Risk Management & Financial Regulation
Final Examination

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Please write entirely your answers. The correction of exercises will be available in the next release of the lecture notes.

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel II?
3. How is calculated the capital requirement with the internal model-based approach in Basel III?

3 Credit risk

1. What is the definition of the default in Basel II?
2. Describe the standard approach (SA) to compute the capital requirement in Basel III.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?
4. Explain the concept of CCF? Why is it difficult to estimate a CCF?

4 Liquidity risk

1. What is the difference between market and funding liquidity risk? Give an example.
2. Describe the liquidity coverage ratio (LCR).
3. Describe the net stable funding ratio (NSFR).

5 Interest rate risk in the banking book (IRRBB)

1. Define the concepts of EVE and NII.
2. Describe the standardized approach to calculate the capital charge of IRRBB.

6 Credit default swaps

We consider a CDS 6M with five-year maturity and \$1 mn notional principal. The recovery rate $\mathcal{R}$ is equal to 60% whereas the spread $\mathcal{s}$ is equal to 400 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart.
2. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection seller A if the reference entity defaults in three years and one month?
3. What is the P&L of the protection buyer B if the reference entity defaults in one year and three months? Same question if the reference entity defaults in three years and one month?
4. What is the relationship between $\mathcal{s}$, $\mathcal{R}$ and λ? What is the implied one-year default probability at the inception date?
5. Two years later, the CDS spread has decreased and is equal to 100 bps. Estimate the new default probability. The protection seller A decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 2.786.

7 Backtesting of the value-at-risk

We consider an individual, whose wealth is equal to \$1 000. He invests 20% in stock A, 20% in stock B, 20% in stock C and 40% in cash. We assume that the current prices are $P_A(t) = \$50$, $P_B(t) = \$20$ and $P_C(t) = \$100$. Let $R_A(t+1)$, $R_B(t+1)$ and $R_C(t+1)$ be the returns of A, B and C for the next year. We assume that:

$$\begin{pmatrix} R_A(t+1) \\ R_B(t+1) \\ R_C(t+1) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} 0.2^2 & 20\% \times 0.2^2 & 0 \\ 20\% \times 0.2^2 & 0.2^2 & -20\% \times 0.2^2 \\ 0 & -20\% \times 0.2^2 & 0.2^2 \end{pmatrix} \right)$$

This means that expected returns are equal to $\mu_A = \mu_B = \mu_C = 10\%$ whereas the volatilities are equal to $\sigma_A = \sigma_B = \sigma_C = 20\%$. Moreover, the cross-correlations are $\rho_{A,B} = +20\%$, $\rho_{A,C} = 0\%$ and $\rho_{B,C} = -20\%$. Finally, we assume that the one-year return r of the cash is equal to 1%.

1. Write the stochastic value $P(t+1)$ of the portfolio for the next year in terms of future prices $P_A(t+1)$, $P_B(t+1)$ and $P_C(t+1)$. Deduce the expression of $P(t+1)$ in terms of future returns $R_A(t+1)$, $R_B(t+1)$ and $R_C(t+1)$. Find the expression of the P&L $\Pi = P(t+1) - P(t)$ with respect to the current value of the portfolio.
2. What is the expected profit of the portfolio at time $t+1$? Deduce the expected return of this investment.
3. Find the volatility $\sigma(\Pi)$ of the P&L?
4. Deduce the one-year expected shortfall with a 97.5% confidence level.
5. What does this result become if $\mu_A = \mu_B = \mu_C = 0\%$? Same question if $\mu_A = \mu_B = \mu_C = 20\%$. Comment on these results.
6. We now assume that $\mu_A = \mu_B = \mu_C = 10\%$. Calculate the one-year value-at-risk with a 99% confidence level.
7. Deduce the value-at-risk for the one-day holding period.
8. One year later, we have observed that the 10 smallest daily returns among the year (which contains 260 trading days) are:

s	1	2	3	4	5	6	7	8	9	10
R_s	−2.5%	−1.3%	−1.1%	−1%	−1%	−0.9%	−0.8%	−0.7%	−0.7%	−0.61%

Calculate the historical value-at-risk with a 99% confidence level for the one-day holding period.

Table 1: Probability $\Pr\{X \leq x\}$ in % where $X \sim \mathcal{B}inomial\,(260, p)$

	p										
x	1.0%	1.1%	1.2%	1.3%	1.4%	1.5%	1.6%	1.7%	1.8%	1.9%	2.0%
0	7.331	5.637	4.333	3.330	2.559	1.965	1.509	1.159	0.889	0.682	0.523
1	26.583	21.938	18.017	14.734	12.004	9.746	7.889	6.368	5.127	4.118	3.300
2	51.767	45.416	39.540	34.186	29.372	25.092	21.324	18.035	15.186	12.735	10.639
3	73.644	67.874	62.021	56.219	50.579	45.188	40.110	35.386	31.042	27.088	23.520
4	87.842	83.922	79.564	74.865	69.927	64.852	59.736	54.667	49.717	44.948	40.409
5	95.185	93.061	90.474	87.439	83.992	80.183	76.076	71.738	67.242	62.659	58.056
6	98.337	97.382	96.106	94.478	92.479	90.106	87.367	84.286	80.895	77.238	73.363
7	99.492	99.125	98.587	97.841	96.852	95.589	94.029	92.160	89.976	87.484	84.698
8	99.862	99.738	99.541	99.243	98.815	98.229	97.455	96.466	95.240	93.760	92.013
9	99.966	99.929	99.865	99.759	99.596	99.355	99.015	98.551	97.942	97.163	96.194
10	99.992	99.983	99.964	99.930	99.874	99.785	99.651	99.457	99.185	98.817	98.335

9. Calculate the historical expected shortfall with a 97.5% confidence level for the one-day holding period.

10. We compare the results of Questions 7 and 8. How many exceptions do you observe during the last year for the Gaussian value-at-risk? What is the theoretical probability to observe this event? Explain the calculus. What can you conclude about the ex-post implied confidence level of the Gaussian value-at-risk?

8 The bivariate Pareto copula

We consider the bivariate Pareto distribution:

$$\begin{aligned} \mathbf{F}(x_1, x_2) &= 1 - \left(\frac{\theta_1 + x_1}{\theta_1}\right)^{-\alpha} - \left(\frac{\theta_2 + x_2}{\theta_2}\right)^{-\alpha} + \\ & \left(\frac{\theta_1 + x_1}{\theta_1} + \frac{\theta_2 + x_2}{\theta_2} - 1\right)^{-\alpha} \end{aligned}$$

where $x_1 \geq 0$, $x_2 \geq 0$, $\theta_1 > 0$, $\theta_2 > 0$ and $\alpha > 0$.

1. Show that the marginal functions of $\mathbf{F}(x_1, x_2)$ correspond to univariate Pareto distributions.
2. Find the copula function associated to the bivariate Pareto distribution.
3. Deduce the copula density function.
4. Show that the bivariate Pareto copula function has no lower tail dependence, but an upper tail dependence.
5. Do you think that the bivariate Pareto copula family can reach the copula functions $\mathbf{C}^-$, $\mathbf{C}^\perp$ and $\mathbf{C}^+$? Justify your answer.
6. Let X_1 and X_2 be two Pareto-distributed random variables, whose parameters are (α_1, θ_1) and (α_2, θ_2).
   (a) Show that the linear correlation between X_1 and X_2 is equal to 1 if and only if the parameters α_1 and α_2 are equal.
   (b) Show that the linear correlation between X_1 and X_2 can never reached the lower bound -1.
   (c) Build a new bivariate Pareto distribution by assuming that the marginal distributions are $\mathcal{P}(\alpha_1, \theta_1)$ and $\mathcal{P}(\alpha_2, \theta_2)$ and the dependence is a bivariate Pareto copula function with parameter α. What is the relevance of this approach for building bivariate Pareto distributions?

9 Calculation of the effective expected positive exposure

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N_0, μ and σ be the notional, the expected return and the volatility of the underlying contract. We assume that $e(t) = N_0 X$ where X follows a log-normal distribution $\mathcal{LN}\left(\mu t, \sigma^2 t\right)$:

$$\Pr\{X \leq x\} = \Phi\left(\frac{\ln x - \mu t}{\sigma\sqrt{t}}\right)$$

1. Calculate the peak exposure $\mathrm{PE}_\alpha(t)$ and the expected exposure $\mathrm{EE}(t)$, and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.
2. We assume that $\mu \ll 1$ and $\sigma \ll 1$. Find an approximation of $\mathrm{EEPE}(0;t)$ and show that $\mathrm{EEPE}(0;t)$ is a linear function of μ, σ^2 and t. How can you justify this approximation?
3. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N_0 is equal to \$1 mn, the maturity T is one year, the volatility σ is set to 50% and μ is equal to 0.
   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.
   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[1]. What is the implied risk weight?
   (c) What does this result become if the bank manages the credit risk with the SA approach and the counterparty credit risk with an internal model in the case where the credit rating of the counterparty is **A+**.

10 Estimation of the loss severity distribution

We consider a sample of n individual losses $\{x_1, \ldots, x_n\}$. We assume that they can be described by the Pareto distribution $\mathcal{P}(\alpha, x^-)$ defined by:

$$\Pr\{X \leq x\} = 1 - \left(\frac{x}{x_-}\right)^{-\alpha}$$

where $x \geq x_-$ and $\alpha > 0$.

1. Find the maximum likelihood estimator $\hat{\alpha}_{\mathrm{ML}}$.
2. Calculate the first two moments of X.
3. Deduce the GMM conditions for estimating the parameter α.
4. Find the estimator $\hat{\alpha}_{\mathrm{MM}}$ of the method of moments using the first moment.
5. We now assume that the losses $\{x_1, \ldots, x_n\}$ have been collected beyond a threshold H meaning that $X \geq H$.
   (a) Compute the conditional distribution $\Pr\{X \leq x \mid X \geq H\}$.
   (b) Find the maximum likelihood estimator $\hat{\alpha}_{\mathrm{ML}}$.
   (c) Find the estimator $\hat{\alpha}_{\mathrm{MM}}$ of the method of moments using the first moment.
6. Application: $x_1 = 1\,274$, $x_2 = 854\,646$, $x_3 = 48\,180$, $x_4 = 686\,806$, $x_5 = 100\,539$, $x_6 = 34\,831\,239$, $x_7 = 39\,442$, $x_8 = 94\,818$, $x_9 = 1\,469$,and $x_{10} = 31\,528$. We have $\sum_{i=1}^{10} x_i = 36\,689\,941$ and $\sum_{i=1}^{10} \ln x_i = 113.608$.
   (a) We assume that $x_- = 100$. Compute $\hat{\alpha}_{\mathrm{ML}}$ and $\hat{\alpha}_{\mathrm{MM}}$.
   (b) We assume that $H = 1\,000$. Compute $\hat{\alpha}_{\mathrm{ML}}$ and $\hat{\alpha}_{\mathrm{MM}}$.
   (c) Which estimator (ML or MM) is the most relevant? Explain why one estimator is largely better than the other?

[1] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.