Risk Management & Financial Regulation
Final Examination

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Please write entirely your answers. The correction of exercises will be available in the next release of the lecture notes.

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel III?
3. What are the differences with the Basel II internal model-based approach?

3 Credit risk

1. What is the definition of the default in Basel II?
2. Describe the standard approach (SA) to compute the capital requirement.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?
4. Explain the concept of CCF? Why is it difficult to estimate a CCF?

4 Counterparty credit risk (CCR) and credit value adjustment (CVA)

1. Define the concept of counterparty credit risk. What differences do you make between the CCR capital charge in Basel II and the CVA capital charge in Basel III?
2. How is calculated the CCR capital requirement? Explain why the exposure-at-default (EAD) has to be estimated.
3. How is calculated the CVA capital requirement?

5 Interest rate risk in the banking book (IRRBB)

1. Define the concepts of EVE and NII.
2. Describe the standardized approach to calculate the capital charge of IRRBB.

6 Market risk of a long-only portfolio

We consider an investment universe, which is composed of two stocks A and B. The current prices of the two stocks are respectively equal to \$500 and \$1000. Their volatilities are equal to 20% and 15% whereas the cross-correlation is equal to $\dfrac{11}{24} \approx 45.83\%$. The portfolio is composed of 2 stocks A and 1 stock B.

1. Gaussian risk measure

   (a) We note $P_A(t)$ (resp. $P_B(t)$) the price of Stock A (resp. B) at time t. Give the expression of the P&L Π of the portfolio between $t-1$ and t with respect to $R_A(t)$ and $R_B(t)$, which are the stock returns between $t-1$ and t.

   (b) Calculate the variance of Π.

   (c) Calculate the Gaussian value-at-risk at the 99% confidence level for a ten-day time horizon.

   (d) Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.

2. Historical risk measure
   The ten worst returns of the portfolio (expressed in %) among the last 300 historical scenarios are the following:

s	1	2	3	4	5	6	7	8	9	10
R_s	-4.6	-3.3	-3.1	-2.9	-2.7	-2.6	-2.5	-2.3	-1.7	-1.1

   (a) Calculate the historical value-at-risk at the 99% confidence level for a ten-day time horizon.

   (b) Calculate the historical expected shortfall at the 97.5% confidence level for a ten-day time horizon.

3. CAPM risk measure
   We assume that:

$$\begin{cases} R_A(t) = \beta_A R_M(t) + \varepsilon_A(t) \\ R_B(t) = \beta_B R_M(t) + \varepsilon_B(t) \end{cases}$$

   where $R_M(t) \sim \mathcal{N}\left(0, \sigma_M^2\right)$ is the market return, and $\varepsilon_A(t) \sim \mathcal{N}\left(0, \tilde{\sigma}_A^2\right)$ and $\varepsilon_B(t) \sim \mathcal{N}\left(0, \tilde{\sigma}_B^2\right)$ are the idiosyncratic risks. Moreover, $R_M(t)$, $\varepsilon_A(t)$ and $\varepsilon_B(t)$ are independent.

   (a) Calculate the variance of Π. Deduce the expression of the expected shortfall at the confidence level α.

   (b) We assume that $\tilde{\sigma}_A$ and $\tilde{\sigma}_B$ are fixed. Find a condition on β_A and β_B such that the expected shortfall is minimum.

   (c) We assume that β_A and β_B are given. Find a condition on $\tilde{\sigma}_A$ and $\tilde{\sigma}_B$ such that the expected shortfall is minimum.

   (d) We assume that $\sigma_M = 10\%$. Show that:

$$\beta_A \beta_B = \frac{33}{24}$$

   Find the values of β_A and β_B such that the expected shortfall is minimum in case b).

7 Risk contribution in the Basel II model

We consider a portfolio of I loans. We denote L the portfolio loss:

$$L = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathrm{LGD}_i \times \mathbb{1}\left\{\boldsymbol{\tau}_i \leq M_i\right\}$$

We can show that, under some assumptions $(\mathcal{H})$, the expectation of the portfolio loss conditionally to the factors $X_1, \ldots, X_m$ is:

$$\mathbb{E}\left[L \mid X_1, \ldots, X_m\right] = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathbb{E}\left[\mathrm{LGD}_i\right] \times \mathrm{PD}_i\left(X_1, \ldots, X_m\right) \tag{1}$$

1. Explain the different notations: EAD_i, LGD_i, $\boldsymbol{\tau}_i$, M_i and PD_i.
2. How do we obtain the expression (1)? What are the necessary assumptions ($\mathcal{H}$)? What is an infinitely fine-grained portfolio?
3. Define the credit risk contribution.
4. Define the *expected loss* (EL) and the *unexpected loss* (UL). Show that the VaR measure is equal to the EL measure under the previous hypothesis ($\mathcal{H}$) if the default times are independent of the factors.
5. Write the expression of the loss quantile $\mathbf{F}^{-1}(\alpha)$ when we have a single factor $X \sim \mathbf{H}$. Why this expression is not relevant if at least one of the exposures EAD_i is negative? What do you conclude for the management of the credit portfolio?
6. In the Basel II model, we assume that the loan i defaults before the maturity M_i if a latent variable Z_i goes below a barrier B_i:
$$\boldsymbol{\tau}_i \leq M_i \Leftrightarrow Z_i \leq B_i$$
We consider that $Z_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$ where Z_i, X and ε_i are three independent Gaussian variables $\mathcal{N}(0,1)$. X is the factor (or the systematic risk) and ε_i is the idiosyncratic risk. Calculate the conditional default probability.
7. Calculate the quantile $\mathbf{F}^{-1}(\alpha)$.
8. What is the interpretation of the correlation parameter ρ.
9. The previous risk contribution was obtained considering the assumptions ($\mathcal{H}$) and the framework of the default model defined in Question 6. What are the implications in terms of Pillar II?

8 The Gumbel copula

We consider the bivariate probability distribution:

$$\mathbf{F}(x_1, x_2) = \exp\left(-\left(\left(\frac{1}{x_1}\right)^{\theta_1} + \left(\frac{1}{x_2}\right)^{\theta_2}\right)^{1/\theta}\right)$$

where $x_1 \geq 0$, $x_2 \geq 0$, $\theta_1 > 0$, $\theta_2 > 0$ and $\theta > 1$.

1. Show that the marginal functions of $\mathbf{F}(x_1, x_2)$ correspond to extreme value distributions[1].
2. Show that the copula function associated to the bivariate probability distribution is the Gumbel copula:
$$\mathbf{C}_\theta(u_1, u_2) = \exp\left(-\left((-\ln u_1)^\theta + (-\ln u_2)^\theta\right)^{1/\theta}\right)$$
3. Show that the Gumbel copula family is positively ordered:
$$\theta' \geq \theta \Rightarrow \mathbf{C}_{\theta'} \succ \mathbf{C}_\theta$$
where $\succ$ denotes the concordance ordering.
4. Show that[2]:
$$u_1 > u_2 \Rightarrow \lim_{\theta \to \infty} \mathbf{C}_\theta(u_1, u_2) = u_2$$
Deduce that:
$$\mathbf{C}^\perp \prec \mathbf{C}_\theta \prec \mathbf{C}^+$$

[1] We remind that the EV distributions include the Gumbel distribution $\mathbf{F}(x) = \exp\left(-e^{-x}\right)$, the FrÃ©chet distribution $\mathbf{F}(x) = \exp\left(-x^{-\alpha}\right)$ and the Weibull distribution $\mathbf{F}(x) = \exp\left(-(-x)^\alpha\right)$.

[2] Hints. Rewrite the function $g(v_1, v_2) = \left(v_1^\theta + v_2^\theta\right)^{1/\theta}$ as follows:

$$g(v_1, v_2) = v_2\left(1 + \left(\frac{v_1}{v_2}\right)^\theta\right)^{1/\theta}$$

5. Verify that $\mathbf{C}_\theta$ is an extreme value copula.

9 Credit spreads

We consider a CDS 3M with two-year maturity and \$1 mn notional principal. The recovery rate $\boldsymbol{\mathcal{R}}$ is equal to 40% whereas the spread $\boldsymbol{s}$ is equal to 150 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection buyer B if the reference entity defaults in one year and two months?

2. What is the relationship between $\boldsymbol{s}$, $\boldsymbol{\mathcal{R}}$ and λ? What is the implied one-year default probability at the inception date?

3. Seven months later, the CDS spread has increased and is equal to 450 bps. Estimate the new default probability. The protection buyer B decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 1.189.

10 Calculation of the effective expected positive exposure

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N and σ be the notional and the volatility of the underlying contract. We assume that $e(t) = N\sigma\sqrt{t}X$ where X follows an exponential distribution $\mathcal{E}(\lambda)$:

$$\Pr\{X \leq x\} = 1 - \exp(-\lambda x)$$

1. Show that the probability distribution of $e(t)$ is also an exponential distribution.

2. Calculate the peak exposure $\mathrm{PE}_\alpha(t)$ and the expected exposure $\mathrm{EE}(t)$, and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

3. Why the counterparty credit risk is a decreasing function of λ?