Analysing
Sovereign Risk
for Portfolio
Management
Decisions

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Managing Sovereign Risk Through a Risk-Budgeting Approach

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Outline

Some issues on the asset management industry
(after the 2008 financial crisis)

Transition in the investment industry

- Concentration of assets under management
- Risk aversion of large institutional investors becomes higher
- Funding ratios are smaller (weakness of retirement systems)
- Pressure for more transparency and robustness
- Risk management as important as performance management

Transition in the passive indexation

- Robustness of market-cap indexation?
- Equity indexes = trend-following strategy / Issue in bond indexing
- Lack of portfolio construction rules ⇒ Risk concentration
- Alternative-weighted indexation = passive indexes where the weights are not based on market capitalization

Some issues on the asset management industry
(after the 2008 financial crisis)

Emergence of heuristic solutions

- Strong criticism of Markowitz optimization
- 2011: success of minimum-variance, erc, mdp, msr and risk parity strategies $\Rightarrow$ These portfolio constructions only depend on risks, not on expected returns
- Special cases of the risk budgeting approach

$\Rightarrow$ Risk budgeting allocation = widely used by market practitioners (multi-asset classes, strategic asset allocation, equity portfolios)

Objective of this talk

- What could we say about risk budgeting allocation from a theoretical point of view?
- How to use the risk budgeting approach to build alternative-weighted indexes for bonds?

The risk budgeting approach

- Definition
- Main properties
- Some popular RB portfolios
- RB portfolios vs optimized portfolios

Three methods to build a portfolio

1. Weight budgeting (WB)
2. Risk budgeting (RB)
3. Performance budgeting (PB)

Ex-ante analysis

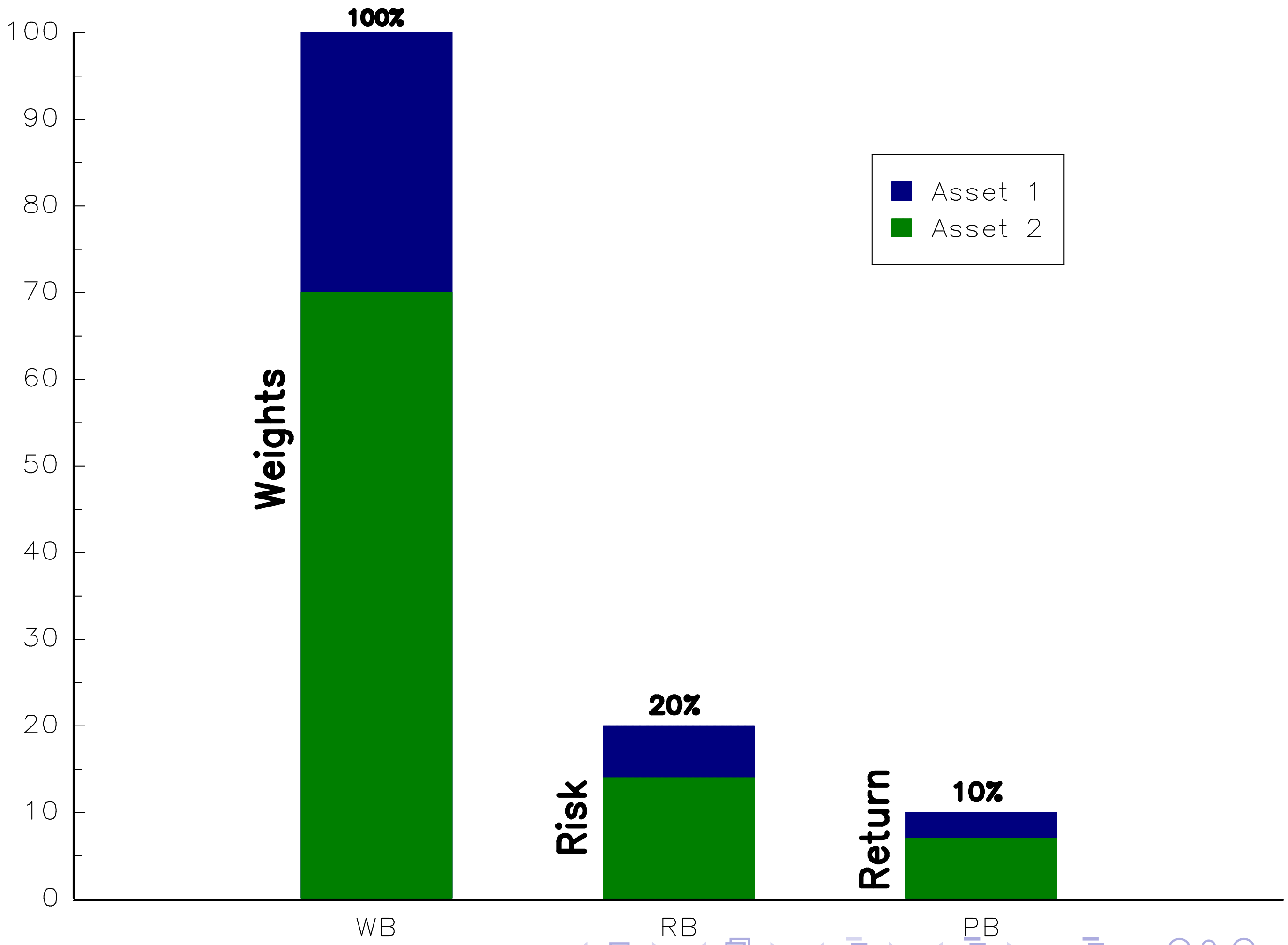
$\neq$

Ex-post analysis

Important result

RB = PB

Figure: The 30/70 rule



Weight budgeting versus risk budgeting

Let $x = (x_1, \ldots, x_n)$ be the weights of n assets in the portfolio. Let $\mathscr{R}(x_1, \ldots, x_n)$ be a coherent and convex risk measure. We have:

$$\begin{aligned} \mathscr{R}(x_1, \ldots, x_n) &= \sum_{i=1}^{n} x_i \cdot \frac{\partial \mathscr{R}(x_1, \ldots, x_n)}{\partial x_i} \\ &= \sum_{i=1}^{n} \mathrm{RC}_i(x_1, \ldots, x_n) \end{aligned}$$

Let $b = (b_1, \ldots, b_n)$ be a vector of budgets such that $b_i \geq 0$ and $\sum_{i=1}^{n} b_i = 1$. We consider two allocation schemes:

1. Weight budgeting (WB)

$$x_i = b_i$$

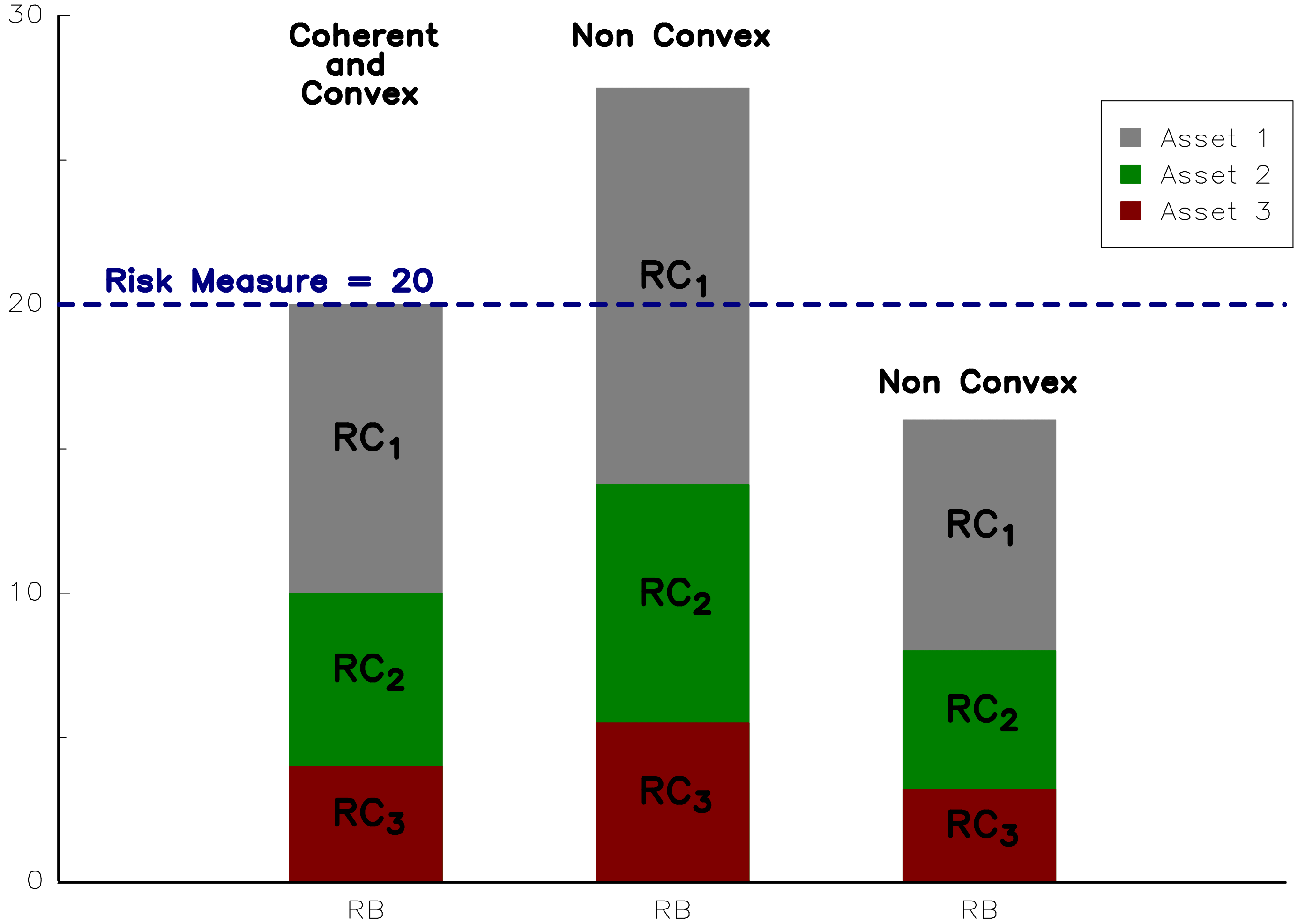
2. Risk budgeting[1] (RB)

$$\mathrm{RC}_i = b_i \cdot \mathscr{R}(x_1, \ldots, x_n)$$

[1] The ERC portfolio is a special case when $b_i = 1/n$.

Importance of the coherency and convexity properties

Figure: Risk Measure = 20 with a 50/30/20 budget rule



Application to the volatility risk measure

Let Σ be the covariance matrix of the assets returns. We assume that the risk measure $\mathscr{R}(x)$ is the volatility of the portfolio $\sigma(x) = \sqrt{x^\top \Sigma x}$. We have:

$$
\begin{aligned}
\frac{\partial\, \mathscr{R}(x)}{\partial\, x} &= \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} \\
\mathrm{RC}_i(x_1,\ldots,x_n) &= x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \\
\sum_{i=1}^n \mathrm{RC}_i(x_1,\ldots,x_n) &= \sum_{i=1}^n x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} = x^\top \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} = \sigma(x)
\end{aligned}
$$

The risk budgeting portfolio is defined by this system of equations:

$$
\left\{
\begin{array}{l}
x_i \cdot (\Sigma x)_i = b_i \cdot \left(x^\top \Sigma x\right) \\
x_i \geq 0 \\
\sum_{i=1}^n x_i = 1
\end{array}
\right.
$$

An example

Illustration

- 3 assets
- Volatilities are respectively 30%, 20% and 15%
- Correlations are set to 80% between the 1st asset and the 2nd asset, 50% between the 1st asset and the 3rd asset and 30% between the 2nd asset and the 3rd asset
- Budgets are set to 50%, 20% and 30%
- For the ERC (Equal Risk Contribution) portfolio, all the assets have the same risk budget

Weight budgeting (or traditional) approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	**50.00%**	29.40%	14.70%	70.43%
2	**20.00%**	16.63%	3.33%	15.93%
3	**30.00%**	9.49%	2.85%	13.64%
Volatility			20.87%	

Risk budgeting approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	31.15%	28.08%	8.74%	**50.00%**
2	21.90%	15.97%	3.50%	**20.00%**
3	46.96%	11.17%	5.25%	**30.00%**
Volatility			17.49%	

ERC approach

Asset	Weight	Marginal Risk	Risk Contribution Absolute	Risk Contribution Relative
1	19.69%	27.31%	5.38%	**33.33%**
2	32.44%	16.57%	5.38%	**33.33%**
3	47.87%	11.23%	5.38%	**33.33%**
Volatility			16.13%	

An example

Computing the risk contribution

The parameters are[2]:

$$x = \begin{pmatrix} 50 \\ 20 \\ 30 \end{pmatrix} \times 10^{-2}, \ \Sigma = \begin{pmatrix} 9.00 & 4.80 & 2.25 \\ 4.80 & 4.00 & 0.90 \\ 2.25 & 0.90 & 2.25 \end{pmatrix} \times 10^{-2}$$

Then, we compute the following quantities:

$$\Sigma x = \begin{pmatrix} 6.1350 \\ 3.4700 \\ 1.9800 \end{pmatrix} \times 10^{-2}, \ x \odot \Sigma x = \begin{pmatrix} 3.0675 \\ 0.6940 \\ 0.5940 \end{pmatrix} \times 10^{-2}$$

$$\sqrt{x^\top \Sigma x} = 20.8698\%, \ \frac{x \odot \Sigma x}{\sqrt{x^\top \Sigma x}} = \begin{pmatrix} 14.6982 \\ 3.3254 \\ 2.8462 \end{pmatrix} \times 10^{-2} = \begin{pmatrix} 14.70\% \\ 3.33\% \\ 2.85\% \end{pmatrix}$$

[2] We have $(\Sigma)_{i,j} = \rho_{i,j} \times \sigma_i \times \sigma_j$. For example, $(\Sigma)_{1,2} = 80\% \times 30\% \times 20\% = 4.80\%$.

An example

Another way to compute the risk contribution

The marginal risk for the first asset is:

$$\frac{\partial \mathscr{R}(x)}{\partial x_1} = \lim_{\varepsilon \to 0} \frac{\sigma(x_1 + \varepsilon, x_2, x_3) - \sigma(x_1, x_2, x_3)}{(x_1 + \varepsilon) - x_1}$$

If $\varepsilon = 1\%$, we have:

$$\frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq \frac{21.1639\% - 20.8698\%}{1\%} = 29.4050\% \text{ and } x_1 \times \frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq 14.7025\%$$

If $\varepsilon = 0.1\%$, we have:

$$\frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq \frac{20.8992\% - 20.8698\%}{1\%} = 29.3974\% \text{ and } x_1 \times \frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq 14.6987\%$$

If $\varepsilon = 0.01\%$, we have:

$$\frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq \frac{20.8728\% - 20.8698\%}{1\%} = 29.3966\% \text{ and } x_1 \times \frac{\partial \mathscr{R}(x)}{\partial x_1} \simeq 14.6983\%$$

RB is (a little) more complex than ERC

Let us consider the two-asset case. Let ρ be the correlation and $x = (w, 1 - w)$ be the vector of weights. The ERC portfolio is:

$$w^* = \frac{1}{\sigma_1} \Big/ \left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2} \right)$$

The RB portfolio with $(b, 1 - b)$ as the vector of risk budgets is:

$$w^\star = \frac{(b - 1/2)\rho\sigma_1\sigma_2 - b\sigma_2^2 + \sigma_1\sigma_2\sqrt{(b - 1/2)^2 \rho^2 + b(1 - b)}}{(1 - b)\sigma_1^2 - b\sigma_2^2 + 2(b - 1/2)\rho\sigma_1\sigma_2}$$

It introduces some convexity with respect to b and ρ.

Table: Weights $w^\star$ with respect to some values of b and ρ

	b	$\sigma_2 = \sigma_1$ 20%	50%	70%	90%	$\sigma_2 = 3 \times \sigma_1$ 20%	50%	70%	90%
ρ	−99.9%	50.0%	50.0%	50.0%	50.0%	75.0%	75.0%	75.0%	75.0%
	−50%	41.9%	50.0%	55.2%	61.6%	68.4%	75.0%	78.7%	82.8%
	0%	33.3%	50.0%	60.4%	75.0%	60.0%	75.0%	82.1%	90.0%
	25%	29.3%	50.0%	63.0%	80.6%	55.5%	75.0%	83.6%	92.6%
	50%	25.7%	50.0%	65.5%	84.9%	51.0%	75.0%	85.1%	94.4%
	75%	22.6%	50.0%	67.8%	87.9%	46.7%	75.0%	86.3%	95.6%
	90%	21.0%	50.0%	69.1%	89.2%	44.4%	75.0%	87.1%	96.1%

Some analytical solutions

- The case of uniform correlation[3] $\rho_{i,j} = \rho$
  - ERC portfolio ($b_i = 1/n$)

$$x_i(\rho) = \frac{\sigma_i^{-1}}{\sum_{j=1}^n \sigma_j^{-1}}$$

  - RB portfolio

$$x_i\left(-\frac{1}{n-1}\right) = \frac{\sigma_i^{-1}}{\sum_{j=1}^n \sigma_j^{-1}},\ x_i(0) = \frac{\sqrt{b_i}\sigma_i^{-1}}{\sum_{j=1}^n \sqrt{b_j}\sigma_j^{-1}},\ x_i(1) = \frac{b_i\sigma_i^{-1}}{\sum_{j=1}^n b_j\sigma_j^{-1}}$$

- The general case

$$x_i = \frac{b_i\beta_i^{-1}}{\sum_{j=1}^n b_j\beta_j^{-1}}$$

where β_i is the beta of the asset i with respect to the RB portfolio.

[3] The solution is noted $x_i(\rho)$.

The RB portfolio is a minimum variance (MV) portfolio subject to a constraint of weight diversification

Let us consider the following minimum variance optimization problem:

$$x^{\star}(c) = \arg\min \sqrt{x^{\top}\Sigma x}$$
$$\text{u.c.} \quad \begin{cases} \sum_{i=1}^{n} b_i \ln x_i \geq c \\ \mathbf{1}^{\top} x = 1 \\ x \geq \mathbf{0} \end{cases}$$

- if $c = c^{-} = -\infty$, $x^{\star}(c^{-}) = x_{\mathrm{MV}}$ (no weight diversification)
- if $c = c^{+} = \sum_{i=1}^{n} b_i \ln b_i$, $x^{\star}(c^{+}) = x_{\mathrm{WB}}$ (no variance minimization)
- $\exists c^{0} : x^{\star}(c^{0}) = x_{\mathrm{RB}}$ (variance minimization and weight diversification)

$\Longrightarrow$ if $b_i = 1/n$, $x_{\mathrm{RB}} = x_{\mathrm{ERC}}$ (variance minimization, weight diversification and perfect risk diversification[4])

[4] The Gini coefficient of the risk measure is then equal to 0.

The RB portfolio is located between the MV portfolio and the WB portfolio

- The RB portfolio is a combination of the MV and WB portfolios:

$$\partial_{x_i} \sigma(x) = \partial_{x_j} \sigma(x) \quad \text{(MV)}$$
$$x_i / b_i = x_j / b_j \quad \text{(WB)}$$

$$x_i \partial_{x_i} \sigma(x) / b_i = x_j \partial_{x_j} \sigma(x) / b_j \quad \text{(RB)}$$

- The volatility of the RB portfolio is between the volatility of the MV portfolio and the volatility of the WB portfolio:

$$\sigma_{\mathrm{MV}} \leq \sigma_{\mathrm{RB}} \leq \sigma_{\mathrm{WB}}$$

With risk budgeting, we always diminish the volatility compared to the weight budgeting

$\Rightarrow$ For the ERC portfolio, we retrieve the famous relationship:

$$\sigma_{\mathrm{MV}} \leq \sigma_{\mathrm{ERC}} \leq \sigma_{1/n}$$

Existence and uniqueness

- If $b_i > 0$, the solution exists and is unique.
- If $b_i \geq 0$, there may be several solutions.
- If $\rho_{i,j} \geq 0$, the solution is unique.

An example with 3 assets: $\sigma_1 = 20\%$, $\sigma_2 = 10\%$, $\sigma_3 = 5\%$ and $\rho_{1,2} = 50\%$.

$\rho_{1,3} = \rho_{2,3}$	Solution		1	2	3	$\sigma(x)$
-25%	$\mathcal{S}_1$	x_i	20.00%	40.00%	40.00%	6.63%
		$\partial_{x_i}\sigma(x)$	16.58%	8.29%	0.00%	
		RC_i	50.00%	50.00%	0.00%	
	$\mathcal{S}_2$	x_i	33.33%	66.67%	0.00%	11.55%
		$\partial_{x_i}\sigma(x)$	17.32%	8.66%	-1.44%	
		RC_i	50.00%	50.00%	0.00%	
	$\mathcal{S}'_1$	x_i	19.23%	38.46%	42.31%	6.38%
		$\partial_{x_i}\sigma(x)$	16.42%	8.21%	0.15%	
		RC_i	49.50%	49.50%	1.00%	
25%	$\mathcal{S}_1$	x_i	33.33%	66.67%	0.00%	11.55%
		$\partial_{x_i}\sigma(x)$	17.32%	8.66%	1.44%	
		RC_i	50.00%	50.00%	0.00%	

Optimality of risk budgeting portfolios

If the RB portfolio is optimal (in the Markowitz sense), the ex-ante performance contributions are equal to the risk budgets:

Black-Litterman Approach

Budgeting the risk = budgeting the performance
(in an ex-ante point of view)

Let $\tilde{\mu}_i$ be the market price of the expected return. We have:

$$x_i \cdot \tilde{\mu}_i \propto x_i \cdot \frac{\partial\, \sigma(x)}{\partial\, x}$$

In the ERC portfolio, the (ex-ante) performance contributions are equal. The ERC portfolio is then the less concentrated portfolio in terms of risk contributions, but also in terms of performance contributions.

Optimality of risk budgeting portfolios

Proof

We consider the quadratic utility function $\mathcal{U}(x) = x^\top \mu - \frac{1}{2}\phi x^\top \Sigma x$ of Markowitz. The portfolio x is optimal if the vector of expected returns satisfies this relationship:

$$\partial_x \mathcal{U}(x) = 0 \Leftrightarrow \tilde{\mu} = \frac{1}{\phi}\Sigma x$$

If the RB portfolio is optimal, the performance contribution PC_i of the asset i is then proportional to its risk contribution (or risk budget):

$$\begin{aligned}
\mathrm{PC}_i &= x_i \tilde{\mu}_i \\
&= \frac{1}{\phi} x_i (\Sigma x)_i \\
&= \frac{\sqrt{x^\top \Sigma x}}{\phi} \cdot \frac{x_i (\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \\
&\propto \mathrm{RC} \\
&\propto b_i
\end{aligned}$$

Optimality of risk budgeting portfolios

An example

$$\sigma = \begin{pmatrix} 10\% \\ 20\% \\ 30\% \\ 40\% \end{pmatrix} \quad \text{and} \quad \rho = \begin{pmatrix} 1.0 & & & \\ 0.8 & 1.0 & & \\ 0.2 & 0.2 & 1.0 & \\ 0.2 & 0.2 & 0.5 & 1.0 \end{pmatrix}$$

Example 1

b_i	x_i	$\partial_{x_i}\,\sigma(x)$	RC_i	$\tilde{\mu}_i$	PC_i
20.0	40.9	7.1	20.0	5.2	20.0
25.0	25.1	14.5	25.0	10.5	25.0
40.0	25.3	23.0	40.0	16.7	40.0
15.0	8.7	25.0	15.0	18.2	15.0

Example 2

b_i	x_i	$\partial_{x_i}\,\sigma(x)$	RC_i	$\tilde{\mu}_i$	PC_i
10.0	35.9	5.3	10.0	5.0	10.0
10.0	17.9	10.5	10.0	9.9	10.0
10.0	10.2	18.6	10.0	17.5	10.0
70.0	36.0	36.7	70.0	34.7	70.0

Generalization to other convex risk measures

If the risk measure is coherent and satisfies the Euler principle (convexity property), the following properties are verified:

1. Existence and uniqueness
2. Location between the minimum risk portfolio and the weight budgeting portfolio
3. Optimality

Some heuristic portfolios as RB portfolios

The EW, MV, MDP and ERC portfolios could be interpreted as (endogenous) RB portfolios.

	EW	MV	MDP	ERC
b_i				
PC_i	β_i	x_i	$x_i \sigma_i$	$\frac{1}{n}$

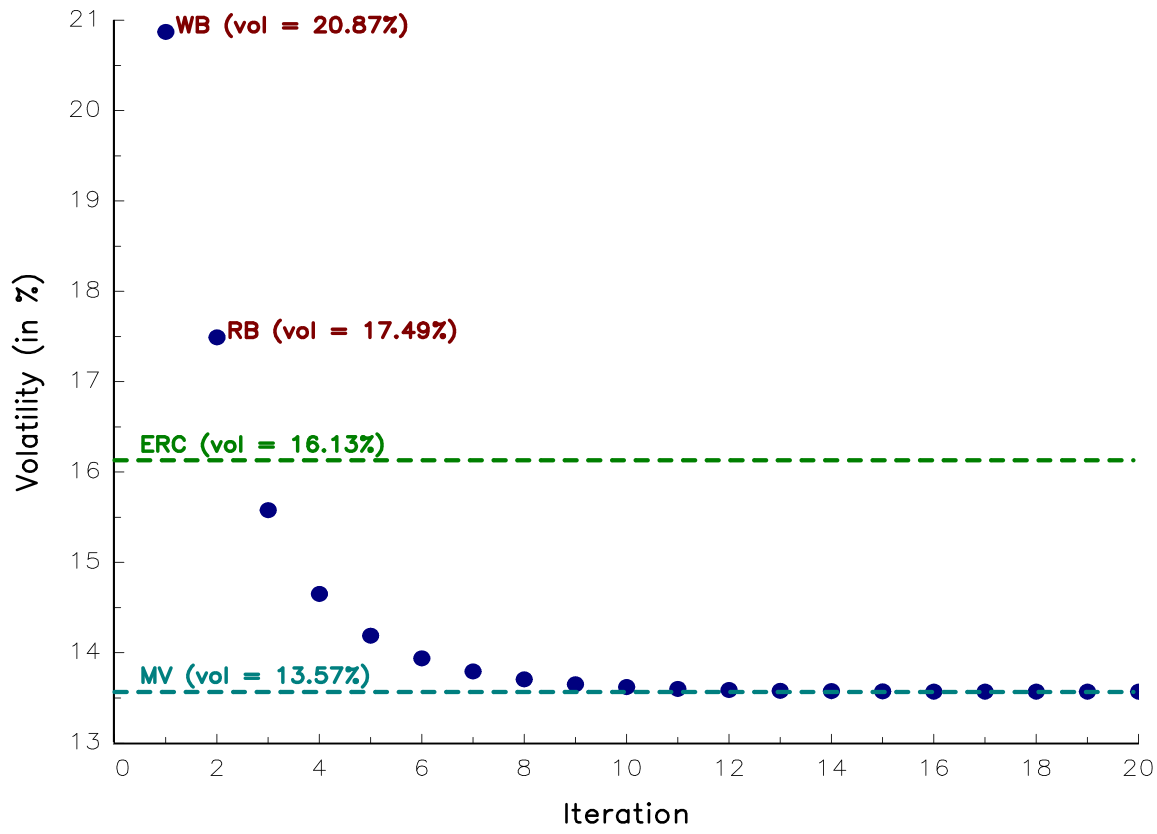
MV and MDP portfolios are two limit portfolios (explaining that the weights of some assets could be equal to zero).

Some heuristic portfolios as RB portfolios

MV portfolio as a limit portfolio

Let us consider an iterated portfolio $\left(x_1^{(t)}, \ldots, x_n^{(t)}\right)$ where t represents the iteration. The portfolio is defined such that the risk budget $b_i^{(t)}$ of the asset i at iteration t corresponds to the weight $x_i^{(t-1)}$ at iteration $t-1$. If the portfolio $\left(x_1^{(t)}, \ldots, x_n^{(t)}\right)$ admits a limit when $t \longrightarrow \infty$, it is equal to the minimum variance portfolio.

Figure: Illustration with the example of Slide 9



What is the problem with optimized portfolios?

> *"The indifference of many investment practitioners to mean-variance optimization technology, despite its theoretical appeal, is understandable in many cases. The major problem with MV optimization is its tendency to maximize the effects of errors in the input assumptions. Unconstrained MV optimization can yield results that are inferior to those of simple equal-weighting schemes" (Michaud, 1989).*

Optimal solutions are of the following form:

$$x^{\star} \propto \Sigma^{-1}\mu$$

The important quantity is then the information matrix:

$$\mathcal{I} = \Sigma^{-1}$$

The eigendecomposition of $\mathcal{I}$ is:

$$V_i(\mathcal{I}) = V_{n-i}(\Sigma) \quad \text{and} \quad \lambda_i(\mathcal{I}) = \frac{1}{\lambda_{n-i}(\Sigma)}$$

What is the problem with optimized portfolios?

An illustration

$\mu_1 = \mu_2 = 8\%$, $\mu_3 = 5\%$, $\sigma_1 = 20\%$, $\sigma_2 = 21\%$, $\sigma_3 = 10\%$ and $\rho_{i,j} = 80\%$.

The eigendecomposition of the covariance and information matrices is:

Asset / Factor	Covariance matrix Σ 1	2	3	Information matrix $\mathcal{I}$ 1	2	3
1	65.35%	−72.29%	−22.43%	−22.43%	−72.29%	65.35%
2	69.38%	69.06%	−20.43%	−20.43%	69.06%	69.38%
3	30.26%	−2.21%	95.29%	95.29%	−2.21%	30.26%
Eigenvalue	8.31%	0.84%	0.26%	379.97	119.18	12.04
% cumulated	88.29%	97.20%	100.00%	74.33%	97.65%	100.00%

⇒ It means that the first factors of the information matrix correspond to the last factors of the covariance matrix. For example, if we consider 100 assets, the 1^{st} factor of $\mathcal{I}$ is the 100^{th} factor of Σ, the 2^{nd} factor of $\mathcal{I}$ is the 99^{th} factor of Σ, etc.

⇒ Shrinkage is then necessary to eliminate the noise factors, but is not sufficient!

What is the problem with optimized portfolios?

An illustration

With the previous example, the optimal portfolio is $(38.3\%, 20.2\%, 41.5\%)$ for a volatility of 15%. The corresponding risk contributions are 49.0%, 25.8% and 25.2%.

1. MVO: the objective is to target a volatility of 15%.
2. RB: the objective is to target the budgets $(49.0\%, 25.8\%, 25.2\%)$.

What is the sensitivity to the input parameters?

ρ			70%	90%		90%	
σ_2					18%	18%	
μ_1							9%
MVO	x_1	38.3%	38.3%	44.6%	13.7%	0.0%	56.4%
	x_2	20.2%	25.9%	8.9%	56.1%	65.8%	0.0%
	x_3	41.5%	35.8%	46.5%	30.2%	34.2%	43.6%
RB	x_1	38.3%	37.7%	38.9%	37.1%	37.7%	38.3%
	x_2	20.2%	20.4%	20.0%	22.8%	22.6%	20.2%
	x_3	41.5%	41.9%	41.1%	40.1%	39.7%	41.5%

$\Rightarrow$ MVO is very sensitive compared to RB. Could you imagine the issue with 100 assets?

Managing sovereign credit risk in bond portfolios

- Bond portfolio management
- Measuring risk contributions in sovereign bond portfolios
- Comparing debt indexation, fundamental indexation and risk-budgeting indexation
- Risk-budgeting versus active management

Time to rethink the bond portfolios management

Two main problems:

1. Benchmarks = debt-weighted indexation (the weights are based on the notional amount of the debt)
2. Fund management driven by the search of yield with little consideration for **credit risk** (carry position $\neq$ arbitrage position)

$\Rightarrow$ Time to rethink bond indexes? (Toloui, 2010)

We need to develop a framework to measure the credit risk of bond portfolios with two goals:

1. managing the credit risk of bond portfolios;
2. building alternative-weighted indexes.

For the application, we consider the euro government bond portfolios. The benchmark is the Citigroup EGBI index[5].

[5] This index is very close to the EuroMTS index.

Ratings or spreads?

Figure: Some popular measures of country risk

Country	S&P Rating[1]	Euromoney Country Risk[2]		Opacity Score[3]	CDS Spread (in bp)	
		Score	Rank		01/09/11	04/10/11
Austria	AAA	84.01	13	16	123	186
Belgium	AA+	77.81	19	19	249	309
Finland	AAA	86.96	8	9	64	85
France	AAA	80.90	16	23	163	201
Germany	AAA	84.98	11	17	76	122
Greece	CCC	52.38	65	31	2,291	5,736
Ireland	BBB+	62.33	43	15	781	726
Italy	A	71.20	30	36	384	487
Netherlands	AAA	86.67	9	24	80	117
Portugal	BBB-	61.35	44	25	957	1,167
Spain	AA	66.71	36	26	376	391
Norway	AAA	93.44	1		44	52
Switzerland	AAA	90.31	3	22	58	79
Denmark	AAA	89.21	4	15	100	153
Sweden	AAA	88.74	5	14	54	66
Canada	AAA	87.17	7	20		
United Kingdom	AAA	80.22	17	18	76	102
United States	AA+	82.10	15	22	52	52
Japan	AA-	74.66	25	25	102	155

[1] Sep. 15th 2011 [2] March 2011 [3] 2009, Milken Institute

Choosing the right measure of credit risk

- Volatility of price returns $\neq$ a good measure of credit risk
- Correlation of price returns $\neq$ a good measure of contagion
- A better measure is the yield spread, but its computation can be difficult as it needs to first define a reference risk-free rate.

$\Rightarrow$ One of the best measure is the CDS spread (it does not depend on the currency, the yield curve or the duration).

The SABR CDS model

Let $S_i(t)$ be the spread of the i^{th} issuer. We have:

$$\mathrm{d}S_i(t) = \sigma_i^S \cdot S_i(t)^{\beta_i} \cdot \mathrm{d}W_i(t)$$

Moreover, we assume that the correlation between the brownian motions $W_i(t)$ and $W_j(t)$ is $\Gamma_{i,j}$.

Calibration of the β_i parameter

We assume that we observe spreads at some given known dates $t_0, \ldots, t_n$. Let $S_{i,j}$ be the observed spread for the i^{th} country at date t_j. The log-likelihood function for the i^{th} country is:

$$\begin{aligned}\ell &= -\frac{n}{2}\ln 2\pi - n\ln\sigma_i^S - \frac{1}{2}\sum_{j=1}^{n}\ln\left(t_j - t_{j-1}\right) - \\ &\quad \beta_i\sum_{j=1}^{n}\ln S_{i,j-1} - \frac{1}{2}\sum_{j=1}^{n}\frac{\left(S_{i,j}-S_{i,j-1}\right)^2}{\left(\sigma_i^S S_{i,j-1}^{\beta_i}\right)^2\left(t_j - t_{j-1}\right)}\end{aligned}$$

Figure: Results for the period January 2008-August 2011

Country	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES	Average
estimate	0.996	1.017	0.816	0.786	0.899	1.070	0.836	1.157	0.793	1.013	1.148	0.957
std-dev.	1.10%	2.00%	1.60%	1.60%	2.00%	1.10%	0.70%	1.70%	0.90%	1.10%	2.10%	1.45%

$\Rightarrow$ We assume that $\beta_i = 1$ (ML estimation is then easy to compute).

Statistics as of January 3rd, 2008

Table: CDS levels $S_i(t)$, volatilities σ_i^S and correlations $\Gamma_{i,j}$

Country	Spread	Volatility	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	6	56.6%	100%										
Belgium	13	65.8%	54%	100%									
Finland	6	103.9%	27%	34%	100%								
France	7	50.1%	55%	60%	42%	100%							
Germany	6	68.8%	51%	43%	23%	50%	100%						
Greece	21	60.6%	42%	37%	28%	46%	43%	100%					
Ireland	15	76.5%	33%	35%	41%	37%	27%	22%	100%				
Italy	21	48.7%	47%	50%	37%	60%	47%	60%	35%	100%			
Netherlands	6	81.7%	22%	38%	32%	29%	17%	4%	29%	22%	100%		
Portugal	17	56.6%	38%	48%	23%	45%	44%	58%	29%	58%	12%	100%	
Spain	20	67.6%	35%	49%	22%	33%	34%	34%	28%	41%	30%	41%	100%

Statistics as of March 1st, 2012

Table: CDS levels $S_i(t)$, volatilities σ_i^S and correlations $\Gamma_{i,j}$

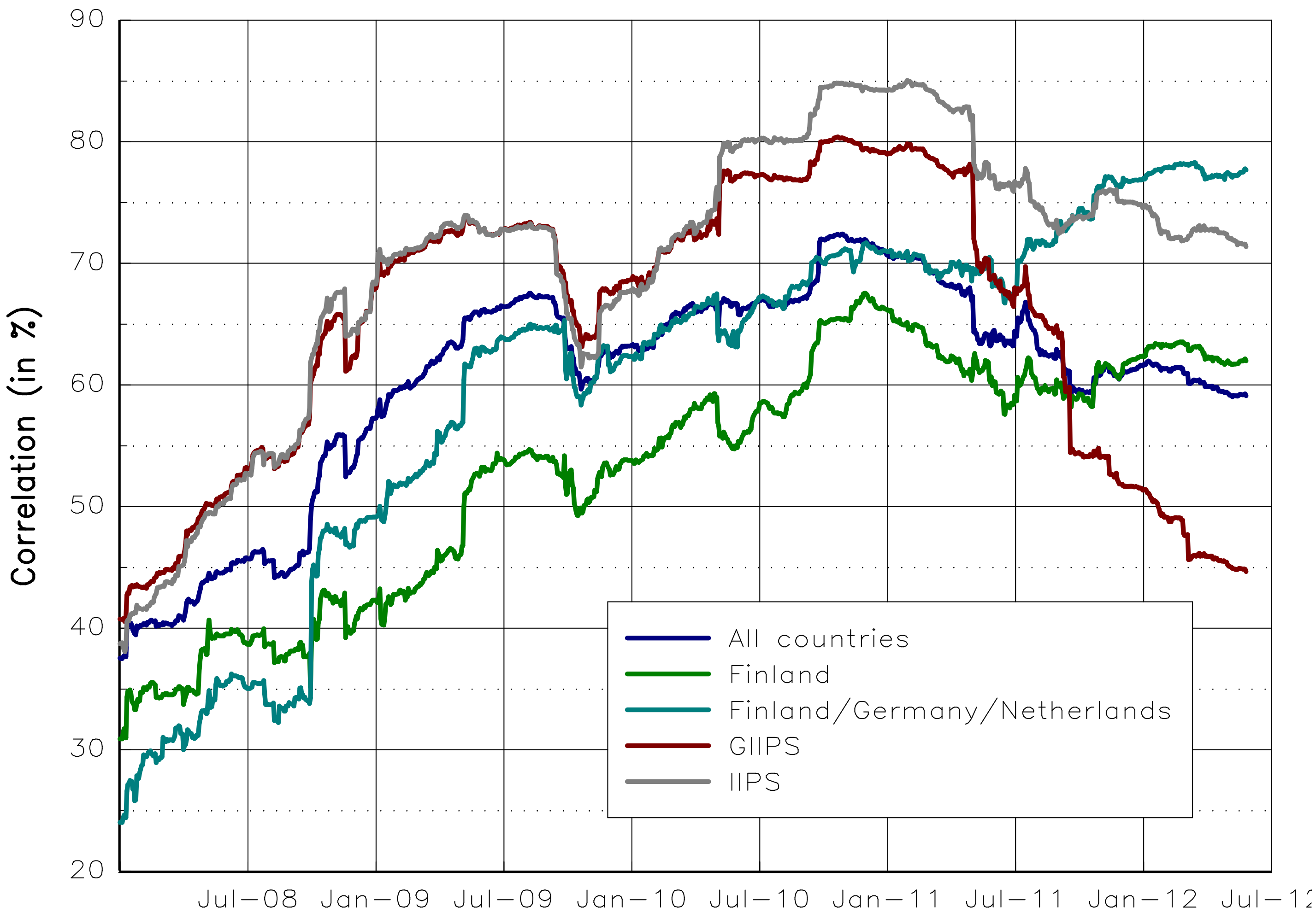
Country	Spread	Volatility	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	158	73.5%	100%										
Belgium	223	73.1%	80%	100%									
Finland	64	68.8%	75%	75%	100%								
France	166	70.9%	87%	85%	78%	100%							
Germany	76	66.0%	82%	78%	73%	86%	100%						
Greece	8,871	163.4%	9%	12%	9%	6%	6%	100%					
Ireland	581	51.9%	62%	72%	57%	67%	66%	16%	100%				
Italy	356	74.2%	74%	86%	72%	80%	73%	11%	71%	100%			
Netherlands	94	67.7%	79%	79%	78%	85%	83%	6%	64%	74%	100%		
Portugal	1,175	56.1%	55%	66%	50%	60%	57%	15%	79%	67%	54%	100%	
Spain	356	72.5%	74%	80%	66%	75%	69%	9%	69%	81%	66%	64%	100%

Statistics as of May 31st, 2012

Table: CDS levels $S_i(t)$, volatilities σ_i^S and correlations $\Gamma_{i,j}$

Country	Spread	Volatility	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	196	71.9%	100%										
Belgium	282	70.5%	80%	100%									
Finland	89	67.2%	73%	75%	100%								
France	219	70.7%	88%	84%	76%	100%							
Germany	102	65.7%	83%	79%	73%	86%	100%						
Greece	8,751	366.5%	3%	4%	4%	2%	-1%	100%					
Ireland	726	49.9%	62%	71%	55%	65%	65%	7%	100%				
Italy	563	71.8%	74%	85%	72%	79%	74%	4%	71%	100%			
Netherlands	128	68.4%	78%	78%	78%	82%	82%	3%	61%	73%	100%		
Portugal	1,185	56.2%	55%	63%	47%	58%	55%	5%	81%	65%	48%	100%	
Spain	599	68.9%	75%	79%	66%	75%	70%	1%	68%	82%	66%	61%	100%

Evolution of the correlation matrix



Computing the credit risk measure for one bond

Let $B(t, D_i)$ be a zero-coupon risky bond of maturity (or duration) D_i. We have:

$$B(t, D_i) = e^{-(R(t)+S_i(t))\cdot D_i}$$

with $R(t)$ the risk-free rate and $S_i(t)$ the credit spread. It comes that:

$$\mathrm{d}\ln B(t, D_i) = -D_i \cdot \mathrm{d}R(t) - D_i \cdot \mathrm{d}S_i(t)$$

If we define the risk measure as the (integrated) volatility of the hedging (CDS) portfolio, we obtain:

$$\sigma_i^B = D_i \cdot \sigma_i^S \cdot S_i(t)$$

where D_i is the duration of the bond i, $S_i(t)$ is the CDS level of the corresponding issuer and σ_i^S is the volatility of the CDS.

σ_i^B is the sovereign credit risk measure for one bond.

Computing the credit risk measure for one bond

An example

Table: Input parameters (01/03/2012)

Country	$S_i(t)$	σ_i^S
Germany	76 bps	66.0%
France	156 bps	70.9%
Italy	356 bps	74.2%

Table: Computing σ_i^B

	Country	N_i	D_i	σ_i^B
#1	Germany	12	8.2	4.11%
#2	France	15	7.1	8.36%
#3	Italy	16	6.5	17.17%
#4	Italy	8	5.9	15.58%

We consider 4 bonds:

- one German bond (notional = 12 M€, duration = 8.2 years);
- one French bond (notional = 15 M€, duration = 7.1 years);
- two Italian bonds (notional = 16 and 8 M€, duration = 6.5 and 5.9 years).

For the German bond, we have:

$$\sigma_i^B = 8.2 \times 0.66 \times 0.0076 = 0.04113$$

Computing the credit risk measure for a bond portfolio

Let $w = (w_1, \ldots, w_n)$ be the weights of bonds in the portfolio. If we define the risk measure as the volatility of the hedging (CDS) portfolio, we obtain:

$$\begin{aligned}\mathscr{R}(w) &= \sigma\left(\sum_{i=1}^{n} w_i \cdot D_i \cdot \mathrm{d}S_i(t)\right) \\ &= \sqrt{w^\top \Sigma w} \\ &= \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \Sigma_{i,j}}\end{aligned}$$

with $\Sigma_{i,j} = \Gamma_{i,j} \cdot \sigma_i^B \cdot \sigma_j^B$ and $\Gamma_{i,j}$ is the correlation between the CDS relative variations of issuers corresponding to the bonds i and j.

$\mathscr{R}(w)$ is the (integrated) volatility of the CDS basket which would perfectly hedge the credit risk of the bond portfolio.

$\mathscr{R}(w)$ is the sovereign credit risk measure for the bond portfolio.

Computing the credit risk measure for a bond portfolio

Remark

The CDS basket is purely virtual. We never buy the CDS basket. We only use it to compute the sovereign credit risk measure.

Remark

$\mathscr{R}(w)$ depends on 3 "CDS" parameters $S_i(t)$, σ_i^S and $\Gamma_{i,j}$ and two "portfolio" parameters w_i and D_i.

Computing the credit risk measure for a bond portfolio

Example with bonds #1, #2 and #3

We have:

$$\Gamma = \begin{pmatrix} 1.00 & & \\ 0.86 & 1.00 & \\ 0.73 & 0.80 & 1.00 \end{pmatrix} \text{ and } w = \frac{1}{43}\begin{pmatrix} 12 \\ 15 \\ 16 \end{pmatrix} = \begin{pmatrix} 27.9\% \\ 34.9\% \\ 37.2\% \end{pmatrix}$$

It comes that:

$$\begin{aligned} \mathscr{R}^2(w) &= 0.279^2 \times 0.0411^2 + 0.349^2 \times 0.0836^2 + 0.372^2 \times 0.1717^2 + \\ & \quad 2 \times 0.279 \times 0.349 \times 0.86 \times 0.0411 \times 0.0836 + \\ & \quad 2 \times 0.279 \times 0.372 \times 0.73 \times 0.0411 \times 0.1717 + \\ & \quad 2 \times 0.349 \times 0.372 \times 0.80 \times 0.0836 \times 0.1717 \\ &= 0.9689\% \end{aligned}$$

We deduce that:

$$\mathscr{R}(w) = \sqrt{0.9689\%} = 9.8433\%$$

Computing the credit risk measure for a bond portfolio

Example with bonds #1, #2, #3 and #4

We have:

$$\Gamma = \begin{pmatrix} 1.00 & & & \\ 0.86 & 1.00 & & \\ 0.73 & 0.80 & 1.00 & \\ 0.73 & 0.80 & \boxed{\color{red}{\mathbf{1.00}}} & 1.00 \end{pmatrix} \text{ and } w = \frac{1}{51}\begin{pmatrix} 12 \\ 15 \\ 16 \\ 8 \end{pmatrix} = \begin{pmatrix} 23.5\% \\ 29.4\% \\ 31.4\% \\ 15.7\% \end{pmatrix}$$

It comes that[6]:

$$\Sigma = \begin{pmatrix} 16.9 & 29.6 & 51.6 & 46.8 \\ 29.6 & 69.8 & 114.8 & 104.2 \\ 51.6 & 114.8 & 294.8 & 267.6 \\ 46.8 & 104.2 & 267.6 & 242.9 \end{pmatrix} \times 10^{-4}$$

We deduce that:

$$\mathcal{R}(w) = \sqrt{w^\top \Sigma w} = 10.6892\%$$

[6] We have:

$$\Sigma_{1,2} = 0.86 \times 0.0411 \times 0.0836 = 29.549 \times 10^{-4}$$

Defining the risk contribution

Our credit risk measure $\mathscr{R}(w) = \sqrt{w^\top \Sigma w}$ is a convex risk measure. It means that:

$$\begin{aligned}\mathscr{R}(w_1, \ldots, w_n) &= \sum_{i=1}^{n} w_i \cdot \frac{\partial\, \mathscr{R}(w_1, \ldots, w_m)}{\partial\, w_i} \\ &= \sum_{i=1}^{n} RC_i\end{aligned}$$

We can then break the risk measure down into n individual sources of risk.

The risk contribution RC_i is an increasing function of the parameters D_i, $S_i(t)$ and σ_i^S.

Bond indexation schemes

Debt weighting

It is defined by[a]:

$$w_i = \frac{\mathrm{DEBT}_i}{\sum_{i=1}^n \mathrm{DEBT}_i}$$

[a] Two forms of debt-weighting are considered : DEBT (with the 11 countries) and DEBT* (without Greece after July 2010). This last one corresponds to the weighting scheme of the EGBI index.

Alternative weighting

1. Fundamental indexation
   The GDP-weighting is defined by:

$$w_i = \frac{\mathrm{GDP}_i}{\sum_{i=1}^n \mathrm{GDP}_i}$$

2. Risk-based indexation
   The DEBT-RB and GDP-RB weightings are defined by:

$$b_i = \frac{\mathrm{DEBT}_i}{\sum_{i=1}^n \mathrm{DEBT}_i}$$

$$b_i = \frac{\mathrm{GDP}_i}{\sum_{i=1}^n \mathrm{GDP}_i}$$

Some results for the EGBI index

Figure: EGBI weights and risk contributions

Country	July-08		July-09		July-10		July-11		March-12		June-12	
	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC
Austria	4.1%	1.7%	3.6%	7.7%	4.1%	2.3%	4.3%	1.5%	4.2%	3.0%	4.3%	2.6%
Belgium	6.2%	6.1%	6.5%	5.1%	6.3%	5.7%	6.4%	6.5%	6.3%	6.6%	6.2%	6.1%
Finland	1.2%	0.4%	1.3%	0.5%	1.3%	0.2%	1.6%	0.2%	1.6%	0.3%	1.6%	0.3%
France	20.5%	9.8%	20.4%	13.2%	22.2%	15.1%	23.1%	13.3%	23.2%	19.0%	23.2%	17.6%
Germany	24.4%	6.1%	22.3%	13.0%	22.9%	6.0%	22.1%	5.3%	22.4%	7.3%	22.4%	7.0%
Greece	4.9%	11.4%	5.4%	8.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ireland	1.0%	1.3%	1.5%	4.3%	2.1%	3.3%	1.4%	5.4%	1.7%	2.3%	1.7%	2.0%
Italy	22.1%	45.2%	22.4%	29.5%	23.4%	38.7%	23.1%	38.5%	22.1%	39.7%	21.8%	42.0%
Netherlands	5.3%	1.7%	5.3%	4.1%	6.1%	1.6%	6.2%	1.2%	6.2%	2.6%	6.5%	2.5%
Portugal	2.4%	3.9%	2.3%	2.3%	2.1%	6.3%	1.6%	6.6%	1.4%	3.0%	1.7%	2.6%
Spain	7.8%	12.4%	9.1%	11.8%	9.6%	20.9%	10.3%	21.3%	10.8%	16.2%	10.7%	17.2%
Sovereign Risk Measure	0.70%		2.59%		6.12%		4.02%		8.62%		12.16%	

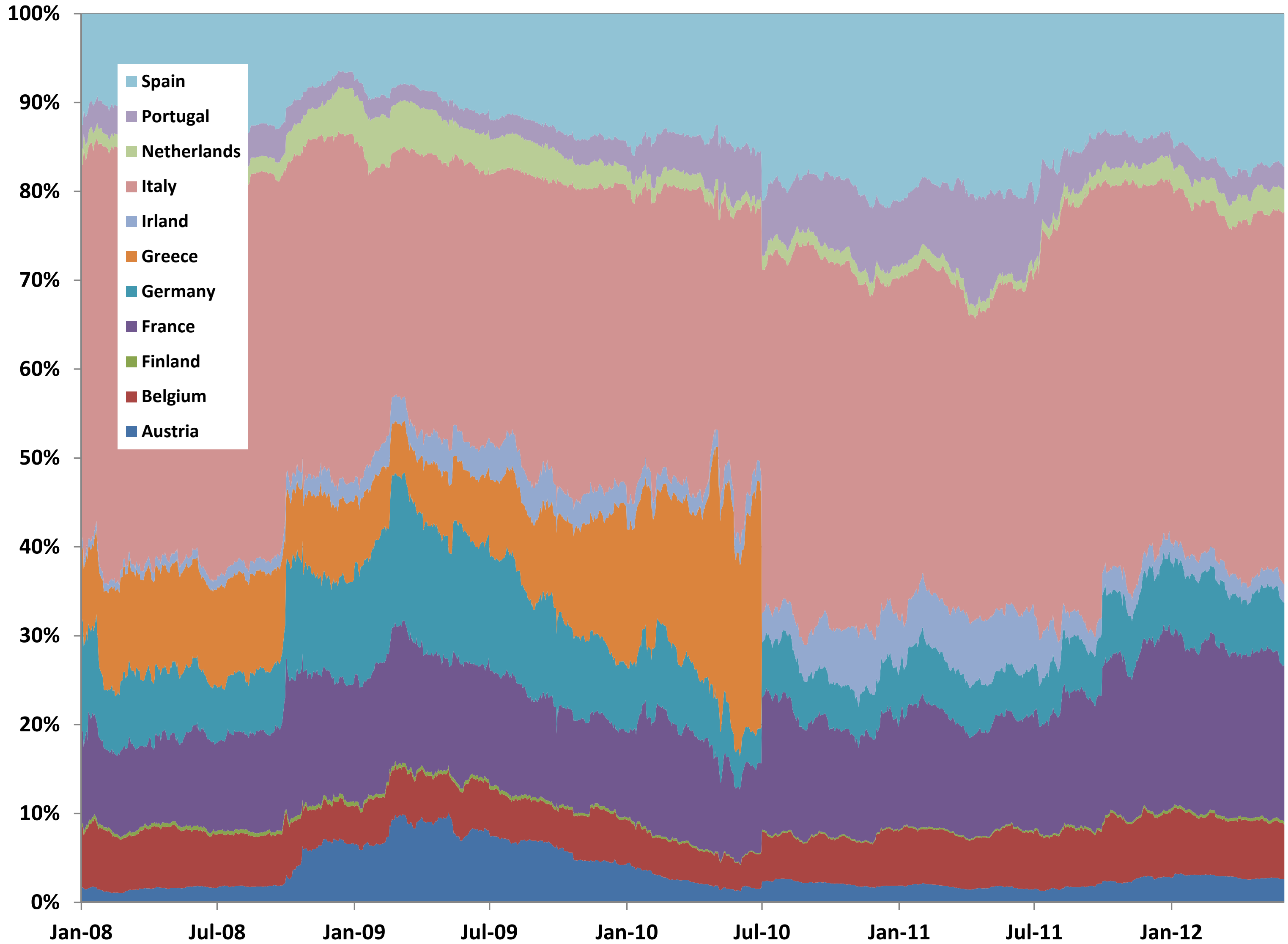
⇒ Small changes in weights but large changes in risk contributions.

⇒ The sovereign credit risk measure has highly increased (the largest value 12.5% is obtained in November 25th 2011).

⇒ If we think that the EGBI portfolio is optimal, we expect that 60% of the performance will come from Italy and France.

Some results for the EGBI index

Evolution of the risk contributions



GDP indexation

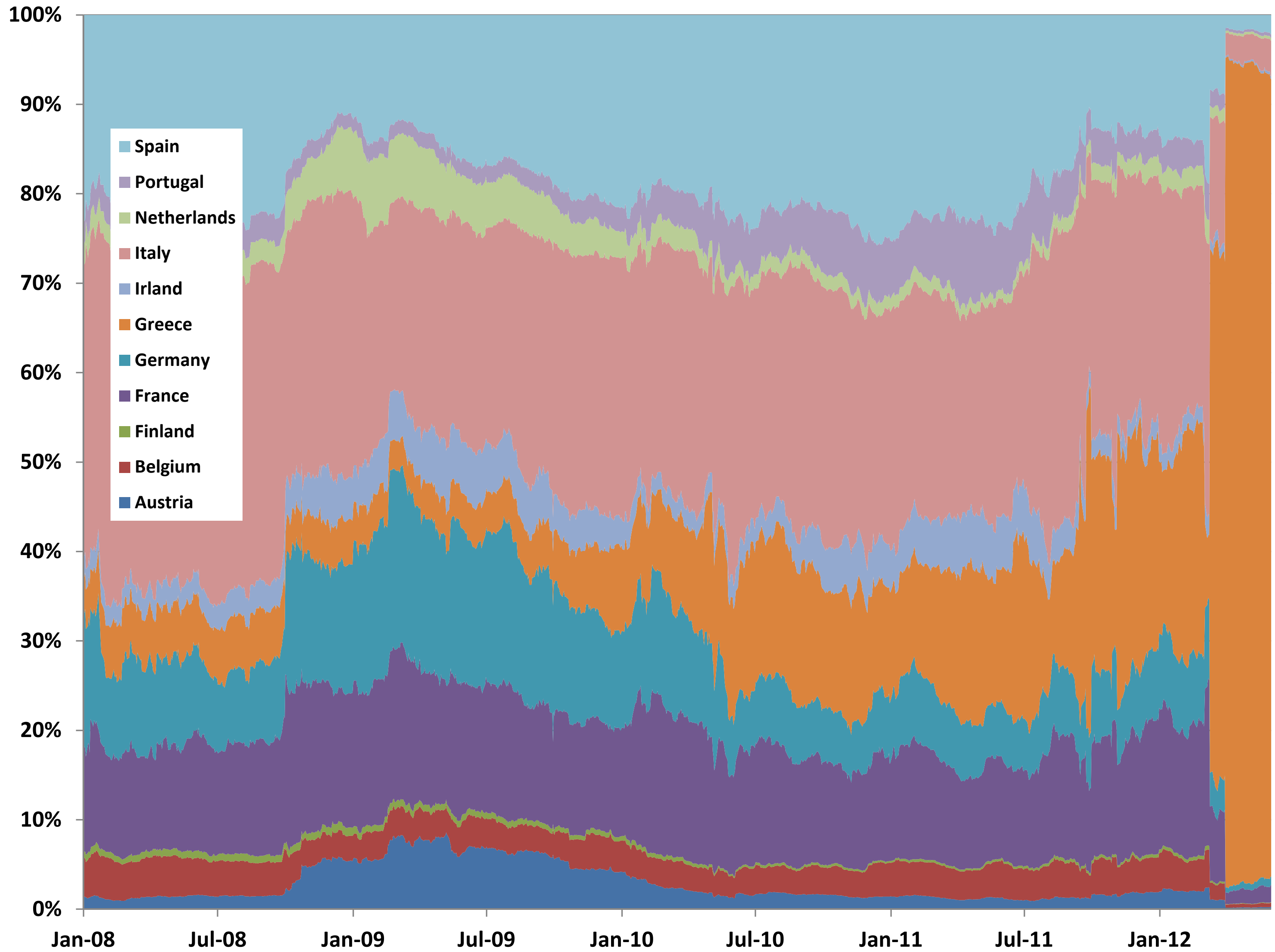
Figure: Weights and risk contributions of the GDP indexation

Country	July-08		July-09		July-10		July-11		March-12		June-12	
	GDP	RC	GDP	RC	GDP	RC	GDP	RC	GDP	RC	GDP	RC
Austria	3.1%	1.4%	3.1%	7.0%	3.1%	1.7%	3.2%	1.0%	3.4%	2.0%	3.4%	0.3%
Belgium	3.8%	4.0%	3.8%	3.2%	3.9%	3.3%	4.0%	3.5%	4.0%	3.4%	4.0%	0.5%
Finland	2.0%	0.8%	1.9%	0.7%	2.0%	0.3%	2.1%	0.3%	2.1%	0.4%	2.1%	0.1%
France	21.2%	11.2%	21.5%	14.9%	21.4%	13.4%	21.5%	10.6%	21.7%	14.0%	21.7%	1.9%
Germany	27.4%	7.6%	27.2%	17.0%	27.7%	6.7%	27.9%	5.8%	27.8%	7.2%	27.8%	1.0%
Greece	2.6%	6.2%	2.7%	4.4%	2.6%	15.7%	2.4%	19.8%	2.4%	26.9%	2.4%	89.1%
Ireland	2.0%	3.0%	1.9%	5.6%	1.8%	2.6%	1.7%	5.9%	1.7%	1.9%	1.7%	0.3%
Italy	17.4%	37.5%	17.3%	23.5%	17.2%	25.8%	17.0%	23.9%	17.1%	24.6%	17.1%	4.0%
Netherlands	6.5%	2.5%	6.5%	5.3%	6.5%	1.6%	6.6%	1.2%	6.5%	2.1%	6.5%	0.3%
Portugal	1.9%	3.3%	1.9%	2.0%	1.9%	5.3%	1.9%	6.7%	1.9%	3.4%	1.9%	0.4%
Spain	12.0%	22.6%	12.0%	16.5%	11.8%	23.7%	11.8%	21.4%	11.6%	14.1%	11.6%	2.2%
Sovereign Risk Measure	0.64%		2.47%		6.59%		4.56%		9.41%		34.92%	

$\Rightarrow$ RC of Debt and GDP indexations are different, but sovereign credit risk measures are similar (until April 2012).

GDP indexation

Evolution of the risk contributions



GDP-RB indexation

Figure: Weights and risk contributions of the GDP-RB indexation

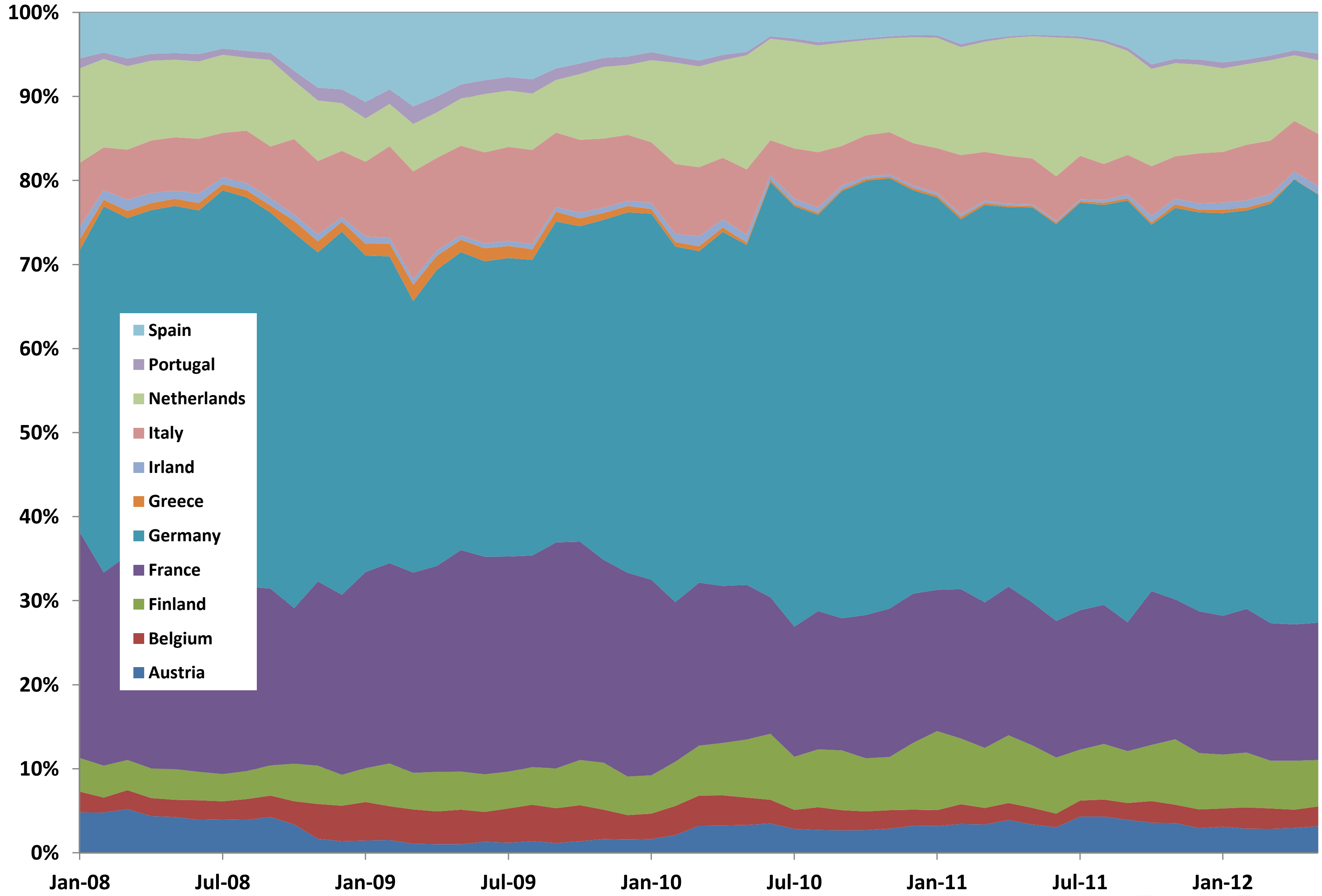
Country	July-08		July-09		July-10		July-11		March-12		June-12	
	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights
Austria	3.1%	3.9%	3.1%	1.2%	3.1%	2.9%	3.2%	4.2%	3.4%	2.8%	3.4%	3.2%
Belgium	3.8%	2.1%	3.8%	4.1%	3.9%	2.2%	4.0%	1.9%	4.0%	2.4%	4.0%	2.4%
Finland	2.0%	3.2%	1.9%	4.4%	2.0%	6.3%	2.1%	6.0%	2.1%	5.7%	2.1%	5.5%
France	21.2%	22.0%	21.5%	25.6%	21.4%	15.5%	21.5%	16.5%	21.7%	16.4%	21.7%	16.3%
Germany	27.4%	47.8%	27.2%	35.5%	27.7%	50.0%	27.9%	48.7%	27.8%	49.9%	27.8%	51.0%
Greece	2.6%	0.7%	2.7%	1.4%	2.6%	0.2%	2.4%	0.2%	2.4%	0.3%	2.4%	0.1%
Ireland	2.0%	0.8%	1.9%	0.6%	1.8%	0.6%	1.7%	0.2%	1.7%	0.8%	1.7%	0.9%
Italy	17.4%	5.3%	17.3%	11.2%	17.2%	6.0%	17.0%	5.2%	17.1%	6.4%	17.1%	6.2%
Netherlands	6.5%	9.2%	6.5%	6.7%	6.5%	12.8%	6.6%	14.0%	6.5%	9.5%	6.5%	8.8%
Portugal	1.9%	0.7%	1.9%	1.6%	1.9%	0.4%	1.9%	0.2%	1.9%	0.6%	1.9%	0.8%
Spain	12.0%	4.2%	12.0%	7.7%	11.8%	3.1%	11.8%	2.9%	11.6%	5.1%	11.6%	4.9%
Sovereign Risk Measure	0.39%		2.10%		3.25%		1.91%		5.43%		7.43%	

$\Rightarrow$ RB indexation is very different from WB indexation, in terms of weights, RC and credit risk measures.

$\Rightarrow$ The dynamics of the GDP-RB is relatively smooth (monthly turnover $\simeq 7\%$, $\max = 20\%$, $\min = 1.8\%$).

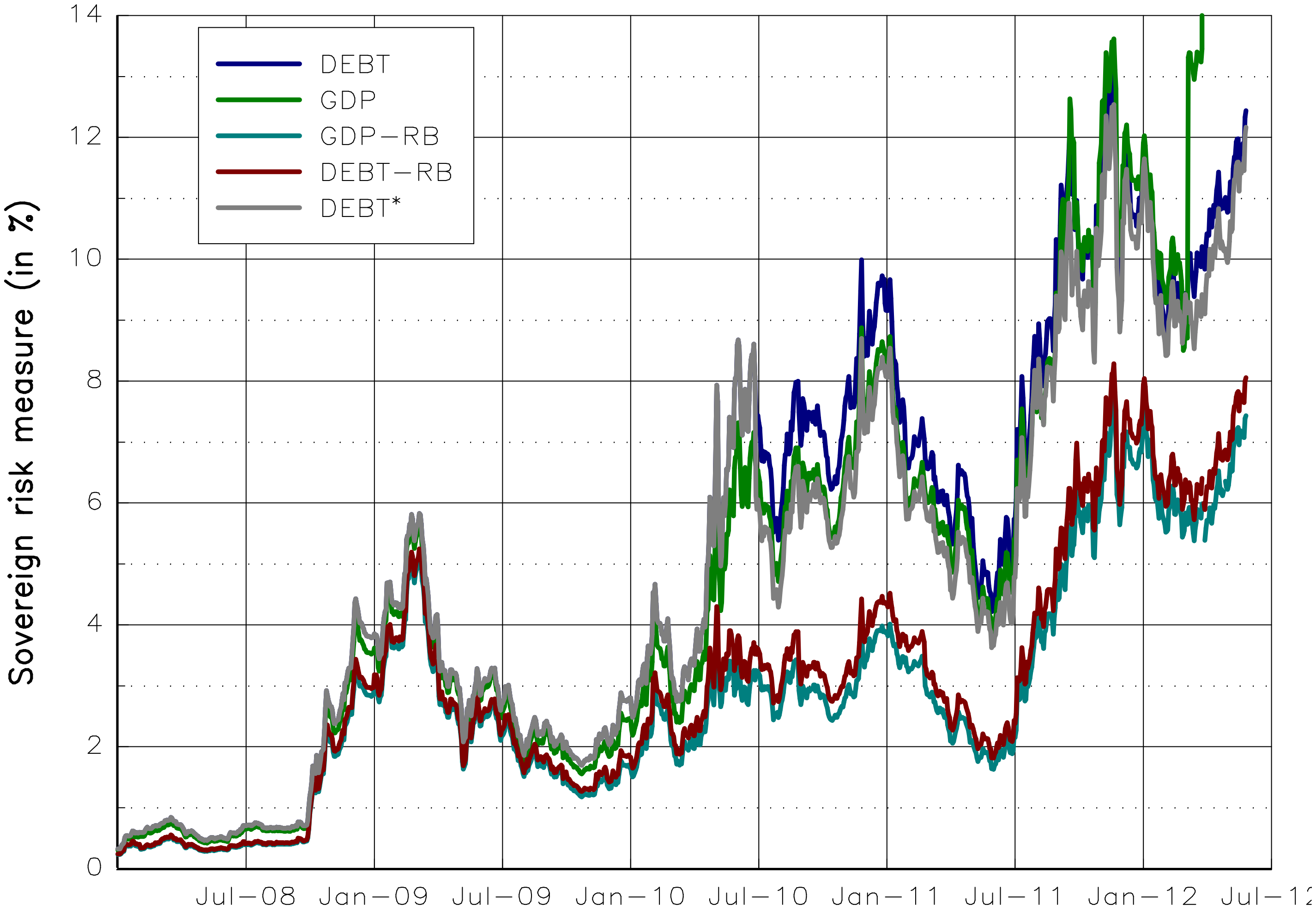
GDP-RB indexation

Evolution of weights



Comparison of the indexing schemes

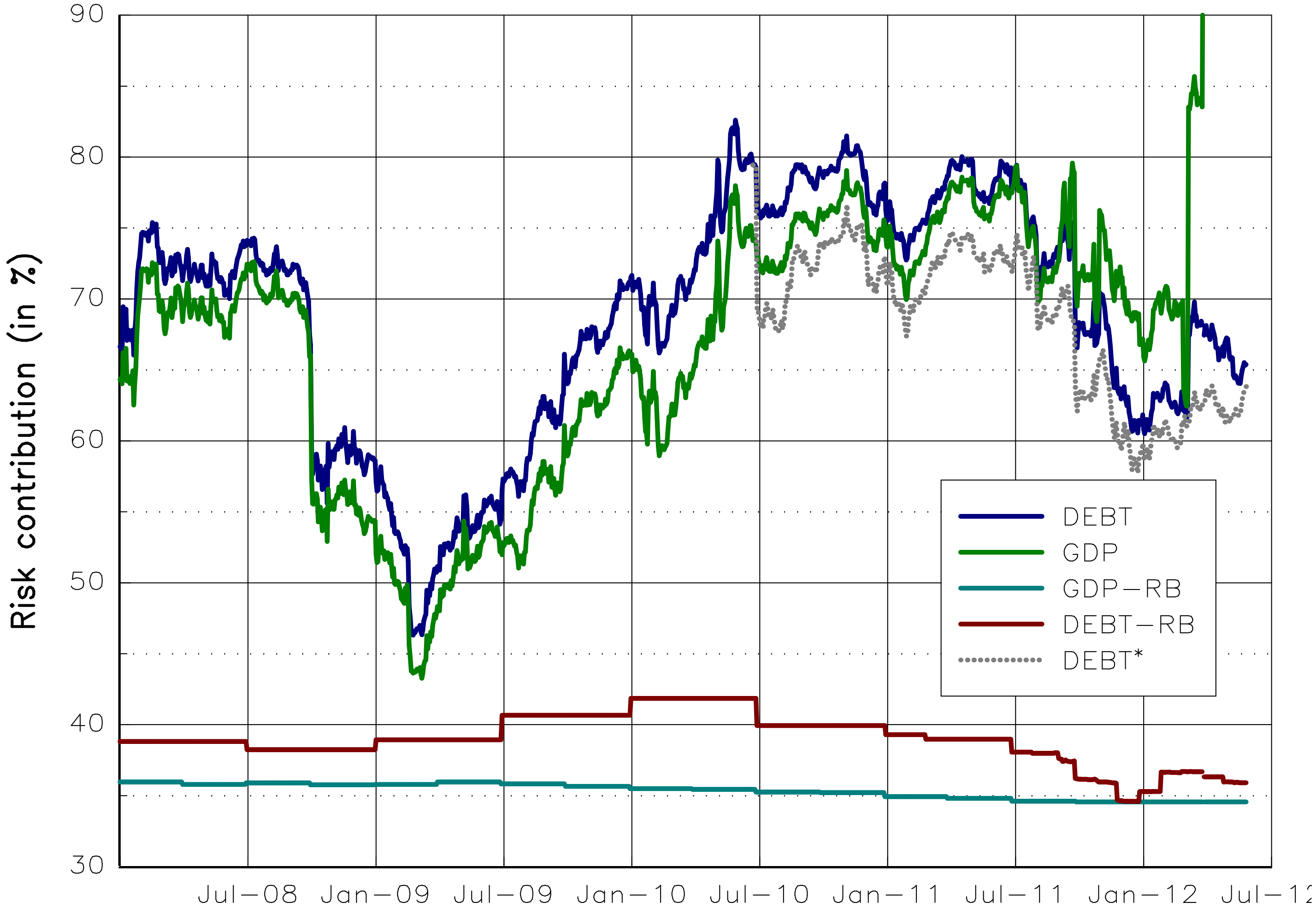
Evolution of the risk measure



⇒ We verify that the risk measure of the RB indexation is smaller than the one of the WB indexation.

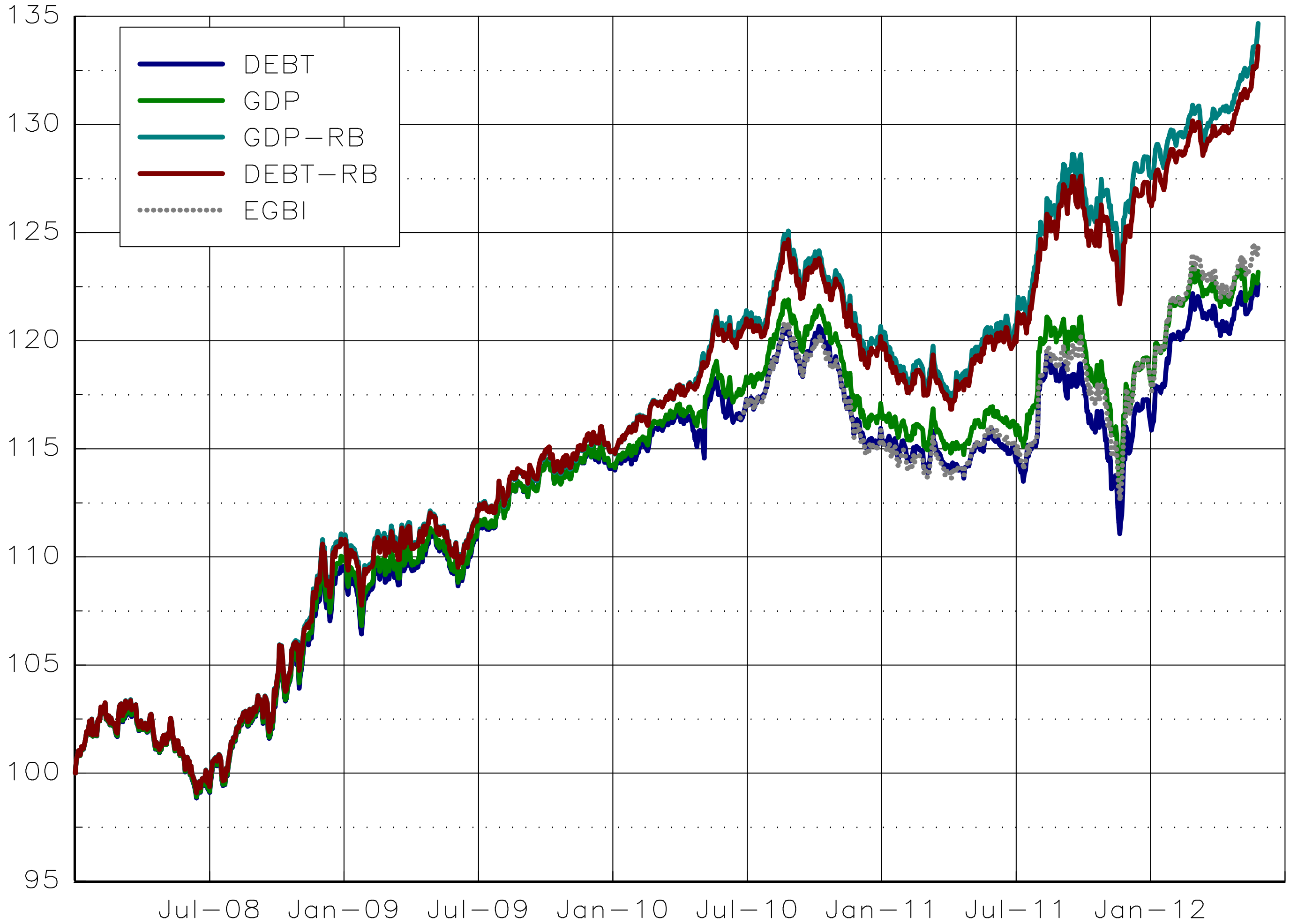
Comparison of the indexing schemes

Evolution of the GIIPS risk contribution



Comparison of the indexing schemes

Performance simulations



⇒ RB indexation / WB indexation = better performance, same volatility and smaller drawdowns.

Comparison with active management

- Database: Morningstar
- Category: Bond EURO Government
- 209 funds[7]

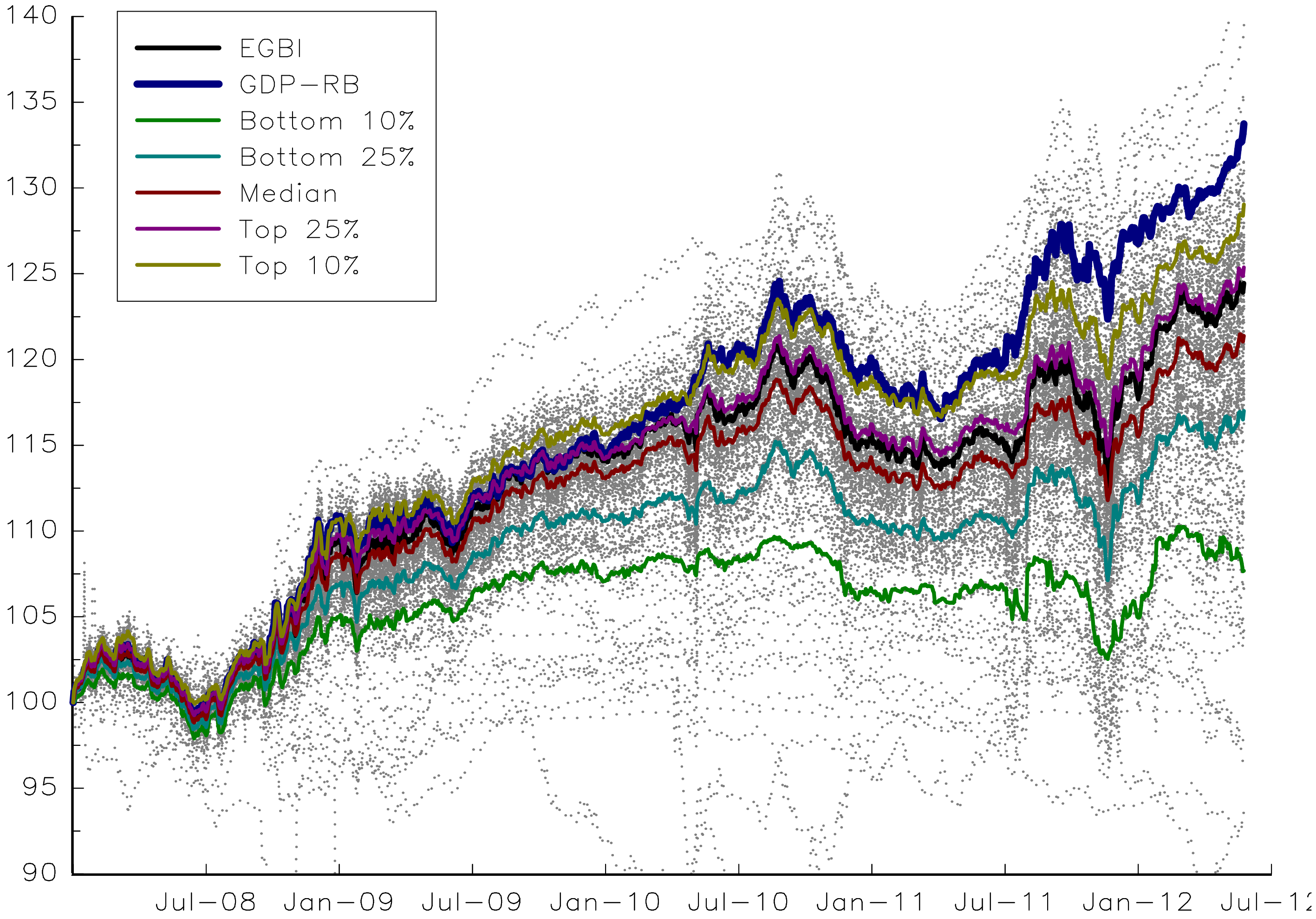
The Academic Rule[8]:

$$\text{Average Performance of Active Management} = \text{Performance of the Index} - \text{Management Fees}$$

$\Rightarrow$ Implied fees for Bond EURO Government: 57 bps / year[9]

[7] We don't take into account the survivorship bias.
[8] There is a large literature on this subject, see e.g. Blake *et al.* (1993).
[9] This figure was only 36 bps / year for the period 01/2008 - 08/2011.

Comparison with active management



#(funds > GDP-RB) = 5

Perf. of GDP-RB Index†

=

Perf. of Top 10%
+ 66 bps / year

† Transaction costs = 15 bps / year

Conclusion

- Risk-budgeting approach = a better approach than portfolio optimization / new theoretical results (Bruder and Roncalli, 2012).

- Credit risk must be managed in bond portfolios.
- Debt-weighted indexation has some limits.

- Fundamental indexation is not enough and must be completed with risk-based methods.
- Risk-budgeting approach is a good compromise between managing the performance and managing the risk.

- Lyxor Asset Management will launch a series of bond indexes based on risk-budgeting techniques.
  - The Lyxor SmartIX Risk Balanced EMU Government Bonds Index is already available (Ticker: RBEGBI Index).

For Further Reading

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Managing Sovereign Credit Risk in Bond Portfolios.
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Journal of Indexes Europe, 1(4), November 2011.

S. Maillard, T. Roncalli, J. Teiletche.
The Properties of Equally Weighted Risk Contribution Portfolios.
Journal of Portfolio Management, 36(4), Summer 2010.

Appendix

- Rediscovering the sovereign credit risk
- The eurozone crisis

Major financial crises

- 200 sovereign external defaults and 68 internal defaults since 1800
- 1982 (Mexico, Latin America)
- 1997 (Asia, Russia)
- 2001 (Turkey, Argentina)

Sovereign credit risk concerns generally emerging markets:

Argentina (1982-83, 1989, 1995, 2001), Brazil (1982, 1987, 1990, 1994, 1998-99), Mexico (1982, 1989, 1994-95), Nigeria (1987, 1989, 1996, 1999), Pakistan (1983, 2000), Philippine (1983, 1997), Turkey (1982, 1994, 1999, 2001), etc.

⇒ Costs may be huge: Argentina (55% of GDP, 1980-82), Japan (10% of GDP, 1990s), Chile (41% of GDP, 1981-87), Israel (30% of GDP, 1977-83), United States (3% of GDP, 1980s), Sweden (10% of GDP, 1990-93), etc.

Source: Crockett (1997), Wyplosz (1999).

Definition

The country risk encompasses three types of risk:

- External debt (e.g. Russian crisis)
- Banking system (e.g. U.S. savings and loan crisis)
- Financial crisis (e.g. Subprime crisis)

Problem with sovereign risk lending (Kindleberger, 1939, 1978)

1. Overenthusiasm
2. Loss of confidence
3. Credit rationing

Instability as a public bad: market failures (Wyplosz, 1999)

1. Moral hazard
2. Adverse selection
3. Multiple equilibria (bad/good)

Specificity of sovereign debt: it takes a long time to deleverage (e.g. Canada, 2000)

The syndrome of this-time-is-different

The essence of this-time-is-different syndrome is simple. It is rooted in the firmly held belief that ***financial crises are things that happen to other people in other countries at other times****; crises do not happen to us, here and now. We are doing things better, we are smarter, we have learned from past mistakes. The old rules of valuation no longer apply. The current boom, unlike the many booms that preceded catastrophic collapses in the past (even in our country), is built on sound fundamentals, structural reforms, technological innovations, and good policy. Or so the story goes [...] Unfortunately, a* ***highly leveraged economy*** *can unwittingly be sitting with its back at the edge of a financial cliff for many years before chance and circumstance provoke a* ***crisis of confidence that pushes it off****.*

Source: Reinhart and Rogoff (2009).

Debt crisis or Euro crisis?

- 2001-2007: No major financial crises
- 2007-2008: Subprime crisis
- September 2008: Bankruptcy of Lehman Brothers
- April 2010: Greece sovereign debt rating is cut to BB+
- August 2011: Downgrade of US government debt

Theory of debt sustainability

- Ponzi scheme
- Bubbles as payoffs at infinity
- Instability of the debt ratio dynamics (primary balance, real interest rate & output)

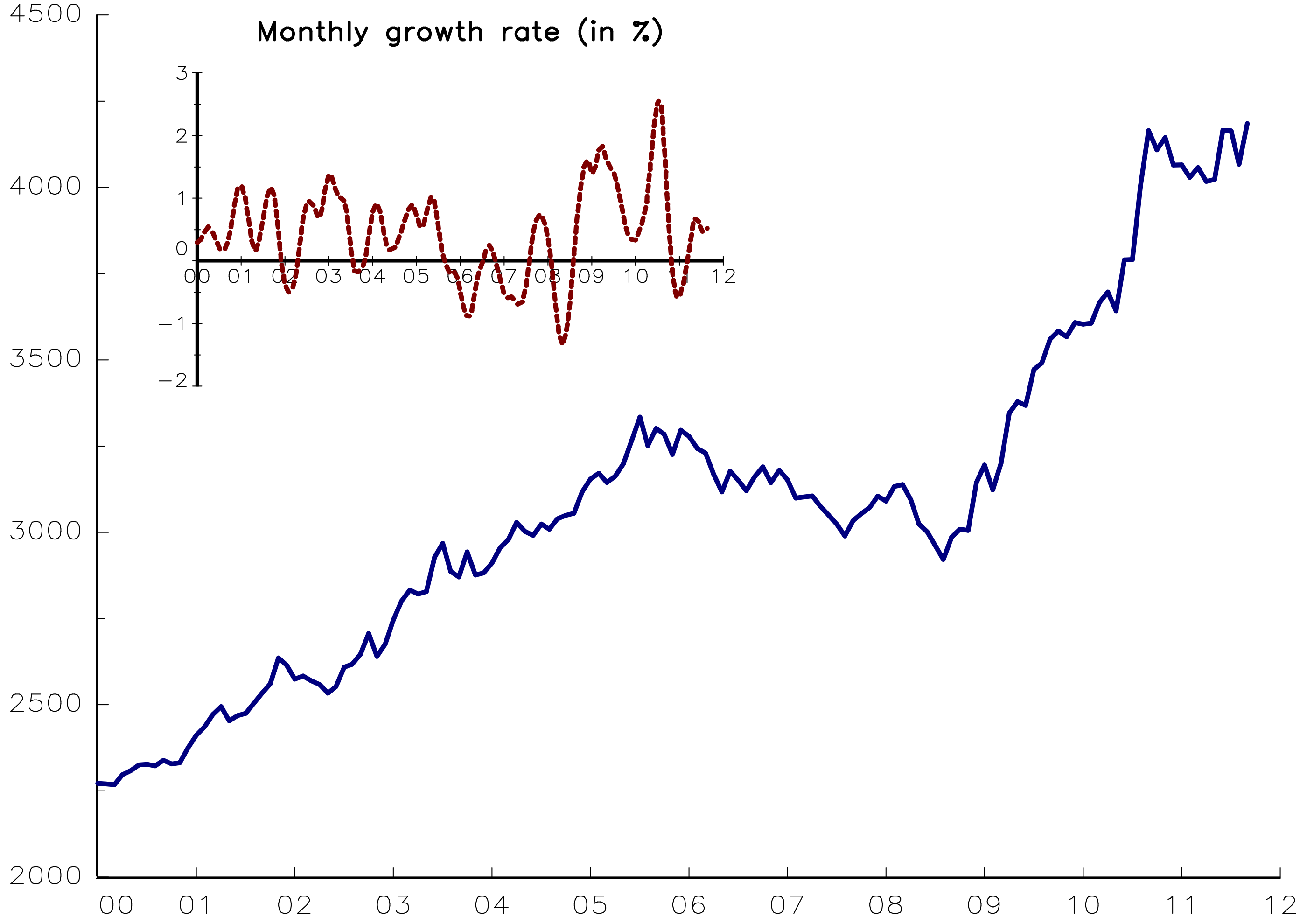
Theory of optimum currency area

It describes the optimal characteristics for the creation of a currency (e.g. a monetary union)

Is the eurozone an optimal currency area?

The eurozone debt

Evolution of the debt market value (in MEUR)



Source: Datastream.

Exposure of investors to the eurozone risk

From domestic bonds to non-domestic bonds (since 2000) ⇒ Diversification of sovereign bond portfolios

- Institutional investors
- Pension funds
- Retail ("contrat en euro")

Figure: Detention of the public debt in 2008

Country	France	Germany	Greece	Italy	Portugal	Spain	UK
France	29.2%	9.1%	16.8%	9.4%	26.6%	12.9%	3.4%
Germany	5.9%	26.5%	9.5%	6.2%	16.3%	13.1%	2.3%
Italy	3.1%	3.0%	6.7%	47.4%	4.5%	1.8%	0.9%
Japan	4.1%	5.1%	2.1%	2.2%	1.1%	1.2%	2.1%
UK	4.0%	4.4%	3.2%	3.3%	3.6%	4.1%	65.3%
US	2.4%	2.8%	0.3%	0.6%	0.2%	1.9%	4.8%
Domestic	29.2%	26.5%	22.0%	47.4%	1.3%	36.4%	65.3%
Non-resident	70.8%	73.5%	78.0%	52.6%	98.7%	63.6%	34.7%

Source: Broyer and Bruner (2010).