How Quantitative Methods Can Help To Understand Some Asset Management Problems?[1]

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[1]The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.

Outline

The Problem

An index I_t at time t is defined by:

$$I_t = \sum_{i=1}^{n} w_{i,t} P_{i,t}$$

where $w_{i,t}$ and $P_{i,t}$ is the weight and the price of the i^{th} asset at date t.

We are interested in two types of weights:

- Weights can depend on prices:

$$w_{i,t} = f\left(P_{i,t}\right)$$

- Weights are not linked to prices:

$$w_{i,t} \perp P_{i,t}$$

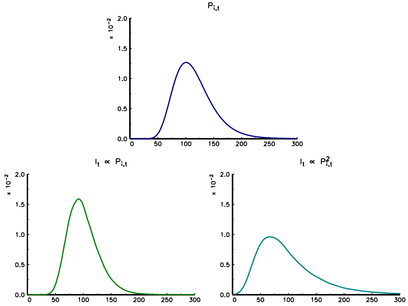
The Problem

Some weighting schemes are not good

If we suppose that we have $w_{i,t} = \varpi_{i,t} P_{i,t}$, we obtain:

$$I_t = \sum_{i=1}^{n} \varpi_{i,t} P_{i,t}^2$$

If prices are log-normal distributed, what is the distribution of I_t ?



Pros and Cons of Market-cap Indexation

Pros of market-cap indexation

- A convenient and **recognized approach** to participate to broad equity markets.
- **Management simplicity**: low turnover & transaction costs.

Cons of market-cap indexation

- Trend-following strategy: momentum bias leads to bubble risk exposure as weight of best performers ever increases.
  ⇒ Mid 2007, financial stocks represent 40% of the Eurostoxx 50 index.
- Growth biais as high valuation multiples stocks weight more than low-multiple stocks with equivalent realised earnings.
  ⇒ Mid 2000, the 8 stocks of the technology/telecom sectors represent 35% of the Eurostoxx 50 index.
  ⇒ 2¹/₂ years later after the dot.com bubble, these two sectors represent 12%.
- Concentrated portfolios.
  ⇒ The top 100 market caps of the S&P 500 account for around 70%.
- Lack of risk diversification and high drawdown risk: no portfolio construction rules leads to concentration issues (e.g. sectors, stocks).

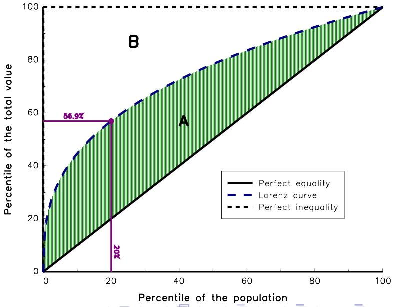
Statistical Measures of Concentration

- The Lorenz curve $\mathscr{L}(x)$
  It is a graphical representation of the concentration. It represents the cumulative weight of the first $x\%$ most representative stocks.

- The Gini coefficient
  It is a dispersion measure based on the Lorenz curve:

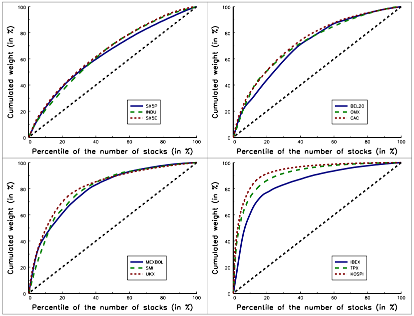
$$G = \frac{A}{A+B} = 2\int_0^1 \mathscr{L}(x)\,\mathrm{d}x - 1$$

  G takes the value 1 for a perfectly concentrated portfolio and 0 for the equally-weighted portfolio.



Concentration of Equity Indexes (December 31, 2009)

Index	Gini	$\mathscr{L}(x)$ 10	$\mathscr{L}(x)$ 25	$\mathscr{L}(x)$ 50
SX5P	0.27	23	45	68
INDU	0.29	21	42	71
SX5E	0.31	24	45	71
BEL20	0.41	28	51	79
OMX	0.44	33	57	79
CAC	0.47	34	58	82
DAX	0.47	29	58	84
HSI	0.51	39	63	83
AEX	0.51	34	62	85
NDX	0.53	47	66	82
NKY	0.59	47	69	87
MEXBOL	0.59	44	68	89
SMI	0.60	41	71	90
SPX	0.63	52	73	89
UKX	0.63	49	76	89
SXXE	0.64	52	76	90
HSCEI	0.64	53	77	90
SXXP	0.67	57	78	90
IBEX	0.69	61	81	91
TPX	0.82	74	90	97
KOSPI	0.86	81	94	98



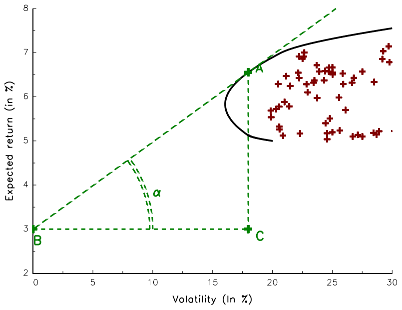
(*) In the case of the SX5P Index, 10% of stocks (respectively 25% and 50%) represent 23% of weight in the index (respectively 45% and 68%).

Main argument of passive management :

The Market Cap Index = The Tangency Portfolio

In the modern portfolio theory of Markowitz, we maximize the expected return for a given level of volatility:

$$\max \mu(w) = \mu^\top w \quad \text{u.c.} \quad \sigma(w) = \sqrt{w^\top \Sigma w} = \sigma^\star$$



- The optimal portfolio is the tangency portfolio.
- Main problem: the solution is very sensitive to the vector of expected returns ⇒ the solution is not robust.
- If the market cap index is the optimal portfolio, it means that expected returns are persistent.

- Academic research has illustrated that Capitalization-weighted indexes are not tangency portfolios.
- Dynamics of cap-weighted indexes = dynamics of price-weighted indexes (e.g. Nikkei and Topix indexes).

Alternative-Weighted Indexation

Alternative-weighted indexation aims at building passive indexes where the weights are not based on market capitalization.

Two sets of responses:

1. Fundamental indexation $\Rightarrow$ promising alpha.
2. Risk-based indexation $\Rightarrow$ promising diversification.

Two ways of using risk-based indexation:

1. Substitute as the capitalized-weighted index.
2. Complement to the capitalized-weighted index.

Portfolio Construction

Equally-weighted (1/n)

Most Diversified Portfolio (MDP)

Minimum-variance (MV)

Equal-Risk Contribution (ERC)

Notations

Let w be the vector of weights, μ the vector of risk premia (e.g. expected returns) and Σ the covariance matrix of returns. The volatility of the portfolio is:

$$\sigma(w) = \sqrt{w^\top \Sigma w}$$

wheras its expected return is:

$$\mu(w) = w^\top \mu$$

The 1/n Portfolio

We have:

$$w_i = \frac{1}{n}$$

Some properties

- It is the less concentrated portfolio:

$$G_w = 0$$

- It is a contrarian strategy.
- It has a take-profit scheme.

The Minimum-Variance Portfolio

The problem is: $w^{\star} = \arg\min \sqrt{w^{\top}\Sigma w}$ u.c. $\mathbf{1}^{\top}x = 1$ and $\mathbf{0} \leq x \leq \mathbf{1}$.
In the short-selling case, the lagrangian function is:

$$f(w;\lambda_0) = \sigma(w) - \lambda_0 \left(\mathbf{1}^{\top}w - 1\right)$$

The solution $w^{\star}$ verifies the following system of first-order conditions:

$$\begin{cases} \partial_x f(w;\lambda_0) = \frac{\partial\,\sigma(w)}{\partial\, w} - \lambda_0 \mathbf{1} = 0 \\ \partial_{\lambda_0} f(w;\lambda_0) = \mathbf{1}^{\top} w - 1 = 0 \end{cases}$$

We have:

$$\frac{\partial\,\sigma(w)}{\partial\, w_i} = \frac{\partial\,\sigma(w)}{\partial\, w_j} = \sigma(w) \quad \text{for all } i,j$$

In the case of no-short selling, write the Kühn-Tucker conditions and we have:

$$\boxed{\frac{\partial\,\sigma(w)}{\partial\, w_i} = \frac{\partial\,\sigma(w)}{\partial\, w_j} \quad \text{for all } w_i \neq 0, w_j \neq 0}$$

The MDP/MSR Portfolio

Let $D(w)$ be the diversification ratio:

$$D(w) = \frac{\sqrt{w^\top \tilde{\Sigma} w}}{\sqrt{w^\top \Sigma w}} = \frac{w^\top \sigma}{\sqrt{w^\top \Sigma w}}$$

where $\tilde{\Sigma}$ is the covariance matrix with $\tilde{\Sigma}_{i,j} = \sigma_i \sigma_j$ (all the correlations are equal to one). We have $D(w) \geq 1$. The MDP portfolio is defined by:

$$\begin{aligned} w^\star &= \arg\max D(w) \\ \text{u.c.} \quad & \mathbf{1}^\top x = 1 \text{ and } \mathbf{0} \leq x \leq \mathbf{1} \end{aligned}$$

Remark

If we assume that the Sharpe ratio is the same for all the assets – $\mu_i - r = s \times \sigma_i$, we obtain:

$$\text{sh}(w) = \frac{w^\top \mu - r}{\sqrt{w^\top \Sigma w}} = s \times D(w)$$

Maximizing $D(w)$ is equivalent to maximize $\text{sh}(w)$.

The ERC Portfolio

The Euler decomposition gives us:

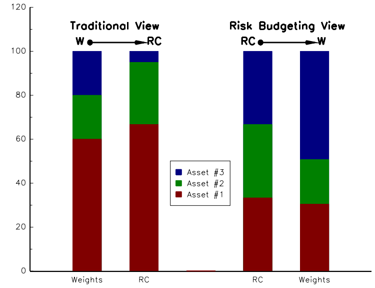
$$\sigma(w) = \sum_{i=1}^{n} w_i \times \frac{\partial \sigma(w)}{\partial w_i} = \sum_{i=1}^{n} \mathrm{RC}_i$$

The idea of the ERC strategy is to find a risk-balanced portfolio such that the risk contribution is the same for all assets of the portfolio:

$$\mathrm{RC}_i = \mathrm{RC}_j \quad \text{for all } i,j$$

An example

3 assets.
Volatilities are respectively 20%, 30% and 15%.
Correlations are set to 60% between the first and second asset, and 10% for the third assets.



Traditional View

w_i	MR_i	RC_i	in %
60.0%	18.8%	11.3%	66.7%
20.0%	23.9%	4.8%	28.3%
20.0%	4.3%	0.9%	5.0%
Volatility		16.9%	

Risk Budgeting View

w_i	MR_i	RC_i	in %
48.5%	17.7%	8.6%	**60.0%**
13.2%	21.7%	2.9%	**20.0%**
38.3%	7.5%	2.9%	**20.0%**
Volatility		14.3%	

ERC View

w_i	MR_i	RC_i	in %
30.4%	15.2%	4.6%	**33.3%**
20.3%	22.7%	4.6%	**33.3%**
49.3%	9.3%	4.6%	**33.3%**
Volatility		13.8%	

Comparison of the 4 Methods

Equally-weighted (1/n)

- Weights are equal
- Easy to understand
- Contrarian strategy with a take-profit scheme
- The least concentrated in terms of weights
- Do not depend on risks

Most Diversified Portfolio (MDP)

- Also known as the Max Sharpe Ratio (MSR) portfolio of EDHEC
- Based on the assumption that sharpe ratio is equal for all stocks
- It is the tangency portfolio if the previous assumption is verified
- Sensitive to the covariance matrix

Minimum-variance (MV)

- Low volatility portfolio
- The only optimal portfolio not depending on expected returns assumptions
- Good out of sample performance
- Concentrated portfolios
- Sensitive to the covariance matrix

Equal-Risk Contribution (ERC)

- Risk contributions are equal
- Highly diversified portfolios
- Less sensitive to the covariance matrix (than the MV and MDP portfolios)
- Not efficient for universe with a large number of stocks (equivalent to the 1/n portfolio)

Comparison of the 4 Methods

In terms of bets

$$\exists i : w_i = 0 \quad \text{(MV - MDP)}$$
$$\forall i : w_i \neq 0 \quad \text{(1/n - ERC)}$$

In terms of risk factors

$$
\begin{array}{rcll}
w_i & = & w_j & \text{(1/n)} \\
\dfrac{\partial\,\sigma(w)}{\partial\,w_i} & = & \dfrac{\partial\,\sigma(w)}{\partial\,w_j} & \text{(MV)} \\
w_i \times \dfrac{\partial\,\sigma(w)}{\partial\,w_i} & = & w_j \times \dfrac{\partial\,\sigma(w)}{\partial\,w_j} & \text{(ERC)} \\
\dfrac{1}{\sigma_i} \times \dfrac{\partial\,\sigma(w)}{\partial\,w_i} & = & \dfrac{1}{\sigma_j} \times \dfrac{\partial\,\sigma(w)}{\partial\,w_j} & \text{(MDP)}
\end{array}
$$

Backtest with the DJ Eurostoxx 50 Universe

Backtesting rules

Monthly rebalancing of the weights.
The covariance matrix used for simulations is the empirical covariance matrix based on a rolling observation period of 1 year.
All indexes are price index (PI).
The study period is January 1993 – December 2009.

	CW	MV	ERC	MDP	1/n
Performance	6.39	8.08	10.30	12.63	9.22
Volatility	22.41	17.65	20.66	20.00	22.43
Sharpe	0.29	0.46	0.50	0.62	0.41
Volatility of TE		14.85	5.98	13.19	4.37
IR		0.11	0.65	0.47	0.65
Drawdown	66.88	55.89	56.84	49.95	61.79
Skewness (monthly)	−0.50	−1.06	−0.55	−0.58	−0.45
Kurtosis (monthly)	3.87	5.31	4.42	4.25	4.70
Skewness	0.06	2.12	0.24	3.44	0.08
Kurtosis	8.63	59.59	11.05	90.58	9.71
Correlation	100.00	75.00	94.66	81.24	98.10

Composition in % (January 2010)

	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
TOTAL	6.1		2.1		2			5.0	
BANCO SANTANDER	5.8		1.3		2				
TELEFONICA SA	5.0	31.2	3.5		2	10.0		5.0	5.0
SANOFI-AVENTIS	3.6	12.1	4.5	15.5	2	10.0	10.0	5.0	5.0
E.ON AG	3.6		2.1		2				1.4
BNP PARIBAS	3.4		1.1		2				
SIEMENS AG	3.2		1.5		2				
BBVA(BILB-VIZ-ARG)	2.9		1.4		2				
BAYER AG	2.9		2.6	3.7	2	2.2	5.0	5.0	5.0
ENI	2.7		2.1		2				
GDF SUEZ	2.5		2.6	4.5	2		5.4	5.0	5.0
BASF SE	2.5		1.5		2				
ALLIANZ SE	2.4		1.4		2				
UNICREDIT SPA	2.3		1.1		2				
SOC GENERALE	2.2		1.2	3.9	2		3.7		5.0
UNILEVER NV	2.2	11.4	3.7	10.8	2	10.0	10.0	5.0	5.0
FRANCE TELECOM	2.1	14.9	4.1	10.2	2	10.0	10.0	5.0	5.0
NOKIA OYJ	2.1		1.8	4.5	2		4.8		5.0
DAIMLER AG	2.1		1.3		2				
DEUTSCHE BANK AG	1.9		1.0		2				
DEUTSCHE TELEKOM	1.9		3.2	2.6	2	5.7	3.7	5.0	5.0
INTESA SANPAOLO	1.9		1.3		2				
AXA	1.8		1.0		2				
ARCELORMITTAL	1.8		1.0		2				
SAP AG	1.8	21.0	3.4	11.2	2	10.0	10.0	5.0	5.0

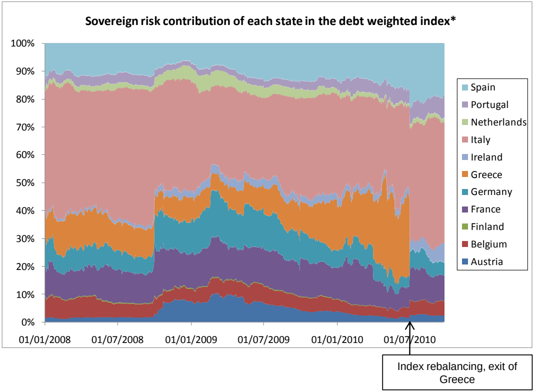
	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
RWE AG (NEU)	1.7	2.7	2.7		2	7.0		5.0	
ING GROEP NV	1.6		0.8	0.4	2				
DANONE	1.6	1.9	3.4	1.8	2	8.7	3.3	5.0	5.0
IBERDROLA SA	1.6		2.5		2	5.1		5.0	1.2
ENEL	1.6		2.1		2			5.0	2.9
VIVENDI SA	1.6	2.8	3.1	4.5	2	10.0	5.9	5.0	5.0
ANHEUSER-BUSCH INB	1.6	0.2	2.7	10.9	2	2.1	10.0	5.0	5.0
ASSIC GENERALI SPA	1.6		1.8		2				
AIR LIQUIDE(L')	1.4		2.1		2			5.0	
MUENCHENER RUECKVE	1.3		2.1	2.1	2		3.1	5.0	5.0
SCHNEIDER ELECTRIC	1.3		1.5		2				
CARREFOUR	1.3	1.0	2.7	1.3	2	3.7	2.5	5.0	5.0
VINCI	1.3		1.6		2				
LVMH MOET HENNESSY	1.2		1.8		2				
PHILIPS ELEC(KON)	1.2		1.4		2				
L'OREAL	1.1	0.8	2.8		2	5.5		5.0	5.0
CIE DE ST-GOBAIN	1.0		1.1		2				
REPSOL YPF SA	0.9		2.0		2			5.0	
CRH	0.8		1.7	5.1	2		5.2		5.0
CREDIT AGRICOLE SA	0.8		1.1		2				
DEUTSCHE BOERSE AG	0.7		1.5		2				1.9
TELECOM ITALIA SPA	0.7		2.0		2				2.5
ALSTOM	0.6		1.5		2				
AEGON NV	0.4		0.7		2				
VOLKSWAGEN AG	0.2		1.8	7.1	2		7.4		5.0
Total of components	50	11	50	17	50	14	16	20	23

Conclusion

- Risk-based indexation historically posts better risk-adjusted performance than capitalization-weighted indexation.
- It is a promising way for investors to gain access to a well-diversified and diversifying exposure (or beta) to broad equity markets.
- Some practical issues:
  1. Turnover managing;
  2. Market price impact minimizing;
  3. Transparency (passive indexation or active strategy?);
  4. Understanding the style bias (small caps, growth, sectors, etc.);
- Existence of professional solutions (indexes/mutual funds).

And Bonds?

- Market-cap indexation based on outstanding amount of debt.
- Ignores risk dimension (e.g. sovereign risk, country risk, etc.).



November 2010

	EGBI Weight	Risk contribution
Italy	23.5%	42.4%
Spain	9.7%	18.6%
France	22.2%	11.2%
Portugal	2.1%	8.3%
Ireland	2.1%	6.8%
Belgium	6.5%	4.7%
Germany	22.9%	4.5%
Austria	4.1%	2.0%
Netherlands	5.7%	1.4%
Finland	1.3%	0.2%
Greece	0.0%	0.0%
Spain + Italy + Portugal + Ireland	37.4%	76.0%

The 3 Profiles

Fund Profiles

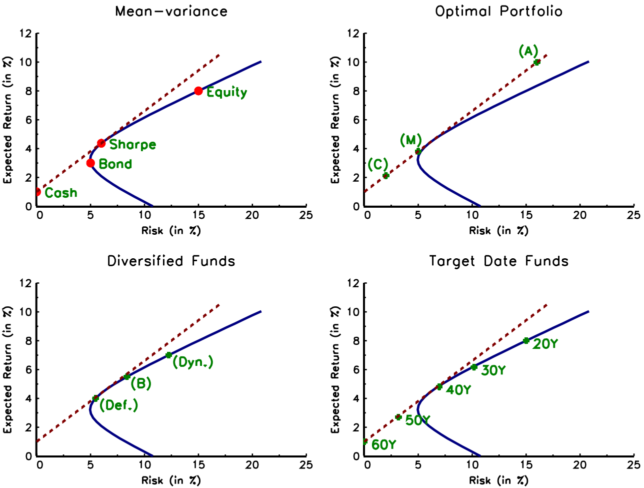
1. Dynamic (20% of bonds and 80% of equities)
2. Balanced (50% of bonds and 50% of equities)
3. Defensive (80% of bonds and 20% of equities)

Investor Profiles

1. Aggressive (high risk tolerance)
2. Moderate (medium risk tolerance)
3. Conservative (low risk tolerance)

Relationship with Portfolio Theory

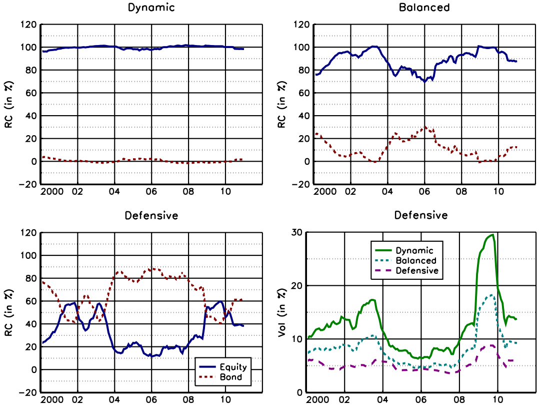
The asset allocation puzzle



Diversification effects

Risk contribution

We consider a backtest with MSCI World (hedged in EUR) and EuroMTS 10Y-15Y.



- Deleverage of an equity exposure
- Diversification in weights $\neq$ Risk diversification
- No mapping between fund profiles and investor profiles

Some Partial Answers

1. Cash is a risky asset in the long term.
2. Bonds have not the same maturity.
3. Stochastic income.

See Campbell and Viciera (2002), Bajeux-Besnainou *et al.* (2003), Cocco *et al.* (2005) or Munk and Sørensen (2010).

Main Result

We consider a universe of n assets. We denote by μ the vector of their expected returns and by Σ the corresponding covariance matrix. We specify the optimization problem as follows:

$$\min \frac{1}{2} w^\top \Sigma w$$

$$\text{u.c.} \begin{cases} \mathbf{1}^\top w = 1 \\ \mu^\top w \geq \mu^\star \\ w \in \mathbb{R}^n \bigcap \mathscr{C} \end{cases}$$

where w is the vector of weights in the portfolio and $\mathscr{C}$ is the set of weights constraints. We define:

- the unconstrained portfolio $w^\star$ or $w^\star(\mu, \Sigma)$:

$$\mathscr{C} = \mathbb{R}^n$$

- the constrained portfolio $\tilde{w}$:

$$\mathscr{C}\left(w^-, w^+\right) = \left\{w \in \mathbb{R}^n : w_i^- \leq w_i \leq w_i^+\right\}$$

Main Result

Theorem

Jagannathan and Ma (2003) show that the constrained portfolio is the solution of the unconstrained problem:

$$\tilde{w} = w^{\star}\left(\tilde{\mu}, \tilde{\Sigma}\right)$$

with:

$$\begin{cases} \tilde{\mu} = \mu \\ \tilde{\Sigma} = \Sigma + \left(\lambda^{+} - \lambda^{-}\right)\mathbf{1}^{\top} + \mathbf{1}\left(\lambda^{+} - \lambda^{-}\right)^{\top} \end{cases}$$

where λ^{-} and λ^{+} are the Lagrange coefficients vectors associated to the lower and upper bounds.

Main Result

Interpretation

We have $\tilde{\Sigma}_{i,j} = \Sigma_{i,j} + \Delta_{i,j}$ with:

$(\Delta)_{i,j}$	w_i^-	$\left]w_i^-, w_i^+\right[$	w_i^+
w_j^-	$-\left(\lambda_i^- + \lambda_j^-\right)$	$-\lambda_j^-$	$\lambda_i^+ - \lambda_j^-$
$\left]w_j^-, w_j^+\right[$	$-\lambda_i^-$	0	λ_i^+
w_j^+	$\lambda_j^+ - \lambda_i^-$	λ_j^+	$\lambda_i^+ + \lambda_j^+$

The perturbation $\Delta_{i,j}$ may be negative, nul or positive.

1. For the volatility, we obtain $\tilde{\sigma}_i = \sqrt{\sigma_i^2 + \Delta_{i,i}}$.

w_i^-	$\left]w_i^-, w_i^+\right[$	w_i^+
$\tilde{\sigma}_i < \sigma_i$	$\tilde{\sigma}_i = \sigma_i$	$\tilde{\sigma}_i > \sigma_i$

2. For the correlation, we obtain $\tilde{\rho}_{i,j} = \dfrac{\rho_{i,j}\sigma_i\sigma_j + \Delta_{i,j}}{\sqrt{\left(\sigma_i^2 + \Delta_{i,i}\right)\left(\sigma_j^2 + \Delta_{j,j}\right)}}$.

$\Rightarrow$ Similar to the **Black-Litterman** approach.

Proof for the Global Minimum Variance Portfolio

We define the Lagrange function as $f(w;\lambda_0) = \frac{1}{2} w^\top \Sigma w - \lambda_0 \left(\mathbf{1}^\top w - 1\right)$ with $\lambda_0 \geq 0$. The first order conditions are $\Sigma w - \lambda_0 \mathbf{1} = 0$ and $\mathbf{1}^\top w - 1 = 0$. We deduce that the optimal solution is:

$$w^\star = \lambda_0^\star \Sigma^{-1}\mathbf{1} = \frac{1}{\mathbf{1}^\top \Sigma \mathbf{1}} \Sigma^{-1}\mathbf{1}$$

With weights constraints $\mathscr{C}(w^-, w^+)$, we have:

$$f\left(w;\lambda_0,\lambda^-,\lambda^+\right) = \frac{1}{2} w^\top \Sigma w - \lambda_0 \left(\mathbf{1}^\top w - 1\right) - \lambda^{-\top}\left(w - w^-\right) - \lambda^{+\top}\left(w^+ - w\right)$$

with $\lambda_0 \geq 0$, $\lambda_i^- \geq 0$ and $\lambda_i^+ \geq 0$. In this case, the first-order conditions becomes $\Sigma w - \lambda_0 \mathbf{1} - \lambda^- + \lambda^+ = 0$ and $\mathbf{1}^\top w - 1 = 0$. We have:

$$\tilde{\Sigma}\tilde{w} = \left(\Sigma + \left(\lambda^+ - \lambda^-\right)\mathbf{1}^\top + \mathbf{1}\left(\lambda^+ - \lambda^-\right)^\top\right)\tilde{w} = \left(2\tilde{\lambda}_0 - \tilde{w}^\top \Sigma \tilde{w}\right)\mathbf{1}$$

Because $\tilde{\Sigma}\tilde{w}$ is a constant vector, it proves that $\tilde{w}$ is the solution of the unconstrained optimisation problem with $\lambda_0^\star = \left(2\tilde{\lambda}_0 - \tilde{w}^\top \Sigma \tilde{w}\right)$.

Examples

Table: Specification of the covariance matrix Σ (in %)

σ_i	$\rho_{i,j}$			
15.00	100.00			
20.00	10.00	100.00		
25.00	40.00	70.00	100.00	
30.00	50.00	40.00	80.00	100.00

Given these parameters, the global minimum variance portfolio is equal to:

$$w^{\star} = \begin{pmatrix} 72.742\% \\ 49.464\% \\ -20.454\% \\ -1.753\% \end{pmatrix}$$

Table: Global minimum variance portfolio when $w_i \geq 10\%$

$\tilde{w}_i$	λ_i^-	λ_i^+	$\tilde{\sigma}_i$	$\tilde{\rho}_{i,j}$			
56.195	0.000	0.000	15.000	100.000			
23.805	0.000	0.000	20.000	10.000	100.000		
10.000	1.190	0.000	19.671	10.496	58.709	100.000	
10.000	1.625	0.000	23.980	17.378	16.161	67.518	100.000

Table: Global minimum variance portfolio when $0\% \leq w_i \leq 50\%$

$\tilde{w}_i$	λ_i^-	λ_i^+	$\tilde{\sigma}_i$	$\tilde{\rho}_{i,j}$			
50.000	0.000	1.050	20.857	100.000			
50.000	0.000	0.175	20.857	35.057	100.000		
0.000	0.175	0.000	24.290	46.881	69.087	100.000	
0.000	0.000	0.000	30.000	52.741	41.154	79.937	100.000

Table: MSR portfolio when $0\% \leq w_i \leq 40\%$ and $\mathrm{sh}^{\star} = 0.5$

$\tilde{w}_i$	λ_i^-	λ_i^+	$\tilde{\sigma}_i$	$\tilde{\rho}_{i,j}$			
40.000	0.000	0.810	19.672	100.000			
40.000	0.000	0.540	22.539	37.213	100.000		
0.000	0.000	0.000	25.000	46.970	71.698	100.000	
20.000	0.000	0.000	30.000	51.850	43.481	80.000	100.000

We obtain:

$$\widetilde{\mathrm{sh}} = \begin{pmatrix} 0.381 \\ 0.444 \\ 0.5 \\ 0.5 \end{pmatrix}$$

Application to the DJ Eurostoxx 50

Backtest with monthly rebalancing and one-year empirical covariance matrix.

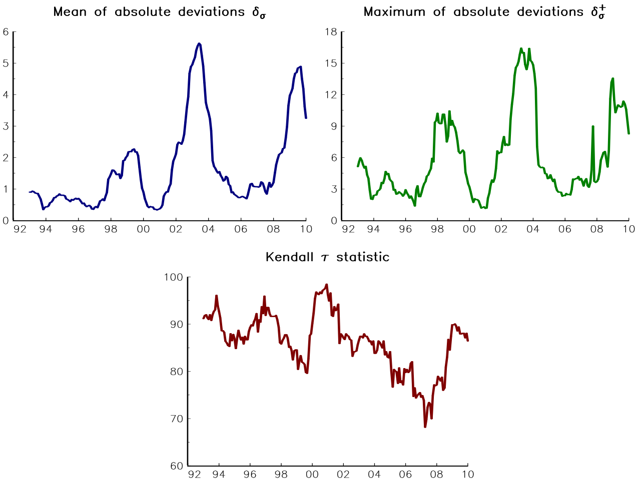
Lower bound is set to 0% and upper bound is set to 5%.

We define:

- the mean absolute deviations as $\delta_\sigma = \frac{1}{n}\sum_{i=1}^{n} \left|\tilde{\sigma}_{i,t} - \sigma_{i,t}\right|$ for the volatility and $\delta_\rho = \frac{2}{n(n-1)}\sum_{i>j} \left|\tilde{\rho}_{i,j,t} - \rho_{i,j,t}\right|$ for the correlation;
- the maximum of absolute deviations as $\delta_\sigma^+ = \max_i \left|\tilde{\sigma}_{i,t} - \sigma_{i,t}\right|$ for the volatility and $\delta_\rho^+ = \max_{i,j} \left|\tilde{\rho}_{i,j,t} - \rho_{i,j,t}\right|$ for the correlation;
- The peak-over-threshold frequency: $\pi_\rho(x) = \frac{2}{n(n-1)}\sum_{i>j} \mathbf{1}\left\{\left|\tilde{\rho}_{i,j,t} - \rho_{i,j,t}\right| > x\right\}$.

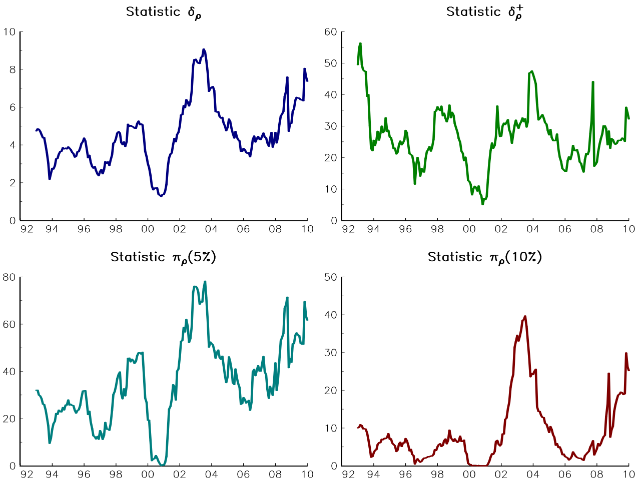
Application to the DJ Eurostoxx 50

Impact (in %) on the volatilities for the MIN portfolio



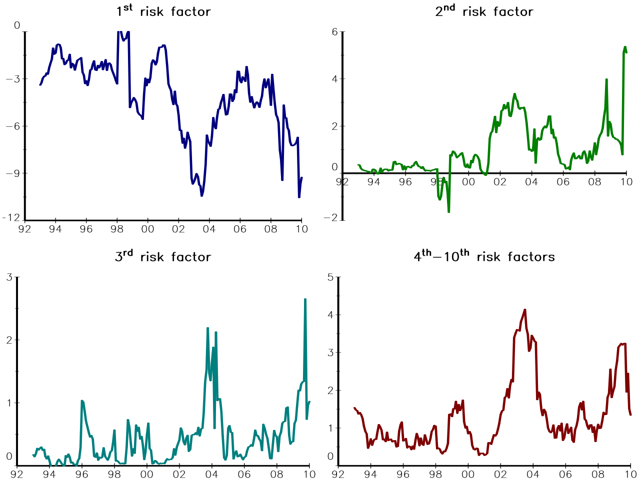
Application to the DJ Eurostoxx 50

Impact (in %) on the correlations for the MIN portfolio



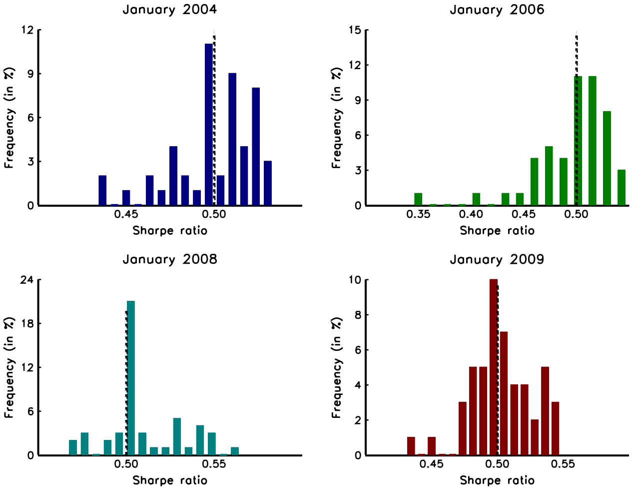
Application to the DJ Eurostoxx 50

Impact (in %) on the risk factors for the MIN portfolio



Application to the DJ Eurostoxx 50

Density of the implied Sharpe ratio for the MSR portfolio



Option Trading & Asset Management strategies

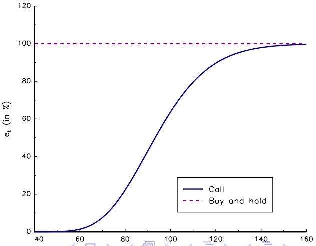
Option Trading strategy $\neq$ Asset Management strategy

- Arbitrage theory
- Asset prices probability distribution (risk-neutral / historical)
- Management style (formula-based / systematic / discretionary)
- Mathematical tools (Itô calculus / statistics)

A long-position in a call option is equivalent to a long position in the underlying:

$$e_t = \Delta_t$$

$\Rightarrow$ A call option = trend-following (long-only) strategy.
$\Rightarrow$ The cost of a call option = call premium.



Decomposing a Strategy into a Payoff and a Cost Function

We consider a systematic strategy where the number n_t of invested shares depends on the price asset $S_t : n_t = f(S_t)$. Let X_t be the value of the strategy (or the fund). The risky exposure and the dynamic of the fund are given by :

$$\begin{aligned} e_t &= n_t \frac{S_t}{X_t} = f(S_t) \frac{S_t}{X_t} \\ \mathrm{d}X_t &= f(S_t)\,\mathrm{d}S_t \end{aligned}$$

If we assume that $\mathrm{d}S_t = \mu(S_t)\,\mathrm{d}S_t + \sigma_t S_t\,\mathrm{d}S_t$, Bruder and Gaussel (2010) show that :

$$X_T = \underbrace{X_0 + \int_{S_0}^{S_T} f(S)\,\mathrm{d}S}_{F(S_T)} - \underbrace{\frac{1}{2}\int_0^T \partial_S f(S_t)\, S_t^2 \sigma_t^2\,\mathrm{d}t}_{C_T}$$

Example: Stop loss, Take profit, etc.

Application to a Trend-following Strategy

1. *Long only* $f(S_t) = mS_t$:

$$F(S_T) = S_0 + \frac{1}{2}m\left(S_T^2 - S_0^2\right)$$

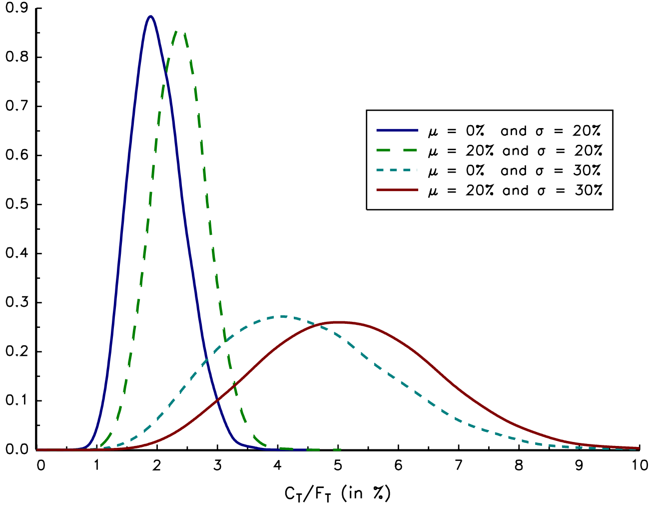
2. *Long short* $f(S_t) = m(S_t - S^\star)$:

$$F(S_T) = S_0 + \frac{1}{2}m\left((S_T - S^\star)^2 - (S_0 - S^\star)^2\right)$$

For these two cases, the cost function is:

$$C_T = \frac{1}{2}m\int_0^T S_t^2\sigma_t^2\,\mathrm{d}t \geq 0$$

This simple model explained some stylized facts of the CTA strategy (leverage, volatility, trends, long-term / short-term, etc.)


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
C_T/F_T (in %)
μ = 0% and σ = 20%
μ = 20% and σ = 20%
μ = 0% and σ = 30%
μ = 20% and σ = 30%

Some Issues

Market Timing

- Behaviorial (speculation) model
- No risk premium

Tactical Asset Allocation

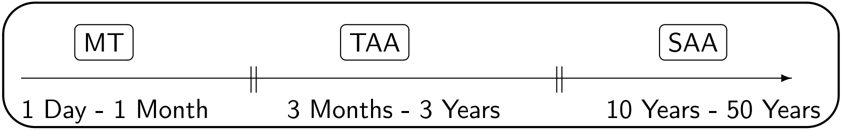
- Business cycle
- Time-varying risk premium

Lucas (1978), Campbell and Cochrane (1999).

Strategic Asset Allocation

- Growth model
- Stationary risk premium

Solow (1956), fundamental approach.



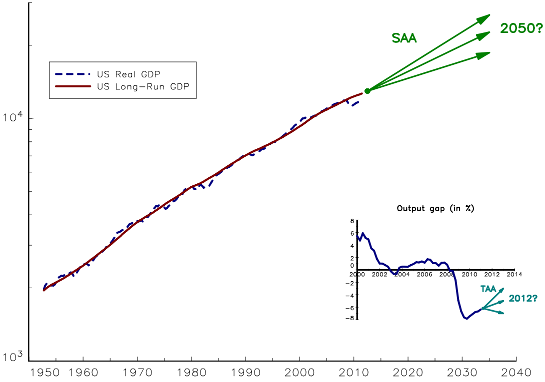
Our Approach

1. A comprehensive framework
2. Distinction between TAA and SAA
3. Based on economic models (Solow model, Golden rule, Philipps curve, Okun's law, NAIRU, etc.)
4. Understanding the concept of risk premium (and its link to the cointegration theory)
5. Sensitivity and Scenario analysis

Economic Modeling of Asset Returns

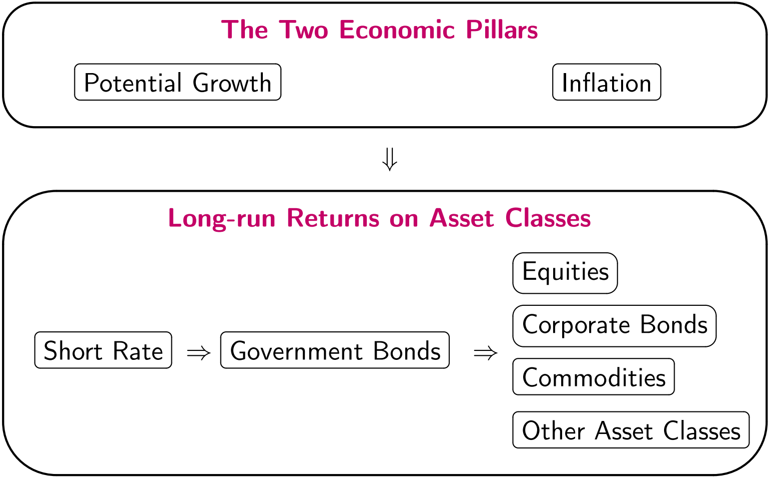
Economic Pillars

- Economic growth
  1. Solow model
  2. Golden rule
- Monetary policy and inflation
  1. Phillips curve
  2. NAIRU



Economic Modeling of Asset Returns

Asset Return and Risk Premium



Economic Modeling of Asset Returns

Short Rates

We propose to derive long-run short rates r_∞ from the lower bound of the normative Golden rule:

$$r_\infty = g_\infty + \pi_\infty$$

where g_∞ is the long-run real potential output growth and π_∞ is the long-run inflation.

	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	8.7%	6.1%	6.4%	4.5%	4.7%	4.8%
EURO	8.0%	4.8%	4.0%	3.6%	3.7%	3.8%
JAPAN	6.4%	2.9%	0.9%	2.4%	2.5%	2.6%
PACIFIC	14.5%	7.2%	5.5%	5.6%	5.3%	5.2%
EM				9.8%	9.3%	9.1%

Economic Modeling of Asset Returns

Sovereign Bonds

The long-run value of the nominal bond yield R_{∞}^{b} is equal to:

$$R_{\infty}^{b} = \mathscr{R}_{\infty}^{\mathrm{b}} + \pi_{\infty}$$

where $\mathscr{R}_{\infty}^{\mathrm{b}}$ is the long-run real bond yield $\mathscr{R}_{\infty}^{\mathrm{b}}$ and π_{∞} is the long-run inflation.

To estimate $\mathscr{R}_{\infty}^{\mathrm{b}}$, we consider the following regression model:

$$\mathscr{R}_{t}^{\mathrm{b}} = \beta_0 + \beta_1 \mathfrak{r}_t + \beta_2 \sigma_t^{\pi} + \beta_3 (B/Y)_t + \varepsilon_t$$

where $\mathfrak{r}_t$ is the real short rate, σ_t^{π} is the inflation risk and $(B/Y)_t$ is the government balance on output ratio (proxy for debt risk).

Economic Modeling of Asset Returns

Risky Bonds

The long-run bond yield R_{∞}^{cr} is equal to:

$$R_{\infty}^{\mathrm{cr}} = R_{\infty}^{\mathrm{b}} + s_{\infty}^{\mathrm{cr}}$$

where R_{∞}^{b} is the US long-run bond yield and s_{∞}^{cr} is the long-run spread.

For the investment grade and high yield spreads, the regression model is:

$$s_t^{\mathrm{cr}} = \beta_0 + \beta_1 \sigma_t^e + \beta_2 g_t + \varepsilon_t$$

where σ_t^e denotes the equity volatility and g_t is the output growth. For the emerging bond spread, the regression model becomes:

$$s_t^{\mathrm{cr}} = \beta_0 + \beta_1 \sigma_t^e + \beta_2 \left(CA/Y\right)_t + \varepsilon_t$$

where $\left(CA/Y\right)_t$ is the current account on output ratio.

Economic Modeling of Asset Returns

Bonds

Table: Economic forecast of the 10-year bond yield

	2010	2020	2030	2050
Sovereign bonds				
US	2.8%	4.9%	5.1%	5.1%
EURO	2.6%	4.5%	4.7%	4.8%
JAPAN	1.1%	3.3%	3.5%	3.6%
PACIFIC	5.5%	6.5%	6.3%	6.2%
EM	5.5%	9.4%	10.1%	10.7%
Corporate bonds				
IG US	6.5%	6.3%	6.4%	6.5%
IG EURO	3.5%	4.8%	5.0%	5.1%
HY US	7.8%	10.2%	10.3%	10.3%
HY EURO	7.8%	10.1%	10.2%	10.2%

Economic Modeling of Asset Returns

Bonds

Expected returns of bonds are deduced from the economic forecast of the 10-year bond yield using a sensitivity/duration hypothesis.

	1980-1990	1990-2000	2000-2010	2020	2030	2050
			Sovereign bonds			
US	11.5%	9.3%	6.1%	1.9%	3.5%	4.3%
EURO	8.4%	8.2%	5.5%	1.8%	3.2%	4.0%
JAPAN		7.3%	2.5%	0.0%	1.7%	2.6%
PACIFIC		12.5%	6.8%	5.5%	6.1%	6.2%
EM		14.2%	10.0%	5.6%	7.6%	9.0%
			Corporate bonds			
IG US		8.0%	6.8%	6.1%	6.2%	6.3%
IG EURO			4.0%	3.7%	4.3%	4.6%
HY US		11.0%	7.0%	8.9%	9.6%	9.9%
HY EURO			4.0%	8.6%	9.4%	9.8%

Economic Modeling of Asset Returns

Equities

The long-run equity return is equal to:

$$R^{e}_{\infty} = R^{b}_{\infty} + \mathscr{R}^{e}_{\infty}$$

where R^{b}_{∞} is the long-run bond yield and $\mathscr{R}^{e}_{\infty}$ is the equity excess return.

The regression model is:

$$\mathscr{R}^{e}_{t+10} = \beta_0 + \beta_1 \,\mathrm{PE}_t + \beta_2 R^{b}_{t} + \varepsilon_t$$

where PE_t is the price earning ratio and R^{b}_{t} is the 10-year bond yield.

	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	15.2%	18.3%	−1.2%	9.2%	8.4%	9.1%
EURO	12.8%	16.8%	0.4%	9.7%	8.2%	8.7%
JAPAN	20.1%	−0.5%	−3.4%	8.8%	4.9%	5.6%
PACIFIC		14.0%	8.7%	14.7%	9.1%	9.5%
EM		8.4%	14.0%	10.7%	10.4%	10.8%

Economic Modeling of Asset Returns

Other assets

1. Small cap
2. Commodities
3. Hedge funds
4. Real estate
5. Foreign exchanges

- Liquidity risk
- Globalization / Convergence
- Ressources / Consumption

Economic Modeling of Volatility and Correlation

Volatility and Correlation

Volatility

- Historical figures
- Mean-reverting properties
- Macro-economic volatility
- Tail risks

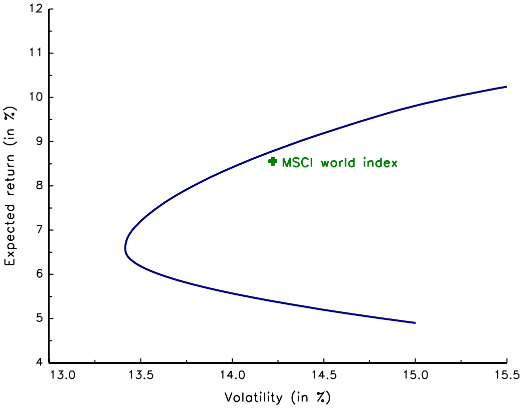
Correlation

- Historical figures
- Time-varying correlations
- Flight-to-quality & globalization
- Inflation regime $\Rightarrow$ bond-stock correlation

Strategic Asset Allocation in Practice

Strategic Equity Portfolio

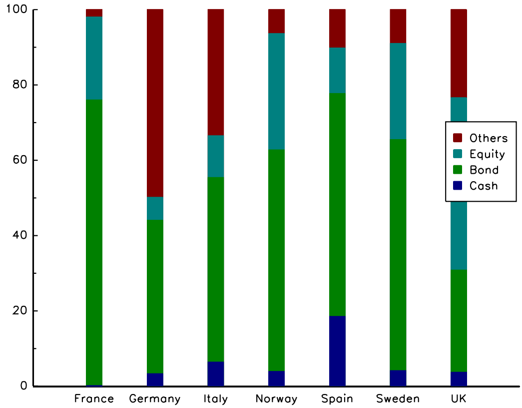
- US ($\simeq$)
- EURO ($-$)
- JAPAN ($--$)
- PACIFIC ($+$)
- EM ($++$)



Strategic Asset Allocation in Practice

Bond-Equity Allocation Policy

Figure: Average allocation of European pension funds



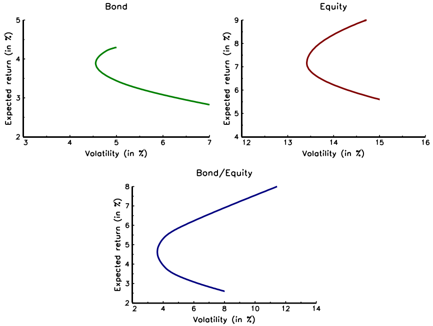
Strategic Asset Allocation

Bond-Equity Allocation Policy

VOL	Weights		ER
	Bond	Equity	
3.6%	82.8%	17.2%	4.6%
4.0%	76.2%	23.8%	5.3%
4.5%	69.5%	30.5%	5.6%
4.6%	68.1%	31.9%	5.7%
5.0%	64.5%	35.5%	5.9%
5.5%	60.5%	39.5%	6.1%
6.0%	56.9%	43.1%	6.2%
8.0%	43.4%	56.6%	6.9%
10.0%	30.5%	69.5%	7.5%
12.0%	18.0%	82.0%	8.2%
15.0%	0.0%	100.0%	9.1%

Standard risk-aversion for long-term investors: $\gamma = 5$.

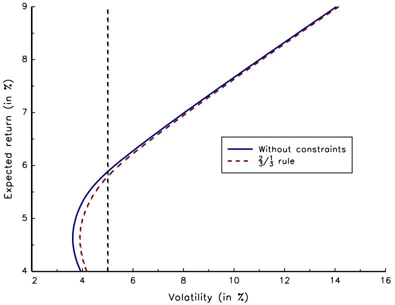
Strong diversification effect.



Strategic Asset Allocation in Practice

The Place of Alternative Investments

- Alternative assets = substitute of equities (not of bonds).
- The $^2/_3 - ^1/_3$ rule (for risk-seeking long-term investors).
- Liquidity risk $\Longrightarrow$ Tactical asset allocation.



Sensitivity and Scenario analysis

Economic scenario

- Expectation → Probability
- Stress scenario

Risk premium

- Confidence intervals
- Scenario analysis

Table: Coefficient estimates for bond regressions

	Study Period	Constant	r_t	σ_t^π	$(B/Y)_t$	R^2
US	1982–2009	0.008 (2.006)	0.59 (3.63)	0.67 (1.51)	−0.11 (−1.00)	0.82
EURO	1982–2009	0.007 (1.988)	0.47 (4.15)	2.03 (2.07)	−0.10 (−0.84)	0.94
JAPAN	1982–2009	0.011 (3.379)	0.66 (5.57)	0.21 (1.82)	−0.05 (−1.10)	0.85
PACIFIC	1982–2009	0.017 (1.212)	0.47 (3.96)	0.15 (0.19)	−0.32 (−2.42)	0.69

Conclusion

Asset Management $\Longrightarrow$ some complex problems:

- Benchmarking
- Portfolio allocation
- Long-term risks
- SAA vs TAA
- Momentum strategies
- Statistical arbitrage
- etc.

Quantitative methods = a tool to understand (and sometimes to solve) these problems.

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