Risk Management & Financial Regulation
Final Examination

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Remark 1 *The correction of exercises will be available in the next release of the lecture notes.*

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation ?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel II?
3. How is calculated the capital requirement with the internal model-based approach in Basel 2.5?

3 Credit risk

1. What is the definition of the default in Basle II?
2. Describe the standardized approach (SA) to compute the capital requirement.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?

4 Counterparty credit risk (CCR) and credit value adjustment (CVA)

1. Define the concept of counterparty credit risk.
2. What differences do you make between the CCR capital charge in Basel II and the CVA capital charge in Basel III?
3. How is calculated the CCR capital requirement? Explain why the exposure-at-default (EAD) has to be estimated.
4. How is calculated the CVA capital requirement?

5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to calculate the capital charge.
3. Describe the loss distribution approach (LDA) to calculate the capital charge.

6 Expected shortfall of an equity portfolio

We consider an investment universe, which is composed of two stocks A and B. The current prices of the two stocks are respectively equal to \$100 and \$200. Their volatilities are equal to 25% and 20% whereas the cross-correlation is equal to -20%. The portfolio is long of 4 stocks A and 3 stocks B.

1. Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.
2. The eight worst scenarios of daily stock returns among the last 250 historical scenarios are the following:

s	1	2	3	4	5	6	7	8
R_A	-3%	-4%	-3%	-5%	-6%	$+3\%$	$+1\%$	-1%
R_B	-4%	$+1\%$	-2%	-1%	$+2\%$	-7%	-3%	-2%

   Calculate then the historical expected shortfall at the 97.5% confidence level for a ten-day time horizon.

7 Risk contribution in the Basle II model

We consider a portfolio of I loans. We denote L the portfolio loss:

$$L = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathrm{LGD}_i \times \mathbb{1}\left\{\boldsymbol{\tau}_i \leq M_i\right\}$$

We can show that, under some assumptions $(\mathcal{H})$, the expectation of the portfolio loss conditionally to the factors $X_1, \ldots, X_m$ is:

$$\mathbb{E}\left[L \mid X_1, \ldots, X_m\right] = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathbb{E}\left[\mathrm{LGD}_i\right] \times \mathrm{PD}_i\left(X_1, \ldots, X_m\right) \tag{1}$$

1. Explain the different notations: EAD_i, LGD_i, $\boldsymbol{\tau}_i$, M_i and PD_i.
2. How do we obtain the expression (1)? What are the necessary assumptions $(\mathcal{H})$? What is an infinitely fine-grained portfolio?
3. Define the credit risk contribution.
4. Define the *expected loss* (EL) and the *unexpected loss* (UL). Show that the VaR measure is equal to the EL measure under the previous hypothesis $(\mathcal{H})$ if the default times are independent of the factors.
5. Write the expression of the loss quantile $\mathbf{F}^{-1}(\alpha)$ when we have a single factor $X \sim \mathbf{H}$. Why this expression is not relevant if at least one of the exposures EAD_i is negative? What do you conclude for the management of the credit portfolio?
6. In the Basle II model, we assume that the loan i defaults before the maturity M_i if a latent variable Z_i goes below a barrier B_i:
   $$\boldsymbol{\tau}_i \leq M_i \Leftrightarrow Z_i \leq B_i$$
   We consider that $Z_i = \sqrt{\rho} X + \sqrt{1-\rho}\varepsilon_i$ where Z_i, X and ε_i are three independent Gaussian variables $\mathcal{N}(0,1)$. X is the factor (or the systematic risk) and ε_i is the idiosyncratic risk. Calculate the conditional default probability.
7. Calculate the quantile $\mathbf{F}^{-1}(\alpha)$.
8. What is the interpretation of the correlation parameter ρ.
9. The previous risk contribution was obtained considering the assumptions $(\mathcal{H})$ and the framework of the default model defined in Question 6. What are the implications in terms of Pillar II?

8 Credit spreads

We consider a CDS 3M with two-year maturity and \$1 mn notional principal. The recovery rate $\mathcal{R}$ is equal to 40% whereas the spread $\mathcal{s}$ is equal to 150 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection buyer B if the reference entity defaults in one year and two months?
2. What is the relationship between $\mathcal{s}$, $\mathcal{R}$ and λ? What is the implied one-year default probability at the inception date?
3. Seven months later, the CDS spread has increased and is equal to 450 bps. Estimate the new default probability. The protection buyer B decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 1.189.

9 Parametric estimation of the loss severity distribution

1. We assume that the severity losses are log-normal distributed: $X_i \sim \mathcal{LN}\left(\mu, \sigma^2\right)$.
   (a) Show that the density function of the log-normal probability distribution is:
   $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)$$
   (b) Deduce the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.
   (c) Calculate the maximum likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$.
   (d) We assume that the losses $\{x_1, \ldots, x_n\}$ were collected beyond a threshold H. Calculate the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.
2. We assume now that $X_i \sim \mathcal{LL}(\alpha, \beta)$ with:
   $$\mathbf{F}(x; \alpha, \beta) = \frac{(x/\alpha)^\beta}{1 + (x/\alpha)^\beta}$$
   (a) Find the density function.
   (b) Deduce the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$.
   (c) Show that the ML estimators satisfy the following first-order conditions:
   $$\left\{\begin{array}{l} \sum_{i=1}^n \mathbf{F}\left(x_i; \hat{\alpha}, \hat{\beta}\right) = n/2 \\ \sum_{i=1}^n \left(2\mathbf{F}\left(x_i; \hat{\alpha}, \hat{\beta}\right) - 1\right) \ln x_i = n/\hat{\beta} \end{array}\right.$$
   (d) What becomes the log-likelihood function of the sample $\{x_1, \ldots, x_n\}$ if we assume that the losses were collected beyond a threshold H?

10 Calculation of CVA and DVA measures

We consider an OTC contract with maturity T between Bank A and Bank B. We denote by $\mathrm{MtM}(t)$ the risk-free mark-to-market of Bank A. The current date is set to $t = 0$ and we assume that:

$$\mathrm{MtM}(t) = N\sigma\sqrt{t}X$$

where N is the notional of the OTC contract, σ is the volatility of the underlying asset and X is a random variable, which is defined on the support $[-1, 1]$ and whose density function is:

$$f(x) = \frac{1}{2}$$

1. Define the concept of positive exposure $e^{+}(t)$. Show that the cumulative distribution function $\mathbf{F}_{[0,t]}$ of $e^{+}(t)$ has the following expression:

$$\mathbf{F}_{[0,t]}(x) = \mathbb{1}\left(0 \leq x \leq \sigma\sqrt{t}\right) \cdot \left(\frac{1}{2} + \frac{x}{2N\sigma\sqrt{t}}\right)$$

   where $\mathbf{F}_{[0,t]}(x) = 0$ if $x \leq 0$ and $\mathbf{F}_{[0,t]}(x) = 1$ if $x \geq \sigma\sqrt{t}$.

2. Deduce the value of the expected positive exposure $\mathrm{EpE}(t)$.

3. We note $\boldsymbol{\mathcal{R}}_B$ the fixed and constant recovery rate of Bank B. Give the mathematical expression of the CVA.

4. We consider the following result:

$$\int_0^T \sqrt{t}e^{-\lambda t}\,\mathrm{d}t = \frac{\gamma\left(\frac{3}{2}, \lambda T\right)}{\lambda^{3/2}}$$

   where $\gamma(s, x) = \int_0^x t^{s-1}e^{-t}\,\mathrm{d}t$ is the lower incomplete gamma function. Show that the CVA is equal to:

$$\mathrm{CVA} = \frac{N\left(1 - \boldsymbol{\mathcal{R}}_B\right)\sigma\gamma\left(\frac{3}{2}, \lambda_B T\right)}{4\sqrt{\lambda_B}}$$

   when the default time of Bank B is exponential with parameter λ_B and interest rates are equal to zero.

5. By assuming that the default time of Bank A is exponential with parameter λ_A, deduce the value of the DVA without additional computations.