Risk-Based Investing & Asset Management
Final Examination

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Remark 1 ***The final examination on risk parity is composed of 4 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file by question. Concerning risk[1] and performance[2] decompositions, present the results as follows:***

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			✓	

Asset	x_i	μ_i	$\mathcal{PC}_i$	$\mathcal{PC}_i^\star$
1				
2				
$\vdots$				
n				
$\mu(x)$			✓	

[1] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^\star$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

[2] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, μ_i is the expected return, $\mathcal{PC}_i$ is the nominal (ex-ante) performance contribution, $\mathcal{PC}_i^\star$ is the relative performance contribution and $\mu(x)$ is the expected return of the portfolio.

1 Risk-based portfolios

We consider an investment universe with 4 assets. We assume that their expected returns are 3%, 6%, 7% and 8%„ and that their volatilities are 5%, 10%, 15% and 20%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & \\ 50\% & 100\% & & \\ 20\% & 20\% & 100\% & \\ 50\% & 50\% & 80\% & 100\% \end{pmatrix}$$

The return of the risk-free asset is equal to 2%.

1. We consider long-short portfolios x with $\sum_{i=1}^{n} x_i = 1$.
   (a) Compute the minimum variance portfolio.
   (b) Calculate the MVO portfolio if we target an ex-ante volatility of 6%.
   (c) Calculate the MVO portfolio if we target an ex-ante volatility of 12%.
   (d) Calculate the MVO portfolio if we target an ex-ante expected return of 5%.
   (e) Compare the four optimized portfolios in terms of expected return, volatility and Sharpe ratio.
2. We restrict the analysis to long-only portfolios meaning that $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0$.
   (a) Compute the minimum variance portfolio.
   (b) Calculate the MVO portfolio if we target an ex-ante volatility of 6%.
   (c) Calculate the MVO portfolio if we target an ex-ante volatility of 12%.
   (d) Calculate the MVO portfolio if we target an ex-ante expected return of 5%.
   (e) Compare the four optimized portfolios in terms of expected return, volatility and Sharpe ratio.
   (f) What is the impact of the long-only constraint?
3. We consider long-only portfolios. For each portfolio, compute the volatility decomposition.
   (a) Determine the equally weighted (EW) portfolio.
   (b) Compute the minimum variance (MV) portfolio.
   (c) Calculate the most diversified portfolio (MDP).
   (d) Find the ERC portfolio.
4. The objective of the investor is to achieve a Sharpe ratio equal to $^1/_2$. Compute the implied risk premia of the four assets for each risk-based portfolio. Comment on these results.
5. Draw the 4 previous risk-based portfolios on the efficient frontier. Comment on these results.

2 Regularizing portfolio optimization

We consider an investment universe with 5 assets. We assume that their expected returns are 4%, 6%, 7%, 8% and 10%,, and that their volatilities are 6%, 10%, 11%, 15% and 15%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 50\% & 100\% & & & \\ 20\% & 20\% & 100\% & & \\ 50\% & 50\% & 80\% & 100\% & \\ 0\% & -20\% & -50\% & -30\% & 100\% \end{pmatrix}$$

We note Σ the covariance matrix.

1. In what follows, we consider the impact of weight constraints on portfolio optimization.

   (a) Compute the long/short minimum variance portfolio $x^\star(\Sigma)$.

   (b) Compute the constrained MV portfolio $\tilde{x}^\star(\Sigma)$ when $0\% \leq x_i \leq 100\%$. Estimate the Lagrange coefficient associated to each constraint and deduce the Jagannathan-Ma shrinkage covariance matrix $\tilde{\Sigma}$. What is the impact on the volatilities and the cross-correlations? Verify that $x^\star\left(\tilde{\Sigma}\right) = \tilde{x}^\star(\Sigma)$.

   (c) Compute the constrained MV portfolio $\tilde{x}^\star(\Sigma)$ when $10\% \leq x_i \leq 40\%$. Estimate the Lagrange coefficient associated to each constraint and deduce the Jagannathan-Ma shrinkage covariance matrix $\tilde{\Sigma}$. What is the impact on the volatilities and the cross-correlations? Verify that $x^\star\left(\tilde{\Sigma}\right) = \tilde{x}^\star(\Sigma)$.

2. We restrict the analysis to long-only portfolios meaning that $\sum_{i=1}^n x_i = 1$ and $x_i \geq 0$.

   (a) What is ridge regression?

   (b) Define the Herfindahl index $\mathcal{H}(x)$. What are the two limit cases of $\mathcal{H}(x)$? What is the interpretation of the statistic $\mathcal{N}(x) = \mathcal{H}^{-1}(x)$?

   (c) We consider the following optimization problem $(\mathcal{P}_1)$:

   $$\begin{aligned} x^\star(\lambda) &= \arg\min \frac{1}{2} x^\top \Sigma x + \lambda x^\top x \\ & \text{u.c.} \left\{ \begin{array}{l} \sum_{i=1}^n x_i = 1 \\ x_i \geq 0 \end{array} \right. \end{aligned}$$

   What is the link between this constrained optimization program, the ridge regression and the weight diversification based on the Herfindahl index?

   (d) Solve Program $(\mathcal{P}_1)$ when λ is equal respectively to 0, 0.0001, 0.001, 0.01, 0.05, 0.10 and 10. Compute the statistic $\mathcal{N}(x)$. Comment on these results.

   (e) What becomes the portfolio $x^\star(\lambda)$ when $\lambda \to \infty$? Give an analytical proof.

3. We consider long/short portfolios.

   (a) What is lasso regression?

   (b) We consider the following optimization problem $(\mathcal{P}_2)$:

   $$\begin{aligned} x^\star(\lambda) &= \arg\min \frac{1}{2} x^\top \Sigma x + \lambda \sum_{i=1}^n |x_i| \\ & \text{u.c.} \sum_{i=1}^n x_i = 1 \end{aligned}$$

   What is the link between this constrained optimization program and the lasso regression?

(c) Solve Program ($\mathcal{P}_2$) when λ is equal respectively to 0, 0.0001, 0.001, 0.01, 0.05, 0.10 and 10. Comment on these results.

(d) What becomes the portfolio $x^{\star}(\lambda)$ when $\lambda \to \infty$? How do you explain this result?

(e) We assume that the investor holds an initial portfolio x_0 defined as follows:

$$x^{(0)} = \begin{pmatrix} 10\% \\ 15\% \\ 20\% \\ 25\% \\ 30\% \end{pmatrix}$$

We consider the optimization problem ($\mathcal{P}_3$):

$$\begin{aligned} x^{\star}(\lambda) &= \arg\min \frac{1}{2} x^{\top} \Sigma x + \lambda \sum_{i=1}^{n} \left| x_i - x_i^{(0)} \right| \\ & \text{u.c.} \sum_{i=1}^{n} x_i = 1 \end{aligned}$$

Solve Program ($\mathcal{P}_3$) when λ is equal respectively to 0, 0.0001, 0.001, 0.0015 and 0.01. Compute the turnover of each optimized portfolio. Comment on these results. What becomes the portfolio $x^{\star}(\lambda)$ when $\lambda \to \infty$? How do you explain this result?

3 Smart beta

We consider the universe of the Eurostoxx 50 Index at the end of December 2014. The Excel file contains the time series of stock prices from December 2013 to December 2014 and the asset weights in the Eurostoxx 50 Index.

1. (a) Estimate the annualized covariance matrix of stock returns using a lag window of 260 trading days.

   (b) Perform a sampling of the Eurostoxx 50 Index by selecting 40 stocks. Give the composition of the portfolio and compute the ex-ante tracking error volatility.

   (c) Same question if we select 30 stocks.

   (d) Same question if we select 10 stocks.

2. (a) We consider the portfolio invested in the 10 largest stocks of the Eurostoxx 50 Index. Find the portfolio which minimizes the tracking error volatility.

   (b) Compare Solutions 2(a) and 1(d). Comment these results.

3. (a) Estimate the beta, the correlation and the tracking error volatility of the EW portfolio with respect to the Eurostoxx 50 Index.

   (b) Compute the ERC portfolio. Deduce then the beta, the correlation and the tracking error volatility of this portfolio.

   (c) Same question if we consider the long-only minimum variance (MV) portfolio.

   (d) Same question if we consider the long-only most diversified portfolio (MDP).

   (e) Comment these results.

4 Factor investing

We consider the risk model based on the following risk factors:

- MKT: the market risk factor
- SMB: the size risk factor (Small minus Big)
- HML: the value risk factor (High minus Low)
- WML: the momentum risk factor (Winners minus Losers)

1. (a) Define the previous four risk factors.
   (b) Download the monthly returns of the US risk factors from Kenneth French's website:

   `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

   (c) Compute the annual return, the volatility, the Sharpe ratio, the maximum drawdown of these risk factors for the period January 2005 – December 2014.

2. We consider an equally-weighted portfolio of the three factors SMB, HML and WML. This portfolio is rebalanced every month.

   (a) Compute the annual return, the volatility and the Sharpe ratio of the EW portfolio.
   (b) We remind that the three factors are long/short. Which leverage should we apply in order to obtain an ex-post volatility which is equal to the volatility of the MKT portfolio.
   (c) What is the realized tracking error volatility of this leveraged portfolio with respect to the MKT portfolio? Deduce its information ratio.

3. For each risk factor, download the monthly returns of 2×3 sorted portfolios from Kenneth French's website.

   (a) Explain what are the 2×3 sorted portfolios.
   (b) Compute the annual return, the volatility and the Sharpe ratio of the portfolios SH (Small High), BH (Big High), SW (Small Winners) and BW (Big Winners). Comment these results.

4. We consider the monthly returns of four mutual funds provided in the EXCEL file. The study period is January 2010 – December 2014.

   (a) For each mutual fund, estimate the exposure on the risk factor MKT. What proportion of return variance is explained by this factor? Estimate the alpha.
   (b) For each mutual fund, estimate the exposure on the four risk factors MKT, SMB, HML and WML. What proportion of return variance is explained by these four factors? Estimate the alpha.
   (c) For each mutual fund, perform the style regression using the portfolios SH (Small High), SL (Small Low), BH (Big High), BL (Big Low), SW (Small Winners), SLO (Small Losers), BW (Big Winners) and BLO (Big Losers). What proportion of return variance is explained by these 8 factors? Estimate the alpha.
   (d) Comment these results.