60
12	4.00	3.00	6.00	7.14	11.60

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The conditional quantile solution

We could also define the conditional stress scenario $\mathbb{S}(Y) = q_{\alpha}(\mathbb{S}(X))$ as the solution of the quantile regression:

$$\Pr\left\{Y_{t} \leq q_{\alpha}\left(\mathbb{S}\right) \mid X_{t} = \mathbb{S}\right\} = \alpha$$

The solution is given by:

$$\begin{split} \mathbb{S}(Y) &= q_{\alpha}(\mathbb{S}) \\ &= \mathbf{F}_{y}^{-1}\left(\mathbf{C}_{2|1}^{-1}\left(\mathbf{F}_{x}\left(\mathbb{S}(X)\right),\alpha\right)\right) \end{split}$$

 \Rightarrow See HFRM, Section 14.2.3.2, pages 912-915

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Reverse stress testing

Reverse stress test "means an institution stress test that starts from the identification of the pre-defined outcome (e.g. points at which an institution business model becomes unviable, or at which the institution can be considered as failing or likely to fail) and then explores scenarios and circumstances that might cause this to occur"

 In stress testing, extreme scenarios of risk factors are used to test the viability of the bank:

$$(\mathbb{S}(\mathcal{F}_1),\ldots,\mathbb{S}(\mathcal{F}_m)) \Rightarrow \mathbb{S}(L(w)) \Rightarrow \begin{cases} D=0 & \text{if } \mathbb{S}(L(w)) < C \\ D=1 & \text{otherwise} \end{cases}$$

 In reverse stress testing, extreme scenarios of risk factors are deduced from the bankruptcy scenario:

$$D = 1 \Rightarrow \mathbb{RS} \left(L(w) \right) \Rightarrow \left(\mathbb{RS} \left(\mathcal{F}_1 \right), \dots, \mathbb{RS} \left(\mathcal{F}_m \right) \right)$$

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Reverse stress testing

We recall that:

$$L(w) = \ell(\mathcal{F}_1, \ldots, \mathcal{F}_m; w)$$

The reverse stress scenario \mathbb{RS} is the set of risk factors that corresponds to the stressed loss $\mathbb{RS}(L(w))$:

$$\mathbb{RS} = \{ (\mathbb{RS}(\mathcal{F}_1), \dots, \mathbb{RS}(\mathcal{F}_m)) : \ell (\mathbb{S}(\mathcal{F}_1), \dots, \mathbb{S}(\mathcal{F}_m); w) = \mathbb{RS}(L(w)) \}$$

 \Rightarrow Not a unique solution

Mathematical solution

We can use the following optimization program

$$\begin{aligned} (\mathbb{RS}(\mathcal{F}_1), \dots, \mathbb{RS}(\mathcal{F}_m)) &= \arg \max \ln f \left(\mathcal{F}_1, \dots, \mathcal{F}_m \right) \\ \text{s.t.} \quad \ell \left(\mathbb{S}(\mathcal{F}_1), \dots, \mathbb{S}(\mathcal{F}_m); w \right) = \mathbb{RS}\left(L(w) \right) \end{aligned}$$

where $f(x_1, \ldots, x_m)$ is the probability density function of the risk factors $(\mathcal{F}_1, \ldots, \mathcal{F}_m)$

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Reverse stress testing

We assume that $\mathcal{F} \sim \mathcal{N}(\mu_{\mathcal{F}}, \Sigma_{\mathcal{F}})$ and $L(w) = \sum_{j=1}^{m} w_j \mathcal{F}_j = w^{\top} \mathcal{F}$. The optimization problem becomes:

$$\mathbb{RS}(\mathcal{F}) = \arg\min\frac{1}{2}\left(\mathcal{F} - \mu_{\mathcal{F}}\right)^{\top} \Sigma_{\mathcal{F}}^{-1} \left(\mathcal{F} - \mu_{\mathcal{F}}\right)$$

s.t. $w^{\top} \mathcal{F} = \mathbb{RS}\left(L(w)\right)$

The Lagrange function is:

$$\mathcal{L}(\mathcal{F};\lambda) = \frac{1}{2} \left(\mathcal{F} - \mu_{\mathcal{F}} \right)^{\top} \Sigma_{\mathcal{F}}^{-1} \left(\mathcal{F} - \mu_{\mathcal{F}} \right) - \lambda \left(w^{\top} \mathcal{F} - \mathbb{RS} \left(L(w) \right) \right)$$

The first-order condition is $\Sigma_{\mathcal{F}}^{-1} (\mathcal{F} - \mu_{\mathcal{F}}) - \lambda w = \mathbf{0}$. It follows that $\mathcal{F} = \mu_{\mathcal{F}} + \lambda \Sigma_{\mathcal{F}} w$, $w^{\top} \mathcal{F} = w^{\top} \mu_{\mathcal{F}} + \lambda w^{\top} \Sigma_{\mathcal{F}} w$, $\lambda = (\mathbb{RS} (L(w)) - w^{\top} \mu_{\mathcal{F}}) / w^{\top} \Sigma_{\mathcal{F}} w$ and:

$$\mathbb{RS}(\mathcal{F}) = \mu_{\mathcal{F}} + \frac{\Sigma_{\mathcal{F}} w}{w^{\top} \Sigma_{\mathcal{F}} w} \left(\mathbb{RS}(L(w)) - w^{\top} \mu_{\mathcal{F}} \right)$$

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Reverse stress testing

Another approach for solving the inverse problem is to consider the joint distribution of \mathcal{F} and L(w):

$$\begin{pmatrix} \mathcal{F} \\ L(w) \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_{\mathcal{F}} \\ w^{\top} \mu_{\mathcal{F}} \end{pmatrix}, \begin{pmatrix} \Sigma_{\mathcal{F}} & \Sigma_{\mathcal{F}} w \\ w^{\top} \Sigma_{\mathcal{F}} & w^{\top} \Sigma_{\mathcal{F}} w \end{pmatrix} \right)$$

The conditional distribution of \mathcal{F} given $L(w) = \mathbb{RS}(L(w))$ is Gaussian:

$$\mathcal{F} \mid L(w) = \mathbb{RS}(L(w)) \sim \mathcal{N}(\mu_{\mathcal{F}|L(w)}, \Sigma_{\mathcal{F}|L(w)})$$

We know that the maximum of the probability density function of the multivariate normal distribution is reached when the random vector is exactly equal to the mean. We deduce that:

$$\mathbb{RS}\left(\mathcal{F}\right) = \mu_{\mathcal{F}|L(w)} = \mu_{\mathcal{F}} + \frac{\Sigma_{\mathcal{F}}w}{w^{\top}\Sigma_{\mathcal{F}}w} \left(\mathbb{RS}\left(L\left(w\right)\right) - w^{\top}\mu_{\mathcal{F}}\right)$$

Univariate stress scenarios Joint stress scenarios Conditional stress scenarios Reverse stress testing

Reverse stress testing

Example

We assume that $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$, $\mu_{\mathcal{F}} = (5, 8)$, $\sigma_{\mathcal{F}} = (1.5, 3.0)$ and $\rho(\mathcal{F}_1, \mathcal{F}_2) = -50\%$. The sensitivity vector w to the risk factors is equal to (10, 3)

The stress scenario is the collection of univariate stress scenarios at the 99% confidence level:

$$\begin{array}{rcl} \mathbb{S}\left(\mathcal{F}_{1}\right) &=& 5+1.5\cdot\Phi^{-1}\left(99\%\right)=8.49\\ \mathbb{S}\left(\mathcal{F}_{2}\right) &=& 8+3.0\cdot\Phi^{-1}\left(99\%\right)=14.98 \end{array}$$

The stressed loss is then equal to:

$$S(L(w)) = 10 \cdot 8.49 + 3 \cdot 14.98 = 129.53$$

Univariate stress scenarios Joint stress scenarios Conditional stress scenarios Reverse stress testing

Reverse stress testing

We assume that the reverse stressed loss is equal to $129.53 \Rightarrow$ we deduce that $\mathbb{RS}(\mathcal{F}_1) = 10.14$ and $\mathbb{RS}(\mathcal{F}_2) = 9.47$

Remark

The reverse stress scenario is very different than the stress scenario even if they give the same loss. In fact, we have $f(\mathbb{S}(\mathcal{F}_1), \mathbb{S}(\mathcal{F}_2)) = 0.8135 \cdot 10^{-6}$ and $f(\mathbb{RS}(\mathcal{F}_1), \mathbb{RS}(\mathcal{F}_2)) = 4.4935 \cdot 10^{-6}$, meaning that the occurrence probability of the reverse stress scenario is more than five times higher than the occurrence probability of the stress scenario

Univariate stress scenarios Joint stress scenarios Conditional stress scenarios Reverse stress testing

Reverse stress testing

In the general case, we consider the following optimization problem:

$$\begin{aligned} & (\mathbb{RS}\left(\mathcal{F}_{1}\right),\ldots,\mathbb{RS}\left(\mathcal{F}_{m}\right)) &= \arg\max\ln f\left(\mathcal{F}_{1},\ldots,\mathcal{F}_{m}\right) \\ & \text{s.t.} \quad \ell\left(\mathbb{S}\left(\mathcal{F}_{1}\right),\ldots,\mathbb{S}\left(\mathcal{F}_{m}\right);w\right) \geq \mathbb{RS}\left(L\left(w\right)\right) \end{aligned}$$

and we use the Monte Carlo simulation method to estimate the reverse stress scenario

Hard to implement in practice!

Exercises

• Exercise 14.3.1 – Construction of a stress scenario with the GEV distribution

References



Basel Committee on Banking Supervision (2017)

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