Risk Management & Financial Regulation
Final Examination

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Please write entirely your answers.

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel II?
3. How is calculated the capital requirement with the internal model-based approach in Basel III?

3 Credit risk

1. What is the definition of the default in Basel II?
2. Describe the standard approach (SA) to compute the capital requirement in Basel III.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?
4. Explain the concept of CCF? Why is it difficult to estimate a CCF?

4 Interest rate risk in the banking book (IRRBB)

1. Define the concepts of EVE and NII.
2. Describe the standardized approach to calculate the capital charge of IRRBB.

5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to calculate the capital in Basel II charge.
3. Describe the loss distribution approach (LDA) to calculate the capital charge.
4. How is the capital requirement calculated in Basel III?

6 Market risk of a market neutral portfolio

Consider an investment universe consisting of two stocks A and B. The current prices of the two stocks are \$50 and \$100, respectively. Their volatilities are 35% and 25%, respectively, while their cross-correlation is equal to 20%. The portfolio is long 4 stocks A and short 1 stock B.

1. Gaussian risk measure

   (a) We note $P_A(t)$ (resp. $P_B(t)$) the price of Stock A (resp. B) at time t. Calculate the P&L Π of the portfolio between $t-1$ and t and give its expression with respect to $R_A(t)$ and $R_B(t)$, which are the stock returns between $t-1$ and t.

   (b) Calculate the variance of Π.

   (c) Calculate the Gaussian value-at-risk at the 99% confidence level for a ten-day time horizon.

   (d) Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.

2. Historical risk measure
   The ten worst PnL of the portfolio (expressed in \$) among the last 300 historical scenarios are the following:

s	1	2	3	4	5	6	7	8	9	10
Π_s	-15.6	-12.3	-10.5	-6.6	-7.7	-6.8	-5.8	-5.6	-5.6	-5.4

   (a) Calculate the historical value-at-risk at the 99% confidence level for a ten-day time horizon.

   (b) Calculate the historical expected shortfall at the 97.5% confidence level for a ten-day time horizon.

3. CAPM risk measure
   We assume that:

$$\begin{cases} R_A(t) = \beta_A R_M(t) + \varepsilon_A(t) \\ R_B(t) = \beta_B R_M(t) + \varepsilon_B(t) \end{cases}$$

   where $R_M(t) \sim \mathcal{N}\left(0, \sigma_M^2\right)$ is the market return, and $\varepsilon_A(t) \sim \mathcal{N}\left(0, \tilde{\sigma}_A^2\right)$ and $\varepsilon_B(t) \sim \mathcal{N}\left(0, \tilde{\sigma}_B^2\right)$ are the idiosyncratic risks. Moreover, $R_M(t)$, $\varepsilon_A(t)$ and $\varepsilon_B(t)$ are independent. We assume that $\beta_A = 0.50$, $\beta_B = 1.0$, $\sigma_M = 20\%$, $\tilde{\sigma}_A = 35\%$ and $\tilde{\sigma}_B = 15\%$.

   (a) Calculate the expression of the P&L Π.

   (b) Calculate the variance of the P&L Π.

   (c) Calculate the Gaussian value-at-risk at the 99% confidence level for a ten-day time horizon.

   (d) Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.

4. Why is this portfolio market neutral? Do we have less risk than the long portfolio of the 4 stocks A?

7 Credit default swaps

We consider a CDS 6M with five-year maturity and \$1 mn notional principal. The recovery rate $\mathcal{R}$ is equal to 50% whereas the spread $\mathcal{s}$ is equal to 500 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart.

2. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection seller A if the reference entity defaults in one year and one month?

3. What is the P&L of the protection buyer B if the reference entity defaults in two years and three months? Same question if the reference entity defaults in three years and one month?

4. What is the relationship between $\mathcal{s}$, $\mathcal{R}$ and λ? What is the implied one-year default probability at the inception date?

5. Two years later, the CDS spread has decreased and is equal to 200 bps. Estimate the new default probability. The protection seller A decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 3.524.

8 Calculation of the effective expected positive exposure

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N_0, μ and σ be the notional amount, the expected return and the volatility of the underlying contract. We assume that $e(t) = N_0 X$ where X follows a log-normal distribution $\mathcal{LN}(\mu t, \sigma^2 t)$:

$$\Pr\{X \leq x\} = \Phi\left(\frac{\ln x - \mu t}{\sigma\sqrt{t}}\right)$$

1. Define the concept of counterparty credit risk. Give two examples.

2. How does the CCR capital charge relate to the credit risk capital charge?

3. Calculate the peak exposure $\mathrm{PE}_\alpha(t)$ and the expected exposure $\mathrm{EE}(t)$, and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

4. We assume that $\mu \ll 1$ and $\sigma \ll 1$. Find an approximation of $\mathrm{EEPE}(0;t)$ and show that $\mathrm{EEPE}(0;t)$ is a linear function of μ, σ^2 and t. How can you justify this approximation?

5. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N_0 is equal to \$1 mn, the maturity T is one year, the volatility σ is set to 50% and μ is equal to 0.

   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.

   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[1]. What is the implied risk weight?

   (c) What is the result if the bank manages the credit risk using the SA approach and the counterparty credit risk using an internal model in the case where the credit rating of the counterparty is **A+**?

9 Parametric estimation of the loss severity distribution

We assume that the severity losses are log-normal distributed: $X_i \sim \mathcal{LN}(\mu, \sigma^2)$.

1. Show that the density function of the log-normal probability distribution is:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)$$

2. Deduce the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.

3. Calculate the maximum likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$.

4. We assume that the losses $\{x_1, \ldots, x_n\}$ were collected beyond a threshold H. Calculate the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.

[1] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

10 The Normal copula function

1. Let $\tau = (\tau_1, \tau_2)$ be a random vector with distribution $\mathbf{F}$. We assume that τ_1 and τ_2 are two exponential default times with parameters λ_1 and λ_2. We also assume that the copula function $\mathbf{C}$ of the random vector τ is Normal with parameter ρ.

   (a) Let $U = (U_1, U_2)$ be a random vector with copula $\mathbf{C}$. We note Σ the matrix defined as follows:

   $$\Sigma = \left(\begin{array}{cc} 1 & \rho \\ \rho & 1 \end{array} \right)$$

   Compute the Cholesky decomposition of Σ. Deduce an algorithm to simulate U.

   (b) We remind that the bivariate cumulative density function of the gaussian standardized vector $X = (X_1, X_2)$ with correlation ρ is:

   $$\Phi(x_1, x_2; \rho) = \int_{-\infty}^{x_1} \Phi\left(\frac{x_2 - \rho x}{\sqrt{1-\rho^2}}\right) \phi(x)\, \mathrm{d}x$$

   Deduce the expression of the Normal copula by considering the change of variable $u = \Phi(x)$. Calculate then the conditional copula function $\mathbf{C}_{2|1}$.

   (c) Describe the simulation algorithm based on conditional distributions. Apply this algorithm to the Normal copula. Show that this algorithm is equivalent to the simulation algorithm based on the Cholesky decomposition.

   (d) How to simulate $\tau = (\tau_1, \tau_2)$ from the random variates (u_1, u_2) generated by one of the two previous algorithms.

2. We consider some special cases of ρ.

   (a) Show that if $\rho = 1$, we have the following relationship:

   $$\tau_1 = \frac{\lambda_2}{\lambda_1} \tau_2$$

   Deduce that the linear correlation between τ_1 and τ_2 is equal to $+1$:

   $$\rho \langle \tau_1, \tau_2 \rangle = +1$$

   (b) What becomes this relationship when $\rho = -1$? Deduce that the linear correlation between τ_1 and τ_2 is not equal to -1:

   $$\rho \langle \tau_1, \tau_2 \rangle > -1$$